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IN TWO SECTIONS—SECTION TWO

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of the

A.S.M.E.

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(Including Indexes to Publications)

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Modern Applications of Overfire Air

By H. C. Carroll,[1] Chicago, Ill.

Overfire air supplied by steam jets has in the past proved to be an economical and effective means for improving combustion in furnaces. More recently high-pressure air jets supplied by power-driven fans have been employed with great success. After stating the requirements, this paper deals with typical applications of overfire air to furnaces equipped with various types of stokers. The use of the portable ultimate-gas-analysis machine in studying combustion conditions in furnaces and in making heat-balance tests is also described.

For many years use has been made of overfire air in furnaces of all kinds to assist in the completion of combustion within the furnace. Earlier applications were mostly to eliminate smoke in hand-fired furnaces burning bituminous coal where steam jets were employed to furnish the necessary velocity of injection. Even recently some applications of overfire air with steam jets have proved both economical and effective.

More recently, however, high-pressure air jets supplied by power-driven fans have been employed in furnaces to provide turbulence to prevent stratification; to increase heat releases in furnaces of limited volume; to effect increased steam production economically; to replace refractory arches designed to mix lean and rich gases; to reduce the combustible in the fly ash when coal is burned in suspension; and to burn refuse more rapidly and smokelessly.

Overfire-air application of this type might be defined as follows:

"The application of an independent source of air, other than normally applied to the furnace, in such manner, volume, and intensity as to provide the proper turbulence and necessary oxygen to perfect the complete combustion of volatile gases and carbon from the fuel within the limits of the furnace."

Portable Ultimate-Analysis Testing Machine

In attempting to determine the extent and loss of unburned gases in the flue gases of steam-generating units, more especially when bituminous coal is burned, the ordinary Orsat giving the usual volumes of CO_2, O_2, and CO was found to be inadequate. Also the collection of flue gases to be sent to the laboratory for ultimate analysis, besides requiring extreme care and much time, was found to be costly and unsatisfactory. Therefore it was decided to take the laboratory to the power plant where a portable ultimate-analysis machine is being used which is so arranged that it can be easily dismantled in three sections and transported in three ordinary-sized boxes specially built for the purpose.

By referring to Fig. 1, showing the portable gas analyzer set up for test, it will be noted that there are two burettes; one being used for the larger volumes such as carbon dioxide and oxygen; and the other one being graduated to 0.02 ml for measuring the small quantities of unburned gases such as hydrogen, carbon monoxide, and methane.

By using the ultimate-gas-analysis machine, it is possible to determine a complete analysis of the flue gases, securing the percentages of CO_2, illuminants, O_2, CO, H_2, and CH_4 present in the gases. The CO_2 is determined by absorption in KOH; the illuminants by absorption in fuming H_2SO_4; the O_2 by absorption in pyrogallol; the CO and H_2 by combustion over CuO heated to 300 C; and the CH_4 by combustion over platinum wire heated to 900 C.

Naturally, the benefits of any change in design of a steam-generating unit will be reflected either in its efficiency and/or in the cost of operation. In the case of distribution of air, an ultimate gas analysis is not a prerequisite to a proper judgment of the performance of a unit. However, if an exact knowledge can be obtained of the composition of the flue gases at various points in its flow through the unit, the engineer then has at his command the data to judge adequately the degree of stratifica-

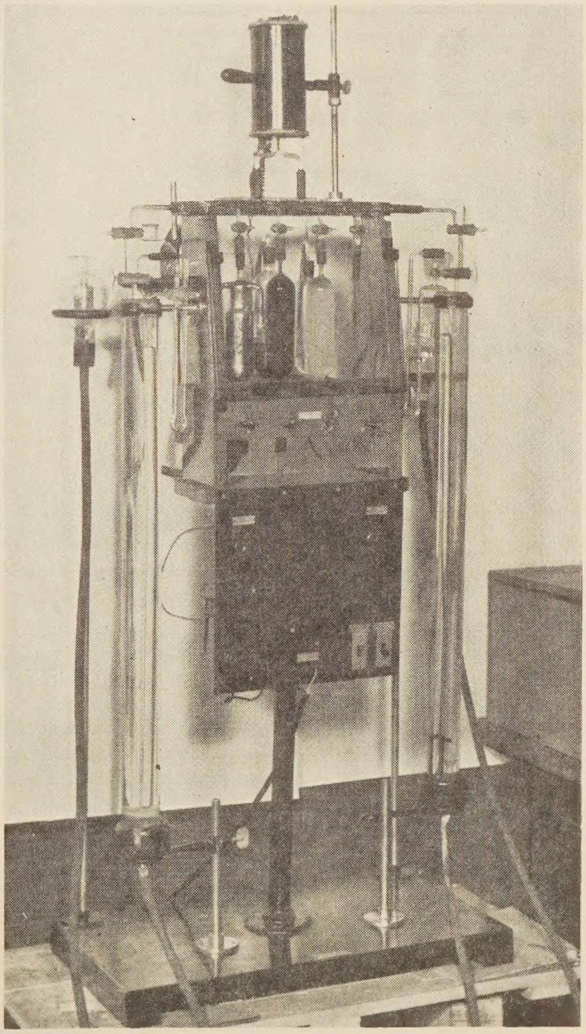

Fig. 1 Portable Ultimate Gas Analyzer

[1] Head, Mechanical Engineering Department, Commercial Testing and Engineering Company. Mem. A.S.M.E.

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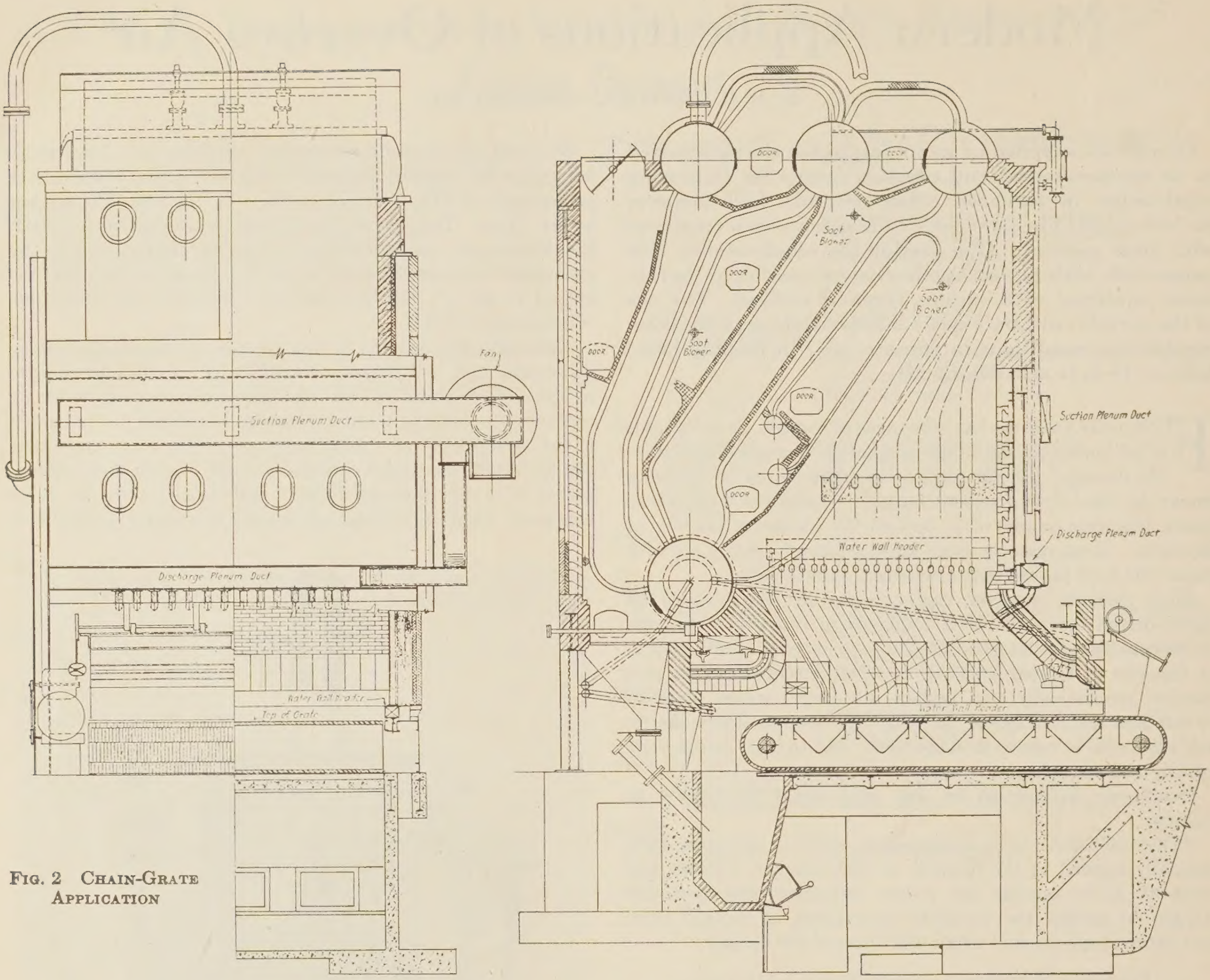

Fig. 2 Chain-Grate Application

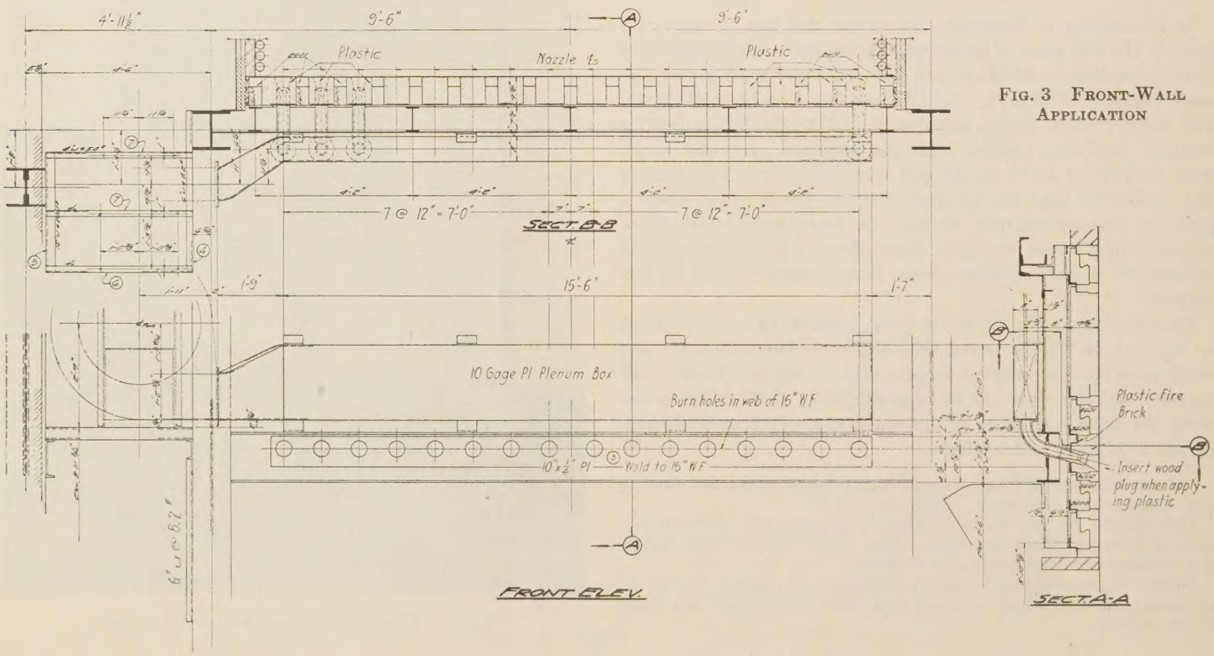

Fig. 3 Front-Wall Application

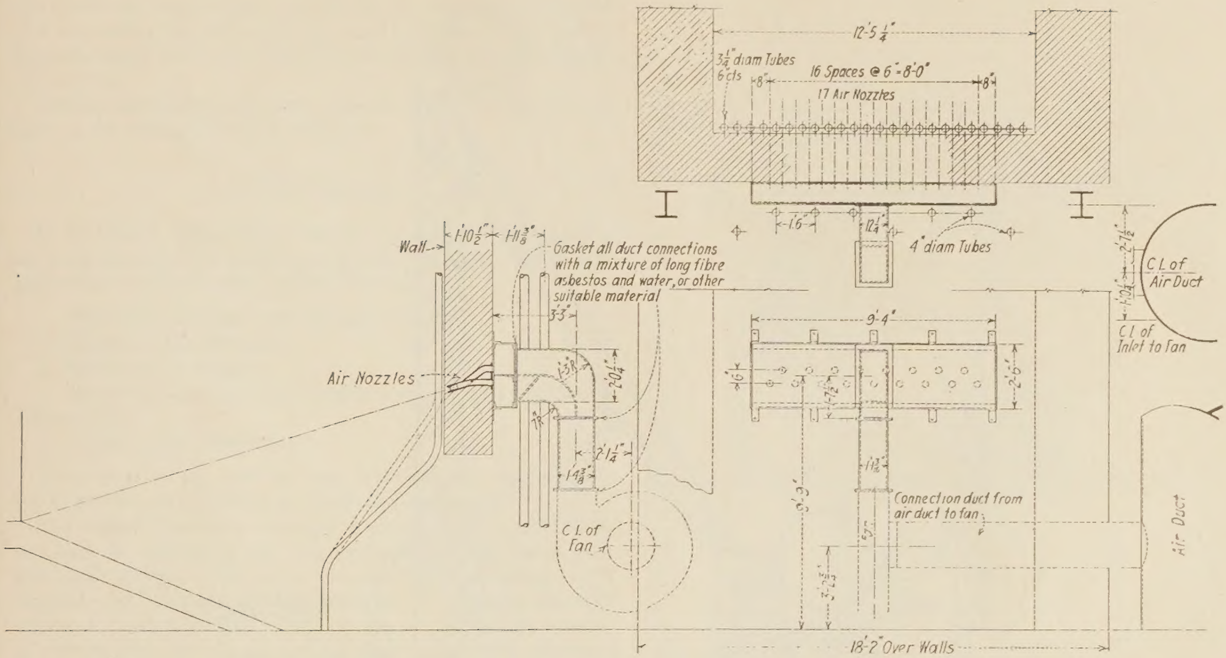

Fig. 4 Rear-Wall Application

tion taking place and the effect of the composition of the gases on the losses in the unit caused by unburned constituents in the flue gases. Information will also be obtained on such phenomena as secondary combustion and its attendant problems.

Again, when making heat-balance tests on steam-generating units, if the exact composition of the gases is not known, a complete heat balance cannot be correctly calculated. As just mentioned, we have found the ordinary Orsat to be inadequate for this purpose, even for determining carbon monoxide with exactness. For this reason, we are of the opinion that many heat balances do not reflect the true conditions of operation of the unit, at least in so far as distribution of losses is concerned.

The possibility of learning what is happening to the flue gases as they travel from the fuel bed to the boiler outlet, therefore offers an intriguing challenge to the test engineer as well as a means to the end of aiding the study of the effects of overfire air.

Anyone familiar with precise test procedure always appreciates the difficulties of obtaining accurate and exact data upon which no statistical bias will exist in the final conclusions. These difficulties are usually magnified in proportion to the desired degree of precision; and for any approach to a correct interpretation of the unburned constituents in a flue gas, a high order of precision in analysis and a careful design of experiment are very definite requirements, because of the very small percentages of unburned gases existing in a flue gas.

Results Before and After Applying Overfire Air

As an example of what these small quantities translate to in terms of loss in efficiency, the results, given in Tables 1 and 2, of an investigation made before and after overfire air was applied indicate both the effects of overfire air and the magnitude of losses because of incomplete combustion of the flue gases. The unit on which the investigation was made was a Springfield cross-drum straight-tube boiler of 819 hp, equipped with a Foster Wheeler economizer, superheater, and preheater, fired by a Westinghouse 7-retort underfeed stoker having 151.9 sq ft of grate surface and 359 ft of rear-waterwall protection.

TABLE 1 GAS ANALYSIS

	Without overfire air, per cent	With overfire air, per cent
Carbon dioxide (CO_2)	13.56	14.27
Illuminants	Not determined	
Oxygen (O_2)	5.04	4.35
Hydrogen (H_2)	0.10	0.01
Carbon monoxide (CO)	0.10	0.08
Methane (CH_4)	0.32	0.05
Total	19.12	18.76

TABLE 2 HEAT LOSSES, PER CENT OF AVAILABLE HEAT

	Without overfire air, per cent	With overfire air, plenum-box press 17.5 in., per cent
Heat loss due to:		
Incomplete combustion of carbon	0.43	0.32
Incomplete combustion of hydrogen	0.32	0.03
Incomplete combustion of hydrocarbons	3.67	0.49
Total heat loss due to unburned gases	4.42	0.84

Applications of Overfire Air

Fig. 2 shows the application of overfire air to a chain-grate stoker, the air being taken from the front air-cooled walls of the setting and walls above the waterwalls at a temperature of approximately 160 F. This 1000-hp unit formerly was equipped with standard refractory arches and refractory walls which were removed and replaced by an inclined-front refractory wall and side waterwalls and stoker water headers.

Air is supplied by a separate overfire motor-driven fan through 14 Venturi nozzles at the angle as shown in Fig. 2. The nozzles are made of heat-resisting metal; and approximately 5 per cent overfire air is supplied to the furnace at 10 in. static pressure and a temperature of 160 F.

With these changes, the unit was capable of producing 25 per cent more steam and gave an increased efficiency of 8 per cent, mostly realized in reduction of loss of unburned gases to the stack. An efficiency of 78.5 per cent was realized on this unit continuously at a rating of 200 per cent without smoke.

All types of coal tributary to the plant with ash content up to 15 per cent can be burned without ignition difficulty.

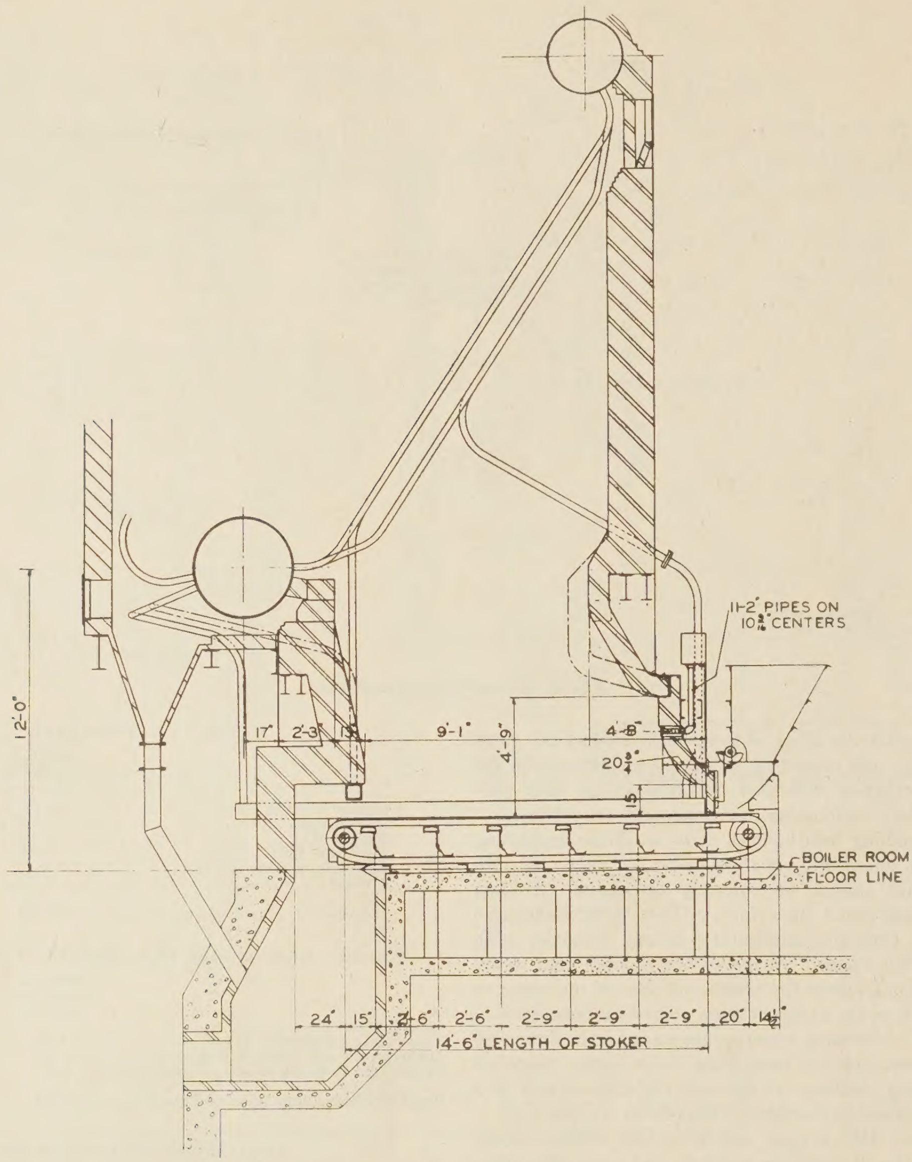

Fig. 5

Fig. 3 is a design for the front-wall application of an underfeed stoker where the plenum box is located in the front above the hopper, and the fan using room air is located on the side of the boiler.

With this 100,000-lb steam-generating unit, with 16 Venturi nozzles supplying overfire air at 16 in. static pressure and operating at 110,000 lb of steam per hour, we were able to eliminate a smoking condition which with certain coals was usual in operation.

The investigation of the performance of this unit brought out some other interesting details in that formerly without overfire air severe enough slagging occurred on the second bank of tubes to cause the unit to be shut down in 2 or 3 months. This condition was eliminated with overfire air.

Fig. 4 shows the application of overfire high-pressure air to the rear of the stoker which in some cases is essential in order to secure the best results. In this installation a rear-wall application was used with preheated air at 300 F.

A divided plenum box was provided, in order to secure less air at the lower ratings and yet maintain turbulence. One half the number of nozzles enter the upper part of the plenum box, and the remainder enter the lower box, one damper being supplied to facilitate cutting off half the air without disturbing the static pressure.

This application was made in order to obtain an increase of rating up to 300 per cent without losses of unburned carbons up the stack, which were excessive before its application. This was in connection with high-volatile Midwestern coal, a static pressure of 16 in. prevailing.

Unfortunately, except perhaps in the case of pulverized-coal burning, the application of overfire air has been made more on a rule-of-thumb basis than on a basis governed by a scientific knowledge of all the phenomena involved. It is the author's opinion that although each installation, involving losses in the flue gases, is a problem in itself, created by furnace design, burning equipment, the kind of fuel used, extent of refractory,

air-cooled and water-cooled surfaces, certain fundamental principles concerning pressure, quantity, and methods of application might be applicable to most installations, if these basic principles were known.

Perhaps this presentation may serve to clarify the thought on some of the vital phases of this subject.

Discussion

F. X. GILG.[2] The primary function of high-velocity overfire air is to minimize the smoke and fly-ash nuisance. Its proper application results in other benefits, such as:

1 Wide open furnaces with minimum arches.
2 Higher stoker capacities.
3 Reduction of slagging.
4 Improved ignition even though furnaces are completely water-cooled.
5 Higher stoker efficiencies.

These benefits are real and very welcome, but the primary job of overfire air is to clear up the stack discharge.

The author has very properly pointed out the difficulties involved in measuring the unburned-combustible losses and also their extent. Test data are important, necessary, and desirable for comparison. His example of increasing the efficiency of a unit from 70.5 to 78.5 per cent by proper application of overfire air is an indication of the possible extent of unburned combustible losses. This is more an indication of improper design or operation, originally, rather than a real measure of the effectiveness of overfire air. The unit described should have been running at 78 per cent efficiency in the first place. However, it does emphasize the possibilities of a simple inexpensive method of correcting for original faulty design or operation, thereby reducing excessive smoke and carry-over.

B&W CHAIN GRATE STOKER
STIRLING BOILER, REFRACTORY FURNACE
INDIANA COAL 2100° A.F.T.

COAL BURNING RATE A.F. LB PER SQ FT	35
EXCESS AIR @ BOILER OUTLET	43%
% ASH IN COAL TO STACK	3.31%
DUST LOADING, GR. PER CF. GAS @ 32F	.17
COMBUSTIBLE IN FLY ASH	26.8%
" " ASH PIT REFUSE	10.6%
LOSS IN EFFICIENCY DUE TO FLY ASH	.13%
" " " " ASH PIT REFUSE	1.27%
STOKER EFFICIENCY	98.6 %
	100.00%

FIG. 6

In discussing comparable stoker performances, boiler efficiencies should not be used as a guide, because this introduces the question of the design of boiler above the stoker. Some boilers are more efficient than others, and to avoid introducing unrelated factors, stoker comparison and performance should be limited to burning rates, excess air, and unburned combustible matter.

Our experience with overfire air is confined mostly to chain-grate stokers, although we are also using it for other methods of fuel burning. Three installations have been chosen upon which we have test data to show the effectiveness of overfire air in eliminating smoke and (*a*) increasing stoker capacity; (*b*) increasing stoker efficiency; (*c*) eliminating furnace slagging; (*d*) general effect on unit performance.

Proper application of overfire air on the chain-grate-stoker-fired Stirling boiler, shown in Fig. 5, permitted higher capacities, increasing the burning rate from 40 to 48 lb per sq ft without smoke even though the front arch was shortened.

[2] Mechanical Engineer, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

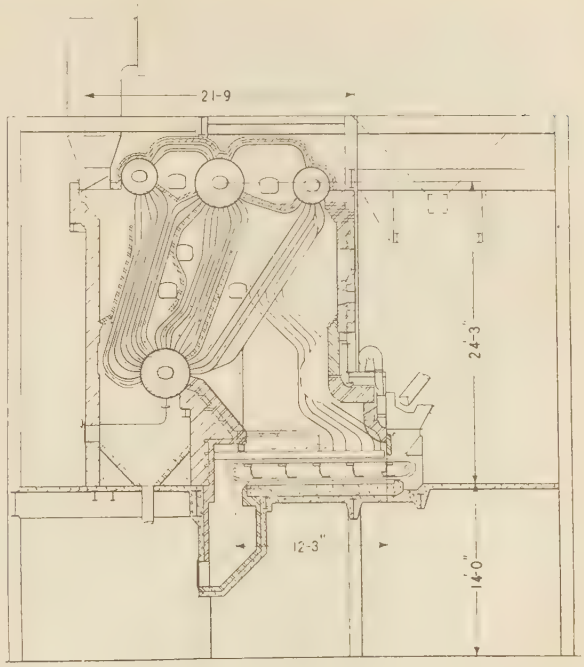

FIG. 7

B&W CHAIN-GRATE STOKER
STIRLING BOILER REFRACTORY FURNACE
W. VA. COAL

	WITHOUT O.F. AIR	WITH O.F. AIR
COAL BURNING RATE A.F. LB PER SQ FT	28.5	28.2
EXCESS AIR @ BOILER OUTLET %	49.7	42.0
COMBUSTIBLE IN FLY ASH %	35.7	17.8
% ASH IN COAL TO STACK	2.56	3.23
DUST LOADING GR. PER CU FT @ 32°	.227	.131
FLY ASH LOSS % EFFICIENCY	.24	.06
ASH PIT LOSS % "	2.28	2.36
STOKER LOSSES	2.52	2.42
INCREASE IN DRY GAS LOSS DUE TO HIGHER EXCESS AIR	.5	
TOTAL STOKER LOSSES	3.02	2.42

FIG. 8

Fig. 6 shows the stoker performance of the same unit operating on Indiana coal at a 35-lb burning rate with high-velocity overfire air. With 43 per cent excess air at the boiler outlet (13 per cent CO_2), the stoker losses were 1.27 per cent in the ash pit, 0.13 per cent in fly ash. Only 3.31 per cent of the ash in the coal went up the stack, resulting in a dust loading of 0.17 g per cu ft of flue gas at 32 F.

Proper application of overfire air, in the case illustrated in Fig. 7, permitted burning rates as high as 40 lb without smoke, with a shortened arch as shown. We have some test data with and without overfire air.

Fig. 8 shows a comparison of stoker performance with and without overfire air, but without smoke in either case. The principal effects at the 28.5-lb burning rate were a lower excess air at the boiler outlet, lower fly-ash loss, and a net pickup of 0.6 per cent in efficiency with overfire air. With an ultimate gas analyzer, as described by the author, undoubtedly we could

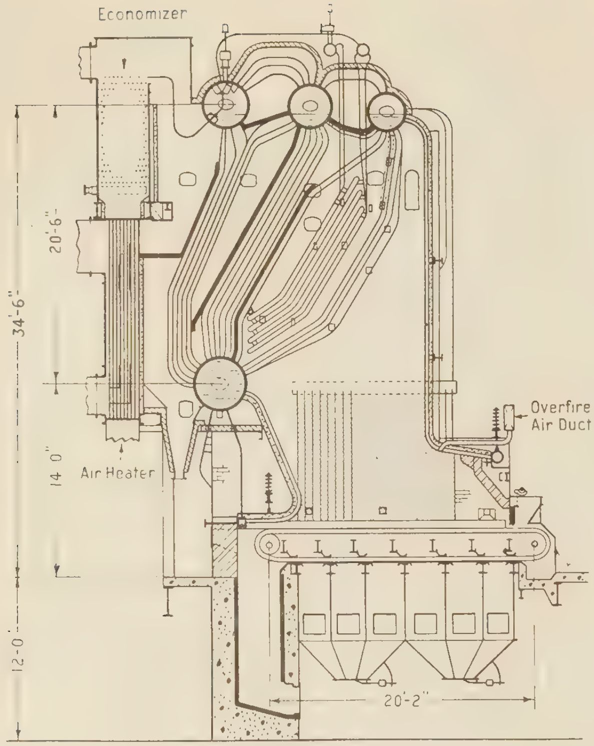

Fig. 9

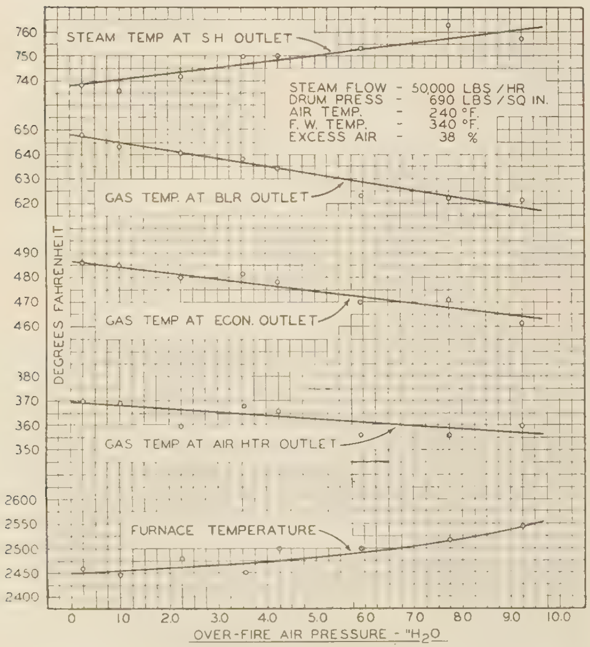

Fig. 10

have detected a further increase in efficiency with overfire air, due to a reduction in unburned-combustible gases which cannot be detected by an ordinary Orsat apparatus.

Fig. 9 shows a stoker-fired unit with overfire air through the water-cooled arch. Previous similar boilers in this same plant had the overfire air injected through the upper part of the refractory ignition arch. There was a marked absence of furnace and boiler-tube slagging on the later units, as compared to the original units, which had to be hand-lanced every shift. Changing the location of the overfire-air nozzles to conform with the later units eliminated the necessity for hand-lancing. This experience is valuable in that it shows the importance of proper location of overfire-air nozzles.

A series of tests was run on this unit with increasing overfire-air pressure, the results being shown in Fig. 10. With the same operating conditions, the effect of increasing overfire-air pressures is to give better absorption in the boiler and superheater, which is a good indication of improved complete combustion in the furnace proper. Generally speaking, by increasing the overfire-air pressure from 0.23 to 9.26 in., the steam temperature increased from 738 to 757 F, the boiler outlet-gas temperature decreased 24 deg, and the air-heater outlet-gas temperature decreased 9 deg. The hot spot in the furnace as read by an optical pyrometer increased 87 deg. The optical-pyrometer readings are submitted only as an indication of increased temperature in the active combustion zone of the furnace as a result of the turbulence created in the furnace by the high-velocity overfire-air nozzles.

We have test data on a large cross-drum boiler, fired by an underfeed stoker after overfire air was installed through the rear wall, with air at 17-in. pressure. In this case, use of overfire air resulted in raising the "no smoke" limit of this boiler from 260,000 to 330,000 lb per hr with an increase in efficiency of 3 per cent at 325,000 lb per hr, by reducing the cinder loss from 7.2 to 4.2 per cent.

In conclusion, may we point out the desirability of further discussions on this subject of high-velocity overfire air as an active aid in improving combustion conditions in stoker-fired units. There is much to be learned about the location and direction of overfire air and its effect on slagging and steam temperature.

W. L. Lundy.[3] Although overfire air has been used in furnaces many years for securing all of the advantages mentioned by the author, there are still thousands of steam plants, ranging in size from small hand-fired to large stoker installations, which have either never installed overfire-air equipment, or have unsuccessfully tried some kind of equipment and have later removed it, or have misapplied it and are not securing the full benefit therefrom.

The present emergency should give impetus to any fuel-saving device which will secure increased output and result in a higher availability factor for the boiler unit as installed.

Nondefense industries are somewhat handicapped by not being able to secure equipment such as fans, motors, or small turbines, but many are still in a position to make up steam jets from salvaged materials for the purpose of jetting overfire air into their furnaces.

Some benefit may be secured in the case of small hand-fired furnaces by merely leaving the firing door slightly open for a minute or so after each charge of coal.

The writer had the following experience, while conducting a test on a small Scotch marine boiler being fired with high-volatile Midwestern coal. The load on the boiler was quite heavy and required the firing of a few scoops of coal every few minutes. When the firing door was closed immediately after charging the coal, heavy black smoke was continuously discharged from the stack. After a few observations to determine the length of time required for leaving the firing door slightly

[3] Steam Engineer, Kimberly-Clark Corporation, Neenah, Wis. Mem. A.S.M.E.

open after the charge of coal, it was possible to continue the test without visible smoke heavier than a slight haze.

The operation of natural-draft chain-grate furnaces can be improved to some extent simply by cutting a few holes in the front mixing arch to admit some overfire air. Further benefits can be secured by judiciously using a small jet of steam through pipes placed in the center of the holes for induction of air and creation of turbulence.

Good judgment must be used and observations and tests made of the effect of the application of overfire air to any furnace. The ultimate-gas-analysis machine, described by the author, is the equipment required to secure the data for obtaining the greatest benefit from overfire-air application. Carefully conducted heat-balance tests will show high unaccounted for losses, unless such a machine is used, whenever unburned hydrogen or hydrocarbons are present in the flue gases. In any case, it is necessary to run comparative tests, with and without overfire air, since it is possible to obtain net results which will show on the wrong side of the ledger.

There are three fundamental principles which must be kept in mind in the application of overfire air to any furnaces using coal as fuel whether it be a stoker-fired furnace, where most of the air comes through the fuel bed, or a pulverized-fuel furnace where all the secondary or tertiary air might be classified as overfire. These three principles are as follows:

1 The overfire air must be applied in a manner not to retard initial ignition.

2 The overfire air should be applied in sufficient quantities and cause sufficient turbulence or mixing, while the gases are entering the high-temperature zone of the furnace, in order to consume the hydrocarbons being distilled from the coal.

3 The turbulent-burning fuel should not impinge against the furnace walls, arch, or heating surface.

A few years ago, the writer investigated the cause of excessive furnace maintenance and the cause of the production of heavy smoke at a utility plant using high-volatile Midwestern coal. This plant had eight 1726-hp Babcock & Wilcox cross-drum boilers, operating at 350 psi and 690 F steam temperature. Each boiler was fired by two Babcock & Wilcox forced-blast chain-grate stokers each 21 ft 3 in. long × 9 ft 6 in. wide, with a division wall between the stokers.

The furnaces were entirely refractory-lined and had a furnace volume of 41,014 cu ft, the ratio of furnace volume to boiler heating surface 2.3 to 1, and ratio of furnace volume to grate surface 9.93 to 1. The furnaces had both front and rear arches with a throat between the arches of 8 ft 3 in. The first row of the lower deck of boiler tubes was 18 ft 4 in. above the operating-floor level.

Each boiler was operated at its maximum capacity of 200 per cent of rating, which was limited by the capacity of the induced-draft fan. The furnaces were smoking very badly, and the brick maintenance had been very high due to slag erosion of the furnace walls and arches.

This plant was using 2300 cfm of overfire air in each furnace or approximately 7 per cent of the total air requirement. The overfire air was jetted into the furnace through two 4-in. pipes by means of small steam jets on the center line of the pipes. These pipes were installed on opposite sides of the furnace at the center line of the throat between the front and rear arch.

It was observed that blue flames flickered on and off between the superheater elements which were located between decks of the boiler tubes. A gas-sampling tube was placed in this location. Samples of the flue gas by Orsat analysis showed low CO_2, high oxygen, and an appreciable amount of carbon monoxide. This was a case of an insufficient quantity of overfire air for the particular coal and wrong location of the jets to obtain proper mixing. The air was also being admitted too far away from the point at which the volatiles were leaving the coal.

It was decided to double the quantity of overfire air and to locate one jet at the front corners of each of the two stokers approximately 14 in. above the grate. The jets were placed parallel with the grate at such an angle with the vertical that the center lines crossed at a point directly below the nose of the front arch at the center line of the grate.

Observations and gas analysis were made on one furnace with the new jets located as described and the old jets also in place as noted. The results are given in Table 3.

TABLE 3 COMPARISON OF FURNACE DATA WITH AND WITHOUT OVERFIRE AIR

Jets in use	Side and front	Old side	New front	No jets
Gate height, in.	6	6	6	6
Steam flow, per cent rating	195	198	198	190
Air flow, per cent rating	193	193	200	197
Flue-gas temperature, F	597	597	593	595
CO_2 recorder last pass	12.6	10.5	12.4	10.0
CO_2 (Orsat) first pass, per cent	12.4	8.5	15.5	8.8
O_2	6.7	11.0	3.6	10.0
CO	0.1	0.3	0.0	0.4
Steam pressure, psi	357	354	353	337
Steam temperature, F	710	699	720	690
Furnace temperature, F[a]	2650	2660	2550	2660

[a] Under front arch above second compartment.

It will be noted from Table 3 that the furnace temperature under the arch was reduced at least 100 F with the new jets as compared with any of the other conditions. This would indicate a local reduction in CO_2 percentage at this point in the furnace with overfire air from the new jets, although the analysis of the first-pass gases actually showed an increase in CO_2 at the point of sampling in the superheater location. It will also be noted that the steam temperature was increased 21 F with the new jets, compared with the old jets, and 30 F increase compared with no jets. This increase in steam temperature improved the water rate of the turbines. The decrease in excess air, elimination of carbon monoxide, and indicated reduction in unburned hydrocarbons resulted in a decided increase in efficiency of the plant.

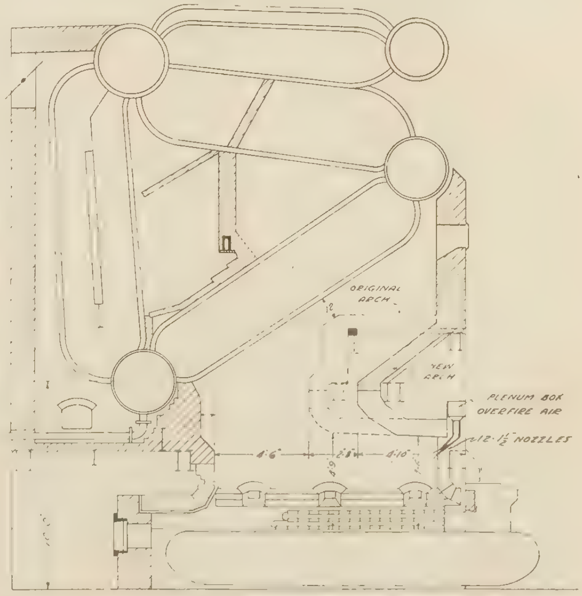

FIG. 11 KIDWELL 613-HP BOILER EQUIPPED WITH HARRINGTON STOKER

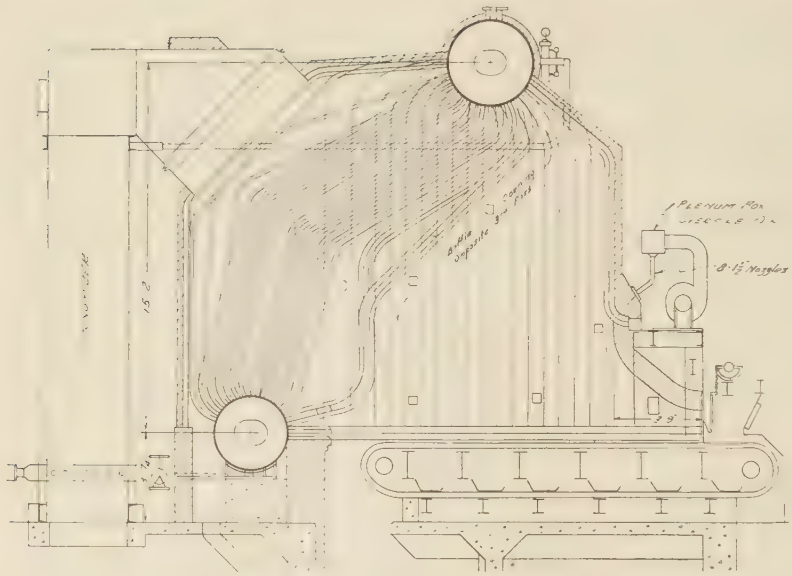

Fig. 12 Babcock & Wilcox Integral-Furnace Boiler, 35,000-Lb per Hr Steam Capacity, Equipped With Chain-Grate Stoker

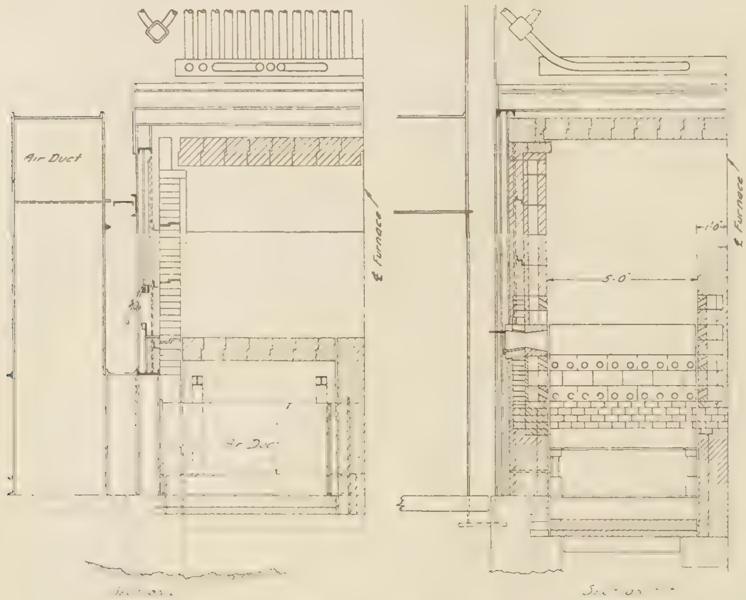

Fig. 13 Wet-Bark-Burning Furnace

One per cent of the steam output of the boiler was required for the steam jets in this installation.

The high division wall between the stokers was eliminated to reduce the brick maintenance.

Fig. 11 shows progressive changes made in the past 3 years in the application of overfire air to an old forced-blast Harrington traveling-grate installation under a 613-hp boiler, having a furnace too small for the boiler and stoker. The original furnace had only 800 cu ft of volume. The small furnace resulted in high furnace maintenance, if the steam output of the boiler exceeded 125 per cent of rating.

The nose of the arch was first moved back 1 ft and the roof above the arch sloped up to the front wall instead of being flat. This increased the furnace volume approximately 15 per cent. Steam jets were installed at the front corners of the furnace to induce overfire air and cause mixing of the gases.

Evaporation tests were run when using Pittsburgh and Midwestern coal; the results showed increases of 4 and 6 points in efficiency, respectively. Further increases in boiler load resulted in similar high furnace maintenance as obtained before installation of the jets.

The nose of the arch was again moved back 1 ft 8 in., and a fan, having a capacity of 1800 cfm of 70 F air at 14 in. pressure, was installed to supply two furnaces. It was found that only 1 to $1^1/_2$ in. pressure was required to accomplish the desired results in efficiency. Higher pressures caused impingement of gases on the arch, which is still too long. The load on these units has increased still further, but the furnace maintenance is lower than originally.

It is the intention to move the nose of the arch back another 1 ft 9 in. and use a higher pressure of the overfire air for mixing the gases instead of retaining any part of the arch for that purpose.

Fig. 12 shows the application of overfire air to a new Babcock & Wilcox integral-furnace type F boiler installed in the same plant as the foregoing units. This is a 35,000-lb per hr boiler with 1450 cu ft of furnace volume, fired by an 11 × 12-ft Babcock & Wilcox chain-grate stoker. The overfire-air fan supplies 4850 lb per hr of 80 F air at 8 in. pressure.

It will be noted that the projected length of the arch is only 3 ft 9 in. and that the mixing of the gases is dependent upon the high-pressure overfire-air jets, together with the baffle behind the first row of boiler tubes. This baffle is open on the right side of the furnace in a width of approximately 4 ft, and from a line approximately 4 ft above the grate to the steam drum of the boiler. The gases from the furnace pass through this baffle opening and flow to the opposite side of the furnace behind this baffle and enter the first pass of the boiler on the left side of the setting.

This unit was placed in operation early in June, 1942.

Fig. 13 shows the design of a furnace installed in one of the writer's company's plants for burning wet bark containing 60 to 75 per cent moisture.

The furnace is installed under a pulverized-fuel furnace, the gases from both passing through a 90,000-lb per hr boiler, operating at 470 psi and 700 F.

This furnace is designed so that all or any part of the air may be admitted either through the grate or through the openings in the side walls under forced draft. The retort side walls are constructed with Furnace Economy Company's type C walls with type E walls in the combustion chamber. Air from the type E walls enters the combustion chamber through fifteen 3-in. cast-iron pipes across the drop-nose arch.

These furnaces have been in operation for two full years without any refractory wall or grate maintenance, using air tem-

peratures of 500 F or more. A few arch tiles around the thimble of the bark spout have had to be replaced because of warpage of the thimble.

Evaporations of 1.5 to 1.7 lb of steam per lb of 60 per cent moisture green balsam bark are obtained in this installation. The furnace has a capacity for burning over 30,000 lb of 60 per cent moisture bark per hr. These evaporation figures are obtained in normal operation of the furnace for a 6-hr period with bark quantities varying from 20,000 to 45,000 lb per hr, then off for a 6-hr period. The evaporation theoretically possible is 1.8 lb of steam per lb of 60 per cent moisture bark, assuming 14 per cent CO_2, 1 per cent combustible loss, and radiation and other losses of 3 per cent.

It is estimated that normally over 80 per cent of the air for burning the bark is passed through the openings in the walls of the retorts. All of this air cannot be considered as overfire, since most of the openings in the side walls are kept covered with bark for best results.

In his paper the author mentions that "except perhaps in the case of pulverized-coal burning, application of overfire air has been made more on a rule-of-thumb basis than on a basis governed by scientific knowledge of all the phenomena involved." It has been the writer's experience that pulverized-coal burning should not be excepted from the rule-of-thumb basis. Every new pulverized-fuel installation must pass through the initial cut-and-try procedure, in order to obtain optimum results, due to the wide variation in coals, furnace design, burner location, range in capacity desired, and the changes in burner and pulverizer design.

In reference to the fundamentals concerning pressure, quantity, and methods of application of overfire air it seems that variations in pressure of from 1 to 16 in. are necessary. The required pressure depends principally upon the furnace design and burning equipment installed. The quantity of overfire air required varies from 5 to 20 per cent in chain- or traveling-grate installations, depending principally upon the volatile matter in the coal and the rate at which the volatile matter leaves the particular coal. The lower percentage prevails for the lower-volatile and slow-burning coals, with the higher percentages applying to the high-volatile and freer-burning coals.

Spreader stokers and pulverized-fuel equipment require considerably more overfire air, and underfeed stokers much less than chain grates. The method of application depends upon local plant conditions, size of units, and many of the other variables involved. It is essential that the air mix with the volatile gases before the hydrocarbons have had an opportunity to crack down to soot, or at least while they are in the process of cracking. The steam vapor present in the overfire-air mixture, when using steam jets, may have some beneficial effect on the burning of the hydrocarbons, which does not obtain, when fans are used.

W. S. Major.[4] This paper supplements all too meager information available on the effect of high-pressure overfire air on multiple-retort stoker-fired furnaces. The test results disclose interesting economy and appear to warrant further investigation along the same lines.

We have done considerable work of this nature, both on this continent and in Great Britain. British coals, particularly, are apt to smoke and in some cases overfire air up to 20 in. pressure has been employed advantageously. However, some questions have been raised as to whether the use of overfire air formed a crutch, embodying a means for eliminating smoke, which could be corrected to better advantage by more care in operation. Experience of the writer's company has indicated that, in certain cases, overfire air discharged immediately above the stoker construction along the front wall is of some advantage and has corrected what would otherwise be a furnace sensitive to smoking. In such instances, air pressure no higher than that available in the wind box has been employed. A header is located in the front wall which supplies air to nozzles spaced across the entire furnace. These discharge air to the fuel bed approximately 2 ft above the coal-feeding rams. By means of dampers the air supply may be varied, and pressures up to the maximum of the wind box employed.

Our engineers have done considerable experimental work along different lines where they have recirculated relatively large quantities of combustion gases introduced into the furnaces at low static pressures. The points of introduction into the furnace are governed by the particular furnace design and gas travel. Our work so far has been confined to the introduction of air either through the front or rear walls. The gases are withdrawn from some readily accessible point or points between the last pass of the boiler and the entrance of the gases into the exit breeching or induced-draft fan.

Some of our work indicates that a relatively large quantity of gases at low pressure is equal to and possibly more effective than a small quantity of overfire air at high static pressure. By using recirculated combustion gases there is greater latitude in the amount of overfire supply.

The use of recirculated flue gas reduces the load on the induced-draft fan and also provides some thermal gain by avoiding the addition of an amount of heat equal to that required to raise the flue gas from room temperature to the temperature of the gas.

In many cases, in our opinion, the use of overfire air is not justified with a multiple-retort stoker, and this means should not be employed until it has been found that proper shaping of the fuel bed will not provide satisfactory combustion. Two fundamental requirements for proper combustion on a stoker of this type are equal stroking of pushers in adjacent retorts and adjustment on the individual pushers to shape the fuel bed from the front to the rear properly. To enable the operator to shape the fuel bed front to rear, we find that adjustments of the individual pushers within $\pm 1/8$ in. are essential. Efforts have have been made to use checkerboard dampers as a substitute for correct pusher movements, but such a solution is a poor substitute for proper stroking.

L. R. Stowe.[5] If 5 or 10 per cent of the total air supply is injected over the fire and this air is used to good advantage, it follows that if this added or secondary air is shut off there will result a furnace loss of some 5 or 10 per cent because of the incomplete burning of the materials in passage through the furnace.

The foregoing sentence states an obvious fact and will be universally accepted as commonplace. However, in a heat balance, if someone sets up a loss of 10 per cent because of incomplete combustion in the furnace, the experts will say it could not happen—not in a modern furnace.

However, if we sugar-coat the statement and set the loss up as "unaccounted for," it is accepted, because that is a part of our habit of bad thinking.

With the combustion art being as advanced as it is supposed to be, isn't it about time we began to account for the unaccountable?

These furnace losses which can wholly or in part be corrected by proper overfire-air injection are responsible for that quite generally misunderstood item unaccounted for. Aside from the fact that these furnace losses are actually just slightly larger

[4] American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

[5] Laclede Stoker Company, Chicago, Ill.

than that old unaccounted for item, the two, i.e., furnace loss and unaccounted for, are identical.

The author describes an elaborate apparatus for detecting these losses. Aside from apparatus of this character, requiring a highly developed technique for its manipulation, there is no instrument for detecting these furnace losses. Therefore, when we conduct a boiler test, these losses in the presence of all our thermometers, scales, gages, and Orsats can take place without being detected. It is really not strange that some folks subconsciously associate unaccounted for with things mysterious.

No, we cannot see these losses, and what is more, no one brags about them; they get no publicity whatsoever. These are some reasons why furnace losses of high magnitude are far more prevalent than is generally conceded.

Regardless of firing equipment, these losses are sure to occur in every furnace in the country unless their nature is fully understood and skillful means are adopted to avoid them. What are these means?

1 A firing equipment of high order whose performance is not lowered by poor physical and chemical properties of the coal.

2 A wholly adequate overfire-air system producing penetrating and sustained turbulence.

Let us examine some data on firing equipment of the very lowest order, in which there might be established some relationship between coal analysis and furnace losses.

The writer has computed the furnace losses for about 400 or more hand-fired U. S. Geological Survey tests. As long as these tests continue to supply information do not consider them out of date.

These tests have been divided into 21 major groups. Within very close limitations, each group satisfies several important conditions, i.e., the tests of each group are alike as to heat release, capacity developed, gas flow per unit time, and air flow or CO_2. However, while one group may be restricted to 100 per cent rating, 35,000 Btu heat release, and 12 per cent CO_2, some other group may be restricted to 130 per cent rating, 45,000 Btu heat release, and maybe only $8^1/_2$ per cent CO_2.

Next, each of the 21 major groups is divided into 3 subgroups, one subgroup having low furnace losses or high furnace efficiency, one subgroup having high furnace losses or low furnace efficiency, and then an intermediate group.

Table 4 gives some data on these groupings. In order to study extremes, the intermediate group has been discarded. So we have 21 cases each varying considerably from the others as to general operating conditions, but each within its own group, limited very closely to specific conditions, saves as to fuel and furnace efficiency.

The headings are sufficiently descriptive to avoid necessity of repeating them here. Attention is called particularly to column 14, emphasizing the point brought out by the author that the showing of the Orsat is of no value in indicating furnace losses. True, 17 cases out of 21 (see bottom line) showed higher CO losses with the subgroup of low furnace efficiencies. But whereas the reported CO losses are usually less than 2 per cent in the low subgroups, the actual total losses due to incomplete combustion of all materials in flight was about 13 per cent or worse. Furthermore, until a furnace loss of about 10 per cent is reached, there is no correlation between the CO readings and the furnace losses. The author's statement that we need better means for analyzing gas is quite definitely correct.

Smoke starts immediately to indicate most forms of furnace loss (see column 15), and in general can be more definitely

TABLE 4 COMPARISON OF DATA FOR HAND-FIRED FURNACES COMPUTED FROM U. S. GEOLOGICAL SURVEY TESTS

Main group no.	Number of tests in sub group	Furnace effic. high & low groups	Difference or range in furnace eff.	Percent volatile proximate analysis			Percent volatile moisture and ash free basis			Volatile carbon (total carbon, ultimate, minus fixed carb. prox. all mois. & ash free)			Sulphur ultimate moisture and ash free basis			Hydrogen moisture and ash free basis		
1	2	3	4	5	A	B	6	A	B	7	A	B	8	A	B	9	A	B
					↑	↓		↑	↓		↑	↓		↑	↓		↑	↓
1	3 1	92.3 83.0	9.3	33.9 37.9	4.0		38.2 47.9	9.7		23.3 28.4	5.1		.37 .52	.15		5.23 5.63	.40	
2	5 3	92.1 81.0	11.1	32.0 36.0	4.0		47.2 44.2		3.0	21.3 24.6	3.3		1.48 4.47	2.99		5.10 5.53	.43	
3	3 3	92.3 86.6	5.7	31.7 34.7	3.0		35.2 44.0	8.8		21.3 23.2	1.9		1.22 4.07	2.85		5.35 5.41	.06	
4	6 5	94.2 84.0	10.2	32.6 36.0	3.4		38.2 19.0		19.2	21.8 24.2	2.4		1.11 3.10	1.99		5.28 5.39	.11	
6	9 8	96.4 86.3	10.1	34.8 34.7		0.1	38.5 42.0	3.5		22.8 23.1	0.3		1.56 3.08	1.52		5.37 5.35		.02
7	7 5	95.7 86.4	9.3	31.4 34.5	3.1		37.1 43.0	5.9		19.2 23.1	3.9		1.53 2.03	.50		5.22 5.50	.28	
9	6 12	98.2 86.4	11.8	33.2 32.1		1.1	38.1 40.2	2.1		22.7 21.6		1.1	1.98 3.64	1.66		5.33 5.32		.01
10	14 11	96.2 85.1	11.1	32.0 33.1	1.1		39.5 41.5	2.0		20.6 23.9	3.3		3.75 2.56		1.19	5.27 5.44	.17	
11	3 4	97.9 89.2	8.7	32.2 .42		8.0	41.0 31.3		9.7	21.3 15.1		6.2	1.96 2.45	.49		5.29 4.88		.41
13	7 4	98.1 87.5	10.6	.83 33.1	4.8		35.4 42.5	7.1		18.4 21.7	3.3		3.52 4.57	1.05		5.22 5.34	.12	
14	13 7	94.9 84.8	10.1	33.3 33.2		0.1	41.2 40.2		1.0	22.9 22.8		0.1	3.10 4.01	.91		5.33 5.34	.01	
15	9 3	97.6 86.9	10.7	29.6 34.2	4.6		37.0 42.0	5.0		20.2 22.7	2.5		2.34 2.63	.29		5.25 5.33	.08	
18	9 5	97.6 86.2	11.4	29.3 31.8	2.5		38.0 40.0	2.0		20.2 20.9	0.7		2.17 3.08	.91		5.28 5.16		.12
18A	4 2	97.2 92.6	4.6	28.7 34.7	6.0		35.5 4[illegible].3	10.8		19.0 23.3	4.3		2.16 6.12	3.96		5.20 5.53	.33	
19	8 9	99.4 87.8	11.6	30.0 33.3	3.3		37.0 44.0	7.0		19.8 23.7	3.9		2.11 4.09	1.98		5.18 5.26	.08	
20	5 2	98.6 86.8	11.8	32.2 37.3	5.1		39.3 50.0	10.7		22.5 26.0	3.5		1.22 2.55	1.33		5.49 5.23		.26
22	5 4	95.7 85.1	10.6	24.8 33.5	8.7		32.6 43.7	11.1		16.5 23.4	6.9		3.54 5.48	1.94		5.07 5.38	.31	
23	6 5	98.2 84.3	13.9	26.8 31.7	4.9		35.0 40.2	5.2		17.4 25.7	8.3		3.27 4.33	1.06		5.04 5.08	.04	
24	5 4	99.9 89.6	10.3	33.0 32.3		0.7	39.0 43.0	4.0		21.9 22.6	0.7		1.92 3.54	1.62		5.36 5.36	=	=
27	3 3	101.6 87.7	13.9	24.9 36.0	11.1		30.5 48.3	17.8		15.9 25.9	10.0		3.10 6.15	3.05		5.03 5.48	.45	
28	4 3	99.7 91.7	8.0	22.6 35.2	12.6		30.2 45.0	14.8		14.9 25.8	10.9		4.32 2.85		1.47	4.93 5.62	.69	
21	237	← CASES →			16	5		17	4		18	3		19	2		15½	5½

Main group no.	Available hydrogen col 9 minus 1/8 of col. 12			"Factor 1" vol. 6 × totl. v. ÷ avail. H. (col. 7 ÷ col. 10 × col 6)			"Factor 2" % oxigen moisture and ash free basis			"Factor 3" oxigen ÷ carbon moist., ash free basis			Percent heat loss due to CO as conventionally reported			Smoke percent black Ringelmann chart		
1	10	A	B	11	A	B	12	A	B	13	A	B	14	A	B	15	A	B
		↑	↓		↑	↓		↑	↓		↑	↓		↑	↓		↑	↓
1	4.38 4.65	27		2.30 2.92	62		6.83 7.86	1.03		.080 .097	.017		.88 2.61	1.73		30.0 43.4	13.4	
2	4.07 4.20	13		2.47 2.71	24		8.25 10.28	2.03		.090 .132	.042		1.07 4.18	3.11		27.1 29.5	2.4	
3	4.64 4.24		.40	1.63 2.27	64		5.87 9.33	3.46		.066 .115	.049		.50 .90	.40		19.0 24.0	5.0	
4	4.28 4.53	25		1.94 2.20	26		8.06 6.83		1.23	.090 .082		.008	1.00 2.10	1.10		34.2 35.1	9	
6	4.39 4.16		23	2.05 2.19	14		7.82 8.49	.67		.087 .097	.010		.70 1.60	.90		19.4 28.5	9.1	
7	4.25 4.14		.11	1.71 2.45	74		7.92 10.93	3.01		.095 .137	.042		.80 2.70	1.90		13.0 34.0	21.0	
9	4.50 4.30		20	5.02 2.14		2.88	6.78 8.22	1.44		.072 .102	.030		.70 1.00	.30		21.0 22.0	1.0	
10	4.27 4.28	.01		1.96 2.17	21		8.52 10.03	1.51		.106 .126	.020		.80 1.60	.80		18.0 26.1	8.1	
11	3.64 3.93	29		2.40 1.50		90	10.50 7.64		2.86	.132 .094		.038	1.20 .80		.40	15.0 7.0		8.0
13	4.35 4.17		.18	1.62 2.20	58		7.01 9.37	2.36		.087 .118	.031		.61 1.30	.69		10.4 30.1	19.7	
14	4.17 4.15		.02	2.34 2.48	14		9.27 10.21	.94		.115 .129	.014		.71 2.20	1.49		14.1 22.3	8.2	
15	4.30 4.16		.14	2.37 2.29		8	7.74 9.41	1.67		.091 .117	.026		1.10 .90		.20	16.0 28.6	12.6	
18	4.20 3.96		.24	1.91 2.35	44		8.69 9.58	.89		.107 .121	.014		.70 .40		.30	6.0 12.3	6.3	
18A	3.33 4.48	1.15		1.67 2.52	85		7.49 10.08	2.59		.091 .130	.039		1.90 1.30		.60	5.2 11.0	5.8	
19	4.30 4.04		26	1.82 3.04	1.22		7.99 9.28	1.29		.097 .119	.022		.70 .90	.20		10.4 33.0	22.6	
20	4.51 3.34		1.17	1.93 4.26	2.33		7.83 15.12	7.29		.093 .201	.108		.30 .80	.50		11.0 13.0	2.0	
22	4.29 4.29	=	=	1.26 2.46	1.20		6.25 8.72	2.47		.076 .111	.035		.50 1.40	.90		7.0 26.2	19.2	
23	4.12 3.99		.13	1.60 2.58	98		7.40 8.73	1.33		.091 .118	.027		.80 1.60	.80		12.4 19.4	7.0	
24	4.38 4.07		.31	1.96 2.43	47		7.90 10.29	2.39		.094 .129	.035		.30 .80	.50		27.0 26.1		.09
27	4.40 4.36		.04	1.20 2.39	1.19		5.01 9.00	3.99		.059 .116	.057		.60 .80	.20		11.0 35.0	24.0	
28	4.15 4.32	.17		1.24 2.86	1.62		6.31 10.43	4.12		.077 .119	.042		.00 .20	.20		5.0 8.2	3.2	
21		7½	13½		18	3		19	2		19	2		17	4		19	2

correlated to furnace losses than can Orsat determinations. Note that in 19 cases out of 21, the higher smoke readings are associated with the low furnace efficiencies.

Columns 5, 6, and 7 show the volatile reported on various bases. On the bottom line is the score. On a proximate basis, 16 cases out of 21 reveal higher volatile with the low furnace efficiencies; on a moisture- and ash-free basis, in 17 cases out of 21 higher volatile is associated with the low-furnace-efficiency subgroups; and under the volatile-carbon data, 18 cases out of 21 associate higher volatile with low furnace efficiency.

Now let us note two elements from the ultimate coal analysis. Regarding oxygen, column 12, in 19 cases out of 21 higher oxygen of the coal is associated with the low-furnace-efficiency groups; and, in column 8, sulphur, in 19 cases out of 21 higher sulphur is associated with the low-efficiency group.

Contrary to some opinions, the hydrogen in the coal (columns 9 and 10) cannot as yet be so definitely related to these furnace losses. Of course hydrogen occasions a loss in its own right, which must not be confused with these losses that are due to a failure to completely burn all materials in flight.

Finally, columns 11 and 13, "mathematical factors," reported by some to be indicators of the difficulties encountered in burning coal, are no more valuable in revealing information than are the simple elements from which they are made up.

If a low order of firing equipment is being employed, which allows these elements of high volatile, high sulphur, and high oxygen to cause a furnace loss, the first thing to do, in order to have the advantages of overfire air fully realized, is to employ a higher order of firing equipment.

The foregoing is rather the reverse side of the picture of overfire air. Now let us take the same 3 subgroups of high, low, and medium furnace efficiency and study the air-supply condition from the same angle as the author has used.

In Fig. 14, we use all three subgroups of high, medium, and low furnace efficiency (see ordinates), and take each subgroup separately to plot all the furnace efficiencies of that group against CO_2 or diminishing air supply. Note that, as the CO_2 grows better and better, furnace efficiency grows worse and worse, because of mounting furnace losses, due to restricted air.

The net result on over-all efficiencies (not shown here) is a meager gain of about 3½ per cent in the entire improvement from 7.2 per cent CO_2 to 12 per cent CO_2.

You see what we are up against? Small excess air is highly desirable in order to make the boiler efficient as a heat absorber, but the smaller the excess air the higher the furnace loss, unless something drastic is done about it. Observe the right-hand end of the lowest curve, Fig. 14; 17 per cent of the heat value of the fuel wasted by the so-called "unaccounted for" loss. This loss can definitely be labeled furnace loss due to inadequate air supply and inadequate turbulence. About 15 per cent more air injected over the fire and accompanied by a penetrating and sustained turbulence would help that condition considerably.

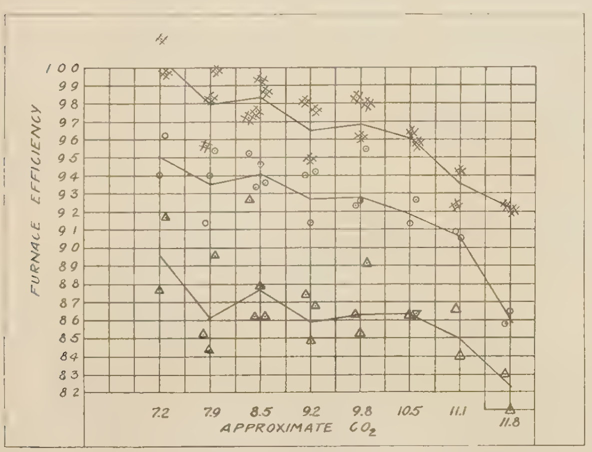

Fig. 14 Furnace Efficiencies Versus CO_2 or Diminishing Air Supply

It might be enlightening to mention that these same boilers were later provided with larger furnaces and fired with both underfeeds and chain grates, but still without adequate overfire air. The excess air came down and the boiler, as a heat absorber, performed better, the over-all efficiency also improving slightly. However, the very fact that the air supply was still further reduced caused the furnace losses to persist—and definitely.

Fig. 14 illustrates what is taking place in thousands of plants every hour and demonstrates by actual data the need for the kind of work the author is doing.

H. M. Toombs.[6] Rather than go directly into a discussion of the value of jets for the introduction of secondary or overfire air, it might be well to examine the entire jet field and see if there is really anything of value to be obtained from the use of any kind of a jet.

We are inclined to use almost any kind of jet for practically any purpose and thus make a mistake which discourages us from further effort. In other words, since first impressions are lasting, if the design is poor or the application wrong, which is found to be true in so many cases, then we just condemn that piece of equipment and have no further use for it.

An operating man will not continue to use equipment which gives him trouble. But let him install something that makes the job easier and he becomes wedded to it for life. The writer does not mean to intimate that an engineer is lazy, but he knows from experience that all he has to do is wait a little while and some problem bobs up for solution. This, of course, makes his job interesting, but a multiplicity of such items will give him gray hair.

According to Bernoulli's theorem, when the pressure in a gas stream of constant volume is increased, the velocity is increased, and when the pressure is decreased, the velocity decreases in direct proportion. So if you wish to convert pressure head to kinetic energy, you force the gas through a small hole. This procedure forms the basis for measuring the flow of gases, liquids, steam, air, and water by means of an orifice meter. These are also measured by a Venturi tube which is a very common type of jet, in everyday use, and one upon which exact dependence is placed. We would not think of questioning the accuracy of measurements made with a calibrated jet.

All operating engineers are familiar with the injector. Railroad locomotive engineers bank their lives on its unfailing action. That is a jet, and a very wonderful jet it is, because it takes steam at boiler pressure and increases the pressure to the point of being able to put water back into the boiler again. In stationary work, when a centrifugal pump fails or a reciprocating pump becomes steam-bound, the injector jet is used to get the water into the boiler. That jet is a real pump.

Ethyl gasoline goes through a combining jet in which is placed the lead mixture. Jets are used for water heaters. Then there is the thermal compressor which is finding application in many new fields. It takes exhaust steam and boosts the pressure so that it can be used in places requiring a medium heat head. Paper mills and rubber plants (not synthetic rubber) use these compound steam boosters. Vacuum pans, filtration projects, sludge removal, all use jets for an endless variety of jobs. And why? Because the jet is simple in design, reliable in service,

[6] Combustion and Refrigeration Specialties, Oak Park, Ill.

and operates with cheap readily available steam. It is compact, has no moving parts, and the wear is comparatively slight.

By such means a high pressure is enabled to move a lower pressure to an intermediate pressure, just as Bernoulli had stated. Therefore, if we want a compact cheap air compressor for introducing secondary or overfire air, why not use a steam jet? It will do things for us but like every other piece of mechanical equipment it has limitations.

When a jet is applied to a furnace, it is used as an air compressor, to boost air or oxygen from atmospheric pressure to a sufficiently high pressure to break through the rising stream of gas from a fuel bed, caused by the natural stack effect in the furnace, or in combination with a forced blower.

Probably, there is no mechanical appliance in the world that has so many different designs as an industrial boiler furnace. Designs have been constantly changing and they keep right on changing, in the name of efficiency, increased heat release, and smokeless combustion. Years ago, furnaces were designed with unbelievably long arches in the front; today they are just as long, in the rear. Then there are furnaces with an abbreviated arch and some with no arches at all. How is it possible to determine in advance that they are going to work and work smokelessly, even with the advice and aid of our smoke departments?

We have not changed our fundamental ideas because air and oxygen are still needed at a combustion temperature, if fires are to burn smokelessly. With increased emphasis and education on these two basically sound theorems we have met with good success.

When a fire burns, and the fuel is being fed from a grate as in the case of a natural-draft chain installation, the coal ignites from the top of the pile and starts to burn down. If all of the air is supplied from the bottom or through the grate, free air will not catch up with the rapidly releasing gas. It would therefore seem reasonable to believe that a certain percentage of air could be supplied to the top of the fire and there combine more rapidly and quickly with the combustion gases. The thought is self-evident to those who have watched a banked fire being brought up, with corner air jets laying a solid blanket of air over the surface of the fuel bed as it travels toward the waterback. Without the jets, the fire is sluggish and dead. With the jets, it is bright and lively with small flames shooting upward over the entire surface; hence the advocates of secondary or overfire air.

Air is very difficult to control; it will always go a way that was not intended. Once it is released from the confining walls of a tube or pipe, natural forces swing into action, and we can control it no further. If the air is needed over the center of the fuel bed, then there must be sufficient force in the stream to overcome the stack action of the rapidly rising gases and cut right through them so that the air is deposited where we want it to go. Jets will do this. Steam jets can be designed that will make air whirl, fan out, cup, or pierce. However, the jets and nozzle all require separate designs; no two can be exactly alike.

Today, in an effort to speed up many industrial processes, recourse is made to agitation. For example, in the field of refrigeration, we have found that by increasing the brine velocity through a shell cooler, which is nothing more than a horizontal return-tubular boiler, the capacity or heat absorption of the cooler is doubled. Boilers which steam rapidly have the fastest circulation. Pumps are being used on boilers to increase the water velocity in the tubes and secure faster wiping action of the tube surfaces. The results obtained by installing pumps on hot-water boilers and heating radiators are well known.

Overfire air will mix the gas streams and prevent stratification of unburned gases, eliminating stray combustion in the tube banks of the boiler. Steam jets have been designed and installed to recirculate the combustion gases over the fire without the introduction of any additional overfire air, and engineers seem to believe that considerable advantage may be obtained by so doing.

The question of steam costs invariably arises whenever jets are discussed. Often more argument is expended in this discussion than effort is made to fix steam leaks in the plant. If steam jets cost as much as some engineers say they do, then steam leaks must run into fabulous sums.

However, this question is again one of economical design. Now, a steam jet is not a hole in a piece of pipe. Exhaustive tests have been made on jet equipment, and excellent economies have been realized, but there is still room for improvement. There is no one living who can assemble stock fittings and make an economical job. It must be built in accordance with the fundamental principles of Bernoulli.

First, we must determine exactly what we wish to accomplish with the air, then decide how much air is required, and finally attempt to supply this air in the most economical manner.

Assume a chain grate 10 $\times$ 10 ft, burning 30 lb of coal per sq ft per hr; or 3000 lb of coal per hr total, which is at the rate of 50 lb per min. Assume that 14 lb of air or 200 cu ft are required to burn 1 lb of coal. To burn 50 lb of coal per min thus requires 10,000 cu ft of air per min. This is quite a sizable amount of air. To guarantee that it will be uniformly mixed with each pound of combustion gases is questionable. A $^5/_{32}$-in. steam jet correctly designed and with 100 psi steam pressure at the nozzle supplies better than 850 cu ft per min. Two jets fired tangentially would use 0.8 $\times$ 60 or 228 lb of steam per hr, and if the coal cost is 20 cents per 1000 lb of steam, the expense would be $4^1/_2$ cents per hr or about one dollar per day, which does not seem excessive for the many advantages derived.

However, this cannot be done with a piece of pipe. It is necessary that the steam nozzle be of correct proportions, that it be correctly placed in the combining tube, and that the induction tube and tailpiece be of correct design.

N. E. Werner.[7] It is the writer's opinion that the quantity of air to be used as overfire air, since its principal use is to create turbulence and agitation, should be held within the range of from 3 to 7 per cent of the total air for combustion required at the maximum continuous steam output of the unit. This overfire air should be introduced at a reasonably high velocity, anywhere from 10,000 to 20,000 fpm depending upon the turbulence required. Uniformly, a separate overfire-air fan must be used to obtain the required static pressure corresponding to the velocity desired.

When the desired velocity in feet per minute has been determined somewhere between the limits just outlined, the pressure head required in the plenum box will be

$$p = d\left(\frac{v}{1096.4}\right)^2 \ldots\ldots \quad [1]$$

where

p = pressure in inches H_2O
d = density of air, lb per cu ft
v = velocity, fpm

Equation [1] is derived from the basic equation

$$v^2 = 2\,gh$$

The quantity of overfire air on a weight basis, somewhere between 3 and 7 per cent of the total air for combustion, is then converted to a volumetric basis as follows

[7] Chief Engineer, Bigelow-Liptak Corporation, Detroit, Mich

$$Q = \frac{W \times S}{60} \quad \text{[2]}$$

where

Q = quantity of overfire air, cfm
W = weight of overfire air, lb per hr
S = specific volume, cu ft per lb

From the known velocity and quantity given, the area of each nozzle is readily obtained from the following formula

$$a = \frac{144\,Q}{CvN} \quad \text{[3]}$$

where

a = area of each nozzle, sq in.
Q = quantity of overfire air, cfm
C = orifice coefficient (see Fig. 15a)
v = velocity, fpm
N = number of nozzles

The nozzles can be spaced conveniently across the furnace width approximately on 12 in. centers, keeping the end nozzles on both sides approximately 12 in. away from the furnace side walls.

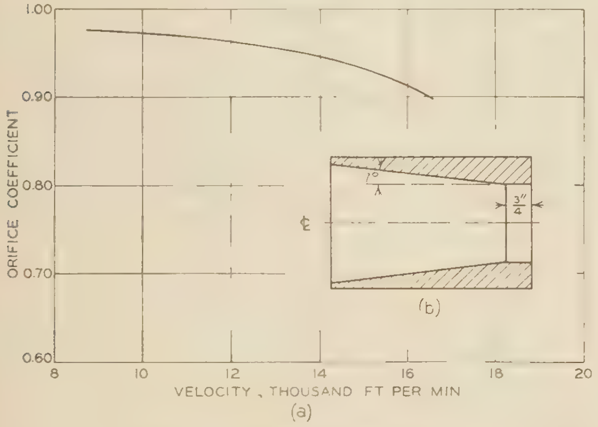

FIG. 15(a) VALUES OF ORIFICE COEFFICIENT C; (b) ORIFICE DESIGN

The number of nozzles to be used can be determined by design in this general manner.

Fig. 15(a) shows the orifice coefficient C, on the ordinate for varying velocities in thousands of feet per minute on the abscissa, for the type of nozzles commonly employed for this work.

The nozzle tips should be designed of a high grade of cast iron, preferably Meehanite HC material, with a very gradual slope to the orifice in the order of approximately 7 deg with the center line of the orifice. The orifice itself should be approximately 3/4 in. long and be parallel with the center line of the nozzle, as shown in Fig. 15(b).

A divided plenum box can be utilized in order to secure a lesser quantity of overfire air at lower operating rates and still maintain substantially the same velocity for continued adequate furnace agitation. In this design the nozzles are staggered, one projecting from each division of the plenum box so that when one division of the plenum box is dampered, only the nozzles from the other division or divisions are operating. In this manner the pressure head in the various divisions of the plenum box can be maintained substantially the same throughout the lesser operating rates, but the quantity of overfire air will be reduced since the nozzles projecting from the dampered sections are not operating. The design of the nozzles for this arrangement is carried out in the same manner as outlined, and the quantities of overfire air are reduced only by the direct reduction of the area of the nozzles.

JOS. HARRINGTON.[8] Thirty years ago a large number of boiler tests showed quite clearly that dense smoke from a chain-grate setting was accompanied by an efficiency drop of about 5 per cent. While we did not know just why this was so, nor just what to do about it, the fact seemed to be clearly proved.

Through all the intervening years, engineers have been conscious of this, but it remained for the author to bring research methods to the point where accurate analyses could be made.

Recent experience by the writer leads to the belief that, while smoke is always accompanied by hydrocarbon loss, the latter may occur with what we are pleased to call a "clean" stack. Two careful tests of recent date illustrate this. Both large pulverized-coal jobs, they were practically duplicates except in the amount of excess air supplied for combustion. In one, the CO_2 was held down to 14 per cent and an efficiency obtained of 88 per cent. In the other, the CO_2 was a full 16 per cent, with an efficiency of 83 per cent. This great difference could be accounted for in only one way, i.e., a hydrocarbon loss.

It would be incorrect to stop at this point, however, and allow the impression to prevail that 16 per cent of CO_2 is *per se* too much. The qualifying phrase, "in this furnace at the rate of firing on this test," should be used, and this qualification must be appended to any statement about the optimum CO_2.

The clear deduction is that the most efficient percentage of CO_2 is a function of the fuel, furnace design, rate of heat release, and means employed for effecting complete combustion within the furnace. Conversely, the popular idea that maximum CO_2 and highest efficiency always go together is definitely wrong.

The author has done a fine job, not only in developing the apparatus necessary for field analyses, but in showing how to detect and evaluate the elusive hydrocarbon and hydrogen losses. In this, he has also shown how to appraise the value of the correct application of overfire air.

It is not merely more air that a smoky furnace requires. In fact, there is usually plenty of air in every furnace. It is a matter of mixing the rich and lean strata found in every furnace in some degree. Overfire-air jets should be for this purpose primarily, rather than simply to put more air into the furnace. Consequently, they should be carefully located and directed to effect the maximum mixing of the gases. A few high-pressure jets are better than many which just let some air ooze in.

It is the writer's opinion that before long overfire-air application will be an integral part of every furnace design. The author has performed a great service in developing the ways and means for evaluating correctly the effect of such application.

C. C. PLUMMER.[9] Generally speaking, there are two major functions for the use of overfire air. The first would be to obtain secondary combustion, and the second to obtain additional turbulence in the furnace by the use of high-velocity air.

There are many instances where secondary combustion may be more harmful than otherwise, and also great care must be taken in the use of high-velocity overfire air to avoid the possibility of soft slag impingement on furnace walls or heating surfaces.

With all types of solid-fuel firing where thick fuel beds are developed, either by hand-firing or by the use of certain types of overfeed stokers, usually of a natural-draft kind, or by the use of underfeeds or chain grates, it is possible to consider the

[8] Northern Illinois Coal Corporation, Chicago, Ill.
[9] Hoffman Combustion Engineering Company, Chicago, Ill.

use of more or less overfire air. Favorable consideration of the use of overfire air in the instances cited should be based upon a careful survey of the combustion processes for each individual job, as well as the general arrangement of furnace walls and boiler heating surfaces, flame travel, furnace volume, and similar factors, which might determine the desirability of overfire air, and whether it can be utilized beneficially or otherwise.

If the character of the fuel bed, and the introduction of primary air for combustion contribute to conditions in which there are two separate effective processes of combustion, this would definitely indicate the desirability of secondary air, if it can be utilized successfully. It would be presumed that a certain coking process would prevail, liberating distillates into the furnace above the fuel bed, in such quantities as to indicate that at least part of the combustible volatile might not come in contact with sufficient oxygen to accomplish complete combustion. The introduction of additional oxygen throughout the furnace, together with an increased turbulence of furnace gases, would promote secondary combustion, resulting in higher efficiency and presumably a better stack condition.

In order to accomplish the desired results, the section of the furnace taken in a horizontal plane should be of sufficient area so that partially burned particles of solid matter, entrained in the gas stream, would not be driven against the furnace walls in a soft state, thus permitting heavy slag deposits.

Likewise, the distance from the fuel bed to the first boiler heating surface should be sufficient, so that the combination of carbon and oxygen, as a result of secondary air, would not take place close enough to the heating surfaces, and the temperature at that point would be below the ignition point. Secondary combustion taking place close to the boiler heating surface will promote slagging of boiler tubes, and if the flame travel from secondary combustion is around the boiler tubes a smoky stack usually results.

It is usually possible to study individual jobs and to determine whether favorable results can be obtained from the use of overfire air in small or greater quantities, or whether better results can be accomplished by utilizing thinner fuel beds and mixtures of coal and air as a result of better control of forced draft through the tuyères and the fuel bed. Another consideration is the utilization of turbulence in the fuel bed, so as to accomplish the highly desirable result of complete combustion low in the furnace with short-flame ignition, thus avoiding the necessity for any appreciable amount of overfire air.

The Firite spreader stoker with the Hoffman tube tuyère grate is a good example of a combination of free oxidation in the fuel bed, complete combustion low in the furnace, as evidenced by short-flame combustion, and continuous turbulence in the fuel bed, due to the tuyère design. All of these factors collectively eliminate the necessity for overfire air, although they do not necessarily preclude its use under certain specific conditions.

The standard design of the writer's company does not call for any overfire air for the purpose of secondary combustion, or for the purpose of additional turbulence in the furnace. A small amount of secondary air injected through the stoker throat piece is used for the purpose of cooling the lip of the throat plate which is the only part of the interior of this machine which is exposed to radiant heat from the furnace. The total amount of air thus used will average not over 1 per cent of the amount going through the stoker grates.

Occasionally there may be an instance where one of our stokers has been installed with the total projected grate area selected for a favorable combustion rate when operating the boiler at around 200 per cent of rating. Later developments may call for much higher ratings, which automatically increase the combustion rates per square foot of grate per hour and lead to some thickening of the fuel bed. In an instance of this kind, a careful investigation might develop the desirability of additional overfire air introduced into the furnace above the fuel bed, and with suitable directional velocity, in order to increase the time element in the furnace. Possibly, this would also promote some secondary combustion of fuel-bed distillates and, in some instances, of gas-entrained carbon particles.

There is a definite and predictable time element required for complete combustion of solid particles in the furnace gases, depending upon particle sizes. With any given installation of our equipment, with any given fuel analysis, we can make accurate predictions of the potential unburned condition of various particle sizes of solid matter entering the first tube bank. An analysis of this kind would immediately disclose the desirability of overfire air, the point in the furnace at which it should be introduced, the direction and static head desirable, and the quantity which would do the job.

Spreader-type stokers not adequately protected from radiant temperatures by water cooling, and using overfire air for cooling the coal-feeding units, in most instances, set up secondary-combustion processes in the furnace which are more detrimental than beneficial. This is evidenced by the fact that observations, in a considerable number of instances, have disclosed that with the same percentage of CO_2 at the top of the furnace the temperatures in the second and third boiler passes are anywhere from 60 F to 120 F higher than with spreader stokers installed under comparable conditions. It is believed that this condition is well understood by superheater manufacturers. We have found that they require additional elements when using convection-type superheaters with the spreader stoker to get the desired superheat. We have also found that they are able to maintain the desired superheat with this stoker equipment much more closely than where large quantities of secondary air are required with some other stoker types.

Author's Closure

From the discussions presented, the author believes that the importance of overfire air is beginning to be appreciated in connection with many kinds of fuel and many types of installation. However, the need for more research, investigations, tests, and results is emphasized.

As Mr. Gilg has brought out the fact that the possibilities are greatest where faulty design exists, it is also being worked into the original modern design of furnaces to provide turbulence and increased combustion rates, especially in large furnaces.

Its application to burning wet bark is almost fundamental if capacity of consumption of the refuse is important, as well as low maintenance and smokeless operation.

It is the author's conclusion that in modern furnace design, covering the numerous methods of firing, more and more attention is being given to the use of overfire air as an integral part of the design, and that there is much more to be learned about its application and results.

The author desires to thank the discussers for their contributions and feels certain that the investigation of this subject will be continued.

Some Mechanical Properties of Plastics and Metals Under Sustained Vibrations

By B. J. LAZAN,[1] GREENWICH, CONN.

A new oscillatory-type testing machine was developed for determining the mechanical properties of materials under alternating torsional stress. The damping capacity and dynamic modulus of rigidity of both plastics and metals were evaluated by the use of this machine and studies are reported[2] of how these properties are affected by sustained cyclic stress below the endurance limit and also at impending fatigue failure. A similar dynamic testing machine was built for applying alternating direct stress, and parallel studies were made under axial loading conditions. The mechanical properties of selected materials in static tension, compression, and torsion were also determined to supplement the dynamic tests. The wide deviations observed between the static and dynamic moduli of elasticity for plastics are analyzed and are associated with the damping capacity of the material. The significance of these deviations, as related to the repeated constant-deflection type of fatigue test on plastics, is discussed. Experimental data on the damping capacities, dynamic moduli of elasticity, and some static mechanical properties are presented for mild steel, Duralumin, grade X laminated bakelite, laminated-canvas phenolic, and methyl-methacrylate plastic.

Statement of Problem

In a large proportion of modern machine and structural members, vibratory forces are superimposed upon static loadings. These dynamic components of the resultant stress affect the mechanical behavior of the materials of construction and cause a reduction in their load-resisting properties. Higher speeds in present-day machinery and transportation have amplified the importance of dynamic forces as a factor in design. Studies made in England by Aitchison showed that 95 per cent of the failures in automobile parts were caused by dynamic forces (1).[3] Hardly any machine-part failures investigated by Roos of the Swedish Materials Testing Laboratory could be attributed to static forces alone; 80 per cent of the failures being caused by repeated stress, the remaining 20 per cent involving impact.

Much research has been stimulated by the need for accurate knowledge of the dynamic mechanical properties of materials. However, most of this work has been confined to one phase of the general problem, i.e., that of evaluating fatigue strengths or resistance to fracture under alternating stress. Although the fatigue strengths are, perhaps, the most valuable properties of materials, other dynamic characteristics are significant in many engineering applications. For example, under the forced resonant vibrations excited by wind, a copper overhead cable of low tensile and fatigue strength may be more durable than a light metal alloy of higher strength but lower damping capacity (12). Similarly, an engine crankshaft or an airplane propeller of high-damping-capacity material may outlast another of low damping capacity, even though its static and fatigue strengths may be relatively low.

The dynamic mechanical properties which affect the serviceability of structural and machine parts follow:

(*a*) The fatigue strengths define the points of fracture.

(*b*) The damping capacities are associated with the peak stresses caused by near-resonant vibrations, the heat produced in a material during cyclic stress, the noise produced by meshing gears and other machine parts, "dynamic ductility," the "whirling" of rotating shafts, and the vibration insulation of a material.

(*c*) The dynamic moduli of elasticity influence the natural frequencies of vibration and the alternating stress caused by a known cyclic strain.

(*d*) The effects of superimposed vibratory stress on the stress-strain and strain-time (creep) relationships are important in many engineering designs where excessive deformation constitutes failure. In heat engines, for example, very small clearance must be maintained for efficient operation.

The special apparatus and methods developed for this work have been used to secure a limited amount of data on all of the dynamic properties mentioned. However, this paper is restricted to a discussion of the damping capacities and dynamic moduli of elasticity of materials under both pure torsional and pure longitudinal vibrations.

Apparatus and Technique

Basic Experimental Arrangement. The basic experimental setup is illustrated in Figs. 1 and 14 (*a*). The oscillator O is attached to the bottom of test specimen S by means of gripping chuck C_b. Chuck C_t, securely attached to massive frame F, supports the specimen-oscillator combination. Since frame F is rigid and massive, the top of the specimen is practically fixed and the specimen-oscillator assembly behaves as a system with a single degree of freedom subjected to the forced vibrations of oscillator O. Vibrations at any proximity to resonance may be produced and controlled.

The controlled vibratory stress induced in the specimen constitutes a fatigue test. This cyclic stress is determined from inertia force produced by the vibrating mass. The dynamic moduli of elasticity are computed from the frequency of the resonant or near-resonant vibrations. The damping capacities are evaluated from the knowledge of the oscillator force and amplitude of vibration at or near resonance. A quantitative analysis of these three properties, with equations, appears in section D of the appendix.

Dynamic tests under many types of stresses may be performed with this apparatus. If the oscillator O, in Fig. 1, is adjusted to produce alternating torsional couples T_o, the specimen-oscillator system vibrates as a torsional pendulum and induces alternating shearing stress in the specimen. If the oscillator is set to produce alternating longitudinal forces F_o along the axis of the specimen,

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[2] This paper was prepared from a thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy at The Pennsylvania State College. The tests dealing with plastics were sponsored by an A.S.M.E. Engineering Foundation Grant.

[3] Numbers in parentheses refer to the Bibliography.

Contributed by the Rubber and Plastics Subdivision of the Process Industries Division, and presented at the Semi-Annual Meeting, Cleveland, Ohio, June 8–10, 1942, of The American Society of Mechanical Engineers.

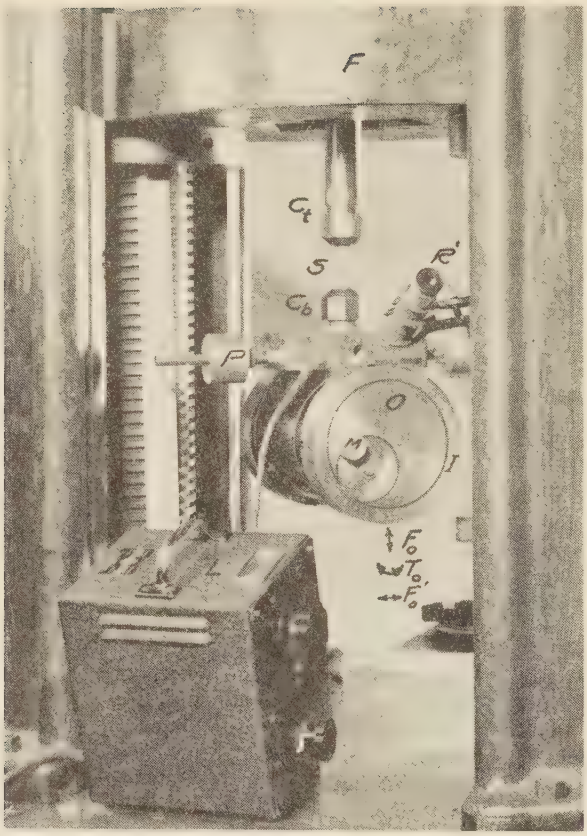


Fig. 1 Universal Dynamic Testing Machine

then direct dynamic stress results. Lateral forces F_o' cause the specimen-oscillator system to vibrate as a cantilever beam and thereby subject the specimen to alternating bending stresses.

The apparatus is therefore capable of producing the three most common types of dynamic stresses, i.e., tension-compression, torsion, and bending, in the pure form. Any combinations of these three types of pure vibrations may also be produced by the "hypocyclic oscillator" to be described under that heading.

The vertical specimen arrangement, Fig. 1, is satisfactory for most torsional-vibration tests and low-capacity direct-stress tests. However, when large specimens are tested under direct dynamic stress, the vibration of the supporting frame becomes quite pronounced. Not only are these frame vibrations troublesome, but they signify a source of energy loss which would distort the data on the damping capacities and dynamic moduli of elasticity. Therefore, when the dynamic forces involved are large, a variation of the basic experimental arrangement to be explained in the following section is employed.

Horizontal Arrangement. Figs. 2 and 14 (*b*) illustrate a direct-stress dynamic testing machine, in which the specimen S is in a horizontal position. The roller R and rail T arrangement enable the masses M_o and M_o' to move freely in a horizontal direction. During the vibration, induced by the oscillator O bolted to the mass M_o, the two masses always moved in opposite directions at any instant of time. The nodal position of the vibration, which always remains stationary, is near the center of the specimen (refer to section C of appendix). Parts on opposite sides of the nodal section vibrate in antiphase, and the specimen is thereby subjected to alternating direct stress. The rails and rollers are made of an alloy steel, heat-treated to a Brinell hardness exceeding 500, and ground to a smooth accurate finish. The extreme hardness reduces the energy loss caused by hysteresis damping at the rail-roller contacts, and the ground finish minimizes the vibrational energy radiated through the rails to the floor.

This horizontal setup has the following important advantages over other fatigue machines, especially when concerned with high-capacity work.

1 No massive frame or other structural units are required. The only parts subjected to the full alternating force are the specimen and its gripping chucks. The floor and rails need support only the constant dead weight of the apparatus, and a special foundation is not required.

2 Slight difficulty is encountered because of eccentric loading of specimens. This eccentricity, which is usually a significant source of error in many direct-stress fatigue machines, may be caused by the nonlinear motion of the specimen chucks, such as occurs in a beam-type machine, or an eccentric center of pressure of the applied load.

The fatigue machine, shown in Fig. 2, reduces both of the foregoing causes of eccentricity because all the motions involved are linear. Furthermore, all elements of the machine are symmetrical about the axis of the specimen and are subject to easy and accurate adjustment for concentricity.

3 The room available for the test specimen or structure is unlimited. The floor rails may be moved apart in order to accommodate a specimen of any length, and with suitable pedestals for

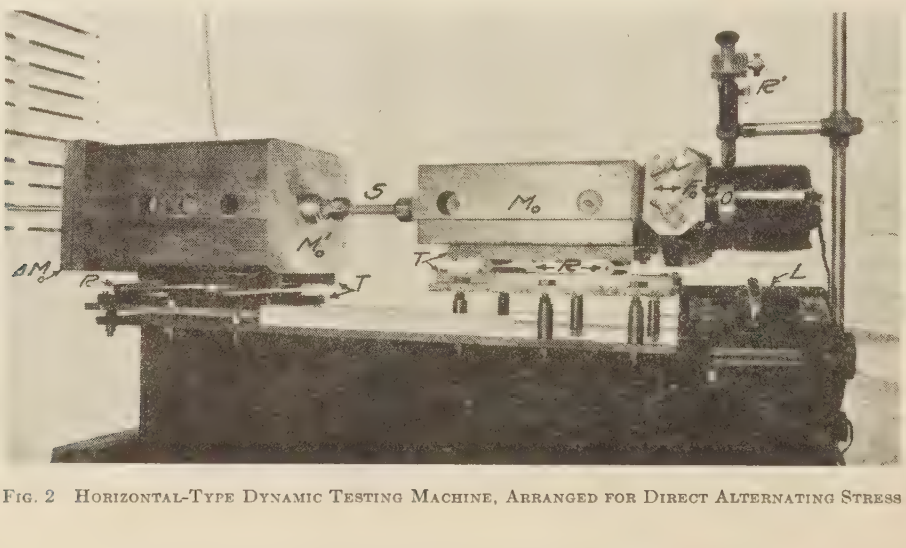


Fig. 2 Horizontal-Type Dynamic Testing Machine, Arranged for Direct Alternating Stress

raising the rails specimens of any width or height may be inserted.

4 If the vibrating system is operated near resonance, large amplifications in the applied force may be realized. Amplification factors exceeding 200 have been observed in this machine.

A new ±150,000-lb direct-stress fatigue machine of this horizontal type which utilizes the resonant-vibration principle is now under consideration.

5 The damping capacities and dynamic moduli of elasticity may be evaluated.

It may be desirable to replace the left-hand weight M'_o by a fixed support, but dynamic forces will then be transmitted to the foundation. For low-capacity machines, it may be more convenient to hang the mass M_o from wires, rather than to support it on rollers.

An experimental arrangement, similar to Fig. 2, may also be used for large-capacity torsional-fatigue tests. In this case the masses, M_o and M'_o, which may be flywheels, would be supported by bearings concentric with the specimen. The two masses would vibrate torsionally in antiphase about the axis of the specimen and thus subject the specimen to alternating torsional forces. This experimental arrangement was not used, however, since the basic setup of Fig. 1 was satisfactory for all the torsional work undertaken.

Descriptions of the main parts of the experimental arrangement, including the two oscillators, will be given.

Centrifugal-Force Oscillator. The oscillator in the horizontal arrangement of Fig. 2 employs the centrifugal force of eccentrically supported rotating masses. In this oscillator, which utilizes an old principle (9, 10), but is of a new compact design, an eccentric mass is attached to the ends of each of two shafts rotating in opposite directions. If the eccentrics are set as shown in Fig. 2 (where only one end of each shaft is visible) the vertical components of the centrifugal force mutually cancel each other and leave only horizontal sinusoidal force F_o.

By turning two diagonally opposite eccentrics through 180 deg, the oscillator can also produce pure torsional vibrations.

As the project expanded in scope, certain limitations appeared in the centrifugal-force oscillator, and the hypocyclic oscillator was therefore developed.

Hypocyclic Oscillator.[4] The principle of this newly developed oscillator is illustrated in Fig. 3. Pinion B meshes with and revolves within stationary internal gear A so that the center of the pinion moves in the circular path C-C'-C''. During its clockwise revolution, pinion B rotates about its own center C counterclockwise. Since the diameter of the pinion is one half that of the internal gear, any point on the pitch circle of pinion B moves with linear sinusoidal motion. Thus point P moves along the horizontal straight line 1-2-1-3-1 and any general pitch point G moves along inclined line 4-6-4-5-4. If a small mass (see M in Fig. 3) is attached at any pitch point, such as P or G, it produces linear sinusoidal force along the direction of the motion.

In Fig. 3 electric motor E has a flange mounting and shaft extension on either end, to which is attached identical ring assemblies R and R'. The motor drives the rotating disks which carry pinions B and B', and the eccentric masses M and M' move with linear sinusoidal motion. The direction of the straight-line motions of these masses and the corresponding line of action of the linear sinusoidal force are adjusted by meshing the proper teeth of pinion B and internal gear A.

The hypocyclic oscillator illustrated in Figs. 1 and 3 can produce many types of pure or combined dynamic force merely by proper presetting of pinion B within gear A; for example:

[4] The patent on this oscillator, which is pending, has been licensed to The Baldwin-Southwark Corporation, Philadelphia, which company is producing it commercially under the name of the "Lazan Mechanical Oscillator."

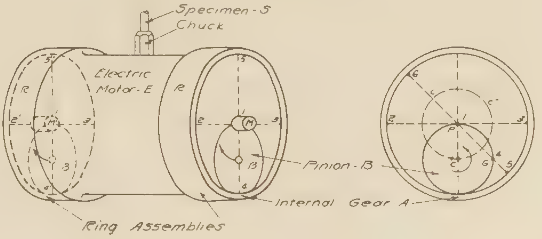


FIG. 3 PRINCIPLE OF HYPOCYCLIC OSCILLATOR

(a) To produce alternating bending stress in the test specimen or structure S, Fig. 1, pinions B and B' are set as shown in Fig. 3 so that masses M and M' oscillate horizontally in phase in paths 1-2-1-3-1 and 1′-2′-1′-3′-1′, respectively.

(b) To produce longitudinal vibrations or alternating direct stress, pinions B and B' are set so they mesh at 3 and 3′, respectively, and the inertia masses remain in the position shown. Masses M and M' therefore move vertically in phase in paths 1-4-1-5-1 and 1′-4′-1′-5′-1′, respectively.

(c) To produce torsional vibrations pinion B is meshed with gear A at point 5 so that mass M and pinion B' remain in the position shown in Fig. 3. Masses M and M' therefore move horizontally in antiphase in paths 1-3-1-2-1 and 1′-2′-1′-3′-1′, respectively.

(d) A state of combined torsion and bending, combined torsion and direct stress, or combined bending and direct stress in any relative magnitude is produced by setting the pinion so the inertia masses move in a predetermined inclined direction and/or at suitable phase angles.

To regulate the magnitude of the force produced by this oscillator, any one or several of a geometric series of masses similar to M may be screwed in place. In a new design, not used in the present work, the force may be continuously regulated while the oscillator is in operation.

The frequency of oscillation may be altered, even if the motor E has a constant speed, by inserting suitable change gears between the motor and ring assembly.

The hypocyclic oscillator weighs about 50 lb, including its $^1/_8$-hp, 1800-rpm, synchronous electric motor, which alone weighs 25 lb. The latest design can produce forces exceeding 500 lb, or torques greater than 2500 in-lb; however, a larger motor may be required for some applications.

Use of Near-Resonance Vibrations. The specimen-oscillator vibrating systems shown in Figs. 1 and 2, may be operated at any proximity to resonance. Near-resonance vibrations were used for two reasons:

(a) To permit a more accurate determination of the damping capacities and the dynamic moduli of elasticity (refer to equations in section D of appendix).

(b) To increase the force-producing capacity of the dynamic testing machine (9). The amplitude of forced vibration produced by an alternating force of constant magnitude (such as the force of an oscillator) may increase more than 200 times as a vibrating system approaches resonance. This vibration amplification is illustrated in Fig. 4, and factor A_v and other symbols are defined in section A of appendix.

Synchronous-Motor Drive for Resonance Control. Because of the steepness of the resonance curve near resonance, an oscillator motor with excellent speed regulation is required, in order to control reasonably the force on the test specimen. For example, experimental curve B of Fig. 4 shows that for a material possessing low damping capacity, a 1 per cent change in oscillator frequency near resonance may cause a 500 per cent change in amplification factor A_v and the stress in the test specimen. If the vibrating system is operated away from resonance, or if high damping

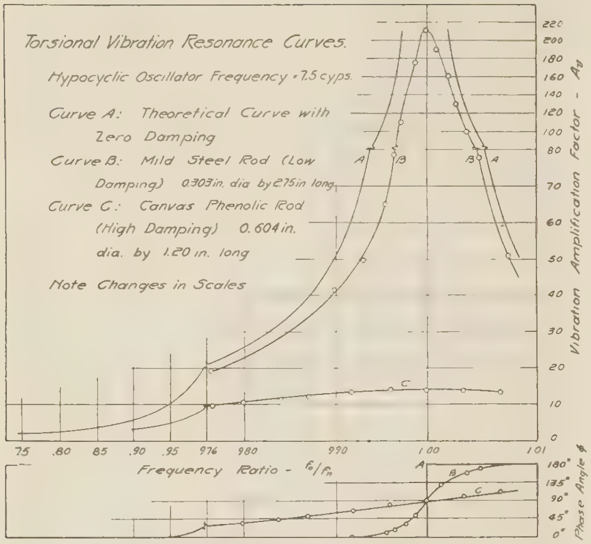


Fig. 4 Resonant-Vibration Curves

capacity is present in the system (see curve C in Fig. 4), then the amplitude of vibration does not vary so critically with oscillator frequency.

The speed of the usual electric motor is influenced by such factors as variation in motor load, line voltage, and heat transients, which disturbances often cause a speed fluctuation exceeding 5 per cent. With special apparatus it is possible to reduce this fluctuation to within 1 per cent in some cases. With the ordinary synchronous motor, however, the speed remains essentially constant.

Method of Tuning Vibrating System to Resonance. Since the oscillator frequency f_o can be changed only in steps with change gears, the natural frequency f_n must be altered to tune the vibrating system to resonance.

The natural frequency of the torsional vibrating system of Figs. 1 and 14 (a), is adjusted by changing the moment of inertia I_o of the oscillator assembly (see Appendix, Equation [2]) by turning poise P on the threaded rod.

The natural frequency of the longitudinal vibrating system of Figs. 2 and 14 (b) is changed by adding masses $\Delta M_o'$ to the left end of M_o' (see Appendix, Equation [3]).

A phase indicator is built in each oscillator (see I, Fig. 1) to show the phase angle between sinusoidal force of the oscillator and displacement of the vibrating system. The position on the resonance curve, Fig. 4, may be determined by reading this phase indicator as explained in the following section.

Measuring Apparatus and Technique. The amplitude of vibration of a body or of an ink spot on a specimen is measured by a microscope-stroboscope technique (see R' and L in Figs. 1 and 2). In many cases a light needle scratch is placed on the vibrating body and, under oblique white illumination, the path of motion can be seen and quickly measured microscopically, without stroboscopic light.

To determine the phase angle of the vibration, the stroboscope is set at nearly twice the frequency of the vibration so that one spot on the oscillator generally appears to be two, always moving in opposite directions over the amplitude of the vibration. When the two spots cross, appearing as one spot, the vibration is at its mean position. At that instant, the phase angle is read on the phase indicator (see I in Fig. 1) illuminated by the same stroboscopic light.

Specimens and Gripping Devices. Nearly all tests reported in this paper were performed on uniform unmachined rods and tubes with outside diameters from $^1/_2$ to $^3/_4$ in. without fillets or specially machined test section.

Standard collet chucks for turret lathe and milling machine[5] (see C_t and C_b in Fig. 1) were used to grip the test specimen. The friction bond between the specimen and collet over $1^1/_2$ in. of specimen length was sufficient for these tests.

One desirable feature of the collet-chuck grip is that it can accommodate as a test specimen any uniform tube and rod without specially machined ends or test section. This is advantageous because specimens are easier to make, long gage lengths may be employed, and plastic and other rods with special surface coatings may be tested in an undisturbed condition. The fact that very few fractures occurred at the grips is indicative of the effectiveness of the chucks in avoiding stress concentration.

Test Procedure and Calculations

The specific procedure followed during any given test was determined by the requirements of the test. However, the general manipulations of the apparatus were somewhat similar for all tests, and the following sample procedure is intended to bring out the general operating details.

Sample Procedure for Torsional-Vibration Study. The specimen, a $^3/_4$-in-diam laminated-canvas-phenolic rod, was left in the as-received condition without disturbing the surface. On the basis of natural-frequency Equation [2] of the Appendix, the hypocyclic oscillator was set at 7.5 cycles per sec with suitable change gears, and the moment of inertia of the oscillator assembly was set at 700 lb-in.^{2} by moving poises P in Fig. 1 to a predetermined position.

The apparatus was assembled as shown in Fig. 1 with a small eccentric mass M screwed in place. The oscillator was turned on, and the phase angle, read as explained previously, was about 30 deg. Since resonant vibrations were desired, the poise P was turned further out on its threaded rod, thereby increasing the moment of inertia of the oscillator and making the phase angle approach 90 deg or resonance. When the moment of inertia I_o reached 731 lb-in.^{2}, the system was at resonance and the amplitude of vibration was read with scale microscope R' as explained before. Immediately after this reading was taken, which required about 3 min, the oscillator was stopped and a larger eccentric mass was screwed in place. Again the phase and amplitude were read and the adjusting-to-resonance process repeated. Successively larger eccentric masses were screwed in place to produce increasingly higher stresses in the specimen.

The alternating torque exerted by the oscillator was computed for each setting by the equation[6]

$$\begin{aligned} T_o &= (\text{oscillator force}) \times (\text{torque arm}) \\ &= (Me\omega_o^2)d \end{aligned}$$

The dynamic modulus of rigidity and the resonance-amplification factor A_r were determined by means of equations given in section D of the Appendix.

Procedure for Longitudinal Vibrations. The test procedure during longitudinal-vibration studies was similar to that just described for torsional vibrations, except that control masses $\Delta M_o'$ in Fig. 2 were used to adjust to any proximity to resonance. The damping capacity and dynamic modulus of elasticity were computed from equations given in section D of the Appendix.

Static Tension, Compression, and Torsion Tests. Several selected specimens were tested under static tension and compression in a screw-driven, hydraulic weighing testing machine. The crosshead speed was maintained at 0.02 in. per min throughout all tests. The load-deflection curves were autographically recorded with an O. S. Peters recording unit.

[5] The manufacturer of these chucks is The Universal Engineering Company of Frankenmuth, Mich.

[6] Symbols used are given in the Nomenclature, section B, appendix.

The mechanical properties under static torsion were determined in a dead-weight type torsion machine, a description of which is beyond the scope of this paper. The load was applied at such a rate as to cause fracture in about an hour.

Atmospheric Conditions During Test. Since the properties of plastics are affected by temperature and humidity, it is desirable to test all plastics in a controlled atmosphere (4, 5). However, during the fatigue tests, the temperature rise due to hysteresis damping is probably large enough to mask all other effects caused by atmospheric conditions. In this preliminary work therefore no attempt was made to condition the specimen prior to testing or to control the temperature and humidity during testing. Nevertheless, sling psychrometer readings were taken during the tests so that the relative humidity could be computed.

Summary, Analysis, and Significance of Test Data

Experimental Curves. Figs. 5 to 9 show the experimental data plotted with either direct or torsional stress as abscissas and with both the resonance-amplification factor A_r (analyzed in the following section) and the dynamic modulus

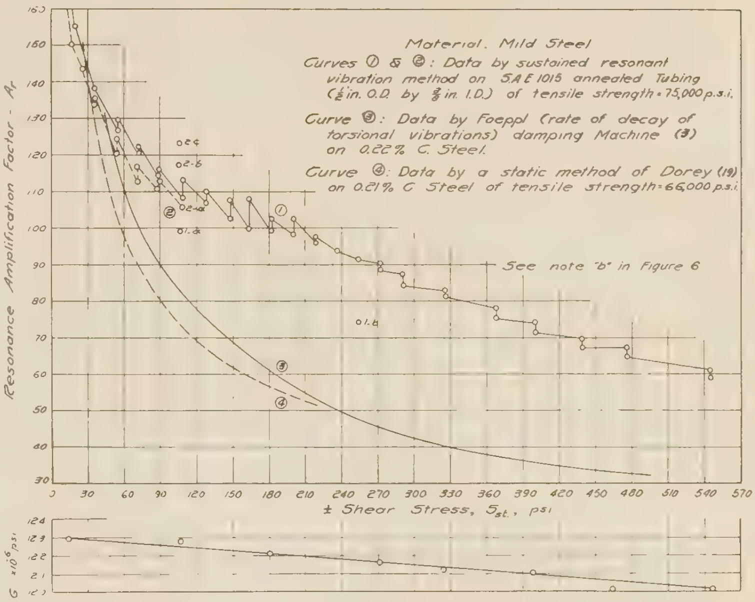


Fig. 5 Resonance-Amplification Factor and Dynamic Modulus of Rigidity of Mild Steel Under Torsional Vibrations

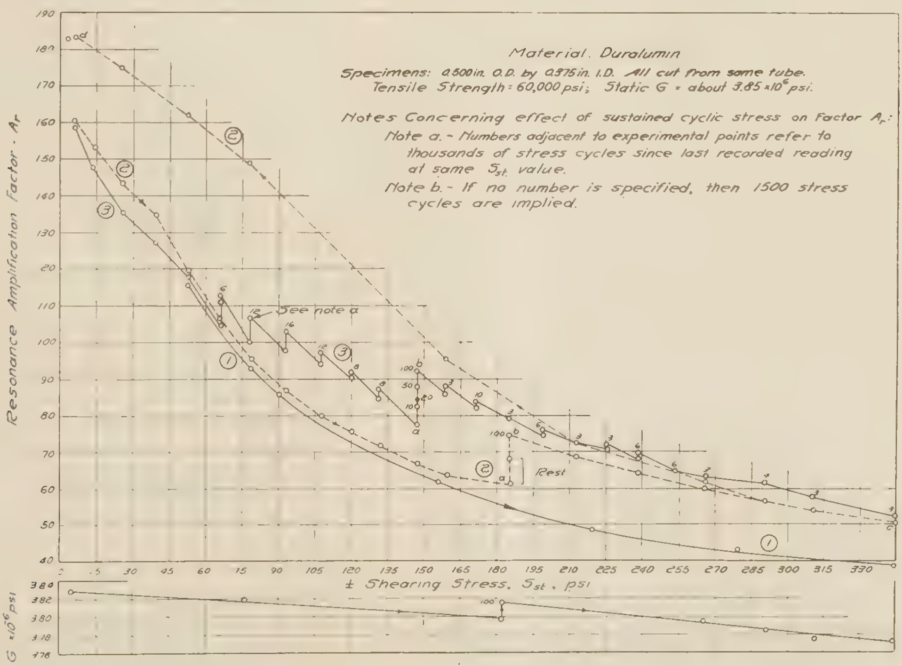


Fig. 6 Resonance-Amplification Factor and Dynamic Modulus of Rigidity of 17 S-T Duralumin Under Torsional Vibrations

of elasticity (discussed in a subsequent section) as ordinates.

In Figs. 5 and 6, the abscissa S_{st} is the shearing stress which would exist in the test specimen, if the maximum value T_o of the oscillator torque were applied statically. The dynamic shearing stress S_s at any abscissa S_{st} equals the product of S_{st} and A_r. The abscissa S_{st} was used in these figures so that any changes in the ordinates A_r and G produced by cyclic stress may be represented by vertical discontinuities in the experimental curves. The thousands of stress cycles since the last recorded point on these vertical portions of the curves are indicated by the numbers adjacent to the experimental points (see notes *a* and *b* of Fig. 6).

In Figs. 7 to 9 no attempt is made to illustrate the effect of cyclic stress on ordinates A_r and G, and the abscissa is the dynamic stress S_s or S_d for the torsional and longitudinal cases, respectively.

Resonance-Amplification Factor A_r. In the following discussion the terms "damping capacity" and "resonance-amplification factor" (a quantitative and reciprocal measure of damping capacity) are used in accordance with the definitions given in section A of the Appendix.

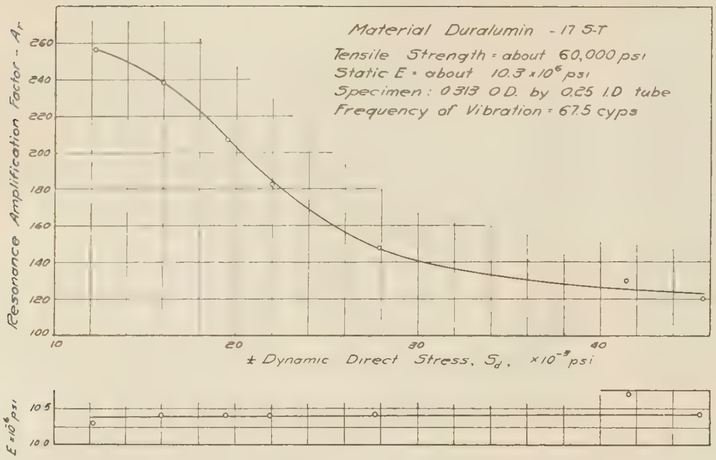


FIG. 7 RESONANCE-AMPLIFICATION FACTOR AND DYNAMIC MODULUS OF ELASTICITY OF DURALUMIN UNDER LONGITUDINAL VIBRATIONS

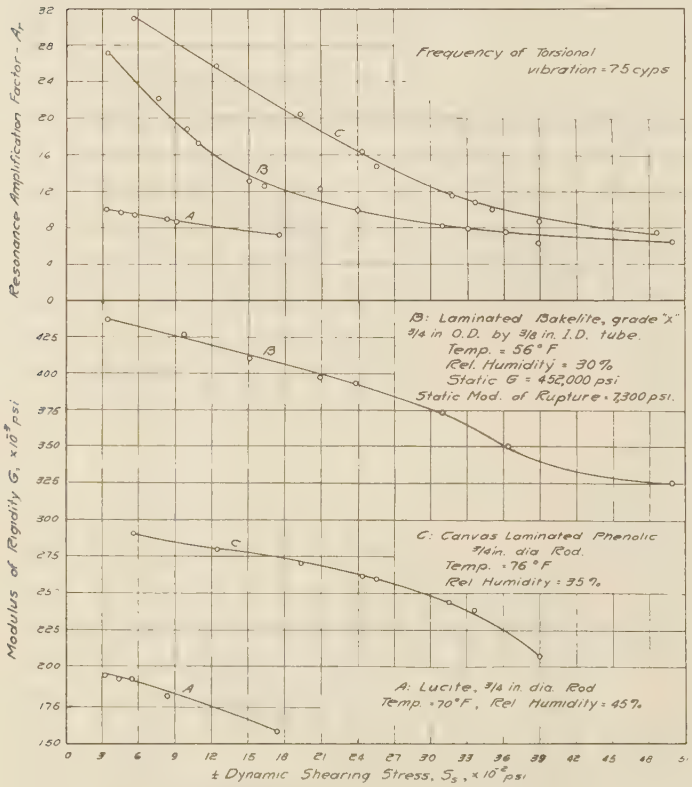


FIG. 8 RESONANCE-AMPLIFICATION FACTOR AND DYNAMIC MODULUS OF RIGIDITY OF PLASTICS UNDER TORSIONAL VIBRATIONS

Fig. 5 shows the resonance-amplification factor of mild-steel specimens. Curves 1 and 2 were obtained with the apparatus described in this paper; curves 3 and 4 were derived from damping-capacity data secured by two different accepted methods and recomputed by Equations [23] and [29], section D of the Appendix, for conversion to ordinate A_r. A fair correlation between the various data exists at low stress, but the deviation increases as the stress increases. The differences in the mild steels used for the various tests may account for some of the deviation, but the "cold-working" effect which will be discussed offers a more probable explanation.

The resonance-amplification factor for the plastics of Figs. 8 and 9 is about $^1/_{10}$ that of metals. Thus for certain engineering applications, plastics may be superior to metals as illustrated by the following numerical example:

If an alternating force excites resonant vibrations, the dynamic stress produced is the product of the resonance-amplification factor A_r and the magnitude of the exciting stress. Under direct stress the endurance limit of duralumin, a strong aluminum alloy, is about 15,000 psi, at which stress the resonance-amplification factor, given in Fig. 7, is 242. Thus, if a duralumin structural member is subjected to longitudinal resonant vibrations, the magnitude of the exciting force cannot exceed $(15{,}000/242) = 62$ psi of cross-sectional area. If this same structural member were made of canvas laminated plastic (with an endurance limit of about 5000 psi, at which stress factor A_r in Fig. 9 is 11) the exciting force could attain a value of $(5000/11) = 464$ psi of cross-sectional area before fatigue failure would occur.

The significance of high damping capacity (or low resonance-amplification factor) in reducing resonant-vibration stresses is further clarified in Fig. 10, which is a replot of Figs. 7 and 9 to different co-ordinates. The superior damping capacity of the plastic, which may more than compensate for its low fatigue strength, is thus a desirable feature in aircraft and other members subjected to vibration-inducing forces. However, the duralumin member may be more durable than the plastic member, if static loads are superimposed or if nonresonant vibrations are excited, or if other damping agents are present in the vibrating system.

The cold-working accompanying sustained cyclic stress may greatly affect the damping capacity of a material. The vibration-decay method (12) and the static method (19) of determining damping capacity cannot readily reveal the effects of the cold working since these methods subject the specimen to only a few cycles of stress and thus yield only the properties of the more or less virgin material. However, in the resonant-vibration method, used in the present work, any number of stress cycles can be applied and the cold working may cause an increase in the resonance-amplification factor.

The vertical discontinuities in curves 1 and 2 of Fig. 5 illustrate the effect of sustained cyclic stress in factor A_r. Because of the accumulative increase in A_r, the rise of curves 1 and 2 above 3 and 4 is reasonable.

A more obvious example of the manner in which cold working may increase the resonance-amplification factor is shown in Fig. 6, curves 1, 2, and 3, for different specimens from the same piece of Duralumin tubing. The specimen of curve 1, tested quite rapidly in order to keep the cold working to a minimum, is the lowest of the three. The specimen represented by curve 2 was tested rapidly to point *a*, then subjected to a large number of cycles which raised the resonance-amplification factor to point *b*, and was then continued to point *c*. As the stress in this specimen was gradually decreased, the return curve *c-d* was considerably higher than the original curve. The specimen of curve 3 was subjected to numerous cycles of stress along its entire length, and therefore it is above the other two curves. Portion *a-b* of curve 3 illustrates the asymptotic manner in which the resonance-amplification factor A_r increases with number of cycles of stress. Although A_r increases rapidly with early cyclic stress, it gradually ap-

proaches a more or less fixed value as the number of cycles becomes large.

Since sustained vibrations may increase the resonance-amplification factor, the calculated stress at resonance based on rate-of-vibration decay or static test data may be low and therefore unsafe.

Although sustained vibrations at low and medium stresses increase the resonance-amplification factor, vibrations at stresses well above the endurance limit may decrease this factor for steel. Sufficient work has not yet been done to correlate these changes in factor A_r with the endurance limit or structural damage caused by overstressing.

Damping capacity of materials as reported by different observers differ widely, depending upon the method of observation (8). An important reason for the experimental scatter is explained as follows:

The observed damping capacity of a material obtained by any dynamic method depends upon the vibrational-energy losses in the entire system. These losses are due to hysteresis within the specimen and to extraneous work done at the supports and loose parts, slippage between specimen and chucks, and general vibration radiation. If these extraneous losses are not small in comparison with the hysteresis energy absorbed, which in itself is rather small for metals, then the values observed for the damping capacity of the material will be too large.

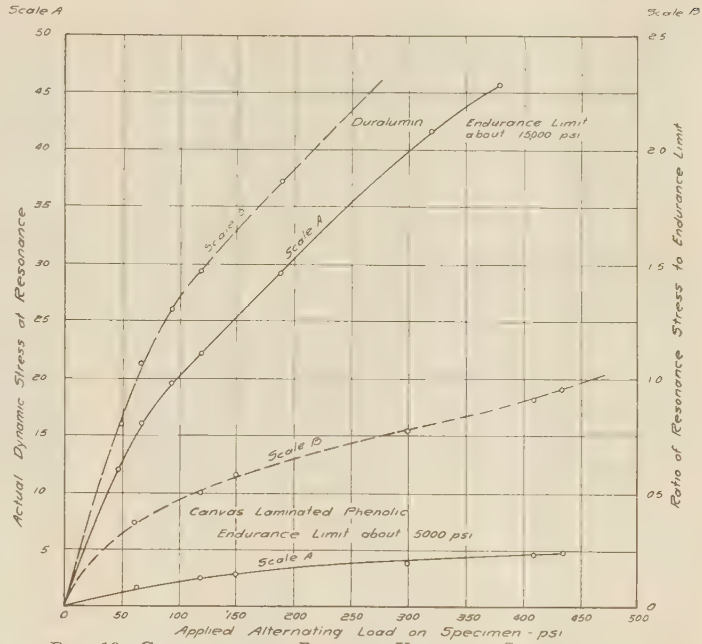


FIG. 10 COMPARISON OF RESONANT-VIBRATION STRESSES IN A PLASTIC AND IN DURALUMIN

Numerous precautions were taken in this study to minimize the extraneous energy losses and other sources of error. While there is no simple method of evaluating the effectiveness of these precautions, the data agree with other test values of damping capacity (tests on magnesium alloys, not reported herein, reasonably check reference 12, and the bakelite data agree well with reference 15). Since damping capacity is highly structure-sensitive, a close comparison is not to be expected.

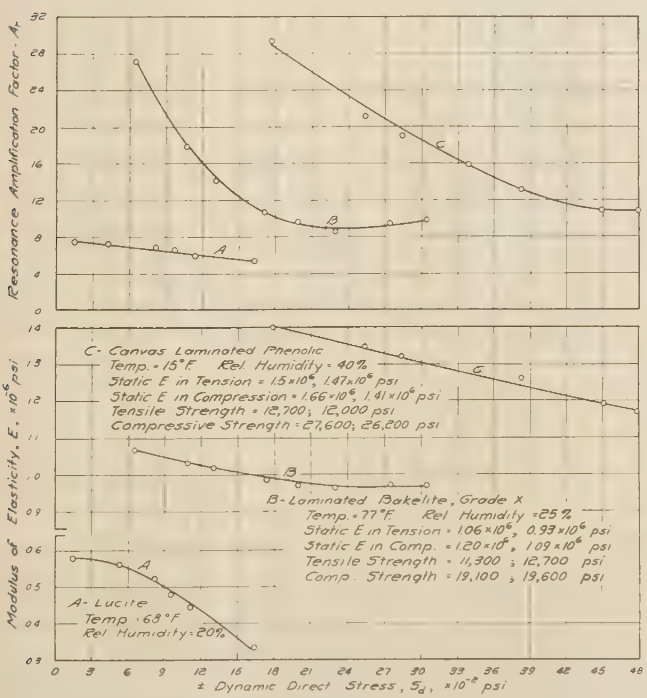


FIG. 9 RESONANCE-AMPLIFICATION FACTOR AND DYNAMIC MODULUS OF ELASTICITY OF PLASTICS UNDER LONGITUDINAL VIBRATIONS

Dynamic Modulus of Elasticity. The dynamic modulus of elasticity of a material, or the ratio of stress to strain effective during a vibration, is usually lower than the static modulus. The four main reasons for this deviation are temperature increases caused by damping, hysteresis-loop distortions, cold working, and thermodynamic effects, which will be discussed in that order.

1 Materials often become very warm under cyclic stress because their damping capacity dissipates part of the vibrational energy as heat. Since the mechanical properties of materials are a function of specimen temperature, the dynamic modulus of elasticity deviates from the static value.

Most metals possess comparatively low damping capacity, and furthermore their moduli are rather insensitive to temperature changes. The temperature of iron and steel, for example, may be increased about 100 deg above room temperature before a 1 per cent change in the modulus results (14). Thus the dynamic moduli of elasticity of metals should not be greatly influenced by the temperature effect.

Plastics and many other nonmetallic materials, on the other hand, possess high damping capacity and therefore heat up considerably under cyclic stress (16). Fig. 11 shows the equilibrium temperature of the specimen as a function of dynamic stress for a phenolic laminated plastic. Furthermore, the mechanical properties of nonmetals are highly sensitive to temperature change (17). For example, the modulus of elasticity in tension of the average phenolic plastic decreases about 30 per cent as the temperature is increased from —38 to 78 F (13). The dynamic modulus of elasticity of a plastic should, therefore, be lower than the static modulus, especially at high stress. Most of the decrease in the dynamic moduli for the plastics shown in Figs. 8 and 9 is probably due to the temperature effect.

2 A material exhibiting a curved stress-strain diagram or a measurable hysteresis loop in a given stress range does not have a constant static modulus of elasticity. The variation in the static modulus during a cycle of stress depends upon the area of the hysteresis loop, or damping capacity, and therefore increases with magnitude of stress.

The dynamic modulus of elasticity for a given stress cycle is

some average of the variable static modulus over this cycle. Therefore, materials possessing high damping capacity should have a lower dynamic than static modulus of elasticity (measured on the upward branch of the load-deflection curve) and also show a decrease in the dynamic modulus as the stress increases. The small reduction in the moduli of the metals as the stress increases, and part of the large drop observed in the plastics, are probably associated with the hysteresis-loop effect.

3 The cyclic stress accompanying a dynamic test disturbs the structure of a material by cold working or "fragmentation." In metals cold working causes a relatively minor change in the modulus of elasticity, as shown in Fig. 6. In plastics, however, high cyclic stress may evidently cause considerable fragmentation in the structure of the material, and the modulus of elasticity may be greatly reduced. The modulus of rigidity of the canvas laminated plastic, C of Fig. 8, for example, was reduced from 414,000 to 337,000 psi (temperature effects corrected) by alternating torsional stress of ± 3000 psi maintained for about 40,000 cycles.

4 The determination of the static load-deflection curve of a material proceeds slowly enough to permit the assumption that the specimen is always at room temperature. The static modulus of elasticity is thus essentially the isothermal or constant temperature value, if the room temperature is assumed constant.

During the longitudinal vibrations, however, the alternating strains occur too rapidly to allow thermal equilibrium to be attained, and the vibration is adiabatic, or constant heat in nature. This adiabatic state requires the specimen to go through cyclic temperature changes at the same frequency as the vibration; cooling down with increasing volume (or stretching) and heating up when compressed. These temperature changes cause contraction and expansion in the specimen, and the effective or adiabatic modulus is slightly higher than the isothermal value (18).

For the materials considered in this paper the adiabatic modulus is less than 1 per cent higher than the isothermal modulus.

The total deviation between static and dynamic moduli of elasticity caused by all four factors mentioned is but a few per cent for the metals tested. The fact that the dynamic modulus of metals decreases slightly with stress is of some practical importance in spring design. If very accurate instruments and a special technique are employed, static tests also show a decrease in modulus with stress (6) of the same order of magnitude as dynamic tests.

In plastics, however, the first three factors discussed reduce the dynamic modulus considerably as the stress increases. Figs. 8 and 9 show that the static modulus of elasticity of a plastic may be 40 per cent higher than its dynamic modulus. However, if the experimental curves of these figures are extrapolated to zero stress, the dynamic moduli are within a few per cent of the static values.

The dynamic method of determining the moduli of elasticity of materials is precise and sensitive although it does not require delicate or elaborate apparatus (20). The dynamic testing apparatus used was sensitive within 0.1 per cent and the probable error of an observation is about 1 per cent if proper precautions are observed.

The Fatigue Test. The concept of a variable dynamic modulus of elasticity in plastics is closely associated with a proper interpretation of fatigue-testing results. A discussion of this association follows.

The two most common types of fatigue-testing machines used on plastics are the R. R. Moore rotating-beam type and the Krouse plate or sheet type (13, 5).

The Krouse machine is of the repeated constant-deflection type in which the end of a cantilever plate is oscillated by a cam-and-arm mechanism. In using this machine, the tacit assumption is made that the static modulus equals the dynamic value, which is contrary to the experimental data of Figs. 8 and 9. On the basis of the observed deviation between the static and dynamic moduli, the error involved in the calculated alternating stress produced by the Krouse machine, or any other repeated constant-deflection type of machines, may exceed 40 per cent.

The endurance to a specific alternating deflection may be more important than the endurance to a specific stress in some types of springs, for example. Thus, the constant-deflection type of machine may yield valuable information; but the results of a repeated-deflection fatigue test on plastics should not be given in terms of stress.

The variable dynamic modulus of elasticity also introduces some inaccuracies in the R. R. Moore machine. The stress in the rotating-beam type of machine is computed by the simple bending equation $S_d = Mr/I$, which does not involve the modulus of elasticity directly. However, in the derivation of this equation, it is assumed that the stress is proportional to the distance from the neutral axis of bending, which in turn assumes that the modulus of the material is constant at all stresses. On the basis of the observed decrease in dynamic modulus of elasticity with stress, Fig. 9, the outer highly stressed fibers will have a lower effective modulus during the cyclic stress.[7] Therefore, it may be concluded that the fatigue strength determined by the rotating-beam test cannot be safely used in design problems involving direct stress. The difference between the rotating-beam fatigue strength and the direct-stress fatigue strength is similar in nature, but probably not so great in magnitude, as the difference between the static modulus of rupture in bending and the static tensile or compressive strength.

For reasons similar to those cited the fatigue strength obtained from torsion tests on solid rods is higher than that under uniform shear. This source of error is reduced by using tubular specimens.

The foregoing discussion was referred to plastics because the effects involved are particularly large for materials of high damping capacity. However, metals are also affected to a limited extent (see comparison of direct-stress fatigue strengths with rotating-beam fatigue strengths in reference 2).

A quantitative analysis of the significance of the variable dynamic moduli of elasticity as related to the fatigue testing of plastics is now under way.

During a fatigue test, the specimen is well above room temperature, as indicated by Fig. 11. Thus, the fatigue strength determined is not that at room temperature but rather at some elevated temperature dependent upon the rate of heat generated within the specimen and that lost to the surroundings (16).

The fatigue strengths of metals are changed only slightly by the temperature increase (2) which occurs during cyclic stress.

The fatigue strengths of plastics are highly temperature-sensitive. For example, the fatigue limit of methyl-methacrylate resin is 2000 psi at 78 F and 4800 psi at —38 F. Thus, in testing plastics such variables as frequency, size of specimen, rate of heat

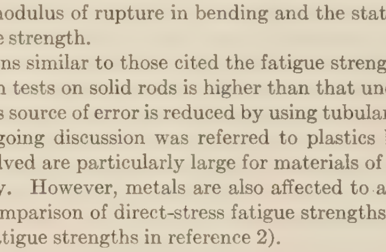
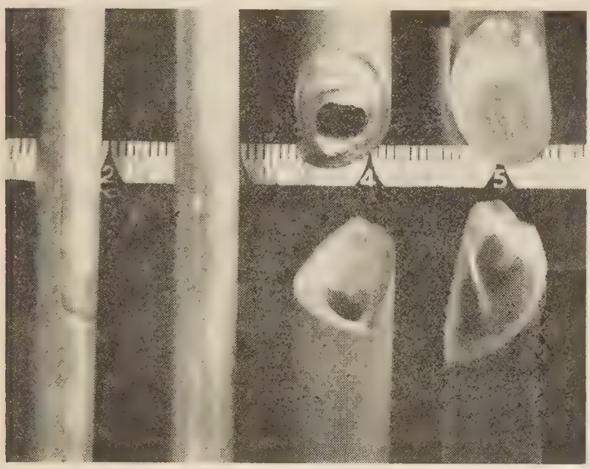


FIG. 11 TYPICAL TEMPERATURE-INCREASE CURVE FOR A PLASTIC UNDER CYCLIC STRESS

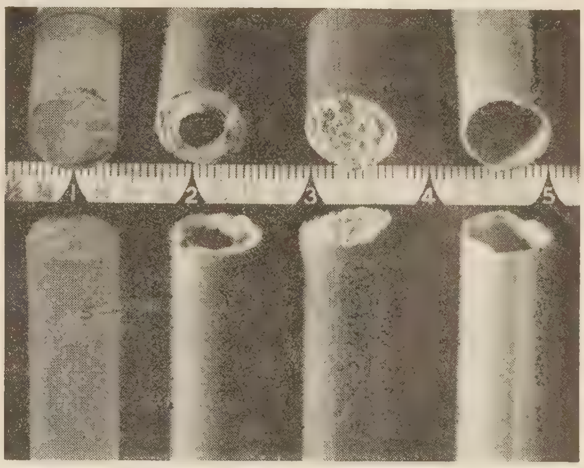
[7] The change in modulus with increasing stress, or distance from the neutral axis, is probably not as large as illustrated in Fig. 9, because, after thermal equilibrium is attained, each point in the test specimen will have about the same temperature regardless of the local stress.



A B C D

Fig. 12 Torsional-Vibration Fatigue Failures

(*A*, steel tubing; *C*, bakelite tubing; *B*, nickel tubing; *D*, lucite rod.)



D C E F

Fig. 13 Longitudinal-Vibration Fatigue Failures

(*C*, bakelite tubing; *E*, canvas phenolic; *D*, lucite rod; *F*, aluminum tubing.)

loss from the specimen, and other factors affecting the specimen temperature will influence the observed fatigue strengths.

The behavior of Lucite (methyl-methacrylate resin), during a fatigue test, illustrates the effect of temperature on the properties of plastics. The Lucite specimen *D*, Fig. 13, was subjected to direct alternating stress in the horizontal arrangement of Fig. 2. The stress was above the endurance limit, and at the working frequency of 67.5 cycles per sec the specimen warmed up to about 160 F. After about 500,000 cycles of stress, one spot on the specimen became hotter than the rest, probably reaching 250 F, and localized plastic flow soon caused a bulge or swelling there. Further stress cycles slowly increased the size of the swelling (see *S* in Fig. 13, specimen *D*), which soon reached a limiting size and stabilized. Measurement showed that most of the deformation in the specimen occurred in this swelling. After about 300,000 more cycles of stress, before actual fracture occurred at the first swelling, and even though the specimen was fan-cooled, a second swelling started about 3 in. away from the first, but again the specimen did not fracture. Finally, after 100,000 cycles additional, the specimen fractured, but not at either original swelling.

It appears that high temperature at the point of potential or impending fatigue failure may in some way strengthen the material. Perhaps the mechanism of this strengthening process is similar to the heat-treating effect in metals.

The foregoing observations partially explain the unusual behavior of notched Lucite fatigue specimens. Tests show (4) that the endurance limit of a 60-deg notched fatigue specimen is 30 per cent greater than that of an unnotched specimen of equal cross-sectional area. Although the greater strength of the notched specimen appears inconsistent with the usual understanding of stress concentration, the heat-treatment effects at the root of the notch may strengthen the material enough to make the difference.

In order to study general behavior of the damping capacity and dynamic moduli of elasticity before fatigue failure, several fatigue tests were run. Miscellaneous broken specimens are shown in Figs. 12 and 13. In nearly all cases observed an abrupt drop in both the resonance-amplification factor and dynamic modulus of elasticity occurred before any macroscopic cracks could be observed in the test specimen.

Conclusions

(*a*) The two oscillatory-type dynamic testing machines developed for this work provide a simple and effective means of studying the mechanical properties of materials under complete reversals of both direct stress and torsional stress.

These machines overcome three limitations in the rate-of-vibration-decay method of measuring damping capacity; i.e., the effects of cyclic stress may be continuously determined, the damping capacity of any material under direct alternating stress may be evaluated, and materials or structures with very high damping capacity may be accurately studied.

Furthermore, a continuous record of the dynamic modulus of elasticity may be kept and complete fatigue tests may be run with these machines.

(*b*) The resonance-amplification factor for the plastics tested was about $^1/_{10}$ that of the metals.

(*c*) The stress induced by resonant vibrations in a structural member is proportional to the product of the exciting force and the resonance-amplification factor. Therefore, structural plastics with a low fatigue limit (about $^1/_3$ that of Duralumin), and a much lower resonance-amplification factor, may actually be more durable than metals in members subject to large vibration-exciting forces.

In some aircraft parts the superior vibration-damping ability and low density of plastics (about 0.5 that of aluminum and 0.18 that of steel) may make them an acceptable substitute for metals.

(*d*) Sustained cyclic stress below the endurance limit increased the resonance-amplification factor of the metals tested as much as 25 per cent. Although the increase was rapid at first, the resonance amplification changed very little after about 100,000 cycles of stress.

(*e*) A pronounced reduction in both the resonance-amplification factor and the dynamic modulus of elasticity occurred at impending fatigue failure in all materials tested. This reduction was usually evident before any macroscopic fatigue cracks could be observed.

(*f*) For most of the metals tested the dynamic modulus of elasticity, under both direct and torsional stress, decreased a few per cent as the alternating stress increased. The dynamic moduli were within a few per cent of the static moduli.

(*g*) For the plastics tested, the dynamic modulus of elasticity under both direct and torsional stresses decreased as much as 40 per cent as the magnitude of the alternating stress increased. However, the dynamic modulus at zero stress, obtained by extrapolating the experimental curves showing dynamic modulus

versus stress, was within a few per cent of the static modulus.

(*h*) Since the dynamic modulus of elasticity may deviate considerably from the static modulus, most repeated-constant-deflection types of fatigue machines do not give reliable results on plastics.

(*i*) The high damping capacity of plastics causes a large amount of heat to be developed within the specimen during the fatigue test. The resulting rise in temperature may greatly influence the mechanical behavior of the plastic.

Acknowledgments

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Appendix

A—Definitions of Terms

The "dynamic modulus of elasticity" refers to the stress-to-strain ratios effective during a vibration.

The "damping capacity" measures the ability of a material to absorb energy while being subjected to cyclic stress. Its magnitude is proportional to the area within the hysteresis loop caused by inelastic action.

The "resonance-amplification factor A_r" of a material is a reciprocal function of its damping capacity. Quantitatively, it is the ratio of the total force F_s (or torque T_s) in a specimen under resonant vibrations to the amplitude of the exciting force F_o (or torque T_o). Materials displaying a large hysteresis loop possess a high damping capacity and a low resonance-amplification factor.

B—Nomenclature

The following nomenclature is used in succeeding sections:

A = cross-sectional area of test specimen, sq in.
a = amplitude of linear sinusoidal vibration of oscillator, in.
A_v = vibration-amplification factor = ratio of internal force in a specimen during a vibration to exciting force
= F_s/F_o for longitudinal vibrations
= T_s/T_o for torsional vibrations
A_r = resonant-vibration-amplification factor = A_v at resonance
D_d = hysteresis damping capacity in direct stress, or energy absorbed per cycle of vibration, in-lb per cycle per cu in.
D_s = hysteresis damping capacity in torsional stress, or energy absorbed per cycle of vibration, in-lb per cycle per cu in.
E = modulus of elasticity in direct stress, psi
e = amplitude of motion of oscillator eccentric masses, in.
f_n = natural frequency of vibrating system, cycles per sec
f_o = frequency of forced vibrations, induced by the oscillator cycles per sec = $\omega_o/2\pi$
F_o = maximum value or amplitude of alternating oscillator force, lb
F_s = internal force (total stress) in test specimen = $A_v F_o$, lb
G = modulus of elasticity in shear, or modulus of rigidity, psi
I_o = moment of inertia of oscillator assembly, lb-in-sec^2
J = polar moment of inertia of cross section of test specimen, in.^{4}
L = effective length of test specimen, in.
M = total mass of oscillator rotating eccentrics, lb-sec^2 per in.
M_o = total mass of oscillator assembly, lb-sec^2 per in.
r = radius of test specimen, in.
S_d = direct stress in either tension or compression, psi
S_{st} = stress that would be caused by torque T_o (or force F_o) if applied statically
= $T_o r/J$ for torsion, psi
= F_o/A for direct stress, psi
S_s = shearing stress induced by torsion, psi
T_o = maximum value or amplitude of alternating oscillator torque, in-lb
T_s = internal torque in test specimen = $A_v T_o$, in-lb
δ_d = logarithmic decrement in longitudinal vibration
δ_s = logarithmic decrement in torsion
θ = amplitude of torsional vibration of oscillator, radians
ϕ = phase angle between rotating vector, representing alternating force produced by eccentric masses M and vector representing oscillator vibration, deg
ψ_d = specific damping capacity in direct stress
ψ_s = specific damping capacity in torsional stress
ω_o = frequency of forced vibrations induced by oscillator, radians per sec
ω_n = natural frequency of vibrating system, radians per sec

C—Equations of Natural Frequency of Vibration (7)

(*a*) Longitudinal vibration of system in Fig. 14 (*a*)

$$\omega^2_n = AE/M_oL \quad [1]$$

expressed in radians per second.

(*b*) Torsional vibrations of system in Fig. 14(*a*)

$$\omega^2_n = GJ/I_oL \quad [2]$$

expressed in radians per second.

(*c*) Longitudinal vibration of system in Fig. 14(*b*)

$$\omega^2_n = [AE]/[L_oM_oM_o'/(M_o + M_o')] \quad [3]$$

expressed in radians per second.

Section *n-n* in the test specimen is the nodal position of the vibration, which remains stationary during the vibration. If L is the length of the specimen between *n-n* and the mass M_o, then Equation [1] applies to Fig. 14(*b*).

Comparing Equations [1] and [3]

$$L = L_o[M_o'/(M_o + M_o')] \quad [4]$$

= Equivalent fixed-end specimen

D—Equations of Stress, Dynamic Moduli of Elasticity, Vibration-Amplification Factor, and Damping Capacities

The equations to be given are derived for the vibrating system, shown in Figs. 1 and 14(*a*). Identical equations apply to the horizontal setup of Figs. 2 and 14(*b*) if L is considered the effective length of specimen between the nodal section *n-n* and mass M_o.

Dynamic Stress in Vibrating Specimen:

1 Longitudinal (Direct) Stress: The total internal stress F_s in the test specimen is caused by (*a*) the inertia force resulting from the vibration of the mass of the complete oscillator assembly and (*b*) the oscillator force F_o, which leads the displacement by phase angle ϕ.

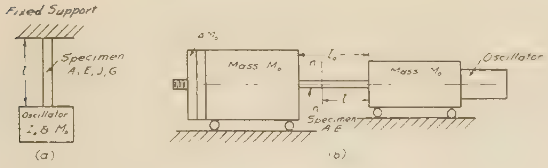

Fig. 14 The Vibrating Systems
(a, Vertical arrangement; b, horizontal arrangement.)

The internal force F_s in the specimen is largest when the displacement a is a maximum. If the two forces mentioned are combined at this instant

$$F_s = M_o a\omega^2_o + F_o \cos\phi \quad [5]$$

Therefore, the maximum unit direct stress is

$$S_d = F_s/A = (M_o a\omega^2_o + F_o \cos\phi)/A \quad [6]$$

2 Torsional (Shear) Stress: By a derivation similar to that just given

$$T_s = I_o\theta\omega^2_o + T_o \cos\phi \quad [7]$$

and

$$S_s = (I_o\theta\omega^2_o + T_o \cos\phi)r/J \quad [8]$$

The computation of the dynamic shearing stress S_s involves the equation $S_s = T_s r/J$, which equation is subject to the inaccuracies discussed under "Fatigue Test."

Dynamic Modulus of Elasticity:

$$E = S_d L/a = (L/A)\,[M_o\omega^2_o + (F_o \cos\phi)/a] \quad [9]$$

$$G = T_s L/J\theta = S_s L/r\theta = (L/J)\,[I_o\omega^2_o + (T_o \cos\phi)/\theta] \quad [10]$$

Vibration Amplification Factor:

1 Longitudinal Vibrations: By definition: $A_v = F_s/F_o$

$$\therefore A_v = (M_o a\omega^2_o + F_o \cos\phi)/F_o = (M_o a\omega^2_o/F_o) + \cos\phi \quad [11]$$

When $\phi = 90$ deg (resonance), A_v becomes the resonance-amplification factor A_r. At any other phase angle, the equivalent resonance-amplification factor at the same deflection a may be found from Equation [21], in which

$$A_r = A_v/\sin\phi = [(M_o a\omega^2_o/F_o) + \cos\phi]/\sin\phi \quad [12]$$

2 Torsional Vibrations:

$$A_v = T_s/T_o = (I_o\theta\omega^2_o/T_o) + \cos\phi \quad [13]$$

$$A_r = [(I_o\theta\omega^2_o/T_o) + \cos\phi]/\sin\phi \quad [14]$$

Damping Capacity:

1 Longitudinal Vibrations: The work done by a sinusoidal force on a vibration of the same frequency is

$$W = \pi a F_o \sin\phi \quad [15]$$

expressed in inch-pounds per cycle (7).

Since this work is dissipated by hysteresis damping D_d in the specimen

$$W = D_d AL \quad [16]$$

$$\therefore W = \pi a F_o \sin\phi = D_d AL \text{ in-lb per cycle}$$

$$\text{and } D_d e = \pi a F_o \sin\phi/AL \quad [17]$$

expressed in inch-pounds per cycle per cubic inch, where D_d is the damping capacity associated with deflection a.

2 Torsional Vibrations:

$$D_s = \pi\theta T_o \sin\phi/AL \quad [18]$$

expressed in inch-pounds per cycle per cubic inch.

Specific Damping Capacity:

$$\psi = \frac{\text{Energy dissipated per cycle by hysteresis damping}}{\text{Elastic strain energy at maximum stress}}$$

1 Longitudinal Vibrations:

$$\psi_d = (D_d AL)/(F_s a/2)$$

Using Equation [17]

$$\psi_d = (2\pi \sin\phi)/(F_s/F_o) = (2\pi \sin\phi)/A_v \quad [19]$$

where ψ_d is the specific damping capacity at a stress associated with deflection a.

At resonance, $\sin\phi = 1$ and A_v becomes A_r; hence

$$\psi_d = 2\pi/A_r \quad [20]$$

Therefore, from Equations [19] and [20], for any given deflection a, the equivalent resonance-amplification factor is

$$A_r = A_v/\sin\phi \quad [21]$$

2 Torsional Vibrations:

$$\psi_s = (2\pi \sin\phi)/A_v \quad [22]$$

$$\psi_s = 2\pi/A_r \quad [23]$$

Logarithmic Decrement δ:

If a vibrating system is isolated so that there are no extraneous energy losses or input, then each vibration will become successively smaller. If X_n and $X_{(n+1)}$ are the amplitudes of two successive vibrations, then by the definition of the logarithmic decrement

$$\delta = \log_e (X_{n+1}/X_n) \quad [24]$$

(see reference 3).

The relationships among the terms δ, ψ, and A_r will be determined.

By the definition of ψ given under "Specific Damping Capacity"

$$\psi = k(X_n^2 - X^2_{n+1})/kX_n^2$$

where k is a constant for a given specimen.

$$\psi = 1 + (X_{n-1}/X_n)^2$$

$$\therefore X_{n+1}/X_n = \pm\sqrt{1-\psi}$$

and

$$\log_e (X_{n-1}/X_n) = \delta = {}^1/_2 \log_e (1-\psi)$$

$$\therefore \psi = 1 - (e^{2\delta})$$

$$\psi = 1 - [1 + 2\delta + (4\delta^2/2) + \ldots + (2^n\delta^n/n!)]$$

$$\psi = -2[\delta + \delta^2 + (2\delta^3/3) + \ldots] \quad [25]$$

Equation [25] may be simplified, the degree of simplification depending upon the magnitude of δ

$$\psi = -2\delta \quad [26]$$

(error < 1 per cent if $\delta < 0.01$, as occurs in many metals).

If δ is large, as in the case of plastics, then the following more accurate equation is used:

$$\psi = -2(\delta + \delta^2) \quad [27]$$

(error < 1 per cent if $\delta < 0.12$, as occurs in most materials of construction).

Equations [26] has been shown to apply to viscous damping (which differs considerably from hysteresis damping) with fair accuracy (7, 8), but the foregoing equations are more general.

To secure a relationship between A_r and δ, either Equation [20] or [23] is substituted in Equations [26] and [27] as follows:

$$A_r = -\pi/\delta \quad [28]$$

(error < 1 per cent if $\delta < 0.01$).

$$A_r = -\pi/(\delta + \delta^2) \quad [29]$$

(error < 1 per cent if $\delta < 0.12$).

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Discussion

W. N. Findley.[8] The author has done some very interesting work with his ingenious oscillator. As a result of his tests, he reached the following conclusions: "For the plastics tested the dynamic modulus of elasticity under both direct and torsional stresses decreased as much as 40 per cent as the magnitude of the alternating stress increased;" and "Since the dynamic modulus of elasticity may deviate considerably from the static modulus, most repeated constant-deflection types of fatigue machines do not give reliable results on plastics." Likewise, a change in dynamic modulus was reported for metals but the change was relatively small compared to that for the plastics. In the opinion of the writer a change in "dynamic" modulus of plastics with stress does not necessarily mean that the results of tests with a repeated constant-deflection type machine would not be very valuable. The unusual character of some of the results reported in the paper prompted a study of the equations from which the "dynamic" modulus of elasticity was computed. This study raised certain questions which lead the writer to wonder whether the equations warrant the conclusions stated.

Indeed, unless such questions as those to be mentioned can be clarified, one must conclude that the stress in materials such as plastics under dynamic loading cannot be determined with any more certainty from an oscillator type of machine than from a repeated-constant-deflection type of machine, and that the same method of computing stress would have to be used with the oscillator as that used with the repeated-constant-deflection machine.

Unfortunately, the author did not indicate the complete development of the equations or the source from which they were obtained, nor the limitations imposed on the solution for the different cases considered.

A complete analysis of the motion of the hypocyclic oscillator as applied to torsion testing is full of complicating factors whose relative importance in the evaluation of the motion and calculation of the modulus of elasticity is not easily determined. Some of these factors are the following: What is the effect of a nonlinear hysteresis damping or a nonlinear stress-strain relation (such as may be the case for plastics) on the geometry of motion of the system? What is the effect of the gyroscopic action of the motor contained in the oscillator on the motion? What is the effect of turbulent-air damping, of the three-dimensional character of the motion of the oscillating eccentric weights, etc.? While these questions have been raised in connection with torsion testing, many of them require consideration also for the axial machine, particularly when the amplitude of motion of the oscillating head of the machine is relatively large as in the case of tests of materials of low modulus such as plastics.

Thus, it would seem unsafe to set down Equations [5] and [7] of the paper by inspection, as the author's presentation seems to imply, because it is necessary to know what part damping and other factors may play in this relationship. This can be determined with certainty only by a fundamental analysis beginning with the equation of motion of the system.

The following discussion will not attempt to present a complete analysis of the system but merely to point out some of the difficulties involved in applying a torsion-pendulum type of machine to the cases considered by the author.

The general equation of motion for a system with a single degree of freedom, such as a torsion pendulum, may be written as follows

$$I\ddot{\theta} = -f(\theta, \dot{\theta}) + F(t)$$

[8] Associate in Theoretical and Applied Mechanics, College of Engineering, University of Illinois, Urbana, Ill. Jun. A.S.M.E.

where I is the moment of inertia of the moving mass, exclusive of the small oscillating eccentric weights and neglecting the mass of the spring (or specimen); $\ddot{\theta}$ is the angular acceleration of the mass at the time t; $f(\theta, \dot{\theta})$ is the relationship between the resisting torque of the spring and the angle of twist, the internal torque (including damping); $F(t)$ is the applied torque (in this case it is the torque caused by the inertia force of the oscillating eccentric weights), expressed as a function of time.

If the system is perfectly elastic $f(\theta, \dot{\theta}) = k\theta$ where k is a constant; and if $F(t)$ is a simple harmonic function, such as $T \sin \omega t$, then the solution for the steady state of vibration is of the form $\theta = A \sin \omega t$, where A is the amplitude of vibration and may be expressed in terms of the constants T, ω, k, I.[9] Two other terms in the solution need not be considered for reasons given in the reference. If the torque-twist relation, $f(\theta, \dot{\theta})$, is expressible by an equation of the form $f(\theta, \dot{\theta}) = k\theta + c\dot{\theta}$ (the so-called viscous-damping case), and $F(t) = P \sin \omega T$, as before, the solution for the steady state of vibration is of the form $\theta = A' \sin (\omega t - \phi)$ where A' is the amplitude of vibration and may be expressed in terms of T, ω, k, I, c.[10]

It will be noticed that the oscillation, in the steady state, for both of these cases is simple harmonic. However, all other cases (the nonlinear type) for which the writer has seen solutions do not yield simple harmonic motion. [11, 12] In only two other cases is an exact solution known; the case of Coulomb damping $f(\theta, \dot{\theta}) = \pm F$,[13] and the case of combined Coulomb and viscous damping, $f(\theta, \dot{\theta}) = c\dot{\theta} \pm F$.[13] No exact solution for the general case of solid friction is known.

Let us now examine by way of example the author's Equation [7]

$$T_s = I_0\theta\omega_0^2 + T_0 \cos \phi$$

This equation is correct for the case of "viscous damping," as just described, when the disturbing force $F(t) = T \sin \omega t$. It is one of a pair of simultaneous equations relating the variables θ and ϕ for the case of viscous friction. The other equation is $T_0 \sin \phi + c\theta\omega$, where c is the constant appearing in the viscous-friction case.[14] However, certain questions arise concerning the validity of this equation when used in connection with the problems considered by the author.

In Equation [7] the angular acceleration used in computing the "inertia" torque was $\theta\omega_0^2$ which was correct if the motion of the oscillator was simple harmonic, but is not necessarily correct for other types of motions. As mentioned, the only type of damping or hysteresis for which the motion is truly simple harmonic is the type for which the "viscous-damping" equation may be used. Thus in order to use Equation [7] with sufficient accuracy to compute the modulus of elasticity of the material, it would be necessary to show for every material tested either that the internal damping is of the "viscous" type and all other damping is negligible, or that the acceleration at the maximum displacement is equal to that which would result from a sinusoidal motion of the same amplitude, and that no other terms would be required in Equation [7], even for the nonviscous type of damping.

Of course a "dynamic modulus" or "pseudo modulus" may be calculated by putting values in the equation even though the damping is not viscous in type but the result thus obtained from an incorrect equation could not be expected to have any real significance in defining "the ratio of stress to strain effective during a vibration."

Perhaps the author has determined that all internal hysteresis may be represented by the viscous-damping type of equation, as just given, or that, regardless of the type of damping, the angular acceleration in question is equal to that which would obtain for viscous damping and that no other terms are required in Equation [7]. If this has been determined, the analysis leading to this conclusion would be a very valuable addition to the paper. Indeed a clarification of such points as this would be necessary if the results are to be taken without question.

The author computes the shearing modulus from Equation [7] by multiplying by $L/(J\theta)$ to get Equation [10]; that is, the modulus is computed from the equation for elastic torsion of a circular shaft, $G = S_sL/(J\theta)$. It is well known that this equation is correct only under conditions for which stress is proportional to strain. If the dynamic stress-strain curve of some of the materials tested shows a well-defined hysteresis loop, as the author's work and that of others indicate, then the relationship between stress and strain is continually changing at every position during the cycle of stress. If this be true Equation [10] could not be expected to yield reliable results. Indeed, if a hysteresis loop is present, there is some question as to what relationship, if any, there might be between the single value of G computed from Equation [10] and the "dynamic modulus" defined by the author as "the ratio of stress to strain effective during a vibration."

An examination of the data presented by the author indicates a possibility that the equation used in his computation of modulus may not have been applicable to some of the cases he considered. Fig. 7 of the paper, longitudinal vibrations of Dural, shows no change in modulus over a wide range of stress, while Fig. 6, torsional vibrations of the same material, shows about a 2 per cent drop in modulus over a much smaller range of stress. A possible explanation might be as follows: The amplitude of motion of the oscillator body probably would be considerably greater in the torsion machine than in the longitudinal machine. Consequently the first term of Equation [7] would be of greater importance relative to the second term than the first term of Equation [5] relative to its second term. Hence, the effect of any deviation from simple harmonic motion would be more pronounced in the results of torsion tests than axial tests. Similarly this effect may explain the fact that a comparison of Figs. 8 and 10 for plastics shows that the observed percentage change in modulus for plastics was about twice as great in the case of the torsion machine as in the axial machine, for comparable values of stress.

A possible method of checking the validity of Equations [5] or [7] would be to perform tests, on each material, with different lengths of specimen, but at the same stress and same frequency. Thus all conditions would be constant except the amplitude of motion. If the method of calculation is correct the computed modulus should be the same for all lengths of specimens.

It may be of interest to note that, in the use of the repeated-constant-deflection type of machine for fatigue tests of plastics,[15] the writer has not observed a "temperature rise due to hysteresis damping . . . . large enough to mask all other effects caused by atmospheric conditions." In tests of cellulose-acetate and phenolic molding material, the specimen temperature was not observed to rise more than 8 F for stresses up to the endurance limit of the acetate (about 20 per cent of the ultimate strength), or 11 F for the phenolic (about 60 per cent of the ultimate strength), at a speed of 1720 cycles per min in still air.

In closing, the writer would like to ask whether e was intended

[9] "Vibration Problems in Engineering," by S. Timoshenko, second edition, D. Van Nostrand Co., New York, N. Y., 1937, p. 14.

[10] Ibid., p. 38.

[11] Ibid., p. 57, and chapters 2 and 3.

[12] "Mechanical Vibrations," by J. P. Den Hartog, first edition, McGraw-Hill Book Company, New York, N. Y., 1934, chapt. 8, pp. 330–374.

[13] "Forced Vibrations With Combined Coulomb and Viscous Friction," by J. P. Den Hartog, Trans. A.S.M.E., vol. 53, 1931, paper APM-53-9.

[14] Ref. 9, p. 43.

[15] Refer to author's Bibliography (16).

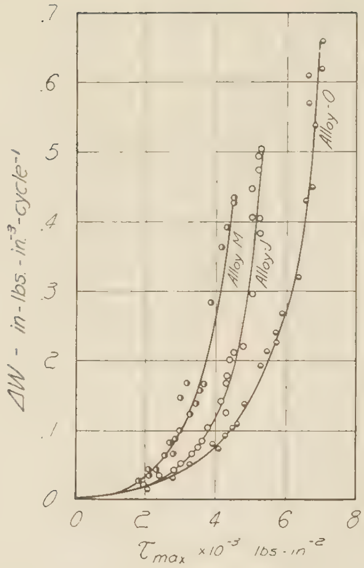

FIG. 15 DAMPING CAPACITY OF MAGNESIUM ALLOYS M, J, AND O[17] IN TORSION, OBTAINED BY METHOD OF SUSTAINED VIBRATIONS
(ΔW is energy dissipated per cubic inch during one complete reversal of stress. Materials were subjected to cyclic stresses before testing.)

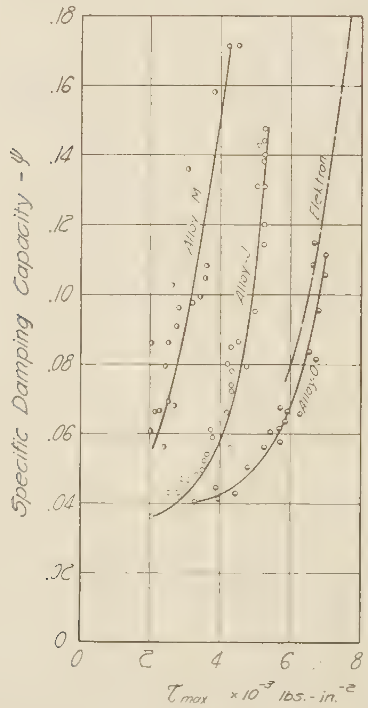

FIG. 16 SPECIFIC DAMPING CAPACITY OF MAGNESIUM ALLOYS M, J, AND O IN TORSION, OBTAINED BY METHOD OF SUSTAINED VIBRATIONS; AND OF ELEKTRON METAL, OBTAINED BY "RATE OF DECAYING VIBRATIONS" METHOD

by the author to be the absolute amplitude or the amplitude relative to the oscillator body? And was I_0 intended to include the mass of the eccentric weights or not?

A. J. YORGIADIS[16] AND A. U. KUTSAY.[16] The methods and equipment developed by the author represent the work of several years, and constitute an important contribution to the science of materials testing.

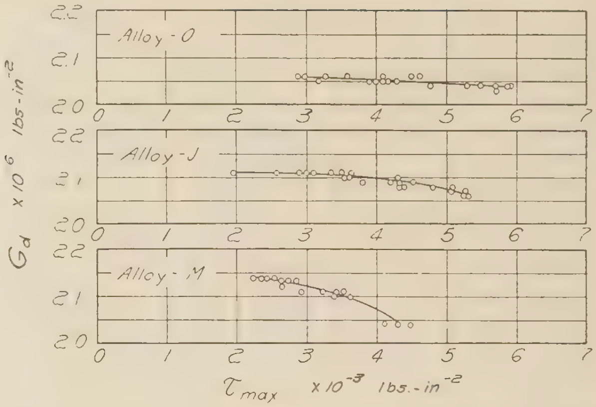

FIG. 17 VARIATION OF DYNAMIC MODULUS OF RIGIDITY WITH MAXIMUM SHEAR STRESS FOR MAGNESIUM ALLOYS M, J, AND O

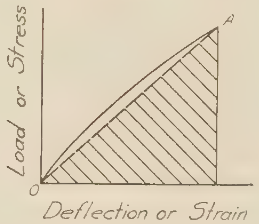

FIG. 18 DIFFERENCE BETWEEN ACTUAL ELASTIC ENERGY, AREA UNDER STRESS-STRAIN CURVE; AND CALCULATED ELASTIC ENERGY, AREA UNDER SECANT LINE
(Area under secant line is shaded portion.)

The writers have been using this equipment for the past year to determine the dynamic properties of some magnesium alloys, which tests are still in progress. It is on the basis of this experience that they desire to make the following remarks:

The testing machines described in the paper and shown in Figs. 1 and 2 can be advantageously used for fatigue tests of constant stress amplitude. When the material to be tested has stable dynamic properties, namely, its dynamic modulus and damping capacities are not affected by the repeated stresses, then advantage can be taken of resonance amplification, a very important characteristic in large-capacity testing. For materials having dynamic properties that are influenced by repeated stress cycles, a constant stress amplitude of the vibrating system is obtained by operating at speeds far below resonance, where the oscillator force is in phase with the displacement. The force in the specimen is then equal to that produced by the oscillator, which is of constant amplitude.

The author explains in detail the use of the oscillator as a machine to determine the dynamic moduli and damping capacity of materials under sustained vibrations. In Fig. 15 of this discussion are shown test data on the damping capacity of solid magnesium rods[17] in torsion, obtained by the writers by using the universal dynamic testing machine, Fig. 1 of the paper. These values were used to calculate the specific damping capacity ψ, plotted in Fig. 16 of this discussion. The scatter in this second group of curves is attributed to the simplifying assumptions made

[16] Research Assistant, Engineering Experiment Station, The Pennsylvania State College, State College, Pa.

[17] For the various magnesium alloys, refer to "Dowmetal Magnesium Alloys Handbook," The Dow Chemical Company, Midland, Mich., Oct., 1937.

in calculating ψ from the experimental data, and this will be further discussed later on. Also in Fig. 16 are plotted test data of a structural magnesium alloy known as Elektron metal, obtained by O. Föppl on his "rate of vibration decay" type of torsional damping machine.[18] Since damping capacity is a highly sensitive property in which large discrepancies are possible, the correlation indicates that the two types of machines yield results that are in agreement. Furthermore, the measuring of damping by sustained vibrations is a more direct method than the "rate of vibration decay" method, since in practice it is the properties of materials under repeated vibrations that are of greater importance.

When it comes to measuring the effect of the number of cycles of repeated stress on the damping capacity and modulus of elasticity, the method described in the author's paper is far superior to previously attempted techniques. Normally two machines have to be used for tests of this nature, as was done by O. Föppl[19] in experiments on the variation of the damping capacity of steel with the number of stress cycles.

The specimen was subjected to repeated loads on a fatigue machine and was removed periodically to have its damping capacity measured on a decaying vibrations damping machine. The handling of the specimen back and forth from one machine to the other for every reading and the resulting difference in gripping were possible causes of error not present in the proposed machine, which performs all these functions on the same setup.

Under item 2 of the section on "Dynamic Modulus of Elasticity" in the author's paper, mention is made of the fact that materials possessing high damping capacity show a decrease in the dynamic modulus as the stress increases. Not only do the magnesium tests substantiate this, but they indicate a possibility of a correlation existing between these two inelastic properties of materials. In Fig. 17 of this discussion are plotted the dynamic moduli of rigidity of the three magnesium alloys whose damping capacities are shown in Figs. 15 and 16. Alloy M with the highest damping has the greatest decrease in modulus, while alloy O with the least damping has the least decrease in modulus with increasing stress.

The equations in the Appendix of the author's paper contain simplifying assumptions. These assumptions are justified for a large number of materials and for a wide range of stresses; but like all such assumptions they set a limit to the usable range of the formulas, beyond which the error introduced becomes significant.

Equations [1] and [2] of the paper are two forms of the expression for the natural frequency of a single-degree-of-freedom system, with no damping and with an elastic restoring force proportional to the displacement. In the systems under consideration, these conditions are not rigorously attained, since damping is always present, and the stress-strain relations are not absolutely straight lines; i.e., the modulus is not constant over the whole range of stresses. As a result, the resonance curves obtained in practice are not symmetrical on both sides of resonance, but there is a slight shift of the peak toward lower frequencies, a well-known characteristic for vibrations of systems whose spring constant decreases as the deflection increases.[20] In addition, the displacement-time relationship deviates to a certain extent from a simple harmonic motion, rendering the first terms of Equations [5] and [7], and Equations [15] and [18] of the paper, slightly inexact. For example, in Equation [5], the acceleration at maximum displacement is taken as $a\omega_0^2$; also, in Equation [15], the work of the sinusoidal force is taken as $\pi a F_0 \sin\phi$. These are only identically true in case of simple harmonic motion.

In the derivations of ψ, Equations [19] and [25], the elastic energy is taken as $F_s a/2$ and KX^2, respectively. In both of these cases, this energy represents the area under the secant line OA (see Fig. 18) and not the actual elastic energy which is the area under the load-deflection or stress-strain curve.

Nevertheless, there is a justification for these simplifying assumptions. The true relations seem to be so complex that, even if available, their use would be too tedious. Furthermore, for most materials, and in particular for metals, the discrepancies introduced are probably of the order of magnitude of a fraction of 1 per cent. For materials with relatively high damping capacity, and whose moduli decrease with stress, the higher the stress, the larger is the error due to the formulas. The writers have noticed in their tests that a possible way of estimating the magnitude of this error is by comparing two values either of damping or modulus, at the same stress, one above resonance, and the other below resonance.

Concerning the magnesium alloys tested, it was found that the equations were satisfactory for nearly all the usable ranges of stresses of the alloys, even though magnesium is a metal with high damping, several times that of steel or aluminum.

The author is to be congratulated on presenting such an outstanding and versatile paper. This new method of approach is welcomed by all those who have struggled with the problem of simplifying fatigue testing and experimental determination of dynamic characteristics of materials.

Author's Closure

In the process of condensing the paper to reasonable length, some of the more theoretical aspects of the equations defining near-resonant vibrations were omitted. Thus only the final form of the equations could be given in most cases; limited space prohibited a discussion of most of the assumptions and the consequent limitations of the solutions. However, the several inquiries justify a more detailed consideration of the accuracy of the equations.

Mr. Findley inquires about the exact meaning of several symbols and also wonders how the vibration of the oscillator body during a test affects the motion of the eccentrics and the resultant oscillator force. The following explanation will clarify several symbols and also show that the effective force produced by the eccentrics of the oscillator is independent of the motion of the oscillator body during the forced vibration. Although the equations are derived for the hypocyclic oscillator under longitudinal vibrations, similar expressions hold for the centrifugal-force oscillator, and the conclusions are valid for torsional vibrations.

Before considering the nature of the true oscillator force, it is expedient to amplify Appendix B with the following definitions:

${}_tF_0'$ = true alternating force produced by oscillator due to absolute acceleration of oscillator eccentrics M (distortions in motion of eccentrics M, due to vibration of oscillator body, are considered), lb

F_0' = maximum value or amplitude of true oscillator force ${}_tF_0'$, lb

${}_tF_0$ = apparent alternating force produced by oscillator, due to acceleration of eccentrics M, relative to oscillator body (distortion in motion of eccentrics, due to vibration of oscillator body, is neglected), lb

F_0 = maximum value or amplitude of apparent oscillator force ${}_tF_0$, lb

e = eccentricity or amplitude of motion of eccentric M if oscillator is stationary, in.

[18] "Die Dämpfung der Werkstoffe," by O. Föppl, *Zeitschrift des Vereines deutscher Ingenieur*, vol. 74, 1930, pp. 1391–1394.

[19] "The Practical Importance of the Damping Capacity of Metals, Especially Steels," by O. Föppl, *Journal of the Iron and Steel Institute*, vol. 134, 1936, pp. 393–455.

[20] "Vibration Problems in Engineering," by S. Timoshenko, second edition, D. Van Nostrand Co., New York, N. Y., 1937, chapt. 2, p. 145, Fig. 93.

- e = pitch radius of internal gear A (Fig. 3 of paper) of hypocyclic oscillator
- M_0' = mass of oscillator excluding eccentrics M, lb-sec² per in.
- M_0 = mass of oscillator, including eccentrics M $= (M_0' + M)$, lb-sec² per in.
- I_0 = moment of inertia of oscillator, including eccentrics M, lb-in-sec²
- ϕ' = true phase angle between rotating vector representing true oscillator force ${}_tF_0'$ and rotating vector representing oscillator vibration $\bar{a}$ (see Fig. 19 of this closure), deg
- ϕ = phase angle between rotating vector, representing apparent oscillator force ${}_tF_0$ and rotating vector, representing oscillator vibration $\bar{a}$ (see Fig. 19), deg

The inertia force produced by a vibrating mass such as eccentrics M is

$$ {}_tF_0' = M(d^2x/dt^2) \qquad [30] $$

where x is the position of the mass at any instant of time (see Fig. 19a).

If the oscillator housing remains stationary during operation, and if the motor speed is constant, then

$$ (d^2x/dt^2) = d^2(e \sin \omega_0 t)/dt^2 = -e\omega_0^2 \sin \omega_0 t $$

$$ \therefore {}_tF_0 = -Me\omega_0^2 \sin \omega_0 t \qquad [31] $$

and

$$ F_0 = Me\omega_0^2 \qquad [32] $$

However, the oscillator housing is not stationary during a dynamic test as just assumed, but it vibrates. This vibration alters the path of motion of the eccentrics and, therefore, changes their acceleration and corresponding inertia forces, as will be discussed.

Fig. 19(a) represents the motion of the eccentric M of the hypocyclic oscillator in which M vibrates sinusoidally in straight path f-g-f-h-f of amplitude e relative to the oscillator housing. Simultaneously, line n-n, the vertical diameter of internal gear A (Fig. 3 of the paper), vibrates with amplitude a at the same frequency but at a lagging phase angle ϕ. If the vibration $\bar{a}$, of the oscillator housing is sinusoidal, an assumption which will be discussed later, the motions may be represented by the time-displacement diagram of Fig. 19(b), and the vector sum $\bar{b}$ is the absolute motion of eccentric M. The vibrations may also be represented by the vector diagram of Fig. 19(c) in which the horizontal projection of rotating vector OA, of length $\bar{a}$, represents the motion of the oscillator housing, and the horizontal projection of vector OE, of length $\bar{e}$, represents the vibration of eccentric M relative to the housing. Thus the absolute position x of the eccentric M at any instant of time is determined from the horizontal projection of the vector sum $\bar{b}$ of the two components of motion as follows

$$ x = e \sin \omega_0 t + a \sin (\omega_0 t - \phi) $$

If the foregoing equation is substituted in Equation [30] and simplified

$$ {}_tF_0' = -M\omega_0^2 \sqrt{e^2 + 2ae \cos \phi + a^2} \sin (\omega_0 t - \alpha) $$

where angle α is defined in Figs. 19(b and c).

Thus ${}_tF_0'$ is the true, but not necessarily the effective, alternating force produced by the oscillator, the maximum value or amplitude of which is

$$ F_0' = M\omega_0^2 \sqrt{e^2 + 2ae \cos \phi + a^2} \qquad [33] $$

If the ratio of a to e is very small, which is true in all cases of longitudinal vibration undertaken and in most cases of torsional vibration, then Equation [33] for the true oscillator force F_0' practically equals Equation [32] for the apparent oscillator force F_0. Even if the ratio of a to e is not small the apparent oscillator

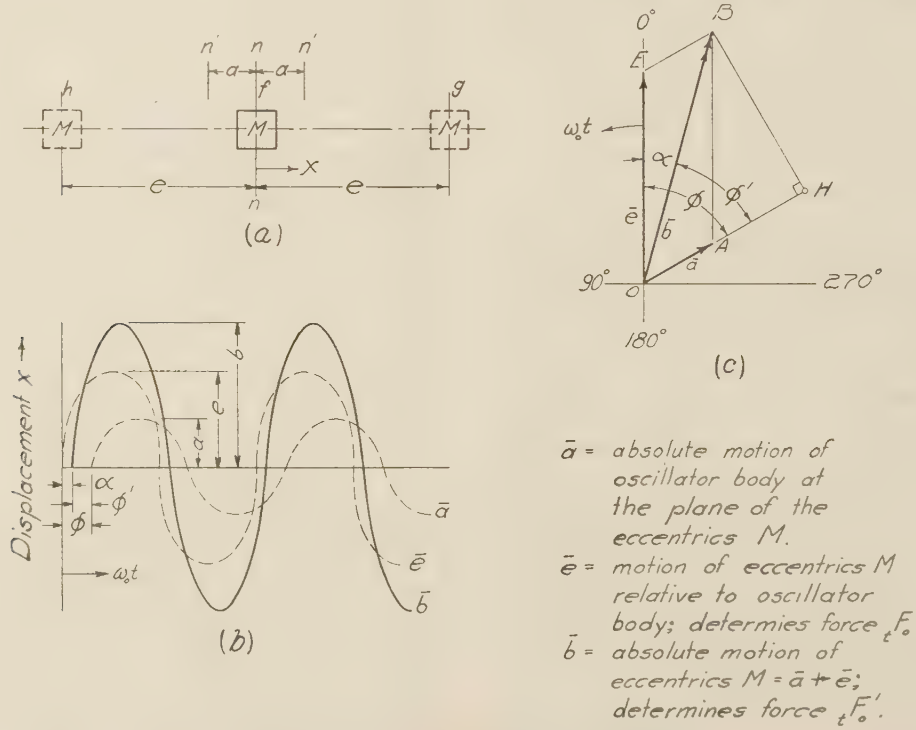

FIG. 19 DETAILS OF THE MOTION OF ECCENTRICS M

force F_0 is still correctly used in the equations of the Appendix. The proof of this statement follows:

The equations given in the Appendix generally involve the product of the oscillator force and a function of the phase angle, for example, $F_0 \sin \phi$ or $F_0 \cos \phi$, rather than the oscillator force alone. It is therefore advisable to investigate the correctness of these functions rather than the correctness of the individual terms.

Equation [15] of the paper will be considered first. The correct expression for the work done should read

$$W = \pi a F_0' \sin \phi'$$

However, from Fig. 19(*c*) of this closure

$$\sin \phi' = BH/OB = e \sin \phi / \sqrt{e^2 + 2ae \cos \phi + a^2} \ldots [34]$$

By combining Equations [33] and [34]

$$F_0' \sin \phi' = \left(M\omega_0^2 \sqrt{e^2 + 2ae \cos \phi + a^2}\right) \left(e \sin \phi / \sqrt{e^2 + 2ae \cos \phi + a^2}\right) = (M\omega_0^2 e) \sin \phi$$

and using Equation [32]

$$F_0' \sin \phi' = F_0 \sin \phi \ldots\ldots\ldots\ldots\ldots\ldots [35]$$

Thus the true oscillator force F_0' (or torque T_0') multiplied by the sine of the true phase angle ϕ' equals the apparent oscillator force F_0 (or torque T_0) multiplied by the sine of the apparent phase angle ϕ. This simplification was used in Equation [15] of the paper and elsewhere, since it is easier to compute F_0 and furthermore the oscillator phase-angle meter indicates the apparent phase angle rather than the true angle.

It may similarly be shown that the function $F_0 \cos \phi$ was used correctly in Equation [5] of the paper and elsewhere. A more basic form of Equation [5] is

$$F_s = M_0' a\omega_0^2 + F_0' \cos \phi'$$

From Fig. 19(*c*)

$$\cos \phi' = OH/OB = (a + e \cos \phi)/ \sqrt{e^2 + 2ae \cos \phi + a^2} \ldots [36]$$

By combining Equations [36] and [33]

$$F_0' \cos \phi' = \left(M\omega_0^2 \sqrt{e^2 + 2ae \cos \phi + a^2}\right) (a + e \cos \phi)/ \sqrt{e + 2ae \cos \phi + a^2} = Ma\omega_0^2 + F_0 \cos \phi \ldots [37]$$

However, $M_0 = M_0' + M$

$$\therefore F_s = M_0' a\omega_0^2 + Ma\omega_0^2 + F_0 \cos \phi = M_0 a\omega_0^2 + F_0 \cos \phi$$

Thus, the function $F_0 \cos \phi$ was correctly employed in the Appendix.

Some of the complications in oscillator testing, mentioned by Mr. Findley, have more academic than practical importance. For example, the calculated value of the gyroscopic action of the motor during torsional vibrations of the hypocyclic oscillator is less than 0.1 lb-ft, and insignificant torque. Furthermore, the alternating gyroscopic torque occurs about a horizontal axis perpendicular to the motor shaft and no vibration about this axis could be detected during the tests.

Similarly, the effect of turbulent air damping on the observed value of the dynamic modulus of elasticity is of little practical importance. In some tests, not reported in the paper, different frequencies were employed so that the "air friction" and other effects could be determined by extrapolation, and no significant correction factor was observed.

An important theoretical aspect of the problem of oscillator testing near resonance relates to the nature of the hysteresis-damped vibration. Unfortunately an exact solution to this problem apparently has not been developed, and its development is beyond the scope of the paper. However, the situation is not as pessimistic as Mr. Findley pictures it; by utilizing simple, undamped vibration theory and extensions thereof, rather than the exact and probably complex theory, oscillator testing can still yield valuable data of sufficient accuracy for most engineering applications. Stated differently, the basic force, Equations [5] and [7] of the paper, which are exact for the undamped vibration, are sufficiently accurate for the hysteresis-damped vibration, as will be discussed.

First the character of hysteresis damping should be understood. Unlike other types of damping, such as viscous or Coulomb, no definite damping force is associated with hysteresis damping. Rather, the logical concept for hysteresis damping is one of energy absorbed per cycle of vibration by virtue of noncoincidence of the upward and downward branch of the load-deflection curve. Consequently, no damping force should appear in Equations [5] and [7].

The correctness of the oscillator-force term of Equations [5] and [7] has already been discussed.

All that remains therefore as a possible source of significant error is in the term designating the inertia force resulting from the vibration of the mass (or moment of inertia) of the complete oscillator assembly. In this term the concept that force is the product of the mass (or moment of inertia) and the acceleration, is exact; however, the acceleration $a\omega_0^2$ (or $\theta\omega_0^2$) is exact only for sinusoidal vibrations, as pointed out by the discussers. The proximity of the hysteresis-damped vibration to a sine wave was partially investigated, when the research project was first begun, by observing experimentally the oscillator displacement as a function of phase angle and a close approximation to a sine wave was indicated for the materials tested.

Mr. Findley has suggested that a possible method of checking the validity of Equations [5] and [7] for a given material is to run tests at the same stress and at the same frequency, but at different lengths of specimens and, therefore, different amplitudes of vibration. Essentially, this procedure amounts to changing the relative magnitude of the two terms of Equations [5] and [7] without changing their sum. The same change in relative magnitude was accomplished in most of the tests reported by changing the mass (or moment of inertia) of the vibrating oscillator and consequently changing the phase angle. In spite of the wide range of phase angles utilized the moduli of elasticity calculated essentially to the same value for a given stress, which indicates the validity of the Equations [5] and [7].

Several questions were raised relative to the simplicity of fatigue testing by the oscillator method. It should be pointed out that if only fatigue data on small or medium specimens are desired it is simpler to operate the oscillator-vibrating system far below resonance. If the oscillator frequency is less than one-tenth of the natural frequency of the system, then the first term of the dynamic stress Equations [5] and [7] is less than one per cent of the total; also the cosine of the phase angle is unity, and thus the alternating force on the specimen practically equals the oscillator force. Therefore, the test procedure during a below-resonance fatigue test is merely to set the oscillator force at a predetermined value and start the vibrations; no further readings of force or amplitude and no further adjustments are necessary.

Before closing, several miscellaneous comments raised by the discussers should be answered.

In the fourth paragraph of the paper, under "The Fatigue Test," the author agrees with Mr. Findley that the repeated-constant-deflection type of fatigue machine, such as the Krouse machine, may give valuable data, but that such data should not be stated in terms of stress.

The author also agrees with Mr. Findley, in the sixth paragraph under "The Fatigue Test," and elsewhere, that the equations of torsion of solid shafts are exact only for perfectly elastic materials. In fact, many tests were run on tubes in order to increase the accuracies of the torsion equations.

The author cannot agree with Mr. Findley's reason why the dynamic modulus decreases more rapidly under increasing torsional stress than under increasing direct stress. It should be observed that, contrary to Mr. Findley's assertion, the relative magnitude of the two terms in Equation [5] approximately equals the relative magnitude of the two terms in Equation [7] for a given phase angle; the amplitude of vibration may be larger in the torsional case but the frequency is correspondingly smaller.

The reason for the scatter in the data on the Duralumin specimen is that the use of a small gage length in this particular test did not permit accurate measurement of the amplitude of vibration. Certainly, for a material with as low a damping capacity as Duralumin, the deviation of the load-deflection curve from a straight line is only a fraction of 1 per cent.

The fact that the observed percentage change in modulus for plastics was about twice as great in torsion as in direct stress for comparable values of stress does not imply inconsistency in the equation employed. Rather, since the torsional fatigue limit of most materials is much less than the direct-stress fatigue limit, and since a decreasing modulus may possibly be associated with the inelastic action that causes fatigue failure, the larger decrease in modulus for the torsional case appears logical.

Mr. Findley, in discussing the significance of the temperature rise which occurs during a fatigue test of plastics, states that he has observed a maximum increase of only 11 deg at stresses up to the endurance limit. However, in determining the fatigue strengths corresponding to a fairly low number of stress cycles he found[21] temperature increases as high as 28 deg. Furthermore, Mr. Findley used a machine similar to the Krouse type in which very few fibers of the flexed plate received the maximum stress. Thus, the temperature increase would normally be less than in a direct-stress (or torsional) type of fatigue test in which all (or a significant portion) of the fibers are subjected to the maximum alternating stress.

[21] See reference (16) in the Bibliography at the end of the paper.

Messrs. Yorgiadis and Kutsay have presented some very significant data on the damping capacity of magnesium alloys which should be of value in the controversy of magnesium versus aluminum in aircraft construction. If the data of Fig. 16 of the discussion are recalculated by means of Equation [23], the average resonance-amplification factor for the magnesium at stresses near the endurance limit is about 45, whereas the resonance-amplification factor for Duralumin of Fig. 6 of the paper is about 100 at its endurance limit. This superior damping capacity of magnesium is of great significance in aircraft parts subjected to steady or transient resonant vibrations.

The nonsymmetrical resonant curves, caused by the nonlinear load-deflection curve of the test specimen, may be observed to a slight degree in Fig. 4 of the paper. This nonsymmetry was more pronounced in plastics and other materials of high damping capacity, which agrees with the observations of Messrs. Yorgiadis and Kutsay.

In discussing equations which involve the elastic energy in the specimen, Messrs. Yorgiadis and Kutsay distinguish between the area under the curve OA of Fig. 18 and the area under the straight line OA. However, the elastic energy that should be used appears to be a matter of definition: (*a*) should it be the work done in stressing the specimen up to point A (proportional to the area under curve OA); or (*b*) should it be the energy released by the specimen when the load A is removed (proportional to the area under the downward branch of the load-deflection curve, which would probably lie below straight line OA); or (*c*) should it be some average of these two energies just mentioned (which may be proportional to the area under the straight line OA)? Although it is customary in many problems to define the elastic energy in a stressed specimen by (*b*), the definition (*c*) was employed in this paper for reasons of simplicity.

The author is very grateful to the discussers for their aid in clarifying certain aspects of the problem of near-resonant vibration.

Elastic Properties of Curved Tubes

By IRWIN VIGNESS,[1] WASHINGTON, D. C.

A theory of flexibility of pipe bends perpendicular to the plane of the bend has been established. Experimental results are given in this paper verifying the derived bending equations. Results show the pipes to be more flexible than expected from the application of the "rod" theory. A flexibility factor is obtained which is identical with that found for the bending of pipe in the plane of its bend. The increased flexibility is caused by a distortion of the cross section of the pipe. There is no change in the torsional rigidity of the curved tube as compared with the rod theory. Transverse stresses similar to those caused by bending in the plane of the bend are set up in the pipe wall. Longitudinal stresses are concentrated at their point of maximum value and are of greater magnitude than expected from ordinary theory. The theory of bending of two-dimensional pipe systems in the plane of their bends is given additional experimental verification.

Nomenclature

The nomenclature used in the paper and in Appendix 1 is as follows:

- E = modulus of elasticity
- I = cross-sectional moment of inertia of a tube with respect to a diameter
- ρ = radius of curvature, in a plane perpendicular to a plane containing the tube bend, caused by an applied bending moment
- $h = \dfrac{tR}{r^2}$
- t = wall thickness
- R = radius of curvature of tube bend
- r = mean radius of tube
- K = rigidity factor
- M_b = bending moment according to bar theory
- M = bending moment
- ω_r = change of tube radius
- θ = angular distance separating a radius of the tube cross section which is under consideration from a radius that is perpendicular to the plane of the tube bend
- S_L = longitudinal stress
- B = longitudinal-stress multiplication factor
- S_t = transverse stress
- γ = transverse-stress multiplication factor

Introduction

It has long been known that tube bends may be more flexible in the plane of the bend than similarly bent rods of the same cross-sectional moments of inertia. Th. von Kármán (1)[2] and Hovgaard (2) have developed a theoretical explanation for this effect. Experimental work (2, 3, 4, 5) is in fair agreement with their theory. Differences which have occurred are attributable to lack of knowledge of the moments and forces applied at the ends of the pipe.

For three-dimensional pipe systems, arrangements of piping can be devised in which the measured strains and end reactions are not in reasonable agreement with theoretical results. The differences can easily be between 50 and 100 per cent (4), as will be demonstrated later in this paper. For most systems, as encountered in practice, the difference between theoretical and experimental results is about 15 to 25 per cent (4, 6, 7),[3] and much of this difference can be attributed to lack of rigidity of the measuring apparatus.

The importance of the discrepancy between measured and calculated end reactions should not be minimized because of the small differences encountered in most practical cases. The reasons the differences are small are that the increased flexibility occurs only in bends and only because of forces and moments acting perpendicular to the bends. This flexibility is usually but a small component of the total involved in the bending of the pipe, and can, for some systems, be changed by a factor of perhaps 100 per cent without greatly affecting the end reactions. If the pipe is more flexible than is accounted for by ordinary theory, there must be a different stress distribution within the pipe wall from that which is assumed by ordinary theory. A possible concentration of stress may occur. This concentration of stress may be as important in pipe systems having small differences between the theoretical and experimental values of end reactions as in those having large differences.

Discussion of Principles That Cause Increased Flexibility of Curved Tubes or Beams

Cross-Sectional Deformation Theory of Bending. It is generally assumed in calculations that curved tubes, when bent by forces or moments acting perpendicular to the plane of the bend, have the same flexibility properties as similarly bent rods having the same cross-sectional moments of inertia as the tubes. It will be shown that this is not a permissible assumption. Neither the bending properties of the tube nor the distribution of stress within the walls of the tube agree with those calculated, or assumed, in rod theory.

A general cross-sectional deformation theory of bending is proposed which is a generalization of the cases developed by von Kármán (1), Hovgaard (2) and Timoshenko (8), for bending in the plane of the bend, and of the case of bending a tube perpendicular to the plane of its bend, as is developed in this paper. This theory states that curved beams of any cross section are more flexible to bending than would be calculated by ordinary bar theory. The increase of flexibility depends, among other things, on the shape and ease of deformation of the beam cross section. When the forces required to distort the cross section are large, the difference between an accurate theory and ordinary bar theory becomes negligible. However, when the forces required to distort the cross section are small, bar theory may not be a sufficiently good approximation. This is the case for thin-walled curved tubes. In general this is the case for thin-walled curved

[1] Naval Research Laboratory, Anacostia Station.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Power Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers.

[3] Particular attention should be given to a discussion of the paper of Hill (6) by Rossheim, Markl, and Andrews (7), in which is stated that the resultant forces or moments, rather than their components, are preferable as a basis of comparison. This avoids placing undue emphasis on the smaller components.

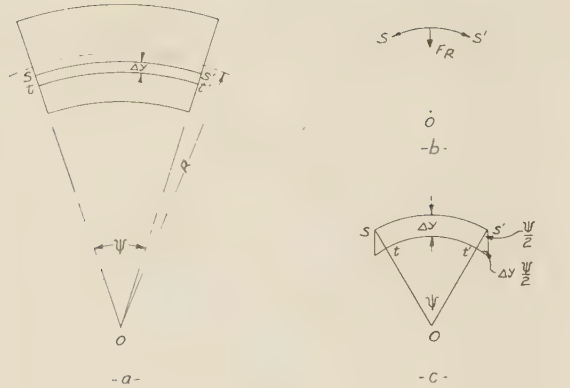

Fig. 1 Bending of a Section of a Curved Beam

(*a*) Plane section of a curved beam. Radius of curvature of a center line is R; ss' and tt' are filaments of the beam that run parallel with its central axis. The filaments subtend a small angle ψ.
(*b*) Forces acting on filament ss'; F_R is a component of force acting in the direction of the radius of curvature.
(*c*) An exaggerated view of the displacement of ss'. Values are valid only for small values of ψ and Δy.

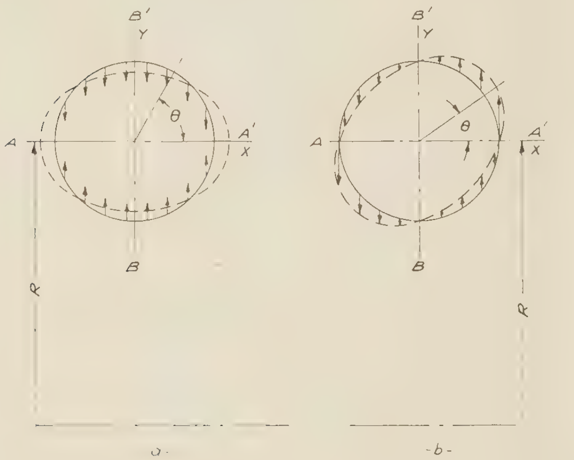

Fig. 2 Deformation of the Cross Section of a Pipe at a Bend of Radius R

(*a*, The bending moment acts in the plane of the bend tending to shorten the radius of curvature. *b*, The bending moment acts perpendicular to the plane of the bend and causes longitudinal compression on the side $BA'B'$.)

beams of any cross-sectional shape, although the geometry of the cross section is an important parameter. It should be emphasized that the strength of such a beam may be considerably less (8) than expected from bar theory.

The physics involved in the bending of a section of a curved beam can be visualized by considering a small section of a curved beam, having a radius of curvature R, and subtending a small angle ψ, as shown in Fig. 1(*a*). The shape of the cross section of the beam need not be considered. A filament ss' is chosen which runs parallel with the central axis of the beam. If the beam is stressed, so that the filament is caused to have either a tensile or a compressive stress, the filament will have a component of force, Fig. 1(*b*), along the radius of curvature R. This component of force will cause the filament to move a distance Δy along R. A filament tt' is separated from ss' by a distance Δy along R and subtends the same angle as ss'. The difference of length of tt' and ss' is then $\Delta y\psi$ (for small values of ψ and Δy), as shown in Fig. 1(*c*). However, if filament ss' should be forced to position tt', by a tensile stress as shown in Fig. 1(*b*), the excess length $\Delta y\psi$ will relieve the strain on this filament and distribute it to others. As all of the filaments that make up the beam tend to relieve themselves in the same manner, the beam will always bend more than is predicted by ordinary theory.

The change of longitudinal-stress distribution due to the shifting of the filaments along R cannot easily be predicted from general considerations. Transverse stresses that cause the deformation of the cross section are always introduced.

Fig. 2(*a*) shows the manner in which the cross section of a pipe at a bend will distort when acted upon by a moment that tends to decrease the radius of curvature of the pipe bend. The solid circular line represents the cross section before stress, and the dotted oval line shows the section after deformation. Center line AA' is contained in the neutral plane. There is a longitudinal tensile stress in the upper half $AB'A'$, hence the filaments (see Fig. 1) will be forced toward the center of curvature of the pipe bend as illustrated by the arrows. The lower section of the pipe is under longitudinal compression; therefore the filaments will be forced away from the center of curvature of the pipe bend.

Fig. 2(*b*) represents the same cross section of the pipe bend. A moment is applied that acts perpendicular to the plane of the bend so that compressional longitudinal stress occurs on the side $BA'B'$ and tension on the opposite side. Center line BB' is contained in the neutral plane. The cross section is circular before the application of stress. After the application of stress it assumes a shape as represented by the dotted oval, the major axes of which are 45 deg to the plane containing the pipe bend.

The physical picture given would tend to show that a curved beam will be more flexible than indicated by the rod theory for bending in any plane. However, for torsional stress, for the case just considered, there is no resultant longitudinal force, and the torsional rigidity of a curved tube is, therefore, the same as that calculated by the rod theory.

Preliminary Experiments

The contribution of any component part of a three-dimensional piping system to the flexibility properties of the whole system may not be large, and the bending of this part by but one component of a bending moment may be yet smaller. Therefore, if only the pipe bends were affected, and if only their bending, caused by one component of bending, were in error, an error of considerable magnitude could be made in the calculation of the flexibility of some components of the pipe without greatly affecting its over-all flexibility properties. If such were the case and end reactions were the only consideration, an error, or a poorly approximate theory, would be permissible. If, however, the theory approximation neglects cross-sectional deformation, it also neglects possible concentrations of stress within the walls of the pipe. This may be an important consideration.

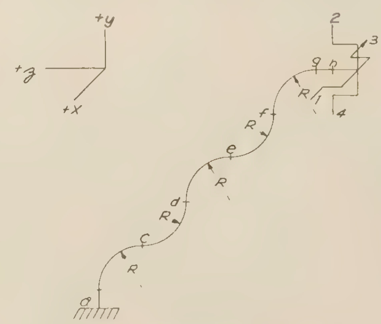

Fig. 3 Pipe or Rod No. 40

(Rod diameter, 0.498 in.; outside diameter of pipe, 0.753 in.; inside diameter, 0.688 in.; $ab = gh = 1$ in., $R = 3.1$ in. A force P, Q, or N is applied at h. P, Q, and N are directed along the x, y, and z axes, respectively. Displacements caused by the forces are observed at points 1, 2, 3, and 4 by means of a traveling microscope; a plane containing these points also contains h.)

A pipe and a similar rod, Fig. 3, were constructed so that, by means of a suitable application of forces at its ends, the effect of different flexibility components could be observed without being masked by general flexibility properties. The fixed end of the pipe, or rod, was clamped rigidly to a heavy steel bedplate. The section of the pipe end contained within the clamp was plugged with a tightly fitting steel rod that just penetrated to the end of the clamp support, which was the theoretical fixed end of the pipe. The free end of the pipe or rod, h, was acted upon by suitable forces and the displacements were observed by means of a traveling microscope. In order that the angular displacements might be determined, projections were fastened on an extension of the free end and their movements along the co-ordinate directions were observed. The calculated values of displacements and rotations of point h were determined by the method of Karelitz and Marchant (9). A rigidity factor $K = 0.493$ was used for bending of the pipe in the plane of the bend.

Tables 1 and 2 provide a comparison between measured and calculated values of displacements and rotations of point h. Terms Δx, Δy, and Δz, ϕyz, ϕzx, and ϕxy are translations and rotations, respectively, in the indicated co-ordinate direction or plane. The displacement of the free end of the pipe, or rod, is caused by a co-ordinate force P, Q, or N.

TABLE 1 END DISPLACEMENTS OF PIPE NO. 40, CAUSED BY FORCES APPLIED AT END OF PIPE

Force	Type	Displacement in inches or radians: Measured	Calculated	Difference	Difference, per cent
$P = 1$ lb	ϕzx	0.002865	0.001820	+0.001045	+57.4
$Q = N = 0$	ϕxy	0.002480	0.001515	+0.000965	+63.7
	Δx	0.06001	0.03737	+0.02264	+60.6
$Q = 1$ lb	ϕyz	0.00305	0.003268	—0.000218	— 6.7
$N = P = 0$	Δy	0.03626	0.03706	—0.00080	— 2.2
	Δz	0.03129	0.03328	—0.00199	— 5.9
$N = 1$ lb	ϕyz	0.00265	0.00285	—0.00020	— 7.0
$P = Q = 0$	Δy	0.0304	0.03320	—0.0028	— 8.4
	Δz	0.0272	0.03010	—0.0029	— 9.6

TABLE 2 END DISPLACEMENTS OF ROD NO. 40, CAUSED BY FORCES APPLIED AT ONE END

Force	Type	Displacement in inches or radians: Measured	Calculated	Difference	Difference, per cent
$P = 1$	ϕzx	0.002825	0.002882	—0.000057	—2.0
$Q = N = 0$	ϕxy	0.002427	0.002397	+0.000030	+1.3
	Δx	0.06065	0.0592	+0.00145	+2.5
$Q = 1$	ϕyz	0.00269	0.002655	+0.000045	+1.7
$P = N = 0$	Δy	0.0313	0.0305	+0.0008	+2.6
	Δz	0.0282	0.0274	+0.0008	+2.9
$N = 1$	ϕyz	0.002320	0.002318	+0.000002	+0.1
$P = Q = 0$	Δy	0.0275	0.0274	+0.001	+0.4
	Δz	0.02493	0.02495	—0.00002	—0.1

For bending of the pipe in the plane of the bend, which is caused by forces Q and N, the agreement between measured and calculated values is good. The differences, less than 10 per cent, can be accounted for by the uncertainty of pipe curvature near the numerous points of inflection. The curvature near these points must be greater than the assumed value of R, and therefore, the pipe should be less flexible than calculated. For bending perpendicular to the plane of the bend, due to force P, the pipe is about 60 per cent more flexible than calculated. This is far greater than any possible experimental error and can only be due to an error in calculation or incompleteness of the mathematical theory.

The very good agreement between measured and calculated results for the rod indicates sufficiently good experimental technique and also demonstrates that the mathematical theory applied breaks down when used for tubing.

The mathematical theory used assumes no cross-sectional distortion for bending perpendicular to the plane of the bend. If such a distortion occurs and is the cause of the mathematical breakdown of the magnitude measured, it should be an easily measurable quantity. This was found to be so, and the distortion was similar to that shown in Fig. 2(*b*).

Theoretical Results

The development of a theory which takes into account cross-sectional distortion for bending of curved tubes in a direction perpendicular to the plane of the bend is given in Appendix 1. The principal results obtained from this development are summarized in this section.

(*a*) Rigidity Factor:

"Bar" bending theory assumes that the stress, in a beam subjected to bending, is linearly proportional to the distance from the neutral axis. Curved tubes, or curved beams, with easily deformable cross sections do not generally obey bar theory but are more flexible in the plane of the bend and perpendicular to the plane of the bend than would be predicted from such a theory. If tubes of circular cross section are considered, bar theory gives the moment resisting bending as

$$M_b = \frac{EI}{\rho} \qquad [1]$$

while the "deformable-cross-section" theory or "tube" theory gives the resisting moment as

$$M = \frac{(12h^2 + 1)}{(12h^2 + 10)} \frac{EI}{\rho} = KM_b \qquad [2]$$

which defines a rigidity factor K as

$$K = \frac{12h^2 + 1}{12h^2 + 10} \qquad [3]$$

The rigidity factor is identical to that derived by von Kármán (1) and Hovgaard (2) for bending in the plane of the bend. It may now be applied generally to the flexural rigidity (bending rigidity) of tubes. The torsional rigidity is the same for bar theory and deformable-cross-section theory.

(*b*) Cross-Sectional Deformation:

The change of tube radius, due to a bending moment acting perpendicular to the plane of the bend is (Equation [56], Appendix 1)

$$\omega_r = \frac{-12rRM \sin 2\theta}{EI\,(12h^2 + 1)} \qquad [4]$$

It is this type of deformation that causes the transverse stress within the pipe walls. Quantitative experimental measurements will be made of this deformation rather than of the more difficult measurements of transverse strain. A verification of either quantity would constitute a check of the theory of both.

(*c*) Longitudinal Stress:

The longitudinal stress for bending perpendicular to the plane of the bend is given by the equation

$$S_L = \frac{4rM}{I}\left[\frac{(3h^2 - 2)\cos\theta + 3\cos^3\theta}{12h^2 + 1}\right] \qquad [5]$$

This is entirely different in form from what would be expected from the bar theory where, for bends of radii of several pipe diameters, the maximum longitudinal stress is

$$S_{L\ \mathrm{bar}} = \frac{Mr}{I} \qquad [6]$$

The equation for longitudinal stress is maximum when θ is zero, and the maximum value of longitudinal stress is

$$S_{L\,\max} = \frac{Mr}{I}\left[\frac{12h^2+4}{12h^2+1}\right] = B\,\frac{Mr}{I} \qquad [7]$$

where B is called the longitudinal-stress multiplication factor and is equal to

$$B = \frac{12h^2+4}{12h^2+1} \qquad [8]$$

It can be seen by inspection that this maximum stress is always greater than predicted by the bar theory and, for small values of h, it may be several times greater.

(*d*) Transverse Stress:

The transverse stress that occurs in the pipe wall, due to deformation of the pipe cross section, is of the same form (see Fig. 10), whether the bending be in the plane, or perpendicular to the plane, that contains the pipe bend. The points of corresponding stress for the two directions of bending are separated by 45 deg on the circumference of the cross section. The transverse stress varies linearly in going through the wall of the pipe, from a maximum on one surface to an equal and opposite maximum on the opposite surface. Its value at the wall surface is

$$\left.\begin{aligned} S_t &= \frac{18h}{12h^2+1}\,\frac{Mr}{I}\sin 2\theta \\ &= \gamma\,\frac{Mr}{I}\sin 2\theta \end{aligned}\right\} \qquad [9]$$

which defines the transverse-stress multiplication factor[4] as

$$\gamma = \frac{18h}{12h^2+1} \qquad [10]$$

(*e*) Torsion Caused by Bending in Plane Perpendicular to Plane of Bend:

An examination of the qualitative representation of forces, causing the distortion of the cross section, as shown in Fig. 2(*b*), indicates that these forces cause a torsion stress tending to twist the pipe. It is shown in Appendix 1, Equations [53] to [55], that this torsional moment is

$$M_T = \frac{M}{R} \qquad [11]$$

per unit length of pipe. It is to be noted that this equation is independent of the wall thickness of the pipe.

Experiments and Results

The preliminary experiments have indicated an increased flexibility and a cross-sectional deformation of curved tubes when the tubes are bent in a direction perpendicular to a plane containing a section of the curved tube being considered. Theoretical considerations have given a quantitative relation between the stresses, strains, and the geometry of the pipe system. Quantitative experimental determinations will now be given to check the theoretical results. These experiments fall into three classes:

(*a*) A determination of the translations of a free end of a pipe, due to given applied forces and moments, when the opposite end of the pipe is fixed.

(*b*) A determination of the change of pipe diameters in a curved section of pipe that is subjected to bending perpendicular to the plane of the bend.

(*c*) A measurement of the longitudinal strain in a curved section of pipe that is subjected to bending perpendicular to the plane of the bend.

[4] Bibliography reference (11), p. 651.

Class (a) End Displacements. Pipe bends of 90 and 180 deg were constructed as shown in Fig. 4(*a*) and (*b*). The value of the rigidity factor K was adjusted by choosing a suitable combination of radius of curvature and wall thickness. For unity rigidity factor a rod was used. The tubes were 0.75 in. OD, and the radius of curvature for different tubes varied from about 2.25 in. to 6 in. The rods were about 0.5 in. diam and had curvatures of about 3 to 6 in. The radius of curvature for a given rod or tube was constant throughout the bend within about 2 per cent which was considered the accuracy of measurement.

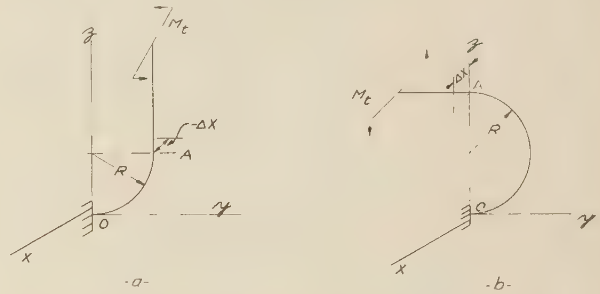

Fig. 4 Tube Bends OA, of Radius of Curvature R, Fixed at O
(A torsional moment is applied at A. Term Δx is the displacement perpendicular to the plane of the bend. a is a 90-deg bend, and b is a 180-deg bend.)

Displacements of the free end (point A of Fig. 4*b*) of the 180-deg pipe bends due to a torsional moment are given in Table 3. The corresponding displacements were calculated by means of Equation [97], of Appendix 2, for $\alpha = \phi = \pi$. For tube theory, the appropriate value of the rigidity factor K was used. For rod theory the value of K was taken as unity. A similar tabulation for 90-deg pipe bends (Fig. 4*a*) is given in Table 4.

TABLE 3 DISPLACEMENT OF FREE END OF A 180-DEG PIPE BEND, PERPENDICULAR TO PLANE OF BEND, DUE TO A TORSIONAL MOMENT APPLIED AT FREE END

	Displacements (mils per in-lb) determined by:			Per cent difference between measured and calculated values in terms of:			
				Measured value		Calculated value	
Rigidity factor, K	Measurement	Bar theory	Tube theory	Bar theory	Tube theory	Bar theory	Tube theory
0.492	0.3214	0.235	0.340	—26.9	+5.7	+36.7	—5.5
0.520	0.289	0.211	0.314	27.0	8.55	37.0	8.0
0.70	0.571	0.526	0.625	7.9	9.4	8.6	8.7
1.00	0.370	0.366	0.366	1.1	1.1	1.1	1.1

TABLE 4 DISPLACEMENT OF FREE END OF A 90-DEG BEND, PERPENDICULAR TO PLANE OF BEND, DUE TO A TORSIONAL MOMENT APPLIED AT FREE END

	Displacements (mils per in-lb) determined by:			Per cent difference between measured and calculated values in terms of:			
				Measured value		Calculated value	
Rigidity factor, K	Measurement	Bar theory	Tube theory	Bar theory	Tube theory	Bar theory	Tube theory
0.364	0.0564	0.0185	0.0688	—67.2	+22.0	+205.	—18.0
0.625	0.0923	0.0541	0.0963	41.5	9.3	70.0	4.2
0.678	0.1235	0.0784	0.361	21.3	10.2	34.0	9.2
0.795	0.181	0.1392	0.196	27.0	8.3	37.0	7.7
0.952	0.0412	0.0372	0.0401	4.8	1.7	10.7	1.7
1.00	0.158	0.1582	0.1582	0.1	0.1	0.1	0.1
1.00	0.0412	0.0411	0.0411	0.2	0.2	0.2	0.2

Tables 3 and 4 show that the tubes are considerably more flexible than predicted by ordinary bar theory and slightly more rigid than calculated by tube theory. Both theories as applied to rods ($K = 1$) agree with experiment. The reason that the tubes are slightly more rigid than when calculated from the tube theory is accounted for by the prevention of distortion of the tube cross section at and near the ends of the bends by the adjoining straight sections, or by the clamp holding the fixed end of the tube. This effect is especially predominant for bends of short radius of curvature such as the first pipe ($K = 0.364$) of Table 4. The radius of curvature of this pipe was 2.25 in. The tube cross

section was held circular at the fixed end, at which point theory, for this case, assumes the distortion to be the greatest. As this distortion was partially prevented for some distance around the bend, the tube should be stiffer than indicated by tube theory (see Fig. 13). This is as indicated in Table 4.

Percentage differences between theory and experiment are given in terms of both the measured value and of the theoretical values. As the deflections are usually calculated, the percentages in terms of the calculated values are most significant. For a comparison in terms of a common magnitude, the percentage differences in terms of the measured values are given.

Fig. 7 provides a comparison between experimental and theoretical values of the rigidity constant. The experimental values were obtained by solving Equation [97] of Appendix 2 for K and using the same and similar experimental data to that included in Tables 3 and 4. Again it is seen that the experimental rigidity constant is slightly higher than calculated because of the effects just mentioned.

Class (b) Cross-Sectional Distortion. A pipe, having the shape and dimensions as shown in Fig. 5, was constructed. The pipe was contained in a single plane. One end of the pipe was fixed and the other end was acted upon by a force perpendicular to the plane containing the pipe. Changes of pipe diameter, caused by this force, were measured at different sections around the pipe bend.

The diameter changes at points a, b, and c are given in Table 4. The applied force was 14.93 lb. Points a and b were chosen some distance from the tangents so that they would not interfere with the measured cross-sectional distortion. Calculated corresponding changes of diameter were obtained by use of Equation [4]. Only the maximum changes (for θ = 45 deg) were considered as the changes in diameter as a function of θ will be given later for a larger-diameter pipe.

TABLE 5 MEASURED AND CALCULATED MAXIMUM CHANGES OF PIPE DIAMETER FOR A CHANGE OF APPLIED STRESS AT F[a]

Position	Bending moment, in-lb	Diameter change, in. Measured	Calculated
a	280	0.0048	0.0047
b	267	0.0042	0.0045
c	16.5	0.0009	0.0003

[a] Refer to Fig. 5.

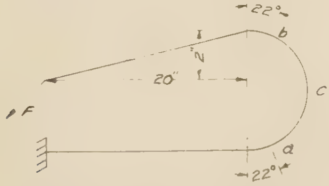

FIG. 5 PIPE NO. 45

(Radius of curvature = 3.1 in.; mean radius = 0.3578 in.; wall thickness = 0.0365 in.; outside diameter = 0.752 in.)

The accuracy of measurement of the diameter changes was about ±0.5 mil. The results are in satisfactory quantitative agreement. It is evident from the small distortion at point c that the torsional component produces no deformation of the cross section, and that the flexibility increase is only due to the bending component of moment. It is interesting to note that the major and minor axes of the "ellipse" are shifted by 90 deg in passing through the point of zero bending moment near c.

Pipe No. 45 was too small in diameter to permit accurate measurements of the change of diameter as a function of θ. Pipe No. 59 was set up as shown in Fig. 6. Fig. 11 illustrates the change of radius as a function of θ for a cross section in the pipe bend near a. The curve is a sine wave, the maximum value of which has been adjusted for best fit with the experimental points. The

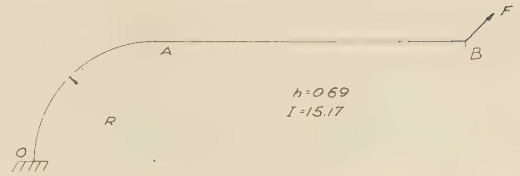

FIG. 6 PIPE NO. 59

(Radius of curvature = 19.18 in.; mean radius = 2.662 in.; average wall thickness = 0.255 in.; average outside diameter = 5.58 in. Section OA is a quarter bend and AB is a tangent; AB is 67 in.)

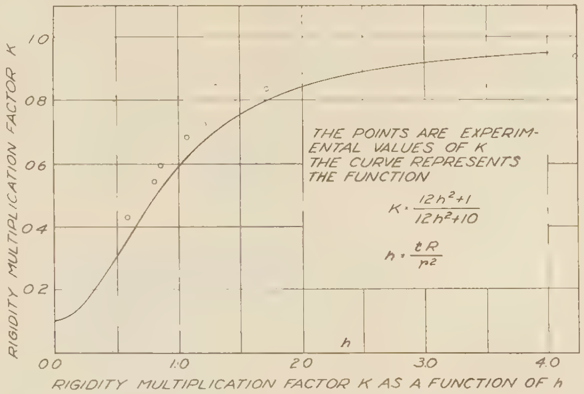

FIG. 7 EXPERIMENTAL AND THEORETICAL VALUES OF RIGIDITY FACTOR K

experimental points fall closely on the sine curve which indicates that the single-term solution (Equation [40], Appendix 1) is sufficiently accurate. Additional terms added to this solution have been shown mathematically not to change the flexibility constant by more than 1 per cent for ordinary values of h.

The cross section of pipe No. 59 had been flattened in the bend so that the diameter in the plane of the bend was about 0.4 in. shorter than its perpendicular diameter. This noncircular cross section and a nonuniformity of wall thickness were expected to cause variations between theoretical and experimental values. These appear more pronounced in measurements given later of longitudinal strain but in no case do they cause serious discrepancies.

There is a distortion of the pipe cross section in the tangent adjacent to the bend. This is forced by the distortion of the cross section in the bend, and it tends to inhibit the distortion in the bend. As theory neglects this effect, the pipe system should be slightly more rigid than predicted by tube theory. No correction for this increased rigidity has been applied for bending in the plane of the bend, and experiments indicate that none is required for bending perpendicular to the plane of the bend. Fig. 7 indicates the magnitude of the discrepancy caused by the inhibition of cross-sectional distortion, and Fig. 13 shows how the distortion is prevented adjacent to the tangent.

Fig. 10 illustrates the transverse stress which results from the cross-sectional distortion that has been shown to exist. For ductile materials the transverse stress can usually be neglected as the stresses are very localized.[5] The stresses pass from a positive maximum to a negative maximum in going through the thickness of the pipe wall.

Class (c) Longitudinal Stress. The longitudinal strain was measured, by means of a Tuckerman strain gage, on the surface of pipe No. 59 (see Fig. 6). For a cross section of the bend near A, values proportional to the measured strain values are shown in

[5] Bibliography reference (11), p. 652.

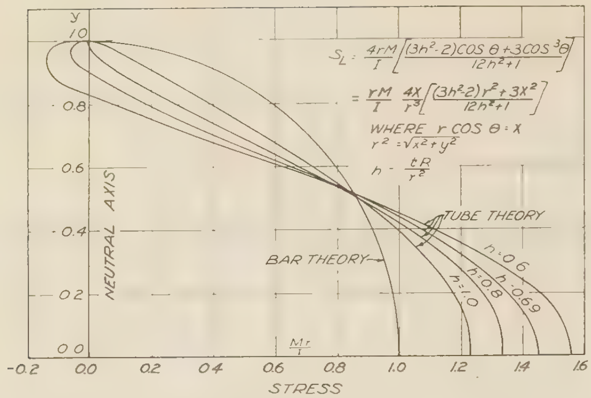

Fig. 8 Longitudinal-Stress Distribution in Wall of Curved Pipe, for Various Values of h, Caused by Bending Perpendicular to Plane Containing Pipe Bend

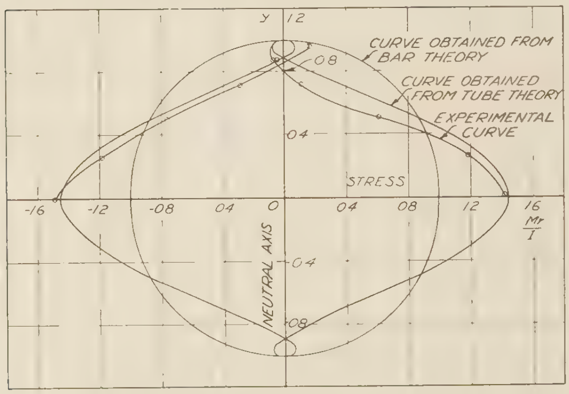

Fig. 9 Longitudinal-Stress Distribution in Wall of Curved Pipe, Caused by Bending Perpendicular to Plane Containing Pipe Bend

(Tensile stress is shown to the right of the Y axis, compressive, to the left.)

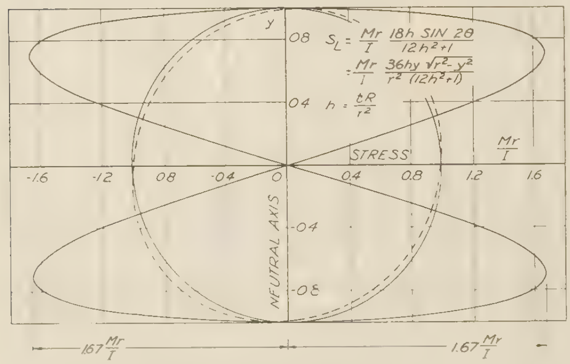

Fig. 10 Transverse-Stress Distribution in Outside Wall of Curved Pipe, Caused by Bending Perpendicular to Plane of Bend

(The stress curve for the pipe wall shown in the upper and lower right quadrants is contained in the upper right and lower left quadrants, respectively. The closed circle and the dotted oval represent the pipe cross section before and after distortion.)

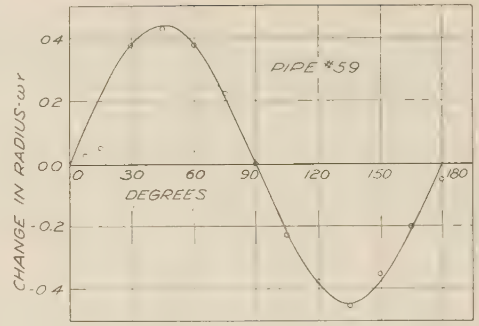

Fig. 11 Change of Cross-Sectional Pipe Radius in a Bend as a Function of Position Around Circumference

(The curve is a sine wave of a vertical scale best adjusted to fit the experimental points. The zero-deg diameter is perpendicular to the plane of the bend.)

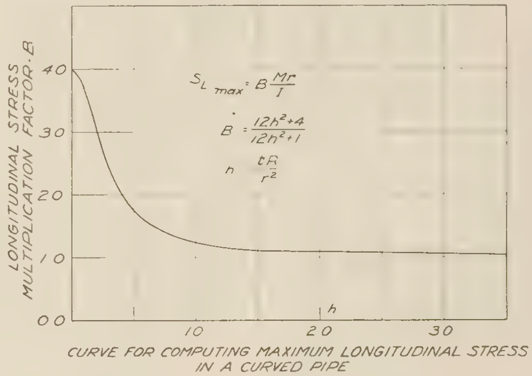

Fig. 12 Maximum Longitudinal-Stress Multiplication Factor for Bending Perpendicular to Plane of Bend

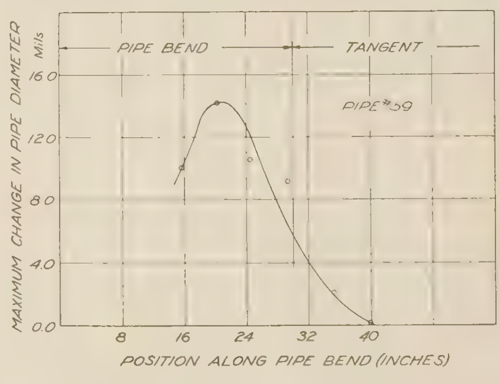

Fig. 13 Maximum Changes in Diameter of Pipe Adjacent to Junction of Pipe Bend and Tangent Caused by Bending Perpendicular to Plane of Bend

Fig. 9. These are shown as the experimental curve. The average value of the experimental maximum stress (or strain) has been adjusted to agree with the corresponding theoretical value. The theoretical curve is for the same value of h that should apply to the experimental pipe. The difference between the two curves can be accounted for by the nonuniform wall thickness and noncircular cross section of the pipe. The stress distribution is quite similar for these two cases and unlike that expected from bar theory. A concentration of stress occurs at the maximum value. Near the neutral axes, the motion Δy (see Figs. 1 and 2b) more than compensates for the strain required for the bending and therefore causes the stress to become reversed in sign.

Fig. 8 shows the theoretical longitudinal-stress distribution for several values of h for one quadrant of a pipe cross section. Maximum values of longitudinal stress are given in Fig. 12.

Application of Tube Theory and End-Reaction Determinations for Bending Perpendicular to Plane of Bend

In piping, for purposes of calculation, the systems can be considered in terms of their component sections. These sections are usually taken as plane quarter bends, tangents, or other simple geometric shapes. The Piping Handbook[6] considers piping systems of quarter bends and tangents and determines the displacement, due to given forces and moments, of the free end of each of these parts when the opposite end is fixed. The theory developed in this report will cause changes in displacements as determined by handbook methods. Corrections which apply for the bending of pipe bends perpendicular to the plane of the bend will be given for quarter bends. These are the handbook cases Nos. 8, 9, and 10, and will be so numbered here. General solutions and flexibility properties of plane bends with a constant radius of curvature are given in Appendix 2.

In the following cases a right-hand system of co-ordinates is used with its origin at the free end of the pipe. A displacement v of the pipe is in the y direction, which is perpendicular to the plane of the bend, and is positive when in the positive y direction (refer to Fig. 21 of Appendix 2). Rotations β_{zy} and γ_{yz} are in the planes indicated and are positive when their axes are along the positive directions of the z and x axes, respectively. Similarly, moments are positive when their axes are in a positive co-ordinate direction, and forces are positive when directed in the positive co-ordinate direction.

The term C represents torsional rigidity and, for Poisson's ratio equal to 0.30, the value of C is

$$C = \frac{EI}{1.3}$$

Case No. 8:

A quarter bend is fixed at one end and is acted upon at the opposite end by a force P perpendicular to the plane of the bend, Fig. 14.

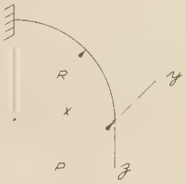

FIG. 14 QUARTER BEND, ACTED UPON BY FORCE PERPENDICULAR TO PLANE OF BEND

(a) $$\beta_{zy} = \frac{PR^2}{EI}\left[0.279 - \frac{0.7854}{K}\right]$$

Term β_{zy} is a rotation in a plane perpendicular to the free end of the bend. The direction of rotation causes a deflection of the free end in the direction of the force

(b) $$\gamma_{yz} = \frac{-PR^2}{EI}\left[\frac{0.5}{K} + 0.65\right]$$

Term γ_{yz} is a rotation in a plane tangent to the free end of the bend and perpendicular to the plane of the bend. The direction of rotation causes a deflection of the free end in the direction of the force.

The displacement

(c) $$v = \frac{PR^3}{EI}\left[0.4632 + \frac{0.7854}{K}\right]$$

is the total displacement and is in the direction of the applied force.

[6] Bibliography reference (11), p. 676.

Case No. 9:

A quarter bend is fixed at one end and acted upon at the free end by a moment. The moment is contained in a plane perpendicular to the plane of the bend and tangent to the pipe at its free end.

(a) $$\beta_{zy} = \frac{MR}{EI}\left[\frac{0.5}{K} - 0.65\right]$$

Term β_{zy} is a rotation in a plane perpendicular to the free end of the bend. For values of $K > \frac{0.5}{0.65}$ the direction of rotation causes a deflection of the free end to the right along the moment arm, as shown in Fig. 15. For values of $K < \frac{0.5}{0.65}$ the direction of rotation is reversed.

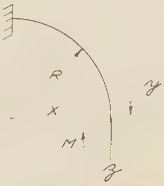

FIG. 15 QUARTER BEND, ACTED UPON BY A BENDING MOMENT PERPENDICULAR TO PLANE OF BEND

(b) $$\gamma_{yz} = \frac{MR}{EI}\left[\frac{0.7854}{K} + 1.021\right]$$

Term γ_{yz} is a rotation in a plane that is tangent to the pipe at its free end and that is perpendicular to the plane of the bend. The direction of rotation is in the same sense as the moment and is such as to give a deflection to the left along the moment arm, as shown in Fig. 15.

(c) The displacement

$$v = -\frac{MR^2}{EI}\left[\frac{0.5}{K} + 0.65\right]$$

is the total deflection of the free end of the pipe and is to the left along the moment arm as shown.

Case No. 10:

A quarter bend is fixed at one end and is acted upon at the other end by a torsional moment. The moment is contained in a plane perpendicular to the free end of the bend, Fig. 16.

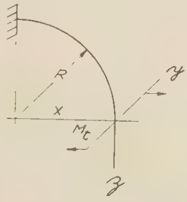

FIG. 16 QUARTER BEND, ACTED UPON BY A TORSIONAL MOMENT AT FREE END

(a) $$\beta_{xy} = \frac{M_tR}{EI}\left[\frac{0.7854}{K} + 1.021\right]$$

Term β_{xy} is a rotation in a plane perpendicular to the free end of the pipe and in the same directional sense as the applied moment, i.e., the deflection is to the left along the moment arm, as shown in Fig. 16.

(b) $$\gamma_{yz} = \frac{M_tR}{EI}\left[\frac{0.5}{K} - 0.65\right]$$

Term γ_{yz} is a rotation in a plane tangent to the free end of the pipe and perpendicular to the plane of the pipe bend. For values of $K > \frac{0.5}{0.65}$ the direction of rotation is such as to cause a deflection of the free end to the right along the moment arm as shown. For values of $K < \frac{0.5}{0.65}$ the direction of rotation is reversed.

(c) $$v = \frac{M_tR^2}{EI}\left[0.279 - \frac{0.7854}{K}\right]$$

The displacement v is to the left along the moment arm as shown. The foregoing cases generate to those given in the Piping Handbook (11) when K is made unity.

Summary

An experimental and theoretical investigation of the bending of pipe bends has been undertaken. It has been shown that for bending in the plane of the bend the present theory is adequate. However, for bending in a plane perpendicular to the plane of the bend, the present theory is not sufficient. A "cross-sectional deformation" theory or "tube" theory has been developed for bending perpendicular to the plane of the bend. This theory can be considered as an extension of the tube theory ordinarily applied for bending in the plane of the bend. The same rigidity factor applies for either type of bending moment. Experiments indicate that the theory, as developed for bending perpendicular to the plane of the bend, is of about the same accuracy as for bending in the plane of the bend.

A concentration of stress occurs in the walls of the pipe that should be taken into account in pipe design. The maximum value of longitudinal stress for bending perpendicular to the plane of the bend, as determined by tube theory, is always greater than that determined by rod theory, although the location of the position of maximum value is the same for both theories. The maximum longitudinal stress for tube theory can be nearly 4 times that determined by rod theory (Fig. 5), but for usual piping systems this stress is about 1.3 to 1.5 times that determined by rod theory. The same consideration of transverse stresses should be taken for bending perpendicular to the plane of the bend as is taken for bending in the plane of the bend as the maximum value of stress is the same in either case.

Acknowledgments

The author expresses his appreciation to Dr. Ross Gunn, of the Naval Research Laboratory, for his continued interest in this problem, and to Mr. Eugene Pardue who has done much of the experimental work.

Appendix 1

Bending of Pipe Bends Perpendicular to Plane of Bend

It has been shown by von Kármán (1) and Hovgaard (2) that the bending of pipe bends, caused by moments acting in the plane of the bend, is accompanied by a deformation of the circular pipe section. This deformation causes the pipe section to become elliptical (nearly) in shape, with the principal axes of the ellipse in the plane of the bend and perpendicular to the plane of the bend. The change of the cross-sectional moment of inertia, due to the deformation, is small but the increase of flexibility may be large. The increase of flexibility is caused by the deformation moving material that is under tension nearer to the center of curvature of the pipe bend, and by moving that which is under compression farther from the center of curvature of the pipe bend. This relieves the stress in the material in proportion to the distance moved and causes a nonlinear distribution of stress as a function of distance from the neutral axes.

When a moment, that acts perpendicular to a plane containing a pipe bend, is applied to the pipe bend, a similar deformation of the circular cross section of the pipe takes place. The oval formed in this case, however, has its principal axes at 45 deg to the plane containing the bend. The change of cross-sectional moment of inertia is small, although the increase of flexibility may be large. The explanation for the increase of flexibility and change of stress distribution is exactly as given in the previous paragraph.

In calculating stresses, strains, and end reactions of piping systems, only the rod theory has been applied for all piping configurations, except for bending curved pipes in the plane of their bend. There has been no report to the author's knowledge regarding the increased flexibility of pipe bends in a direction perpendicular to the plane of the bend or in the change of stress distribution in the pipe wall, as compared with rod theory. Equations have been derived and experimentally verified by the use of small-scale pipe systems which give the rigidity factor and stress distribution for bending of pipes in a direction perpendicular to the plane of the bend.

Fig. 17 shows a section of a pipe bend having a radius of curvature R, length ψR, wall thickness t, and mean pipe radius r. A right-handed rectangular co-ordinate system is taken with its origin at the center of the pipe, the y axis along the radius of curvature of the bend, and the z axis along the center line of the tube. Positive directions are those shown in the figure. A filament of the pipe wall ss', parallel with the axes of the pipe, is shown in detail in Fig. 18. The angle θ, in Fig. 17, represents the angular distance of this filament from the x axis.

Assume a longitudinal force, F, acting on filament ss'. If p is the force per unit area then

$$dF = ptrd\theta \qquad [12]$$

As the filament is curved, there is a component of force f_R, along the radius of curvature of the pipe bend. For small values of ψ

$$df'_R = -ptr\psi d\theta \qquad [13]$$

Term p is positive when the filament is in tension and f'_R is positive when in the positive direction of the y axis.

A distortion of the pipe wall, due to f'_R, will cause the filament to become displaced a distance Δy (Fig. 19) along the radius of curvature of the pipe bend. If the pipe is subjected to a bending stress in any plane such that filament ss' is subjected to a longitudinal stress, the filament will move to position s_1s_1' and the strain of the filament will be relieved by an amount

$$\Delta L_R = \psi\Delta y \qquad [14]$$

where ΔL_R might be termed an apparent elongation of the filament, due to a displacement of the filament in the R direction.

The length of the neutral axis subtending the angle ψ is

$$L_n = R\psi \qquad [15]$$

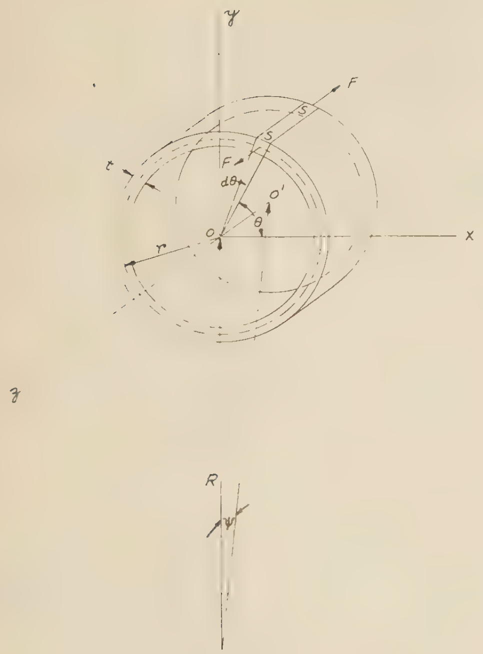

Fig. 17 Section of a Pipe Bend

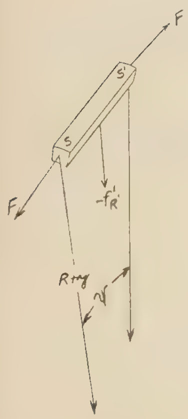

Fig. 18 Forces Acting on Filament ss' of Pipe Wall

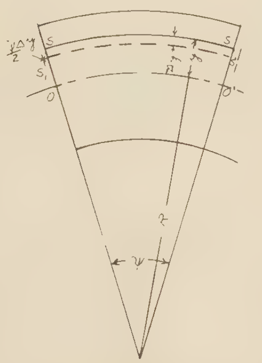

Fig. 19 Displacement of Filament ss' Toward Center of Curvature

A plane perpendicular to the plane containing the pipe bend is shown in Fig. 20. The curved lines represent the pipe after it has been bent to a radius of curvature ρ by a moment acting perpendicular to the plane of the bend.

The length of the neutral axis subtending the same angle as the filament ss' is

$$L_n = \rho\Phi \qquad [16]$$

If R and ρ are large compared with r and if the angles ψ and Φ are small, one can equate Equations [15] and [16] to obtain

$$R\psi = \rho\Phi \qquad [17]$$

If there were no shifting of position of ss' along the radius of curvature of the pipe bend, the change of length of the filament in bending to radius of curvature ρ (Fig. 20) would be

$$\Delta L\rho = x\Phi \text{ (nearly)} \qquad [18]$$

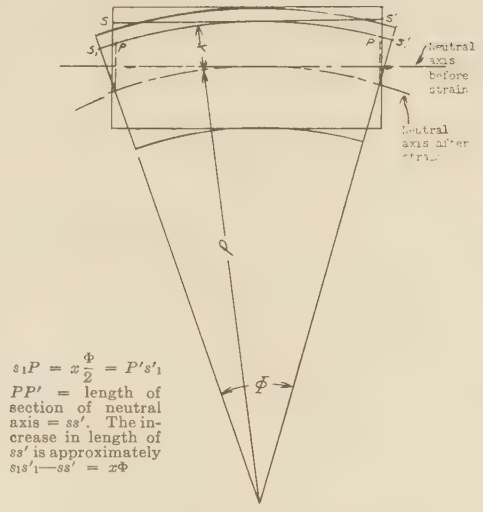

Fig. 20 Section of Pipe Containing Filament ss' With Plane of Bend Perpendicular to Paper

(Straight lines are projections of pipe before bending and curved lines are projections of section after bending to a radius of curvature ρ.)

However, due to shifting the filament along R, the actual change of length will be

$$\Delta L = \Delta L\rho + \Delta L_R \qquad [19]$$

or

$$\Delta L = x\Phi + \psi\Delta y \qquad [20]$$

The change of length of the filament per unit length of the corresponding neutral axis is

$$\epsilon = \frac{\Delta L}{L_n} = \frac{x}{\rho} + \frac{\Delta y}{R} \qquad [21]$$

The stress on the filament is

$$p = E\epsilon = E\left(\frac{x}{\rho} + \frac{\Delta y}{R}\right) \qquad [22]$$

where E is the modulus of elasticity.

The radial force (along R) on the filament as found by combining Equations [13] and [22] is

$$df'_R = -tEr\left(\frac{x}{p} + \frac{\Delta y}{R}\right)\psi d\theta \qquad [23]$$

If the radial force per unit length of pipe is taken, the relation $\psi R = 1$ must be satisfied and

$$df_R = -\frac{tEr}{R}\left(\frac{x}{\rho} + \frac{\Delta y}{R}\right)d\theta \qquad [24]$$

A procedure similar to that used by von Kármán (1) will now be followed. Let w_r be the radial displacement of the filament ss' and w_t be the tangential displacement, then

$$\Delta y = w_t \cos\theta + w_r \sin\theta \qquad [25]$$

as

$$x = r\cos\theta \qquad [26]$$

Equation [24] becomes

$$\epsilon = \frac{1}{R}\left(\frac{rR}{\rho}\cos\theta + w_t\cos\theta + w_r\sin\theta\right) \ldots\ldots [27]$$

The tangential and radial displacements are related by the equation

$$\frac{dw_t}{d\theta} + w_r = 0 \ldots\ldots [28]$$

So Equation [27] can be written

$$\epsilon = \frac{1}{R}\left(\frac{rR}{\rho}\cos\theta + w_t\cos\theta - \frac{dw_t}{d\theta}\sin\theta\right) \ldots\ldots [29]$$

An equation giving the relation between the displacement Δy of the filament ss' and the angle θ can be obtained by minimizing the potential energy due to the bending of a pipe section. The condition of bending considered will be that perpendicular to the plane of the pipe bend. The strain energy consists of two parts:

(*a*) That caused by the deformation of the pipe cross section.
(*b*) That caused by bending of the pipe.

The fact that the pipe has a curvature perpendicular to the direction of bending is not considered in part (*b*).

The energy due to deformation of the pipe section is

$$V_1 = \int_0^{2\pi} \frac{M_1{}^2 r d\theta}{2EI_1} \ldots\ldots [30]$$

where

$$I_1 = \frac{t^3}{12} \ldots\ldots [31]$$

is the moment of inertia of a section of the pipe wall of unit width and

$$M_1 = \frac{-EI_1}{r^2}\left(\frac{d^2w_r}{dt^2} + w_r\right) \ldots\ldots [32][7]$$

is the moment acting on the pipe wall.

Combine Equations [28], [30], and [32] to obtain

$$V_1 = \frac{Et^3}{24r^3}\int_0^{2\pi}\left(\frac{d^3w_t}{d\theta^3} + \frac{dw_t}{d\theta}\right)^2 d\theta \ldots\ldots [33]$$

The energy due to bending of the pipe is

$$V_2 = \frac{1}{2}\int_0^{2\pi} \epsilon p dA \ldots\ldots [34]$$

where p = force per unit area acting on a filament having cross-sectional area dA and causing an elongation ϵ.

Now

$$p = E\epsilon \ldots\ldots [35]$$

$$dA = trd\theta \ldots\ldots [36]$$

and Equation [29] gives a value for ϵ, so

$$V_2 = \frac{Etr}{2R^2}\int_0^{2\pi}\left(\frac{rR}{\rho}\cos\theta + w_t\cos\theta - \frac{dw_t}{d\theta}\sin\theta\right)^2 d\theta. [37]$$

The total strain energy is

[7] Bibliography reference (10), p. 459.

$$V = V_1 + V_2 = \frac{Etr}{2R^2}\left\{\int_0^{2\pi}\left(\frac{rR}{\rho}\cos\theta + w_t\cos\theta - \frac{dw_t}{d\theta}\sin\theta\right)^2 d\theta + \frac{t^2R^2}{12r^4}\int_0^{2\pi}\left(\frac{d^3w_t}{d\theta^3} + \frac{dw_t}{d\theta}\right)^2 d\theta\right\} \ldots [38]$$

From physical considerations, it can be seen that the relation between w_t and θ must be of the form

$$w_t = \Sigma_n C_n \cos 2n\theta \ldots\ldots [39]$$

If the first term of this series is used for an approximate solution one obtains

$$w_t = C\cos 2\theta \ldots\ldots [40]$$

This solution and its derivatives substituted in Equation [38] give

$$V = \frac{Etr}{2R^2}\left\{\int_0^{2\pi}\left(\frac{rR}{\rho}\cos\theta + C\cos 2\theta\cos\theta + 2C\sin 2\theta\sin\theta\right)^2 d\theta + \frac{t^2R^2C^2}{12r^4}\int_0^{2\pi}(8\sin 2\theta - 2\sin 2\theta)^2 d\theta\right\} \ldots [41]$$

as $(\cos 2\theta\cos\theta = 2\cos^3\theta - \cos\theta)$ and $(\sin 2\theta\sin\theta = 2\cos\theta - 2\cos^3\theta)$ so

$$V = \frac{Etr}{2R^2}\int_0^{2\pi}\left\{\left[\left(\frac{rR}{\rho} + 3C\right)\cos\theta - 2C\cos^3\theta\right]^2 + \frac{3t^2R^2C^2}{r^4}\sin^2 2\theta\right\} d\theta \ldots [42]$$

which after integration becomes

$$V = \frac{\pi Etr}{2R^2}\left[\left(\frac{rR}{\rho}\right)^2 + \left(\frac{rR}{\rho}\right)3C + (5 + 6h^2)\frac{C^2}{2}\right] \ldots [43]$$

where

$$h = \frac{tR}{r^2} \ldots\ldots [44]$$

To minimize the potential energy with respect to C let

$$\frac{dV}{dC} = 0 = \frac{3rR}{\rho} + (5 + 6h^2)C \ldots\ldots [45]$$

or

$$C = \frac{-rR}{\rho}\left(\frac{3}{5 + 6h^2}\right) \ldots\ldots [46]$$

so

$$w_t = \frac{-rR}{\rho}\left(\frac{3}{5 + 6h^2}\right)\cos 2\theta \ldots\ldots [47]$$

$$w_r = \frac{-rR}{\rho}\left(\frac{6}{5 + 6h^2}\right)\sin 2\theta \ldots\ldots [48]$$

and

$$\Delta y = \frac{-rR}{\rho}\left(\frac{3}{5 + 6h^2}\right)(\cos 2\theta\cos\theta + 2\sin 2\theta\sin\theta) \ldots [49]$$

The moment resisting bending of the pipe section is

$$M = \int_0^{2\pi} r\cos\theta\, pdA \ldots\ldots [50]$$

By the use of Equations [29], [35], and [36] one obtains

$$M = \frac{tEr^3}{\rho}\int_0^{2\pi}\cos\theta\left[\cos\theta-\left(\frac{3}{5+6h^2}\right)(\cos 2\theta\cos\theta + 2\sin 2\theta\sin\theta)\right]d\theta = \frac{tEr^3}{\rho}\int_0^{2\pi}\left[\cos^2\theta-\left(\frac{3}{5+6h^2}\right)(3\cos^2\theta-2\cos^4\theta)\right]d\theta = \frac{\pi tEr^3}{\rho}\left[1-\frac{9}{10+12h^2}\right] \ldots [51]$$

$$M = \frac{\pi tEr^3}{\rho}\left[\frac{12h^2+1}{12h^2+10}\right] = \frac{EI}{\rho}\left[\frac{12h^2+1}{12h^2+10}\right] \ldots [52]$$

The bar theory of bending gives

$$M_b = \frac{EI}{\rho} \ldots [53]$$

So Equation [52] becomes

$$M = KM_b \ldots [54]$$

where

$$K = \frac{12h^2+1}{12h^2+10} \ldots [55]$$

Term K is identical to the rigidity factor determined for bending in the plane of the bend.

If the value of ρ as determined from Equation [52] is used to eliminate ρ from Equation [48], the value

$$w_r = \frac{-12rRM \sin 2\theta}{EI(12h^2+1)} \ldots [56]$$

is obtained that gives the change of tube radius due to a bending moment acting perpendicular to the plane of the bend (refer to Fig. 11).

The transverse stress that occurs in the pipe wall, due to the deformation of the pipe cross section, varies linearly from a maximum of one sign at the inner surface to a maximum of the opposite sign at the outer surface. The transverse stress on the surface of the pipe is

$$S_t = \frac{M_1 t}{2I_1} \ldots [57]$$

By the use of Equations [31], [32], and [56], one obtains

$$S_t = \frac{-18hrM \sin 2\theta}{I(12h^2+1)} = -\gamma\frac{Mr}{I}\sin 2\theta \ldots [58]$$

$$\gamma = \frac{18h}{12h^2+1} \ldots [59]$$

may be called the transverse-stress multiplication factor and is identical to that used for bending in the plane of the pipe bend. However, the points of maximum stress are separated by 45 deg for the two types of bending.

The longitudinal stress

$$S_L = E\epsilon \ldots [60]$$

can be determined in the following form by the use of Equations [27], [47], [48], and [52]

$$S_L = \frac{4rM}{I}\left[\frac{(3h^2-2)\cos\theta + 3\cos^3\theta}{12h^2+1}\right] \ldots [61]$$

The maximum value of this stress always occurs at $\theta = 0$ and is equal to

$$S_L = \frac{rM}{I}\left(\frac{12h^2+4}{12h^2+1}\right) \text{ (see Figs. 8, 9, and 12)} \ldots [62]$$

An interesting minimum is observed for $\cos^2\theta = (2-3h^2)/9$ at which value the longitudinal stress is

$$S_{L\,\min} = \frac{-8\,rM}{9I}\,\frac{(2-3h^2)^{3/2}}{(12h^2+1)} \ldots [63]$$

For values of $h^2 < \frac{2}{3}$ the value of stress reverses as the neutral axis is approached. For values of $h^2 \geq \frac{2}{3}$ there is no reversal or negative minimum (see Figs. 8 and 9).

The twisting moment per unit length of pipe is

$$M_T = \int_0^{2\pi} x\,df_R = \int_0^{2\pi} r\cos\theta\,df_R \ldots [64]$$

By Equation [24] and eliminating Δy and x one obtains

$$M_T = \frac{tEr^3}{\rho R}\int_0^{2\pi}\cos\theta\left[\cos\theta - \frac{3}{5+6h^2}(\cos 2\theta\cos\theta + 2\sin 2\theta\sin\theta)\right]d\theta \ldots [65]$$

But this is Equation [51] divided by R so the twisting moment per unit length of pipe is

$$M_T = \frac{M}{R} \ldots [66]$$

Appendix 2

End Displacements of Curved Tube Bent Out of Its Plane of Curvature by Moments Acting at One End

Cross-sectional deformation theory will be applied to circularly bent tubes. The tubes are fixed at one end and the forces or moments are applied at the free end. Three cases will be considered, when the action applied is:

(*a*) A force perpendicular to the plane of curvature. The same procedure and nomenclature will be used as in the similar case for bars by Timoshenko (10).

(*b*) A moment acting in a plane tangent to the free end of the bend and perpendicular to the plane of curvature of the bend.

(*c*) A torsional moment acting in a plane perpendicular to the free end of the bend.

Fig. 21 shows a circularly curved tube AB, fixed at A and

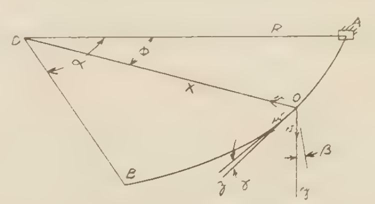

Fig. 21 Pipe AB of Constant Radius of Curvature R Fixed at A and Acted Upon by Force or Moment at B

(The translations and rotation of a cross section at any point O are indicated by u, v, w, and β and γ for directions along the x, y, z axes and rotations in the xy and yz planes, respectively. The initial position of the pipe is in the xz plane.)

acted upon by a force or a moment at B. The radius of curvature of the tube is OD. For any tube cross section O, a system of rectangular co-ordinates is taken with its origin at the center of the tube at O, and with its x, y, and z axes directed along the radius of curvature, perpendicular to the plane of curvature, and along the tangent of the tube at O, respectively. The positive directions are as shown in the figure. Terms u, v, and w, are displacements of the cross section in the respective co-ordinate

directions. Terms β and γ are rotations of the cross section in the x,y and y,z planes. Positive values of rotations exist when the axes of rotation are in positive co-ordinate directions. Forces are positive when in a positive co-ordinate direction, and moments are positive when their axes are in positive directions. Angle α is subtended by AB and ϕ by AO.

Case (a). If $1/\rho$ and $1/R_2$ are the radii of curvature of the tube at O in the y,z and x,z planes after deformation, and θ is the angle of twist per unit length, then if $1/R$ is the initial radius of curvature of the tube, the following relations show the moments causing the deformation

$$\frac{EI_x}{\rho} = M_x \quad [67]$$

$$EI_y\left(\frac{1}{R_2} - \frac{1}{R}\right) = M_y \quad [68]$$

$$C\theta = M_z \quad [69]$$

It can also be shown (10) that

$$\frac{1}{\rho} = \frac{\beta}{R} - \frac{d^2v}{ds^2} \quad [70]$$

$$\frac{1}{R_2} = \frac{1}{R} + \frac{u}{R^2} + \frac{d^2u}{ds^2} \quad [71]$$

$$\theta = \frac{d\beta}{ds} + \frac{1}{R}\frac{dv}{ds} \quad [72]$$

where ds is an increment of length along the axis of the tube and

$$ds = Rd\phi \quad [73]$$

If a force P is applied in a positive direction at B it is seen that

$$M_x = -PR\sin(\alpha - \phi) \quad [74]$$

$$M_y = 0 \quad [75]$$

$$M_z = PR[1 - \cos(\alpha - \phi)] \quad [76]$$

If KEI is substituted for EI_x and the foregoing equations solved as by Timoshenko (10), the value of displacement is

$$v = \frac{PR^3}{C}[\phi - \sin\phi - \sin\alpha(1 - \cos\phi)] + \frac{PR^3}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\phi\cos(\alpha - \phi) - \sin\phi\cos\alpha] \quad [77]$$

and

$$\beta = \frac{PR^2}{C}[\phi - \sin\alpha + \sin(\alpha - \phi)] - \frac{v}{R} \quad [78]$$

$$\gamma = -\frac{du}{Rd\phi} = -\frac{PR^2}{C}[1 - \cos\phi - \sin\alpha\sin\phi] - \frac{PR^2}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\cos(\alpha - \phi) + \phi\sin(\alpha - \phi) - \cos\alpha\cos\phi] \quad [79]$$

Case (b). If a moment M with its axis directed along the radius of curvature in a positive direction be applied at B, the moment resulting at O will be

$$M_x = M\cos(\alpha - \phi) \quad [80]$$

$$M_y = 0 \quad [81]$$

$$M_z = M\sin(\alpha - \phi) \quad [82]$$

Combine these equations with Equations [67] through [72] and substitute KEI for EI_x to get

$$M\cos(\alpha - \phi) = \frac{KEI}{R^2}\left[\beta R - \frac{d^2v}{d\phi^2}\right] \quad [83]$$

$$-M\sin(\alpha - \phi) = \frac{C}{R^2}\frac{d(\beta R + v)}{d\phi} \quad [84]$$

Equation [84] can be integrated directly and if subjected to the conditions that when

$$\phi = 0 \text{ then } \beta = v = 0$$

becomes

$$\beta R = \frac{R^2M}{C}[\cos\alpha - \cos(\alpha - \phi)] - v \quad [85]$$

Substitute Equation [85] into [83] to eliminate βR

$$\frac{d^2v}{d\phi^2} + v = \frac{MR^2}{C}\cos\alpha - MR^2\left[\frac{1}{KEI} + \frac{1}{C}\right]\cos(\alpha - \phi) \quad [86]$$

Integrate Equation [86] and require that when $\phi = 0$ then $v = 0$ and $\frac{dv}{d\phi} = 0$. The displacement then is

$$v = \frac{MR^2}{C}[1 - \cos\phi]\cos\alpha + \frac{MR^2}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\phi\sin(\alpha - \phi) - \sin\alpha\sin\phi] \quad [87]$$

The rotations of the cross section at O are

$$\beta = \frac{RM}{C}[\cos\alpha - \cos(\alpha - \phi)] - \frac{v}{R} \quad [88]$$

$$\gamma = -\frac{MR}{C}\cos\alpha\sin\phi - \frac{MR}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\sin(\alpha - \phi) - \phi\cos(\alpha - \phi) - \sin\alpha\cos\phi] \quad [89]$$

Case (c). If a torsional moment M be applied at B so that its axis is tangent to the tube at B and directed away from A, then the moments resulting at O will be

$$M_x = M\sin(\alpha - \phi) \quad [90]$$

$$M_y = 0 \quad [91]$$

$$M_z = M\cos(\alpha - \phi) \quad [92]$$

Again, as in case (b), by combining these equations with Equations [67] through [72], the following equations are obtained

$$M\sin(\alpha - \phi) = \frac{KEI}{R^2}\left(\beta R - \frac{d^2v}{d\phi^2}\right) \quad [93]$$

$$M\cos(\alpha - \phi) = \frac{C}{R^2}\frac{d}{d\phi}(\beta R + v) \quad [94]$$

Integrate Equation [94] and impose conditions that when

$$\phi = 0 \text{ then } v = \beta = 0 \text{ to obtain}$$

$$\beta R = \frac{MR^2}{C}[\sin\alpha - \sin(\alpha - \phi)] - v \quad [95]$$

Eliminate βR from Equation [93] by means of Equation [95]

$$\frac{d^2v}{d\phi^2} + v = \frac{MR^2}{C}\sin\alpha - MR^2\left[\frac{1}{KEI} + \frac{1}{C}\right]\sin(\alpha - \phi) \quad [96]$$

Integrate Equation [96] and impose conditions that when

$$\phi = 0 \text{ then } v = 0 \text{ and } \frac{dv}{d\phi} = 0$$

The displacement of the cross section at 0 is

$$v = \frac{MR^2}{C}[1 - \cos\phi]\sin\alpha + \frac{MR^2}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\cos\alpha\sin\phi - \phi\cos(\alpha - \phi)] \quad \ldots\ldots\ldots [97]$$

and the rotations at 0 are

$$\beta = \frac{MR}{C}[\sin\alpha - \sin(\alpha - \phi)] - \frac{v}{R} \quad \ldots\ldots [98]$$

$$\gamma = \frac{-MR}{C}\sin\alpha\sin\phi - \frac{MR}{2}\left[\frac{1}{KEI} + \frac{1}{C}\right][\cos\alpha\cos\phi - \cos(\alpha - \phi) - \phi\sin(\alpha - \phi)] \quad \ldots\ldots\ldots [99]$$

By setting $\alpha = \phi = \frac{\pi}{2}$ in the solutions of v, β, and γ, the values are obtained for end deflections of quarter bends as applied to the Piping Handbook cases Nos. 8, 9, and 10.

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Discussion

William Hovgaard.[8] This paper brings the difficult problem of three-dimensional pipe bends one important step nearer to its complete solution.

The writer has given considerable study to this problem and in 1935 and 1937 read two papers[9,10] before this Society giving an algebraical solution based on the theory of deflection by bending.

In that solution account was taken of the effects of forces and couples acting normal to the plane of the bends, but in so doing the theory of the bending of solid curved rods was applied; the distorting effect on the tube section, with which this paper deals, was not included.

Much attention was given to the determination of the relative importance of the terms in the expressions for rotation and displacement caused by bending of the pipe bends out of their plane. These terms, being apparently small, were referred to as "secondary" terms.

The papers [9,10] were accompanied by a numerical calculation for a certain typical three-dimensional pipe system and, finally, elaborate calculations were carried out for the same system with various lengths of tangents, both for the case when the secondary terms were included and for the case when they were neglected. The results are presented in a table[11] giving the terminal reaction forces and couples.

It was found from this and several other calculations that, in such pipe systems as ordinarily occur in the propulsive machinery of destroyers, the secondary terms for any length of tangents were so small that they could be neglected.

Still the writer felt some doubt of the completeness of the solution and in the concluding remarks to the discussion on his paper,[10] he made the following statements:[12]

"The algebraic method is straightforward, and the only point which needs to be cleared up is the behavior of curved pipes when subject to forces and couples acting normal to their plane. . . . the few tests which have been made so far on this problem do not seem to check with the formulas. It is desirable that more complete tests be made, ..."

It is fortunate that this problem has now been taken up by the author, who has shown that the flexibility of bends perpendicular to their plane is greater than expected from the application of the "rod" theory, on account of distortion of the cross action of the pipe, not hitherto taken into account.

The solution presented is very complete from the mathematical point of view, and is corroborated by some preliminary experiments. It does not seem quite clear, however, what the influence is quantitatively of the deformation of the pipe section pointed out by the author.

It would be of great interest if a calculation in accordance with the author's theory could be made for the pipe bend used in the numerical example of the writer's papers,[9,10] in which case the results could be compared with those given in the table referred to.[11]

The value of such a theoretical study would be much enhanced by an experimental test of the same pipe bend, the complete dimensions of which are given in the writer's first paper.[9]

Unfortunately, the writer has not the time or facilities available for undertaking such work himself, but it is suggested as a suitable thesis for students in a technical university.

Arthur McCutchan.[13] The author is to be commended both for his intelligent approach to the problem and for the realistic

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[9] "Stresses in Three-Dimensional Pipe Bends," by William Hovgaard, Trans. A.S.M.E., vol. 57, 1935, pp. 401–415; discussion, Trans. A.S.M.E., vol. 58, 1936, pp. 391–400.

[10] "Further Studies of Three-Dimensional Pipe Bends," by William Hovgaard, Trans. A.S.M.E., vol. 59, 1937, pp. 647–650; discussion, Trans. A.S.M.E., vol. 60, 1938, pp. 596–605.

[11] Bibliography reference (10), p. 650.

[12] "Further Studies of Three-Dimensional Pipe Bends," by William Hovgaard, author's closure to discussion, Trans. A.S.M.E., vol. 60, 1938, p. 605.

[13] Engineer, Engineering Division, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

manner in which he has fitted his findings into the general theory of flexibility of pipe bends.

Ever since the tests described by Messrs. E. T. Cope and E. A. Wert[14] were made, the writer has sought a rational explanation for the greater flexibility of curved pipe acted on by forces and moments perpendicular to the plane of the bend. In discussing discrepancies between theory and results obtained on those tests, Dr. S. Timoshenko suggested that there must be some unknown "form factor" which would need to be applied to secure the desired agreement between test and theory. A test program was initiated by H. E. Mayrose under Dr. Timoshenko's direction which substantiated the previously observed fact that the deflections due to sidewise loading were greater than predicted by existing theory. The results of these tests were reported by Professor Mayrose in his discussion[15] of a paper by Prof. Hovgaard.[9] Although Professor Mayrose suggested that the K factor used to correct for flattening of the cross section in formulas for two-dimensional pipe bends might be applied also to three-dimensional problems, he did not observe, nor attempt to measure, distortion of the pipe cross section when subjecting bends to sidewise loading. The fact that the tubes which he tested were small, 1 in. OD and less, may have served to obscure the flattening of the cross section at 45 deg to the plane of the bend.

It would appear, therefore, that the author has made a real contribution to knowledge of the behavior of pipe bends through his discovery that flattening of the pipe cross section occurs under sidewise loading.

Unfortunately, incorporating the K factor in equations for sidewise loading further complicates the already involved problem of determining the flexibility of piping lying in three planes. In order to appraise the effect of including the K factor in the equations for sidewise loading, the illustrative three-dimensional problem,[16] given in the "Piping Handbook," was recalculated. The resultant of the end reactions was found to be reduced by less than 3 per cent, when the so-called secondary rotations were neglected. However, the rotations in the complementary planes assume much greater values when the K factor is applied so this comparison is not a fair measure of the differences to be expected. For $K = 0.5$, the rotation in the plane perpendicular to the free end in Case 9 becomes $^1/_7$ of the rotation in the plane tangent to the free end rather than $^1/_{12}$ as when $K = 1$. This secondary rotation increases with reduction in value of K, until for $K = 0.1$, the secondary rotation is approximately $^1/_2$ as great as the principal rotation. The same changes take place in Case 10 where torsion acts on the end of a quarter bend.

The relative magnitude of the rotations in planes perpendicular and tangent to the free end caused by a sidewise force, Case 8, is greatly affected by the value of K. For $K = 1$, the rotation in the plane perpendicular to the free end is approximately $^1/_2$ as great as in the tangent plane, while for $K = 0.1$, the relations are reversed and the rotation in the plane perpendicular to the free end is 1.3 times the rotation in the plane tangent to the free end.

In attempting to apply these changes to Cases 8, 9, and 10 (Piping Handbook), the writer has made the abscissas of the bending-moment diagrams in the graphoanalytical method equal to the algebraic sum of the coefficients of rotation in planes perpendicular and tangent to the free end rather than to base them on rotation in the principal plane of action as at present.

It would be helpful if the author would include an example illustrating the application of the revised cases to a three-dimensional problem. Two 60-in-radius arcs of 12-in nominal diam $^1/_2$-in-wall pipe, disposed in planes at right angles to each other with no connecting straight pipe, would appear to be a good test case, since it would seem that two of the reacting forces should be in exact balance.

In view of the results so far obtained by the writer there is some uncertainty as to just how the revised cases can be combined to best advantage to give a complete solution of a three-dimensional problem. If the author finds it possible to include a solution of such a three-dimensional problem in his closure it should prove most helpful.

[14] "Load-Deflection Relations for Large, Plain, Corrugated, and Creased Pipe Bends," by E. T. Cope and E. A. Wert, Trans. A.S.M.E., vol. 54, 1932, FSP-54-12, pp. 115–143.

[15] Discussion by H. E. Mayrose, of "Stresses in Three Dimensional Pipe Bends" by William Hovgaard, Trans. A.S.M.E., vol. 58, 1936, pp. 395–397.

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D. B. Rossheim[17] and A. R. C. Markl.[18] This paper makes a valuable contribution to the understanding of thin curved tubes, and assumes particular significance in view of the excellent test data adduced in support of the theoretical development. That the flexibility and stress-intensification factors are identical for flexure in and transverse to the plane of curvature should not cause surprise; on the contrary, it is surprising that the fact has escaped attention until the present time. Fig. 22 of this discussion clearly establishes that transverse bending (diagram b) represents no more than an intermediate condition between the two extremes of bending in the plane illustrated by (a) and (c), which show, respectively, the distortion produced by moments tending to close the bend (increase curvature) and open the bend (decrease curvature).

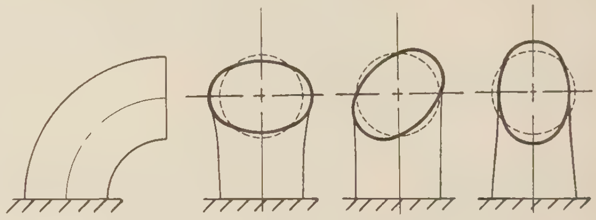

Fig. 22 Distortion in Pipe Bend

(Undistorted side view is shown at left of bend. a, Distortion of bend caused by moment tending to decrease radius of curvature of bend. b, Distortion of bend caused by moment acting perpendicular to plane of bend. c, Distortion of bend caused by moment tending to increase radius of curvature of bend.)

Taking advantage of the attention which those interested in expanding present-day knowledge of thermal-expansion effects in pipe will give to this paper, and without detracting in any way from the importance of the author's contribution to this subject, we feel it opportune to raise some of the most pressing unsolved problems, in the hope that further thought and study will be stimulated thereby:

1 *Limitation of Formulas.* Two approaches are available for the calculation of toroidal shapes, the thin-tube theory, which the author expands in this paper, and the curved-bar theory. Neither is complete, the former neglecting the displacement of the neutral axis toward the center of curvature, and the latter disregarding cross-sectional distortion. The one is nearer to the truth for thin tubes and the other probably more suited to sharp-radius thick-walled elbows. A definition of the limits of each is needed, or preferably a new approach combining all significant effects.

2 *Modifications Due to Pressure.* The thin-tube theory considers an external bending moment only; in actual applications, either internal or external pressure and some axial loading and shear are involved in addition to bending. Since the increased

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[18] M. W. Kellogg Company, New York, N. Y.

flexibility of curved pipe is due to its ovalization under flexure, which would be opposed by internal pressure, the question arises whether and to what extent the ovalization persists under internal pressure; also, whether it is accentuated by external pressure.

3 *Stability Considerations.* The distortion and stress redistribution, attendant to bending of a curved pipe, can be expected to affect its collapse resistance, both under external pressure and due to sustained axial forces or moments.

4 *Significance of Calculated Stresses.* Assuming that the stress-intensification factors can be properly evaluated, their significance with regard to ultimate failure still has to be established. Stresses are localized and, depending upon the capacity of other parts of the system to absorb overload, they should or should not be compared with normal design stresses. For steady stress, Hovgaard[19] has expressed the opinion that a design to the elastic limit will be safe. For cyclic conditions, Dennison[20] and the writers[21] have made tests providing a rough indication of the limits of safety. In this connection, it is noted that fatigue tests on bars do not even truly predict the endurance of straight pipe; also, that there is evidence that straight pipe has somewhat greater flexibility than indicated by the bar theory, a phenomenon which is as yet unexplained.

5 *Other Allied Problems.* Without intention to discourage, it is noted that there are numerous other effects pertinent to the subject of flexibility of piping, such as those in miter bends, at tees or laterals, in flanges, etc., about which practically nothing is known and which are, for this reason, often disregarded entirely by the uninitiated. Even a crude evaluation of these effects would do much to advance the basis for a proper piping-design practice.

[19] "Further Research on Pipe Bends," author's Bibliography (2), p. 292.

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Author's Closure

The practical engineer's first question, when a refinement of theory is introduced, is how greatly will the new theory affect the results as formerly derived. The answer to this question can only be given with exactness for specific cases. A general answer must necessarily be qualified. For many piping arrangements the contributions of transverse bending of bends play but a small part in determining the flexibility of the system. In such cases large changes of the transverse flexibility of the bends will have little effect on the flexibility of the system. This is the case with most piping systems.

Mr. McCutchan has found that for the illustrative three-dimensional problem given in the "Piping Handbook," the resultant of the end reactions is changed by less than 3 per cent by the introduction of the transverse-rigidity factor. The author has calculated the same problem by the method of Karelitz and Marchant,[22] in which secondary rotations have been included, and has found the resultant end reactions to be changed by less than 3 per cent. Evidently in cases of this type neither secondary rotations nor flexibility changes of transverse bending greatly affect the flexibility of the system. Piping systems that

[22] Bibliography (9).

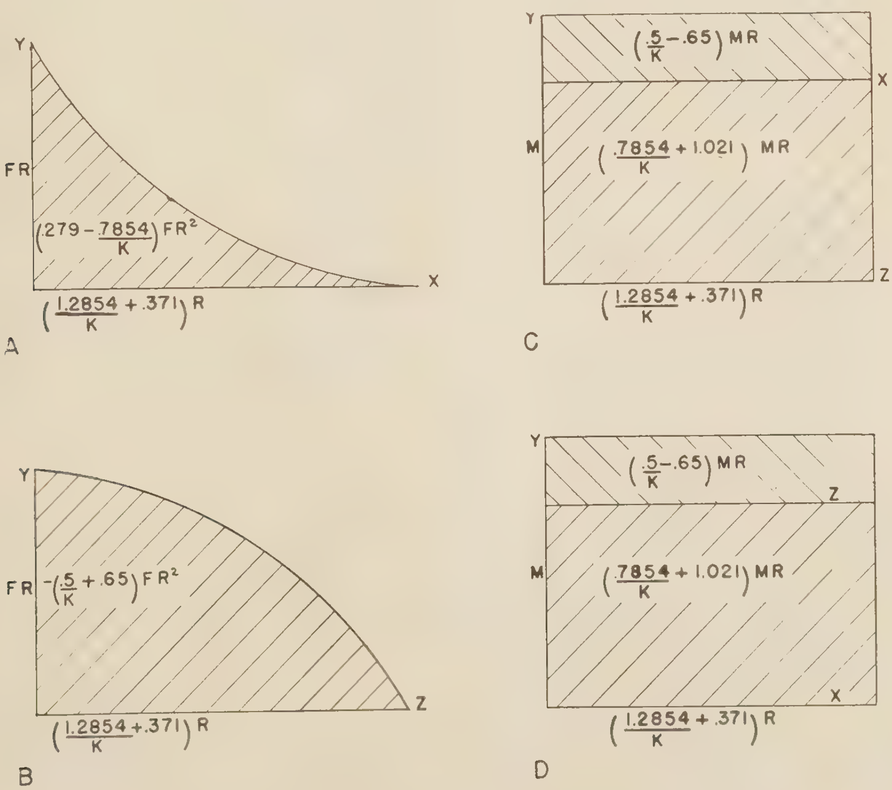

Fig. 23 Bending-Moment Diagrams for "Piping Handbook" Cases 8, 9, and 10
(Diagrams *A* and *B* apply to case 8; diagrams *C* and *D* apply to cases 9 and 10, respectively.)

contain a good percentage of their lengths in bends and that have large transverse moments along these bends are the only types in which large changes of end reactions need be expected.

It is hoped that this will give an idea of the quantitative influence of the transverse-rigidity factor as desired by Dr. Hovgaard. The stress concentration, caused by cross-sectional deformation, should be considered even though end-reaction changes are small. This is particularly true in view of the uncertain behavior of alloys used for high-temperature high-pressure piping at different stress combinations.

The application of the transverse-rigidity factor requires a negligible amount of additional work for most analytical methods of determining pipe end reactions. An appreciable added difficulty is encountered in applying it to the graphoanalytical method illustrated in the "Piping Handbook." In this latter case the abscissas of the bending-moment diagrams are taken, as stated by McCutchan, as the sum of the two rotations involved. For the handbook Cases 8, 9, and 10 the bending-moment diagrams are given in Fig. 23 of this closure, to supplement the illustrations given in the paper (Figs. 14, 15, and 16). If secondary rotations are to be neglected, the minor rotations in these diagrams can be omitted.

The additional unsolved problems put forth by Rossheim and Markl emphasize that considerable research must be accomplished before fundamental engineering knowledge catches up with present engineering practice as applied to piping. Rather than tending to discourage, a listing of these problems should provide greater incentive to labor. Many problems to be encountered along these lines are capable only of empirical solutions. Methods of measurements have lately been so improved that good results should be forthcoming. Perhaps a most important problem to be added to the list of Rossheim and Markl is the measurement of strains in installed piping systems which are caused by the temperature and pressure changes under service conditions. Such measurements are by no means impossible.

In Fig. 10 of the paper, a correction, in which S_L should be changed to S_t, should be noted. The curve for this figure is plotted for $h = 0.8$.

Instrumentation in the Study of Diesel Combustion

By E. W. Landen,[1] Chicago, Ill., and L. A. Blanc,[2] Peoria, Ill.

Physical apparatus and technique have been developed for studying the fundamental processes of combustion within the cylinders of Diesel engines. The study of engine performance actually should begin with the energy changes taking place in the combustible material. For a better understanding of the procedure followed in such tests the authors describe the "window technique" developed for the optical study of combustion. In this system a quartz port is introduced into the cylinder wall through which, for example, a spectrograph may be used to observe the flame. Another method for studying combustion and its effects on the cylinder is by means of the Edgerton camera, or high-speed continuously moving film. The technique of measuring pressures within the combustion chambers with the cathode-ray oscilloscope is described. The most important results of the study reported in this paper are the records of flame duration and pressure obtained.

A DIESEL engine may be studied not only by performance tests in the field and laboratory but also by attempting to obtain information about the fundamental processes within the engine. Early development work consisted largely of trying a number of practical designs. By the process of elimination, and the accumulation of experience, improvements were made. Physical apparatus may be employed to investigate the fundamental processes within the engine. The performance of a Diesel engine depends upon energy changes taking place in the combustible material within the cylinder. The radiation emitted is characteristic of energy changes and the temperature of the flame; therefore important information should be obtained from the radiation. This knowledge can be used in future engine design.

The Diesel test engine used in the present work is a $3^3/_4$-in. bore, 5-in-stroke, four-cycle, single-cylinder engine which develops 12 hp at 1800 rpm. Different types of engines are possible by merely interchanging block and head assemblies. A direct-current dynamo is used to start the engine and to absorb the power delivered. The dynamo permits continuous speed control from 0 to 1800 rpm and continuous load control from idle to 12 hp.

Window Technique

A prerequisite for any optical study of combustion is a transparent port through the cylinder wall or engine head. Since quartz is heat-resistant, has a low coefficient of expansion and high tensile strength, it is an ideal material through which the explosions in the engine chamber may be observed. It is transparent to the visible and ultraviolet and the transparency extends somewhat into the infrared region of the spectrum. Quartz windows should be mounted in some type of removable plug, in preference to mounting them directly in the engine wall. The plug can then be taken out for renewing windows and cleaning soot from the surface which is in contact with the burning gases.

Quartz may either be cemented into a mount or held in with a suitable gasket material. Cementing quartz into a plug is a standard practice for providing the engine wall with a transparent window. Since invar and quartz have practically the same coefficient of expansion within the temperature range of 20 to 350 C, the quartz can be cemented directly into invar without danger of cracking the window as the engine warms up and cools. Porcelain cement has been found excellent for binding the quartz and invar together. In certain cases, porcelain cement may be diluted slightly with asbestos fibers which makes it more plastic. Windows may be mounted with the outer surface flush with the rim of the retainer when using the cement technique. The cross-sectional diagram, shown in Fig. 1(*a*), illustrates a window cemented into a plug. The technique for mounting the window in this manner is described. An invar plug with the desired shape has a seat at *B* on which the quartz rests and a

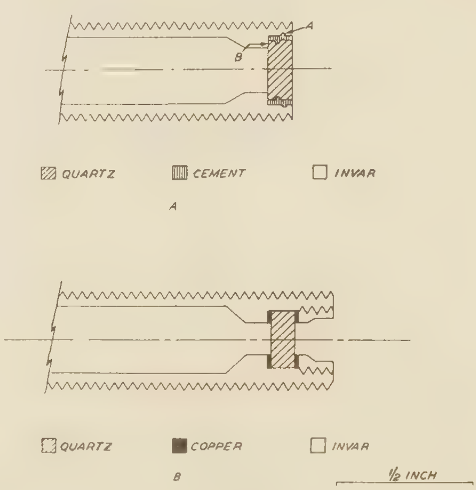

Fig. 1 Quartz Window Cemented in Plug *A* and Held by Copper Gaskets *B*

recess as shown in Fig. 1(*a*) at *A*. The quartz is then ground to fit the invar seat at *B* until a good vacuum-tight joint is obtained, using the sound of a Hyvac pump as a criterion. The Hyvac pump holds the window in position while the cement is applied. The pumping is continued until it is set and the entire assembly placed in an oven at 150 C overnight. The cement makes a mechanical bond, holding the window in position, and the ground invar-quartz joint prevents gas leakage when pressure is applied to the quartz window.

The quartz may also be held between machined surfaces with a soft gasket material separating it from the invar or iron surfaces in the retaining plug. Very soft copper washers $^1/_{64}$ in. thick have been found to prevent the quartz from cracking and they

[1] Armour Research Foundation.
[2] Caterpillar Tractor Company.

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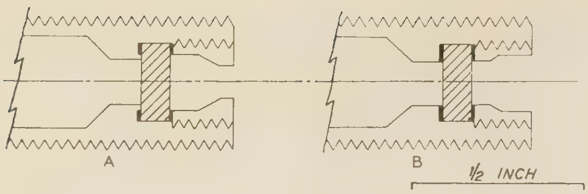

Fig. 2 Window With Two Types of Shields to Prevent Soot Collection on Window

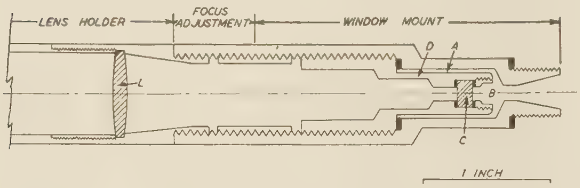

Fig. 3 Window Construction in Which Soot Collection on Window Can Be Controlled

produce a tight seal. In making the washers, two metal punches are used, one size for the inner diameter and a larger size for the outer diameter. These washers are flattened between steel plates. Finally, they are heated to a cherry red and dropped into a beaker of cold water. Fig. 1(*b*) shows a cross section of a window assembly using copper washers.

Quartz windows which are fastened in the cylinder wall or engine head and kept at the temperature of the cooling jacket become sooty after a number of explosions in the chamber. Experiments were tried, placing a shield in front of the window. The types of shields used are shown in Fig. 2, *A* and *B*. They are necessarily small to reduce effects on the normal combustion of the engine. Type *A* is slightly more effective in delaying the soot formation on the quartz, but it collimates the light beam considerably. The solid angle is largest in type *B*, other things being equal, permitting a greater cone of light to pass through the window.

One window mounting was made in which streams of nitrogen were directed toward the window, producing a washing effect. This proves to be slightly beneficial in delaying, but does not remedy the soot formation.

A new window type was used in which the temperature of the window may be controlled to some extent. The window also has provision for side passing the main air blast on combustion, causing the major portion of the soot to collect in places other than on the window. This window construction is shown in cross section in Fig. 3. The small air chamber *A* permits the carbon-infected air to be compressed, producing a movement of the air at *B* which blows the soot away from the window *C*. The side wall *D* may be made thin and long, or thick and short, depending upon the window temperature desired. The light transmitted by the window is collimated by the small fused-quartz lens *L*.

This type of window has been in operation in a direct-injection-type Diesel engine for days with no sign of soot settling on the window. In the main chamber of the precombustion-chamber-type Diesel engine, no soot appeared on the window. The same window, when used in the precombustion chamber, did form a film which stayed on the window during operation. The thickness of the film changed with the operating conditions of the engine.

It is of interest to note that it is possible by focusing the image of the window on a screen to observe the deposition and removal of such soot particles as may remain on the window long enough to be detected by the eye.

Observations With the Spectrograph

A burning fuel in the open atmosphere emits a characteristic flame. The flame is to be observed with a spectrograph to determine wave lengths of emitted light, either in the form of a band spectrum or continuous radiation. For observing the spectra of the engine, a large Littrow-type quartz spectrograph is used. The optics of the spectrograph are crystal quartz and the lens has a focal length of approximately 180 cm. The numerical aperture of the lens is about 25. The entire spectrum from 2000 A to 8000 A can be recorded on three settings of a 4 × 10-in. plate, and the adjustment permits the recording of any particular spectral region. A drum camera shown in Fig. 4 can be placed on the spectrograph in place of the plateholder. When using the drum camera, the spectrograph is placed along the engine as shown in Fig. 5. The drum of the camera is connected directly to the crankshaft of the engine by means of a jack shaft and disk coupling.

The apparatus for studying the radiation from the combustion of the Diesel fuel oil represented in Fig. 5 shows the beam of light coming from the engine window W. This collimated beam from W is reflected by mirrors M_1 and M_2 so it passes into the spectrograph slit S through the lens L. The radiation from the burning charge exposes the film on the drum camera during the rotation of the crankshaft. Thus, the spectrum is recorded during the entire combustion cycle of the engine. This has the advantage of resolving the spectrum into crank-angle intervals as burning progresses.

The spectrum obtained from the flame when photographed by the drum camera shows a continuous radiation in the visible and

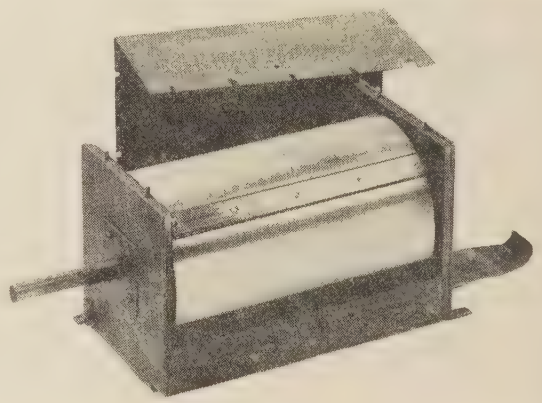

Fig. 4 Drum Camera

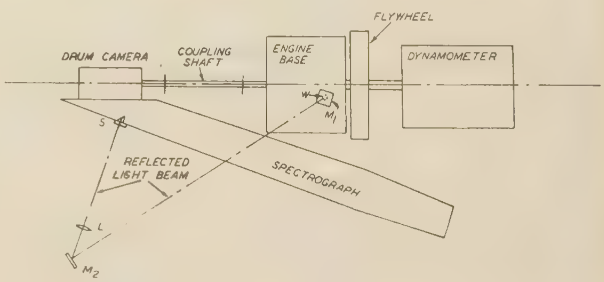

Fig. 5 Diagrammatic Sketch of Apparatus for Taking Spectrograms With Drum Camera

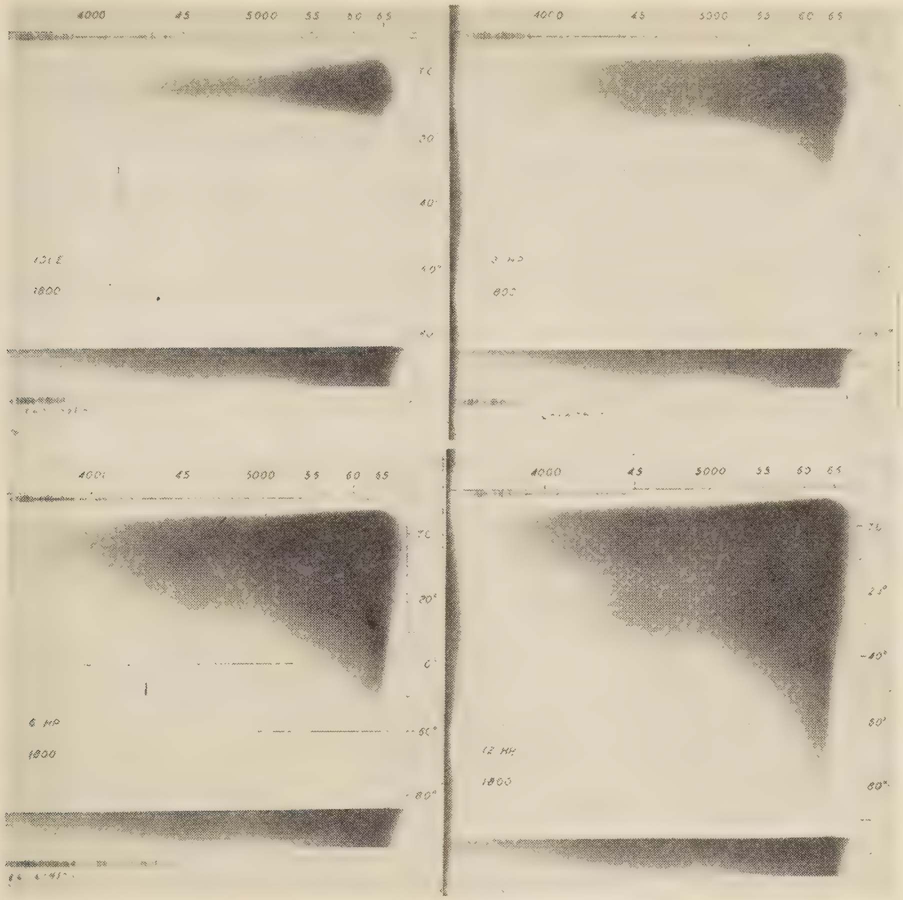

FIG. 6 SPECTROGRAMS OF DIRECT-INJECTION-ENGINE FLAME

ultraviolet region. This is interrupted by the water-vapor band spectrum from 3300 A to 3064 A. Spectrograms of the direct-injection-engine flame are shown in Fig. 6. Only the visible region is shown in these spectrograms.

An interesting point can be brought out here which demonstrates the sensitivity of the spectrographic method. One type of fuel oil used had a slight trace of copper, which was present after refining. The persistent spectral lines at 3248 A and 3274 A were observed in spectrograms, while standard chemical analysis failed to show the presence of copper.

The spectrum can be recorded on a photographic plate on which the entire radiation accumulates on a given exposure. A sector disk synchronized with the crankshaft can be used to expose any crank-angle interval desired. Mechanical considerations of driving a synchronized sector disk or drum camera, combined with the desired angular resolution, determine which method is desired. A prism-type ultraviolet-transmitting spectrograph with short-focal-length optics is usually not adaptable to a drum camera because of the curvature of the focal plane; hence a sector disk would be necessary.

Spectrograms of individual flames may be obtained by a simple small spectrograph, using a pinhole-aperture window in the engine head for the spectrograph slit and a moving film for recording the spectrum. The resolving power of such an instrument is determined by the size of the pinhole and the optics used. Lenses of small numerical aperture and a large prism must be used, which are consistent with a relatively flat focal plane. Distribution of the spectrogram in crank-angle degrees may be made by moving the film rapidly. Intensity of radiation is a problem in this method, but with the high-luminosity Diesel flame, spectrograms of individual flames are possible. These spectrograms, because of the short exposure and low intensity of the shorter wave lengths, will not extend far into the ultraviolet.

It is possible that spectra in the infrared region may give additional information on the combustion. Absorption spectroscopy, another fruitful field, can be accomplished with the drum-camera spectrograph and a suitable source of continuous radiation.

FLAME-TRACE PHOTOGRAPHY

The distribution over crank angle of the luminous flame in a Diesel engine can be measured photographically. A high-speed continuously moving film (Edgerton camera) is satisfactory for photographing successive flame traces. With suitable reduction pulleys, the film speed may be controlled from a small value up to as high as 80 fps. The desired observation determines the film speed.

A spark between pointed tungsten electrodes marks crank angles on the film. Two electrodes were attached to the flywheel, which is grounded. A third electrode is insulated from, but rigidly attached to, the engine. As the flywheel rotates, the grounded electrodes pass the insulated electrode, discharging a condenser. The condenser C, in Fig. 7, is kept charged by a variable-voltage supply capable of supplying voltages up to 6000 v. The light

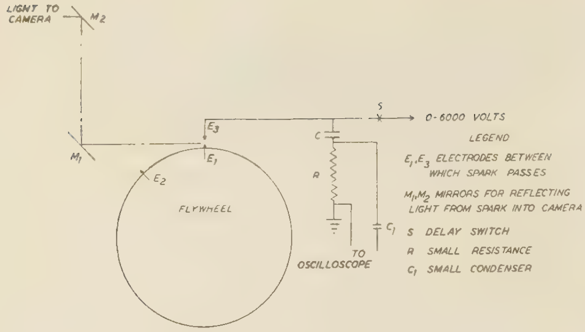

Fig. 7 Method of Producing Timing by Photographing Spark and Producing an Impulse on Oscilloscope for Marking Crank Angle on Pressure Record

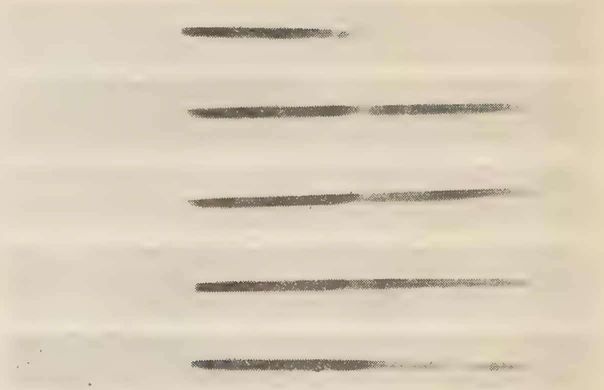

Fig. 8 Examples of Flame Traces

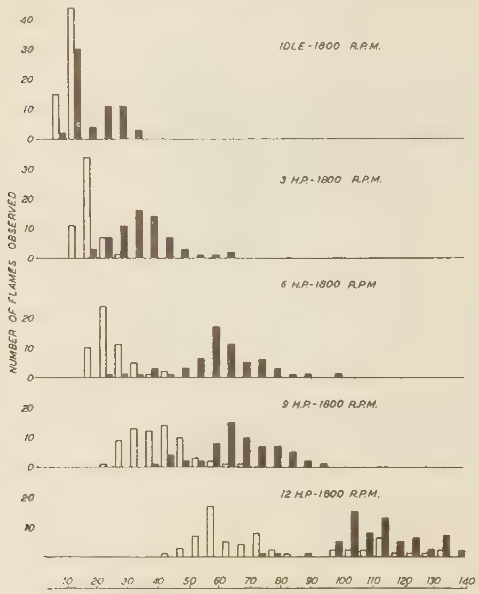

Fig. 9 Grouping of Flames in 5-Deg Crank-Angle Intervals
(Data obtained on direct-injection engine.)

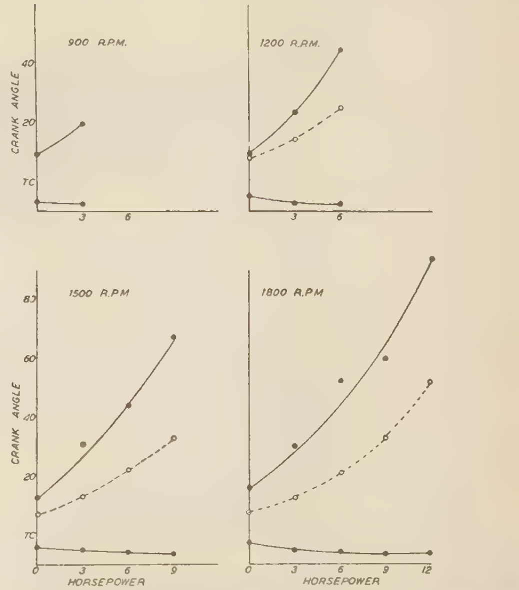

Fig. 10 Average Lengths of Intense Portions of Flame and Entire Flame

from the spark is reflected properly and focused with a lens so a sharp image is formed on the moving photographic film. This shows up as a small black dot on the film, marking the crank angle at that point. The power supply has a switch connected to the film driving motor of the high-speed camera which starts the series of sparks after the film is in motion. There is also a resistor supplying a small impulse for marking crank angles on a cathode-ray oscilloscope when this is desired.

A rectangular slit in front of the window is placed in the engine head so the image of the slit on the photographic film is perpendicular to the movement of the film. Changes in luminosity are then recorded on the film as variations of film densities. The size of the slit and the speed of the film may both be used to control the exposure on the film. The point image of the spark is adjusted by movement of the mirrors to line up with the image of the slit on the film at rest.

Data obtained by using the flame-trace photography may be correlated with spectrograms obtained by using the drum camera on the spectrograph. Flames from the direct-injection-type Diesel engine were photographed and it was found that the flame, in most cases, could be divided into two parts. An intense flame with high luminosity occurs early in the flame life, followed by a flame of low luminosity. Examples of the flame traces are shown in Fig. 8, the engine operating at one half rated load. The duration of the intense part of the flame and the entire flame are measured in terms of crank-angle degrees. Fig. 9 is a plot of flame duration versus the number of flames observed on a direct-injection engine. The clear bars represent the duration as measured by the initial high-density trace and the colored bars the entire length of the flame trace.

The average lengths of the intense portion of the flame and the entire flame are plotted in Fig. 10. The ordinates represent the crank angle and the abscissas the brake-horsepower output; each engine speed is plotted on one set of axes. The solid line represents the end of the entire flame and the broken line represents the end of the high luminous part of the flame.

The spectrum of the direct-injection engine, as photographed with the drum camera, is shown by a diagrammatic sketch in Fig. 11. Consider the spectrogram to be made up of parts A and B. Part A is of high luminosity showing the spectrum which extends into the ultraviolet. Part B shows only a short region of the visible spectrum. At low loads, part A is very pronounced and B is small. Intermediate loads represented in Fig. 11 show

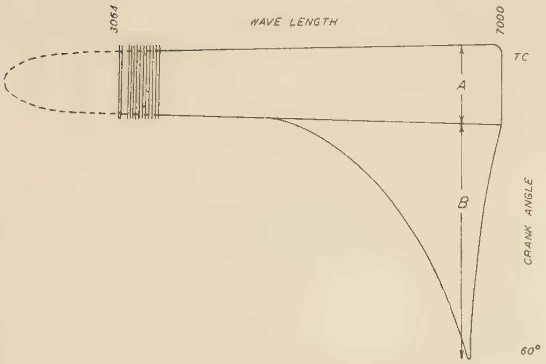

FIG. 11 SCHEMATIC DIAGRAM OF SPECTROGRAM

the parts A and B distinctly. At full-load operating conditions, parts A and B are not clearly defined in either the spectrograms or the flame traces.

Study of Pressure and Knock

The pressure within the combustion chambers is measured by amplifying the electrical impulses set up in a quartz crystal. These amplified impulses are shown on a cathode-ray-oscilloscope screen. The quartz-crystal pressure pickup is a standard type 307A R.C.A. unit. A 3-stage amplifier having a frequency response 1 to 17,000 is used to amplify the impulses from the quartz crystal. The amplified signal is fed directly to the vertical plates of the cathode-ray tube and the amplifier has a calibration scale which facilitates measurement of absolute pressures within the combustion chambers of the engine.

Any oscilloscope with a medium persistent green screen is suitable for visual observation. For photographing the pressure records, either a short persistent blue screen tube or an oscilloscope with an intensifying screen is found to be more satisfactory. The horizontal sweep in the oscilloscope circuit is usually suitable for a time axis. This plots the pressure as ordinates and the crank angle as abscissas directly on the oscilloscope screen. Crank angles are marked by some type of impulse timer connected to the crankshaft. An angular-sweep potentiometer is also useful when it is desired to use crank angle instead of a time axis. This apparatus makes it possible to change the angular rotation into a linear motion on the horizontal sweep of the oscilloscope screen. It has the advantage in that angles can be calibrated statically and the observations made dynamically. Such an apparatus is now available commercially.

Pictures of a large number of successive pressure diagrams are possible. A 35-mm movie camera geared properly to the crankshaft of the Diesel engine permits photographing the cathode-ray-tube screen as the pressure diagram is traced out. A series of successive pressure records show changes in pressure from one explosion to the next; also, the high-frequency wave, associated with engine knock, is superimposed on the pressure wave which may be observed in successive explosions.

Simultaneous Flame and Pressure Records

Striations observed in the flame traces show that the flame changes in luminosity at a high frequency which may indicate the presence of pressure waves in the system. In order to determine any correlation between this variation in luminosity of the flame and changes in the pressure record, it is necessary to record flame traces and pressure records simultaneously.

Flame traces can be photographed simultaneously with the corresponding pressure records, using the high-speed camera and the apparatus described in the preceding section. Each spark image photographed with the flame trace also shows up as a small impulse on the pressure diagram. The high-speed camera, by means of a relay system, starts the series of sparks shortly after the film has begun to move so it is possible to find the first flame trace with spark images adjacent. This flame trace corresponds to the first pressure record in the series showing the impulse markings. After the first flame trace and pressure record have been established, the others can be numbered consecutively.

Conclusion

Experiments designed to use physical principles and physical apparatus are significant in view of adopting those in order to get as much information as possible on the combustion process. Probably the most important development described herein is the window technique because any optical investigation requires the transmission of light to be reasonably constant throughout a given exposure. Of most immediate interest to the engineer are the records of flame duration and pressure obtained as described herein.

The Effect of Wood Structure Upon Heat Conductivity

By F. F. WANGAARD,[1] SEATTLE, WASH.

Shrinkage and swelling, mechanical properties, and working qualities of wood are all related to the orientation of structural units of the cell wall. The effect of fibrillar orientation upon the heat-conducting properties of wood indicates a new technique for the selection of various qualities of wood. The differential thermal conductivity of wood in the longitudinal and transverse directions is shown to be due chiefly to the anisotropic character of the fibrils, and deviations from anticipated transverse-conductivity values, as well as longitudinal-transverse heat-conductivity ratios, are closely related to fibrillar orientation. In this paper application of longitudinal-transverse conductivity ratios is proposed particularly for the detection of mild compression wood.

One of the properties of wood which contributes to its wide usefulness is its relatively high thermal resistance. All woods, however, are not equally resistant to the flow of heat and, while recognition of the general dependence of heat conductivity upon the density of wood may often be sufficient, heat-conductivity data of a precise nature are required for many purposes. Calculations of heat loss in various types of construction and studies of heat transfer in kiln drying and wood preservation illustrate the diversity of application of these data.

Previous investigations have confirmed the basic relationships which exist between heat conductivity, specific gravity, and moisture content, and a fairly accurate prediction of heat conduction transversely through the various woods which have been studied can now be made. However, the same studies have suggested that other factors undoubtedly operate to influence the heat conductivity of wood. The effects of different directions of grain and of macroscopic and microscopic structural features upon conductivity have been investigated previously in an attempt to account for the uncorrelated variation remaining after the relationship between transverse heat conductivity, specific gravity, and moisture content had been determined.

In the present work an attempt has been made, through an intensive study of the wood of Douglas fir, to account more fully for the variation about anticipated conductivity values. Since all of the physical and mechanical properties of wood exhibit a more or less pronounced variation about normal anticipated values, it is reasonable to assume that any basic characteristic of wood which is found to exert an influence divergent from normal upon heat conductivity may also play a similar role with regard to other properties of wood. The determination of such influences may possibly lead to the development of new means for the recognition or selection of abnormal material. Incidental to the primary objective of this study was the determination of heat-conductivity data for several species to serve as a check upon the heat-conductivity-specific-gravity and moisture-content relationship.

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REVIEW OF THE LITERATURE

Effect of Specific Gravity and Moisture Content

Recent determinations of heat conduction in wood have been based to a large extent upon various modifications of the hot-plate principle. Using the latter method Griffiths and Kaye (1),[2] Van Dusen (2), Rowley (3), the author (4), and MacLean (5) have obtained heat-conductivity constants for a large number of timbers, and the influences of specific gravity and moisture content upon the heat conductivity of wood have been indicated by these workers as well as by Kollmann (6). The author (4) has shown the relationship which exists between heat conductivity and the two variables, specific gravity and moisture content, by means of an alignment chart, based upon more than 500 determinations of transverse heat conductivity in the wood of 49 species, Fig. 1. Recently, MacLean (5) derived an equation intended to permit the calculation of thermal conductivity on the basis of the same variables.

Effect of Direction of Grain

Investigations by many early workers revealed a rate of conduction of heat parallel with the grain of wood several times greater than that in the transverse direction. More recently Griffiths and Kaye (1) obtained absolute results which showed longitudinal-conductivity values approximately twice as great as those obtained from measurements of heat flow directed at right angles to the fiber axis. Kollmann (6) has indicated that for most common woods longitudinal conductivity is somewhat less than 3 times as great as tranverse conductivity.

One of the earlier experiments bears particularly upon the present problem. Following the method introduced by de Senarmont for the study of relative conductivities along the structural axes of crystals, Knoblauch (7) demonstrated that the ratio of longitudinal to transverse conductivity for several woods varied between extreme limits of 1.5 to 1 and 3.2 to 1.

Finck (8) tested a number of fibrous insulating materials using a small hot-plate apparatus. Although wood was not included among the test materials, his results are of interest because of similarity in fiber structure. He observed that conductivity parallel with the fibers, when these showed orientation in a specimen, was greater than that in the direction perpendicular to the fibers in the ratio of about 3 to 1. This, Finck ascribed to a lower internal-surface resistance between fiber and air when the fibers were arranged parallel with the direction of heat flow. He concluded that this arrangement permitted the high conductivities of the fibers to exert a maximum influence on the total conductivity of the material.

Kollmann (6) and Forsaith (9) have attributed the greater conductivity along the grain in wood as compared with that across the grain to the minute structure of the cell wall. The cell walls of wood, in common with those of other higher plants, seemingly consist essentially of long chains of glucose-anhydride units which compose cellulose. Each of the polysaccharide chains is oriented in a direction parallel or nearly parallel with the longitudinal axis of the cell, and is bound by internal forces.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Parallel chains, however, are held together by forces of a different type. Kollmann assumed that the forces acting within the polysaccharide chains are greater than the forces which bind parallel chains together. From the fundamental viewpoint, that the conductivity of heat is a manifestation of an equalization of molecular energy and that a strong binding force offers less resistance to such energy transmission than one which is weaker, he has explained the greater conductivity of heat in the longitudinal direction.

Several investigators have discussed the influence of direction of grain, i.e., radial and tangential, upon the transverse conductivity of heat in wood. Griffiths and Kaye (1) found that radial conductivity usually exceeded that in the tangential direction by some 5 to 10 per cent. Rowley (3) concluded that, in species with strongly marked springwood and summerwood, tangential conductivity was somewhat greater than radial conductivity, but that no appreciable difference existed between conductivities in these two directions in woods which possess uniformity of structure throughout the annual ring.

Statistical analyses of results obtained by the writer (4) indicated that radial conductivity was significantly greater than tangential conductivity in the hardwoods generally, whereas in the softwoods no superiority in the radial direction was detectable. These results, confirming the conclusion of Griffiths and Kaye, in so far as the hardwoods are concerned, may be reasonably explained on the basis of the influence exercised by the wood rays. These rays, the cells of which are oriented in the radial direction, comprise an average volume approximately twice as great in hardwoods as compared with softwoods; hence, their influence upon the conductivity of heat would be displayed more prominently in the former group of woods. MacLean (5) has lately found no significant difference between radial and tangential conductivity in the wood of Douglas fir.

Effect of Gross Wood Structure

In an attempt to account for departures from normal conductivities, the influence of gross wood structure upon the transverse conductivity of heat has been explored (4). If variables in structure such as type, size, and disposition of the longitudinal cells have an effect upon this physical property, an appreciable difference might be anticipated between the conductivities at like densities and moisture contents of woods offering such a contrast in structure as do the hardwoods and softwoods. Analysis of the data from 194 tests on hardwoods and 355 softwood tests separately obtained by Rowley and the author failed to show a significant difference between the conductivity values of two such diverse groups of woods.

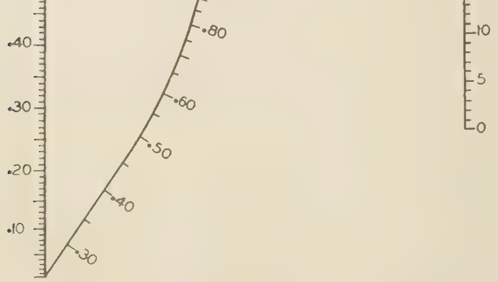

Fig. 1 Alignment-Chart Representation of Relationship Between Transverse Heat Conductivity, Specific Gravity, and Moisture Content of Wood

(Specific gravity based on oven-dry weight of wood and volume at the prevailing moisture content. Moisture-content percentage, based on weight of moisture and oven-dry weight of wood.)

Nearly all investigators have presented in their results a constant value for conductivity, or conductivity-density relationships, for each kind of wood studied. Each kind of wood is characterized by a distinct combination of structural characteristics, although the influence of factors peculiar to any one species as such is relatively slight. This conclusion was reached by Rowley (3), and the density-conductivity relationship expressed by Kollmann (6), together with the high correlation index of the relationship, shown in Fig. 1, confirms the lesser role played by the structural factors characteristic of individual species. The author (4) has presented average test results for individual species in the form of deviations from the normal conductivity-specific-gravity and moisture-content relationship. These results indicate that individual species with few exceptions vary in conductivity within comparatively narrow limits about estimated values. Table 1 shows the extent of average species deviations from the relationship presented in Fig. 1, together with the number of tests upon which each value is based.

Although in discussing the conductivity-specific-gravity relationship, Van Dusen (2) states, "The deviation . . . from a smooth curve is doubtless due to a great extent to the varying amount of resinous substances present," the author (4) was unable to confirm this statement. No evidence of variation in heat conductivity because of chemical composition of wood was detected, although the species which were examined showed considerable diversity in composition. The increased density of extremely resinous specimens of the southern pines, for example, appeared to counteract the additional conducting ability of the resin itself. MacLean (5) on the other hand has argued that the higher extractive content of heartwood of certain species, including the oaks and sugar maple, may account for their abnormal conductivity. A wide variation in conductivity shown in MacLean's tests of southern cypress was also attributed to a probable variation in extractive content, although in tests conducted on the same species the author found variation in cell-wall structure to be the principal cause of deviation from anticipated conductivity.

Effect of Cell-Wall Structure

The cell walls of plant fibers have long been known to consist of an aggregation of fibrils more or less oriented in the direction of the long axis of the fiber. Ritter (10) has stated that the fibrils in the outer layer of the secondary cell wall in wood fibers are oriented nearly at right angles to the fiber axis while those in the remaining layers of the secondary wall are spirally arranged and vary in orientation from 0 to 30 deg with the same axis. Pillow and Luxford (11) have pointed out that the angle at which the orientation of the fibrils diverges from the longitudinal axis of the cells in coniferous woods is characteristic of certain types of wood and may serve as a criterion of their strength properties in comparing more than one specimen of the same species. These authors refer to the angle at which the fibrils are inclined with reference to the longest axis of the fibers as the slope of the fibrils.

Bailey (12) and Bailey and Berkley (13) have stated that the long axes of pit orifices may often serve as an indication of fibrillar orientation in the central layer of the secondary cell wall, but caution against the use of this indicator in the deter-

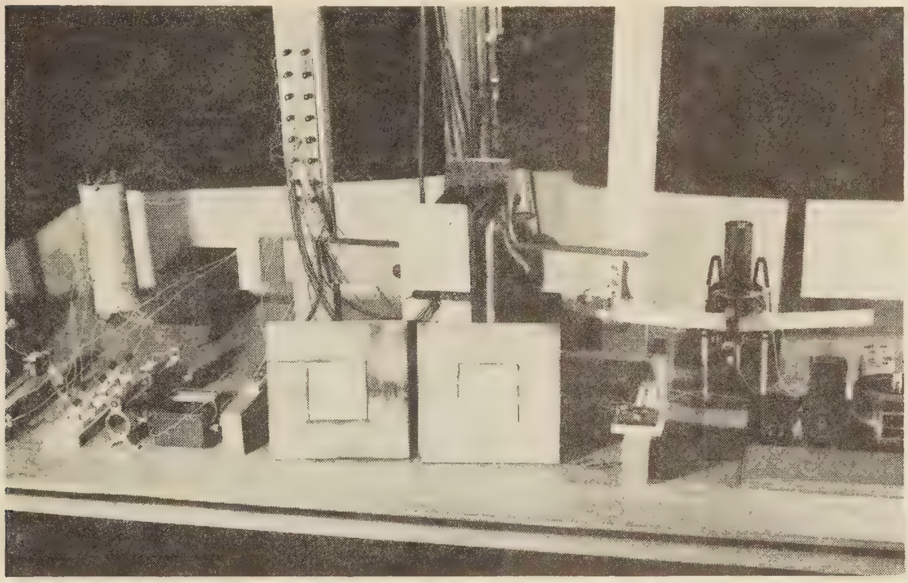

FIG. 2 A PAIR OF TRANSVERSE-CONDUCTIVITY TEST SPECIMENS MOUNTED IN GUARD RINGS READY FOR TESTING

mination of fibrillar orientations of the thin inner and outer layers of the secondary wall.

Several relationships between fibrillar orientation and wood properties have been indicated in the literature on the subject. Robinson (14) and Koehler (15) have presented evidence that tension fractures are related to fibrillar orientation. Maby (16) found that a low fibrillar slope corresponds with low longitudinal shrinkage of wood upon drying. On the basis of a small number of tests on normal wood and compression wood of loblolly pine and Douglas fir, Pillow and Luxford (11) determined that strength-density ratios are clearly dependent upon the slope of the fibrils. Garland (17) has shown significant correlations between fibrillar angle and strength-density and stiffness-density indexes in a large number of tests of commercial shortleaf pine timber, and Schrader (18) has shown a similar relationship for Douglas fir. Indirectly measured effects of fibrillar angle upon resonance of wood have been reported by Knoblauch (7) and Lark-Horovitz (19).

The greater portion of the deviations from the heat-conductivity-specific-gravity and moisture-content relationship for wood has been found (20) to be induced by the orientation of the fibrils. Transverse heat conductivity in coniferous woods appeared to be subnormal when the fibrils were oriented at an angle less than 13 to 14 deg, and greater than normal when the orientation of the fibrils diverged more than this amount from the long axis of the fibers.

EXPERIMENTAL WORK

Transverse Conductivity of Heat

In all essential features the apparatus used for the determination of transverse heat conductivity corresponded to that recommended as a standard for the testing of insulating materials by the United States Bureau of Standards (2). As in previous tests (4), the standard hot plate was modified to permit the use of test specimens 5-in-square, rather than the conventional 12-in-square size. Fig. 2 indicates the appearance of the testing equipment and shows the placement of duplicate test specimens in hardwood guard rings.

Specimens were prepared from air-seasoned boards of the eight species marked with an asterisk in Table 1. While the test material of most of these species was limited to that obtainable from one or two boards, the Douglas-fir specimens were selected especially to provide a wide range of structure from the standpoint of density and rate of growth. Direction of grain and number of annual rings per inch were noted for each specimen. Moisture content was calculated as percentage of moisture on the basis of the oven-dry weight of the specimen. The specific gravity of each test specimen was calculated on the basis of oven-dry weight and volume at the time the conductivity determination was made. Conductivity k was expressed in Btu per sq ft per deg F per in. per hr.

Longitudinal Conductivity of Heat

Reference has been made to the previous application to wood of the method of de Senarmont. The simplicity of this experimental procedure and the facility with which results are obtained offer a marked contrast to the time-consuming hot-plate method used in the first phase of this study. If a source of heat is applied at one point on the plane surface of isotropic material, heat will be conducted equally in all directions from that point and a series of concentric circular isotherms will be established. When heat is applied similarly to a surface in the plane of two structural axes of anisotropic material, the flow of heat will usually not be equal in the direction of the different axes, and the isotherms will consequently be elliptical in shape. The ratio of the conductivities in the directions of two such axes is proportional to the square of the ratio of the corresponding axes of the isothermal ellipse.

One specimen of each test-pair, which had been employed previously for the determination of transverse heat conductivity, provided the test material for this study of conductivity ratios. At the time of test, the moisture content of the wood ranged from 6 to 8 per cent. An edge of the specimen was covered with a uniform thin film of paraffin and this surface was then dusted with talcum powder. A pointed brass rod, $^1/_8$ in. diam, was heated in a Bunsen-burner flame and firmly pressed to the paraffin-covered surface.

The major axis of the elliptical figure formed by the melted paraffin in each instance corresponded to the longitudinal axis

of the wood, and the minor axis to a transverse direction of the grain. Measurement of the axes of the elliptical figure to an accuracy of 0.01 in. was accomplished through the use of a finely graduated steel rule and a strong reading lens. The ratio of longitudinal to transverse conductivity obtained for each specimen represents an average of two separate determinations.

Examination of Cell-Wall Structure

The study of cell-wall structure with respect to heat conductivity was limited to the wood of a single species. Douglas fir was chosen because of its outstanding commercial importance, as well as for the fact that its relatively simple structure permits of a reasonably effective sampling procedure in collecting measurements of the type to be described in the following paragraphs.

Reference has been made previously to the fact that fibrillar orientation exerts a pronounced influence upon certain mechanical and physical properties of wood including the conductivity of heat. The object of this phase of the study was to obtain a quantitative measure of this structural feature in specimens the heat-conducting properties of which were known. From small blocks which were cut from the moisture-content samples, sections for microscopic examination were prepared in the usual way by the use of a sliding microtome. Temporary water mounts of these sections were examined under the microscope. Examination of these sections to determine fibril slope or angular declination from the longitudinal axis of the tracheids, in the major portion (middle layer) of the cell wall was based upon measurement of the angle of orientation shown by pit apertures in radial sections. In Douglas fir, at least, the ray crossings are most opportune for such a purpose, since in this portion of the tracheid wall the pit aperture is more or less elongated even in the springwood portion of the annual growth, whereas the apertures of the bordered pit-pairs between contiguous tracheids in the springwood are commonly circular and therefore of no significance in the determination of fibrillar orientation. In the summerwood where the bordered pit-pairs display a slitlike aperture, repeated measurements completely substantiated the angular values obtained from measurement of ray-crossing pits in that portion of the annual ring.

Fig. 4 Late Springwood and Early Summerwood Section Indicating Average Fibril Angle of 34 Deg

(Photomicrographs showing ray crossings of Douglas fir and variation in fibrillar orientation as indicated by pit apertures; ×650.)

Fibril angle was measured by means of an eyepiece micrometer with a movable crosshair which was aligned in a direction parallel with the long axes of the pit apertures. A pointer fixed to the eyepiece indicated the angular rotation of the eyepiece from the longitudinal direction on a graduated circular scale which was mounted on the upper end of the microscope tube. The variable orientation of fibrils in the springwood and summerwood portions of the annual ring is shown in the photomicrographs, Figs. 3 and 4. This variation necessitated a weighting of the mean values obtained separately for springwood and summerwood on the basis of the relative amounts of wood substance contained in those portions of the wood. The relative number of cells and average cell-wall thickness in each portion of at least two annual rings served as the basis of weighting.

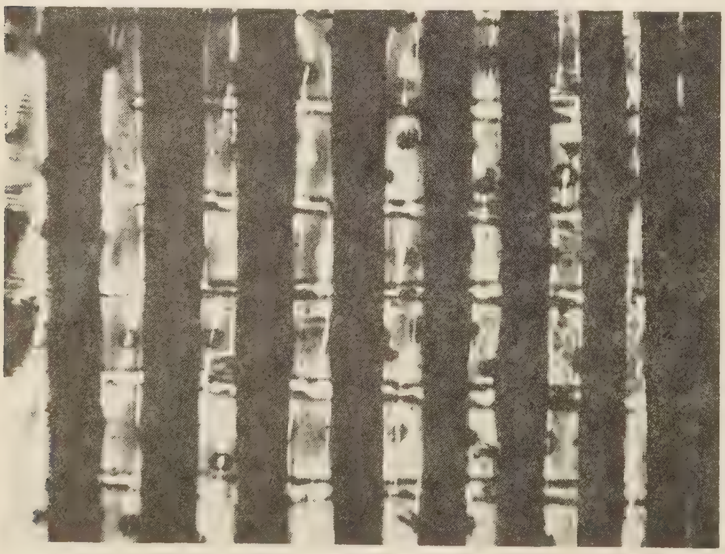

Fig. 3 Summerwood Section Indicating Average Fibril Angle of 14 Deg

EXPERIMENTAL RESULTS

Transverse-Conductivity Data

Table 2 shows the results of 39 heat-conductivity determinations which were made on Douglas fir at a mean temperature of 80 F. These results were first compared on the basis of the relationship previously found to exist between transverse heat conductivity, specific gravity, and moisture content of wood shown in the alignment chart, Fig. 1. Column 7 in Table 2 entitled "anticipated conductivity" represents the conductivity value which would be anticipated from a consideration of specific gravity and moisture content of the test specimens. In column 8 the deviation of the actual test value from anticipated conductivity is expressed as a percentage of anticipated conductivity value. For final results of similar treatment of data in 22 tests of seven additional species, see Table 1.

TABLE 1 VARIATION BETWEEN SPECIES IN TRANSVERSE HEAT CONDUCTIVITY[a]

Species	Average percentage deviation from alignment-chart values	Number of tests
Red oak *Quercus* sp. L.	5.2	27
Southern red oak *Quercus rubra* L.	— 0.9	12
White oak *Quercus* sp. L.	3.2	22
Chestnut *Castanea dentata* (Marsh.) Borkh.	— 3.2	7
Paper birch *Betula papyrifera* Marsh.	— 5.2	5
Western paper birch* *Betula papyrifera occidentalis* Sarg.	— 5.0	2
Yellow birch *Betula* sp. L.	— 6.5	36
American elm *Ulmus americana* L.	5.5	7
Hackberry *Celtis occidentalis* L.	0.5	3
Bitternut hickory *Hicoria cordiformis* (Wangh.) Brit.	— 0.2	1
Butternut *Juglans cinerea* L.	— 6.4	2
Cucumber magnolia *Magnolia acuminata* L.	2.9	2
Tulip poplar *Liriodendron tulipifera* L.	— 4.8	8
Hop hornbeam *Ostrya virginiana* (Mill.) K. Koch	—16.0	1
Blue beech *Carpinus caroliniana* Walt.	5.5	3
Basswood *Tilia glabra* Vent.	7.2	3
Black willow *Salix nigra* Marsh.	3.9	1
Northern black cottonwood* *Populus trichocarpa hastata* Henry	- 5.3	3
Sassafras *Sassafras variifolium* (Salisb.) Ktze.	16.1	1
Oregon myrtle *Umbellularia californica* (Hook. and Arn.) Nutt.	7.6	1
Silver maple *Acer saccharinum* L.	4.4	2
Striped maple *Acer pennsylvanicum* L.	— 0.8	3
Bigleaf maple *Acer macrophyllum* Pursh.	10.8	5
Bigleaf maple* *Acer macrophyllum* Pursh.	3.6	4
Hard maple *Acer* sp. L.	2.7	8
Soft maple *Acer* sp. L.	— 1.8	6
Black gum *Nyssa sylvatica* Marsh.	6.5	3
Pacific dogwood* *Cornus nuttallii* Aud.	— 9.6	2
Red ash *Fraxinus pennsylvanica* Marsh.	— 0.6	4
White ash *Fraxinus* sp. L.	0.7	9
Balsa *Ochroma lagopus* Swartz.	0.9	6
Loblolly pine *Pinus taeda* L.	3.2	7
Shortleaf pine *Pinus echinata* Mill.	6.2	43
Longleaf pine *Pinus palustris* Mill.	— 0.4	17
Western white pine *Pinus monticola* Dougl.	— 0.6	4
Northern white pine *Pinus strobus* L.	1.2	27
Limber pine *Pinus flexilis* James.	— 1.4	5
Sugar pine *Pinus lambertiana* Dougl.	— 4.3	6
Norway pine *Pinus resinosa* Ait.	2.7	16
Lodgepole pine *Pinus contorta latifolia* Engelm.	— 0.7	5
Ponderosa pine *Pinus ponderosa* Dougl.	4.0	41
Western larch *Larix occidentalis* Nutt.	— 2.0	7
Lowland white fir *Abies grandis* Lindl.	— 6.7	13
White fir *Abies* sp. L.	— 8.0	9
Silver fir* *Abies amabilis* (Dougl.) Forbes.	—18.2	3
Engelmann spruce *Picea engelmanni* (Parry) Engelm.	—14.4	3
Sitka spruce* *Picea sitchensis* (Bong.) Carr.	—17.6	2
Sitka spruce *Picea sitchensis* (Bong.) Carr.	— 8.1	8
Western hemlock *Tsuga heterophylla* (Raefn.) Sarg.	— 9.0	28
Eastern hemlock *Tsuga canadensis* (L.) Carr.	3.0	8
Douglas fir *Pseudotsuga taxifolia* (Lamb.) Brit.	— 9.3	19
Douglas fir* *Pseudotsuga taxifolia* (Lamb.) Brit.	— 7.7	39
Southern cypress *Taxodium distichum* (L.) Rich.	1.4	57
Redwood *Sequoia sempervirens* (Lamb.) Endl.	6.4	9
Western red cedar *Thuja plicata* D. Don.	— 1.8	18
Southern white cedar *Chamaecyparis thyoides* (L.) B.S.P.	1.8	1
Alaska yellow cedar* *Chamaecyparis nootkatensis* (Lamb.) Spach.	5.3	6

[a] This table (with the exception of those items marked with an asterisk which result directly from the present study) is reproduced with permission of the American Society of Heating and Ventilating Engineers. It is based upon tests conducted independently by F. B. Rowley and the author.

TABLE 2 RESULTS OF TRANSVERSE-HEAT-CONDUCTIVITY DETERMINATIONS ON DOUGLAS FIR

Specimen number	Specific gravity	Moisture content, per cent	Number of rings per inch	Direction of grain	Transverse conductivity, k	Anticipated conductivity, k	Deviation, per cent
10AA′	0.59	8.1	4	Oblique	0.931	1.007	— 7.5
10BB′	0.61	6.4	4	Oblique	0.911	1.007	— 9.5
11AA′	0.59	8.1	9	Oblique	0.967	1.007	— 4.0
12AA′	0.49	9.4	5	Oblique	0.728	0.865	—15.8
13AA′	0.48	9.6	5	Oblique	0.795	0.848	— 6.2
14AA′	0.64	8.4	18	Radial	0.805	1.083	—25.7
15AA′	0.47	7.5	6	Tangential	0.717	0.798	—10.2
16AA′	0.45	7.2	16	Oblique	0.688	0.781	—11.9
16BB′	0.47	8.1	17	Oblique	0.723	0.819	—11.7
17AA′	0.46	7.7	20	Radial	0.700	0.799	—12.4
17BB′	0.47	6.9	17	Radial	0.705	0.804	—12.3
18AA′	0.56	9.3	5	Tangential	0.912	0.973	— 6.3
18BB′	0.55	9.6	5	Tangential	0.885	0.961	— 7.9
19AA′	0.57	9.1	12	Tangential	0.836	0.987	—15.3
19BB′	0.57	8.9	12	Tangential	0.842	0.981	—14.2
20AA′	0.55	9.5	4	Tangential	0.929	0.961	— 3.3
20BB′	0.56	9.5	4	Tangential	0.943	0.978	— 3.6
21AA′	0.57	9.6	4	Tangential	0.949	0.990	— 4.1
21BB′	0.58	9.3	4	Tangential	0.964	1.010	— 4.6
22AA′	0.52	9.0	14	Radial	0.761	0.903	—15.7
23AA′	0.49	7.8	26	Oblique	0.924	0.842	+ 9.7
23BB′	0.49	8.4	25	Oblique	0.929	0.848	+ 9.6
24AA′	0.38	6.7	30	Radial	0.662	0.679	— 2.5
24BB′	0.37	6.5	31	Radial	0.671	0.671	0.0
25AA′	0.43	6.4	13	Radial	0.734	0.740	— 0.8
25BB′	0.43	6.5	17	Radial	0.732	0.744	— 1.6
26AA′	0.46	6.3	33	Radial	0.700	0.784	—10.7
26BB′	0.43	6.1	34	Radial	0.656	0.740	—11.4
27AA′	0.53	6.7	14	Oblique	0.790	0.886	—10.8
27BB′	0.52	6.8	12	Oblique	0.793	0.874	— 9.2
28AA′	0.59	6.4	17	Radial	0.793	0.981	—19.2
29AA′	0.53	6.3	10	Radial	0.767	0.883	—13.2
30AA′	0.64	6.8	18	Radial	1.002	1.060	— 5.5
30BB′	0.68	6.4	23	Radial	1.045	1.109	— 5.8
31AA′	0.51	6.3	24	Radial	0.787	0.851	— 7.5
31BB′	0.51	5.7	23	Radial	0.769	0.836	— 8.0
32AA′	0.61	6.0	9	Radial	1.025	1.005	+ 2.0
32BB′	0.63	6.2	12	Radial	1.048	1.037	+ 1.1
33AA′	0.50	5.8	22	Oblique	0.723	0.833	—13.2
Species average							— 7.7

Effect Associated With Species

The average deviation from alignment-chart values for each of the species involved in these tests appears in Table 1. Although their reliability may be subject to some doubt because of the small number of specimens tested (with the exception of Douglas fir), the magnitude of these average deviations for individual species compares closely with the results of previous work. Average deviations here are shown to vary from —18.2 per cent for silver fir to +5.3 per cent for Alaska yellow cedar, as compared with extreme limits for average deviations in previous work (4) of —14.4 per cent and +10.8 per cent for two different species when more than a single determination was involved.

Sitka spruce, silver fir, and Douglas fir represent a closely related group of species to which the following statement was previously applied by the writer (4): "In general, coniferous wood of moderate growth from such species as Engelmann spruce, Sitka spruce, western hemlock, lowland white fir, white

fir, and Douglas fir averages from five to fifteen per cent lower in transverse conductivity of heat than would be anticipated from a consideration of density and moisture content alone." In the case of Douglas fir, a representative group of specimens of which was tested, the average deviation for the species is shown to be —7.7 per cent based on 39 tests, whereas 19 tests conducted previously indicated an average deviation for this species of —9.3 per cent. The near equality of these values serves to confirm the statement quoted earlier in this paragraph.

The transverse-conductivity data shown in Table 2 indicate that deviations within an individual species are fully as pronounced as those which for want of further evidence may tentatively be said to be characteristic of individual species. Douglas-fir specimen No. 14 AA′, for example, showed a negative deviation of 25.7 per cent from the alignment-chart value, while specimen No. 23 AA′ of the same species deviated by a positive 9.7 per cent from the predicted value for conductivity. An attempt to correlate these deviations for all of the Douglas-fir specimens, first, with specific gravity, and second, with rate of growth (number of annual rings per inch), met with no success.

For practical purposes, the alignment chart, Fig. 1, appears to offer a reasonably satisfactory method of arriving at an approximate value for transverse heat conductivity of wood of any species on the basis of these additional tests. Somewhat greater accuracy may be obtained by adjusting the alignment-chart value by the average percentage deviation for the individual species in the case of those woods for which this value is available (Table 1). For greater refinement in predicting conductivities, further information is necessary.

Effect of Fibrillar Orientation

Although fibrillar declination has been shown to influence the transverse heat conductivity of wood (20) the evidence available heretofore was insufficient completely to define this relationship. The present study was confined to the Douglas-fir specimens and consisted in the correlation of deviations in transverse conductivity with mean fibrillar slope. Inasmuch as the mean deviation for Douglas fir was determined as —7.7 per cent, this value was designated as normal for the species and the scale of deviations was adjusted accordingly. Fig. 5 shows the straight-line relationship between adjusted-deviation percentage and sine of fibril angle in graph form.

In consideration of these results there naturally arises the question: Is this observed relationship actually one of cause and effect or is fibrillar orientation merely a visible evidence of some other characteristic of the cell wall? May not changes in chemical make-up of the cell wall actually be the cause of the greater transverse heat conductivity at greater fibrillar declinations from the longitudinal axis? The next phase of the study was undertaken in part to answer these questions.

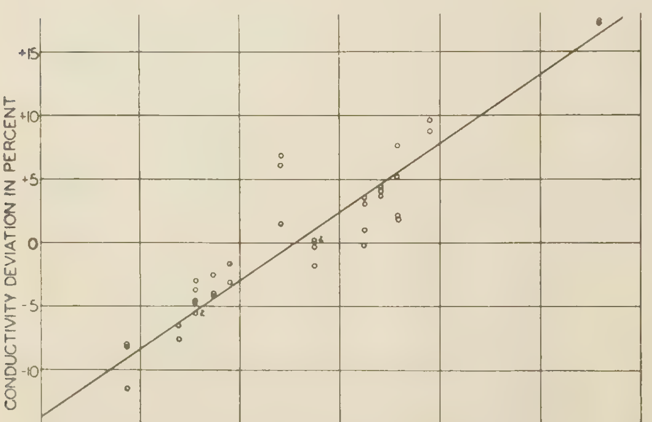

Fig. 5 Effect of Fibril Angle Upon Deviation From Anticipated Transverse-Conductivity Value for Douglas Fir

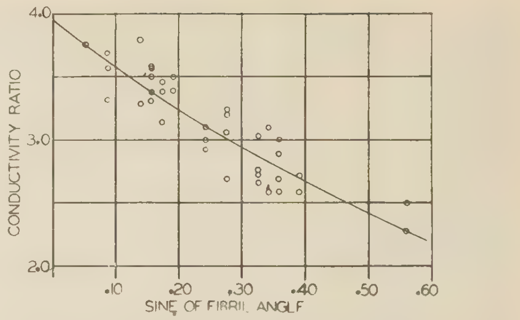

Fig. 6 Effect of Fibril Angle Upon Longitudinal-Transverse Conductivity Ratio for Air-Dry Douglas Fir

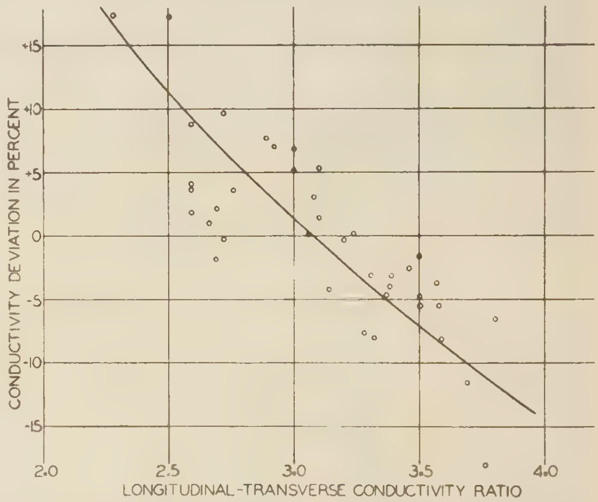

Fig. 7 Relationship Between Deviation From Normal Transverse-Conductivity Value and Longitudinal-Transverse Conductivity Ratio for Air-Dry Douglas Fir

Longitudinal-Transverse Conductivity Ratios

The data resulting from the series of longitudinal-conductivity tests on Douglas fir are presented as longitudinal-transverse conductivity ratios in Fig. 6 which shows the relationship existing between conductivity ratio and sine of fibrillar declination for this species. The curve was fitted to the data by the method of least squares and its correlation index 0.902 indicates a high degree of correlation between these two variables. Over the range of mean-fibrillar-angle values observed in the tests (3 to 34 deg), the ratio of longitudinal to transverse conductivity may be seen to vary in a regular manner from 3.80 to 2.28. This range of conductivity-ratio values is nearly as wide for Douglas fir alone as that found in the entire data covering eight different species.

The correlations shown between deviation from normal transverse conductivity and fibril angle in Douglas fir, Fig. 5, and between conductivity ratio and fibril angle,

Fig. 6, in the same species provide the necessary information for a graphic representation of the relationship existing between conductivity ratio and departure from normal values for transverse conductivity in this species. The curve, Fig. 7, indicates clearly the manner in which conductivity ratios may be employed as an aid in estimating transverse-conductivity values of specimens of Douglas-fir wood.

Conclusions

The applicability to additional species of the previously determined relationship, Fig. 1, between transverse heat conductivity and specific gravity and moisture content for wood in general is confirmed by the results of this study covering a fairly wide range of specific gravity (0.36 to 0.66) in eight different species. Specific gravity and moisture content alone, however, are not sufficient to account completely for observed results. In the present study certain influences induced by characteristics inherent within species are again indicated although their nature is not completely known. Because of the small number of tests on which most of the species averages are based, these could easily be discounted as failing to be truly representative. The larger basis for the —7.7 per cent average departure of Douglas fir, however, cannot be so regarded. Although for practical purposes the tentative acceptance of average deviations for individual species from alignment-chart values is proposed (Table 1), experience suggests that similarly to Douglas fir and other species which have been thoroughly studied in the past further investigation will reveal a wide range of deviations within each species. In Douglas fir, at least, these variations are independent of specific gravity and rate of growth as shown by these data.

Subject to the same variation within species is the ratio of longitudinal and transverse conductivites. Although not appearing in this paper, data collected in this study show that for all species tested there is a general relationship between longitudinal-transverse conductivity ratio and deviation from the alignment-chart value for transverse conductivity. Minimum values of the conductivity ratio are associated with the greatest positive deviations from the alignment chart. This relationship is considerably improved when data of the hardwoods are excluded or, as shown in Fig. 7, when limited to the results of tests on Douglas fir alone.

Finck's (8) explanation for fibrous materials of a greater conductivity parallel with the direction of the fibers, when these were oriented in a specimen, fails to account for these observed facts as the arrangement of the cell walls and air spaces remains constant. The average ratio of longitudinal to transverse conductivity for Douglas fir, for example, is nevertheless very close to the ratio (3 to 1) that he observed. Whatever may be the other influences which produce a difference between longitudinal and transverse conductivities in fibrous materials as determined by Finck, it seems highly probable that the anisotropic character of the fiber walls themselves is at least partly responsible for the observed differences.

That the variation in longitudinal-transverse conductivity ratios is closely associated with the orientation of fibrils in the cell wall of Douglas fir is evidenced by the curve in Fig. 6. The ratio of longitudinal to transverse heat conductivity decreases with an increase in fibril slope. This evidence is in agreement with that obtained from transverse-conductivity determinations, Fig. 5, and shows that the anisotropic character of the fibrils, and of the crystallites of which they are composed, predominantly influences the differential conductivity of wood in the longitudinal and transverse directions. By measurement of the variation in fibril orientation, and the consequent effect of such variation upon the conductivity of heat, the theories that have been advanced by Kollmann (6) and Forsaith (9) to explain the greater longitudinal conductivity are substantiated.

There appears to be a widely held belief that the peculiarities of compression wood may be caused by its abnormal chemical composition. This hypothesis is based upon analyses which show that typical compression wood includes a greater than normal lignin content, whereas the percentage of cellulose is at the same time reduced. In so far as heat conductivity is concerned, an increased percentage of nearly isotropic lignin could not account both for the increased transverse conductivity and the decreased longitudinal conductivity observed in specimens of high fibrillar-angle characteristic of compression wood. Although this fact might be explained on the basis of a reduction in the percentage of longitudinally directed anisotropic cellulose, such an assumption is not valid as the varying orientation of the cellulose fibrils themselves has been proved and is sufficient to account for the observed conductivity effects.

Transverse-conductivity deviations in Douglas fir are so greatly dependent upon variations in fibril slope, Fig. 5, that there is a strong possibility that no other factors operate to influence this property. The data indicate that an average, or normal, fibril declination in Douglas fir is between 14 and 15 deg, as measured from the longitudinal axis. Specimens characterized by a lower fibrillar slope exhibit transverse-conductivity deviations as much as 15 per cent below that of normal wood, and those with a greater fibrillar slope, in contrast, show positive conductivity deviations exceeding those anticipated for normal wood by an equal amount. Even greater positive departures may be anticipated from specimens of typical compression wood. A previous study (20) indicated that similar relationships exist with respect to the wood of other coniferous species.

The influence of fibril angle upon transverse conductivity of heat may be the chief factor responsible for average deviations by individual species recorded in Table 1. That certain species may be generally characterized by a low or high average fibril declination does not conflict with the recognized fact that in any species there will still remain a distribution of angular values about that average. Information available at present is not sufficient to prove conclusively whether other species characteristics are involved.

The close relationship which is shown between conductivity ratio and fibrillar angle for Douglas fir, Fig. 6, permits the substitution of this convenient test for the tedious microscopic measurement of fibril declination in future investigations into other properties of wood dependent upon this particular structural feature.

Compression wood has long been recognized as a serious type of defective wood occurring irregularly in the timber of coniferous species. Pronounced compression wood is weak in proportion to its weight; it shrinks longitudinally and is difficult to work with tools. Extreme types of compression wood are readily recognized at sight but the less pronounced forms are troublesome in manufacturing and wood-using practice. Microscopically, compression wood can be identified by the high fibrillar slope which it exhibits as compared with the low declination of the fibrils in normal wood. Measurement of fibrillar declination and of the heat-conducting properties which have been shown in this study to be closely correlated indicates that variation in fibrillar angle is normal in wood. Specifically in Douglas fir, fibrillar slope and these properties dependent upon fibrillar slope have been found to vary continuously throughout a wide range of values. This aids in confirming evidence from other studies to the effect that compression wood is not a distinctly different type of wood, but merely an example of an extreme form of normal variation.

Microscopic examination of wood specimens for the detection

of compression wood is tedious and in most instances impractical. The close correlation of conductivity ratios with fibril declination makes possible the use of this relationship as an indirect method of detecting compression wood. The continuous nature of the relationships which have been found between fibril angle and various properties of wood indicates the cause of much of the variation in the mechanical properties of so-called "normal" wood and suggests the application of conductivity ratios as a new technique in the selection of various qualities of wood for specialized uses.

The determination of the foregoing relationships has been confined to the thoroughly air-dry wood of Douglas fir. The extension of these principles to this wood at different moisture contents and to additional species, although undoubtedly feasible, requires further investigation.

Summary

Although specific gravity and moisture content of wood are the chief factors affecting transverse conductivity of heat, average departures from this relationship have been determined for a representative group of species and their tentative acceptance for practical purposes is proposed. The orientation of fibrils in the cell walls of wood is shown to account for the greater part of the variation about conductivity values anticipated on the basis of specific gravity and moisture content in Douglas fir. Similar relationships have been found to exist with respect to the wood of other coniferous species; in fact, the influence of fibril angle upon transverse conductivity of heat may be the chief factor responsible for the average deviation from anticipated values found for most, if not all, of the individual species studied.

Variation in longitudinal-transverse conductivity ratio is also closely related to fibril orientation and definitely substantiates theories previously advanced to the effect that the anisotropic character of the fibrils predominantly influences the differential conductivity of wood in the longitudinal and transverse directions. At least in so far as heat conductivity is concerned, the high fibrillar-angle characteristic of compression wood, not abnormal chemical composition, accounts for the abnormal properties of this type of defective wood.

The close relationship shown between conductivity ratio and fibrillar angle for Douglas fir suggests a new technique applicable to future investigations into other properties of wood dependent upon this particular structural feature, heretofore only determinable by tedious microscopic or X-ray measurement. Longitudinal-transverse conductivity ratios are also proposed for application in the selection or segregation of various qualities of wood for specialized uses, particularly for the detection of mild compression wood.

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20 "The Transverse Heat Conductivity of Wood," by Frederick F. Wangaard, Thesis for the Degree of Doctor of Philosophy, New York State College of Forestry, Syracuse, N. Y., 1939.

Discussion

H. L. Henderson.[3] The author points out that earlier researches by other investigators indicated that heat is conducted parallel with the grain at a rate from 2 to 3 times as fast as across the grain or transversely, varying with different species of wood. The reason for this phenomenon is attributed to the theory that the cell walls which run longitudinally consist of long chains of glucose-anhydride units which compose the cellulose. These chains being bound together by molecular energy transmit heat more rapidly in a longitudinal direction.

He points out also that his researches indicate that heat is conducted radially in hardwoods at a substantially faster rate than tangentially, while in softwoods it is the reverse. This is explained by the fact that hardwoods have about twice the wood-ray volume found in softwoods, hence there are more wood cell walls running directly from the center to the outside of the tree to conduct heat.

Cell walls, he says, are composed of aggregates of fibrils running generally in the long axis and these are tilted at varying degrees. When the orientation of these fibrils is less than 14 deg, the rate of heat conductivity is below normal in a transverse direction.

The author goes into considerable detail in explaining his technique in gathering his information and shows several photomicrographs illustrating the difference in fibril angles in springwood and summerwood of Douglas fir. He shows in Table 2, the results of transverse-heat-conductivity determinations in Douglas fir as they are affected by direction of grain, while in Table 1 he shows the variation between a great many species in transverse heat conductivity.

[3] Associate Professor of Forest Utilization, New York State College of Forestry, Syracuse, N. Y.

Several curves picture the effect of fibril-angle deviation from anticipated transverse-conductivity values of Douglas fir, the longitudinal transverse-conductivity ratio and the relationship between deviation from normal transverse-conductivity value and longitudinal transverse conductivity.

In conclusion, he emphasizes that the many tests made on Douglas fir are sufficient to prove that these variations are independent of specific gravity and rate of growth while for the eight other species studied future tests might show more variation from his results due to the small number of tests made.

He minimizes the effect that compression wood might have on heat conductivity and states: "Transverse-conductivity deviations in Douglas fir are so greatly dependent upon variations in fibril slope, Fig. 5, that there is a strong possibility that no other factors operate to influence this property."

Finally, in closing, the author ventures to predict that certain abnormal growths of wood which sometimes make it defective for certain uses might be detected by the heat-conductivity tests instead of the longer and more tedious determinations by microscope or X ray.

Author's Closure

Professor Henderson has summarized the principal points discussed in the original paper. Two statements, however, apparently need clarification. The writer found that radial conductivity was significantly greater than tangential conductivity in hardwoods. In the softwoods the conductivities in these two directions were found to be essentially the same. Professor Henderson states that the author has minimized the effect of compression wood on heat conductivity. A more accurate interpretation would be that evidence was presented to indicate that the abnormal heat-conducting properties of compression wood are the result of its characteristic high fibrillar angle rather than its abnormal chemical composition.

IN TWO SECTIONS—SECTION TWO

Transactions

of the

A.S.M.E.

SOCIETY RECORDS—PART 1

FEBRUARY, 1943

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Foreword

THE Transactions of The American Society of Mechanical Engineers include selected technical papers and reports delivered at meetings of the Society, its Professional Divisions, and its Local Sections, the *Journal of Applied Mechanics* (contributions of the Applied Mechanics Division), certain records of the Society of permanent value, and indexes to its publications.

In order to secure the advantages of timeliness and greater usefulness in issuing these Society Records, the material comprising them is divided into a number of parts, each one of which is mailed as a supplement to one of the regular monthly issues of the Transactions. For 1943, the first of these, the present issue, contains the personnel of the Council and committees for the year. Another, to be issued sometime later in the year, will contain memorial notices of deceased members. The indexes to miscellaneous publications, *Mechanical Engineering,* and to the Transactions themselves, must, necessarily, be issued in 1944, and will probably be mailed as a supplement to the January issue of that year.

In binding the 1943 Transactions, all of these parts of the Society Records will be assembled at the back of the volume as has been customary for several years. To aid in locating references in the bound volumes, the page numbers of the sections containing the *Journal of Applied Mechanics* and the Society Records are preceded by the letters A and RI, respectively.

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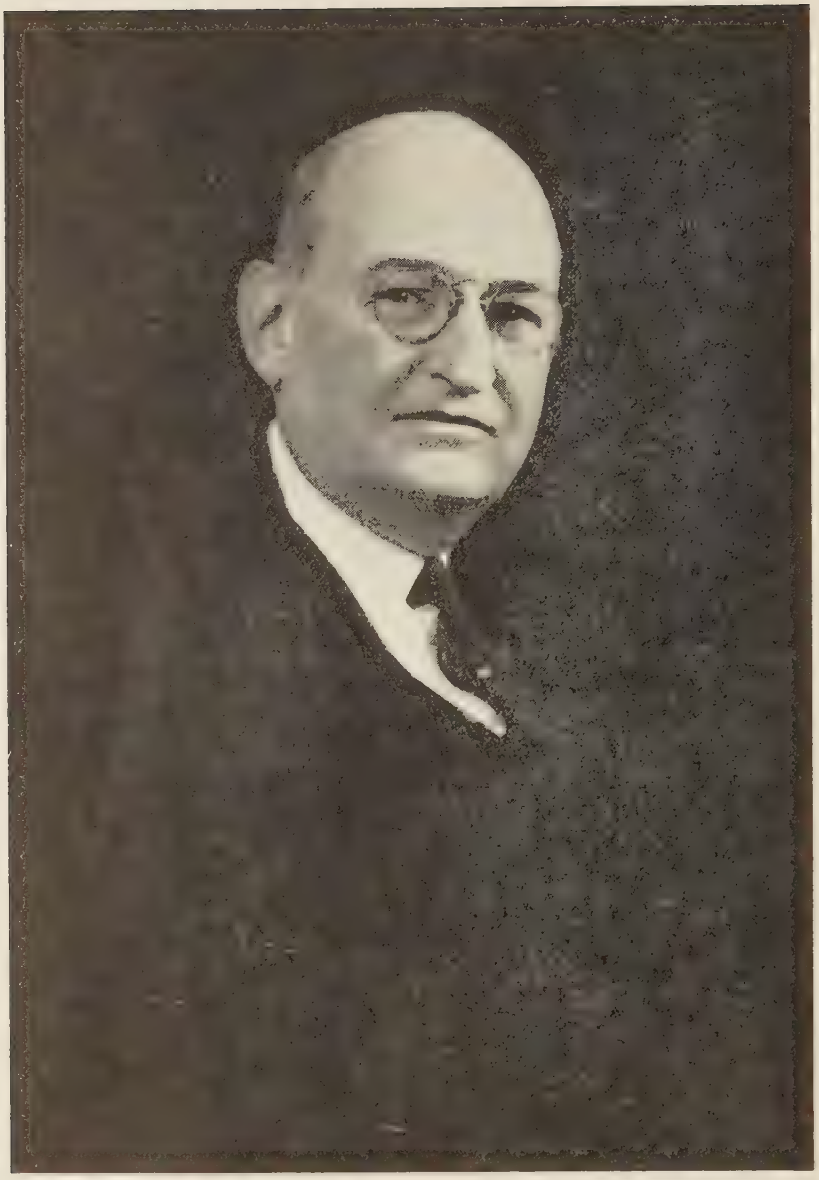

HAROLD V. COES

President of The American Society of Mechanical Engineers
1942-1943

Harold V. Coes

Harold Vinton Coes, President of The American Society of Mechanical Engineers for the year 1942-1943, is a Fellow of the Society and vice-president of Ford, Bacon & Davis, Inc., engineers of New York. He was born in Hyde Park, Mass., and studied at Northeastern Manual Training School, Philadelphia, Pa., and at Massachusetts Institute of Technology, from which he received the degree of Bachelor of Science in 1906.

From 1908 to 1911, Mr. Coes was mechanical engineer and assistant to the president of the Liquid Carbonic Company, Chicago, Ill., where he assisted in the design of, and built and operated the then largest liquid-carbonic-acid plant in the world at Cambridge, Mass. During the same period, in behalf of his company he acted as consulting engineer for the Searchlight Gas Company, acetylene-gas manufacturers. This was followed by the position of manager of the Chicago office and, later, principal industrial engineer of Lockwood, Green & Co., engineers, Boston, Mass. In 1914, he became vice-president and general manager of the Sentinel Manufacturing Company, New Haven, Conn., manufacturers of automatic gas appliances.

During 1917, he joined the staff of Gunn, Richards & Co., New York, industrial engineers, as industrial engineer, in which capacity he reorganized production in munitions plants of Canada. His work was so successful that he was invited to join the staff of Ford, Bacon & Davis, Inc., in the last year of World War I and as industrial engineer assisted in the design of munitions plants and of the Richmond, Va., plant of the Emergency Fleet Corporation for the manufacture of single-ended Scotch marine boilers for troop transports. Representing his company, he served as acting general manager, Platt Iron Works, Dayton, Ohio, manufacturers of whippet tanks, shells, submarine pumps, compressors, etc.

During the period from 1924 to 1928, he was director, vice-president, and general manager of the Belden Manufacturing Company of Chicago. However, preferring professional work, Mr. Coes reassociated himself with Ford, Bacon & Davis, Inc., in 1928 as manager of the industrial department and devoted his attention to problems of administration and industrial management. He served for a year representing his company as director and executive vice-president of Vulcan Iron Works, Wilkes-Barre, Pa., manufacturer of mine hoists, cement machinery, and industrial locomotives. In 1937, he became a partner of the firm and, in 1941, vice-president.

Mr. Coes has been an active member of the Society since 1907, serving as Manager, 1929-1932, and as Vice-President, 1927 and 1932-1934. His committee work includes service on the Professional Divisions Committee, the Materials Handling Division as chairman, the Finance Committee as chairman, Budgeting Policy Committee, the Special Committee on Depreciation, and the Special Committee on War Production. In 1927-1928 he served as vice-chairman of the Chicago Local Section. He represented the A.S.M.E. on the board of United Engineering Trustees, Inc., serving as president and as chairman of the organization's finance committee, and was one of the Society's representatives on the Engineers' Defense Board.

Other national and local professional affiliations include the American Management Association, of which he is chairman of the Finance Committee; Engineering Index, Inc., director; Association of Consulting Management Engineers, of which he is a past-president; Society for Advancement of Management; National Association of Manufacturers; Army Ordnance Association; Montclair Society of Engineers; and American Arbitration Association. For the Seventh International Management Congress, he was vice-chairman of the finance committee. He was one of three Americans selected to attend the International Budgetary Conference on budgetary procedure and control, which was to have been held at Geneva, Switzerland, July, 1930, but the meeting was cancelled and the papers sent to a subsequent meeting but not presented in person.

Mr. Coes is the author of many papers on economics, management, and industrial and marketing subjects, such as "Production Control," the section on "Materials Handling" in the "Cost and Production Handbook," "Depreciation and Obsolescence," and sections in the A.M.A. "Handbook of Business Administration."

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(Personnel of Biography Committee, p. RI-7)

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† Personnel of all Technical Committees, pp. RI-25-38.

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(Personnel of Boiler Code Committees, pp. RI-37-38)

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(*Continued*)

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See also A.S.M.E. Representatives on Other Research Committees, etc., pages RI-26, 31, 34, 35, 38
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A. I. Peterson (1943)
Wallace Clark (1944)
J. M. Talbot (1945)

DANIEL GUGGENHEIM MEDAL FUND, INC.

R. F. Gagg (1943)
E. E. Aldrin (1944)
J. E. Younger (1945)

JOSEPH A. HOLMES SAFETY ASSOCIATION

J. F. Barkley

HOOVER MEDAL BOARD OF AWARD

S. F. Voorhees (1943)
W. L. Batt (1945)
W. H. Kenerson (1947)

INTER-AMERICAN DEVELOPMENT COMMISSION

A. M. Greene, Jr.
W. H. McBryde
C. M. Muchnic

INTER-AMERICAN ENGINEERING COOPERATION

A. M. Greene, Jr.

INTERNATIONAL ELECTROTECHNICAL COMMISSION

U.S. NATIONAL COMMITTEE

H. N. Davis
Paul Diserens
Francis Hodgkinson

Alternate

C. Harold Berry

JOINT CONFERENCE COMMITTEE

(Presidents and Secretaries of Founder Societies)

President, Secretary } *Ex-Officio*

MARSTON AWARD

W. L. Abbott (1945)

NATIONAL BUREAU OF ENGINEERING REGISTRATION

V. M. Palmer

NATIONAL CONFERENCE ON ENGINEERING POSITIONS

W. F. Carhart
W. L. Cisler
H. B. Oatley
R. L. Sackett

NATIONAL FIRE WASTE COUNCIL

J. A. Neale

NATIONAL MANAGEMENT COUNCIL

J. M. Talbot (1944)—Wallace Clark, *Alternate*
J. R. Bangs, Jr. (1945)—J. M. Juran, *Alternate*
A. I. Peterson (1946)—C. W. Lytle, *Alternate*

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

E. G. Bailey (1945)

ALFRED NOBLE PRIZE

A. M. Greene, Jr.

POSTWAR PLANNING

To be appointed

UNITED ENGINEERING TRUSTEES, INC.

Walter Kidde (1943)
K. H. Condit (1944)
H. A. Lardner (1946)

VERMILYE MEDAL ADVISORY COMMITTEE

W. D. Fuller (1947)

WASHINGTON AWARD COMMISSION

W. J. Sando (1943)
C. C. Austin (1944)

WORLD POWER CONFERENCE

EXECUTIVE COMMITTEE, U.S. NATIONAL COMMITTEE

President, *Ex-Officio*

PROFESSIONAL DIVISIONS

ARTICLE B6A, PAR. 16: The Standing Committee on Professional Divisions shall, under the direction of the Council, have supervision of the Professional Divisions of the Society.

STANDING COMMITTEE

W. A. CARTER, *Chairman* (1943)
W. M. SHEEHAN (1944)
J. H. SENGSTAKEN (1945)
J. A. GOFF (1946)
J. R. BANGS (1947)

Junior Adviser

K. J. BERRIAN (1943)

Aeronautic

Organized, 1920

Reorganized, 1941

See Aviation

Applied Mechanics

Organized, 1927

J. H. KEENAN, *Chairman*

EXECUTIVE COMMITTEE

J. H. KEENAN, *Chairman*
J. N. GOODIER, *Secretary*
L. S. JACOBSEN
JESSE ORMONDROYD
H. PORITSKY

Associates

J. P. DEN HARTOG	G. B. PEGRAM
H. L. DRYDEN	R. E. PETERSON
RUPEN EKSERGIAN	E. O. WATERS
J. C. HUNSAKER	B. M. WOODS
A. L. KIMBALL	

Representative on Aviation Liaison Group

J. C. HUNSAKER

Representative, San Francisco Section

W. M. MOODY

Research Secretary

JESSE ORMONDROYD

JOURNAL OF APPLIED MECHANICS

J. M. LESSELLS, *Editor*

SPONSORS

Dynamics, F. M. LEWIS
Elasticity, STEPHEN TIMOSHENKO
Fluid Mechanics, H. W. EMMONS
Lubrication, G. B. KARELITZ *
Plasticity, A. NÁDAI
Strength of Materials, C. R. SODERBERG
Thermodynamics, J. A. GOFF

* Deceased, January 19, 1943.

Aviation

Organized, 1941

(Formerly Aeronautic)

R. F. GAGG, *Chairman*

EXECUTIVE COMMITTEE

R. F. GAGG, *Chairman*
J. E. YOUNGER, *Secretary*
E. E. ALDRIN
A. R. STEVENSON, JR.
J. M. CLARK, *Ex-Officio*

Associate Members

C. H. DOLAN, for the Mid-West
M. J. THOMPSON, for the Southwest

Junior Advisers

B. E. DEL MAR
F. H. HART

ADVISORY COMMITTEE

KARL ARNSTEIN	R. K. LEBLOND
CARL BREER	W. B. MAYO
E. C. CLARKE	P. B. MORGAN
H. M. CRANE	T. A. MORGAN
LOUIS DEFLOREZ	S. A. MOSS
W. F. DURAND	E. A. SPERRY
A. J. GIFFORD	A. R. STEVENSON, JR.
M. B. GORDON	J. G. VINCENT
WILLIAM HOVGAARD	THEODOR VON KÁRMÁN
J. C. HUNSAKER	C. J. WARD
P. G. JOHNSON	E. P. WARNER
C. F. KETTERING	B. M. WOODS
ALEXANDER KLEMIN	ORVILLE WRIGHT

LIAISON WITH OTHER PROFESSIONAL DIVISIONS

Applied Mechanics, J. E. YOUNGER
Diesels, S. ALLEN GUIBERSON, III
Heat Transfer, V. J. SKOGLUND
Hydraulics, THEODOR VON KÁRMÁN
Instruments, E. A. SPERRY
Management, R. E. GILLMOR
Metals, Aluminum Alloys, R. F. TEMPLIN
Metals, Ferrous, F. P. DODS
Plastics, G. P. YOUNG
Production Engineering, R. F. GAGG
Rubber, CHARLES CLEARY
Safety, JEROME F. LEDERER
Wood Industries, ALEXANDER KLEMIN

REPRESENTATIVES OF OTHER BODIES

Aviation Division Research Committee, ALEXANDER KLEMIN
Daniel Guggenheim Medal Board of Award, E. E. ALDRIN, R. F. GAGG, J. E. YOUNGER
Education and Training for the Industries Committee, A. R. STEVENSON, JR.
Institute of Aeronautical Sciences, J. E. YOUNGER

Fuels

Organized, 1920

H. F. HEBLEY, *Chairman*

EXECUTIVE COMMITTEE

H. F. HEBLEY, *Chairman*
D. C. WEEKS, *Secretary*
O. F. CAMPBELL
A. W. THORSON
J. E. TOBEY

Associates

A. R. MUMFORD, *Acting Chairman*
J. F. BARKLEY
J. S. BENNETT, 3RD
T. C. CHEASLEY
W. G. CHRISTY
M. P. CLEGHORN
B. J. CROSS
M. D. ENGLE
D. S. FRANK
E. R. KAISER
T. A. MARSH
M. A. MAYERS
W. E. REASER
R. L. ROWAN
R. A. SHERMAN

COAL TESTING CODE

Organized, 1939, Jointly with A.I.M.E.

R. L. ROWAN, *Chairman*
J. E. TOBEY, *Vice-Chairman*

A.S.M.E. Representatives

H. C. CARROLL	A. R. MUMFORD
R. A. FORESMAN	W. T. REID
R. M. HARDGROVE	R. A. SHERMAN
J. H. KERRICK	L. A. SHIPMAN
T. A. MARSH	A. W. THORSON

DOMESTIC FUELS

M. P. CLEGHORN, *Chairman*
T. C. CHEASLEY
T. A. MARSH
R. A. SHERMAN
J. E. TOBEY

JOINT AWARDS

A.S.M.E. M. D. ENGLE (1943)
A.S.M.E. W. G. CHRISTY (1944)
A.I.M.E. PAUL WEIR (1945)
A.I.M.E. D. R. MITCHELL (1946)
A.S.M.E. H. F. HEBLEY (1947)

MODEL SMOKE LAW

J. F. BARKLEY, *Chairman*
O. F. CAMPBELL
A. G. CHRISTIE
W. G. CHRISTY
T. A. MARSH
T. E. PURCELL
R. A. SHERMAN
R. R. TUCKER

Fuels

(Continued)

PROGRAM

A. W. THORSON, *Chairman*
D. S. FRANK, *Assistant Chairman*
E. R. KAISER, *Junior Member*

REVIEW OF PAPERS

M. D. ENGLE
H. F. HEBLEY
A. W. THORSON
D. C. WEEKS

Graphic Arts

Organized, 1922

EXECUTIVE COMMITTEE

A. E. GIEGENGACK
F. W. HOCH
W. B. LAUGHTON
R. G. MACDONALD
W. M. PASSANO
B. L. SITES
B. D. STEVENS
B. L. WEHMHOFF

Heat Transfer

Organized, 1941

(Formerly Heat Transfer Professional Group, organized 1938)

L. M. K. BOELTER, *Chairman*
H. B. NOTTAGE, *Secretary*

EXECUTIVE COMMITTEE

L. M. K. BOELTER, *Chairman*
R. A. BOWMAN
H. C. HOTTEL
R. H. NORRIS
W. S. PATTERSON

Advisory Associates

T. B. DREW
E. D. GRIMISON
C. E. LUCKE
A. K. SCOTT
J. H. SENGSTAKEN

Junior Representatives

P. R. TRUMPLER
W. R. WYKOFF

STANDING COMMITTEES

COORDINATION

H. B. NOTTAGE, *Chairman*

Liaison Officer for Local Sections

A. K. SCOTT

Representative, San Francisco Section

W. M. MOODY

Representative for Aviation Liaison

R. H. NORRIS

Representatives of Other Divisions

Fuels, B. J. CROSS
Hydraulic, J. D. SCOVILLE
Metals Engineering, W. TRINKS
Oil and Gas Power, F. G. HECHLER
Petroleum, J. D. PETERSON
Power, O. F. CAMPBELL
Process Industries, ARNOLD WEISSELBERG
Railroad, L. H. FRY

Research Secretary

C. F. KAYAN

Members at Large

R. H. HEILMAN
G. L. TUVE

PAPERS

R. A. BOWMAN, *Chairman*
G. A. HAWKINS
C. F. KAYAN
L. B. SCHUELER
A. K. SCOTT

TESTING TECHNIQUE

G. L. TUVE, *Chairman*
B. J. CROSS
R. H. JACKSON
J. H. RUSHTON

THERMO-PHYSICAL PROPERTIES OF MATERIALS

J. F. DOWNIE SMITH, *Chairman*
O. KENNETH BATES
T. H. CHILTON
R. C. H. HECK
MAX JAKOB

SUBCOMMITTEE ON SPECIFIC HEAT OF GASES

MAX JAKOB, *Chairman*
W. L. DE BAUFRE
J. A. GOFF
A. C. GULLIKSON
R. C. H. HECK
R. L. SWEIGERT

TECHNICAL COMMITTEES

AIRCRAFT HEAT TRANSFER

R. H. NORRIS, *Chairman*

DIRECT-FIRED FLUID HEATERS AND BOILERS

L. B. SCHUELER, *Chairman*
JOHN BLIZARD
D. S. FRANK
E. D. GRIMISON
W. J. WOHLENBERG

HEATED OR COOLED ENCLOSURES

C. F. KAYAN, *Chairman*
E. L. KNOEDLER
W. A. TICKNOR

INDUSTRIAL FURNACES AND KILNS

W. TRINKS, *Chairman*
H. C. HOTTEL
W. A. TICKNOR

THEORY AND FUNDAMENTAL RESEARCH

A. P. COLBURN, *Chairman*
L. M. K. BOELTER
T. B. DREW
MAX JAKOB
D. D. STREID

UNFIRED HEAT TRANSFER EQUIPMENT

TOWNSEND TINKER, *Chairman*
R. A. BOWMAN
G. A. HAWKINS
A. C. MUELLER
B. E. SHORT
S. P. SOLING
W. H. THOMPSON

Hydraulic

Organized, 1926

E. B. STROWGER, *Chairman*

EXECUTIVE COMMITTEE

E. B. STROWGER, *Chairman*
L. J. HOOPER, *Secretary*
J. D. SCOVILLE
BRUNSWICH SHARP
R. V. TERRY

CAVITATION

E. B. STROWGER, *Sponsor*
L. F. MOODY, *Chairman*
R. T. KNAPP
J. M. MOUSSON
W. J. RHEINGANS
G. F. WISLICENUS

Representatives of Other Societies

American Society for Testing Materials, F. N. SPELLER
Engineering Institute of Canada, ERNEST BROWN
Institution of Mechanical Engineers, G. S. BAKER

Representative of France

A. TENOT

Representative, San Francisco Section

W. M. MOODY

HYDRAULIC PRIME MOVERS

R. V. TERRY, *Sponsor*
J. F. ROBERTS, *Chairman*
A. ABERLI
E. H. COLLINS
J. P. GROWDEN
L. F. HARZA
P. L. HESLOP
GEORGE JESSUP
F. H. ROGERS
F. SCHMIDT
S. O. SCHOMBERGER
S. H. VAN PATTER

PUMPING MACHINERY

F. G. SWITZER, *Sponsor*
R. L. DAUGHERTY, *Chairman*
B. F. TILLSON
HANS ULMANN

WATER HAMMER

J. D. SCOVILLE, *Sponsor*
S. LOGAN KERR, *Chairman*
N. R. GIBSON
EUGENE HALMOS
L. F. MOODY
R. S. QUICK
E. B. STROWGER

Affiliated Societies and Their Representatives

American Society of Civil Engineers, N. R. GIBSON and FORD KURTZ
American Water Works Association, F. M. DAWSON and L. H. KESSLER

Hydraulic

(Continued)

Associate Members, Representing:

Australia, George Higgins
Brazil, A. W. K. Billings and F. Knapp
Engineering Institute of Canada, R. W. Angus and F. M. Wood
France, Louis Bergeron and Charles Camichel
Germany and Verein deutscher Ingenieure, D. Thoma
Great Britain and Institution of Mechanical Enginers, E. Bruce Ball and A. H. Gibson
Italy, Gaudenzio Fantoli and Albino Pasini
Switzerland, Charles Jaeger and O. Schnyder

Machine Shop Practice

Organized, 1921

Reorganized, 1941

See Production Engineering

Management

Organized, 1920

J. M. Talbot, *Chairman*

EXECUTIVE COMMITTEE

J. M. Talbot, *Chairman*
L. A. Appley, *Vice-Chairman*
G. M. Varga, *Secretary*
Carlos deZafra, *Assistant Secretary*
J. M. Juran
A. I. Peterson
J. A. Willard

Representatives of Local Sections

Atlanta, S. C. Hale
Detroit, J. A. Carlin
Kansas, A. H. Sluss
Louisville, C. D. Eldridge
Metropolitan, M. S. Symon
Milwaukee, B. V. E. Nordberg
New Orleans, P. F. Hoots
Philadelphia, C. S. Gotwals
Rochester, V. M. Palmer
San Francisco, B. A. Gayman
Seattle, H. J. McIntyre
South Texas, V. M. Faires

Representative on Committee on Education and Training for the Industries

Lillian M. Gilbreth

Representative on Aviation Liaison Group

R. E. Gillmor

Representative on War Production Committee

K. H. Condit

Research Secretary

E. H. Hempel

GENERAL COMMITTEE

J. R. Bangs
R. M. Barnes
W. L. Batt
C. W. Beese
F. B. Bell
Wallace Clark
K. H. Condit
Howard Coonley
Carlos deZafra
N. E. Elsas
S. P. Fisher
W. D. Fuller
W. H. Gesell
L. M. Gilbreth
R. E. Gillmor
G. E. Hagemann
C. H. Hatch
E. H. Hempel
P. E. Holden
W. F. Hosford
D. S. Kimball
W. H. Kushnick
J. K. Louden
T. S. McEwan
L. C. Morrow
D. B. Porter
F. E. Raymond
J. W. Roe
E. H. Schell
E. D. Smith
A. R. Stevenson, Jr.
L. W. Wallace
A. Williams
John Younger

COMMITTEE CHAIRMEN

Industrial Marketing, J. R. Bangs
Quality Control, A. I. Peterson
Work Standardization, J. K. Louden

DEPRECIATION STUDIES

P. T. Norton, Jr.

Materials Handling

Organized, 1920

G. E. Hagemann, *Chairman*

EXECUTIVE COMMITTEE

G. E. Hagemann, *Chairman*
C. F. Dietz, *Vice-Chairman*
C. H. Barker, Jr., *Secretary*
A. J. Burke, *Assistant Secretary*
J. A. Jackson
F. J. Shepard, Jr.

Associates

H. E. Blank, Jr.
H. C. Keller
Bernard Lester
M. C. Maxwell
R. H. McLain
F. E. Moore
P. D. Oesterle
Virgil Palmer
M. W. Potts
Sidney Reibel
R. B. Renner
E. D. Smith
H. E. Stocker
G. R. Wadleigh
J. B. Webb

Junior Associates

Cornelius Crowley
E. Z. Gabriel
R. W. Grundman
D. D. Jones

SAFETY CODE FOR PACKAGE CONVEYORS

H. C. Keller, *Chairman*

STANDARD NOMENCLATURE

Chairman to be appointed

STANDARDIZATION OF SHIPMENTS ON SKIDS AND PALLETS

H. E. Stocker, *Chairman*
C. J. Barker, *Secretary*
C. E. Mochrie
P. F. Nydegger
M. W. Potts

Metals Engineering

Organized, 1927

Reorganized, 1940

(Formerly Iron and Steel)

W. Trinks, *Chairman*

EXECUTIVE COMMITTEE

W. Trinks, *Chairman*
R. A. North, *Secretary*
M. J. Dempsey
J. H. Hitchcock
J. H. Romann
M. D. Stone

Associates

J. A. Clauss
G. L. Fisk
S. M. Marshall
P. M. Mueller
D. B. Rossheim
W. M. Sheehan
R. G. Sturm
R. J. Wean
W. R. Webster
S. M. Weckstein
T. H. Wickenden
A. W. Winston

Representative, San Francisco Section

Walter Kassebohn

Oil and Gas Power

Organized, 1921

E. S. Dennison, *Chairman*

EXECUTIVE COMMITTEE

E. S. Dennison, *Chairman*
L. N. Rowley, Jr., *Secretary*
C. E. Beck
H. E. Degler
C. W. Good
E. J. Kates

Associates

G. C. Boyer
R. D. Campbell
M. M. Dana
G. J. Dashefsky
W. L. H. Doyle
W. K. Gregory
F. G. Hechler
P. B. Jackson
B. V. E. Nordberg
M. J. Reed
Lee Schneitter

Junior Representative

C. K. Holland

Research Secretary

Lee Schneitter

Representative, San Francisco Section

E. G. Gothberg

Liaison Representatives

American Society of Naval Architects and Marine Engineers, B. V. E. Nordberg
Aviation Liaison Group, H. E. Degler
Heat Transfer Division, F. G. Hechler
Railroad Division, R. T. Sawyer

HONORS AND AWARDS

E. J. Kates, *Chairman*
C. W. Good
Ernest Nibbs

Oil and Gas Power

(Continued)

OIL ENGINE POWER COST

H. C. Major, *Chairman*
H. C. Lenfest, *Secretary*
L. T. Brown
E. Hale Codding
Malcolm Duncan
J. H. Gallaway
E. J. Kates
A. A. Lyman
A. B. Morgan
M. J. Reed
T. M. Robie
R. T. Sawyer
Lee Schneitter
P. H. Schweitzer
J. B. Sims
H. C. Thuerk
C. A. Trimmer
Stanley Wright

OIL AND GAS POWER CONFERENCES

1944 Meeting, Location Selection Committee

H. E. Degler, *Chairman*
C. E. Beck
W. K. Gregory

PAPERS

G. C. Boyer, *Chairman*
G. J. Dashefsky
P. B. Jackson
R. T. Sawyer

PUBLICITY

L. N. Rowley, Jr.

Petroleum

Organized, 1925

Reorganized, 1937

W. F. Herbert, *Chairman*

EXECUTIVE COMMITTEE

W. F. Herbert, *Chairman*
W. H. Carson, *Secretary*
E. H. Barlow
H. L. Eggleston

Research Secretary

E. E. Ambrosius

Liaison Representative, San Francisco Section

Herman Dishington

Power

Organized, 1920

Theodore Baumeister, Jr., *Chairman*

EXECUTIVE COMMITTEE

Theodore Baumeister, Jr., *Chairman*
J. A. Keeth, *Secretary*
O. F. Campbell
L. M. Goldsmith
J. N. Landis

Liaison Representative, San Francisco Section

E. C. Gothberg

Process Industries

Organized, 1934

William Raisch, *Chairman*

EXECUTIVE COMMITTEE

William Raisch, *Chairman*
T. R. Olive, *Secretary*
J. W. Hunter
A. F. Sperry
Arnold Weisselberg
W. R. Woolrich
J. I. Yellott
F. L. Yerzley

Liaison Officer With Standing Committee on Professional Divisions

J. H. Sengstaken

Research Secretary

Arnold Weisselberg

Other Liaison Representatives

Aviation Group
Industrial Instruments and Regulators Committee, E. A. Sperry
Heat Transfer Division, Arnold Weisselberg
San Francisco Section, Herman Dishington
Society of Automotive Engineers, F. L. Yerzley

COMMITTEE CHAIRMEN

Air Conditioning, C. F. Kayan
Drying, Arnold Weisselberg
Food Processing, G. L. Montgomery
Industrial Instruments and Regulators
E. S. Smith, *Chairman*
J. C. Peters, *Secretary*
Manufactured and Natural Gas, E. J. Devlin
Mechanical Separation, Richard O'Mara
Papers, Awards, and Honors, C. E. Lucke
Program, J. W. Hunter
Sanitation, William Raisch
Sugar, F. M. Gibson
Sulphur, B. E. Short
Vegetable Oils, R. W. Morton

COMMITTEE ON INDUSTRIAL INSTRUMENTS AND REGULATORS

E. S. Smith, *Chairman*
E. D. Haigler, *Vice-Chairman*
J. C. Peters, *Secretary*

Executive Subcommittee

P. G. Exline
R. L. Goetzenberger
P. W. Keppler
W. J. King
E. S. Lee
H. L. Mason
H. F. Moore
G. W. Smith
A. F. Sperry
I. M. Stein
M. J. Zucrow

Production Engineering

Organized, 1941

(Formerly Machine Shop Practice)

Warner Seely, *Chairman*

EXECUTIVE COMMITTEE

Warner Seely, *Chairman*
C. L. Tutt, Jr., *Secretary*
J. M. Alden
Sol Einstein
E. K. Ernest
R. F. Gagg
Erik Oberg

Associates

Hans Ernst
W. J. Hargest
A. M. Johnson
A. F. Murray
E. O. Waters

Railroad

Organized, 1920

J. R. Jackson, *Chairman*

EXECUTIVE COMMITTEE

J. R. Jackson, *Chairman*
E. L. Woodward, *Secretary*
J. G. Adair
K. F. Nystrom
W. C. Sanders
W. M. Sheehan

Research Secretary

F. H. Clark

Representative, San Francisco Section

M. P. Taylor

Rubber and Plastics Group

Organized, 1942

G. M. Kline, *Chairman*

EXECUTIVE COMMITTEE

G. M. Kline, *Chairman*
E. F. Riesing, *Vice-Chairman*
R. A. Boyer, *Secretary*
L. E. Jermy
E. G. Kimmich
J. F. D. Smith
F. L. Yerzley

GENERAL COMMITTEE

A. L. Alves	M. E. Lerner
T. S. Carswell	R. A. North
D. H. Cornell	H. M. Richardson
John Delmonte	R. G. Seaman
J. H. Dillon	J. H. Teeple
F. L. Haushalter	J. C. Travilla, Jr.
G. H. Kaemmerling	W. A. Zinzow

Textile

Organized, 1921

F. L. Bradley, *Chairman*

EXECUTIVE COMMITTEE

F. L. Bradley, *Chairman*
W. B. Heinz, *Vice-Chairman*
W. W. Starke, *Secretary*
M. Earl Heard
J. D. Robertson
E. R. Stall
E. Wadsworth Stone
J. W. Vaughan, Jr.

Associates

A. D. Asbury	Winn Chase
A. W. Benoit	M. A. Golrich, Jr.
W. S. Brown	R. DeVere Hope

Southern Representative

S. B. Earle

SUBCOMMITTEE CHAIRMEN

Activities, W. B. Heinz
Air Conditioning, Wendell Brown
Drying, J. D. Robertson
Lighting, Earle Mauldin
Lubrication, R. W. Vose
Power and Heat Utilization, E. Wadsworth Stone

Wood Industries

Organized, 1921

Sern Madsen, *Chairman*

EXECUTIVE COMMITTEE

Sern Madsen, *Chairman*
A. C. Fegel, *Secretary*
M. J. MacDonald, *Vice-Chairman*
D. R. Gray
N. S. Jones

Associates

C. L. Babcock	E. D. May
P. H. Bilhuber	R. H. McCarthy
H. B. Carpenter	C. B. Norris
F. P. Cartwright	T. D. Perry
G. E. French	A. D. Smith, Jr.
A. W. Keuffel	H. M. Sutton
A. S. Kurkjian	Charles White
J. S. Mathewson	

COMMITTEE CHAIRMEN

Dimensional Limits and Allowances, Sern Madsen
Wood Finishing, M. J. MacDonald

LOCAL SECTIONS

ARTICLE B6A, PAR. 17: The Standing Committee on Local Sections shall, under the direction of the Council, have supervision of the Local Sections of the Society.

STANDING COMMITTEE ON LOCAL SECTIONS

F. L. WILKINSON, JR., *Chairman* (1943)

S. R. BEITLER (1944)
(Alternate for F. W. MARQUIS)
J. A. KEETH (1945)
OLIVER B. LYMAN (1946)
A. R. MUMFORD (1947)

Junior Adviser

F. H. FOWLER, JR. (1943)

REGIONAL GROUP DELEGATES TO ANNUAL CONFERENCES

Terms expire October, 1943

R. M. MATSON, *Speaker for 1942 Conference*, Group VIII
J. B. JONES, *Secretary*, Group IV
J. W. ZELLER, Group I
W. H. LARKIN, 3RD, Group II
CARL SCHABTACH, Group III
A. M. SELVEY, Group V
B. G. ELLIOTT, Group VI
H. T. AVERY, Group VII

Terms expire October, 1944

C. C. AUSTIN, *Speaker for 1943 Conference*, Group VI
F. C. RICHARDSON, *Secretary*, Group I
W. S. GLEESON, Group II
F. S. ERDMAN, Group III
W. R. CHAMBERS, Group IV
R. R. SLAYMAKER, Group V
J. G. MCGIVERN, Group VII
L. J. CUCULLU, Group VIII

AKRON-CANTON

Organized: 1920
Territory: Counties of Richland, Ashland, Medina, Summit, Portage, Wayne, Stark, Holmes, Tuscarawas, Carroll, and Coshocton in Ohio
Place of Meeting: As selected monthly
Number of Members: 189

EXECUTIVE COMMITTEE

O. J. HORGER, *Chairman*
A. E. SHETLER, *Vice-Chairman*
D. H. CORNELL, *Secretary-Treasurer*
M. R. BOWERMAN
R. W. HURSH
M. C. PIERCE
J. H. VANCE
A. G. WALKER

ANTHRACITE-LEHIGH VALLEY

Organized: 1920, as Lehigh Valley; reorganized, 1928, as Anthracite-Lehigh Valley
Territory: Counties of Bradford, Susquehanna, Wayne, Sullivan, Wyoming, Lackawanna, Columbia, Luzerne, Monroe, Pike, Schuylkill, Carbon, Berks, Lehigh, Northampton in Pennsylvania, and Warren in New Jersey
Place of Meeting: One meeting annually at Allentown, Bethlehem, Easton, Hazleton, Pottsville, Reading, Scranton, and Wilkes-Barre
Local Organization: The Engineers' Club of Lehigh Valley
Number of Members: 215

EXECUTIVE COMMITTEE

J. A. GISH, JR., *Chairman*
W. TALLGREN, R. H. PORTER — *Vice-Chairmen*
C. H. FOLMSBEE, *Secretary-Treasurer*

Managers:
C. W. BELL
J. R. CONNELLY
C. R. DIECKMAN
E. A. GORNEY
W. GREACEN
H. F. HATFIELD
J. A. LLOYD
W. G. MCLEAN

Assistant Managers:
W. E. ANDERSON
J. W. BLISS
K. W. BLOOM
L. D. GRABOSKI
S. I. HAMMOND
J. B. LAUDIG
L. E. MYLTING
H. C. SCHWEIKART

ATLANTA

Organized: 1913
Territory: Radius of sixty miles from Atlanta, Ga.
Place of Meeting: Atlanta Athletic Club
Luncheon meeting every Monday at 12:30 p.m. at Atlanta Athletic Club
Number of Members: 94

EXECUTIVE COMMITTEE

JOHN RITTELMEYER, *Chairman*
W. J. MCALPIN, *Vice-Chairman*
J. W. PARKER, JR., *Secretary*
G. F. ALLEN
J. A. DODD
C. L. HUEY
A. C. KEISER
W. J. MCALPIN

BALTIMORE

Organized: 1916
Territory: Radius of thirty miles from Baltimore, Md.
Place of Meeting: Engineers Club of Baltimore
Number of Members: 317

EXECUTIVE COMMITTEE

W. P. HILL, *Chairman*
A. M. GOMPF, *Vice-Chairman*
W. D. BOYNTON, *Secretary-Treasurer*
E. M. BENJES
S. E. BRILLHART
R. C. HINE
HERMAN HOLLERITH, JR.
C. F. MERRIAM
J. H. POTTER

JUNIOR GROUP

H. W. HYDE, *Chairman*
D. H. FAX, *Vice-Chairman*
H. H. SEVERN, *Secretary*
E. M. BENJES
JOHN DOERING
F. J. JEFFERS
H. R. KNUST

BIRMINGHAM

Organized: 1915
Territory: Radius of sixty miles from Birmingham, Ala.
Place of Meeting: Tutwiler Hotel
Number of Members: 83

EXECUTIVE COMMITTEE

T. M. FRANCIS, *Chairman*
H. G. MOUAT, *Vice-Chairman*
A. H. BLAIR, *Secretary*
J. M. GALLALEE
G. M. RUST

BOSTON

Organized: 1909
Territory: Radius of thirty miles from Boston, Mass.
Place of Meeting: Mass. Inst. of Technology
Local Organization: Engineering Societies of New England
Number of Members: 619

BOSTON
(Continued)

Executive Committee

Kerr Atkinson, *Chairman*
G. A. Orrok, Jr., *Vice-Chairman*
R. A. Spence, *Secretary-Treasurer*
H. N. King
S. S. Perry
J. A. Powell

BRIDGEPORT

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Fairfield County, Conn.
Place of Meeting: Stratfield Hotel
Local Organizations: The Bridgeport Tool Engineers Association; The Bridgeport Engineers Club
Number of Members: 164

Executive Committee

Rudolf Beck, *Chairman*
J. M. Lucarelle, *Vice-Chairman*
W. H. Sniffen, *Secretary*
A. W. Hagan, *Treasurer*
J. L. Corcoran
P. A. Ibold
C. N. Hoagland
O. J. Richmond
J. W. Roe
J. D. Skinner
E. R. Spaulding
C. P. Wicks

BUFFALO

Organized: 1915
Territory: Radius of thirty miles from Buffalo, N.Y.
Place of Meeting: Mareen Hotel, Main St. at Utica
Local Organization: Engineering Society of Buffalo
Number of Members: 200

Executive Committee

N. C. Barnard, *Chairman*
C. A. Ross, *Vice-Chairman*
L. R. Burmester, *Acting Secretary*
C. E. Harrington, *Treasurer*
W. M. Kauffmann
C. G. Kiplinger
W. A. Miller
N. S. Snyder
Stephen Wagner
J. L. Yates

CENTRAL ILLINOIS

Organized: 1937
Territory: All the territory in Central Illinois between the following counties on the northern boundary: Bureau, LaSalle, Knox, Stark, Putnam, Marshall, Livingston, Peoria; counties on the southern boundary: Pike, Scott, Morgan, Sangamon, Macon, Piatt, Douglas, and Edgar
Place of Meeting: Hotel Pere Marquette or Caterpillar Show Room
Number of Members: 120

Executive Committee

L. J. Fletcher, *Chairman*
C. H. Casberg, *Vice-Chairman*
R. E. McClain, *Secretary-Treasurer*
J. L. Deffenbaugh, *Assistant Secretary-Treasurer*
W. L. H. Doyle
H. G. Kniese

CENTRAL INDIANA

Organized: 1916
Territory: Radius of eighty miles from Indianapolis, within Indiana
Place of Meeting: Indianapolis Athletic Club
Local Organization: Indiana Engineering Society
Number of Members: 170

Executive Committee

R. B. Holmes, *Chairman*
H. A. Bolz, *Vice-Chairman*
M. E. Bechtold, *Secretary-Treasurer*
R. B. Bass
G. L. Fowler
R. W. Gausmann
D. H. Kelly
D. P. Morse
W. C. Wischmeyer

CENTRAL PENNSYLVANIA

Organized: 1921
Territory: Radius of approximately sixty miles from State College, Pa.
Place of Meeting: State College and Altoona, Pa.
Number of Members: 71

Executive Committee

J. O. P. Hummel, *Chairman*
H. A. Sorensen, *Secretary-Treasurer*
J. S. Doolittle
F. T. Mavis
R. Y. Sigworth
F. C. Stewart
H. C. Thuerk

CHICAGO

Organized: 1913
Territory: Radius of fifty miles from Chicago, Ill.
Headquarters: Mid-West A.S.M.E. Office, Room 1617, 205 West Wacker Drive, Chicago, Ill.
Place of Meeting: Civic Opera Bldg., 20 N. Wacker Dr.
Meetings: Tuesday, 7:30 p.m.
Local Organization: Western Society of Engineers
Number of Members: 899

Executive Committee

J. R. Michel, *Chairman*
H. M. Black, *Vice-Chairmen*
J. P. Magos, *Vice-Chairmen*
J. C. Marshall, *Vice-Chairmen*
V. L. Sherman, *Vice-Chairmen*
P. A. Stephenson, *Vice-Chairmen*
F. B. Orr, *Secretary-Treasurer*
C. C. Austin
R. H. Bacon
C. B. Cole
L. M. Ellison
A. H. Jens
L. M. Johnson
N. R. Kendall
J. S. Kozacka
F. H. Lane
T. S. McEwan
H. L. Nachman
W. H. Oldacre
C. W. Parsons
H. S. Philbrick
Ralph Sargent
Karl Tranzen
R. E. Turner
C. L. Wachs
J. I. Yellott

CINCINNATI

Organized: 1912
Territory: Radius of thirty miles from Cincinnati, Ohio
Place of Meeting: Engineers' Club Rooms, Ninth & Race Sts.
Local Organization: Engineers' Club of Cincinnati
Number of Members: 211

Executive Committee

H. B. Brandt, *Chairman*
Hans Ernst, *Vice-Chairman*
R. L. Smith, *Secretary-Treasurer*
H. L. Haworth
R. S. Hyatt
C. A. Joerger
E. M. Martellotti
J. G. Martin
F. W. Spalding
E. B. Woodruff

CLEVELAND

Organized: 1918
Territory: Counties of Lorain, Cuyahoga, Lake, Geauga, and Ashtabula in Ohio
Place of Meeting: Cleveland Engineering Society Club
Local Organization: Cleveland Engineering Society
Number of Members: 323

Executive Committee

R. R. Slaymaker, *Chairman*
E. R. McCarthy, *Secretary*
A. G. Trumbull, *Treasurer*
F. A. Barnes
F. W. Brooks
L. C. Cole
J. P. Dearasaugh
H. M. Hammond
G. E. Kentis
W. G. Stephan

COLORADO

Organized: 1919
Territory: Entire State of Colorado
Place of Meeting: Albany Hotel, Denver, Colo.
Local Organization: Colorado Engineering Council (Colorado Society of Engineers)
Number of Members: 93

Executive Committee

Durbin Van Law, *Chairman*
H. L. Hartburg, *Vice-Chairman*
G. H. Woelbing, *Secretary-Treasurer*
F. A. Lockwood
N. A. Parker
F. H. Prouty
G. A. Richter
R. F. Throne
J. T. Strate

COLUMBUS

Organized: 1920
Territory: Counties of Union, Delaware, Licking, Madison, Franklin, Fayette, Pickaway, and Ross in Ohio
Place of Meeting: Battelle Memorial Institute and The Ohio State University
Number of Members: 83

Executive Committee

H. M. Blank, *Chairman*
S. M. Marco, *Vice-Chairman*
E. J. Lindahl, *Secretary-Treasurer*
W. L. Davis
C. Z. Gillivan
G. E. Haney
H. R. Limbacher
J. L. Purdy

DAYTON

Organized: 1926
Territory: Counties of Drake, Miami, Champaign, Preble, Montgomery, Greene, and northern part of Butler and Warren in Ohio
Place of Meeting: Engineers' Club of Dayton
Local Organization: Engineers' Club of Dayton
Number of Members: 145

Executive Committee

E. M. Kelly, *Chairman*
W. P. Smith, *Vice-Chairman*
D. C. Kennard, *Secretary*
A. W. Kimmel, *Treasurer*
C. W. Ball
L. R. Bridge
H. M. Gano
J. J. Healy
H. C. Wight

DETROIT

Organized: 1916
Territory: Radius of thirty miles from Detroit, Mich.
Place of Meeting: Place varies
Local Organization: Engineering Society of Detroit
Number of Members: 502

Executive Committee

J. W. Armour, *Chairman*
C. R. Alden, *Secretary-Treasurer*
C. L. Brattin
P. W. Lehman
L. W. Lentz
R. W. Parkinson
J. H. Spurgeon
H. L. Walton
R. K. Weldy
Herbert Wells
T. E. Winkler
A. M. Selvey, *Ex-Officio*

Junior Group

B. H. Webb, *Chairman*
R. F. Bergin
V. A. Russnack
R. W. Vorhees, Jr.

EAST TENNESSEE

Organized: 1922
Territory: All counties in Tennessee east of the west boundary of Scott, Morgan, Cumberland, White, Warren, Coffee, Moore, Franklin; Belle County in Kentucky; and Rossville, Dade, Walker, Cattasa, Whitfield, Murray, Gordon, Chattooga in Georgia
Place of Meeting: Places varies
Local Organization: Chattanooga Engineers Club and Knoxville Technical Club
Luncheon Meeting every Monday noon at Chattanooga Engineers Club
Number of Members: 111

Executive Committee

W. R. Chambers, *Chairman*
W. S. Moorehouse, *Vice-Chairman*
J. Mack Tucker, *Secretary-Treasurer*
A. F. G. Bedinger
W. F. Hamlin
J. A. Hunter
C. B. Kerns
R. T. Mathews
D. C. Sherman

ERIE

Organized: 1917
Territory: Radius of thirty miles from Erie, Pa.
Place of Meeting: Erie County Court House
Number of Members: 95

Executive Committee

E. C. Ims, *Chairman*
G. H. Kaemmerling, *Vice-Chairman*
F. B. Schneider, *Secretary-Treasurer*
G. W. Bach
F. G. Brinig
H. E. Goetz
C. I. Rainesalo
McDonald S. Reed
H. B. Joyce, *Ex-Officio*

FLORIDA

Organized: 1925
Territory: State of Florida
Place of Meeting: Various Cities in State
Local Organization: Florida Engineering Society, Gainesville, Fla.
Number of Members: 82

Executive Committee

W. E. Drew, *Chairman*
John Hunter, *1st Vice-Chairman*
T. H. Gardner, *2nd Vice-Chairman*
R. A. Thompson, *Secretary*
Charles Beensen
A. B. Hale
A. Robert Major
H. N. Ellis, *Student-Chairman*

FORT WAYNE

Organized: 1939
Territory: Counties of LaGrange, Steuben, Noble, DeKalb, Whitley, Allen, Wabash, Huntington, Wells, Adams, Miami, Blackford and Jay in Indiana; Counties of Williams, Defiance, Paulding, Van Wert and Mercer in Ohio
Local Organization: Fort Wayne Engineers' Society
Number of Members: 37

Executive Committee

W. H. Connor, *Chairman*
K. K. Cooper, *Vice-Chairman*
E. S. Buck, *Secretary*
R. P. Loveland, *Treasurer*
W. L. Knaus
F. L. Ruoff

GREEN MOUNTAIN

Organized: 1923
Territory: Entire State of Vermont and neighboring and closely related communities of Claremont and Hanover, N.H.
Place of Meeting: Springfield, Windsor, Vt., and Claremont, N.H.
Local Organization: Vermont Engineering Society
Number of Members: 53

Executive Committee

C. H. Adams, *Chairman*
F. T. Gear, *Vice-Chairman*
F. A. Johnson, *Secretary-Treasurer*
E. D. Clark
H. L. Daasch
D. T. Hamilton
J. B. Johnson

GREENVILLE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Greenville, S.C.
Place of Meeting: Meetings held at Greenville, Clemson College, S.C., Canton, Asheville, and Enka, N.C.
Number of Members: 37

Executive Committee

R. B. Fuller, *Chairman*
R. H. Chapman, *Vice-Chairman*
J. C. Whitehurst, *Secretary-Treasurer*
B. E. Fernow
C. R. Hoey
J. A. McPherson
R. S. Pruitt
W. Paul Tindall
J. W. Vaughan, Jr., *Ex-Officio*

HARTFORD

Organized: 1917, as Branch of Conn. Section; reorganized, 1923; New Britain Section merged with Hartford Section, July 1, 1940
Territory: Hartford County except that portion served by New Britain Section
Place of Meeting: Hartford Electric Light Company
Number of Members: 194

Executive Committee

L. C. Smith, *Chairman*
E. R. Lewis, Jr., Henry Michelsen } *Vice-Chairmen*
G. E. Falk, *Secretary-Treasurer*
J. S. Appleyard
Herbert Burdick
T. F. Cassidy
F. O. Hoagland
R. D. Keller
W. S. Paine
H. F. Ramm
C. H. Richardson
G. E. Stanway
C. C. Stevens
S. J. Teller
R. M. Treat

INLAND EMPIRE

Organized: 1921
Territory: East of Columbia River in State of Washington, and Counties of Okanogan and Benton, and part of Northern Idaho
Place of Meeting: Davenport Hotel, Spokane
Luncheons: Wednesdays at 12:00 noon, Davenport Hotel, Spokane
Local Organization: Associated Engineers of Spokane
Number of Members: 19

Executive Committee

J. G. McGivern, *Chairman*
F. W. Candee, *Vice-Chairman*
N. W. Humphrey, *Secretary-Treasurer*
Alexander Lindsay
L. J. Pospisil
E. B. Parker

ITHACA

Organized: 1936
Territory: Radius of thirty miles from Ithaca plus following cities: Binghamton, Corning, Endicott, Geneva, Painted Post
Place of Meeting: Alternately in different cities of Section territory, as announced
Number of Members: 80

Executive Committee

C. Lyman Wilder, *Chairman*
S. S. Garrett, *Vice-Chairman*
F. S. Erdman, *Secretary-Treasurer*
D. S. Kimball
W. M. Sawdon
M. P. Whitney
N. R. Wickersham

KANSAS CITY

Organized: 1921
Territory: Radius of sixty miles from Kansas City, Mo.
Place of Meeting: University Club
Local Organization: Engineers' Club of Kansas City
Number of Members: 117

Executive Committee

M. A. Durland, *Chairman*
F. R. Applegate, *Vice-Chairman*
R. P. Hahn, *Secretary*
C. E. Pearce, *Treasurer*
E. E. Ambrosius
E. M. Bruzelius, Jr.
H. L. Crain
H. E. Manuel
L. T. Mart
J. R. Stone, *Ex-Officio*

LOUISVILLE

Organized: 1922
Territory: Radius of thirty miles from Louisville, Ky. (includes Lexington, Ky.)
Place of Meeting: University of Louisville, Louisville, Ky.
Local Organization: Engineers and Architects Club
Number of Members: 72

Executive Committee

W. F. Lucas, *Chairman*
J. K. Meyer, *Vice-Chairman*
H. V. Heuser, *Secretary*
C. D. Eldridge, *Treasurer*
N. W. Cummins
H. C. Murphy
J. H. Romann

MEMPHIS

Organized: 1923
Territory: Radius of sixty miles from Memphis, Tenn., and eastern half of Arkansas including all the territory east of a line drawn north and south through the western boundary of the city of Little Rock
Number of Members: 30

Executive Committee

W. H. Roberts, *Chairman*
J. A. Mollino, Jr., *Secretary-Treasurer*
Mack D. Rust

METROPOLITAN

Organized: 1910
Territory: Metropolitan District, New York and New Jersey
Place of Meeting: Engineering Societies Building, 29 West 39th Street, New York, N.Y.
Number of Members: 3,532

Executive Committee

G. J. Nicastro, *Chairman*
H. C. R. Carlson, *Secretary*
Adolf Ehbrecht, *Treasurer*
W. S. Gleeson
G. E. Hagemann
W. H. Larkin
H. E. Martin
P. T. Onderdonk
C. B. Peck
F. D. Carvin, *Ex-Officio*

Junior Group

C. C. Kirby, *Chairman*
F. H. Fowler, Jr., *Vice-Chairman*
F. M. Gibson, Jr., *Secretary*
J. S. Hunter
E. J. Nobles
R. A. Robertson
H. B. Strong

MID-CONTINENT

Organized: 1919
Territory: Entire State of Oklahoma; territory in Arkansas not included in Memphis Section; part of Louisiana; and territory in Texas north of the southern boundaries of the counties of Gaines, Dawson, Bordon, Scurry, Fisher, Jones, and Shackelford
Place of Meeting: Usually Mayo Hotel, Tulsa, Okla.
Luncheon Meetings with Engineers Club of Tulsa, Mondays at 12:00 noon
Local Organization: Engineers Club of Tulsa
Number of Members: 138

Executive Committee

W. L. Ducker, *Chairman*
V. C. Gillon, H. T. Sears, R. H. Teed, G. Wilenzick — *Vice-Chairmen*
J. H. Keyes, *Secretary*
E. H. Parker, *Treasurer*
R. G. Ayers
E. C. Baker
D. O. Barrett
D. A. Cant
R. A. Colgin
Edward Dolezal
C. O. Glasgow
Nathan Janco
Gwynne Raymond

MILWAUKEE

Organized: 1904
Territory: Radius of fifty miles from Milwaukee, Wis.
Place of Meeting: Wisconsin Club
Local Organization: Engineers' Society of Milwaukee
Luncheon Meetings once each month, 3rd Wednesday at Wisconsin Club
Number of Members: 279

Executive Committee

R. J. Smith, *Chairman*
J. L. Martin, *Secretary-Treasurer*
W. D. Bliss
F. H. Dorner, Jr.
T. F. Eserkaln
W. C. Lindemann
J. H. Stanek
LaRue H. Stark

Junior Group

J. H. Stanek, *Chairman*
M. E. Ruess, *Secretary*
J. J. Drinka
R. J. Fobian
Sebastian Judd
J. L. Martin
J. E. Peterman

MINNESOTA

Organized: Minneapolis, 1913; St. Paul. 1913; the two Sections merged, 1934
Territory: Entire State of Minnesota
Place of Meeting: Minnesota Union, Univ of Minnesota
Local Organization: Minneapolis Engineers' Club, Minnesota Federation of Architectural and Engineering Societies
Number of Members: 101

Executive Committee

L. C. Sprague, *Chairman*
W. H. Erskine, *Vice-Chairman*
L. S. Whitson, *Secretary-Treasurer*
Charles Foster
R. W. Jones, Jr.
L. G. Straub
M. S. Wunderlich

NEBRASKA

Organized: 1922
Territory: State of Nebraska, and Council Bluffs, Iowa
Place of Meeting: Lincoln and Omaha
Local Organization: Engineers' Clubs of Lincoln and Omaha
Luncheon Meeting every Wednesday noon at the Omaha Engineers' Club—4th Monday Evening at Lincoln
Number of Members: 35

NEBRASKA
(*Continued*)

Executive Committee

J. K. Ludwickson, *Chairman*
G. A. Rogers, *Vice-Chairman*
E. V. Perkins, *Secretary-Treasurer*
W. L. De Baufre
C. F. Moulton
W. F. Weiland

NEW HAVEN

Organized: 1912, reorganized, 1923
Territory: Portions of New Haven and Middlesex Counties, Conn.
Place of Meeting: Mason Laboratory, Yale University
Number of Members: 84

Executive Committee

F. C. Richardson, *Acting Chairman*
G. H. Eaton, *Vice-Chairman*
F. W. Preston, *Acting Secretary-Treasurer*
A. F. Breitenstein
A. D. Wilcox
L. H. Von Ohlsen, *Ex-Officio*

NEW ORLEANS

Organized: 1916
Territory: All of Louisiana except the northern part allotted to Mid-Continent Section
Place of Meeting: Room 422, St. Charles Hotel
Local Organization: Louisiana Engineering Society
Number of Members: 118

Executive Committee

L. J. Cucullu, *Chairman*
H. W. Waterfall, *Vice-Chairman*
A. M. Hill, *Secretary-Treasurer*
M. C. Abrahm
E. L. Cowan
C. R. Hammett
J. K. Mayer
E. A. McLellan

NORTH TEXAS

Organized: 1922
Territory: All of Texas north of an approximately straight line through Del Rio, Fredericksburg, Georgetown, Cameron, Nacogdoches, and center, including the cities mentioned, and south of north boundaries of the counties of Parmer, Castro, Swisher, Briscoe, Hall, and Childress. Also the City of Texarkana, Ark.
Place of Meeting: Dallas Power & Light Co. Bldg. Auditorium
Local Organization: Technical Club of Dallas
Number of Members: 116

Executive Committee

H. M. Robinson, *Chairman*
D. C. Pfeiffer, *Vice-Chairman*
E. J. Wacker, *Secretary-Treasurer*
J. K. Chattey
P. M. Cordell
J. A. Noyes
F. C. Justice, *Ex-Officio*

Student Chairmen

Hugh Campbell, Southern Methodist University
Harlan Dowell, Texas Technological College

NORWICH

Organized: 1930
Territory: Counties of Tolland, Windham, and New London in Connecticut, and Westerly District in Rhode Island
Place of Meeting: New London Junior College, Pequot Ave., New London, Conn.
Number of Members: 52

Executive Committee

A. W. Luce, *Chairman*
J. S. Leonard, *Secretary-Treasurer*
W. E. Beaney
G. A. Creighton
K. P. Hanson
W. A. Hardy
George Sandor

ONTARIO

Organized: 1917
Territory: Province of Ontario, Canada
Place of Meeting: Hart House, University of Toronto
Number of Members: 167

Executive Committee

H. H. Angus, *Chairman*
S. G. Clarke, *Vice-Chairman*
Ernest Jones, *Secretary-Treasurer*
T. H. Blair
C. R. Davis
H. G. Hill
T. S. Jardine
K. D. Leitch
W. G. McIntosh
A. A. Moline
W. D. Sheldon
A. D. Smith
Fred'k Truman

Junior Group

J. M. Van Winckle, *Chairman*
C. E. Beynon, *Secretary-Treasurer*
W. J. Fraser
A. D. Hogg
D. R. Malcolm

OREGON

Organized: 1919
Territory: State of Oregon and that territory in Washington within a radius of thirty miles from Portland, Ore.
Place of Meeting: Usually Public Service Bldg., Portland, Ore.
Local Organization: Oregon Society of Engineers
Number of Members: 55

Executive Committee

L. T. Hays, *Chairman*
E. P. Weiser, *Vice-Chairman*
M. M. Clayton, *Secretary*
A. J. Chaput, *Treasurer*
A. D. Hughes
G. F. McDougall
A. A. Osipovich
Tom Perry
E. D. Rowan

PENINSULA

Organized: 1923
Territory: West of the east boundaries of the following counties: Emmet, Charlevoix, Antrim, Kalkaska, Missaukee, Clare, Isabell, Gratiot, Clinton, Eaton, Calhoun, and Branch, Mich.
Place of Meeting: Grand Rapids, Mich.
Luncheon Meeting last Thursday noon each month
Local Organization: Engineers' Club of Grand Rapids
Number of Members: 54

Executive Committee

J. M. Gorrie, *Chairman*
Donald McSorley, *1st Vice-Chairman*
K. E. Crowser, *2nd Vice-Chairman*
C. J. Kuenzel, *Secretary-Treasurer*
C. P. Buck
J. R. de Hamer
G. E. Gebben
C. G. Lohmann
G. H. Waring
C. A. Hamilton, *Ex-Officio*

PHILADELPHIA

Organized: 1912
Territory: Counties of Bucks, Montgomery, Chester, Philadelphia, Delaware, Pa., and the State of Delaware
Place of Meeting: Philadelphia Engineers' Club, 1317 Spruce Street, Philadelphia, Pa.
Local Organization: Philadelphia Engineers' Club
Luncheon Meeting every Thursday noon at 12:30 p.m. at Philadelphia Engineers' Club
Number of Members: 1,090

Executive Committee

C. S. Gotwals, *Chairman*
L. N. Gulick, *Vice-Chairman*
F. W. Miller, *Secretary-Treasurer*
J. P. Clark
S. T. Mackenzie
J. J. McCarthy
J. S. Morehouse
M. C. Randall
B. W. Webb
R. W. Worley

Junior Group

W. B. Pegram, *Chairman*
G. G. Martinson, *Vice-Chairman*
R. D. Stapfer, *Secretary*
R. D. Shoaf, *Treasurer*

PIEDMONT—NORTH CAROLINA

Organized: As a Branch, 1923; as a Section 1927; name changed from Charlotte Section to Piedmont—North Carolina, July 1, 1940
Territory: Radius of seventy-five miles from Charlotte, N.C.
Luncheon Meeting every other Monday at 1:00 p.m. at Efirds Department Store Dining Room
Local Organization: Charlotte Engineers Club
Number of Members: 45

Executive Committee

A. G. LeClerc, *Chairman*
I. W. Leggett, *Vice-Chairman*
T. O. Sills, *Secretary-Treasurer*

PIEDMONT—NORTH CAROLINA
(Continued)

F. F. Bahnson
B. L. Cotton
Isaac Fidler
F. Raymond Jackson
R. W. Olive, *Ex-Officio*

PITTSBURGH

Organized: 1920
Territory: Counties bounded by and including Beaver, Butler, Venango, Forest, Jefferson, Indiana, Somerset, Fayette, Greene, and Washington, Pa.
Place of Meeting: Engineers' Society of Western Pennsylvania, William Penn Hotel
Local Organization: Engineers' Society of Western Pennsylvania
Number of Members: 496

Executive Committee

T. J. Barry, *Chairman*
L. E. F. Wahrenburg, *Vice-Chairman*
E. W. Jacobson, *Secretary*
K. F. Treschow, *Treasurer*
R. P. Ambrose
T. G. Estep
J. N. Evans
J. C. Hoar
R. E. Peterson
Kenneth Seaver

PLAINFIELD

Organized: 1921
Territory: Plainfield and territory included between Elizabeth, Bound Brook, Metuchen, and Watchung, N.J.
Place of Meeting: Elizabeth Carteret Hotel, Elizabeth, and Plainfield Masonic Temple, Plainfield
Local Organization: Plainfield Engineers Club, Singer Engineering Society
Number of Members: 211

Executive Committee

G. E. Leavitt, Jr., *Chairman*
P. F. Nydegger, *Vice-Chairman*
W. T. Christopher, *Secretary*
J. D. Potter, *Treasurer*
K. A. Reeve

PROVIDENCE

Organized: 1920
Territory: Radius of thirty miles from Providence, R.I.
Place of Meeting: Providence Engineering Society Building, 195 Angell St., Providence, R.I.
Local Organization: Providence Engineering Society
Number of Members: 168

Executive Committee

J. D. Robertson, *Chairman*
R. M. Scott, *Vice-Chairman*
E. W. Harrington, *Secretary-Treasurer*
J. S. Chafee
J. D. Eldert
G. W. Kelsey
P. N. Kistler
Alfred MacGillivray
A. William Meyer
P. V. Miller
R. L. Wales

RALEIGH

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Raleigh, N.C.
Places of Meeting: North Carolina State College, Raleigh, N.C.; Duke University, Durham, N.C.
Local Organizations: North Carolina Society of Engineers, Raleigh Engineers Club
Number of Members: 35

Executive Committee

F. B. Turner, *Chairman*
F. J. Reed, *Vice-Chairman*
F. C. Bragg, *Secretary*
G. W. Baity
R. B. Rice
R. M. Rothgeb

ROCHESTER

Organized: 1919
Territory: Radius of thirty miles from Rochester, N.Y.
Place of Meeting: Rochester Engineering Society Rooms, Sagamore Hotel
Local Organization: Rochester Engineering Society, Sagamore Hotel
Luncheon Meeting every Tuesday at 12:15 p.m. at Sagamore Hotel
Number of Members: 115

Executive Committee

W. D. Wood, *Chairman*
A. W. Schuster, *Vice-Chairman*
I. G. McChesney, *Secretary-Treasurer*
W. J. Brown
C. A. Elwood
A. W. Moxon
F. D. Punnett
A. E. Schell
S. C. Stacy

ROCK RIVER VALLEY

Organized: 1926
Territory: Thirty miles east and west of Madison, Wis., and extending southward through Rockford, Ill.
Meeting Places: Madison, Wis., Beloit, Wis., and Rockford, Ill.
Local Organization: Rock River Valley Engineering Council
Number of Members: 52

Executive Committee

B. G. Elliott, *Chairman*
K. H. Casson, *Vice-Chairman*
D. W. Nelson, *Secretary-Treasurer*
W. K. Cochran
E. L. Dahlund
C. A. Jacobson
J. C. White

ST. JOSEPH VALLEY

Organized: 1929
Territory: Counties of La Porte, Starke, Pulaski, St. Joseph, Marshall, Fulton, Elkhart, and Kosciusko in Indiana, and Cass and Berrien Counties in Michigan
Place of Meeting: South Bend, Ind.
Number of Members: 52

Executive Committee

L. E. Waddington, *Chairman*
E. T. Cobb, *Secretary*
C. R. Egry
W. D. A. Peaslee
C. C. Wilcox
O. E. E. Zahn

ST. LOUIS

Organized: 1909
Territory: Radius of thirty miles from St. Louis, Mo.
Place of Meeting: Place varies
Local Organization: Engineers' Club of St. Louis
Number of Members: 220

Executive Committee

R. C. Thumser, *Chairman*
C. B. Briscoe, *Vice-Chairman*
H. B. Welge, *Secretary-Treasurer*
Herbert Kuenzel
R. W. Merkle
Albert Vigne

SAN FRANCISCO

Organized: 1910
Territory: All territory north of the northern boundaries of the counties of San Luis Obispo, Kern, and San Bernardino
Place of Meeting: Engineers' Club, 206 Sansome St.
Luncheon Meetings, Tuesdays, California Hotel, Oakland; Thursdays, Engineers' Club, San Francisco
Local Organization: San Francisco Engineers' Club
Number of Members: 420

Executive Committee

G. N. Somerville, *Chairman*
R. B. Plass, *Vice-Chairman*
W. H. Kassebohm, *Secretary-Treasurer*
H. T. Avery
W. S. Everett
Alf Hansen
A. K. Ingraham

Junior Group

E. D. Kane, *Chairman*
E. E. Polomik, *Vice-Chairman*
P. B. Dawson, *Secretary*

SAVANNAH

Organized: 1923
Territory: Radius of 125 miles from Savannah in Georgia
Place of Meeting: Savannah Hotel
Local Organization: Engineers' Council of Savannah Chamber of Commerce
Number of Members: 22

Executive Committee

W. L. Mingledorff, Jr., *Chairman*
A. M. Ormond, *Vice-Chairman*
B. J. Sams, *Secretary-Treasurer*
W. H. Artley
A. P. Keisker
L. C. Roesel

SCHENECTADY

Organized: As a Branch, 1919; as a Section, 1927
Territory: Radius of thirty miles from Schenectady, N.Y.
Place of Meeting: Rice Hall
Number of Members: 259

SCHENECTADY
(Continued)

Executive Committee

Stanford Neal, *Chairman*
D. C. Brecht, A. P. Kellogg, E. L. Thearle, *Vice-Chairmen*
L. A. Gore, *Secretary*
J. E. Ryan, *Treasurer*
H. E. Brunelle
D. R. Edwards
R. G. McAndrew

SOUTHERN CALIFORNIA

Organized: 1915 as Los Angeles Section; reorganized 1941
Territory: South of southern boundaries of following counties: Monterey, Kings, Tulares, and Inyo, Calif.
Place of Meeting: Barker Bros. Store
Local Organization: Technical Societies of Los Angeles
Luncheon Meetings Thursdays at 12:00 noon at Engineers' Club
Number of Members: 568

Executive Committee

J. Calvin Brown, *Chairman*
J. S. Gallagher, *Vice-Chairman*
R. G. Roshong, *Secretary-Treasurer*
D. K. Coyle
B. M. Palm
D. K. Rollow
Edward Timbs
Lloyd Yost
P. L. Armstrong, Jr., *Ex-Officio*

Junior Group

W. H. Shallenberger, *Chairman*
C. M. Sandland, *Secretary*
W. K. Botticher

San Diego Subcommittee

J. L. Bacon, *Chairman*

SOUTH TEXAS

Organized: 1919
Territory: South Texas and the northern part of the State not included in the North Texas Section territory
Place of Meeting: No fixed meeting place, but all meetings held in Houston
Number of Members: 181

Executive Committee

H. F. Moller, *Chairman*
H. E. Degler, *Vice-Chairman*
S. G. Kershner, *Secretary-Treasurer*
John Doggett, Jr.
J. B. T. Downs
V. M. Faires
B. W. Farquhar
H. G. Hiebeler
J. J. King
M. H. Kotzebue
Ralph Neuhaus
C. L. Orr
F. D. Rahm

SUSQUEHANNA

Organized: 1927
Territory: Counties of Cumberland, Dauphine, Lebanon, Adams, York, and Lancaster
Place of Meeting: Engineering Society of York, and at Lancaster Twice a Year
Local Organization: Engineering Society of York and Engineers' Society of Pennsylvania, Harrisburg, Pa.
Number of Members: 85

Executive Committee

S. P. Soling, *Chairman*
Jacob Fisch, *Vice-Chairman*
T. K. Breda, *Secretary*
M. F. Gannett
M. G. Leeson
E. T. P. Neubauer
O. E. Weber

SYRACUSE

Organized: 1920
Territory: Radius of thirty miles from Syracuse, N.Y.
Place of meeting: Ball Room of the Onondaga Hotel
Local Organization: The Technology Club of Syracuse
Number of Members: 84

Executive Committee

D. V. Shetland, *Chairman*
M. F. Williams, *Vice-Chairman*
E. A. Failmezger, *Secretary-Treasurer*
L. E. Potter, *Assistant Secretary-Treasurer*
H. T. Avery
C. F. Dietz
W. E. Hopton
E. R. Jefferson
J. W. Linford
M. B. Moyer
E. K. Rhodes
G. I. Vincent

TOLEDO

Organized: 1920
Territory: Radius of thirty miles from Toledo, Ohio
Place of Meeting: University Club, Toledo, Ohio
Local Organization: Affiliated Technical Societies of Toledo
Number of Members: 57

Executive Committee

Sydney Bevin, *Chairman*
N. W. Dorman, *Vice-Chairman*
N. K. Anderson, *Secretary-Treasurer*
T. L. Hallenbeck
R. F. Hill
B. H. Keyserling
H. O. Kranich
W. C. Schroeder
E. W. Weaver
H. L. Yaryan, Jr.

TRI-CITIES

Organized: 1920
Territory: Radius of thirty miles from Moline, Ill.
Place of Meeting: Rock Island, Ill., Moline, Ill., and Davenport, Iowa.
Luncheon Meeting every Wednesday, Davenport Hotel, 12:00 noon
Number of Members: 80

Executive Committee

K. R. Hodges, *Chairman*
P. E. Anderson, *Vice-Chairman*
C. A. Carlson, *Secretary-Treasurer*
R. M. Barnes
R. A. Cross
J. M. Hartman

UTAH

Organized: 1923
Territory: State of Utah
Place of Meeting: University Club, Salt Lake City
Local Organization: Utah Society of Professional Engineers
Number of Members: 41

Executive Committee

G. W. Carter, *Chairman*
H. L. Zetterman, *Vice-Chairman*
R. D. Baker, *Secretary-Treasurer*
E. L. Koffmann
R. E. Kunkel
J. P. Olgardt

VIRGINIA

Organized: 1919
Territory: State of Virginia
Place of Meeting: Richmond, Norfolk, Blacksburg, Charlottesville, Roanoke, University, Petersburg
Local Organization: Central Virginia Engineers Club of Hampton Roads
Number of Members: 224

Executive Committee

J. B. Jones, *Chairman*
D. A. Roger, *Vice-Chairman*
F. S. Roop, Jr., *Secretary*
W. J. Barber, *Treasurer*
Otis N. Hall
H. R. Hopkins
G. E. Hoppe, Jr.
M. L. Ireland, Jr.
Arthur Roberts, Jr.
H. C. Wintzer

WASHINGTON, D.C.

Organized: 1919
Territory: District of Columbia
Place of Meeting: Auditorium, Potomac Electric Power Co., 10th & E Sts., Washington, D.C.
Number of Members: 453

Executive Committee

J. W. Huckert, *Chairman*
C. E. Miller, *Vice-Chairman*
E. J. Lane, *Secretary-Treasurer*
W. P. Green
Rudolph Michel
W. B. Ensinger, *Ex-Officio*

WATERBURY

Organized: 1917, as a Branch; reorganized as a Section, 1923
Territory: Litchfield County and a portion of New Haven County
Place of Meeting: Elton Hotel
Number of Members: 74

Executive Committee

A. L. Alves, *Chairman*
J. R. Hicks, *Vice-Chairman*
A. W. Miner, *Secretary-Treasurer*
P. E. Petersen
W. C. Schneider
R. S. Sperry
R. S. Storrs
C. M. Warner

WESTERN MASSACHUSETTS

Organized: 1922
Territory: Includes counties of Berkshire, Franklin, Hampden, and Hampshire
Place of Meeting: Highland Hotel, Springfield, Mass.
Local Organization: Engineering Society of Western Massachusetts
Number of Members: 83

Executive Committee

R. A. Packard, *Chairman*
E. Lovell Smith, *Vice-Chairman*
D. A. Bartlett, *Secretary-Treasurer*
J. F. Malcolm
E. Wadsworth Stone
C. F. Dupee, *Ex-Officio*

WESTERN WASHINGTON

Organized: 1919
Territory: State of Washington west of Columbia River with exception of territory included in 30-mile radius of Portland, Ore.
Place of Meeting: Engineers' Club, Seattle, Wash.
Local Organization: Seattle Engineers' Club
Luncheon Meetings daily at noon at Engineers' Club, Seattle
Number of Members: 195

Executive Committee

N. J. Schaal, *Chairman*
G. F. Gayer, *Vice-Chairman*
L. B. Cooper, *Secretary-Treasurer*
F. E. Blumberg
J. P. Butler
D. G. Fassett
C. B. Shovar, *Ex-Officio*

WEST VIRGINIA

Organized: 1925
Territory: State of West Virginia, South of Parallel 39
Place of Meeting: Charleston, W.Va.
Number of Members: 40

Executive Committee

J. G. Miller, *Chairman*
E. L. Hudson, *Vice-Chairman*
J. L. Barker, Jr., *Secretary-Treasurer*
G. J. Huber, Jr., *Assistant Secretary-Treasurer*
M. S. Bloomsburg
A. H. Cannon
H. L. Carspecken
J. F. Malloy
L. B. McQuaide

WORCESTER

Organized: 1915
Territory: Radius of thirty miles from Worcester, Mass.
Place of Meeting: Sanford Riley Hall, Worcester Poly. Inst.
Local Organization: Worcester Engineering Society
Number of Members: 131

Executive Committee

G. H. MacCullough, *Chairman*
H. P. Crane, *Vice-Chairman*
W. F. Beth, *Secretary-Treasurer*
L. R. Ball
L. J. Hooper
C. M. MacMahon
F. W. Mierke
F. E. Strandberg
W. M. Wilcox

YOUNGSTOWN

Organized: 1928
Territory: Counties of Trumbull, Mahoning, and Columbiana in Ohio, and Mercer and Lawrence in Pennsylvania
Place of Meeting: Republic Rubber Co. Club Rooms, Albert St., Youngstown, Ohio
Number of Members: 64

Executive Committee

H. W. Smith, *Chairman*
L. A. Kline, *Vice-Chairman*
C. W. Foard, *Secretary-Treasurer*
F. J. Bowers
C. H. Legler
W. J. Longacher
H. E. Melin

STUDENT BRANCHES

ARTICLE B6A, PAR. 20: The Standing Committee on Relations With Colleges shall, under the direction of the Council, have supervision of the Student Branches of the Society and of such work of the Society as aims to further the education of engineers through the colleges and schools of accepted standing.

STANDING COMMITTEE, RELATIONS WITH COLLEGES

J. I. YELLOTT, *Chairman* (1943)
H. E. DEGLER (1944)
G. L. SULLIVAN (1945)
R. P. REECE (1946)
H. J. BROWN (1947)

A. D. HUGHES, HERBERT KUENZEL, R. H. PORTER, J. W. ZELLER — *Advisory Members* (1943)

C. K. HOLLAND, *Junior Adviser* (1943)

Communicate with Student Branch through Honorary Chairman

Name and Location	Year Authorized	No. of Members †	Chairman	Secretary	Honorary Chairman
Akron, Univ. of, Akron, Ohio	1924	49	Z. N. Harris	R. M. Kallgren	F. S. Griffin
Alabama Polytechnic Inst., Auburn, Ala.	1920	52	W. C. Richter	C. A. Overbey	C. R. Hixon
Alabama, Univ. of, University, Ala.	1931	21	....	....	J. M. Gallalee
Arizona, Univ. of, Tucson, Ariz.	1937	29	F. S. Fiedler	Reece Dunaway	M. L. Thornburg
Arkansas, Univ. of, Fayetteville, Ark.	1910	23	Ralph Dougherty	A. H. Bacher	R. G. Paddock
British Columbia, Univ. of, Vancouver, B.C., Can.	1938	53	Don Bannerman	Walter Goodwin	W. O. Richmond
Brown Univ., Providence, R.I.	1923	15	J. H. Blake	J. T. Toher	P. N. Kistler
Bucknell Univ., Lewisburg, Pa.	1916	37	K. J. Baserman	M. A. Clark	W. D. Garman
California Inst. of Tech., Pasadena, Calif.	1914	41	H. A. Lassen	R. W. Protzen	W. H. Clapp
California, Univ. of, Berkeley, Calif.	1912	85	Les Burgess	Merrick Taylor	R. G. Folsom
Carnegie Inst. of Tech., Pittsburgh, Pa.	1913	65	Gail Mainknecht	Paul Miller	T. G. Estep
Case School of Applied Science, Cleveland, Ohio	1913	100	Edward Koepke	Robert Tuve	W. A. Lynam
Catholic Univ. of America, Washington, D.C.	1922	29	J. D. Gallivan, III	J. J. Gormley	C. O. G. Wittig
Cincinnati, Univ. of, Cincinnati, Ohio	1909	108	Harold Hemstreet	Albert Meyer	C. A. Joerger
Clarkson College of Tech., Potsdam, N.Y.	1930	54	Gerald Zecher	Rolf Neunes	H. A. Weiss
Clemson A.&M. College, Clemson College, S.C.	1921	52	W. H. Parks, Jr.	J. M. Richbourg	B. E. Fernow
Colorado State College of A.&M. Arts, Fort Collins, Colo.	1914	23	J. K. Dickson	G. H. Booton	J. H. Scofield
Colorado, Univ. of, Boulder, Colo.	1914	40	C. F. Drexel	J. N. White	W. F. Mallory
Colorado School of Mines Division, Golden	——	10	Rex Cheek	C. E. Morgan	W. M. Richtman
Columbia Univ., New York, N.Y.	1909				
Management Division	——	1	....	....	C. F. Kayan
Mechanical Division	——	24	....	J. P. Bartels	C. F. Kayan
Connecticut, Univ. of, Storrs, Conn.	1941	40	Norman Olson	J. E. Donahue	K. P. Hanson
Cooper Union Inst. of Tech., New York, N.Y.	1920	49	Emidio Peluso	Robert Leff	W. A. Vopat
Cooper Union Night School of Engineering, New York, N.Y.	1920	25	....	....	E. A. Salma
Cornell Univ., Ithaca, N.Y.	1908	78	R. H. Flack	R. E. Warren	L. T. Wright
Delaware, Univ. of, Newark, Del.	1929	9	A. Q. Mowbray, Jr.	George Lucas	Leo Blumberg
Detroit, Univ. of, Detroit, Mich.	1930	42	Jack Lance	Morley LeGarie	J. J. Uicker
Drexel Inst. of Tech., Philadelphia, Pa.	1920	29	George Darby	Alwynne Clemmer	A. H. Repscha
Duke Univ., Durham, N.C.	1935	34	C. H. Gingher	S. L. Gulledge, Jr.	E. S. Thiess
Florida, Univ. of, Gainesville, Fla.	1926	27	H. N. Ellis	T. B. Pasteur, Jr.	N. C. Ebaugh
George Washington Univ., Washington, D.C.	1924	27	R. W. McCullough	W. H. Hisic	B. J. Cruickshanks
Georgia School of Tech., Atlanta, Ga.	1915	76	R. L. Kreger	R. W. Feagles	R. L. Allen
Idaho, Univ. of, Moscow, Idaho	1925	45	John Gibson	Richard Levering	H. F. Gauss
Illinois Inst. of Tech., Chicago, Ill.	1940	177	J. R. Cronin	R. W. Roberts	J. S. Kozacka
Illinois, Univ. of, Urbana, Ill.	1909	122	G. P. Salnero	R. G. Tuell	P. E. Mohn
Iowa State College, Ames, Iowa	1919	80	Marion Brush	Marvin Winders	R. W. Breckenridge
Iowa, State Univ. of, Iowa City, Iowa	1913	19	J. W. Burnside	Abraham Cohen	I. T. Wetzel
Johns Hopkins Univ., Baltimore, Md.	1917	30	E. F. Vitek	Ferdinand Kuehn	Julian C. Smallwood
Kansas State College, Manhattan, Kan.	1914	44	S. O. Jewett	D. J. Blevins	A. J. Mack
Kansas, Univ. of, Lawrence, Kan.	1909	34	John Beamer	Leon Carlson	E. E. Ambrosius
Kentucky, Univ. of, Lexington, Ky.	1911	32	Ralph Eschborn	H. J. Macke	C. C. Jett
Lafayette College, Easton, Pa.	1919	51	G. M. Betterley	J. S. Attinello	E. W. Nelson
Lehigh Univ., Bethlehem, Pa.	1911	..	A. W. Hemphill, Jr.	....	A. C. Bates
Louisiana State Univ., University, La.	1916	54	Frank Carroll	L. R. Daniel, Jr.	G. F. Matthes
Louisville, Univ. of, Louisville, Ky.	1928	32	John Burns	D. E. Mapother	H. H. Fenwick
Maine, Univ. of, Orono, Maine	1910	62	R. D. McKeen	S. F. Gilman	I. H. Prageman
Marquette Univ., Milwaukee, Wis.	1923	40	J. C. Kemp	A. E. Mundt	J. E. Schoen
Maryland, Univ. of, College Park, Md.	1937	94	G. C. Webster	J. A. Kressinger	W. P. Green
Massachusetts Inst. of Tech., Cambridge, Mass.	1909	104	K. R. Wadleigh	H. L. Shivek	J. A. Hrones
Michigan College of Min. & Tech., Houghton, Mich.	1930	28	Edward Piech	Raymond Fahlen	A. P. Young
Michigan State College, East Lansing, Mich.	1917	95	Peter Duch	P. D. Liebig	J. M. Campbell
Michigan, Univ. of, Ann Arbor, Mich.	1914	58	J. K. Koffel	Norman Jimmerson	R. C. Porter
Minnesota, Univ. of, Minneapolis, Minn.	1913	67	Daniel Schiavone	Robert Melcher	Fulton Holtby
Mississippi State College, State College, Miss.	1926	32	E. W. Pettis, Jr.	R. L. Wilson	O. D. M. Varnado
Missouri School of Mines & Metallurgy, Rolla, Mo.	1930	38	Johnson Wright	Seymour Orlofsky	A. V. Kilpatrick

† As of January 4, 1943.

Name and Location	Year Authorized	No. of Members †	Chairman	Secretary	Honorary Chairman
Missouri, Univ. of, Columbia, Mo.	1909	58	John Simon	Richard Phelan	E. S. Gray
Montana State College, Bozeman, Mont.	1920	17	Edward Kunkel	Cecil Johnson	R. E. Gibbs
Nebraska, Univ. of, Lincoln, Neb.	1909	52	Robert Marcotte	Melvin Hartmann	W. F. Weiland
Nevada, Univ. of, Reno, Nev.	1928	16	Arthur Weller	Robert Rae	J. R. VanDyke
Newark College of Engineering, Newark, N.J.	1924	80	R. D. Hull	Milton Maxwell	H. F. Ritterbusch
New Hampshire, Univ. of, Durham, N.H.	1926	44	F. H. Robbins	G. F. Kelley	E. P. Nye
New Mexico State College of A.&M. Arts, State College, New Mex.	1938	8	Charles Botkin	Jack Howard	A. M. Lukens
New Mexico, Univ. of, Albuquerque, New Mex.	1935	29	Robert Simpers	Ronald Schneider	A. D. Ford, Sr.
New York, College of the City of, New York, N.Y.	1922	78	Howard Judson	Harold Berkol	E. B. Smith
New York Univ. (Day), New York, N.Y.	1909	68	M. P. Kurutz	R. A. Avigdor	A. H. Church
New York Univ. Evening School, New York, N.Y.	1933	59	S. V. Hrankowski	Ravin Weingart	A. H. Church
North Carolina State College, Raleigh, N.C.	1920	75	I. J. Hetherington	J. L. Singer	F. C. Bragg
North Dakota Agricultural College, Fargo, N.D.	1929	23	Rex Sherrett	Ralph Schuricht	A. W. Anderson
North Dakota, Univ. of, Grand Forks, N.D.	1923	17	Palmer Reiten	Bruce Fox	Alex Diakoff
Northeastern Univ., Boston, Mass.					
Division A	1922	166	Douglas C. Handy	Nino Molino	A. E. Whittaker
Division B	——	..	L. K. Scott, Jr.	P. F. Sheridan	A. E. Whittaker
Northwestern Univ., Evanston, Ill.	1935	17	J. W. Switack	G. V. Dodd	B. H. Jennings
Notre Dame, Univ. of, Notre Dame, Ind.	1929	40	John Garceau	H. E. Reilly	C. R. Egry
Ohio Northern Univ., Ada, Ohio	1922	19	E. B. Freyfogel	Charles Browning	J. A. Needy
Ohio State Univ., Columbus, Ohio	1911	52	Robert Keener	Richard Lipp	S. M. Marco
Oklahoma A.&M. College, Stillwater, Okla.	1921	54	Cliff Isbell	Ed Talley	C. M. Leonard
Oklahoma, Univ. of, Norman, Okla.	1917	76	G. P. Wild	James Overbey	E. M. Sims
Oregon State Agricultural College, Corvallis, Ore.	1909	72	Edmund Fearey	Thomas Marshall	A. D. Hughes
Pennsylvania State College, State College, Pa.	1909	78	John Melzer	Boris Osojnak	C. L. Allen
Pennsylvania, Univ. of, Philadelphia, Pa.	1925	29	W. W. Whitmore, Jr.	J. J. Littley	August Ulmann, Jr.
Pittsburgh, Univ. of, Pittsburgh, Pa.	1917	35	R. W. Kahn	H. J. Zerbey	T. G. Beckwith
Polytechnic Inst. of Brooklyn (Day), Brooklyn, N.Y.	1909	34	R. C. Miller	A. D. Dutchman	A. T. Kniffen
Polytechnic Inst. of Brooklyn Evening School, Brooklyn, N.Y.	1909	19	William Angus	Al Block	A. T. Kniffen
Pratt Inst., Brooklyn, N.Y.	1923	103	Frank Milici	William Thompson	K. E. Quier
Princeton Univ., Princeton, N.J.	1926	9	W. A. Hutton	L. R. Payton	C. L. Tutt, Jr.
Puerto Rico, Univ. of, Mayaguez, P.R.	1923	..	....	....	Auturo Davila
Purdue Univ., West Lafayette, Ind.	1909	145	D. M. Badger	Meyer Evanson	R. W. Leutwiler
Queen's College, Kingston, Ont., Can.	1941	48	J. H. Brazier	K. A. McCaffrey	W. A. Wolfe
Rensselaer Polytechnic Inst., Troy, N.Y.	1910	73	D. P. Peterson	J. E. Morrill	M. A. Cook
Rhode Island State College, Kingston, R.I.	1930	29	Eugene D'Aquanno	Walter Kudzma	E. L. Carpenter
Rice Inst., Houston, Tex.	1926	48	R. C. Wittlinger	K. A. Campbell	J. B. T. Downs
Rose Polytechnic Inst., Terre Haute, Ind.	1926	42	Antonio Bogran	Richard Mott	H. C. Gray
Rutgers Univ., New Brunswick, N.J.	1920	59	Christopher Maggio	John Cantwell	J. B. Cejka
Santa Clara, Univ. of, Santa Clara, Calif.	1925	11	Paul Steffen	W. P. Houle	O. F. Zahn
South Dakota State College, Brookings, S.D.	1935	20	Roger A. Miller	J. E. Wendt	L. L. Amidon
Southern California, Univ. of, Los Angeles, Calif.	1929	43	Robert Mannes	J. W. Maston	S. F. Duncan
Southern Methodist Univ., Dallas, Tex.	1933	15	Hugh Campbell	Carl Ivey	Stanley Patterson
Stanford Univ., Stanford University, Calif.	1909	30	Stephen Kline	Henry Metzer	R. A. Seban
Stevens Inst. of Tech., Hoboken, N.J.	1908	25	Jack Howe	Robert Sullivan	E. H. Fezandie
Swarthmore College, Swarthmore, Pa.	1921	28	H. B. McCormick, Jr.	C. A. Cibelius	P. J. Potter
Syracuse Univ., Syracuse, N.Y.	1912	45	D. R. Fisher	V. G. Ward	J. A. King
Tennessee, Univ. of, Knoxville, Tenn.	1923	60	W. K. Stair	J. T. Bailey	F. R. O'Brien
Texas, A.&M. College of, College Station, Tex.	1921	71	Martin Lantau	R. C. Halton	J. G. H. Thompson
Texas Technological College, Lubbock, Tex.	1930	63	C. R. Allters	Eugene Davidson	L. J. Powers
Texas, Univ. of, Austin, Tex.	1921	66	T. R. Casberg	Don Lewis	H. J. Kent
Toronto, Univ. of, Toronto, Ont., Can.	1933	102	J. P. G. Gordon	W. A. Moeser	G. R. Lord
Tufts College, Tufts College, Mass.	1917	55	W. N. Kernander	R. E. Hunt	D. A. Fisher
Tulane Univ. of Louisiana, New Orleans, La.	1933	54	W. B. Rudolf	J. M. Jennings, Jr.	A. M. Hill
U.S. Naval Academy, Postgraduate School, Annapolis, Md.	1925	..	....	....	P. J. Kiefer
Utah, Univ. of, Salt Lake City, Utah	1923	43	Morton Larsen	Dale Worden	M. B. Hogan
Vanderbilt Univ., Nashville, Tenn.	1928	31	Warren Seyfried	John Weydell	R. N. McDonald
Vermont, Univ. of, Burlington, Vt.	1922	12	Joseph Corbett	M. F. Nelson	R. G. Chapman
Villanova College, Villanova, Pa.	1925	51	H. C. Behnke	W. J. Doyle	K. J. Moser
Virginia Polytechnic Inst., Blacksburg, Va.	1915	187	F. S. Childers	F. R. Chase	E. M. Simons
Virginia, Univ. of, University, Va.	1923	34	James Borden	W. W. Willoughby	H. C. Hesse
Washington, State College of, Pullman, Wash.	1920	31	Robert Hohnson	Charles McIntosh	F. W. Candee
Washington Univ., St. Louis, Mo.	1911	38	Robert Bown	Harry Nelgnar	R. R. Tucker
Washington, Univ. of, Seattle, Wash.	1917	34	W. L. Carlson	Oliver Laster	L. B. Cooper
West Virginia Univ., Morgantown, W.Va.	1922	22	James Couch	J. L. Jenkins	H. M. Cather
Wisconsin, Univ. of, Madison, Wis.	1909	42	J. P. Wilson	John McCann	D. W. Nelson
Worcester Polytechnic Inst., Worcester, Mass.	1914	55	Theodore Pierson, III	Herbert Marsh	L. H. Hooper
Wyoming, Univ. of, Laramie, Wyo.	1925	17	J. F. Shutts	Frank Iwatsuki	C. E. Anderson
Yale Univ., New Haven, Conn.	1910	66	J. A. LeVan	C. B. Hart	S. W. Dudley

† As of January 4, 1943.

RESEARCH COMMITTEES

Article B6A, Par. 24: The Standing Committee on Research shall, under the direction of the Council, have supervision of the research activities of the Society.

The first Standing Committee on Research was organized in 1909.

STANDING COMMITTEE

M. D. Hersey, *Chairman* (1943)
Herman Weisberg (1944)
W. R. Elsey (1945)
J. F. D. Smith (1946)
D. L. Lindquist (1947)

LUBRICATION

Appointed October, 1915, to investigate the fundamental problems of lubrication, to formulate results of investigations previously made, and to keep in touch with contemporary research in this field

(Reorganized May, 1936)

G. B. Karelitz, *Chairman* †
S. J. Needs, *Secretary*
A. L. Beall
Oscar Bridgeman
W. E. Campbell
H. A. Everett
A. E. Flowers
J. C. Geniesse
Raymond Haskell
M. D. Hersey
B. F. Hunter
C. M. Larson
F. C. Linn
G. L. Neely
B. L. Newkirk
E. S. Pearce
Ernest Wooler

FLUID METERS

Appointed 1916 to develop the theory of fluid meters of all kinds and to report on the best methods for their installation and use

(Reorganized July, 1926)

R. J. S. Pigott, *Chairman*
J. R. Carlton, *Secretary*
H. S. Bean
S. R. Beitler
R. K. Blanchard
B. O. Buckland
Louis Gess
E. W. Jacobson
A. J. Kerr
T. H. Kerr
I. O. Miner
M. P. O'Brien
W. S. Pardoe
L. K. Spink
R. E. Sprenkle
E. C. M. Stahl
T. R. Weymouth
M. J. Zucrow

STRENGTH OF GEAR TEETH

Appointed in December, 1921, to investigate factors affecting the strength and life of gear teeth

R. E. Flanders, *Chairman*
C. H. Logue, *Secretary*
Earle Buckingham
A. M. Greene, Jr.
C. W. Ham
F. E. McMullen
E. W. Miller
Ernest Wildhaber

† Deceased, January 19, 1943.

CUTTING OF METALS

Appointed in September, 1923, to study the problems of metal cutting, including tool materials, tool design, lubrication, cooling, and speeds and feeds

(Reorganized April, 1942)

Executive Committee (Total personnel 18)

M. F. Judkins, *Chairman*
O. P. Adams, *Secretary*
O. W. Boston
R. C. Deale
Howard Scott

MECHANICAL SPRINGS

Appointed May, 1924, to determine the status of the mechanical-spring art, to promote and conduct necessary and adequate research, and to develop the art to the point of standardization

J. R. Townsend, *Chairman*
C. T. Edgerton, *Secretary*
R. W. Cook
W. T. Donkin
Rupen Eksergian
G. E. Hansen
Benjamin Liebowitz
David Lofts
R. D. Brizzolara (*Alternate*)
D. J. McAdam, Jr.
L. C. Peskin
R. E. Peterson
J. W. Rockefeller, Jr.
B. W. St. Clair
M. F. Sayre
T. R. Weber
Keith Williams
J. K. Wood
F. P. Zimmerli

ELEVATORS

Appointed June, 1924, to study the function and operation of elevator safeties and buffers and their associated mechanisms and to develop methods of test for the approval of elevator safety devices

(Reorganized August, 1940)

D. J. Purinton, *Chairman*
D. L. Lindquist, *Vice-Chairman*
G. H. Reppert (*Alternate*)
J. A. Dickinson, *Secretary*
S. W. Jones, *Ex-Officio*
E. M. Bouton
E. B. Dawson (*Alternate*)
K. A. Colahan
G. P. Keogh
F. Pavlicek (*Alternate*)
J. J. Matson
M. B. McLauthlin
C. R. Callaway (*Alternate*)
W. S. Paine
J. L. Keane (*Alternate*)
C. A. Peters

EFFECT OF TEMPERATURE ON THE PROPERTIES OF METALS

Appointed December, 1924, as a joint research committee of the A.S.T.M. and the A.S.M.E. to encourage the investigation and accumulation of data on the properties of metals used in the mechanic arts at extremely high and low temperatures

N. L. Mochel, *Chairman*
H. J. Kerr, *Vice-Chairman*
J. W. Bolton, *Secretary*
R. H. Aborn
W. H. Armacost
A. B. Bagsar
A. D. Bailey
F. E. Bash
C. L. Clark
E. S. Dixon
F. B. Foley
J. R. Freeman, Jr.
H. J. French
H. W. Gillett
A. J. Herzig
G. F. Jenks
J. J. Kanter
C. E. MacQuigg
E. L. Robinson
J. H. Romann
A. E. White
J. S. Worth
Director, National Bureau of Standards, U.S. Department of Commerce
Representative of Bureau of Ships, U.S. Navy Department

BOILER FEEDWATER STUDIES

Appointed March, 1925, as a Joint Research Committee of the American Boiler Manufacturers Association, American Railway Engineering Association, American Water Works Association, Edison Electric Institute, the American Society for Testing Materials, and the A.S.M.E. to study methods of analysis and treatment of boiler feedwater for stationary and railroad practice

Executive Committee (Total personnel 41)

C. H. Fellows, *Chairman*
R. C. Bardwell, *Vice-Chairman*
J. B. Romer, *Secretary*
A. G. Christie *
R. E. Coughlan
B. W. De Geer
Max Hecht
H. E. Jordan
P. B. Place
S. T. Powell
F. N. Speller
M. F. Stack
G. E. Tate
E. H. Tenney
A. E. White *

CONDENSER TUBES

Appointed May, 1925, to investigate and report on the causes of failure of tubes used in steam condensers and similar heat interchange apparatus

A. E. White, *Chairman*
D. C. Weeks, *Vice-Chairman*
P. A. Bancell
R. A. Bowman
E. S. Bunn

* Official A.S.M.E. representative serving on this committee.

CONDENSER TUBES
(*Continued*)

D. K. Crampton
C. A. Crawford
H. M. Cushing
R. E. Dillon
C. O. Evans
J. R. Freeman, Jr.
V. M. Frost
C. F. Harwood
G. C. Holder
W. C. Holmes
W. B. Price
J. S. Rodgers
M. F. Stack
W. R. Webster
Director, Bureau of Ships, U.S. Navy Department

WORM GEARS

Appointed May, 1927, to investigate certain problems in connection with the action of worm gear drives and to recommend improvements in their design, manufacture, and use

Earle Buckingham, *Chairman*
G. H. Acker
L. R. Buckendale
D. L. Lindquist
A. A. Ross
B. F. Waterman
Representative of Bureau of Ships, U.S. Navy Department

STRENGTH OF VESSELS UNDER EXTERNAL PRESSURE

Appointed June, 1929, to develop reliable design data on the strength of cylindrical and spherical surfaces under external pressure, particularly with reference to jacketed vessels

F. V. Hartman, *Chairman*
W. D. Halsey
M. B. Higgins
A. W. Limont, Jr.
H. E. Saunders
E. E. Shanor
D. B. Wesstrom
F. S. G. Williams
D. F. Windenburg

WIRE ROPE

Appointed April, 1930, to investigate existing rope so that it may be better understood and more effectively used

W. H. Fulweiler, *Chairman*
H. LeR. Brink
D. L. Lindquist
G. W. Martin
A. H. McDougall
B. V. E. Nordberg
W. S. Paine
W. J. Ryan
George Simpson
L. E. Young

CRITICAL PRESSURE STEAM BOILERS

Appointed June, 1931, to study the characteristics of high-pressure forced-circulation steam-generating units

H. L. Solberg, *Chairman*
W. H. Armacost
A. D. Bailey
E. G. Bailey
F. S. Clark
C. H. Fellows
H. J. Kerr
G. A. Orrok
E. C. Petrie
E. L. Robinson
P. W. Thompson

ROLLING OF STEEL (PLASTICITY)

Appointed October, 1938, to study plasticity in the particular field of rolling of steel

A. Nadai, *Chairman*
C. W. MacGregor, *Secretary*
E. C. Bain
C. L. Eksergian
J. H. Hitchcock
G. B. Karelitz †
M. D. Stone
W. Trinks

FURNACE PERFORMANCE FACTORS

Appointed in October, 1941, to collect and rationalize data on the performance of commercially important furnaces as an aid to design and operation

A. R. Mumford, *Chairman*
Alex D. Bailey
  J. R. Michel (*Alternate*)
John Blizard
S. P. Burke
O. F. Campbell
W. A. Carter
B. J. Cross
T. B. Drew
F. G. Ely
A. C. Fieldner
  W. C. Schroeder (*Alternate*)
J. H. Harlow
H. C. Hottel
E. L. Lindseth
W. H. McAdams
A. J. Nerad
E. B. Powell
A. A. Raymond
W. T. Reid
R. A. Sherman

† Deceased, January 19, 1943.

Philip Sporn
A. W. Thorson
Herman Weisberg
W. J. Wohlenberg

FORGING OF STEEL SHELLS

Appointed in October, 1941, to study methods of shell manufacture under modern conditions

M. D. Stone, *Chairman*
W. Trinks, *Projects Director*
John Dierbeck
D. W. Fletcher
W. M. Frame
W. N. Howley
A. F. Macconochie
W. P. Muir
A. Nadai
A. R. Nettenstrom
George Sachs
A. E. Van Cleve
U.S. Army, Ordnance Department
U.S. Navy, Bureau of Ordnance

A.S.M.E. Representatives on Other Research Committees

See also A.S.M.E. Representatives on Other Activities, page RI-9

AMERICAN COORDINATING COMMITTEE ON CORROSION

American Society for Testing Materials

C. H. Fellows
S. L. Kerr

FATIGUE PHENOMENA OF METALS

American Society for Testing Materials

C. T. Edgerton

METALLURGICAL RESEARCH

Advisory Committee to the National Bureau of Standards

C. H. Bierbaum

PROPERTIES OF REFRACTORY MATERIALS

Advisory Committee to the National Bureau of Standards

E. B. Powell

WATER FOR INDUSTRIAL USES

American Society for Testing Materials

J. H. Walker

STANDARDIZATION COMMITTEES

Article B6A, Par. 23: The Standing Committee on Standardization shall advise the Council on the dimensional standardization work of the Society, including relations with the American Standards Association.

The first Standing Committee on Standardization was organized in April, 1911

STANDING COMMITTEE

L. T. Knocke, *Chairman* (1943)
T. E. French (1944)
W. H. Hill (1945)
J. H. Taylor (1946)
E. J. Bryant (1947)

STANDARDIZATION AND UNIFICATION OF SCREW THREADS (B1)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee originally organized in June, 1921. Reorganized in February, 1929*

A.S.M.E. Members (Total personnel, 36)

R. E. Flanders, *Chairman* †
A. M. Houser, *Vice-Chairman* †
Earle Buckingham, *Secretary*
E. J. Bryant
G. S. Case
T. G. Crawford
H. C. E. Meyer
P. V. Miller †
W. C. Mueller
R. H. Perry
G. T. Trundle

SUBCOMMITTEE CHAIRMEN

Special Subcommittee on Revision of American Standard, P. V. Miller

No. 1 on Scope, Arrangement and Editing of American National Standard, R. E. Flanders
No. 4 on Acme and Other Similar Threads, Except Gages, Earle Buckingham
No. 5 on Screw Thread Gages and Inspection, G. S. Case
No. 6 on Threading of General Purpose Nuts, J. S. Davey
No. 7 on Screw Threads for High Temperature Bolting, W. H. Gourlie

PIPE THREADS (B2)

** Joint sponsorship with the American Gas Association. Sectional Committee originally organized in 1913. Reorganized May, 1927*

A.S.M.E. Members (Total personnel, 42)

A. S. Miller, *Chairman*
C. B. LePage, *Acting Secretary*
A. F. Breitenstein †
E. J. Bryant
E. S. Cornell, Jr.
J. J. Crotty
J. J. Harman
A. M. Houser †
P. V. Miller †
F. H. Morehead
W. C. Morris
L. N. Shannon
O. M. Tislarish
Frank Thornton, Jr.
Rowland Tompkins
B. B. Wescott
J. H. Williams

SUBCOMMITTEE CHAIRMEN

No. 1 on Editing and Gaging, A. M. Houser
No. 2 on Taper Pipe Threads, S. B. Terry
No. 3 on Straight Pipe Threads, P. V. Miller
No. 4 on Plumbers' Threads, A. F. Breitenstein
No. 6 on Special Threads for Thin Tubes, C. C. Winter

Special Subcommittee on Tolerances on Thread Elements, E. J. Bryant
Special Editing Subcommittee on Taper Pipe Threads, S. B. Terry
Special Editing Subcommittee on Straight Pipe Threads, P. V. Miller
Special Subcommittee on Truncation, E. J. Bryant

BALL AND ROLLER BEARINGS (B3)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized December, 1920*

A.S.M.E. Members (Total personnel, 20)

W. P. Kennedy, *Vice-Chairman* †
D. E. Batesole †
L. A. Cummings
O. H. Dorer
F. G. Hughes
G. E. Hulse †
W. L. Iliff †
L. F. Nenninger
S. M. Weckstein †
Ernest Wooler

ALLOWANCES AND TOLERANCES FOR CYLINDRICAL PARTS AND LIMIT GAGES (B4)

** Sole sponsorship. Sectional Committee originally organized in June, 1920. Reorganized in September, 1930*

A.S.M.E. Members (Total personnel, 39)

J. E. Lovely, *Chairman* †
F. E. Banfield, Jr.
F. S. Blackall, Jr.
E. J. Bryant
Earle Buckingham †
F. H. Colvin †
T. G. Crawford
R. E. W. Harrison
F. O. Hoagland
E. N. Jacobi
H. C. E. Meyer
P. V. Miller
W. C. Mueller
E. C. Peck †
R. H. Perry
W. C. Schoenfeldt
C. C. Stevens
G. T. Trundle, Jr.

SUBCOMMITTEE CHAIRMAN

No. 1 on Tolerance Systems, R. E. W. Harrison

SMALL TOOLS AND MACHINE TOOL ELEMENTS (B5)

** Joint sponsorship with the National Machine Tool Builders' Association, and the Society of Automotive Engineers. Sectional Committee organized February, 1922*

A.S.M.E. Members (Total personnel, 32)

W. C. Mueller, *Chairman* †
F. O. Hoagland, *Vice-Chairman*
J. B. Armitage
O. W. Boston
E. J. Bryant
Earle Buckingham
F. H. Colvin
S. A. Einstein
F. M. Farmer
V. P. Gill
H. E. Harris
John Haydock
J. P. Laux †
J. E. Lovely †
A. F. Murray †
Erik Oberg †
Frank Thornton, Jr.

Technical Committees

Executive Committee

A.S.M.E. Members (Total personnel, 5)

W. C. Mueller, *Chairman*
F. O. Hoagland, *Vice-Chairman*
H. E. Harris

No. 1 on T-Slots

A.S.M.E. Members (Total personnel, 6)

Erik Oberg, *Chairman*
J. B. Armitage
Harry Cadwallader, Jr.
S. A. Einstein
F. O. Hoagland

No. 2 on Tool-Posts and Tool Shanks

A.S.M.E. Members (Total personnel, 8)

O. W. Boston, *Chairman*
F. S. Blackall, Jr.
Granger Davenport
F. M. Farmer
M. E. Lange

No. 3 on Machine Tapers

A.S.M.E. Members (Total personnel, 19)

E. J. Bryant, *Chairman*
C. B. LePage, *Acting Secretary*
J. B. Armitage
F. S. Blackall, Jr.
Earle Buckingham
F. H. Colvin
B. P. Graves
E. E. Griffiths
H. E. Harris
F. O. Hoagland
J. H. Horigan
L. F. Nenninger

SUBGROUP CHAIRMEN

Steep Tapers Series, S. McMullan
Revision on Slow Taper Standard, E. J Bryant

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* Note: All of these standards committees for which the Society is sponsor or joint sponsor, or on which it has representation, are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representative serving on this committee.

SMALL TOOLS AND MACHINE TOOL ELEMENTS (B5)

(Continued)

No. 4 on Spindle Noses and Collets for Machine Tools

A.S.M.E. Members (Total personnel, 26)

J. E. Lovely, *Chairman*
L. F. Nenninger, *Secretary*
J. B. Armitage
H. W. Faus
B. P. Graves
F. O. Hoagland
A. M. Johnson
M. E. Lange
J. H. Mansfield
L. D. Spence

SUBGROUP CHAIRMEN

No. 1 on Milling Machines, Small and Medium, J. B. Armitage
No. 2 on Large Milling Machines, F. B. Kampmeier
No. 3 on Grinding Machines Spindles, H. J. Griffing
No. 5 on Drilling Machines and Horizontal Boring Machines, S. McMullan
No. 6 on Turning Machines, Including Automatic Screw Machines, Lathes, Automatic Lathes, Turret Lathes, and Automatic Chucking Machines, J. E. Lovely
No. 8 on Correlation of Counter Proposals for Spindle Noses, J. E. Lovely

No. 5 on Milling Cutters

A.S.M.E. Members (Total personnel, 20)

J. B. Armitage
A. N. Goddard
E. E. Griffiths
J. H. Horigan
E. K. Morgan
Erik Oberg
E. D. Vancil

SUBGROUP CHAIRMEN

No. 1 on Profile Cutters, E. D. Vancil
No. 2 on Keyways, J. B. Armitage
No. 3 on Nomenclature, A. C. Danekind
No. 4 on Limits, J. H. Horigan
No. 5 on Formed Cutters, H. C. Hungerford
No. 6 on Hobs (to be appointed)
No. 7 on Inserted Tooth Cutters, J. B. Armitage

No. 6 on Designations and Working Ranges of Machine Tools

A.S.M.E. Members (Total personnel, 19)

John Haydock, *Chairman*
Earle Buckingham
T. H. Doan, Jr.
H. W. Faus
B. P. Graves
J. J. McBride
E. R. Smith

No. 7 on Twist Drill Sizes

A.S.M.E. Members (Total personnel, 6)

W. C. Mueller, *Chairman*
E. E. Griffiths
J. H. Horigan

No. 8 on Jig Bushings

A.S.M.E. Members (Total personnel, 8)

E. E. Griffiths
J. H. Horigan

SUBGROUP CHAIRMAN

Subgroup on Liner Outer Diameters and Tolerances (to be appointed)

No. 9 on Punch Press Tools

A.S.M.E. Members (Total personnel, 14)

D. H. Chason
E. W. Ernest
E. E. Griffiths
H. E. Harris
D. M. Palmer

No. 10 on Forming Tools and Holders

A.S.M.E. Members (Total personnel, 10)

W. C. Mueller, *Chairman* †
E. E. Griffiths
L. D. Spence

No. 11 on Chucks and Chuck Jaws

A.S.M.E. Members (Total personnel, 10)

J. E. Lovely, *Chairman*
F. M. Farmer

SUBGROUP CHAIRMEN

No. 1 on Master Chuck Jaws, J. E. Lovely
No. 2 on Adapters for Air Cylinders, J. E. Lovely

No. 12 on Cut and Ground Thread Taps

A.S.M.E. Member (Total personnel, 7)

J. E. Ennis

No. 13 on Splines and Splined Shafts

A.S.M.E. Members (Total personnel, 16)

J. B. Armitage
R. E. W. Harrison
F. O. Hoagland
J. E. Lovely

No. 17 on Nomenclature for Small Tools and Machine Tool Elements

A.S.M.E. Members (Total personnel, 12)

O. W. Boston, *Chairman and Secretary*
F. S. Blackall, Jr.
F. H. Colvin
H. E. Harris
F. O. Hoagland

Ex-Officio Members

A. N. Goddard
W. C. Mueller

No. 19 on Single-Point Cutting Tools

A.S.M.E. Members (Total personnel, 2)

F. H. Colvin, *Chairman*
O. W. Boston, *Secretary*

No. 20 on Reamers

A.S.M.E. Members (Total personnel, 16)

F. H. Colvin
F. M. Farmer
T. F. Githens
E. E. Griffiths
J. H. Horigan
O. E. Koehler

SUBGROUP CHAIRMAN

No. 1 on Reamer Proposal, C. M. Pond

No. 21 on Tool-Life Tests for Single-Point Tools

A.S.M.E. Members (Total personnel, 10)

O. W. Boston, *Chairman*
E. E. Griffiths
M. F. Judkins
H. L. Moir

GEARS (B6)

* *Joint sponsorship with the American Gear Manufacturers Association. Sectional Committee organized June, 1921*

A.S.M.E. Members (Total personnel, 26)

U. S. Eberhardt, *Chairman*
Earle Buckingham, *Vice-Chairman* †
C. B. LePage, *Acting Secretary*
G. H. Acker
L. H. Fry
C. B. Hamilton, Jr.
D. T. Hamilton
M. R. Hanna
O. A. Leutwiler

SUBCOMMITTEE CHAIRMEN

No. 3 on Nomenclature, D. T. Hamilton
No. 4 on Tooth Form (Spur Gears), W. P. Schmitter
No. 8 on Materials, E. J. Wellauer
No. 9 on Inspection, Granger Davenport

PIPE FLANGES AND FITTINGS (B16)

* *Joint sponsorship with the Heating, Piping and Air Conditioning Contractors National Association, and the Manufacturers Standardization Society of the Valve and Fittings Industry. Sectional Committee organized October, 1921*

A.S.M.E. Members (Total personnel, 50)

C. P. Bliss, *Chairman* †
J. J. Harman, *Secretary*
A. L. Baker
L. W. Benoit †
A. L. Brown
Sabin Crocker
Ferdinand Fink
V. M. Frost
H. E. Haller
J. S. Hess
H. A. Hoffer †
E. L. Hopping
A. M. Houser
C. A. Kelting †
J. R. Kruse
(John Blizard, *Alternate*)
M. B. MacNeille
F. H. Morehead
L. S. Morse, Sr.
Ludwig Skog
J. R. Tanner †
J. H. Taylor
Rowland Tompkins
G. W. Watts
J. H. Williams
J. H. Zink

SUBCOMMITTEE CHAIRMEN

Executive Committee, C. P. Bliss †
No. 1 on Cast Iron Flanges and Flanged Fittings, A. M. Houser
No. 2 on Screwed Fittings, F. H. Morehead
No. 3 on Steel Flanges and Flanged Fittings, C. P. Bliss †
No. 4 on Materials and Stresses, A. M. Houser
No. 5 on Face to Face Dimensions of Ferrous Flanged Valves, J. R. Tanner
No. 6 on Malleable Iron or Steel Brass Seat Unions (to be appointed)
No. 7 on Rating of Pipe Fittings (to be appointed)
No. 8 on Marking of Pipe Fittings, F. H. Morehead

SHAFTING (B17)

** Sole sponsorship. Organized October, 1918*

A.S.M.E. Members (Total personnel, 13)

C. M. Chapman, *Chairman* †
C. B. LePage, *Secretary*
H. C. E. Meyer
L. C. Morrow
G. N. Van Derhoef †
L. W. Williams †

BOLT, NUT, AND RIVET PROPORTIONS (B18)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1922*

A.S.M.E Members (Total personnel, 57)

W. R. Halliday, *Chairman* †
W. C. Stewart, *Secretary*
H. E. Aldrich
F. C. Billings
B. G. Braine
G. S. Case
T. G. Crawford
H. P. Frear
A. M. Houser †
J. W. Huckert
Herman Koester †
W. C. Mueller
S. F. Newman
R. H. Perry
J. R. Tanner †
R. J. Whelan
E. M. Whiting
V. R. Willoughby
(J. J. McBride, *Alternate*)

SUBCOMMITTEE CHAIRMEN

No. 1 on Large and Small Rivets (to be appointed)
No. 2 on Wrench-Head Bolts and Nuts (to be appointed)
No. 3 on Slotted and Recessed Head Screws, F. P. Tisch
No. 4 on Track Bolts and Nuts (to be appointed)
No. 5 on Round Unslotted Head Bolts (Carriage Bolts) (to be appointed)
No. 6 on Plow Bolts (to be appointed)
No. 7 on Body Dimensions and Materials (to be appointed)
No. 8 on Nomenclature, G. S. Case
No. 9 on Socket Head Cap and Set Screws, Herman Koester †

PLAIN AND LOCK WASHERS (B27)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized August, 1925*

A.S.M.E. Members (Total personnel, 40)

Eugene Caldwell	J. J. McBride
T. G. Crawford	H. C. E. Meyer
B. S. Lewis †	W. C. Mueller †
C. H. Loutrel	E. M. Whiting †

SUBCOMMITTEE CHAIRMEN

No. 1 on Plain Washers, W. L. Barth
No. 2 on Spring Washers, E. Cowlin

TRANSMISSION CHAINS AND SPROCKETS (B29)

** Joint sponsorship with the Society of Automotive Engineers, and the American Gear Manufacturers Association. Sectional Committee organized September, 1917. Reorganized December, 1926*

A.S.M.E. Members (Total personnel, 12)

W. J. Belcher	D. B. Perry
Joseph Joy	C. R. Weiss
L. V. Ludy †	G. A. Young
J. F. McCann	

SUBCOMMITTEE CHAIRMEN

No. 1 on Roller Chain Standardization (to be appointed)
No. 2 on Silent Chain Standardization, G. A. Young

CODE FOR PRESSURE PIPING (B31)

** Sole sponsorship. Sectional Committee organized March, 1926. Reorganized December, 1937*

A.S.M.E. Members (Total personnel, 102)

E. B. Ricketts, *Chairman*
R. E. Bryant
L. H. Carr
H. C. Cooper
D. H. Corey
Sabin Crocker
H. D. Edwards
E R. Fish
Charles Fitzgerald
V. M. Frost
T. W. Greene
H. E. Haller
W. D. Halsey
J. S. Haug
H. A. Hoffer
G. G. Hollins
E. L. Hopping
A. M. Houser †
Alfred Iddles †
D. S. Jacobus
T. M. Jasper
C. A. Kelting
E. H. Krieg
G. Sinding Larsen
M. B. MacNeille
G. W. Martin
H. C. E. Meyer
J. W. Moore
(J. D. Capron, *Alternate*)
F. H. Morehead
(J. J. Harman, *Alternate*)
H. H. Morgan
L. S. Morse, Sr.
R. M. Nee
E. W. Norris
C. W. Obert
Geo. A. Orrok
A. L. Penniman, Jr.
C. S. Robinson †
J. H. Romann
D. B. Rossheim
G. W. Saathoff
G. K. Saurwein
Ludwig Skog
H. S. Smith
(H. H. Moss, *Alternate*)
J. R. Tanner
J. H. Taylor
J. H. Vance
H. L. Whittemore
J. H. Williams
T. F. Wolfe

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, Sabin Crocker
No. 2 on Power Piping, Alfred Iddles †
No. 4 on Gas and Air Piping, J. S. Haug
No. 5 on Refrigeration Piping, A. B. Stickney
No. 6 on Oil Piping (to be appointed)
No. 7 on Piping Materials and Identification, F. H. Morehead
No. 8 on Fabrication Details, Ludwig Skog
No. 9 on District Heating Piping, G. K. Saurwein
Special Committee to confer with Boiler Code Subcommittee on Material Specifications, the M.S.S., the A.S.T.M. and Sectional Committee B31 and Sectional Committee B16 to coordinate the several specifications relating to piping, valves and fittings
Special Committee on Pressures and Temperatures Above Those Covered by Code, E. F. Fish

WIRE AND SHEET METAL GAGES (B32)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized November, 1928. Reorganized November, 1939*

A.S.M.E. Members (Total personnel, 37)

A. P. Cottle	J. F. Howe †
E. W. Ernest	F. G. Wilson †

SUBCOMMITTEE CHAIRMAN

Wire and Sheet Metal Gages, H. W. Tenney

SCREWS THREADS FOR HOSE COUPLINGS (B33)

** Sole sponsorship. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 28)

A. L. Brown, *Secretary*
A. F. Breitenstein †
J. J. Crotty
W. L. Curtiss
W. E. Dunham †
J. J. Harman
(F. C. Ernst, *Alternate*)
A. M. Houser
H. C. E. Meyer
J. H. Williams

SUBCOMMITTEE CHAIRMEN

Special Subcommittee to Draft Recommended Specifications (to be appointed)
Subcommittee on Basic Thread Dimensions, D. R. Miller

WROUGHT IRON AND WROUGHT STEEL PIPE AND TUBING (B36)

** Joint sponsorship with the American Society for Testing Materials. Sectional Committee organized April, 1928*

A.S.M.E. Members (Total personnel, 46)

H. H. Morgan, *Chairman*
Sabin Crocker, *Secretary*
J. S. Adelson
H. E. Aldrich
Newell Hamilton
E. L. Hopping
(A. B. Morgan, *Alternate*)
A. M. Houser †
D. S. Jacobus †
(F. S. Clark, *Alternate*) †
J. J. Kanter
H. C. E. Meyer

WROUGHT IRON AND WROUGHT STEEL PIPE AND TUBING (B36)

(*Continued*)

F. H. Morehead
H. B. Oatley †
J. H. Romann
Ludwig Skog †
J. R. Tanner
A. E. White

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, H. H. Morgan
No. 2 on Pipe and Tubing for Low Temperature Service, J. J. Shuman
No. 3 on Pipe and Tubing for High Temperature Service, J. R. Tanner
No. 4 on Materials, F. H. Morehead

PRESSURE AND VACUUM GAGES (B40)

** Sole Sponsorship. Sectional Committee organized July, 1930*

A.S.M.E. Members (Total personnel, 45)

M. D. Engle, *Chairman*
A. W. Lenderoth, *Secretary* †
E. J. Bryant
J. P. Cavanaugh †
Paul Diserens
C. H. Graesser
W. F. Jones
R. J. Kehl
J. C. McCune †
A. H. Morgan
H. B. Reynolds
W. C. Schoenfeldt

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope, M. D. Engle
No. 2 on Definitions, C. F. Schwep
No. 3 on Gage Sizes and Mounting Dimensions, H. B. Reynolds
No. 4 on Accuracy and Test Methods, O. J. Hodge

STOCK SIZES, SHAPES AND LENGTHS FOR HOT AND COLD FINISHED IRON AND STEEL BARS (B41)

** Sole sponsorship. Sectional Committee organized June, 1930*

A.S.M.E. Members (Total personnel, 25)

F. H. Dechant
E. W. Ernest
H. D. Tanner
L. W. Williams †
G. H. Woodroffe

SUBCOMMITTEE CHAIRMEN

No. 1 on Hot Rolled Steel, Henry Wysor
No. 2 on Cold Finished Steels, L. E. Creighton
No. 3 on Hot Rolled Iron (to be appointed)

SPECIFICATIONS FOR LEATHER BELTING (B42)

** Sole sponsorship. Sectional Committee organized February, 1931*

A S.M.E. Members (Total personnel, 24)

H. T. Coates
R. W. Drake †
W. E. Emley
King Hathaway
P. G. Rhoads
G. A. Schieren

SUBCOMMITTEE CHAIRMEN

No. 1 on Standard Specifications (to be appointed)
No. 2 on Recommendations for Selection, Care and Installation, G. A. Schieren

CLASSIFICATION AND DESIGNATION OF SURFACE QUALITIES (B46)

** Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized May, 1932*

A.S.M.E. Members (Total personnel, 68)

R. F. Gagg, *Chairman*
E. J. Abbott
E. J. Bryant
T. G. Crawford
R. C. Deale †
U. S. Eberhardt
S. A. Einstein
W. W. Gilbert
J. J. Harman
R. E. W. Harrison †
F. V. Hartman
F. O. Hoagland
H. J. Holtzclaw
R. T. Kent
H. F. Kurtz
Joseph Manuele
F. C. Spencer
C. C. Stevens
W. C. Stewart
J. S. Tawresey
C. H. Whitaker
Ernest Wooler

SUBCOMMITTEE CHAIRMEN

Special Subcommittee on Revision and Editing, R. F. Gagg
No. 2 on Surfaces Produced by Molds, Dies, Rolls, or Any Other Means of Deforming Materials (to be appointed)
No. 3 on Coated Surfaces, G. B. Hogaboom
No. 4 on Symbols for Indicating Surface Quality on Drawings, T. G. Crawford
No. 5 on Ways, Means and Apparatus for Measuring Quality of Surface (to be appointed)
No. 7 on Standards for Appearance of Surfaces (to be appointed)

COMBUSTION SPACE FOR SOLID FUELS (B50)

** Sole sponsorship. Sectional Committee organized June, 1933*

A.S.M.E. Members (Total personnel, 22)

C. E. Bronson, *Chairman*
W. G. Christy
John Hunter
A. J. Johnson
V. G. Leach †
J. P. Magos
J. F. McIntire
F. L. Meyer
C. A. Reed
John Van Brunt †

SUBCOMMITTEE CHAIRMEN

No. 1 on Purpose and Scope, C. E. Bronson
No. 2 on Combustion and Design, B. M. Guthrie
No. 3 on Warm Air Furnaces, J. H. Manny
No. 4 on Steel Heating Boilers, W. B. Russell
No. 5 on Cast Iron Boilers, J. F. McIntire

SCHEME FOR IDENTIFICATION OF PIPING SYSTEMS (A13)

** Joint sponsorship with the National Safety Council. Sectional Committee organized June, 1922*

A.S.M.E. Members (Total personnel, 35)

E. E. Ashley
W. L. Bunker
Crosby Field
E. L. Hopping
H. L. Miner
H. S. Smith
Frank Thornton, Jr.
Rowland Tompkins
Frank Umbehocker †

SUBCOMMITTEE CHAIRMEN

Executive Committee, A. S. Hebble
Identification by Colors (to be appointed)
Classification, Crosby Field
Identification Markings Other Than Color (to be appointed)
Editing Subcommittee, A. S. Hebble

MINIMUM REQUIREMENTS FOR PLUMBING AND STANDARDIZATION OF PLUMBING EQUIPMENT (A40)

** Joint sponsorship with The American Public Health Association. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 54)

C. B. LePage, *Acting Secretary*
J. F. Carney, *Alternate*
J. J. Crotty
A. M. Houser
G. W. Martin
(A. H. Morgan, *Alternate*)
W. K. McAfee
Thorndike Saville †
W. R. Webster †

SUBCOMMITTEE CHAIRMEN

Research Committee on Plumbing (to be appointed)
No. 1 on Minimum Requirements for Plumbing, T. I. Coe
No. 2 on Staple Vitreous China Plumbing Fixtures, H. R. Van Sciver
No. 3 on Staple Porcelain (All Clay) Plumbing Equipment, H. R. Van Sciver
No. 4 on Enameled Sanitary Ware, A. H Cline, Jr.
No. 5 on Traps (to be appointed)
No. 6 on Brass Plumbing Products (to be appointed)
No. 7 on Brass Fittings for Flared Copper Tubes, F. L. Riggin
No. 8 on Cast Iron Soil Pipe and Fittings (to be appointed)
No. 9 on Gasoline, Oil and Grease Separators (to be appointed)
Joint Committee on Threaded Cast Iron Pipe, F. H. Morehead
No. 11 on Soldered Fittings for Tubing, A. M. Houser
No. 12 on Minimum Air Gaps in Plumbing Systems, W. K. McAfee

ROLLED THREADS FOR SCREW SHELLS OF ELECTRIC SOCKETS AND LAMP BASES (C44)

** Joint sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized March, 1929*

A.S.M.E. Members (Total personnel, 16)

E. J. Bryant †

ROLLED THREADS FOR SCREW SHELLS OF ELECTRIC SOCKETS AND LAMP BASES (C44)

(Continued)

EARLE BUCKINGHAM †
(A. B. MORGAN, *Alternate*)
E. S. SANDERSON †

LETTER SYMBOLS AND ABBREVIATIONS FOR SCIENCE AND ENGINEERING (Z10)

** Joint sponsorship with the American Association for the Advancement of Science, American Institute of Electrical Engineers, American Society of Civil Engineers, and the Society for the Promotion of Engineering Education. Sectional Committee organized January, 1926. Reorganized October, 1935*

A.S.M.E. Members (Total personnel, 44)

S. A. MOSS, *Vice-Chairman* †
K. H. CONDIT
R. E. PETERSON †
(S. R. BEITLER, *Alternate*) †
G. A. STETSON
FRANK THORNTON, JR.
E. P. WARNER

SUBCOMMITTEE CHAIRMEN

Executive Committee, H. M. TURNER
Steering Committee, H. M. TURNER
No. 1 on Letter Symbols and Signs for Mathematics, A. A. BENNETT
No. 2 on Symbols for Hydraulics, J. C. STEVENS
No. 3 on Symbols for Mechanics, R. E. PETERSON †
No. 4 on Symbols for Structural Analysis, ALBERT HAERTLEIN
No. 5 on Symbols for Heat and Thermodynamics, S. A. MOSS †
No. 6 on Symbols for Photometry, E. C. CRITTENDEN
No. 7 on Aeronautical Symbols, G. W. LEWIS
No. 8 on Symbols for Electric and Magnetic Qualtities, EDWARD BENNETT
No. 9 on Symbols for Radio, H. M. TURNER
No. 10 on Symbols for Physics, H. K. HUGHES
No. 11 on Abbreviations for Engineering and Scientific Terms, G. A. STETSON

DRAWINGS AND DRAFTING ROOM PRACTICE (Z14)

** Joint sponsorship with the Society for the Promotion of Engineering Education. Sectional Committee organized July, 1926*

A.S.M.E. Members (Total personnel, 46)

T. E. FRENCH, *Chairman*
H. P. FREAR
F. DER. FURMAN
J. J. HARMAN
A. C. HARPER
E. R. HILL
SAMUEL KETCHUM †
C. W. KEUFFEL
F. R. LANEY
H. B. LANGILLE
RUDOLPH MICHEL, *Alternate*
F. W. MING
W. C. MUELLER
E. B. NEIL
J. W. OWENS
F. C. PANUSKA
ED S. SMITH †
ROWLAND TOMPKINS

SUBCOMMITTEE CHAIRMAN

Subcommittee on Revision, F. G. HIGBEE

GRAPHIC PRESENTATION (Z15)

** Sole sponsorship. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 30)

G. E. HAGEMANN, *Secretary* †
C. M. BIGELOW
WALLACE CLARK
T. E. FRENCH
D. B. PORTER †

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope (to be appointed)
No. 2 on Terminology (to be appointed)
No. 3 on Preferred Practice for Time Series Charts, A. H. RICHARDSON
No. 4 on Engineering and Scientific Graphs, W. A. SHEWHART

SPEEDS OF MACHINERY (Z18)

** Sole sponsorship. Sectional Committee organized May, 1928*

A.S.M.E. Members (Total personnel, 27)

C. M. BIGELOW †
J. F. DAGGETT
R. C. DEALE †
PAUL DISERENS
F. S. ENGLISH
P. G. RHOADS
F. C. SPENCER

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope, A. E. HALL
No. 2 on Questionnaire and Canvass of Industry, F. S. ENGLISH
No. 3—Special Reviewing Committee (to be appointed)

GRAPHICAL SYMBOLS AND ABBREVIATIONS FOR USE ON DRAWINGS (Z32)

** Joint sponsorship with American Institute of Electrical Engineers. Sectional Committee organized April, 1936*

A.S.M.E. Members (Total personnel, 48)

E. E. ASHLEY
J. M. BARNES
T. E. FRENCH †
G. F. HABACH
D. T. HAMILTON
J. J. HARMAN
W. C. MUELLER
L. L. MUNIER
F. C. PANUSKA †
W. C. STEWART
T. R. THOMAS

SUBCOMMITTEE CHAIRMEN

No. 1 on Symbols for Use in Mechanical Engineering, T. E. FRENCH
No. 2 on Symbols for Use in Electrical Engineering, H. W. SAMSON
No. 3 on Abbreviations for Use on Drawings, T. E. FRENCH

DEVELOPMENT OF STATISTICAL APPLICATIONS IN ENGINEERING AND MANUFACTURING

Joint sponsorship with the American Mathematical Society, American Society for Testing Materials, American Statistical Association, and Institute of Mathematical Statistics. Appointed in December, 1929

A.S.M.E. Members (Total personnel, 9)

A. G. ASHCROFT
L. K. SILLCOX †
J. S. TAWRESEY †

A.S.M.E. Representatives on Miscellaneous Standardization Committees

See also A.S.M.E. Representatives on Other Activities, page RI-9

ACOUSTICAL MEASUREMENTS AND TERMINOLOGY

** Sponsor body: Acoustical Society of America*

(J. S. PARKINSON, *Alternate*)
(R. V. PARSONS, *Alternate*)

AERONAUTICS

** Sponsor body: Society of Automotive Engineers*

E. A. SPERRY, JR.

BUILDING CODE REQUIREMENTS FOR LIGHT AND VENTILATION

** Sponsor bodies: Federal Housing Administration, and U.S. Public Health Service*

F. R. SCHERER

CAST IRON AT ELEVATED TEMPERATURES

Subcommittee of American Society for Testing Materials Committee A-3 on Cast Iron

D. B. ROSSHEIM

COAL AND COKE

Committee of American Society for Testing Materials

R. M. HARDGROVE

DEFINITIONS OF ELECTRICAL TERMS

** Sponsor body: American Institute of Electrical Engineers*

C. H. BERRY

DRAINAGE OF COAL MINES

** Sponsor body: American Mining Congress*

O. M. PRUITT

ELECTRIC WELDING APPARATUS

** Sponsor bodies: American Welding Society*

R. E. KINKEAD

FOREST FIRE PROTECTION

Committee of National Fire Protection Association

C. B. White

LOADING PLATFORMS AT FREIGHT TERMINALS AND WAREHOUSES

** Sponsor body: American Trucking Association, Inc.*

M. C. Maxwell

MANHOLE FRAMES AND COVERS

** Sponsor bodies: ASA Telephone Group, and American Society of Civil Engineers*

Anton Hansen

ASA MECHANICAL STANDARDS COMMITTEE

Alfred Iddles, *Chairman* †
(A. L. Baker, *Alternate*) †
J. E. Ennis
E. W. Ernest
F. O. Hoagland
(M. E. Lange, *Alternate*)
E. L. Hopping
F. H. Morehead
(A. M. Houser, *Alternate*)
H. H. Morgan
Frank Thornton, Jr.
H. L. Whittemore, *Alternate*

Executive Committee,
Alfred Iddles
J. E. Ennis

METHODS OF TESTING WOOD

** Sponsor bodies: U.S. Forest Service, and American Society for Testing Materials*

C. M. Bigelow

MISCELLANEOUS OUTSIDE COAL-HANDLING EQUIPMENT

** Sponsor body: American Mining Congress*

Ralph Sargent

PETROLEUM PRODUCTS AND LUBRICANTS

** Sponsor body: American Society for Testing Materials*

R. G. N. Evans
G. B. Karelitz ‡
(H. J. Masson, *Alternate*)
(S. J. Needs, *Alternate*)

PREFERRED NUMBERS

** Special Committee of ASA*

K. H. Condit

RATING OF RIVERS

** Sponsor body: U.S. Geological Survey*

D. W. Mead

REQUIREMENTS FOR THE INSTALLATION OF GAS BURNING EQUIPMENT IN POWER BOILERS

** Sponsor body: American Gas Association*

O. F. Campbell

ROTATING ELECTRICAL MACHINERY

** Sponsor body: ASA Electrical Standards Committee*

(To be appointed)

SPECIFICATIONS FOR CAST IRON PIPE AND SPECIAL CASTINGS

** Sponsor bodies: American Gas Association, American Society for Testing Materials, American Water Works Association, and the New England Water Works Association*

J. E. Gibson
L. R. Howson

SPECIFICATIONS FOR CLEAN BITUMINOUS COAL

** Sponsor body: American Institute of Mining and Metallurgical Engineers*

R. A. Sherman
(E. L. Lindseth, *Alternate*)

SPECIFICATIONS FOR FIRE TESTS OF BUILDING CONSTRUCTION AND MATERIALS

** Sponsor bodies: ASA Fire Protection Group, National Bureau of Standards, and American Society for Testing Materials*

R. C. Parlett

‡ Deceased, January 19, 1943.

SPECIFICATIONS FOR SIEVES FOR TESTING PURPOSES

** Sponsor bodies: American Society for Testing Materials, and National Bureau of Standards*

R. M. Hardgrove

ASA STANDARDS COUNCIL

Alfred Iddles †
J. E. Lovely †
(C. B. LePage, *Alternate*) †
(W. C. Mueller, *Alternate*) †

SPECIAL COMMITTEE ON STANDARDIZATION OF THERBLIGS, PROCESS CHARTS, AND THEIR SYMBOLS

D. B. Porter, *Chairman*
R. W. Allen
R. M. Barnes
C. H. Cox
L. M. Gilbreth
J. M. Juran
H. B. Maynard
J. A. Piacitelli
G. M. Varga

THERMAL INSULATING MATERIALS

Committee of American Society for Testing Materials

R. H. Heilman

U.S. INTERDEPARTMENTAL COMMITTEE ON SCREW THREADS

Earle Buckingham
A. M. Houser
J. W. Huckert

VOLUME WATER HEATING

Committee of American Gas Association

Marc Resek

WIRE ROPES FOR MINES

** Sponsor body: American Mining Congress*

E. S. Wellhofer

POWER TEST CODES COMMITTEES

ARTICLE B6A, PAR. 27: The Standing Committee on Power Test Codes shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Power Test Codes, including the interpretation of such codes.

The first Standing Committee on Power Test Codes was organized in December, 1918, to revise and extend the Power Test Codes which had been formulated by various technical committees appointed to develop particular codes. This work began in 1884.

STANDING COMMITTEE

FRANCIS HODGKINSON, *Chairman* (1944)
A. G. CHRISTIE, *Vice-Chairman* (1946)
J. A. KEENE, *Junior Observer* (1943)

Term expires 1943

LOUIS ELLIOTT
G. A. HORNE
H. B. REYNOLDS
P. W. SWAIN
E. N. TRUMP

Term expires 1944

C. H. BERRY
FRANCIS HODGKINSON
D. S. JACOBUS
L. F. MOODY
E. B. RICKETTS

Term expires 1945

THEODORE BAUMEISTER
P. H. HARDIE
B. V. E. NORDBERG
R. J. S. PIGOTT
M. C. STUART

Term expires 1946

A. G. CHRISTIE
PAUL DISERENS
N. R. GIBSON
GEO. A. ORROK
E. B. POWELL

Term expires 1947

(To be appointed)

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(1) GENERAL INSTRUCTIONS

Appointed December, 1918

Reorganized, 1939

THEODORE BAUMEISTER, *Chairman*
PAUL DISERENS
HENRY KREISINGER
A. R. MUMFORD
R. H. SNYDER
C. R. SODERBERG
M. C. STUART
P. W. SWAIN

(2) DEFINITIONS AND VALUES

Appointed December, 1918

Reorganized, 1936

R. J. S. PIGOTT, *Chairman*
L. J. BRIGGS
W. F. DAVIDSON
L. S. MARKS
G. S. PETERSON
F. G. PHILO
J. C. SMALLWOOD
P. W. SWAIN
A. C. WOOD

(3) FUELS

Appointed December, 1918

W. J. WOHLENBERG, *Chairman*
E. G. BAILEY
H. W. BROOKS
S. B. FLAGG
D. M. MYERS
F. G. PHILO
G. S. POPE
E. B. RICKETTS
E. X. SCHMIDT
NICHOLAS STAHL
E. N. TRUMP

(4) STATIONARY STEAM-GENERATING UNITS

Appointed December, 1918

E. R. FISH, *Chairman*
A. D. BAILEY
M. W. BENJAMIN
B. J. CROSS
MARTIN FRISCH
P. H. HARDIE
R. M. HARDGROVE
ALFRED IDDLES
E. L. LINDSETH
E. L. MCDONALD
E. B. POWELL
R. SHELLENBERGER
R. L. SPENCER

(5) RECIPROCATING STEAM ENGINES

Appointed December, 1918

Reorganized, 1931

A. G. CHRISTIE, *Chairman*
K. S. M. DAVIDSON
HENRIK GREGER
J. A. HUNTER
H. G. MUELLER
B. V. E. NORDBERG
A. V. SAHAROFF
A. G. WITTING

(6) STEAM TURBINES

Appointed December, 1918

C. H. BERRY, *Chairman*
I. E. MOULTROP, *Secretary*
O. D. H. BENTLEY
W. E. CALDWELL
C. B. CAMPBELL
A. G. CHRISTIE
H. P. DAHLSTRAND
V. M. FROST
A. E. GRUNERT
FRANCIS HODGKINSON
S. A. MOSS
R. O. MULLER
T. E. PURCELL
G. B. WARREN

(7) RECIPROCATING STEAM-DRIVEN DISPLACEMENT PUMPS

Appointed December, 1918

R. D. HALL, *Chairman*
E. H. BROWN
J. N. CHESTER
J. E. GIBSON
G. L. KOLLBERG
M. B. MACNEILLE
D. W. MEAD
L. A. QUAYLE

(8) CENTRIFUGAL AND ROTARY PUMPS

Appointed December, 1918

Reorganized, 1936

R. L. DAUGHERTY, *Chairman*
H. E. BECKWITH
R. G. FOLSOM
R. C. GLAZEBROOK
W. B. GREGORY
R. T. KNAPP
J. B. LINCOLN
M. B. MACNEILLE
L. F. MOODY
ARVID PETERSON
F. H. ROGERS
W. C. RUDD
MAX SPILLMAN
F. G. SWITZER
W. M. WHITE
I. A. WINTER

(9) DISPLACEMENT COMPRESSORS AND BLOWERS

Appointed December, 1918

Reorganized, 1935

PAUL DISERENS, *Chairman*
G. T. FELBECK
C. R. HOUGHTON
J. F. HUVANE
R. M. JOHNSON
J. F. D. SMITH

(10) CENTRIFUGAL AND TURBO-COMPRESSORS AND BLOWERS

Appointed December, 1918

Reorganized, 1929

ARVID PETERSON, *Chairman*
M. C. STUART, *Chairman* (Fans)
E. L. ANDERSON
THEODORE BAUMEISTER
C. A. BOOTH
W. H. CARRIER
THOMAS CHESTER
E. D. CURLEY
L. E. DAY
Z. G. DEUTSCH
S. H. DOWNS
P. E. GOOD
J. J. GROB
H. F. HAGEN
PAUL HOFFMAN
F. H. JENKINS

(10) CENTRIFUGAL AND TURBO-COMPRESSORS AND BLOWERS

(Continued)

H. D. Kelsey
A. L. Kimball
W. W. Lawrence
R. D. Madison
L. S. Marks

(12) CONDENSERS, WATER HEATING, AND COOLING EQUIPMENT

Appointed December, 1918

Geo. A. Orrok, *Chairman*
P. H. Hardie, *Secretary*
C. H. Baker, Jr.
J. F. Grace
D. W. R. Morgan
H. B. Reynolds
P. E. Reynolds

(13) REFRIGERATING SYSTEMS

Appointed December, 1918

Reorganized May, 1939

B. H. Jennings, *Chairman* †
A. C. Buensod
(R. W. Waterfill, *Alternate*)
J. C. Consley
(H. B. Pownall, *Alternate*)
R. J. Ewer †
Walter F. Jones †
M. A. Nelson †
A. W. Oakley
C. L. Svenson
Frank Zumbro †

(14) EVAPORATING APPARATUS

Appointed December, 1918

E. N. Trump, *Chairman*
B. N. Bump
E. A. Newhall
H. L. Parr
L. C. Rogers

(15) STEAM LOCOMOTIVES

Appointed December, 1918

W. F. Kiesel
H. B. Oatley
G. E. Rhoads
L. K. Sillcox

(16) GAS PRODUCERS

Appointed December, 1918

C. D. Smith

(17) INTERNAL-COMBUSTION ENGINES

Appointed December, 1918

Reorganized, 1939

Lee Schneitter, *Chairman*
F. H. Dutcher, *Secretary*
J. C. Barnaby

† Official A.S.M.E. representative serving on this committee.

G. C. Boyer
H. E. Degler
W. L. H. Doyle
L. B. Jackson
E. J. Kates
E. C. Magdeburger
B. V. E. Nordberg
Russell Pyles
M. J. Reed
O. D. Treiber

(18) HYDRAULIC PRIME MOVERS

Appointed December, 1918

Reorganized, 1931

S. L. Kerr, *Chairman*
C. M. Allen
L. M. Davis
H. L. Doolittle
W. F. Durand
N. R. Gibson
J. P. Growdon
T. H. Hogg
L. J. Hooper
C. W. Hubbard
E. C. Hutchinson
D. J. McCormack
L. F. Moody
W. J. Rheingans
J. F. Roberts
E. B. Strowger
R. V. Terry
W. M. White

(19) INSTRUMENTS AND APPARATUS

Appointed December, 1918

W. A. Carter, *Chairman*
C. M. Allen
W. C. Andrae
E. G. Bailey
H. S. Bean
L. J. Briggs
J. D. Davis
K. J. De Juhasz
R. E. Dillon
F. M. Farmer
J. B. Grumbein
W. W. Johnson
W. H. Kenerson
E. S. Lee
E. L. Lindseth
Osborn Monnett
S. A. Moss
R. J. S. Pigott
E. B. Ricketts
W. A. Sloan
R. B. Smith
I. M. Stein

(20) SPEED, TEMPERATURE AND PRESSURE RESPONSIVE GOVERNORS

Appointed December, 1921

Reorganized February, 1940

C. R. Soderberg, *Chairman*
C. L. Avery
R. J. Caughey
W. L. H. Doyle
Herbert Estrada
S. N. Fiala
J. R. Hagemann
W. C. Holmes
S. L. Kerr
A. F. Schwendner
Raymond Sheppard
R. B. Smith
H. E. Stickle

(21) DUST SEPARATING APPARATUS

Appointed October, 1934

M. D. Engle, *Chairman*
Ollison Craig, *Secretary*
A. D. Bailey
H. H. Bubar
W. G. Christy
H. O. Croft
J. M. DallaValle
H. O. Danz
H. C. Dohrmann
J. W. Fehnel
H. F. Hagen
P. H. Hardie
C. W. Hedberg
J. H. Leech
H. E. Macomber
H. B. Meller
H. C. Murphy
B. F. Tillson

A.S.M.E. Representatives on Other Technical Committees

See also A.S.M.E. Representatives on Other Activities, page RI-9

DEVELOPMENT OF DEFINITIONS FOR THE NET CALORIFIC VALUE AND GROSS CALORIFIC VALUE OF FUELS

Sponsor body: American Society for Testing Materials

W. J. Wohlenberg

COMMITTEE ON REDEFINING SO-CALLED STANDARD TON OF REFRIGERATION

Sponsor body: American Society of Refrigerating Engineers

G. B. Bright

COMMITTEE ON GASEOUS FUELS

Sponsor body: American Society for Testing Materials

E. X. Schmidt

COAL TESTING CODE COMMITTEE

Joint sponsorship with the American Institute of Mining and Metallurgical Engineers

A. R. Mumford

SPECIFICATIONS FOR PRIME MOVER SPEED GOVERNING

Joint sponsorship with the American Institute of Electrical Engineers

C. L. Avery
Herbert Estrada
J. R. Hagemann
Raymond Sheppard
C. R. Soderberg
A. F. Schwendner

SAFETY COMMITTEES

ARTICLE B6A, PAR. 25: The Standing Committee on Safety shall advise the Council on the activities of the Society having to do with engineering and industrial safety, except the activities of the Boiler Code Committee, for which special provision is made.

The first Standing Committee on Safety was appointed in October, 1921.

STANDING COMMITTEE

A. E. WINDLE, *Chairman* (1943)
To be appointed (1944)
E. R. GRANNISS (1945)
H. W. GABOR (1946)
J. R. CONNELLY (1947)

SAFETY CODE FOR ELEVATORS (A17)

** Joint sponsorship with The American Institute of Architects, and the National Bureau of Standards. Sectional Committee organized November, 1922*

Reorganized July, 1940

A.S.M.E. Members (Total personnel, 49)

O. P. CUMMINGS, *Vice-Chairman*
C. R. CALLAWAY
J. W. DEGEN
D. L. HOLBROOK †
BASSETT JONES
D. L. LINDQUIST
M. B. MCLAUTHLIN
W. S. PAINE
W. H. SEAQUIST, *Alternate* †
S. F. VOORHEES
H. L. WHITTEMORE

SUBCOMMITTEE CHAIRMEN

Emergency Elevator Rules, D. J. PURINTON
Executive Committee, D. J. PURINTON
Existing Elevators, D. J. PURINTON
Inspectors' Manual, K. A. COLAHAN
Mechanical Safety Equipment, D. L. LINDQUIST
Wire Rope, D. J. PURINTON
Working, G. H. REPPERT

SAFETY CODE FOR MECHANICAL POWER-TRANSMISSION APPARATUS (B15)

** Joint sponsorship with the International Association of Industrial Accident Boards and Commissions, and the National Conservation Bureau. Sectional Committee organized February, 1921*

A.S.M.E. Members (Total personnel, 24)

G. M. NAYLOR, *Chairman* †
P. G. RHOADS, *Secretary*
D. C. WRIGHT †
(G. N. VAN DERHOEF, *Alternate*) †

SUBCOMMITTEE CHAIRMEN

No. 1 on Detail Classification of Belts (to be appointed)
No. 2 on Modification of Rule 223 for Cone Pulley Belt (to be appointed)
No. 3 on Mechanical Power Control (to be appointed)
No. 4 on Use of ASA Code Versus State Codes (to be appointed)
No. 5 on Statistics on Place of Occurrence of Accidents (to be appointed)
No. 6 on V-Belt Drives, D. C. WRIGHT

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* Note: All of the safety committees for which the Society is sponsor or joint sponsor, or on which it has representation, are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representative serving on this committee.

SAFETY CODE ON COMPRESSED AIR MACHINERY AND EQUIPMENT (B19)

** Joint sponsorship with the American Society of Safety Engineers—Engineering Section, National Safety Council. Sectional Committee organized May, 1923*

A.S.M.E. Members (Total personnel, 24)

D. L. ROYER, *Chairman*
H. D. EDWARDS
W. J. GRAVES

SAFETY CODE FOR CONVEYORS AND CONVEYING MACHINERY (B20)

** Joint sponsorship with the National Conservation Bureau. Sectional Committee organized November, 1925, reorganized, April, 1937*

A.S.M.E. Members (Total personnel, 53)

D. L. ROYER, *Chairman*
C. T. COLLEY
W. J. GRAVES
M. A. KENDALL †
(N. W. ELMER, *Alternate*) †
P. T. ONDEDONK
C. G. PFEIFFER
R. B. RENNER
F. J. SHEPARD, JR.
J. G. WHEATLEY

SUBCOMMITTEE CHAIRMEN

No. 1 on All Types of Chain Conveyors, Belt Conveyors, Belt Elevators Including Steel Belt, and Screw, Track or Scraper Conveyors, C. G. PFEIFFER
No. 2 on Gravity Conveyors and Chutes, Live Roll Conveyors, H. G. DALTON
No. 3 on Cable-Operated and Cable Flight Conveyors and Cableways, R. MCA. KEOWN
No. 4 on Air, Steam, or Liquid Conveyors, J. J. MCNULTA
No. 5 on Tiering, Piling, and Stacking Conveyors, J. G. WHEATLEY

SAFETY CODE FOR CRANES, DERRICKS, AND HOISTS (B30)

** Joint sponsorship with U.S. Navy Department, Bureau of Yards and Docks. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 53)

LEWIS PRICE †
F. H. SCHWERIN
R. H. WHITE †
H. L. WHITTEMORE

SUBCOMMITTEE CHAIRMEN

Executive Committee, J. C. WHEAT
No. 1 on Overhead and Gantry Cranes, R. H. WHITE
No. 2 on Locomotive and Tractor Cranes, H. H. VERNON
No. 3 on Derricks and Hoists, LEWIS PRICE
No. 4 on Miscellaneous Equipment for Cranes and Hoists, L. W. HOPKINS
No. 5 on Jacks, E. W. CARUTHERS
Editing Committee, H. H. VERNON

A.S.M.E. Representatives on Other Safety Committees

See also A.S.M.E. Representatives on Other Activities, page RI-9

SAFETY CODE FOR ABRASIVE WHEELS

** Sponsor bodies: Grinding Wheel Manufacturers Association of United States and Canada, and International Association of Industrial Accident Boards and Commissions*

J. B. CHALMERS

SAFETY CODE FOR CONSTRUCTION WORK

** Sponsor bodies: The American Institute of Architects, and National Safety Council*

C. H. O'NEIL

COOPERATION WITH OTHER ENGINEERING SOCIETIES

Committee of American Society of Safety Engineers—Engineering Section of National Safety Council

H. L. MINER

ASA SAFETY CODE CORRELATING COMMITTEE

A. W. LUCE
(A. E. WINDLE, *Alternate*)

SAFETY CODE FOR EXHAUST SYSTEMS

** Sponsor body: International Association of Industrial Accident Boards and Commissions*

T. F. HATCH

SAFETY CODE FOR FLOOR AND WALL OPENINGS, RAILINGS AND TOE BOARDS

** Sponsor body: National Safety Council*

A. E. WINDLE

SAFETY CODE FOR FORGING AND HOT METAL STAMPING

** Sponsor bodies: American Drop Forging Institute, and National Safety Council*

C. F. PARK

SAFETY CODE FOR LADDERS

** Sponsor body: American Society of Safety Engineers—Engineering Section of National Safety Council*

H. C. HOUGHTON

SAFETY CODE FOR LAUNDRY MACHINERY AND OPERATION

** Sponsor bodies: American Institute of Laundering, International Association of Governmental Labor Officials, and National Association of Mutual Casualty Companies*

E. J. Carroll

SAFETY CODE FOR LIGHTING FACTORIES, MILLS, AND OTHER WORK PLACES

** Sponsor body: Illuminating Engineering Society*

A. W. Luce

LOW VOLTAGE ELECTRICAL HAZARDS

Special Committee of the American Society of Safety Engineers—Engineering Section of National Safety Council

J. P. Jackson

SAFETY CODE FOR MECHANICAL REFRIGERATION

** Sponsor body: American Society of Refrigerating Engineers*

O. A. Anderson
Crosby Field
E. W. Gallenkamp
W. F. Jones
(A. W. Oakley, *Alternate to all A.S.M.E. Representatives*)

PANEL OF CONSULTANTS TO ADVISE THE MERCHANT MARINE COUNCIL AT U.S. COAST GUARD HEADQUARTERS, WASHINGTON, D.C.

D. S. Jacobus

SAFETY CODE FOR PAPER AND PULP MILLS

** Sponsor body: National Safety Council*

R. L. Weldon

SAFETY CODE FOR POWER PRESSES AND FOOT AND HAND PRESSES

** Sponsor body: National Safety Council*

J. B. Chalmers

SAFETY CODE FOR PREVENTION OF DUST EXPLOSIONS

** Sponsor bodies: National Fire Protection Association, and U.S. Department of Agriculture*

R. M. Ferry

SAFETY CODE FOR PROTECTION OF HEADS, EYES, AND RESPIRATORY ORGANS OF INDUSTRIAL WORKERS

** Sponsor body: National Bureau of Standards*

T. A. Walsh, Jr.
(T. F. Hatch, *Alternate*)

SAFETY CODE FOR PROTECTION OF INDUSTRIAL WORKERS IN FOUNDRIES

** Sponsor bodies: American Foundrymen's Association, and National Founders Association*

H. M. Lane

SAFETY IN QUARRY OPERATIONS

** Sponsor body: National Safety Council*

Redfield Proctor

SAFETY CODE FOR RUBBER MACHINERY

** Sponsor bodies: International Association of Industrial Accident Boards and Commissions and National Safety Council*

E. S. Ault

SPECIFICATIONS AND METHOD OF TEST FOR SAFETY GLASS

** Sponsor bodies: National Bureau of Standards and National Conservation Bureau*

T. A. Walsh, Jr.

SAFETY CODE FOR TEXTILES

** Sponsor body: National Safety Council*

M. A. Golrick, Jr.

SAFETY CODE FOR VENTILATION

** Sponsor body: American Society of Heating and Ventilating Engineers*

T. F. Hatch

SAFETY CODE FOR WALKWAY SURFACES

** Sponsor bodies: The American Institute of Architects, and American Society of Safety Engineers—Engineering Section of National Safety Council*

G. K. Palsgrove

SAFETY CODE FOR WORK IN COMPRESSED AIR

** Sponsor body: International Association of Industrial Accident Boards and Commissions*

L. J. Eibsen

BOILER CODE COMMITTEES

Article B6A, Par. 26: The Special Committee on Boiler Code shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Codes for Pressure Vessels, including the interpretations of these codes.

The first Special Committee on Boiler Code was organized in September, 1911.

SPECIAL COMMITTEE

D. S. Jacobus, *Honorary Chairman*
E. R. Fish, *Chairman*
H. B. Oatley, *Vice-Chairman*
J. W. Shields, *Secretary*
M. Jurist, *Assistant Secretary*
C. A. Adams
H. E. Aldrich
T. B. Allardice
H. C. Boardman
Perry Cassidy
R. E. Cecil
A. J. Ely
V. M. Frost
C. E. Gorton
A. M. Greene, Jr.
W. G. Humpton
J. O. Leech
I. E. Moultrop
C. O. Myers
C. W. Obert
James Partington
D. B. Rossheim
D. L. Royer
Walter Samans
S. K. Varnes
A. C. Weigel

Honorary Members

W. H. Boehm
W. F. Durand
T. E. Durban
C. L. Huston
W. F. Kiesel, Jr.
M. F. Moore
H. LeRoy Whitney

CONFERENCE COMMITTEE

T. R. Archer, Delaware
L. M. Barringer, Seattle, Wash.
J. G. Bollock, St. Joseph, Mo.
B. M. Book, Pennsylvania
E. J. Brock, St. Louis, Mo.
H. S. Brunson, Minnesota
E. S. Carpenter, Rhode Island
L. M. Cave, Maryland
S. Cherrington, Ohio
City Boiler Inspr., Parkersburg, W.Va.
D. E. Chadwick, Kansas City, Mo.
A. L. Colby, Louisiana
A. J. Conway, Indiana
M. A. Edgar, Wisconsin
C. W. Foster, Omaha, Neb.
W. H. Furman, New York
F. D. Garvin, Houston, Tex.
Gerald Gearon, Chicago, Ill.
C. H. Gram, Oregon
J. A. Gregory, Tampa, Fla.
C. W. Harness, Iowa
F. A. Heckinger, Memphis, Tenn.
H. K. Kugel, District of Columbia
P. N. Lehoczky, Ohio
M. L. Lobdell, Washington
E. C. Luster, Miami, Fla.
C. E. McGinnis, Los Angeles, Calif.
H. H. Mills, Detroit, Mich.
J. A. Murdock, Massachusetts
J. D. Newcomb, Jr., Arkansas
W. L. Newton, Oklahoma
F. A. Page, California
L. C. Peal, Nashville, Tenn.
J. D. Reed, Texas
C. Sattler, West Virginia
E. K. Sawyer, Maine
J. F. Scott, New Jersey
C. I. Smith, Utah
Wm. E. Smith, Hawaiian Islands
J. A. Strait, Tulsa, Okla.
John H. Thorpe, Michigan
C. E. Ward, North Carolina

EXECUTIVE COMMITTEE

D. S. Jacobus, *Chairman*
H. E. Aldrich, *Vice-Chairman*
E. R. Fish
V. M. Frost
C. E. Gorton
H. B. Oatley
C. W. Obert
James Partington

SUBCOMMITTEES

Boilers of Locomotives

James Partington, *Chairman*
F. H. Clark
J. M. Hall
H. B. Oatley

Care of Steam Boilers and Other Pressure Vessels in Service

F. M. Gibson, *Chairman*
D. C. Carmichael
V. M. Frost
J. R. Gill
Frank Henry
J. A. Hunter
H. J. Kerr
P. B. Place
S. T. Powell
C. W. Rice
J. B. Romer
W. C. Schroeder
F. G. Straub

Coordination

V. M. Frost, *Chairman*
E. R. Fish
C. W. Obert

Ferrous Materials

D. B. Rossheim, *Chairman*
A. B. Bagsar
L. D. Burritt
E. C. Chapman
C. L. Clark
A. J. Ely
H. J. French
W. R. Grunow
M. B. Higgins
A. M. Houser
W. G. Humpton
A. Hurtgen
T. McLean Jasper
J. J. Kanter
H. J. Kerr
A. B. Kinzel
L. J. Mason
N. L. Mochel
E. L. Robinson
A. P. Spooner
S. K. Varnes
A. E. White
R. L. Wilson

Heating Boilers

J. W. Turner, *Chairman*
C. E. Bronson
J. A. Darts
Wm. Ferguson
C. E. Gorton
L. N. Hunter
W. E. Stark

Material Specifications

J. O. Leech, *Chairman Pro Tem*
Perry Cassidy
A. M. Greene, Jr.
W. G. Humpton
P. J. Smith
A. C. Weigel

Miniature Boilers

C. E. Gorton, *Chairman*
W. H. Furman
G. A. Luck
C. W. Obert

Nonferrous Materials

H. B. Oatley, *Chairman*
J. J. Aull
W. F. Burchfield
D. K. Crampton
J. R. Freeman, Jr.
A. M. Houser
E. F. Miller
Joseph Price
R. L. Templin

Power Boilers

H. E. Aldrich, *Chairman*
T. B. Allardice
Perry Cassidy
E. R. Fish
V. M. Frost
D. L. Royer
A. C. Weigel

Rules for Inspection

(This subcommittee is being reorganized)

Special Design

D. B. Wesstrom, *Chairman*
H. C. Boardman
R. E. Cecil
T. W. Greene
D. B. Rossheim
E. O. Waters
F. S. G. Williams

Unfired Pressure Vessels

E. R. Fish, *Chairman*
C. A. Adams
C. E. Bronson
R. E. Cecil
Paul Diserens
H. S. Smith
D. B. Wesstrom

Welding

Members of A.S.M.E. Boiler Code Committee

James Partington, *Chairman*
O. R. Carpenter
E. C. Chapman

Welding
(*Continued*)

J. H. Deppeler
W. D. Halsey
R. K. Hopkins
J. T. Phillips
L. A. Sheldon

Members of Conference Committee of American Welding Society

C. W. Obert, *Chairman*
C. A. Adams
H. C. Boardman
Walter Samans
A. C. Weigel

SPECIAL COMMITTEES

Approval of New Materials

C. A. Adams, *Chairman*

Clad Vessels

S. K. Varnes, *Chairman*

Extension of Fusion Welding Requirements

H. E. Aldrich, *Chairman*

Feedwater

C. W. Rice, *Chairman*

Issuance of Code Symbol Stamps

C. O. Myers, *Chairman*

Material Specifications for Piping, Valves and Fittings

A. C. Weigel, *Chairman*

Radiographic Examination of Welded Joints

C. A. Adams, *Chairman*

Revision of Section VIII of the A.S.M.E. Boiler Code

E. R. Fish, *Chairman*

Rules for Bolted Flanged Connections

D. B. Wesstrom, *Chairman*

Rules for Dished Heads

H. C. Boardman, *Chairman*

Rules for Openings

T. D. Tifft, *Chairman*

Safety Valve Requirements

H. B. Oatley, *Chairman*

Work of Boiler Code Committee

H. E. Aldrich, *Chairman*

API-ASME COMMITTEE ON UNFIRED PRESSURE VESSELS

Walter Samans, *Chairman*

A.S.M.E. Representatives

R. E. Cecil
E. R. Fish
D. S. Jacobus
T. McLean Jasper
James Partington

A.P.I. Representatives

A. J. Ely
M. B. Higgins
K. V. King
Walter Samans
T. D. Tifft

THE WOMAN'S AUXILIARY TO THE A.S.M.E.

The Woman's Auxiliary to the A.S.M.E. was organized on May 10, 1923, and its Constitution and By-Laws was approved by the Council of the A.S.M.E. on October 27, 1924. The objects of the Auxiliary are to render service to all that pertains to the interest of the profession of mechanical engineering; to cooperate with any committees of the A.S.M.E.; and to assist the sons and daughters of the members of the Society or worthy students of mechanical engineering in obtaining scholarships; and to promote any other objects consistent with the aims or objects of the A.S.M.E.

NATIONAL OFFICERS

President, Mrs. E. C. M. Stahl
First Vice-President, Mrs. R. F. Gagg
Second Vice-President, Mrs. G. W. Farny
Third Vice-President, Mrs. C. M. Sames
Fourth Vice-President, Mrs. Justin J. McCarthy
Fifth Vice-President, Mrs. S. F. Duncan
Recording Secretary, Miss Burtie Haar
Corresponding Secretary, Mrs. Crosby Field
Treasurer, Mrs. A. H. Morgan

STANDING COMMITTEE CHAIRMEN

Student Loan, Mrs. R. F. Gagg
Membership, Mrs. G. E. Hagemann
Calvin W. Rice Scholarship, Mrs. H. N. Davis
Custodian, Miss Burtie Haar

COUNCIL REPRESENTATIVES

James W. Parker
Warren H. McBryde

OFFICERS OF LOCAL SECTIONS

Cleveland

Chairman, Mrs. F. Warren Brooks

Los Angeles

Chairman, Mrs. Edward Timbs

Metropolitan

Chairman, Mrs. J. Noble Landis
First Vice-Chairman, Mrs. Geo. E. Hagemann
Second Vice-Chairman, Mrs. C. H. Young
Third Vice-Chairman, Mrs. Earl Smith
Recording Secretary, Mrs. C. F. Kayan
Corresponding Secretary, Mrs. Waldo M. McKee
Treasurer, Mrs. C. E. Gus

Philadelphia

Chairman, Mrs. Justin J. McCarthy

AWARDS

The following paragraphs deal with the medals, awards, scholarships, and loan funds which come within the jurisdiction of the A.S.M.E. Other awards available to Student Members are listed in *Mechanical Engineering,* February, 1938, page 183. The Society also participates with other engineering societies in a number of joint awards. Further details concerning all the awards may be obtained from the headquarters of the Society.

Honorary Membership, to which persons of acknowledged professional eminence are elected by unanimous vote of Council under the provisions of the By-Laws and Rules. A list of honorary members is given on page RI-43.

Life Membership, which may be conferred by the Council for distinguished service to the Society; or secured by a member by payment for an annuity in accordance with the provisions of the By-Laws.

A.S.M.E. Medal, established by the Society in 1920 to be presented, together with an engraved certificate, for distinguished service in engineering and science. May be awarded for general service in science having possible application in engineering.

Holley Medal, instituted and endowed in 1924 by George I. Rockwood, Past Vice-President of the Society, to be bestowed, together with an engraved certificate, for some great and unique act of genius of engineering nature that has accomplished a great and timely public benefit.

Worcester Reed Warner Medal, established in 1930, provision for which was made in the will of Worcester Reed Warner, Honorary Member of the Society, is a gold medal to be bestowed, together with an engraved certificate, for an outstanding contribution to engineering literature.

Melville Medal, established in 1914 by the bequest of Rear-Admiral George W. Melville, Honorary Member and Past-President of the Society, to be presented, together with an engraved certificate, for an original paper or thesis of exceptional merit, presented to the Society for discussion and publication, to encourage excellence in papers. The medal may be presented annually.

Spirit of St. Louis Medal, established by an endowment fund created in 1929 by citizens of St. Louis, Mo., to be awarded for meritorious service in the advancement of aeronautics. This medal will be awarded at the discretion of the Council of the Society at approximately three-year periods upon the recommendation of its Board of Honors and Awards.

Spirit of St. Louis Junior Award, established in 1938 by an endowment fund created by the General Committee for the 1935 Aeronautic Meeting in St. Louis; a cash award of $50, made every three years, for the best paper on an aeronautic subject presented at any A.S.M.E. meeting during the three-year period either personally by the author (a Junior Member of the Society under thirty years of age) or by a Junior Member designated by him, and submitted to the Committee on Medals within a reasonable period (to be determined by the Committee) after its initial presentation.

Pi Tau Sigma Medal Award, established in 1938, endowed by Pi Tau Sigma, the national honorary mechanical engineering fraternity, to be presented annually, together with an engraved certificate, to a young mechanical engineer for outstanding achievement in his profession within the ten years after graduation from a regular four-year mechanical engineering course of a recognized American college or university. Any mechanical engineering graduate, not more than thirty-five years of age, whose achievement has been all or in part in any field including industrial, educational, political, research, civic, etc., is eligible.

Junior Award, annual cash award of $50, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with an engraved certificate, for the best paper or thesis submitted by a Junior Member.

Charles T. Main Award, annual cash award of $150, established in 1919 from a fund created by Charles T. Main, Past-President of the Society, to be awarded, together with an engraved certificate, to a Student Member of the Society, for the best paper within the general subject of the influence of the profession upon public life. The exact subject is assigned by the Board of Honors and Awards, subject to the approval of the Council, and is announced each year through the Honorary Chairmen of the Student Branches.

Student Awards, two annual cash awards of $25 each, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with engraved certificates, for the best papers or theses submitted by Student Members. The awards for 1932 and subsequent years have been given, one for undergraduate and the other for postgraduate work.

SCHOLARSHIPS AND LOAN FUNDS

Max Toltz: Loan Fund of $15,000 established by Major Max Toltz, former member of the Council of the Society, the income to be used for assistance to Student Members.

John R. Freeman: Fund of $25,000 established in 1926 by John R. Freeman, Past-President of the Society, the income to be used for travel scholarships and research.

Woman's Auxiliary: Educational Loan Fund offered by the Woman's Auxiliary to the Society to assist sons and daughters of members or worthy students of mechanical engineering. Calvin W. Rice Memorial Scholarship Fund for students in mechanical engineering from South America.

RECIPIENTS OF AWARDS

The names of the recipients of the different awards to date are given in the following lists, together with the dates of presentation, and the services or papers for which the awards were made. There were no awards for the years not listed.

A.S.M.E. Medal

1921 Hjalmar Gotfried Carlson, in recognition of the services rendered the Government because of his invention and part in the production of 20,000,000 Mark III drawn steel booster casings used principally as a component of 75-mm high explosive shells, but also used extensively in gas shells and bombs

1922 Frederick Arthur Halsey, for his paper describing the premium system of wage payments presented before the Society at the Providence Meeting in 1891, as the adoption of the methods there proposed has had a profound effect toward harmonizing the relations of worker and employer

1923 John Ripley Freeman, for his eminent service in engineering and manufacturing by his meritorious work in fire prevention and the preservation of property

1926 R. A. Millikan, in recognition of his contributions to science and engineering

1927 Wilfred Lewis, for his contributions to the design and construction of gear teeth

1928 Julian Kennedy, for his services and contributions to the iron and steel industry

1929 William LeRoy Emmet, for his contributions in the development of the steam turbine, electric propulsion of ships, and other power-generating apparatus

1931 Albert Kingsbury, for his research and development work in the field of lubrication

1933 Ambrose Swasey, for his contributions to the advancement of the engineering profession and for his part in the development of the turret lathe and the astronomical telescope

1934 Willis H. Carrier, in recognition of his research and development work in air-conditioning

1935 Charles T. Main, for distinguished achievements in the textile and other industries, in engineering education, and for eminent service to the engineering profession

1936 Edward Bausch, for meritorious mechanical developments in the field of optics

1937 Edward P. Bullard, for outstanding leadership in the development of station-type machine tools

1938 Stephen J. Pigott, for outstanding leadership in marine propulsion and construction

1939 James E. Gleason, for service to the cause of safer and better transportation

1940 Charles F. Kettering, for outstanding inventions and research

1941 Theodor von Kármán, for his brilliance as a teacher, his researches in elasticity and many fields of physics and mechanics, and his distinguished leadership in the fields of aerodynamics and aircraft design

1942 E. G. Bailey, for achievement and leadership in steam and combustion engineering.

Holley Medal

1924 Hjalmar Gotfried Carlson, for his inventions and processes which made possible the timely production of drawn steel booster casings for artillery ammunition, thereby aiding victory in World War I (diploma in recognition of achievements presented in 1921)
1927 Elmer Ambrose Sperry, for achievements and inventions that have advanced the naval arts, including the gyroscope that has freed navigation from the dangers of the fluctuating magnetic compass
1929 Baron Chuzaburo Shiba, for his contributions to knowledge through fundamental research, including the field of aerodynamics, by the development of ultra-rapid kinematographic methods
1934 Irving Langmuir, for his contributions to science and engineering, including the development of gas-filled incandescent lamps, thoriated filament for thermionic emission, atomic hydrogen welding, phase control operation of the thyratron tube, and fundamental research in oil films
1936 Henry Ford, for revolutionary influence through invention and practice on transportation and on mass production methods in manufacturing
1937 Frederick G. Cottrell, for preeminent public service—the invention of electric precipitation—advancement of the science of gas liquefaction—gifts for engineering research
1938 Francis Hodgkinson, for meritorious services in the development of the steam turbine
1939 Carl E. Johansson, in recognition of his pioneer work in the development of basic measuring gages
1940 Edwin Howard Armstrong, for his leadership in the field of radio communication
1941 John C. Garand, for the invention and development of the semi-automatic rifle, which has been adopted by the U.S. Army as the U.S. Rifle, Caliber .30, M1, an outstanding contribution to our national defense
1942 Ernest O. Lawrence, for originating the cyclotron, a unique invention for producing high-speed electrified particles, and for adapting it to research in physics, chemistry, medicine, and the properties of engineering materials.

Worcester Reed Warner Medal

1933 Dexter S. Kimball, for his contributions to efficiency in management as exemplified by his recently revised "Principles of Industrial Organization" and by his many articles, engineering society papers, and public addresses
1934 Ralph E. Flanders, for his contributions to a better understanding of the relationship of the engineer to economic problems and social trends as exemplified by the many papers which he has presented
1935 Stephen Timoshenko, for his contributions to the theory of the design of elastic structures and the treatment of dynamics of moving machinery
1936 Charles M. Allen, for his early and continued hydraulic laboratory work and for the permanent value of the papers on his development of methods of testing large hydraulic turbine installations
1937 Clarence F. Hirshfeld, for his research and contributions to the theory and practice of heat-power engineering as exemplified by books and papers
1938 Lawford H. Fry, for contributions relating to improved locomotive boiler design and utilization of better materials in railway equipment
1939 Rupen Eksergian, for influential papers of permanent value in A.S.M.E. Transactions
1940 William Benjamin Gregory, for distinguished work in hydraulic engineering, which has been the basis for many engineering papers
1941 Richard Vynne Southwell, for his many distinguished services to engineering and science through papers and publications in many fields, including aeronautics, theory of structures, elasticity, and hydrodynamics
1942 Fred H. Colvin, for his contributions to both technical advancement and improvement in management in the metal-working industries, as influenced by more than fifty years of articles and books—particularly American Machinists' Handbook.

Melville Medal

1927 Leon P. Alford, "Laws of Manufacturing Management"
1929 Joseph W. Roe, "Principles of Jig and Fixture Practice"
1930 Herman Diederichs and William D. Pomeroy, "The Occurrence and Elimination of Surge or Oscillating Pressure in Discharge Lines From Reciprocating Pumps"
1931 Arthur E. Grunert, "Comparative Performance of a Pulverized-Coal-Fired Boiler Using Bin System and Unit System of Firing"
1932 Alexey J. Stepanoff, "Leakage Loss and Axial Thrust in Centrifugal Pumps"
1933 William E. Caldwell, "Characteristics of Large Hell Gate Direct-Fired Boiler Units"
1935 Oscar R. Wikander, "Draft-Gear Action in Long Trains"
1936 H. A. Stevens Howarth, "The Loading and Friction of Thrust and Journal Bearings With Perfect Lubrication"
1937 Alfred J. Büchi, "Supercharging of Internal-Combustion Engines With Blowers Driven by Exhaust-Gas Turbines"
1938 Alphonse I. Lipetz, "Air Resistance of Railroad Equipment"
1939 Lester M. Goldsmith, "High-Pressure High-Temperature Turbine-Electric Steamship *J. W. Van Dyke*"
1940 Carl A. W. Brandt, "The Locomotive Boiler"
1941 Roger V. Terry, "Development of the Automatic Adjustable-Blade-Type Propeller Turbine"
1942 J. Kenneth Salisbury, "The Steam-Turbine Regenerative Cycle—An Analytical Approach."

Spirit of Saint Louis Medal

1929 Daniel Guggenheim, founder of the Guggenheim Fund for the Promotion of Aeronautics
1932 Paul Litchfield, for his work in encouraging and sponsoring airship design and construction in this country
1935 Will Rogers, for his splendid, constructive, and unselfish work in the achievement of aviation, and the building up of public confidence in aviation through his articles in the press, over the radio, and from the speaker's platform
1938 James H. Doolittle, for meritorious service in the advancement of aeronautics
1941 John E. Younger, for notable contributions to the science of airplane design, particularly in the conception, analysis, and supervision of the development of the fundamental design principles, requirements, and criteria which first assured the success of the pressure-cabin type of high-altitude airplane.

Spirit of Saint Louis Junior Award

1941 Wilbur W. Reaser, for his paper, "Calculation of the Heat Loss from an Airplane Cabin."

Pi Tau Sigma Medal

1938 Wilfrid E. Johnson, for his development work in the field of refrigeration
1939 John I. Yellott, Jr., in recognition of significant achievements in steam-flow research and engineering education; also contributions on "Supersaturated Steam" and "Condensation of Flowing Steam in Diverging Nozzles"
1940 George A. Hawkins, for significant achievements in high-pressure steam research and engineering education
1941 R. Hosmer Norris, for outstanding achievement in mechanical engineering, particularly in the heat-transfer field
1942 John T. Rettaliata, for outstanding achievement in mechanical engineering.

Junior Award

1915 Ernest O. Hickstein, "Flow of Air Through Thin Plate Orifices"
1916 L. M. McMillan, "The Heat Insulating Properties of Commercial Steam-Pipe Coverings"
1919 E. D. Whalen, "Properties of Airplane Fabrics"
1921 S. Logan Kerr, "Moody Ejector Turbine"
1922 R. H. Heilman, "Heat Losses From Bare and Covered Wrought-Iron Pipe at Temperatures up to 800 Degrees Fahrenheit"
F. L. Kallam, "Preliminary Report on the Investigation of the Thermal Conductivity of Liquids"
1923 S. S. Sanford and Sabin Crocker, "The Elasticity of Pipe Bends"
1924 R. H. Heilman, "Heat Losses Through Insulating Material"
1925 Gilbert S. Schaller, "An Investigation of Seattle as a Location for a Synthetic Foundry Industry"

1927 William M. Frame, "Stresses Occurring in the Walls of an Elliptical Tank Subjected to Low Internal Pressure"
1928 M. D. Aisenstein, "A New Method of Separating the Hydraulic Losses in a Centrifugal Pump"
1929 Arthur M. Wahl, "Stresses in Heavy, Closely Coiled Helical Springs"
1930 Ed Sinclair Smith, "Quantity-Rate Fluid Meters"
1931 M. K. Drewry, "Radiant-Superheater Developments"
1932 Edmond M. Wagner, "Frictional Resistance of a Cylinder Rotating in a Viscous Fluid Within a Coaxial Cylinder"
1933 Townsend Tinker, "Surface Condenser Design and Operating Characteristics"
1934 John I. Yellott, Jr., "Supersaturated Steam"
1935 Stanley J. Mikina, "Effect of Skewing and Pole Spacing on Magnetic Noise in Electrical Machinery"
1936 Harwood F. Mullikan, Jr., "Evaluation of Effective Radiant Heating Surface and Application of the Stefan-Boltzman Law to Heat Absorption in Boiler Furnaces"
1937 Leslie J. Hooper, "American Hydraulic-Laboratory Practice"
1938 Arthur C. Stern, "Separation and Emission of Cinders and Fly Ash"
1940 Robert E. Newton, "A Photoelastic Study of Stresses in Rotary Disks"
1941 John T. Rettaliata, "The Combustion Gas Turbine"
1942 Winston M. Dudley, "Analysis of Longitudinal Motions in Trains of Several Cars."

Charles T. Main Award

1925 Clement R. Brown, Catholic University of America. Subject: "The Influence of the Locomotive on the Unity of the United States"
1926 W. C. Saylor, Johns Hopkins University. Subject: "The Effect of the Cotton Gin Upon the History of the United States During Its First Seventy Years"
1927 No Award. Subject: "Scientific Management and Its Effect Upon the Industries"
1928 Robert M. Meyer, Newark College of Engineering. Subject: "Scientific Management and Its Effect Upon Manufacturing"
1929 No Award. Subject: "The Influence of Engineering on Farm Production"
1930 Jules Podnossoff, Polytechnic Institute of Brooklyn. Subject: "The Value of the Safety Movement in the Industries"
1931 Robert E. Klise, University of Michigan. Subject: "Interchangeability—Its Development and Significance in Industry"
1932 Marshall Anderson, University of Michigan. Subject: "Apprenticeship and Vocational Training"
1933 George D. Wilkinson, Jr., Newark College of Engineering. Subject: "Progress in the Prevention of Smoke and Atmospheric Pollution"
1934 Philip P. Self, Colorado State College. Subject: "Air Conditioning—Its Practicability and Relation to Public Welfare"
1935 G. Lowell Williams, Lafayette College. Subject: "Coordinated Transportation—An Economic Comparison of Railroad, Bus, Truck, Water, and Air Transportation for Long and Short Haul"
1936 No Award. Subject: "Development in the Generation and Distribution of Power and Their Effect Upon the Consumer"
1937 Allan P. Stern, Case School of Applied Science. Subject: "The Influence of the Introduction of Labor Saving Machinery Upon Employment in the United States"
1938 Edward W. Connolly, University of Detroit. Subject: "Economic Limitations in Engineering Design, With Concrete Examples"
1939 James R. Bright, Lehigh University. Subject: "The Economics of Investment in New Manufacturing Equipment—With Concrete Cases"
1940 Frank De Pould, Case School of Applied Science. Subject: "What Has Been the Effect of Technological Advance on Employment?"
1941 John J. Balun, University of Detroit. Subject: "The Need and Possibilities of Participation by Engineers in Public Affairs"
1942 Bernard J. Isabella, Case School of Applied Science. Subject: "The Engineer and Preparation for the Coming Peace."

Student Award

1916 Boynton M. Green, Stanford University, "Bearing Lubrication"
Howard E. Stevens, Rensselaer Polytechnic Institute, "An Investigation of the Dynamic Pressure on Submerged Flat Plates"
M. Adam, Louisiana State University, "The Adaptability of the Internal Combustion Engine to Sugar Factories and Estates"
1917 H. R. Hammond and C. W. Holmberg, The Pennsylvania State College, "Study of Surface Resistance With Glass as the Transmission Medium"
1919 C. F. Leh and F. G. Hampton, Stanford University, "An Experimental Investigation of Steel Belting"
W. E. Helmick, Stanford University, "An Experimental Investigation of Steel Belting"
1920 Howard G. Allen, Cornell University, "Wire Stitching Through Paper"
1921 Karl H. White, University of Kansas, "Forces in Rotary Motors"
Richard H. Morris and Albert J. R. Houston, University of California, "A Report Upon an Investigation of the Herschel Type of Improved Weir"
1923 Charles F. Olmstead, University of Minnesota, "Oil Burning for Domestic Heating"
H. E. Doolittle, University of California, "The Integrating Gate: A. Device for Gaging in Open Channels"
1924 George Stuart Clark, Stanford University, "Two Methods Used for the Determination of the Gasoline Content of Absorption Oils in Absorption Plants"
L. J. Franklin and Charles H. Smith, Stanford University, "The Effect of Inaccuracy of Spacing on the Strength of Gear Teeth"
1925 Harry Pease Cox, Jr., Rensselaer Polytechnic Institute. "A Study of the Effect of End Shape on the Towing Resistance of a Barge Model"
W. S. Montgomery, Jr., and E. Ray Enders, Jr., Pennsylvania State College, "Some Attempts to Measure the Drawing Properties of Metals"
1926 R. E. Peterson, University of Illinois, "An Investigation of Stress Concentration by Means of Plaster of Paris Specimens"
Cecil G. Heard, University of Toronto, "Pressure Distribution Over U.S.A. 27 Aerofoil With Square Wing Tips—Model Tests"
1927 Alfred H. Marshall, Princeton University, "Evaporative Cooling"
Roger Irwin Eby, University of Washington, "Measurement of the Angular Displacement of Flywheels"
1928 Clarence C. Franck, Johns Hopkins University, "Condition Curves and Reheat Factors for Steam Turbines"
1929 Frank Vernon Bistrom, University of Washington, "An Investigation of a Rotary Pump"
William Wallace White, University of Washington, "An Investigation of a Rotary Pump"
1930 Gerard Eden Claussen, Polytechnic Institute of Brooklyn, "High-Temperature Oxidation of Steel"
Harold L. Adams and Richard L. Stith, University of Washington, "A Wind Tunnel for Undergraduate Laboratory Experiments"
1931 Jules Podnossoff, Polytechnic Institute of Brooklyn, "Pressure and Energy Distribution in Multi-Stage Steam Turbines Operating Under Varying Conditions"
1932 H. E. Foster, Jr., University of Tennessee, "Factors Affecting Spray Pond Design" (Undergraduate Award)
William A. Mason, Stanford University, "An Experimental Investigation of the Flame Propagation in Internal-Combustion Engines" (Postgraduate Award)
1933 Hugo V. Cordiano, Polytechnic Institute of Brooklyn, "Thermal Analysis of Lithium-Magnesium System of Alloys" (Undergraduate Award)
James A. Ostrand, Jr., Princeton University, "Sudden Enlargement in the Open Channel" (Postgraduate Award)
1934 H. Reynolds Hudson, Georgia School of Technology, "Dynamic Balance and Functional Utility Applied to Automotive Design" (Undergraduate Award)
1935 Charles P. Bacha, Rutgers University, "The Behavior of Metals Subjected to Combined Stress" (Postgraduate Award)

Robert W. Beal, Oregon State College, "Do Lubricating Oils Wear Out?" (Undergraduate Award)

1936 Leon B. Stinson, Oklahoma Agricultural and Mechanical College, "Polymerized Motor Fuels; Their Economic Significance" (Undergraduate Award)

DeWitt D. Barlow, Jr., Princeton University, "The Critical Speeds of Lateral Vibrations of Shafts with Gyroscopic Effects" (Postgraduate Award)

1937 Gino J. Marinelli, Rensselaer Polytechnic Institute, "Investigation of the Towing Resistance of a Model Submarine Hull" (Undergraduate Award)

1938 Marshall C. Long, Princeton University, "An Investigation Into the Angular Characteristics of an Adjustable Blade Current Meter" (Postgraduate Award)

Donald C. McSorley, Michigan State College, "Humidity Insulation" (Undergraduate Award)

1939 David T. James, Michigan State College, "Bells—Concerning Their Tones" (Undergraduate Award)

1940 George W. Shepherd, Jr., Princeton University, "An Automatic Mechanical Control for Synchronizing Prime Movers" (Postgraduate Award)

Edward D. Rowan, Oregon State College, "Powder Metallurgy" (Undergraduate Award)

1941 G. Walker Gilmer, III, University of Florida, "Center of Pressure Characteristics of a Marconi Yacht Sail" (Undergraduate Award)

1942 Arthur W. McClure, Princeton University, "A Specific Speed Analysis of Turbosuperchargers for Aircraft" (Postgraduate Award)

J. Packard Laird, Princeton University, "An Analysis of Motorcycle Behavior" (Undergraduate Award).

Freeman Travel Scholarship

1927 Herbert N. Eaton
1928 Blake R. Van Leer
1929 Robert T. Knapp
1931 Reginald Whitaker
1932 G. Ross Lord
1933, 1934 H. J. Casey
1935, 1936 Victor L. Streeter

HONORARY MEMBERS

HONORARY MEMBERS IN PERPETUITY

Alexander Lyman Holley, Founder of the Society. Died 1882.

John Edson Sweet, Founder of the Society. Died 1916.

Henry Rossiter Worthington, Founder of the Society. Died 1880.

DECEASED HONORARY MEMBERS

	Elected	Died
Leon Pratt Alford	1941	1942
Horatio Allen	1880	1889
Lorenzo Allievi	1937	1941
Sir William Arrol	1905	1913
Sir John Audley Frederick Aspinall	1911	1937
William Wallace Atterbury	1925	1935
Sir Benjamin Baker	1886	1907
Johann Bauschinger	1884	1893
Sir Henry Bessemen	1891	1898
Sir Frederick Joseph Bramwell	1884	1903
John Alfred Brashear	1908	1920
Gustave Canet	1900	1908
Andrew Carnegie	1907	1919
Daniel Kinnear Clark	1882	1896
Rudolph Julius Emmanuel Clausius	1882	1888
Hutchinson I. Cone	1936	1941
Sir John Goode	1889	1892
Peter Cooper	1882	1883
Charles de Fréminville	1919	1936
Carl Gustaf Patrick de Laval	1912	1913
Rudolph Diesel	1912	1913
Alex Dow	1936	1942
James Dredge	1886	1906
Victor Dwelshauvers-Dery	1886	1913
Thomas Alva Edison	1904	1931
Alexandre Gustave Eiffel	1889	1923
Marshal Ferdinand Foch	1921	1929
Sir Charles Douglas Fox	1900	1921
John Ripley Freeman	1932	1932
John Fritz	1900	1913
Major-General George Washington Goethals	1917	1928
Franz Grashof	1884	1893
Rear-Admiral Robert Stanislau Griffin	1920	1933
Otto Hallauer	1882	1883
Charles Haynes Haswell	1905	1907
Nathanael Greene Herreshoff	1921	1938
Friedrich Gustav Herrmann	1884	1907
Gustav Adolph Hirn	1882	1890
Joseph Hirsch	1889	1901
Ira N. Hollis	1928	1930
Robert Woolston Hunt	1920	1923
Benjamin Franklin Isherwood	1894	1915
Henri Léauté	1891	1916
Erasmus Darwin Leavitt	1915	1916
Henri Le Chatelier	1927	1936
Anatole Mallet	1912	1919
Charles H. Manning	1913	1919
Rear-Admiral George Wallace Melville	1910	1912
The Honorable Sir Charles Algernon Parsons	1920	1931
Charles Talbot Porter	1890	1910
Edwin Jay Prindle	1939	1942
Auguste C. E. Rateau	1919	1930
Sir Edward J. Reed	1882	1906
Franz Reuleaux	1882	1905
Calvin Winsor Rice	1931	1934
Palmer C. Ricketts	1931	1934
Henri Adolphe-Eugene Schneider	1882	1898
Charles M. Schwab	1918	1939
C. William Siemans	1882	1883
Viscount Ei-ichi Shibusawa	1929	1931
Ambrose Swasey	1916	1937
Elihu Thomson	1930	1937
Henry Robinson Towne	1921	1924
Henri Tresca	1882	1885
William Cawthorne Unwin	1898	1933
Samuel Matthews Vauclain	1920	1940
Henry Hague Vaughan	1939	1942
Oskar von Miller	1912	1934
Francis A. Walker	1886	1897
Worcester Reed Warner	1925	1929
George Westinghouse	1897	1914
Sir William Henry White	1900	1913
Sir Alfred Fernandez Yarrow	1914	1932

LIVING HONORARY MEMBERS

	Elected
William Lamont Abbott	1940
Robert W. Angus	1940
Edmund Bruce Ball	1939
William L. Batt	1942
Willis H. Carrier	1942
Mortimer Elwyn Cooley	1928
William Frederick Durand	1934
Charles E. Ferris	1942
Arthur M. Greene, Jr.	1940
Herbert Clark Hoover	1925
Clarence Decatur Howe	1941
Jerome C. Hunsaker	1942
David Schenck Jacobus	1934
Masawo Kamo	1929
Dexter Simpson Kimball	1939
Albert Kingsbury	1940
Charles Thomas Main	1939
George A. Orrok	1936
Grande Ufficiale Ing. Pio Perrone	1920
Harry R. Ricardo	1942
Rear Admiral Samuel Murray Robinson	1941
James A. Seymour	1940
Aurel Stodola	1941
Major General William H. Tschappat	1938
Right Honorable Lord Weir	1920
Major General Charles Macon Wesson	1941
Orville Wright	1918

PAST-PRESIDENTS

Dates in parentheses denote year of death.

ALEXANDER LYMAN HOLLEY, *Chairman of the Preliminary Meeting for Organization of The American Society of Mechanical Engineers* (1882)

1880–1882	ROBERT HENRY THURSTON (1903)
1883	ERASMUS DARWIN LEAVITT (1916)
1884	JOHN EDSON SWEET (1916)
1885	JOSEPHUS FLAVIUS HOLLOWAY (1896)
1886	COLEMAN SELLERS (1907)
1887	GEORGE H. BABCOCK (1893)
1888	HORACE SEE (1909)
1889	HENRY ROBINSON TOWNE (1924)
1890	OBERLIN SMITH (1926)
1891	ROBERT WOOLSTON HUNT (1923)
1892	CHARLES HARDING LORING (1907)
1893–1894	ECKLEY BRINTON COXE (1895)
1895	EDWARD F. C. DAVIS (1895)
1895	CHARLES ETHAN BILLINGS (1920)
1896	JOHN FRITZ (1913)
1897	WORCESTER REED WARNER (1929)
1898	CHARLES WALLACE HUNT (1911)
1899	GEORGE WALLACE MELVILLE (1912)
1900	CHARLES HILL MORGAN (1911)
1901	SAMUEL T. WELLMAN (1919)
1902	EDWIN REYNOLDS (1909)
1903	JAMES MAPES DODGE (1915)
1904	AMBROSE SWASEY (1937)
1905	JOHN RIPLEY FREEMAN (1932)
1906	FREDERICK WINSLOW TAYLOR (1915)
1907	FREDERICK REMSEN HUTTON (1918)
1908	MINARD LAFEVER HOLMAN (1925)
1909	JESSE MERRICK SMITH (1927)
1910	GEORGE WESTINGHOUSE (1914)
1911	EDWARD DANIEL MEIER (1914)
1912	ALEXANDER CROMBIE HUMPHREYS (1927)
1913	WILLIAM FREEMAN MYRICK GOSS (1928)
1914	JAMES HARTNESS (1934)
1915	JOHN ALFRED BRASHEAR (1920)
1916	DAVID SCHENCK JACOBUS
1917	IRA NELSON HOLLIS (1930)
1918	CHARLES THOMAS MAIN
1919	MORTIMER ELWYN COOLEY
1920	FRED J. MILLER (1939)
1921	EDWIN S. CARMAN
1922	DEXTER SIMPSON KIMBALL
1923	JOHN LYLE HARRINGTON (1942)
1924	FREDERICK ROLLINS LOW (1936)
1925	WILLIAM FREDERICK DURAND
1926	WILLIAM LAMONT ABBOTT
1927	CHARLES M. SCHWAB (1939)
1928	ALEX DOW (1942)
1929	ELMER AMBROSE SPERRY (1930)
1930	CHARLES PIEZ (1933)
1931	ROY V. WRIGHT
1932	CONRAD N. LAUER
1933	A. A. POTTER
1934	PAUL DOTY (1938)
1935	RALPH E. FLANDERS
1936	WILLIAM L. BATT
1937	JAMES H. HERRON
1938	HARVEY N. DAVIS
1939	ALEXANDER G. CHRISTIE
1940	WARREN H. MCBRYDE
1941	WILLIAM A. HANLEY
1942	JAMES W. PARKER

TREASURERS

Apr. 1880—Dec. 1881	LYCURGUS B. MOORE *
Dec. 1881—Nov. 1884	CHARLES W. COPELAND (1895)
1894—1925	WILLIAM H. WILEY (1925)
1925—1935	ERIK OBERG
1935—date	WILLIAM D. ENNIS

SECRETARIES

Organization Meeting, 1880	SAMUEL S. WEBBER, JR. (1921)
Acting Secretary, Apr.-Nov. 1880	LYCURGUS B. MOORE *
Nov. 1880—Mar. 1883	THOS. WHITESIDE RAE (1895)
1883—1906	FREDERICK R. HUTTON (1918)
1906—1934	CALVIN W. RICE (1934)
1934—date	CLARENCE E. DAVIES

* Deceased. Year not known.

Index to Society Records, Part 1

The page numbers in this section are preceded by the letters "RI," which are omitted in the following index.

Combined Firing of Coal and Natural Gas on Stoker-Fired Units

By H. L. Crain,[1] Kansas City, Mo.

For reasons of economy, increased capacity, and the possibility of improving operation when only inferior coals were available, the Kansas City Power & Light Company, in 1937, installed natural-gas burners in three of the chain-grate stoker-fired boilers at its Grand Avenue Station, to supplement coal firing. Later, additional boilers were equipped, the last two installations being made in 1941. Over the period of operation, the average use of gas at this station has represented 25 to 30 per cent of the fuel requirements. The reasons leading to the adoption of this combined firing system are outlined in the paper, as well as the details of the installations and the operating results achieved.

The availability of natural gas for purchase on a dump contract at an attractive price was the basic reason for installation of gas-burning equipment in 1937, under three boilers at the Grand Avenue Station of the Kansas City Power & Light Company. Desirable features, other than fuel costs, were considered. It was anticipated that the maximum boiler capacity, with existing draft equipment, could be increased and that dependable boiler capacity could also be increased at times when inferior grades of coal were burned. It was not planned to replace coal with natural-gas fuel but to supplement coal firing with gas by burning both fuels simultaneously. The major portion of the gas was to be burned in the summer; however, some gas was to be available for 10 months per year with 12-month service a possibility. The sale of gas on such a contract was advantageous to the local gas-distributing company, as it would result in an improvement in the load factor, which was high in winter and low in summer due to seasonal use of an unusually large number of domestic-heating customers. The contemplated installation was to be made on Combustion Engineering Company Ladd type steam-generating units of 225,000-lb per hr capacity each. These boilers were designed to burn $1^1/_4$-in. Missouri coal screenings on forced-draft chain-grate stokers of 528 sq ft area (24 ft × 22 ft), using a preheated-air temperature of 350 F (Fig. 1).

Combustion Troubles From Inferior Grades of Coal

Coal was supplied from strip-pit mines in the west central part of Missouri and contained approximately 11 per cent moisture, 17 per cent ash, 3 per cent sulphur, 31 per cent volatile, 41 per cent fixed carbon, and 10,500 Btu per lb as received from the mines. No trouble was experienced when burning coal of this quality. However, inferior grades of coal were received at times, particularly in the winter season when the residential consumption was high and on occasions when new seams were opened and foreign material was unavoidably included in the shipments.

On these occasions clinker trouble was experienced on the stokers. Huge slabs of a heavy fused mass of refuse would be conveyed to the ash pits, resulting in decreased ratings, higher ash-pit loss, and lowered boiler efficiency. It was impossible to clear up the trouble except by reducing the boiler rating further and running the coked fuel bed into the pits. Combustion of this poorer grade of coal also resulted in dirtier heat-absorbing surfaces and increased preheated-air temperature to the stokers, which in turn aggravated the coking process. While it was possible to use preheated-air temperatures of 400 to 450 F with

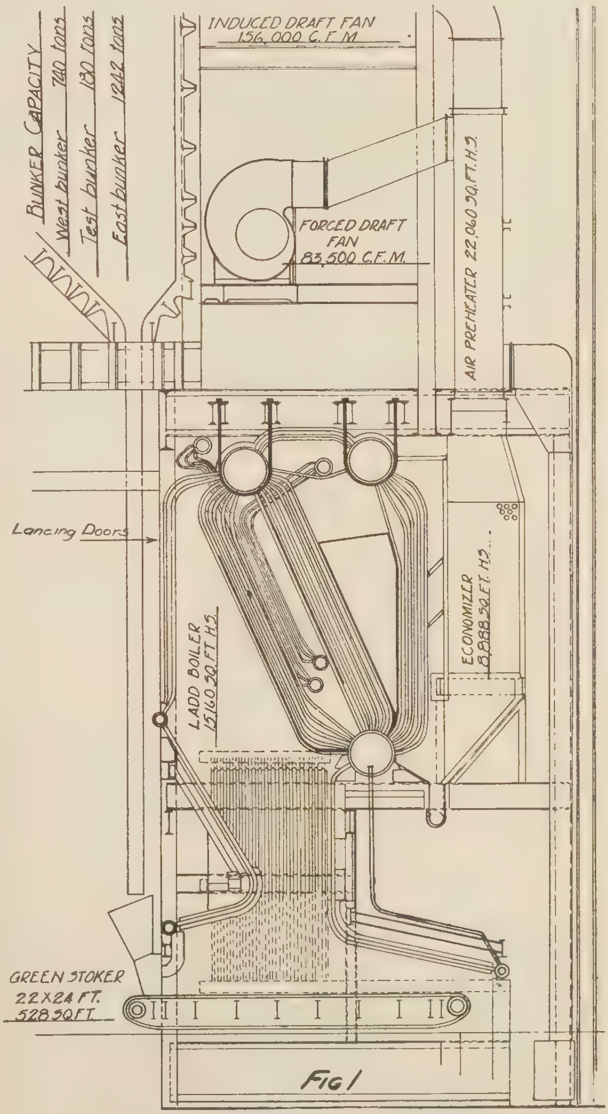

Fig. 1 General Arrangement of Boiler and Equipment, Grand Avenue Station

[1] Plant Results Engineer, Kansas City Power & Light Company. Mem. A.S.M.E.

Presented at the Sixth Annual Meeting of the Fuels Division, of The American Society of Mechanical Engineers, and the Coal Division of the American Institute of Mining and Metallurgical Engineers, St. Louis, Mo., Sept. 30–Oct. 1, 1942.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

the better coal, at times the maximum usable air temperature to prevent coking with the inferior grades did not exceed 300 to 350 F. For better control of air temperature, air by-passes were installed around the air preheaters, and some time later water sprays, discharging onto the stoker just ahead of the coal hoppers, were installed to cool the grates. The combination of air by-passes and water sprays reduced the coking difficulties with no apparent detrimental effect on the stokers but at the expense of stack temperature.

In addition to coking trouble, poor ignition of the inferior-quality coal resulted in lowered dependable boiler capacities. On one occasion three boilers of 225,000-lb per hr capacity each could not pull a 280,000-lb per hr steam load, and a fourth boiler was required to maintain the steam pressure. This was with coal of 9100 to 9500 Btu per lb heat content. While this condition is extreme, the capacities of the boilers have been limited by coal quality to 175,000 lb per hr on numerous occasions. It is not to be inferred that the limitation on maximum ratings existed at all times, since with good-quality coal, ratings of 220,000 lb per hr could be carried. Washed coal gave excellent results and 50 per cent washed coal was very satisfactory; however, it was not always available.

The proposed use of gas fuel was not expected to result in increased capacities and noncoking conditions at all times, as gas was to be purchased on a dump basis and the station was subject to having the gas supply cut off at an hour's notice. Also, there was the possibility in extremely cold weather that gas would not be available for a period of 2 months and at a time when it would be very desirable. However, its use was expected to result in increased dependable capacities, when it was available, by reducing the rate of coal burning on the stokers.

First Gas-Burning Equipment Installed

The first installation of burner equipment consisted of four sets of three burners each, arranged in line across the rear boiler wall, at a height of approximately 11 ft above the stoker and just above the throat of the combustion chamber, Fig. 2. The

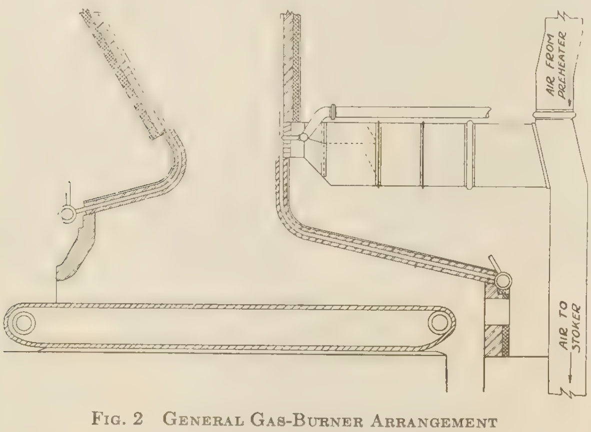

Fig. 2 General Gas-Burner Arrangement

burners were of the fishtail type, made of 3-in. 18-8 stainless-steel tubing, set between the waterwall tubes and directed upward at an angle of $22^1/_2$ deg to the horizontal. There was no provision for premixing of gas and air for combustion ahead of the burners, and the air supply was through sets of stainless-steel louvers surrounding each group of burners. There was a divided-duct arrangement for each group of burners, consisting of one large and one small area with manual damper control for air regulation, Fig. 3.

In operation, the direction of the burners and louvers resulted

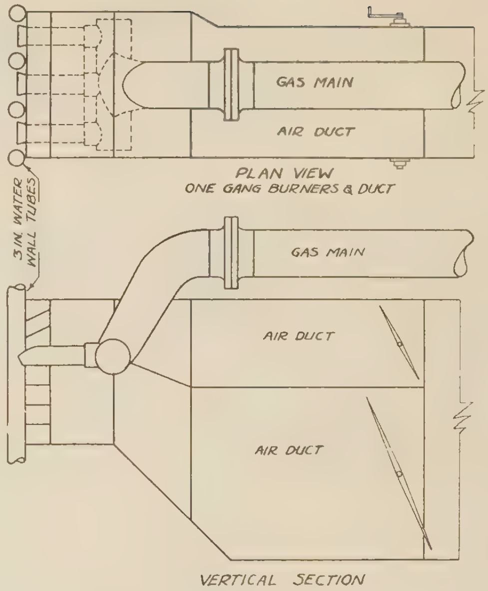

Fig. 3 Burner and Air-Duct Layout

in too high a temperature in the superheater zone, and after numerous trial settings, the burner tips were finally rotated 180 deg to a downward direction, the air louvers being set to direct the air above the burners downward and that below the burners horizontally. This resulted in satisfactory furnace conditions.

Some difficulty was also experienced in gas regulation at low gas flows, which was attributed to large gas-burner area. Consequently in the installation in the second boiler the area was reduced by replacing the twelve 3-in. burners with sixty 1-in. size, giving a reduction in burner area of 45 per cent. This was accomplished by replacing each large burner with a vertical header containing five of the smaller burners. In the original installation, the center burner of each group was blanked off with a corresponding reduction in burner area of $33^1/_3$ per cent. Since both methods improved gas regulation with no distinct advantage of one over the other, the latter method was used in the third boiler. In any of the three boilers, operation may be with one to four sets of burners, but normal operation is with two sets at low gas flows and four sets at high gas flows.

Final Installation Made in 1941

The extension of gas-burning equipment to two additional boilers was made in 1941. These installations were somewhat different from the one originally made, because of construction difficulties and available space. In one boiler, 24 burners were arranged in two groups of 12. There were four horizontal rows of three burners each per group. The two upper rows and the two lower rows were controlled separately, thus allowing the use of either 12 or 24 burners, depending upon the requirements. In the other boiler, the arrangement was with 20 burners in two groups of 10, with two rows of 5 each per group. Upper and lower rows were likewise controlled separately. All burners in both boilers were of the fishtail type made of $1^1/_2$-in. pipe with 18-8 stainless-steel tips welded on the end and directed downward 30 deg to the horizontal. Air supply for combustion was essentially the same as in the first three installations except for changes in duct arrangement required by space limitation.

In all boilers the gas flow is controlled by pressure-balanced regulating valves. Burner pressures range up to 25 in. of water on the first three installations and up to 60 in. on the latter two

which have less burner area than the former. Gas-main pressures are normally from 10 to 15 psig. Gas measurement for billing purposes is made by Wescott concentric-type orifice meters and corrected to a base of 14.65 psia, at 60 F and 0.675 specific gravity. Heat content is approximately 1040 Btu per cu ft as determined with a Junkers-type calorimeter.

Available gas ranges from zero to 30,000,000 cu ft per day total for the Grand Avenue and Northeast Stations and is distributed between the two plants on an economical and practical basis. The maximum flow is equivalent to approximately 1500 tons of coal and represents about 75 per cent of the fuel requirements of the two stations. The maximum gas consumption on any one stoker-fired boiler has been 256,000 cu ft per hr, and the maximum boiler rating obtained, 255,000 lb per hr. This represents 75 per cent of the total fuel requirement being supplied by gas and 25 per cent by coal, which is the minimum quantity of coal burned on the stoker when using gas. The average use of gas on stoker-fired boilers has been 25 to 30 per cent of the total fuel. When the amount of gas is limited, it is used preferably on swing boilers with coal as the base fuel, since swinging with gas gives the faster response to load fluctuations.

Method of Measuring Combustion of Three Fuels

Proper combustion of the combined fuels is gaged by three methods, i.e., smoke observer, periodical flue-gas analysis by Orsat, and curves to determine the desired steam flow–air flow relation with Bailey boiler meters, for variable amounts of gas. Fuel-air ratios have been calculated on the basis that gas can be burned with approximately 15 per cent excess air, and coal on stoker units with approximately 40 per cent. All combustion calculations and operating guides have been based on a fuel-bed thickness of 6 in. and a stoker speed of 7.6 fph. This is the minimum speed at which the stokers can operate, and 6 in. of fuel provides sufficient coal on the stokers for protection against overheating.

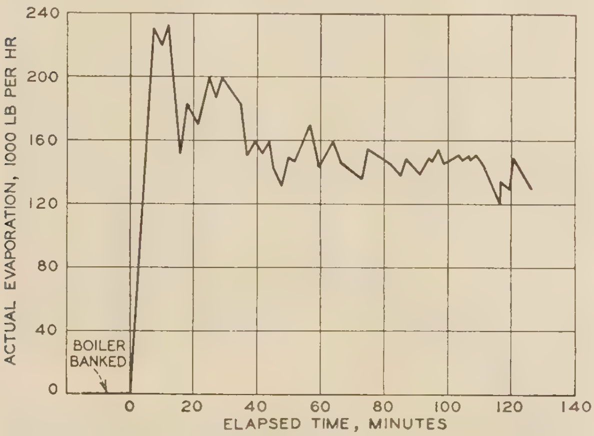

Fig. 4 Rating Pickup From Banked Boiler With Gas

The combined firing of coal and gas is satisfactory in all respects. From a load-handling standpoint it is very flexible and convenient. When gas is available, dependable boiler capacities are increased, and maximum capacities with existing draft equipment are increased 25,000 to 35,000 lb per hr per boiler. The use of gas helps ignition of poor-quality coal and relieves coking troubles by reducing the rate of coal burning and by reducing air temperature to the stoker. The lower air temperature is a result of cleaner steam- and water-heating surfaces. The use of gas has eliminated the necessity of using the air bypasses on the air preheaters. Another favorable item is the reduced time for bringing a boiler on the line in an emergency. Fig. 4 shows the rating pickup with gas, from a banked boiler to 225,000 lb per hr in approximately 7 min. This load on the boiler existed for only a few minutes but could have been maintained much longer if it had been necessary.

Results Attained With Gas Fuel

The burning of 1,350,000,000 cu ft of gas fuel on stoker-fired units in 1941, or 25 per cent of the total fuel requirements at Grand Avenue Station, resulted in the following:

1 No change in average monthly efficiency over burning coal alone.
2 No change in banking losses.
3 Comparable base-fuel costs.
4 Reduced fuel-handling labor costs.
5 Reduced maintenance on fuel-handling and burning equipment.
6 Reduced power consumption on fuel-handling and burning equipment.
7 Increased steam temperature.

There was no reduction in average monthly boiler-operating efficiency, as would normally be expected when burning gas, due to increased hydrogen and moisture losses. However, calculated test efficiency with boilers thoroughly cleaned before tests do show a decrease in boiler efficiency with the use of gas (Fig. 5). The fact that no reduction in operating efficiency was experienced

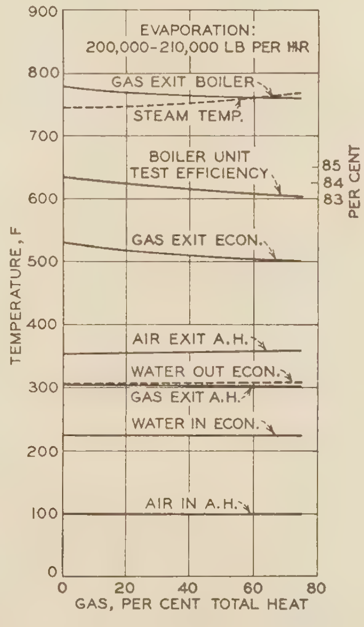

Fig. 5 Test Temperatures and Efficiency When Burning Gas on Clean Boiler

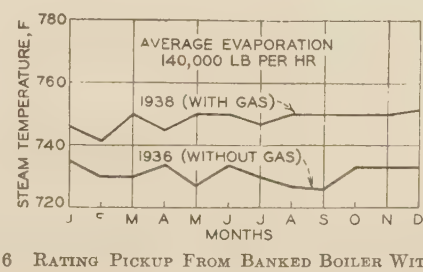

Fig. 6 Rating Pickup From Banked Boiler With Gas

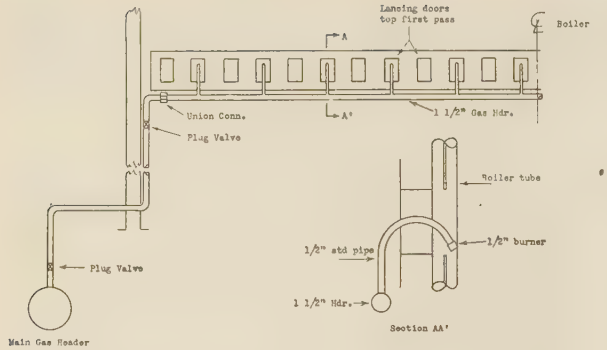

Fig. 7 Experimental Gas-Burner Layout at Superheater

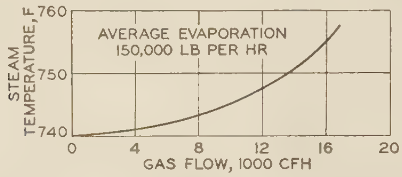

Fig. 8 Effect of Gas Burning at Superheater on Steam Temperature

over periods of time is attributed to the fact that the use of gas resulted in cleaner boilers and better heat transfer as well as lowered ash-pit losses, which offset increased hydrogen losses. At Grand Avenue Station, the practice of holding boilers on a bank with gas fuel is not followed as it is desirable to maintain a bed of coal on the stokers, even during banking periods. However, results obtained at Northeast Station, where gas is used for banking on strictly gas-fired boilers, indicate that a considerable saving can be made in this manner.

One unexpected result of the gas installation was increased steam temperature. In spite of the reduced amount of flue gas obtained by burning gas, the steam temperature increased approximately 15 F at a normal rating of 150,000 lb per hr. Fig. 6 shows the average monthly steam temperature for the years 1936 and 1938, the year previous to and the year following the original gas installations. This increased steam temperature is attributed to the location of the gas burners, which were high enough in the furnace to increase the gas temperature in the superheater zone.

Following the results obtained on steam temperature, it was decided to try the use of small gas burners near the top of the combustion chamber, for steam-temperature control at times when minimum amounts of gas were available. Fig. 7 shows the installation of twelve $^1/_2$-in. burners distributed uniformly across the front of a boiler and projecting through small lancing doors. Air was admitted through the lancing doors for purposes of combustion but there was no control of the amount of air which was undesirable. However, a 15 F rise was obtained in steam temperature at a rating of 150,000 lb per hr when burning 16,000 cu ft of gas per hr (Fig. 8) at the superheater. No gas was used through the main burners during this check. The use of a controlled preheated-air supply should offer even greater possibilities for steam-temperature control. This has not been tried as yet but has been contemplated.

The results attributed to the use of gas fuel discussed in this paper apply only to Grand Avenue Station, and it must be borne in mind that results to be obtained elsewhere, under different conditions, may vary considerably.

While the use of gas fired simultaneously with coal has proved very satisfactory both from an economic and operating standpoint, there is the possibility now that, in the near future, gas will not be available for such usage in this area. However, its use in Grand Avenue Station after the war is contemplated in greater amounts.

Discussion

E. R. Fish.[2] This paper and one by W. C. Schroeder[3] are both pertinent to the present national conditions, but there are certain features in connection with the use of such fuels that merit comment. However, they are not ordinarily given the consideration they deserve.

Where gas and oil have been used as boiler fuels, there have been numerous cases of more or less disastrous furnace explosions, more so in connection with gas than with oil. It has happened all too frequently that for some reason or other explosive gas mixtures have formed in boiler furnaces and have been ignited by some means or other with a resulting explosion. There are many cases where large property damage has been caused, and in all too many cases loss of life and personal injuries have accompanied the explosion.

Oil leaking into a boiler furnace and remaining for a period of time gradually evaporates. If the furnace happens to be hot, the rate of evaporation is greatly increased over the rate that occurs in a cold furnace. It is comparatively easy for an explosive mixture to be formed.

There have been many instances where fuel gas has leaked into a furnace and an explosive mixture formed and from some source or other has become ignited.

With both fuels, it sometimes happens that the flow is interrupted whereby the flame is extinguished, and, when the flow has resumed, ignition was not effected quickly enough to forestall the formation of an explosive mixture which is likely to be ignited later.

In most plants using these fuels more or less elaborate instructions have been given covering the precautions which must be taken in lighting off and describing what should be done in the case of the accidental extinguishing of the fire.

It is not at all unlikely that when combinations of fuel such as are treated in these papers are attempted, the operating personnel will be "solid-fuel-minded" and relatively ignorant of the peculiarities of gaseous or liquid fuels. It is therefore very necessary that particular attention be paid to the additional training and instructions that such personnel should receive if the described combination fuel firing is installed

It seems hardly necessary to do more in this discussion than to draw attention to the possibilities of accidents of the kind mentioned. Operating instructions are readily obtainable through various sources.

Joseph Gershberg.[4] Increase in steam temperature resulting from the combination of coal and gas firing described in the paper presents an interesting contrast with the writer's experience in the use of overfire oil in the stoker-fired boilers at one of the power stations of the company with which he is associated.

To prevent smoke during heavy loads on these boilers, oil was injected through three burners located in the narrow central portion of the rear furnace wall, about 15 ft above the extension grates of the underfeed stoker. Whenever this was done, the

[2] Chief Engineer, Boiler Division, The Hartford Steam Boiler Inspection and Insurance Company, Hartford, Conn. Fellow, A.S.M.E.

[3] "Use of Mixtures of Oil and Coal in Boiler Furnaces," by W. C. Schroeder, *Mechanical Engineering*, November, 1942, pp. 793–798, 804.

[4] Division Engineer, Brooklyn Edison Company, Inc., Brooklyn, N. Y. Mem. A.S.M.E.

steam temperature would drop rather excessively below the normal value obtained for the same load without the use of overfire oil. This behavior is believed to be due to the more complete combustion of the coal "volatiles" near the fuel bed, produced by their mixing with the lean gases drawn toward them from above the ashpit by the high-velocity jets of the burning oil. Secondary combustion at the inlet to the superheater perhaps is greatly reduced if not entirely eliminated. Under such conditions, for the same total heat released by the fuel, more heat will be absorbed by the water envelope of the furnace, with the result that steam will be produced at a greater rate but with a reduced steam temperature.

Viewed similarly, the case of the paper seems to lose its contradictory aspect and rather supports the writer's experience. According to the author, the original upward direction of the burner tips and air louvers caused a too-high steam temperature, which was reduced by the subsequent 180-deg rotation of the tips and louvers. In the first instance, the combustion of the high-volatile matter from the coal and of the natural gas quite likely was not completed in the furnace. As a result, less steam was evaporated and its temperature was raised high by the superheater swept over by gases much hotter than normally. In the second instance, the combustion of gases in the furnace was improved and the duration of these gases inside the furnace was prolonged. Therefore, more heat was absorbed by the water envelope of the furnace, resulting in more steam evaporated with a lowered steam temperature. In comparing again this instance with that when no gas is burned, we observe that the difference in steam temperatures is brought about by the fact that the burning of the gas at the throat of the combustion chamber releases hotter gases at a higher level of the furnace. This condition, coupled with the possible upward push given by the gas and air jets at the throat, shortens the duration of hotter gases inside the furnace with the result that less steam of higher temperature is produced than when coal alone is burned.

Further corroboration of this view is given in the paper by the marked raising of the steam temperature with twelve small gas burners projecting through lancing doors at the top of the combustion chamber. Here 15 F increase in steam temperature is obtained while burning 16,000 cu ft of gas per hr which represents approximately only 8 per cent of the total heat, as compared with about 70 per cent required, according to Fig. 5 of the paper, for the same steam-temperature rise.

To the author's explanation of no reduction in operating efficiency, it may be well to add the beneficial effect of reduction in cinder loss when gas is used. At 200,000 lb per hr evaporation, coal when used alone is burned at about 50 lb per sq ft of grate surface. This is rather a high rate of burning for the type of coal used and is likely to produce a heavy cinder emission.

Author's Closure

The author wishes to thank those who participated both in the written and oral discussion of this paper.

Mr. Fish's remarks on the hazards incident to the burning of liquid and gaseous fuels is very timely. Too many precautions cannot be taken to eliminate conditions which might cause serious disasters when burning these fuels.

In the installations covered in this paper, each gas header contains two sets of plug cocks between which are located the regulating valve and atmospheric vent. The vent line is piped to the outside of the building. Shutoff valves and metering equipment are also located outside the building. When a boiler using gas is taken out of service, both the plug cocks and the regulating valve are closed, and the vent line is opened to the atmosphere. Before workmen enter the setting, it is purged for 10 min with the induced-draft fan. Regulations prohibit smoking within the boiler setting. Just previous to lighting the fire in a cold boiler, it is again purged for 10 min. Since gas fuel is admitted to the furnace only when a coal fire exists in the furnace, torches are not used for lighting the gas burners. As long as the boiler is hot, gas pressure is maintained at the regulating-valve inlet.

Mr. Gershberg's discussion of the phenomena involved in the combustion of gas-fired overstokers, indicates that relative absorption of heat by superheater and boiler surfaces, and consequent steam-temperature rise, is a function of the level in the furnace at which combustion is completed. The author agrees with this and believes the type of burner used is partly responsible for the delayed combustion of the gas. The use of high-velocity turbulent burners would probably have caused more complete combustion at a lower level and less increase in steam temperature.

In regard to the rate of coal burning, the furnaces and stokers were designed to burn approximately 60 lb per sq ft of grate area per hr. Cinder emission is not unusually heavy at the higher rates of coal burning, with Missouri coals. However, it is considerable with some Arkansas and Oklahoma coals.

As noted in this paper, the probable discontinuance of the use of gas fuel in this installation has now become a reality as a result of the fuel-conservation program in this country.

Experience in the Use of Electrostatic Fly-Ash Precipitators

By IRVIN G. McCHESNEY,[1] ROCHESTER, N. Y.

This paper discusses several years' experience with Cottrell electrostatic fly-ash precipitators. Early rod-curtain electrodes were modified following model studies carried out by the Research Corporation, Bound Brook, N. J. The improved design uses punched plates for collecting electrodes. The solutions of other operating problems, including distribution of gas, flue corrosion, and the design of by-pass ducts, are considered.

THE rod-curtain-type precipitator, developed by the Research Corporation, was an improvement over the older concrete-type precipitator, reducing cost, weight, and size. After initial success with the rod-curtain type in the cement industry, a full-scale experimental installation was made at Station No. 3 of the Rochester Gas & Electric Corporation. The initial installation, put in operation in December, 1935, was rated at 106,000 cfm of flue gas at a temperature of 325 F. Since 1935, five installations have been made at Station No. 3 with a total capacity of 650,000 cfm.

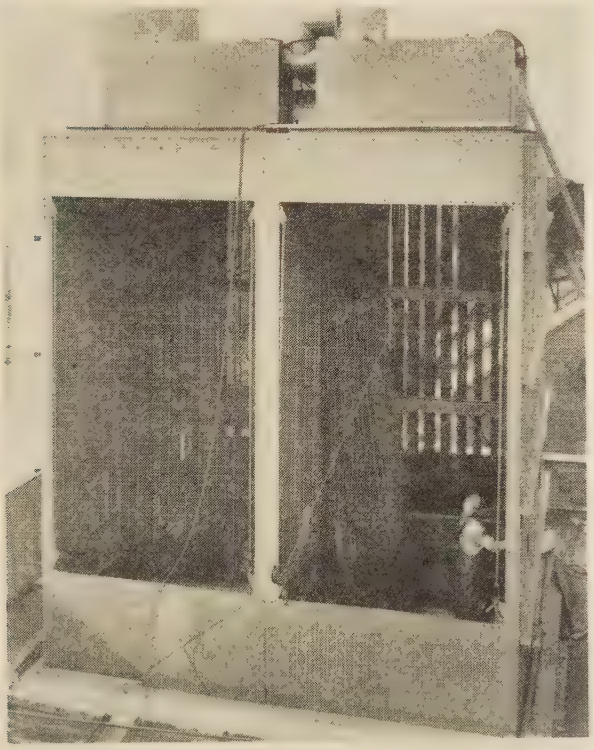

Fig. 1 Internal Construction of a Rod-Curtain-Type Cottrell Precipitator

The internal construction of the precipitator is shown in Fig. 1 which is from a photograph taken during erection. Eight rod-curtain frames are suspended across each half of the precipitator. There are three banks of frames in the direction of gas flow, one in the front third, one in the middle, and one in the rear third. The ionizing wires are suspended between the tube frames. An external view of a completed installation is shown in Fig. 2.

Early Performance

The initial performance of the rod-curtain precipitator was below guarantee. Poor performance was credited to arcing at high load and "blowing-through" of fly ash.

Some of the arcing was due to swaying of the high-tension frames at high load. More rigid supports improved the condition. Other attempts were made to reduce arcing by varying the voltage applied to various groups of ionizing wires. Half- and full-wave energizing were used. Some improvement was made by the installation of surge suppressors; also careful setting of all electrode clearances was found to be essential.

The greater amount of blowing-through occurred in the lower part of the precipitator where heavy ash concentrations were found. Early attempts were made to reduce blowing-through by improving the gas distribution to the precipitator. Slight difficulty was experienced in equalizing the gas flow in the horizontal breeching at the left of Fig. 2. There was considerable difficulty, however, in getting reasonably good gas distribution in the expanding section between the horizontal run of breeching and the inlet face of the precipitator. Three groups of vanes were installed at the inlet of the expanding section. These vanes distributed the gases horizontally and vertically. Air-velocity measurements were made across the face of the precipitator and vane settings were adjusted until fairly even distribution was ef-

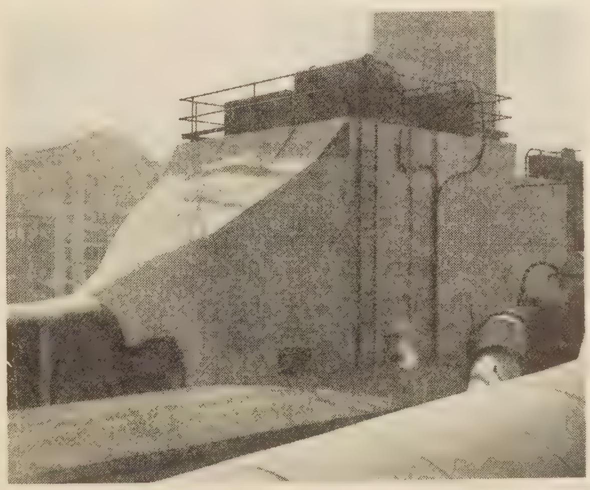

Fig. 2 Cottrell Precipitator Installation at Station No. 3, Showing Flue Connections With Weatherproof Insulation

[1] Test Engineer, Rochester Gas & Electric Corporation. Jun. A.S.M.E.

Contributed by the Fuels Division and presented at the Fall Meeting, Rochester, N. Y., Oct. 12–14, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

fected. The results due to improvement in gas distribution were inconclusive.

Model Precipitator Tests

Model experiments were carried out in the spring of 1938 by C. W. Hedberg and his associates of the Research Corporation. A $^1/_{20}$ scale model was installed in a small duct by-passing the full-scale precipitator. This model had a rated capacity of 11,500 cfm. Fly-ash concentrations were normally in the order of 3 to 4 grains per cu ft of gas. During periods when soot blowers were operated the ash concentrations increased to 12 grains per cu ft.

Many experiments were conducted varying the design of the electrodes, improving baffling to prevent gas flow through the ash hoppers, and varying the voltage applied to various banks of ionizing wires.

In early experiments the middle and rear banks of rod curtains were covered with 5-mesh wire screen. Later, corrugated plates were used to cover the front bank of rod curtains. In the final design, all the rod curtains were covered on both sides with perforated plates. The greatest single gain in performance was made as a result of improvement in baffling the bottom of the precipitator to prevent gas flow through the hopper and subsequent re-entrainment of ash.

Model efficiencies increased from 77 per cent at 10,500 cfm in the original design to above 90 per cent at 12,000 cfm after improvements in electrode design and baffling had been made.

Arcing Studies

Experiments were made to determine the effect of arcing. Half-wave energizing greatly reduced arcing but there was little if any gain in efficiency attributable to the elimination of a normal amount of arcing. Other experiments to control arcing and improve collection included reducing the energizing voltage in the front bank of electrodes and stepping up the voltage progressively in the middle and rear banks.

At the conclusion of model experiments, three tests were made at 13,450 cfm and 325 F with an average efficiency of 93 per cent. These improvements indicated in model experiments were carried over to the full-scale installation with gratifying results. After changes were made in May, 1938, the performance under normal operating conditions averaged somewhat above 90 per cent efficiency.

Corrosion of Breechings

Precipitators were installed on the roof of the plant with uninsulated breechings. The breechings were fairly well protected from wind by ventilating monitors. The short section of breeching connecting the precipitator to the induced-draft fan corroded through in 3 years of service. The breeching was $^3/_{16}$ in. thick and more than 20 holes were found in it. All exposed breeching on Nos. 1 and 2 boilers was carefully inspected. In certain sections, including the stack connection, corrosion was serious.

Tests were conducted to determine the most satisfactory method of preventing further corrosion of breechings. Some of the factors causing corrosion are listed in order of importance, as follows:

1 Temperature of gases.
2 Weather—snow, rain, wind, and low temperature.
3 Amount of exposed flue area.
4 Presence of charged particles in the gas stream.

Gas temperatures leaving the economizer ranged from 225 F at one-third load to 325 F at full load. A temperature drop from the economizer to the precipitator of 40 to 50 deg occurred, varying with the boiler load and weather conditions. An area of 3000 sq ft of breeching was exposed in this connection. From these observations minimum gas temperatures were well in excess of the dew point. Breeching-metal temperatures and gas temperatures at the metal surface were largely dependent upon wind and weather, the most adverse conditions being snow and rain. Other conditions of low load, low outdoor temperature, and high winds were accountable for several periods when the breeching-metal temperature dropped below the dew point of the flue gases.

Special apparatus was developed for dew-point measurement. Testing for dew-point temperatures was carried out over a period of several weeks with widely varying load conditions. Dew-point observations ranged from 110 F to a maximum of 115 F. The coal burned during the period of testing was Pittsburgh bituminous with a sulphur content of from $1^1/_2$ to 2 per cent.

The presence of charged particles in the gas stream leaving the precipitator had an effect on the rate of corrosion. Grounding of particles on the precipitator-outlet breeching were observed to cause current flow through the breeching metal. Corrosion of breeching metal was accelerated in the immediate vicinity of the precipitator outlet.

Flue corrosion was entirely stopped by covering the breeching with No. 16 gage galvanized iron. The sheeting was fastened across the expansion joints of the breeching to form a smooth covering. The application is well illustrated in Fig. 2. During an 11-day continuous test, following the application of galvanized sheeting, the breeching-metal temperature reached a minimum of 170 F. Several times, outdoor temperatures of 0 to 10 F were observed during light-load periods. During the test interval, unprotected areas dropped to metal temperatures as low as 80 F. The inner face of unprotected metal was wet with corrosion progressing rapidly.

All exposed breechings were then protected from weather by galvanized sheeting and the corrosion of breechings stopped.

Ash Deposits in Breeching

Early trouble was experienced with fly ash depositing on the floor of the breeching leading to the precipitator. Turning vanes were installed in the elbow connecting the vertical run from the economizer with the horizontal breeching. Gas velocities were equalized throughout the breeching cross section. This increased velocities along the floor of the breeching and reduced ash de-

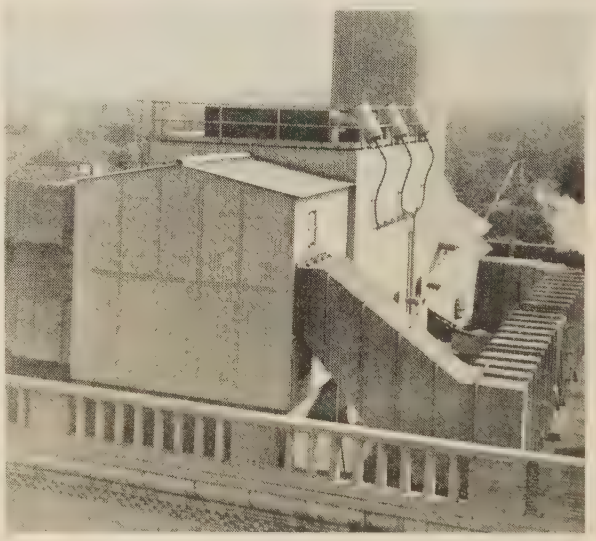

Fig. 3 Cottrell Precipitator Installation at Station No. 8, Showing Gas-Mixing Chamber and Breeching Connections

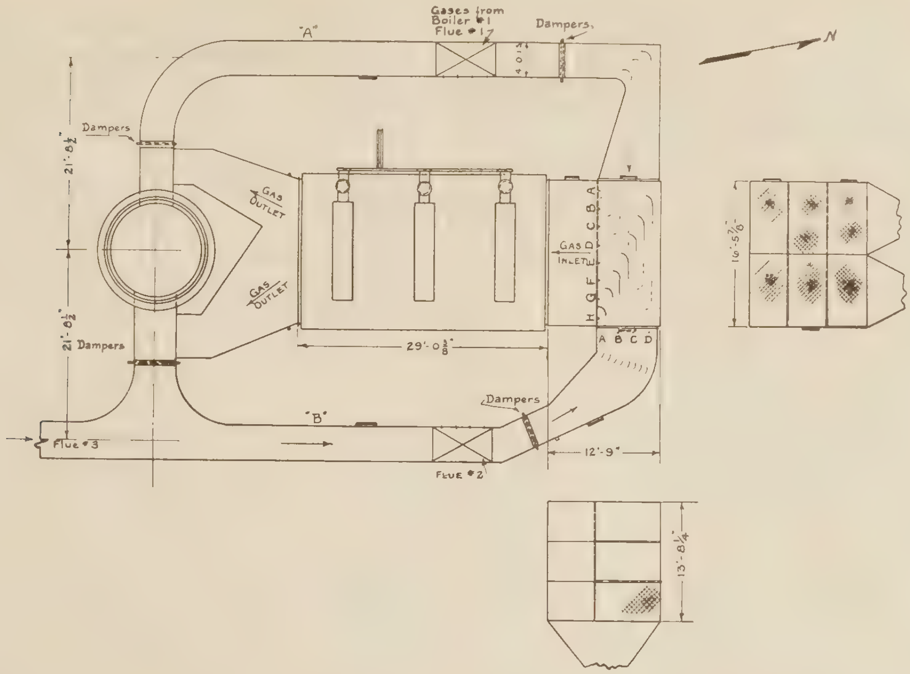

Fig. 5 Plan of Precipitator Installation and Elevations of Gas Mixer at Station No. 8

posits. A hopper bottom was also installed in the expanding section immediately in front of the precipitator. Any adverse effect of ash deposits on the distribution of gas across the face of the precipitator was reduced by the installation of the hopper bottom. Ash deposits have been less troublesome since these changes were made.

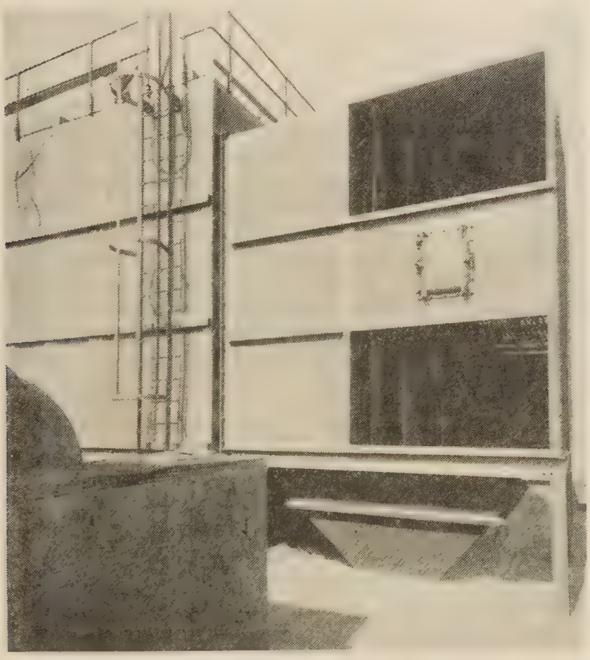

Fig. 4 Details of Gas-Mixing Chamber, Showing Arrangement of Turning Vanes and Punched Plates

Precipitator Installation at Station No. 8

A single precipitator was installed on the roof of the central heating plant at Lawn Street in the fall of 1940. It serves three boilers and has a capacity of 135,000 cfm at 400 F. The installation is shown in Fig. 3. Three ducts are shown entering the gas-mixing chamber in the foreground. The gas then passes through the precipitator to the stack.

Gas enters the mixing chamber at three levels. Ducts from Nos. 2 and 3 boilers enter the upper and lower third from the left. A duct from No. 3 boiler enters the middle third from the right. Punched plates are installed between the upper, middle, and lower thirds of the mixing chamber. Each third is equipped with turning vanes to direct the gases through a punched plate to the precipitator inlet. The arrangement of punched plates and turning vanes in the gas-mixing chamber is shown in Fig. 4. A plan of the breechings, mixing chamber, and precipitator is shown in Fig. 5.

Draft Limitations

Station No. 8 was first put in service in October, 1925, and the third boiler was in service in December, 1927. The precipitator was installed many years after the last boiler. For this reason, it was necessary to locate the precipitator on the roof of the plant between the outlet of the induced-draft fans and the stack. The head loss available for carrying gases through the breeching and precipitator could not exceed the stack draft without developing positive pressures at the fan outlets.

When the precipitator was first put in service, the gas passed through four successive punched plates. These plates were installed to give even distribution of gas to the precipitator. In addition, the gas entering the upper and lower third of the mixing chamber was guided in partitioned ducts. These restrictions caused a positive pressure of 0.40 in. of water in the fan outlet at the roof at 70 per cent of rated gas flow. The capacity of the plant was considerably reduced by placing an additional head on the induced-draft fans.

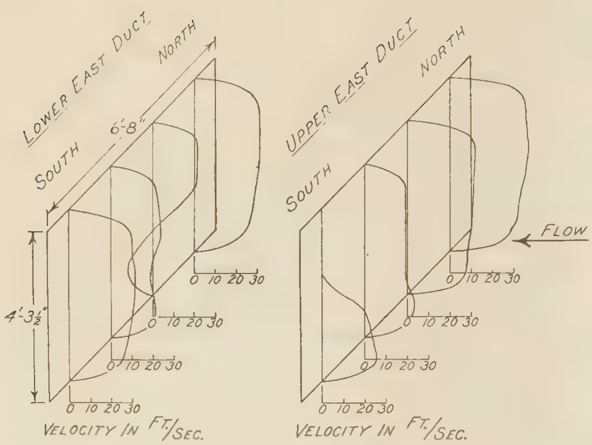

Fig. 6 Initial Velocity Traverses of Upper and Lower Inlets to Gas-Mixing Chamber

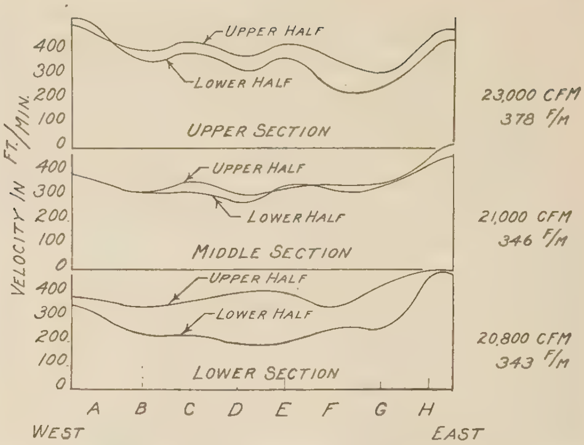

Fig. 8 Initial Velocity Traverses of Gas-Mixing-Chamber Discharge

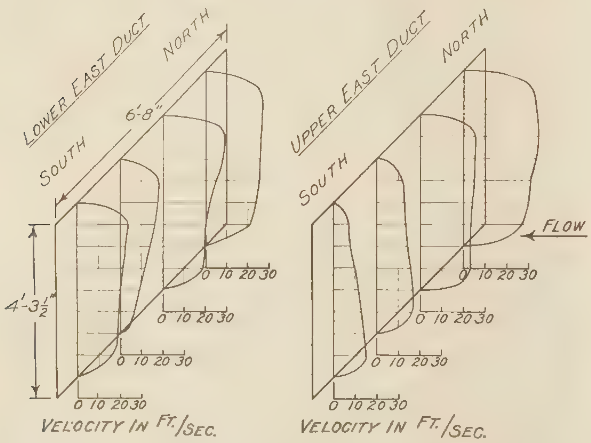

Fig. 7 Final Velocity Traverses of Upper and Lower Inlets to Gas-Mixing Chamber

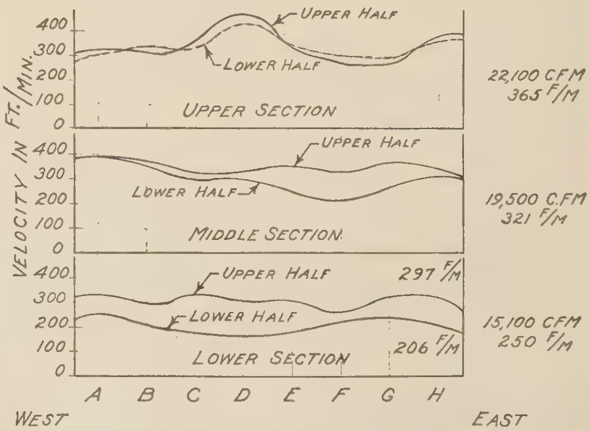

Fig. 9 Final Velocity Traverses of Gas-Mixing-Chamber Discharge

Before the heating season started, all the punched plates were removed from the breeching except one. This was located at the outlet of the gas-mixing chamber approximately 6 ft from the precipitator inlet. Partitions were removed from the breeching, and narrow turning vanes substituted. The removal of the punched plates caused poor gas distribution to the precipitator with consequent loss in operating efficiency. Measurements taken in February, 1941, indicated poor gas distribution with high velocities occurring near the bottom of the precipitator inlet.

Correction of Gas Distribution

After the heating season extensive tests were carried out by the Research Corporation to improve operating efficiency. Following these tests hopper baffles were installed to prevent reentrainment of ash. Velocities and dust concentrations were high in the lower portion of the precipitator. Turning vanes in the breeching and gas-mixing chamber were adjusted to get a fairly even distribution of gas across the inlet of the precipitator with appreciably lower velocities in the lower third.

Distribution was studied by drawing air through the ducts and measuring velocity distribution. The distribution found in the upper and lower ducts is shown in Fig. 6. After adjusting turning vanes and removing minor obstructions the air distribution was improved. Distribution after a final setting is shown in Fig. 7. Velocities were tapered down from the north wall to the south wall to reduce velocities along the east side of the precipitator inlet.

Air distribution was improved by adjustments made in the upper and lower inlet ducts. After these adjustments the distribution at the mixing-chamber outlet is shown in Fig. 8. High velocities still existed along the east side of the precipitator. After several adjustments the final vane settings in the mixing chamber gave satisfactory flow distribution. Distribution following the final vane settings is shown in Fig. 9. Velocities were reduced in the lower third of the precipitator inlet to reduce the amount of ash blown through the precipitator in the region of high concentration.

The precipitator efficiency was greatly improved by the installation of hopper baffles and proper gas distribution. Efficiency tests have not been conducted since these changes were made but the stack is normally very clear, indicating high efficiency.

Positive pressures in inlet breechings have been eliminated up to gas flows in excess of the rated capacity of the precipitator (135,000 cfm). This gas flow corresponds to a steam generation of 220,000 lb per hr.

Removal of Ash From Breechings

Ash accumulation in the inlet breechings seriously affected gas distribution. It was found necessary to provide a series of fixed

air nozzles in the upper and lower inlet breechings. These air nozzles were blown once a day. An air lance has since been substituted for the fixed nozzles. This method is equally effective and reduces the amount of compressed air required. The lance is used through doors in the side of the breeching. Objectionable ash accumulations have been prevented since regular lancing began.

During the heating season of 1941, ash accumulated rapidly in the breeching between the precipitator outlet and the stack. This accumulation was also sufficient to impair gas distribution through the precipitator. The rate of ash accumulation in this breeching has been greatly reduced since the installation of hopper baffles and the readjustment of gas-distributing vanes. This section of breeching requires cleaning after each heating season.

Corrosion of By-Pass Breechings

No. 1 boiler outlet is directly connected through a by-pass breeching to the stack. This breeching, shown as section *A* in Fig. 5, is seldom used. Corrosion was severe in this section during the first heating season. The inner face of the breeching was wet. At several points the metal was entirely gone and very thin over large areas. Near the middle of the section ash had completely filled the breeching. The by-pass was cleaned and patched and the dampers shown near the stack were moved back near the outlet from No. 1 boiler. A small amount of air was allowed to leak into the by-pass. These changes stopped flue corrosion. Ash accumulations still occur but their removal is not a serious operating problem.

Section *B*, by-passing No. 2 boiler to the stack is heated by gases from No. 3 boiler. Corrosion and ash accumulations are less serious in this section. When No. 3 boiler is out of service a small amount of air is allowed to leak into this idle breeching. This procedure has arrested corrosion.

Conclusions

Substitution of punched-plate electrodes for the original rod-curtain design increases precipitator efficiency.

Hopper baffles are essential for the prevention of re-entrainment of ash and subsequent loss of efficiency.

Gas flow to the precipitator should be distributed evenly across the inlet face with some reduction of velocity in the lower levels.

Breechings should be insulated on the outside sufficiently to protect them from weather and maintain metal temperatures above the dew point during operation.

Air should be admitted to idle breechings in sufficient amounts to suppress the dew point and prevent corrosion.

Ash should be removed at frequent intervals to prevent poor gas distribution to the precipitator.

Gas distribution should be a consideration in breeching design. Rapidly expanding sections leading to a precipitator are undesirable. Good distribution in right-angle bends is easily effected by the use of turning vanes.

Discussion

C. W. Hedberg.[2] This timely and well-conceived paper presents a subject which brings out some very interesting factors concerning the operating experiences which were encountered during the early stages of operation of Cottrell electrical precipitators on the particular boilers mentioned in the paper.

The rod-curtain type of precipitator was first installed at Station No. 3 of the Rochester Gas & Electric Corporation in an effort to design a precipitator not necessarily to replace the concrete-plate type, but one that could be used where installation of the concrete plates would be impossible because of space limitations and inadequate structural steel necessary to support the weight. Many concrete-plate types are being installed today. The performance and electrical operation of each type are approximately the same on fly ash.

The shortcomings of rod-curtain collecting electrodes on certain types of fly ash were taken into account by installing a model test unit at Station No. 3. It is gratifying to know that the design factors obtained from this model unit and carried over into numerous commercial precipitators have corroborated these results.

The author has given a general résumé of the development work on fly-ash precipitators which was carried on at Station No. 3. It is believed that this work has resulted in improvements which have been beneficial to other users of this type of equipment, and a large portion of the credit belongs to the Rochester Gas & Electric Company. That company not only pioneered in equipment change, but, as indicated by the author, also co-operated wholeheartedly with the Research Corporation in giving direction to the work and in providing the facilities for carrying it on.

Serious corrosion has occurred on relatively few precipitator installations and usually it has been in localized areas rather than generally throughout the installation. However, conditions for its occurrence exist in varying degrees in almost all installations, so that the matter deserves attention both in initial design and also when equipment is inspected during boiler outages. The author has given the important factors which cause corrosion. As regards the fourth in his list, we are inclined to believe that the reason for accelerated corrosion in the outlet breeching does not lie in the observed current flow through the breeching metal. Charged particles leaving the precipitator collect on the grounded breeching surfaces and give up their charge. The deposit thus formed is permeable to gases and is a good heat insulator, resulting in a metal temperature behind the deposit well below the dew point of the gas which diffuses through the deposit.

The method worked out and applied at Rochester for combating corrosion is a good one. A Gunite lining on the interior surfaces is also very satisfactory.

As pointed out by the author, the precipitator arrangement at Station No. 8 was dictated largely by space limitations. It should be added that this arrangement, employing as it does a single large precipitator unit for handling the combined gases in a central cleaning plant, is a more economical arrangement than a unit system comprising a separate precipitator for each boiler.

Normal gas velocities in a precipitator are substantially lower than normal velocities in breechings, and provision must, therefore, be made to reduce the initial velocity and distribute the gases uniformly through the electrode ducts. Distribution means are usually built into the connections between the precipitator and the breechings rather than as an integral part of the precipitator itself.

The Station No. 8 arrangement presented an unusually difficult problem in this respect. It was further complicated by the necessity for mixing the gases from the several sources in the amounts delivered to the precipitator to avoid stratification both as regards gas velocity and composition which might otherwise cause some portions of the precipitator to operate far less efficiently than others.

The author has described the steps taken to obtain desired distribution without introducing excessive draft loss, and the results show, without question, the importance of such careful distribution in the final performance of the system.

[2] Chief of Technical and Development Department, Research Corporation, Bound Brook, N. J.

O. F. Campbell.[3] The writer would like to have information on the following matters:

1 What effect has the mol per cent of water vapor in the flue gases passing through the precipitators upon the effectiveness of the precipitators?

2 What has been the experience to date of the effectiveness of electrostatic precipitators by the addition of a small amount of some volatile chemical?

It is the writer's understanding that a high mol per cent of water vapor or a small amount of some volatile chemical has tremendously increased the effectiveness of electrostatic fly-ash precipitators.

[3] Combustion Engineer, Sinclair Refining Company, East Chicago, Ind. Mem. A.S.M.E.

The Elimination of Carry-Over Under Steel-Mill Operating Conditions

By H. M. Rivers[1] and W. P. Hill[2]

At the Sparrows Point steel mill an expanded production schedule brought greatly increased demands on the steam-generating facilities, which eventually led to a severe carry-over condition. The authors review the steps taken to correct the causes of carry-over, both chemically, mechanically, and in firing methods. Studies were conducted on various boiler units of the plant, each of which offered somewhat different problems for solution. However, a balanced solution to the carry-over problem was evolved which applies to boiler operation at this plant as a whole, details of which are given. An explanation is also given of the experimental methods used in arriving at an acceptable solution.

An expanded production schedule brought greatly increased demands on all steam-generating facilities at Sparrows Point. As operating conditions become more acute, there developed unmistakable signs of a severe carry-over condition. Valves, traps, and regulating equipment became inoperative. Scale deposits formed on turbine governors and blading. At times, superheat temperatures took sudden dips, indicating surges of carry-over that might have disastrous consequences in steam-consuming equipment. It was imperative that measures be taken to eliminate this heavy entrainment of boiler water in the steam.

Boiler-Water Control

As a first step, boiler-water conditions were reviewed very carefully to make certain they were nonconducive to carry-over. Oily and saponifiable impurities were assured to be absent. Concentrations of alkalinity, phosphate, and sulphite were maintained no higher than necessary to afford adequate protection against scale and corrosion. Blowdown was regulated to keep total solids always in a range consistent with nonfoaming conditions in hundreds of similar boiler plants. In spite of these efforts, carry-over continued. With this failure to get results by usual chemical control methods, it was apparent that major responsibility for the carry-over lay in some other factor which remained to be found and eliminated.

Effects of Firing Methods on No. 10-D Boiler

Carry-over was exceptionally bad from No. 10-D boiler, a 42,175-lb per hr unit with three longitudinal drums and water-walls which discharge into the outer drums as indicated in Figs. 1 and 2. Table 1 records one instance in which steam contamination increased from about 2 ppm up to 1440 ppm with a load increase from 70,000 to 88,000 lb per hr. The offending factor developed to be a variation in the method of firing. Fuel oil and blast-furnace gas are fired manually, the ratio of fuels being dependent upon the amount of gas available. Whenever the gas supply fails, additional fuel oil is fired. Meanwhile, there may be a momentary drop in steam output, followed by a quick rise as the oil takes hold. Sudden increases in oil input were almost invariably accompanied by surges of carry-over. The change back from oil to predominantly gas firing had no significant effect on steam purity, even though the boiler rating might rise considerably if the gas supply returned unexpectedly.

TABLE 1 EFFECT OF LOAD SURGES ON NO. 10-D BOILER BEFORE INSTALLATION OF CURTAIN BAFFLES

Sample no.	Load, lb per hr	Water in glass, per cent	Total solids in steam, ppm
A-1	40000	80	2.2
A-2	60000	40	1.7
A-3	70000	30	1.5
A-4	73000	60	1.8
A-5	80000	30	20.4
B-1	70000	40	1.6
B-2	80000	40	592.0
B-3	88000	40	1440.0
B-4	75000	40	1300.0
B-5	75000	40	286.0
B-6	70000	40	36.6
B-7	68000	40	4.1
C-1	98000	20	392.0
C-2	65000	30	2.1
C-3	70000	30	1.6
C-4	70000	30	1.3

Note: Steam samples taken about 1 min apart.
Total solids in boiler water were 2110 ppm in "A" tests; 2330 ppm in "B" and "C" tests.

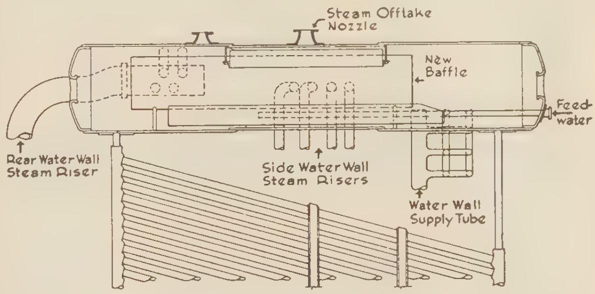

Fig. 1 Longitudinal Section No. 10-C and No. 10-D Boilers, Showing Location of Steam Risers and Steam-Drum Baffling

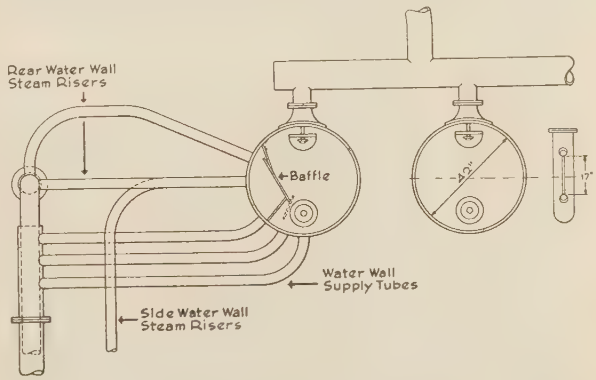

Fig. 2 No. 10-C and 10-D Boilers. Cross Section Showing Location of Steam Risers and Steam-Drum Baffling

[1] Chemical Engineer, Hall Laboratories, Inc., Pittsburgh, Pa.
[2] Steam Engineer, Bethlehem Steel Company, Sparrows Point, Md. Mem. A.S.M.E.

Contributed by the Power Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers.

This relationship between steam purity and firing methods might be explained by differences in the characteristics of gas and oil flames. Blast-furnace gas burns slowly with a long soft flame that often persists beyond the first pass of generating tubes. Oil, on the other hand, gives a short flame which is intensely bright due to particles of incandescent carbon. With oil firing, a greater percentage of the total heat is absorbed by the furnace-wall tubes; with gas firing, less heat is taken up in the furnace walls, more in the convection banks and superheater. Because of these flame characteristics, any change which increases the ratio of oil to gas greatly increases the amount of heat absorbed in the furnace-wall tubes. Since the most severe surges of carry-over accompanied such changes in firing, it was evident that carry-over originated primarily in the waterwalls.

No. 10-D boiler was initially equipped with small perforated baffles opposite the discharge ends of the waterwall steam risers. These were replaced by much larger solid baffles running nearly the length of the drums and reaching from the top of the drum to a point several inches below water level, as indicated in Figs. 1 and 2. Subsequent tests, recorded in Table 2, revealed steam

TABLE 2 EFFECT OF LOAD SURGES ON NO. 10-D BOILER AFTER INSTALLATION OF CURTAIN BAFFLES

Sample no.	Load, lb per hr	Water in glass, per cent	Total solids in steam, ppm
A-1	65000	30	0.4
A-2	60000	30	0.4
A-3	85000	30	0.1
A-4	83000	30	0.4
B-1	25000	40	0.4
B-2	83000	60	0.4
B-3	75000	50	3.9

NOTE: Samples taken about 1 min apart. In test "A" boiler was operating under normal load conditions. In test "B" load was dropped manually to 25,000 lb per hr and raised as quickly as possible by the addition of oil. Due to time lag in cooling coil and conductance cell, the effects of a particular load change appear in the conductance reading made 1 min later.

contamination to be negligible under normal firing and load conditions. A load surge from 25,000 to 83,000 lb per hr increased steam contamination by about 3.5 ppm, as shown by the last figure in the column of total solids, the effect on total solids developing a little later than the load change. Water level, at the time, was higher than normal, and this may have had more effect than the load surge itself.

Critical Ratings and Water Levels of No. 10-C Boiler

On the strength of what curtain baffles did for the No. 10-D boiler, similar baffles were placed in No. 10-C boiler, its twin. Steam purity remained consistently good except at very high ratings and abnormally high water levels, even though load surges as great as 50,000 lb per hr were often encountered. It was found that, for each boiler rating, there was a critical water level which could not be exceeded without causing severe carry-over. The critical condition appeared at lower water levels when ratings became higher, as is clearly indicated in Fig. 3.

No matter how greatly the boiler rating surged, steam purity remained good as long as the limiting peak load was not exceeded. Firing methods had some effect on the limiting-load conditions, critical points appearing at loads about 5000 lb per hr less with oil than with gas. Furnace and draft conditions limit the load on this boiler to a maximum of about 100,000 lb per hr which happens to be the maximum rating which can be carried without impairment of steam purity under any operating conditions. Carry-over is now completely avoided by regulating feedwater and fuel input according to the limits given in Fig. 4.

Presumably, boiler-water concentrations should have some influence on the general carry-over situation. However, in collecting the data from which Fig. 4 was developed, boiler-water solids ranging from 2300 to 3700 ppm were encountered. The

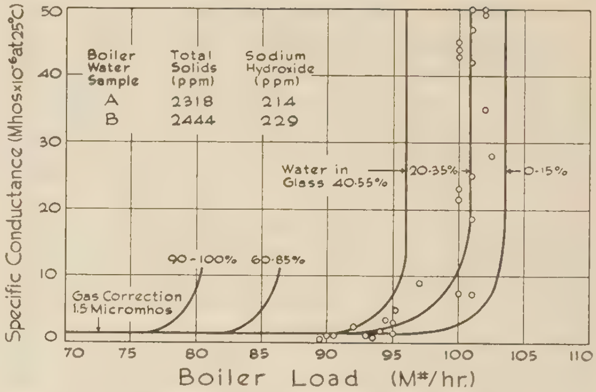

FIG. 3 No. 10-C BOILER. TYPICAL DATA CURVE SHOWING CORRELATION BETWEEN WATER LEVEL, LOAD, AND CONDUCTANCE (Data points are included for only the 20–35 per cent water condition.)

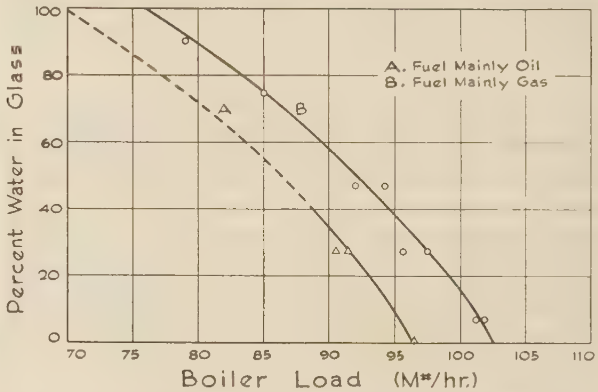

FIG. 4 No. 10-C BOILER. LIMITING LOAD AND WATER-LEVEL CONDITIONS FOR DELIVERY OF STEAM CONTAINING 2 PPM MAXIMUM SOLIDS AT SUPERHEATER INLET

effects, if any, were not noticeable enough to alter any of the conclusions mentioned.

Effects of Load and Firing Methods on No. 9-C Boiler

No. 9-C boiler is a two-drum integral-furnace steam generator, rated at 90,000 lb per hr, equipped with economizer and baffle-type steam purifier, as shown in Fig. 5. Some carry-over occurred with nearly all significant rises in load, most of which were the result of sudden increases in total plant steam demand. Since No. 9-C boiler and its twin No. 9-D boiler are the only ones in the house equipped with automatic combustion control, they must take most of the momentary swings in plant load by themselves; the other boilers, being manually controlled, respond much too slowly to absorb major changes in steam demand.

Whenever the boilerhouse gas supply falls off, the No. 9 boilers respond immediately by increasing their oil consumption enough to compensate for the decrease in gas input to the entire house. Consequently, the No. 9 boilers must often take great surges in fuel input. Because of these operational features, variations in boiler ratings were often severe, with serious carry-over as the usual result.

Particular Significance of Water Level on No. 9-C Boiler

Under ordinary operating conditions, No. 9-C boiler showed little carry-over except when high water levels were unmistakably involved. Periods of high water were induced by load and firing factors, as previously mentioned, or by a peculiar periodicity in

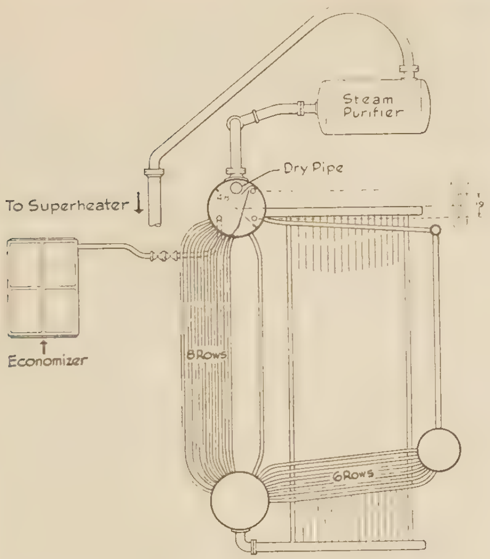

Fig. 5 No. 9-C Boiler. Outline Section Showing Location of Steam-Drum Baffling, Steam Purifier, and Economizer

water-level fluctuations. At times, the water level oscillated in regular cycles with similar swings occurring simultaneously in the steam flow and economizer outlet temperature. The points of maximum water level were usually accompanied by increased steam contamination.

Once the water level commenced fluctuating, it seemed to continue independently of all operating factors. Under typical conditions, the load swings were on the order of 20,000 to 25,000 lb per hr, and often considerably more. Water level fluctuated between plus and minus 6 in. of the normal line, while economizer temperatures varied from 275 to over 415 F, which is the upper limit of the recording instrument. Since maximum temperatures reached in the economizer were higher than the temperature of saturated steam at boiler pressure, opening of the feedwater valve admitted steam and water, rather than water alone, to the boiler. This might cause a momentary rise in boiler output; and due to the delay in admitting feedwater, the water level might continue to drop even after the regulator opened. At about the instant water level reached its lowest point and started upward, steam output and economizer temperature commenced to drop, both eventually reaching minimum values when the feedwater valve closed again due to the rising water level.

This "hunting" of the water level could be arrested somewhat by careful manual control of the water or by giving the boiler a long hard blowdown. There was some evidence, also, that the fluctuations were more severe when boiler-water concentrations were high. It would appear, therefore, that water conditions affected steam purity indirectly by magnifying the fluctuations in water level. However, most of the evidence indicated that economizer temperatures were a far more important factor. The pulsations were noticeably worse with gas fuel than with oil. Since at a given boiler rating the last-pass temperatures are higher with gas firing than with oil, more heat is available to the economizer. Such a condition would logically intensify any water-level fluctuations influenced by economizer outlet temperature. A heavy blowdown might help temporarily by stabilizing economizer temperatures at a lower level. However, the pulsating condition usually returned in a few hours, long before boiler-water concentrations could build up to anywhere near their preblowdown value.

All indications were that the irregular feeding of alternately high- and low-temperature water materially affected boiler output, water level, and consequently steam purity. It was concluded that a quicker acting feedwater regulator might improve conditions materially. A new and much more sensitive regulator was installed, and immediately the fluctuations subsided virtually to unimportance. The pulsating condition returns whenever boiler-water concentrations become exceptionally high, especially if the fuel is predominantly gas. Nevertheless, steam purity re-

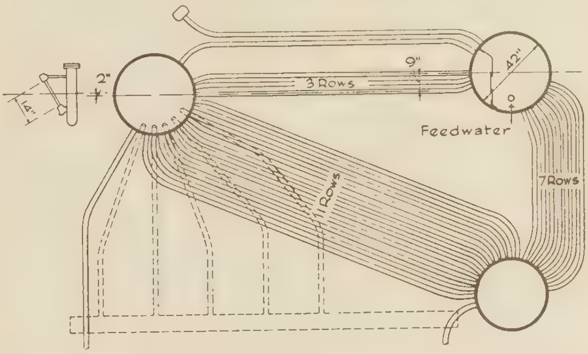

Fig. 6 No. 1 Hot-Strip-Mill Boiler. Outline Section Showing Steam-Drum Baffling and Relative Position of Drums

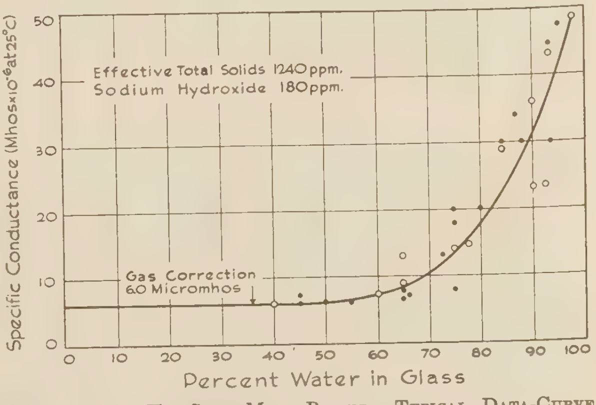

Fig. 7 No. 1 Hot-Strip-Mill Boiler. Typical Data-Curve Sheet, Showing Correlation Between Water Level and Conductance at Boiler Rating of 67,000 Lb per Hr
(Circles denote data obtained under relatively stable test conditions)

TABLE 3 ESTIMATION OF TOTAL SOLIDS CONCENTRATIONS; NO. 1 HOT-STRIP-MILL BOILER

Sample	Total solids, ppm	Chloride, ppm Cl	Sodium hydroxide, ppm NaOH	TS-NaOH (S)	S/Cl	S/K[a]	Corrected chloride	Cl × K	Corrected T.S.	Effective T.S.
A	860	155	152	708	4.6	144	149	760	734	885
B	1271	210	183	1085	5.2	221	215	1030	1058	1240
C	1476	265	256	1220	4.6	248	256	1300	1260	1515
D	1476	265	256	1220	4.6	248	256	1300	1260	1515
E	2062	320	406	1656	5.2	336	328	1570	1613	2020

[a] Average ratio of solids to chlorides (K) is 4.9.
Corrected chloride = (S/K + true Cl) ÷ 2.
Corrected total solids = (Cl × K + true total solids) ÷ 2.
Effective total solids = (corrected total solids + NaOH).

mains consistently good because now it is possible to keep water levels always below the point at which carry-over occurs.

Effects of Load and Water Level in No. 1 Hot-Strip-Mill Boiler

This is a three-drum bent-tube boiler (Fig. 6), rated at 75,000 lb per hr and fired entirely with fuel oil. Whereas the No. 9 and No. 10 boilers receive approximately 2 per cent unsoftened well water for make-up, the hot-strip-mill boilers are fed 100 per cent zeolite-treated well water. All important features of boiler design, boiler operation, and water composition differ greatly from any discussed heretofore.

In estimating the foaming effect of boiler-water constituents, it was assumed that foaming potentialities were proportional to both chloride and total-solids concentrations as determined by usual methods. Although the ratio of solids to chlorides may vary from sample to sample, it will generally stay close to a certain average value. This average solids-chloride value was made use of in calculating the "effective" solids concentrations given in Table 3. Since hydroxide alkalinity has definite foam-aggravating properties of its own, it was considered independently.

Fig. 7 is a typical data-curve sheet showing the correlation between water level and conductance of the condensed steam. A number of such curves, shown in Fig. 8, were obtained for several normal load and water conditions. These factors evidently affect steam purity to an important degree, but only after a certain critical water level has been exceeded. The feedwater regulator was originally set to carry water at about 50 per cent of a glass. Occasionally water level fluctuated into the range at which carry-over takes place and steam purity suffered accordingly. The regulator was reset to maintain water level at about 30 per cent of a glass. Subsequent tests showed no carry-over at all under normal operating conditions.

Relative Importance of Chemical and Mechanical Factors

In all of the foregoing instances, chemical features were undoubtedly secondary to those of a mechanical nature. Good reasons for this were found in the data from No. 9-C boiler, on which especially careful studies were made to correlate boiler-water conditions, water levels, and steam purity at normal boiler ratings.

Fig. 9 is a typical data-curve sheet which shows the relationship between water level and conductance on No. 9-C boiler. The "effective" total-solids and chloride concentrations were estimated in the same manner as were the values in Table 3. In all, fourteen such curves were prepared. Data from these curves were then combined in preparing the family of curves in Fig. 10, which summarizes the relationships between chloride concentrations, water level, and conductance at the steam-purifier inlet. Simultaneous entrainment at the purifier outlet would be about 20 per cent as great.

To verify the accuracy of the assumptions and graphical methods, Fig. 10 was used as the basis for plotting "corrected" curves on the original data-curve sheets. Each corrected curve represents the theoretical steam-purity–water-level relationship corresponding to the "effective" chloride concentration existing during the test (see Fig. 9). In some cases, the original curves deviated appreciably from the corrected curves, agreement being somewhat better for conditions at the purifier outlet. In spite of these deviations, the general correspondence was surprisingly close, usually within the degree of accuracy with which an operator can estimate the amount of water in a gage glass. This reasonable agreement between actual and corrected curves indicates that the basic assumptions and graphical methods were acceptably accurate for the purpose at hand.

Boiler rating influences steam purity to some extent. The specific effects of this variable were not studied in detail, for No. 9-C boiler could not be operated under fixed load conditions. However, from the scattering of experimental points, and from a comparison of many data collected at different average loads, it appears that, other things being equal, carry-over changed about 15 per cent for each 1000 lb per hr over or under 75,500, which is the average of all ratings carried during the tests. For this boiler, if steam to the purifier never contains over 10 ppm of total solids, it is quite certain that steam leaving the purifier will always be of satisfactory quality. Limiting conditions for a maximum of 10 ppm solids at the purifier inlet are given in Fig. 11, which is based on the assumption that contamination increases or decreases by 15 per cent with a load change of 1000 lb per hr. Points taken from Fig. 11 checked exceptionally well with the actual experimental data.

From Fig. 10, it can be seen that changing water level by 1 per cent of a glass affected steam purity to about the same degree as a change of 6.5 ppm in chloride (or about 100 ppm total solids, since the ratio of solids to chloride averaged 15.8 during the period of these tests). Figs. 10 and 11 clearly indicate that for the conditions established water level was by far the most important single factor conducive to carry-over. Boiler ratings, and especially sudden changes in rating, were influential primarily through their effect on water level. Water conditions, in themselves, were of much less importance; otherwise, it is doubtful whether any correlation at all could have been obtained under the wide variety of water conditions encountered. An extremely important feature of boiler design appears to be the area of water

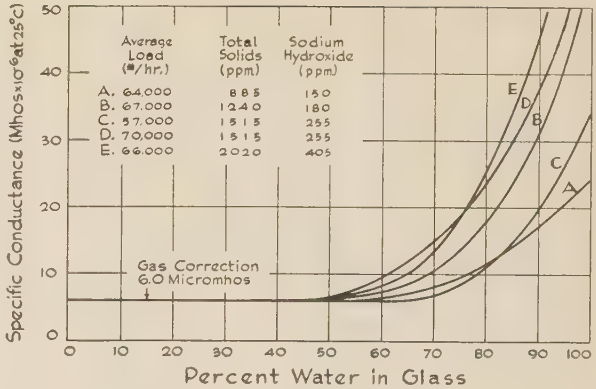

Fig. 8 No. 1 Hot-Strip-Mill Boiler. Correlation Between Water Level and Conductance Under Various Load and Boiler-Water Conditions

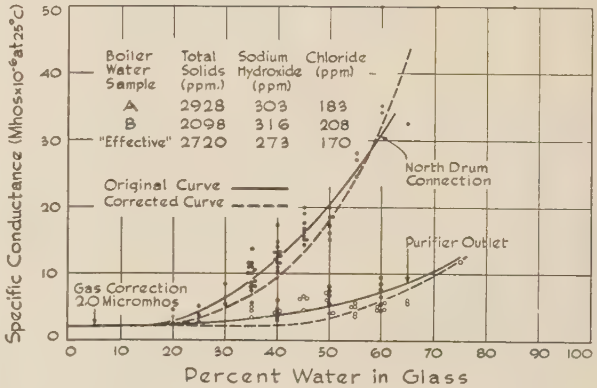

Fig. 9 No. 9-C Boiler. Correlation Between Water Level and Conductance at an Average Load of 77,500 Lb per Hr

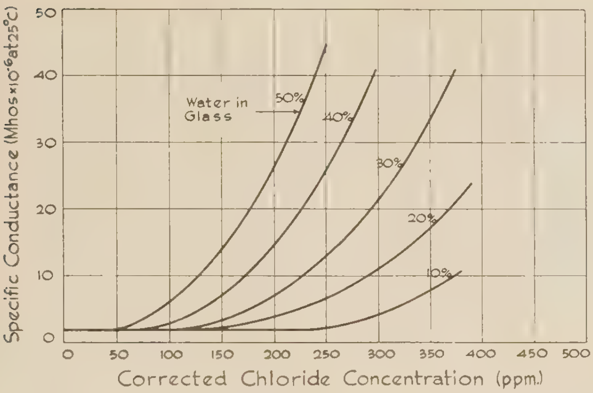

Fig. 10 No. 9-C Boiler. Summary of Fourteen Data-Curve Sheets, Showing Correlation Between Water Level, Chloride Concentration, and Conductance at Purifier Inlet

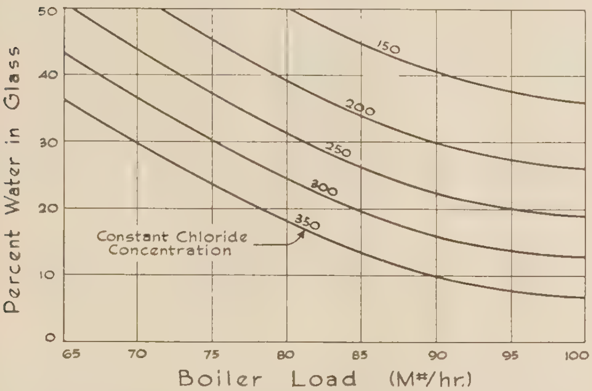

Fig. 11 No. 9-C Boiler. Limiting Load, Water Level, and Chloride Concentrations for Production of 10 Ppm Maximum Solids at Separator Inlet

surface in the steam drums. Greater surface means greater cubical displacement with changes in water level and, consequently, less fluctuation in water level with changes in boiler rating.

A specific example will best illustrate the relative value of chemical measures in carry-over control. Assume, on the basis of Fig. 11, that No. 9-C boiler will safely generate up to 80,000 lb of steam per hr with a boiler-water chloride concentration of 200 ppm (3100 ppm total solids) and a maximum water level of 40 per cent of a glass. Imagine that load conditions require the boiler to deliver 90,000 lb per hr from time to time. According to Fig. 11, this higher rating could be carried without difficulty if chlorides were reduced to 150 ppm (2350 ppm total solids, or 750 ppm less than the previous figure). This would mean increasing the blowdown rate from 3 per cent up to about 4 per cent of the total feed. A much simpler and more direct solution would be simply to carry a little less water in the boiler, about 5 per cent of a glass less, to be exact. It would require only a simple adjustment of the feedwater regulator, and the change would be scarcely noticeable by looking at the gage glass. As a matter of fact, this is precisely what was done on No. 9-C boiler, although the water level was lowered somewhat more than was indicated by this particular example. Carry-over from the purifier outlet now remains negligible under all ordinary load conditions, while water concentrations are free to vary as they will within reasonable limits set up for other chemical control purposes.

The relatively secondary importance of water conditions in all cases was clearly demonstrated by the success of corrective measures primarily mechanical in nature. Extreme carry-over from No. 10-D boiler was largely eliminated before chemical factors were even considered. A wide range of boiler-water concentrations was encountered during the tests on No. 10-C boiler, yet their influence was not great enough to affect the play of other variables involved. Although water conditions on No. 9-C boiler evidently influenced water-level fluctuations to some degree, the effects were readily overcome by closer control over water levels. Four years ago, before zeolite softeners were installed at the hot strip mill, the boilers there carried vastly different load and water conditions than they do now. On the basis of steam-purity tests made at that time, the plant was advised "always to keep the water level below mid-point on the glass," which is essentially what our recent studies affirm.

A Balanced Solution to the Problem as a Whole

Several corrective measures have gone into effect already. Boiler-water conditions are carefully controlled according to safe practice as established by experience in a great number of similar boiler plants. The No. 10 boilers are closely operated with respect to their limiting load and water-level conditions. Highly efficient feedwater regulators have been provided to stabilize water levels on the No. 9 boilers.* Safety valves have been adjusted to release first on the No. 10 boilers, thus lessening the likelihood of carry-over which might result from the popping of either of the No. 9 boilers.

If carry-over continues despite these efforts, consideration may be given to the possibility of its more effective separation by steam purifiers or steam scrubbers. Steam scrubbers would have one disadvantage here because the boiler water contains appreciable calcium-phosphate sludge. As entrained boiler water becomes diluted with feedwater in the scrubber, an adherent sludge would be produced which might eventually foul the scrubber.

There are, of course, possibilities of making further improvements in the steam-drum baffling, and changes of this nature will be made wherever desirable. Additional steam circulators might be installed to reduce steam velocities and thereby encourage better separation of moisture and steam. However, such an installation would involve some rather major changes in boiler design, and so steam circulators are not likely to be considered in the near future.

A careful study of plant operation might reveal several ways to eliminate unnecessary load surges,* unequal distribution of boiler loads, unfavorable firing methods, and erratic water-level control. Although steel-mill loads cannot be "tailor-made" to suit the carry-over whims of one boilerhouse or another, boilerhouse operation might be adapted more satisfactorily to the plant steam demands which must be met. For instance, the No. 9 boilers are extremely sensitive to load changes, while the No. 10 boilers are not. Much could be accomplished, therefore, if automatic combustion control were installed on the No. 10 boilers. They could then absorb the major swings in plant steam demand and fuel supply while the No. 9 boilers remained on relatively base loads.

Appendix

Experimental Methods

Steam samples were collected as nearly as possible in accordance with A.S.M.E. recommendations.[3] The steam flowed continuously and condensed in a copper cooling coil. Specific conductance of the condensed steam was measured by means of a

* See last paragraph of closure.

[3] A.S.M.E. Power Test Code, "Instruments and Apparatus," Part 11, "Determination of Quantities of Steam," 1929.

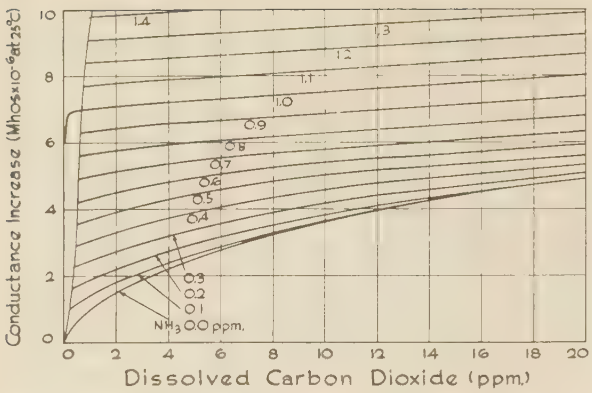

Fig. 12 Effect of Dissolved Ammonia and Carbon Dioxide on Conductance of Steam Condensate

Leeds & Northrup conductivity-resistivity bridge and a Leeds & Northrup "Micromax" strip-chart recorder calibrated over a range of zero to 50 micromhos. The dip cells and flow cells[4] contained electrodes freshly platinized and standardized. Corrections for cell constant and sample temperature were made by means of compensating devices installed in the instruments.

Dissolved carbon dioxide was determined by titration with fresh N/44 sodium-hydroxide solution to the phenolphthalein end point, the operation being carried out in a 100-ml glass-stoppered cylinder. Ammonia was measured by Nesslerization. Corrections for dissolved gases were then estimated from the family of curves in Fig. 12.[5] The gas corrections, as determined analytically, ran always within 0.1 to 0.3 micromho of the minimum value to which the conductance fell under ideal steaming conditions at the time of testing. It was therefore assumed that the gas correction remained relatively constant at this minimum value, and determinations for dissolved gases were run only periodically. The gas content of the steam should remain comparatively uniform, since it depends primarily upon the composition and amount of make-up water and the efficiency of degasification in the feedwater heater. After being corrected for dissolved gases, the conductance values, in micromhos, were multiplied by an empirical factor, 0.65, to obtain ppm total dissolved and suspended solids in the steam.

There are certain unavoidable limitations to this method of correcting for dissolved gases. These can be overcome, presumably, by use of suitable degasifying apparatus ahead of the conductance cell. However, such apparatus would induce fatoo great a lag between time of sampling and response by the measuring instrument. In studies of this nature, it is necessary to determine the critical water level or load at which carry-over of a certain degree occurs, and these variables are difficult to hold constant for any length of time even under ideal test conditions. For this reason, conductance data must be collected as quickly as possible. The present experiments were definitely handicapped, even though the steam samples delayed hardly a minute in passing through the cooling coil and conductance cell. Degasifying apparatus would increase the delay even more and would certainly blur all sharp changes in steam purity.

[4] Leeds & Northrup Catalogue No. 4940.

[5] "Determination of Purity of Steam by Electrolytic-Conductivity Method," by W. B. Gurney, M. C. Schwartz, and T. E. Crossan; discussion by A. Watson, Trans. A.S.M.E., vol. 62, 1940, pp 732–733.

Discussion

J. M. Drabelle.[6] This paper on the elimination of carry-over under steel-mill operating conditions is most timely because of the fact that we are all now operating under increased loading on our boilers and before the present war is over yet higher loadings will be carried.

Based on the experience with the 300,000-lb 650 to 750 F boilers at Cedar Rapids Power Station we have learned as have the authors that carry-over is not due to any one single cause but is a combination of water-level control, chemical control, and proper baffling in the drum of the boiler, particularly at the point of discharge of waterwall risers. Operating engineers facing this problem should approach it on that basis. The authors are to be congratulated on recording actual experiences and what was done, which is always helpful to the operating men.

C. W. Foulk.[7] All the laboratory experiments of the writer confirm the belief that the largest single item in the reduction of carry-over is the maintenance of a certain distance (determined by the salt and sludge content of the boiler water and the rate of evaporation) between water level and steam outlet. The writer once asked a boiler manufacturer why power-plant boilers, at least, were not constructed to give more space at that point. His answer was, the added cost.

J. A. Holmes.[8] This paper on carry-over in steam clearly and logically points out a condition that should be carefully studied by every engineer operating a boiler plant. The authors have emphasized the matter of proper water levels in boilers and the importance of eliminating fluctuating water levels. No matter whether or not the water in the boiler is of the foaming type, water levels will always play a very important part in the operation of boilers and in the reduction of solids in steam.

This subject of water level raises a question that is frequently discussed, i.e., just how does lowering of the water level affect carry-over? Contrary to a popular belief, our experiments in vertical glass tubes and in a small experimental boiler containing glass windows for observation have shown that the height of the water above any steam-generating surface does not appreciably affect the depth of the foam layer on top of the water. In other words, if there is a bank of steam-generating tubes coming into a drum, the depth of foam formed above the water level will be the same, no matter whether the depth of water in the drum is 10 in. or 20 in. Apparently, the drop in water level simply lowers the top of the foam level in equal proportion. It is difficult to visualize this as being true. We can, however, visualize cases where turbulence or surging or even circulation would be affected by a decrease in water depth, and this in turn would affect the foam depth.

In most boiler waters containing as much solids as mentioned in the paper, there will be some foam layer formed on the surface of the boiler water. The problem is to prevent this foam building up to a point where it carries solids into the steam leaving the boiler. Any drop in water level which automatically lowers the foam level, any mechanical device that stabilizes the water level or breaks up the foam, or any chemical treatment that will reduce foam volume will work wonders in reducing the harmful effect of solids in the steam.

[6] Consulting Engineer, Iowa Electric Light and Power Company, Cedar Rapids, Iowa. Mem. A.S.M.E.

[7] Professor Emeritus, Department of Chemistry, The Ohio State University, Columbus, Ohio.

[8] Director of Service, National Aluminate Corporation, Chicago, Ill. Mem. A.S.M.E.

This paper also shows the need for a rapid and easy method of testing for solids in steam and this is best done by conductivity apparatus. As the authors point out, there is apt to be a lag in the collection of the sample and the actual time of carry-over, and this should always be considered in evaluating any results, and, in fact, every effort should be made to reduce this lag as much as possible. This means proper picking of the sample points and as short a sampling line as possible.

The authors are to be complimented on their logical and complete method of attacking a problem that is encountered in so many steel mills. Undoubtedly this work is of value to and will be appreciated by many steel-mill and power-plant engineers.

W. H. Rowand.[9] Some time ago we had an experience similar to that of the authors' in which the objectionable solids carry-over was eliminated by suitable drum-baffle changes without imposing any restrictions on or changes to the operation of the steel-mill equipment.

[9] Service Department, The Babcock & Wilcox Company, New York, N. Y. Jun. A.S.M.E.

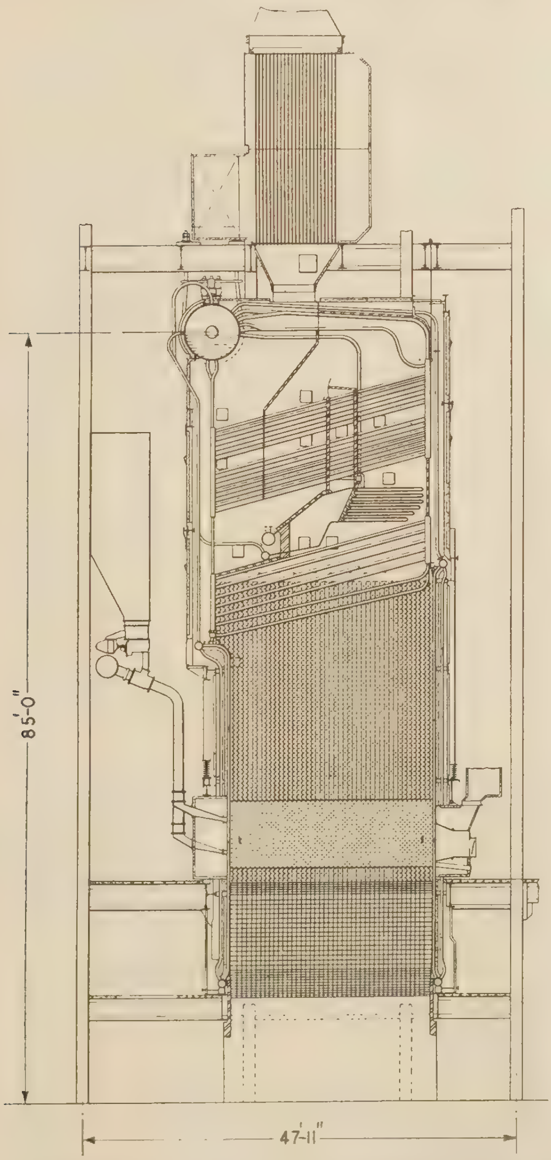

Fig. 13 Cross Section of Boiler Experiencing Objectionable Solids Carry-Over

The unit, which is shown in Fig. 13 of this discussion, was placed in service during the early part of 1937. It is a B&W cross-drum boiler with a water-cooled hopper-bottom furnace, fired with pulverized coal and blast-furnace gas. The maximum capacity is 400,000 lb of steam per hr with 425 lb pressure at the superheater outlet and 725 F temperature.

The steam from this and the other units in the power plant is used in the turboblowers and to furnish electric power for the blooming mills and a large hot continuous-strip mill. The other boilers are base-loaded. This boiler takes all of the load swings by the use of an automatically controlled butterfly valve in the line from the superheater to the main steam header.

The normal load carried by this unit is from 100,000 to 400,000 lb per hr and is an almost continuously swinging one. The load fluctuations vary in magnitude from a small change to as great as 200,000 lb per hr. Increases or decreases in load of from 150,000 to 200,000 lb per hr, covering a period of 15 to 20 sec, are frequently encountered. One period was timed in which a load increase of 150,000 lb per hr covered a time interval of only 5 sec and decreased the same amount in 9 sec.

The boiler receives about 50 per cent make-up of Monongahela River water which is filtered and treated externally in a hot-process lime-soda softener. In the fall of the year, the feedwater to the boiler has as high as 500 to 600 ppm total solids.

The 72-in-diam steam-and-water drum was equipped originally with a steam scrubber, spray washers, and a deflecting baffle, as shown in Fig. 14, and was guaranteed to give 2 ppm solids carry-over with a boiler-water concentration of 1700 ppm total solids. It was found that the limiting boiler-water concentration was between 1700 and 2000 ppm and the limiting water level about 3 in. to 4 in. above normal level, beyond which the carry-over became excessive. Below these limits the carry-over was about 1.5 ppm.

Because of the difficulty of holding the boiler-water concentration below the limiting value particularly in the fall of the year, several superheater tubes failed periodically, due to overheating from solids deposited in them. Finally in the fall of 1939, the condition of the superheater was so bad that it was decided to purchase a new superheater and the then recently developed cyclone separators for the drum internals. These were both installed in January, 1940, and no further trouble has been experienced with the superheater or elsewhere since then.

Fig. 15 shows the arrangement of the cyclone separators in the drum, whereby all of the steam and circulating water entering the drum is passed through the cyclone separators, effecting practically complete separation of the steam and water before they enter the main portion of the drum. The steam scrubber was retained, but the spray washers were eliminated, the feedwater being introduced into the drum below the water level.

With this arrangement, it was determined that the solids carry-over did not exceed 0.5 ppm with boiler-water concentrations up to 6000 ppm, and the limiting water level was found to be 16 in. above normal rather than 3 in. to 4 in., which it had been previously. At this high level it will be noted that the cyclone separators are flooded. Thus the cyclone separators eliminated boiler-water concentration as a limitation and increased the permissible water-level range about 12 in. to 13 in., which provided more than sufficient leeway for the severe operating conditions.

Two bothersome operating conditions were encountered when the unit was first placed in service after installing the cyclone separators. First, the magnitude of the water-level swings during fluctuations in load was large enough to cause frequent blowing of the high- and low-level alarms. The boiler is equipped with a

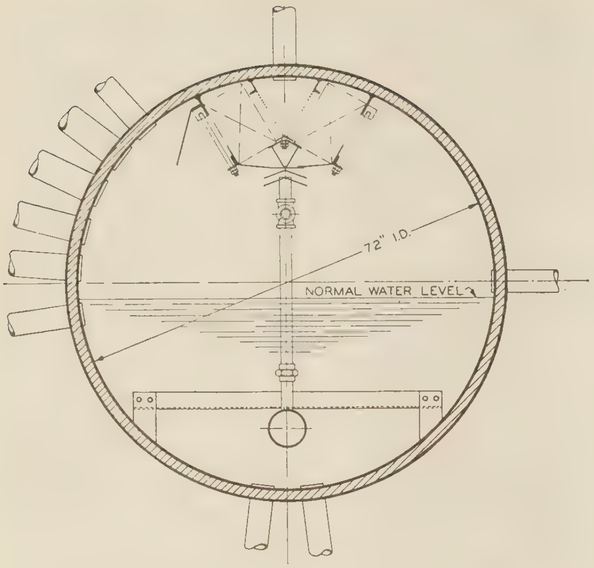

Fig. 14 Steam-and-Water Drum of Boiler

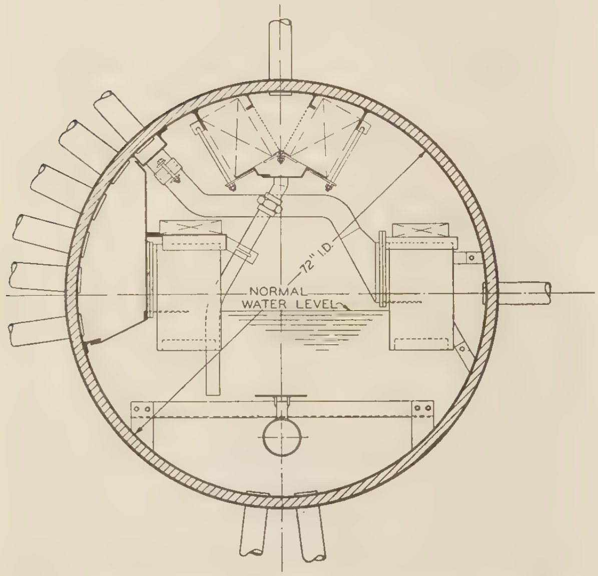

Fig. 15 Arrangement of Cyclone Separators in Steam-and-Water Drum

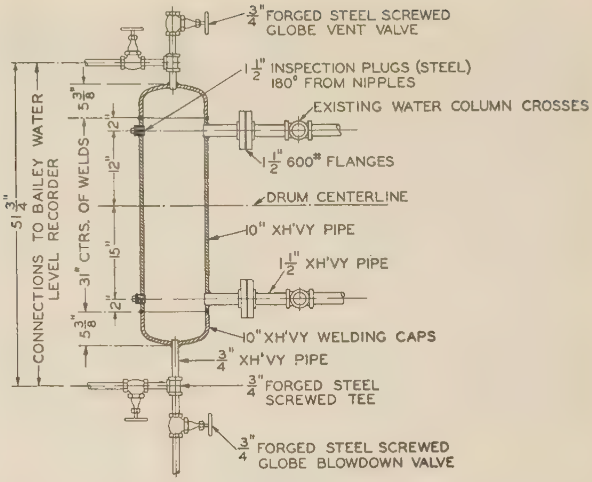

Fig. 16 Surge Tank Installed in Parallel With Water Column

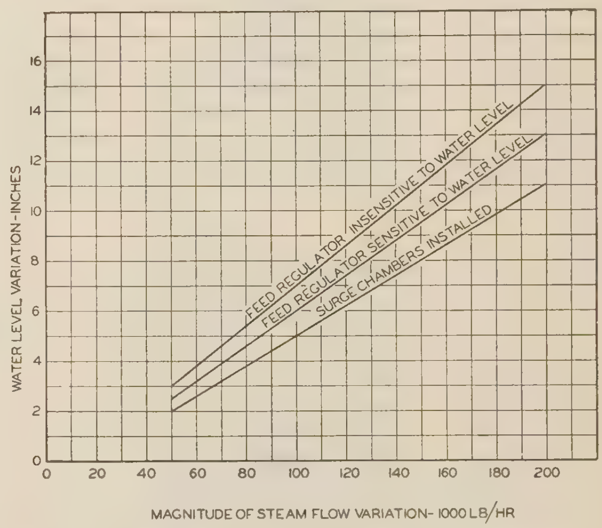

Fig. 17 Water-Level Variations Versus Load Variations up to 200,000 Lb per Hr

Bailey three-element feedwater regulator and, by making the regulator sensitive to water level, it was possible to reduce the frequency of the whistle's blowing greatly. This, however, increased the variation of feedwater flow to the boiler which caused the steam temperature to fluctuate about 40 to 60 F.

Because the large leeway in permissible water-level variation was available in so far as solids carry-over was concerned, a surge tank, shown in Fig. 16, was installed in parallel with each water column, and the feedwater-regulator connections were attached to one of the surge tanks. This effectively reduced the magnitude of the water-level variation in the water column and the water-level impulse to the feedwater regulator. This change reduced the fluctuation in the steam temperature to 20 to 25 deg and the alarm whistles blow only infrequently.

The water-level variations with load variations up to 200,000 lb per hr for these different conditions are shown in Fig. 17.

Table 4, of this discussion, shows the analysis of a typical boiler-water sample after installing the cyclone separators. The high values of caustic, phosphate, silica, and organic matter are noteworthy since they showed no tendency to cause trouble even at these high levels with the cyclone separators installed.

TABLE 4 TYPICAL BOILER-WATER ANALYSIS

	Ppm
Sodium hydroxide	1404
Sodium carbonate	203
Disodium phosphate	243
Sodium sulphate	2232
Sodium chloride	203
Iron and aluminum oxides	31
Silica	93
Sodium nitrate	91
Ether soluble matter	8
Organic matter	502
Suspended matter	466
Total solids	5476

R. C. Ulmer.[10] The results given in this paper are in agreement with the theory of carry-over postulated in a paper by Foulk and Ulmer[11] and verified in a paper by Foulk and Ryznar.[12] Carry-over of the type discussed in the present paper undoubtedly results from the filling of the steam space in the offtake drum with foam and the carrying over of some of this foam with the steam. Gross carry-over of the amount referred to in the present paper was found by Foulk and Ryznar[12] to occur only when the foam layer actually touched the steam-outlet pipe. Therefore, in order to prevent the formation of the carry-over in question, it is necessary to prevent the foam layer from reaching the steam outlet. The most obvious way to accomplish this object is to lower the water level, as was done in the present case.

Lowering the water level does not decrease the thickness of the foam layer but merely lowers it so that it does not come in contact with the steam outlet. There are, however, several factors that actually change the thickness of the foam layer. These are (1) concentration and nature of the total solids in the boiler water, (2) rate of evaporation, and (3) pressure.

As stated in the paper, the effect of concentration on carry-over is small in comparison with the effect of change in water level. This is easily understood because Foulk and Ryznar[12] have shown that, in their test boiler using sodium chloride alone, an increase in concentration of 6000 ppm increased the thickness of foam layer only about $^1/_2$ in. In this test boiler then, a change of 6000 ppm in sodium-chloride concentration had the same effect on carry-over as a change of $^1/_2$ in. in water level.

With respect to the relative effect of different soluble salts, exception is taken to the statement in the paper that hydroxide alkalinity has definite foam-aggravating properties of its own and must be considered independently. Numerous results by Foulk and Ulmer[11] indicate that the effect of soluble salts on carry-over is additive and therefore that the effect of sodium hydroxide is no different from that of other boiler-water salts. In fact, both sodium sulphate and tri-sodium phosphate promote carry-over to a greater extent than does sodium hydroxide. The fact that the concentration of the hydroxide ion can readily be controlled has perhaps focused attention upon this agent.

Rate of evaporation greatly influences the thickness of the foam layer and, consequently, the amount of carry-over. This results from the fact that the number of steam bubbles formed and, hence, the thickness of the foam layer depend upon the rate of evaporation. Sudden increases in the rate of evaporation, such as those mentioned in the paper, cause the formation of more steam bubbles and, consequently, the foam level may build up to such a height that it reaches the steam outlet and causes carry-over.

Changes in steam pressure have a marked effect upon carry-over because the thickness of the foam layer is affected in two ways; (1) the steam bubbles in the foam layer expand or contract depending upon whether there is, respectively, an increase or decrease in pressure and, (2) the number of steam bubbles formed at the heating surface increases greatly if there is a sudden decrease in pressure and decreases if there is a sudden increase in pressure. As there was considerable load fluctuation in the case at hand, it is entirely possible that there was sufficient decrease in steam pressure at times to cause expansion of steam bubbles in already existing foam and the formation of new bubbles, thereby increasing the foam height to such an extent that carry-over occurred.

[10] Research Department, The Detroit Edison Company, Detroit, Mich.

[11] "Solid Matter in Boiler Water Foaming," by C. W. Foulk and R. C. Ulmer, *Industrial and Engineering Chemistry*, vol. 30, 1938, pp. 158–160.

[12] "Foaming of Boiler Water," by C. W. Foulk and J. W. Ryznar, *Industrial and Engineering Chemistry*, vol. 31, 1939, pp. 722–725.

P. B. Place.[13] The authors attacked their carry-over problem systematically, established operating performance, and were rewarded by successful application of their conclusions. In three different types of boilers, they corrected excessive carry-over by three different methods, each method designed to correct an undesirable condition in a particular boiler. The analytical approach to the problem and appreciation of the fact that all carry-over problems are not similar are highly commendable.

The writer appreciates the authors' emphasis on the fact that the corrections were mechanical and not chemical. Chemists are burdened with many carry-over problems which should be solved by the co-operative effort of both chemical and mechanical engineers. The problem is a mutual responsibility. There is a tendency to consider boiler waters as foaming or nonfoaming, suggestive of a sharp demarcation and a critical chemical condition. This is misleading. Probably all boiler waters foam, but the amount of foaming and character of the foam varies with concentration, rating, and pressure. Within a limited range of foaming condition, mechanical changes may control or reduce the foaming to give marked improvement in carry-over but the foaming condition is still present and is primarily a chemical problem.

The authors state that boiler-water conditions were controlled to keep total solids within a range consistent with nonfoaming conditions in hundreds of similar boiler plants. The boiler manufacturers have frequently defended drum-baffle designs as being standard and as having given satisfactory results in hundreds of similar units. Both are justified in their claims, and the uncertain factors are usually the degree of foaming and the ability of the baffle to control the foam.

The carry-over from No. 10-D boiler has all the characteristics of foam-over. The very rapid, excessive, and erratic change in conductivity is typical of foaming, and the authors' statement that the carry-over evidently originated in the waterwalls confirms this. Foam is generated in the boiler at the steam-generating surfaces and is delivered to the drum where it must be controlled and reduced before it fills the drum and flows over into the steam outlet.

Simultaneous demands for greater rates of steam generation, higher concentrations, higher pressures, and cleaner steam put a severe burden on the mechanical engineer, and it is difficult to satisfy one demand without sacrificing to another. The chemical engineer must contrive to attack the problem of foaming and its inhibition.

In the past 2 years the writer has had the opportunity of testing a large number of drum baffles in a low-pressure boiler fitted with windows and electric lights, under a great variety of water and operating conditions. We have found no definite relationship between concentration and foaming. In one case, we had a very badly foaming water with some 3500 ppm of sulphate and reduced the foam to a negligible amount by the addition of some 3000 ppm of chloride. The further addition of sulphate, caustic, and phosphate to give a total concentration of 12,000 ppm did not produce as much foaming as the original 3500 ppm concentration. Concentrations of over 50,000 ppm have been kept under full control, when concentrations of a few thousands could not, under the same operating conditions.

The carry-over in the other two boilers, described in the paper, is indicative of spray and more logically a function of water level. The amount of carry-over is relatively low, increasing steadily with water level, and the data can be plotted in a family of curves. The authors' interpretation of the results appears to be correct, and their corrective measures have given satisfactory results. Although the concentrations in these boilers are comparable with

[13] Research and Development Department, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

that in boiler No. 10-D, it appears that foaming was not excessive because of differences in steam-generating conditions.

All boilers have, of course, some critical water level above which carry-over develops. If foaming is not excessive, this critical water level is often that at which excessive spray develops due to the submerging of active riser or circulator tubes. If foaming is excessive, the unit may have a different critical level for different water conditions, and carry-over results when the foam level is pushed up to a steam circulator by an increase in water level.

Authors' Closure

The authors and discussers appear to agree on two very important points; namely (1) that boiler-water impurities (excepting, of course, organic matter) have a minor function in carry-over, as compared with mechanical factors such as steam-purifying facilities and the height of water level in the steam-disengaging space, and, (2) that carry-over is primarily a mechanical problem, involving water-level control, proper design of steam-drum baffling, and effective steam-purification, and not a chemical one beyond the requirements of intelligent blowdown control.

Mr. Ulmer concludes that the effect of soluble salts is additive and that no one constituent is particularly more troublesome than another. Mr. Place, working under a great variety of water and operating conditions, could find no definite correlation between concentrations and foaming. The authors' data were collected over a period of several months with gaps of many weeks between some of the tests. Although water conditions naturally changed a great deal in the meantime, it was nevertheless possible to get good correlations between steam purity and certain mechanical variables. If the over-all effects of solids were pronounced, or if certain constituents were appreciably more aggresive than others, it is doubtful whether any correlations could ever have been found under the great diversity of conditions dealt with.

Moreover, evidence cited by Messrs. Holmes, Ulmer, and Place, together with the authors' work on No. 9-C boiler and the hot-strip-mill boiler, indicates that the concentrations normally found in practice produce foam layers hardly over 1 in. or so in thickness, and that the thickness depends more upon operational factors than upon the character of the boiler water. It would seem, therefore, that boiler operators would do well to direct most of their attention toward total-solids concentrations and good blowdown control, rather than worry too much about "doctoring" the boiler water chemically to prevent foaming.

The discussers almost unanimously stress water level as the most important single factor conducive to carry-over and imply that water-level control is the starting point in carry-over correction. Second in favor come mechanical changes such as improved steam-drum baffling or, as a last resort, the installation of special steam-purifying equipment. All seem to appreciate the importance of indirect measures which tend to stabilize drum pressures and rates of steam generation.

Since the presentation of the paper, the No. 9 boilers in No. 1 boilerhouse have been equipped with new economizers having the feedwater regulators located on the inlet, rather than on the discharge side. This has helped materially in overcoming some of the water-level fluctuations, such as unavoidably result from locating a feedwater-control valve on the outlet side of a steaming economizer. A recent plant study revealed the fact that the plant demand for high-pressure steam could be stabilized by picking up loads more gradually on the turbogenerators. The effect in easing load surges on the No. 9 boilers has been quite gratifying.

Excess Air and Brake Mean Effective Pressure

By P. H. Schweitzer,[1] State College, Pa.

It is known that there is a definite relation between the excess air and the brake mean effective pressure (bmep) of an internal-combustion engine, but that relation is seldom expressed in a convenient form. A handy formula tying the bmep to the excess air is considered very useful, therefore the author submits the following

$$\text{bmep} = 180\,\frac{0.4}{f} \cdot \frac{14.5}{r_{th}} \cdot \frac{\eta_{sc}}{1 + (\lambda - 1)\eta_{sc}}\,\eta_{vol} \ldots\ldots [1]$$

where f denotes the fuel consumption, lb per bhp-hr, r_{th} the chemically correct air-fuel ratio, η_{vol} the volumetric efficiency, η_{sc} the scavenging efficiency, and λ the excess-air factor, which is defined as the actual air-fuel ratio divided by the chemically correct air-fuel ratio.

Equation [1] has a universal validity for either four- or two-stroke-cycle engines, for carburetor engines, and fuel-injection engines of both compression-ignition, or spark-ignition type, normally aspirated or supercharged engines of all types.

For four-stroke-cycle engines with negligible contamination of the fresh charge by residual gases ($\eta_{sc} = 1$), Equation [1] is reduced to the yet simpler formula

$$\text{bmep} = 180\,\frac{0.4}{f} \cdot \frac{14.5}{\lambda\, r_{th}}\,\eta_{vol} \ldots\ldots\ldots\ldots\ldots [2]$$

The derivation of Equations [1] and [2] is found in the Appendix. Here it is of interest to point out the implications of these formulas and the service they can render to the internal-combustion engineer.

A typical Diesel fuel which is composed of 7 parts of carbon and 1 part of hydrogen requires 14.5 lb of air to burn 1 lb of it completely. With a fuel consumption of 0.4 lb per bhp-hr and a volumetric efficiency of 100 per cent (which is the upper limit for normally aspirated engines), according to Equation [2] the maximum bmep is

$$\text{bmep}_{max} = 180 \text{ psi}$$

This is only obtained, however, with an excess-air factor of $\lambda = 1$. If we have 50 per cent excess air (which is an average figure for Diesel engines), and the volumetric efficiency is $\eta_{vol} = 0.7$,

$$\text{bmep}_{Diesel} = 180\,\frac{0.4}{0.4} \cdot \frac{1}{1.5}\,0.7 = 84 \text{ psi}$$

A carburetor engine with a specific fuel consumption of 0.5 lb per bhp-hr operating with an air-fuel ratio of 14.5 (close to the chemically correct mixture) and a similar volumetric efficiency gives a bmep

$$\text{bmep}_{carb} = 180\,\frac{0.4}{0.5} \cdot \frac{1}{1}\,0.7 = 100 \text{ psi}$$

showing an advantage of 16 per cent over the Diesel. In either type of engine, the bmep is directly proportional to volumetric efficiency and inversely proportional to the specific fuel consumption and excess-air factor.

This is not exactly true in a two-stroke-cycle engine or in an incompletely scavenged four-stroke-cycle engine. Then the scavenging efficiency enters in a manner shown by Equation [1] which is represented by Fig. 1.

Scavenging efficiency is defined as the amount of fresh air in the cylinder charge divided by the total cylinder charge; volumetric efficiency as the total cylinder charge divided by the piston displacement. It is apparent that 180 psi bmep can hardly be exceeded in normally aspirated engines, but with supercharging η_{vol} can be made larger than 1, and the bmep goes up directly with η_{vol}.

If in Equations [1] and [2], the fuel consumption is expressed in pounds per indicated horsepower, the formulas give mean indicated pressures rather than brake mean effective pressures.

Appendix

Derivation of Equation [1]

The horsepower output of an engine is

$$\text{hp} = \frac{\dfrac{V_{disp}}{12} \times \text{bmep} \times \dfrac{2n}{c}}{33{,}000} \ldots\ldots\ldots\ldots\ldots [3]$$

where n is the rpm and c is the number of strokes per cycle.

Naturally, the horsepower is also equal to the total fuel consumption divided by the specific fuel consumption

$$\text{hp} = \frac{F_h}{f} = \frac{V_{pure} \times \rho \times \dfrac{2n}{c} \times 60}{r \times f \times 1728}$$

$$= \frac{V_{pure} \times \rho \times \dfrac{2n}{c} \times 60}{r_{th} \times \lambda \times f \times 1728} \ldots\ldots\ldots\ldots\ldots [4]$$

where V_{pure} is the volume of pure air in the cylinder before combustion (see Fig. 1), r the actual, r_{th} the theoretical ratio, $\lambda = r/r_{th}$ the excess-air factor, and ρ the weight of 1 cu ft of air under NTP conditions.

From Equations [3] and [4]

$$\text{bmep} = 13{,}750\,\frac{\rho}{\lambda \times f \times r_{th}} \cdot \frac{V_{pure}}{V_{disp}}$$

A volume of 1 cu ft of dry air under NTP conditions weighs 0.0765 lb, therefore

$$\text{bmep} = \frac{1050}{\lambda \times f \times r_{th}} \cdot \frac{V_{pure}}{V_{disp}} \ldots\ldots\ldots\ldots [5]$$

It should be realized that V_{pure} is more than that part of the air delivered which is retained in the cylinder. It includes some pure air contained in the residual gas remaining in the cylinder

[1] Professor of Engineering Research, The Pennsylvania State College. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

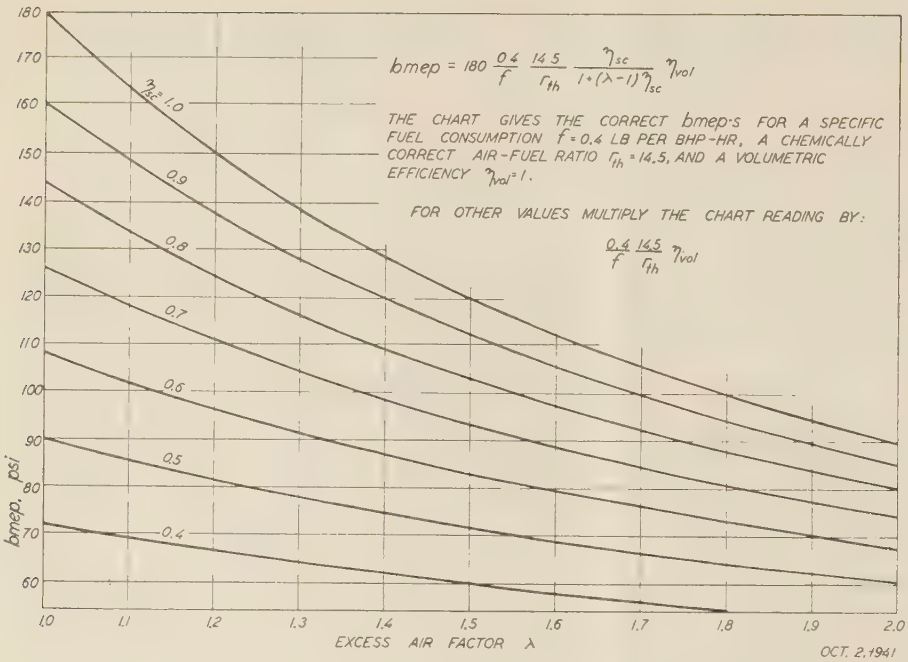


Fig. 1 Relation Between Excess-Air Factor and Brake Mean Effective Pressure

(This chart covers both four-stroke- and two-stroke-cycle engines of all types at any load.)

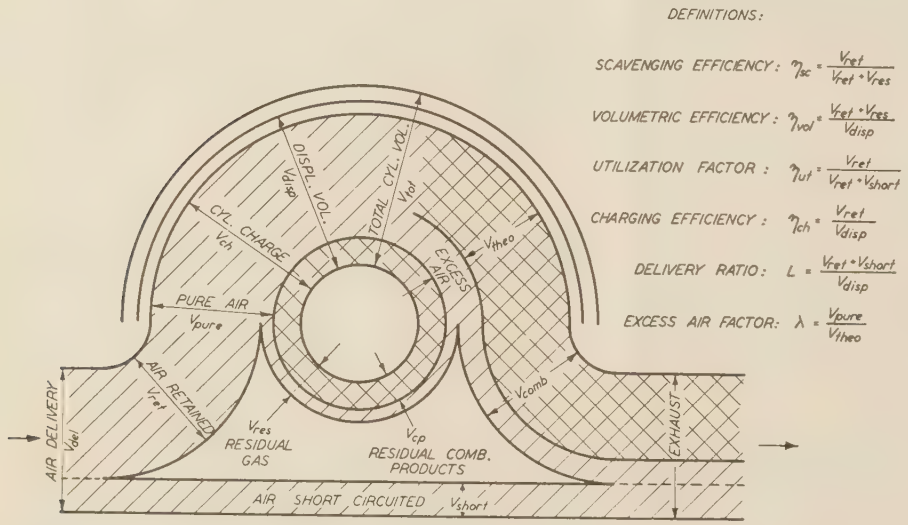


Fig. 2 Diagram Showing Charging Process of Internal-Combustion Engine

(Diagram represents either a two-stroke-cycle engine or a four-stroke-cycle engine with considerable valve overlap. In a four-stroke-cycle engine without valve overlap, the air short-circuited v_{short} is zero and the area below the dash line is missing.)

from the previous cycle. Referring to Fig. 2, the following definitions and relations can be written:

$$\eta_{sc} = \frac{V_{ret}}{V_{ret} + V_{res}} \qquad [6]$$

$$\eta_{vol} = \frac{V_{ret} + V_{res}}{V_{disp}} \qquad [7]$$

$$\lambda = \frac{V_{pure}}{V_{theo}} = \frac{r}{r_{th}} \qquad [8]$$

$$V_{pure} = V_{ret} + V_{res} - V_{cp} \qquad [9]$$

$$\frac{V_{cp}}{V_{res}} = \frac{V_{theo}}{V_{ret}} \qquad [10]$$

In Equations [6] to [10], we have five unknowns: V_{pure}, V_{ret}, V_{res}, V_{cp}, and V_{theo}. These can be solved for V_{pure} for which we get

$$V_{pure} = V_{disp} \frac{\eta_{vol}\ \eta_{sc}\ \lambda}{1 + (\lambda - 1)\eta_{sc}} \qquad [11]$$

Equations [5] and [11] result in

$$\text{bmep} = \frac{1050}{f \times r_{th}} \cdot \frac{\eta_{vol}\ \eta_{sc}}{1 + (\lambda - 1)\eta_{sc}} \qquad [12]$$

which can also be written as

$$\text{bmep} = 180 \frac{0.4}{f} \cdot \frac{14.5}{r_{th}} \cdot \frac{\eta_{sc}}{1 + (\lambda - 1)\eta_{sc}} \eta_{vol} \qquad [13]$$

Discussion

H. M. Jacklin.[2] In discussing this paper, it was thought that the results of a brief study of the performance of approximately 40 Diesel and compression-ignition engines would be interesting. The results of this study appear in Fig. 3 of this discussion. It

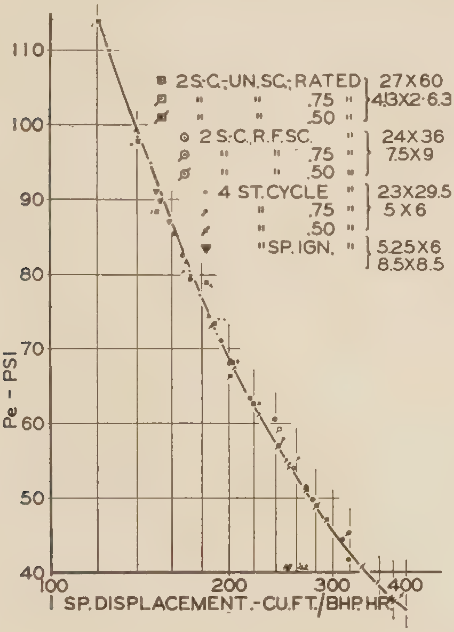


Fig. 3 Relation Between P_E and Specific Displacement for Injection-Type Oil Engines

will be noted that these oil engines have been divided into four classes:

1 Two-stroke cycle with uniflow scavenging.
2 Two-stroke cycle with reversed-flow scavenging.
3 Four-stroke cycle.
4 Two or three engines using spark ignition.

Also, it will be noted that the range of cylinder sizes used in each classification is depicted on the same figure. The data show brake mean effective pressures appearing in the vertical scale and were taken from published data of performance at full load; and these data were plotted against the specific displacement expressed in cubic feet per brake horsepower hour. It should be noted that the specific displacement is simply a measure of the volume displaced by the pistons and is probably the maximum volume of air that any of these engines could use. However, lowered volumetric efficiency seems to make it necessary for certain engines to have a high specific displacement in order to deliver the output. If it be assumed that all the engines were rated with the same actual air-fuel ratio in the cylinder, it would be expected that all points would fall on one line.

Uniflow two-cycle and four-cycle engines have both shown brake mean effective pressures above 110 psi. Following down the curve, it is found that the best reversed-flow two-cycle engine has a brake mean effective pressure of about 82 psi, and that all the rest of the ordinary two-cycle units range downward to close to 40 psi mean effective pressure, with a specific displacement of 320 cfm.

It is thought that these data indicate that the various manufacturers are rating their engines at approximately the same smoke density or, better still, the same actual air-fuel ratio. Further, these data demonstrate that it is entirely possible to show as high brake mean effective pressures with the two-stroke cycle as with the four-stroke cycle when the two-stroke cycle has been properly studied and developed.

It would appear that the relative volumetric efficiency of the various engines in this study would vary somewhat inversely as the specific displacement.

[2] Professor of Mechanical Engineering, Purdue University, Lafayette, Ind. Mem. A.S.M.E.

Ralph Miller.[3] The volume of air taken in on the suction stroke of a four-stroke-cycle Diesel engine is less than the piston displacement, because of the combined effect of pressure loss through the inlet valve and heating of the air during the suction stroke by the hot walls. Contrary to the suggested use of a scavenging efficiency factor, the residual gas in the clearance volume has no effect upon the volumetric efficiency of a four-cycle engine.

This can be proved by calculating the temperature of the mixture of the gas and fresh air in terms of the initial temperatures and then calculating the volumes of the residual gas and the air charge. It will be found that the expansion of the air charge, when raised from atmospheric temperature to the mixture temperature, equals the contraction of the residual gas. It follows that the volumetric efficiency of a four-cycle engine is neither affected by the temperature of the residual gas nor by the volume of the clearance space. The volumetric efficiency is always equal to the efficiency which would be obtained with zero clearance.

The foregoing is based upon the assumption that (1) the pressure at the end of the exhaust stroke is the same as the inlet-manifold pressure, also (2) that the specific heat of the residual gas and the incoming air is the same. Actually, the specific heat of the residual gas is about 3 per cent higher because of the higher temperature and the CO_2; however, the effect on volumetric efficiency is negligible.

Valve overlap does not clear out residual clearance-volume gas, but, if carried to extremes, will reduce the volumetric efficiency. Tests on a 10 × 12-in. 750-rpm, and a $12^1/_2$ × 15-in. 300-rpm engine showed a drop in volumetric efficiency from 87 per cent with 30-deg overlap to 68 per cent with 140-deg overlap; that is, the value V_{del} in Fig. 2 of the paper was only 68 per cent of piston displacement, indicating that the short circuit was reversed.

Although the author correctly assumes that the scavenging efficiency N_{sc} is 1 in a four-cycle engine, his Equation [6] does not permit this assumption. Volumetric efficiency likewise seems to be in error as given in Equation [7]. The volumetric efficiency of a four-cycle Diesel engine is simply the ratio of the volume of air taken in (referred to atmospheric conditions) to the piston displacement. The fresh air in the residual gas amounts to about 1.5 per cent of the piston displacement in an engine with 14:1 compression ratio when operating with 100 per cent excess air or about 75 psi bmep. By adding the percentage of fresh air in the residual gas to the volumetric efficiency, a true expression of the percentage of total air is obtained.

At the present state of the art few, if any, Diesel engines are being rated above 80 psi bmep for heavy-duty continuous service. This limit is imposed by the temperature of the cycle rather than by combustion. Many engines can operate with clear exhaust up to 105 psi bmep but cannot be rated for continuous service above 75 to 80 psi.

To utilize the higher ratings permitted by good combustion efficiency, new materials must be developed for the parts that suffer from increased temperatures, such as cylinder liners,

[3] Worthington Pump & Machinery Corp., Buffalo, N. Y. Mem. A.S.M.E.

pistons and rings, valves, etc., or methods of cooling these parts must be developed far beyond any systems in use today.

Proof of the foregoing statements is given as follows:

Clearance volume .... V_1
Clearance gas weight .... W_1
Clearance gas temperature .... T_1
Piston-displacement volume .... V_2
Weight of displacement volume .... W_2
Temperature of displacement volume .... T_2
Specific heat .... C_p
Temperature of mixture .... T_3

Then

$$\text{Total heat in gas} = W_1 \times T_1 \times C_p$$

$$\text{Total heat in air} = W_2 \times T_2 \times C_p$$

$$\text{Total weight of mixture} = W_1 + W_2$$

$$\text{Total heat of mixture} = (W_1 \times T_1 \times C_p) + (W_2 \times T_2 \times C_p)$$

From this

Temperature of mixture

$$T_3 = \frac{(T_1 \times W_1 \times C_p) + (W_2 \times T_2 \times C_p)}{(W_1 + W_2) \times C_p}$$

$$= \frac{(T_1W_1) + (T_2W_2)}{(W_1 + W_2)} \text{ deg Fahr}$$

If it can be shown that the gas volume at T_3 temperature plus the air volume at T_3 temperature is equal to $V_1 + V_2$, or the clearance volume plus the piston displacement, then it is proved that the residual gas has no influence upon volumetric efficiency.

$$\text{Volume of air at temperature of } T_3 = \frac{V_2 \times T_3}{T_2}$$

$$\text{Volume of gas at temperature of } T_3 = \frac{V_1 \times T_3}{T_1}$$

$$T_3 = \frac{(T_1 \times W_1) + (T_2 \times W_2)}{(W_1 + W_2)}$$

$$W_1 = \left(\frac{V_1}{T_1}\right) K$$

$$W_2 = \left(\frac{V_2}{T_2}\right) K$$

$$T_3 = \frac{\left(\frac{T_1 \times V_1}{T_1}\right) K + \left(\frac{T_2 \times V_2}{T_2}\right) K}{\left(\frac{V_1}{T_1}\right) K + \left(\frac{V_2}{T_2}\right) K} = \frac{\left(\frac{T_1 \times V_1}{T_1}\right) + \left(\frac{T_2 \times V_2}{T_2}\right)}{\left(\frac{V_1}{T_1}\right) + \left(\frac{V_2}{T_2}\right)}$$

$$\frac{\dfrac{(T_1 \times V_1)T_2 + (T_2 \times V_2)T_1}{(T_1 \times T_2)}}{\dfrac{(V_1 \times T_2) + (V_2 \times T_1)}{(T_1 \times T_2)}} = \frac{(T_1 \times V_1 \times T'_2) + (T_2 \times V_2 \times T_1)}{(V_1 \times T_2) + (V_2 \times T_1)}$$

$$= \frac{(T_1 \times T_2)(V_1 + V_2)}{(V_1 \times T_2) + (V_2 \times T_1)}$$

Clearance-gas volume + air volume at T_3 (F)

$$\left(\frac{V_2 \times T_3}{T_2}\right) + \left(\frac{V_1 \times T_3}{T_1}\right) = \frac{(T_1 \times T_3 \times V_2) + (V_1 \times T_3 \times T_2)}{(T_2 \times T_1)}$$

$$= \frac{T_3[(T_1 \times V_2) + (T_2 \times V_1)]}{(T_2 \times T_1)}$$

Substitute for T_3

$$\frac{[(T_1 \times T_2)(V_1 + V_2)] \times [(T_1 \times V_2) + (T_2 \times V_1)]}{(V_1 \times T_2 + V_2 \times T_1)(T_2 \times T_1)} = V_1 + V_2$$

Author's Closure

In his interesting discussion, Mr. Miller furnishes a neat mathematical proof to the effect that the "residual gases in the clearance volume have no effect upon the volumetric efficiency of a four-cycle engine," if the inlet-manifold pressure is equal to the pressure at the end of the exhaust stroke, that means with no supercharge. In turn, he contends that Equation [7] of the paper, defining the volumetric efficiency, and Equation [6], defining scavenging efficiency, are incorrect. Mr. Miller does not suggest that Equations [1] and [2], which give the relation between excess air and brake mean effective pressure were incorrect, and they are considered valid within 0.5 per cent. (That minute error comes from the fact that the volume of the exhaust is about 6 per cent larger than the volume of the air retained.)

The disagreement can be reduced to preferences in the selection of definitions. The definition of volumetric efficiency, given by Mr. Miller as $V_{\text{ret}}/V_{\text{disp}}$ is a popular one and is quite satisfactory as long as it is applied to four-stroke-cycle engines of the common variety. However, that same definition breaks down if applied to two-stroke-cycle engines, or even to four-stroke-cycle engines which have simultaneously considerable clearance volume and considerable excess air. The reason for this is that the power output of an engine is naturally expected to increase if either the volumetric efficiency increases or the scavenging efficiency increases. It would be disturbing if that were not so. But, if we define volumetric efficiency as Mr. Miller suggests

$$\eta'_{\text{vol}} = \frac{V_{\text{ret}}}{V_{\text{disp}}}$$

then Equation (1) will look like

$$\text{bmep} = 180\,\frac{0.4}{f} \cdot \frac{14.5}{r_{\text{th}}} \left(\frac{1}{\dfrac{1}{\eta'_{\text{vol}}} + \dfrac{\lambda - 1}{\eta'_{\text{vol}} + \dfrac{V_{\text{res}}}{V_{\text{disp}}}}} \right)$$

and brake mean effective pressure will increase when V_{res} increases. A definition which gives such results is unsatisfactory.

Only in two cases will this definition of the volumetric efficiency avoid absurdity:

1 If there is no excess air $\lambda = 1$ (carburetor engine), when

$$\text{bmep} = 180 \cdot \frac{0.4}{f} \cdot \frac{14.5}{r_{\text{th}}} \cdot \eta'_{\text{vol}}$$

2 If the residual-gas volume $V_{\text{res}} = 0$, when

$$\text{bmep} = 180 \cdot \frac{0.4}{f} \cdot \frac{14.5}{r_{\text{th}}} \cdot \frac{\eta'_{\text{vol}}}{\lambda}$$

If excess air is present and the residual gas is not negligible, the conventional definition of volumetric efficiency is unsatisfactory.

A consistently satisfactory system of definitions can be arrived at by the following reasoning:

The air delivered into an engine cylinder is split into two parts; i.e., the air short-circuited, which leaves through the exhaust port or valve overlap, and the air retained, which stays in the cylinder after port or valve closure

$$V_{\text{del}} = V_{\text{short}} + V_{\text{ret}}$$

The utilization factor $\eta_{ut} = V_{ret}/V_{del}$ is an index of the amount of air retained, as $(1 - \eta_{ut})$ is the relative amount of air that is short-circuited to the exhaust without participating in the combustion.

The utilization factor is a measure of our success in utilizing the air without much waste. It is largely controlled by the scavenging arrangement.

There is another way to increase the amount of air retained, namely, by increasing the delivered air. The delivery ratio

$$L = \frac{V_{del}}{V_{disp}}$$

is a measure of the air fed into the engine and is predominantly controlled by the capacity of the blower.

The air retained V_{ret}, together with the residual gas V_{res} remaining in the cylinder after scavenging, constitutes the cylinder charge V_{ch}. This charge may be more or less than the displacement volume V_{disp} depending upon the volumetric efficiency, which is

$$\eta_{vol} = \frac{V_{ch}}{V_{disp}} = \frac{V_{ret} + V_{res}}{V_{disp}}$$

This volumetric efficiency is a measure of our success in filling the cylinder, irrespective of the composition of the charge. It is predominantly controlled by the scavenge pressure and the port (valve) size. It can be determined by a pressure gage and a thermometer at any point during the compression stroke.

During combustion, part of the air contained in the cylinder charge burns and part of it, the excess air, is not involved in the attendant chemical reactions. Part of this excess air escapes through the exhaust with the combustion products; and part of it, $V_{res} - V_{cp}$ (V_{cp} represents combustion products in the residual gas), remains in the cylinder and participates in the subsequent cycle. Therefore, the cylinder charge consists of three parts; i.e., the retained portion of the air delivered, part of the combustion products from the preceding cycle, and part of the excess air from the preceding cycle.

The scavenging efficiency is a measure of our success in clearing the cylinder from the residual gases from the preceding cycle and is defined as

$$\eta_{sc} = \frac{V_{ret}}{V_{ch}} = \frac{V_{ret}}{V_{ret} + V_{res}}$$

This efficiency is an indication of the contamination of the air charge. Principally it can be determined by gas analysis.

The fresh air available for combustion is described by the term "charging efficiency"

$$\eta_{ch} = \frac{V_{ret}}{V_{disp}}$$

which is the term frequently called volumetric efficiency in four-stroke-cycle engines. The power output of the engine is roughly proportional to it.

Simple algebra will show that

$$\eta_{ch} = \eta_{sc}\eta_{vol} = \eta_{ut}L$$

This is a consistent and convenient terminology, applicable to both two-stroke- and four-stroke-cycle engines.

Naturally if $V_{res} = 0$, $\eta_{sc} = 1$ and $\eta_{ch} = \eta_{vol}$, and the conventional definition

$$\eta_{vol} = \frac{V_{ret}}{V_{disp}}$$

becomes valid.

Now coming back to Mr. Miller's thesis, that the size of the clearance volume and the temperature of the clearance gas have no influence upon the volumetric efficiency, the author agrees with that thesis under the stipulated conditions (no supercharge) if volumetric efficiency signifies the quantity which in his terminology is designated by "charging efficiency."

This may be made clearer by the following reasoning:

Mr. Miller has found that if one takes V_{disp} volume of air at T_{inl} temperature and V_{cl} volume of air at T_{res} temperature and mixes the two together, without altering the pressures, the resulting temperature will be an intermediate T_{mix} and at that temperature the gas will occupy a volume

$$V_{tot}{}^{T_{mix}} = V_{cl} + V_{disp}$$

because the shrinkage of $V_{cl}{}^{T_{res}}$ will just equal the expansion of $V_{disp}{}^{T_{inl}}$.

This is only possible if the amount of air retained in the cylinder is at every time $V_{disp}{}^{T_{inl}}$, irrespective of how much V_{cl} is or what the temperature T_{res} is, for the following reason: After mixing, the cylinder charge consists of the retained air and the residual gas

$$V_{ch} = V_{ret} + V_{res}$$

where all V's without upper suffix refer to NTP conditions.

This cylinder charge originated from sucking in V_{disp} volume of air at T_{inl} temperature and retaining V_{cl} volume of air at T_{res} temperature.

$$V_{ch} = V_{disp}{}^{T_{inl}} + V_{cl}{}^{T_{res}}$$

Since nothing gets lost out of the clearance gas during mixing $V_{res} = V_{cl}{}^{T_{res}}$, therefore, $V_{ret} = V_{disp}{}^{T_{inl}}$.

Now in our terminology

$$\frac{V_{ret}}{V_{disp}} = \eta_{ch} = \eta_{vol} \times \eta_{sc}$$

which simply means that, when the clearance volume is increased η_{vol} increases, η_{sc} decreases, and the product $\eta_{vol} \times \eta_{sc}$ remains constant.

Power Pulsation Between Synchronous Generators

By TROELS WARMING,[1] MILWAUKEE, WIS.

The power pulsation between synchronous generators depends on how close the system is to resonance with disturbing impulses. These impulses originate, in most cases, from four-cycle engines on the network. The reason is that such engines give a very low disturbing frequency, since an unbalanced torque will give only one impulse for every two revolutions. This frequency will often be close to the natural frequencies of a system of several generators in parallel. Consequently, it is necessary to find these natural frequencies and if we are close to resonance to calculate the power pulsation. In this paper is given a graphical method for finding the natural frequency of any number of generators running in parallel. An estimate is made of the unbalanced torque which may be expected from an internal-combustion engine. Based on this the power pulsation is computed. A simple formula is given for the case with only two generators in parallel, and it is shown how a system of several generators may be reduced to an equivalent system of only two. Of particular interest is the influence of the damping on a vibrating system of this kind. This is illustrated by an example with three generators. According to the customary calculations, the parallel operation in this particular case should be very difficult. The damping, however, changes the form of vibration to an extent that operation is deemed safe.

It is a well-known fact that alternating-current generators run at very nearly constant speed. There is nothing mysterious about this; for instance, it would be easy to change this speed if an adjustment were made on the governors for all the generators on the network. However, it is impossible to change the speed of one generator without changing the speed of all.

The phenomenon, effectuating this condition, is the synchronous torque, which constitutes a flexible coupling between the generator and the network or bus bar. This means that we will get a system of generators, each having a certain mass moment of inertia, and all connected through flexible couplings to a shaft (the bus bar), without any mass, Fig. 1. It is evident that vibrations may appear in this system. For their exact determination, we must know the spring constants, masses, and damping of the system, and frequency and magnitude of the disturbing impulse.

The electric current in the rotor of an alternating-current generator produces a magnetic field which rotates with the engine. In the stator, another magnetic field rotates, but this field is excited by the alternating current and must, consequently, follow the frequency of the bus bar. If this frequency is f cycles per sec, then the magnetic field will rotate f/p revolutions per second, with $2p$ = number of poles.

Normally, these two magnetic fields will rotate at the same speed, except for the vibrations dealt with in this paper. The rotor field is leading about 20 electrical deg at full load. The two magnetic fields will react upon each other with a torque that increases with the phase angle; this torque provides the elastic force for the flexible couplings in Fig. 1.

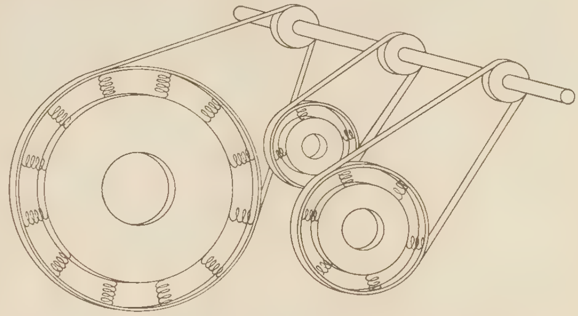


Fig. 1 Three Generators in Parallel Represented by Equivalent Mechanical System

It is this same torque that transmits energy from the shaft to the generator. This means that the output of the engine is proportional to this torque, the speed of the generator being practically constant. In other words the output of the engine increases with the phase angle. The rate of increase is given by the manufacturer of the generator as P_R. Value P_R is the increase in kilowatt output corresponding to an increase of 1 electric radian in the phase angle. For standard generators, the value of P_R at full load is approximately 3 times the kilowatt output. With decreasing load P_R decreases, and at no load the value is approximately 60 per cent of the value at full load.

It is now easy to figure the spring constant or synchronizing torque T_S.

$$T_S = \frac{63{,}000}{n} \cdot P_R \cdot 1.34 \cdot p \text{ lb-in. per mechanical radian}$$

where

$$\text{torque} = 63{,}000 \cdot \frac{\text{hp}}{n}$$

$$n = \text{engine speed, rpm}$$

$$1 \text{ kw} = 1.34 \text{ hp}$$

$$p \text{ electric radian} = 1 \text{ mechanical radian}$$

With f electric cycles per sec, we have $f \cdot 60 = p \cdot n$. The formula then will be

$$T_S = 5{,}060{,}000 \frac{P_R \cdot f}{n^2} \text{ lb-in. per mechanical radian}$$

In case the bus bar has absolutely constant frequency, which in Fig. 1 will correspond to the condition that the shaft is fixed in one position (disregarding the constant rotation), we will find what is called the natural frequency for an infinite system.

$$\omega_0^2 = \frac{T_S}{m} = 5{,}060{,}000 \frac{P_R \cdot f}{n^2} \cdot \frac{386}{WR^2 \cdot 144} = 13{,}600{,}000 \frac{P_R \cdot f}{n^2 \cdot WR^2}$$

$$N_0 = \frac{30}{\pi} \cdot \omega_0 = \frac{35{,}200}{n} \sqrt{\frac{P_R \cdot f}{WR^2}} \text{ vibrations per min}$$

[1] Mechanical Engineer, Nordberg Manufacturing Company. Jun. A.S.M.E.

Presented at the Fifteenth National Meeting of the Oil and Gas Power Division, Peoria, Ill., June 17–19, 1942, of The American Society of Mechanical Engineers.

where n = engine speed, rpm
P_R = kw per electric radian
f = electric cycles per sec
WR^2 = total lb-ft² in unit

In the United States the standard is $f = 60$ electric cycles per sec. This will bring the formula into the following form

$$N_0 = \frac{272{,}000}{n}\sqrt{\frac{P_R}{WR^2}}\text{ vpm}$$

When several generators run in parallel, considerable simplification will be achieved, if all mass moments of inertia and spring constants are referred to a system rotating at $60 \cdot f$ revolutions per minute. This will, in most cases, mean 3600 rpm. Electrical and mechanical degrees then will be identical. In this system we will have

$$T_S = \frac{63{,}000}{60 \cdot f} \cdot P_R \cdot 1.34 \text{ lb-in. rad}^{-1}$$

$$m = WR^2 \cdot \frac{144}{386}\left(\frac{n}{60 \cdot f}\right)^2 \text{ lb-in. sec}^2$$

For $f = 60$ we will get

$$T_S = 23.5 \cdot P_R \text{ lb-in. rad}^{-1}$$

$$m = \frac{WR^2 \cdot n^2}{34{,}700{,}000} \text{ lb-in. sec}^2$$

Modern generators are furnished with damper windings. Their effect is to provide a damping torque proportional to the velocity with which the phase angle changes. This damping is T_D lb-in. sec rad^{-1}. On most generators T_D has a magnitude of between 0.5 and 1.5 per cent of the synchronizing torque T_S. In this paper, an average value $T_D = 0.01\ T_S$ is used. It makes no difference here to which system the values are referred.

The impulses originate, in most cases, from irregularities in the driving machinery. To obtain an estimate of what may be expected from an internal-combustion engine, the following assumption is made: One half of the cylinders firing consecutively have a mean indicated pressure (mip) 10 per cent above the average mip, and the other half have a mip 10 per cent below the average. A greater unbalance is not to be expected, and in case it should occur, there will be justification in demanding a better adjustment of the fuel pumps.

The indicated torque of the engine will then be roughly as

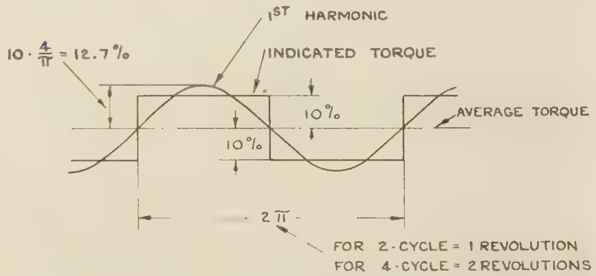


Fig. 2 Calculating Disturbing Impulse

shown in Fig. 2. The result will be a disturbing impulse, $I \sin \omega t = 0.127\ M_I \sin \omega t$, where M_I = mean indicated torque. With a mechanical efficiency of 80 per cent, this will correspond to $I = 0.16\ M_E$, where M_E is the mean effective torque.

This torque must be referred to the same system rotating $60 \cdot f$ rpm

$$I = 0.16 \cdot 63{,}000\ \frac{\text{hp}}{60 \cdot f}\text{ lb-in.}$$

For $f = 60$ we will finally get a disturbing impulse

$$I \sin \omega t = 2.8 \cdot \text{hp} \sin \omega t \text{ lb-in.}$$

where hp = the horsepower of the disturbing unit

$\omega = \frac{\pi}{30} \cdot N_D$

N_D = impulse frequency. For a two-cycle engine, N_D = rpm; for a four-cycle engine $N_D = {}^1/_2$ rpm.

It may be added that other disturbing impulses might occur but in most cases the foregoing one will be predominant.

All necessary data for computing the vibrations have now been described. The appearance of the vibrations, as previously mentioned, is a variation of the phase angle between bus bar and generator rotor. When this angle changes, two things happen: (1) A pulsating torque will be applied to the rotor and stator structures. The resulting stresses might loosen the magnetic coils or have other detrimental effects. (2) The output of the generator will change, and we will get a certain power pulsation between the generators on the network. The result might be flickering of the lights.

The magnitude of the vibrations is usually measured by percentage of power pulsation. If the output varies from $(K + L)$ to $(K - L)$, then the pulsation is $2L$. Most generators are designed so they momentarily can stand a power pulsation of 150 per cent of the rated output. For continuous operation, however, only 66 per cent is tolerated.

The parallel operation of two or more generators can now be investigated. As mentioned before, all values are referred to a rotation at one speed, which usually is 3600 rpm. The relative amplitude s of any one of the generators will determine the torque $s \cdot T_S$ on the unit as well as the power pulsation $2 \cdot s \cdot P_R = \frac{2 \cdot s \cdot T_S}{23.5}$. The bus bar has no capacity for absorbtion of power so $\Sigma s \cdot P_R = 0$. Consequently, we also have $\Sigma s \cdot T_S = 0$ which means that the combined torque on the bus bar from all the generators is 0. Confirmation is thus provided that it is correct to compute the vibrations of the mechanical system, Fig. 1, instead of the real system with electrically connected generators.

Two Generators in Parallel

The conception is quite simple when there are only two generators on the network. The two masses m_1 and m_2 are connected to the bus bar through springs with spring constants T_{S1} and T_{S2}, Fig. 3. So far no damping is considered. The two natural frequencies for an infinite system are found by

$$\omega_1^2 = \frac{T_{S1}}{m_1} \qquad \text{and} \qquad \omega_2^2 = \frac{T_{S2}}{m_2}$$

The bus bar has no mass, so it is evident that the system in Fig. 3 may be replaced by the system in Fig. 4. We have here the same two masses m_1 and m_2 directly connected with a spring having spring constant T_S

$$\frac{1}{T_S} = \frac{1}{T_{S1}} + \frac{1}{T_{S2}}$$

The natural frequency of such a system is found by

$$\omega_0^2 = \frac{T_S(m_1 + m_2)}{m_1 \cdot m_2} = \frac{T_{S1} \cdot \omega_2^2 + T_{S2} \cdot \omega_1}{T_{S1} + T_{S2}}$$

$$N_0 = \frac{30}{\pi} \cdot \omega \qquad\qquad T_S = k \cdot P_R$$

$$N_0 = \sqrt{\frac{P_{R1}\cdot N_2^2 + P_{R2}\cdot N_1^2}{P_{R1} + P_{R2}}} \text{ vibrations per min}$$

where N_1 and N_2 are the two natural frequencies for an infinite system; N_0 will always be between N_1 and N_2.

It is now supposed that an impulse $I \sin \omega t$ acts upon m_1. The two masses then will vibrate with movements $a_1 \sin \omega t$ and $a_2 \sin \omega t$. The relative amplitude is $a_r = a_1 - a_2$. Value a_r will

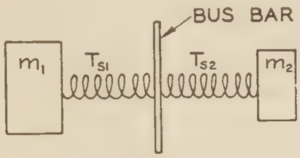


Fig. 3 Two Generators on Network, Represented by Two Masses Connected to Bus Bar Through Springs

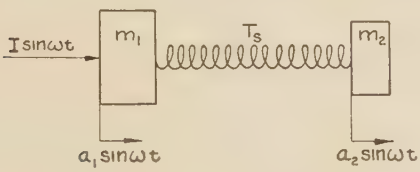


Fig. 4 System of Fig. 3 Replaced by One With Bus Bar Eliminated

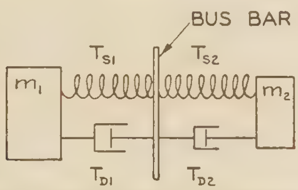


Fig. 5 System of Fig. 3 With Damping Introduced

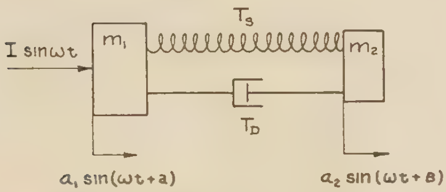


Fig. 6 System of Fig. 5 Replaced by One With Bus Bar Eliminated

determine the torque $T_S\cdot a_r$ and the power pulsation $\frac{2\cdot T_S\cdot a_r}{23.5}$ between the two generators. In order to find a_r we will consider the instant of maximum amplitude. Acceleration force and spring force on m_2 must here total zero

$$T_S\cdot a_r + m_2 a_2 \omega^2 = 0$$

$$a_2 = -a_r \frac{T_S}{m_2\cdot\omega^2}$$

but
$$\omega_0^2 = \frac{T_S(m_1 + m_2)}{m_1\cdot m_2}$$

so
$$a_2 = -a_r \frac{\omega_0^2\cdot m_1}{\omega^2(m_1 + m_2)}$$

and
$$a_1 = a_r + a_2 = a_r \frac{\omega^2(m_1 + m_2) - \omega_0^2\cdot m_1}{\omega^2(m_1 + m_2)}$$

The forces on the system as a whole from acceleration and impulse also must total zero.

$$I + m_1 a_1 \omega^2 + m_2 a_2 \omega^2 = 0$$

With the help of the equations for a_1 and a_2 we then get

$$I + a_r m_1(\omega^2 - \omega_0^2) = 0$$

$$a_r = \frac{I}{m_1\omega^2}\cdot\frac{1}{\frac{\omega_0^2}{\omega^2} - 1}$$

This formula gives the relative movement of the two generators when no damping is considered.

The influence of the damping will be examined at resonance only, which may be represented by the system shown in Fig. 5. In this case, we have as before the two masses m_1 and m_2 connected to the bus bar by springs with spring constants T_{S1} and T_{S2}. In addition, however, we now have damping in the system, the two damping coefficients being T_{D1} and T_{D2} lb-in. sec rad^{-1}.

To simplify this system in the same manner as before, it is assumed that spring constants and damping coefficients for the two generators are proportional

$$\frac{T_{D1}}{T_{S1}} = \frac{T_{D2}}{T_{S2}}$$

This assumption is fairly correct as said proportion in most cases is around 1 per cent. If the assumption is not correct, and an exact analysis is desired, a method must be used similar to the one developed later for several generators in parallel. This means that one of the generators must be replaced by an equivalent generator with the correct damping.

Using the foregoing assumption, it will be seen that if, at certain amplitudes, the two spring forces equal each other, then the two damping forces will also be equal to each other. The bus bar has no mass, so this means that we again can forget about the bus bar and connect the two masses directly, as shown in Fig. 6.

$$\frac{1}{T_S} = \frac{1}{T_{S1}} + \frac{1}{T_{S2}}$$

$$T_D = T_S \frac{T_{D1}}{T_{S1}}$$

When the impulse $I \sin \omega t$ acts upon m_1, the result will be amplitudes $a_1 \sin(\omega t + \alpha)$ and $a_2 \sin(\omega t + \beta)$, as shown in Fig. 6. The forces acting upon m_2 are spring force, damping force, and acceleration force. The vector diagram for these forces is shown in Fig. 7. The angle γ is comparatively small so we have approximately

$$T_S\cdot a_r = m_2\cdot a_2\cdot\omega^2$$

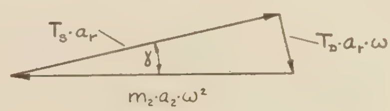


Fig. 7 Vector Diagram of Forces

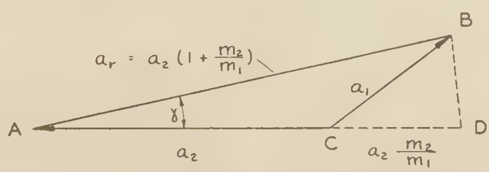


Fig. 8 Vector Diagram of Movements

We consider here only the case of resonance, therefore

$$\omega^2 = \frac{T_S(m_1 + m_2)}{m_1 \cdot m_2}$$

$$a_r = \frac{m_2 \cdot a_2 \cdot \omega^2}{T_S} = a_2\left(1 + \frac{m_2}{m_1}\right)$$

In Fig. 8 is given a vector diagram for the movements. This same diagram may be used as a force diagram for the forces acting on the system as a whole from acceleration and impulse. Each radian then must correspond to a torque of $m_1\omega^2$.

The acceleration force on m_1 is $m_1 \cdot a_1 \cdot \omega^2$ and is represented by $CB = a_1$. The acceleration force on m_2 is $m_2 \cdot a_2 \cdot \omega^2$ and is represented by $DC = a_2\frac{m_2}{m_1}$. The third force DB, which is necessary for equilibrium, is then the impulse which we must have to excite the vibrations

$$I = DB \cdot m_1\omega^2$$

However, the triangle ABD is almost similar to the triangle in Fig. 7, so with approximation we have

$$\frac{AB}{DB} = \frac{T_S}{T_D \cdot \omega}$$

$$I = \frac{a_r \cdot T_D \cdot \omega}{T_S} \cdot m_1 \cdot \omega^2$$

$$a_r = \frac{I}{m_1 \cdot \omega^2} \cdot \frac{T_S}{T_D \cdot \omega}$$

This formula gives the relative movement of the two generators at resonance.

It is now possible to establish a method of procedure for finding the relative movement a_r of two generators in parallel. First find the static movement $a_s = \frac{I}{m_1 \cdot \omega^2}$ caused by an impulse $I \sin \omega t$ on the free mass m_1. Mass m_1 is that of the unit on which the disturbing impulse actually does act. The relative movement is then $a_r = a_s \, . \, F$, i.e., the static movement multiplied by the magnification factor F. Value F equals $\frac{1}{\frac{\omega_0^2}{\omega^2} - 1}$ but cannot exceed the value at resonance $F = \frac{T_S}{T_D \cdot \omega}$. With the average value $T_D = 0.01 \cdot T_S$ we get $F = \frac{100}{\omega}$.

The power pulsation between the two generators now is

$$B = 2 \cdot a_r \cdot P_R$$

$$P_R = \frac{T_S}{23.5} \quad \text{or} \quad \frac{1}{P_R} = \frac{1}{P_{R1}} + \frac{1}{P_{R2}}$$

Example. An example based on an actual installation with a two- and a four-cycle engine in parallel, will now be given. The electric frequency is 60 cycles per sec, so the corresponding formulas are used.

Unit No. 1 is a four-cycle engine, 520 bhp, 350 kw, WR^2 = 470,000 lb-ft², 200 rpm, and P_R = 1880 kw. The natural frequency for an infinite system is

$$N_1 = \frac{272{,}000}{n}\sqrt{\frac{P_R}{WR^2}} = \frac{272{,}000}{200}\sqrt{\frac{1880}{470{,}000}} = 86 \text{ vpm}$$

Unit No. 2 is a two-cycle engine, 1500 bhp, 1000 kw, WR^2 = 186,500 lb-ft², 257 rpm, and P_R = 2290 kw. The natural frequency for an infinite system is

$$N_2 = \frac{272{,}000}{257}\sqrt{\frac{2290}{186{,}500}} = 117 \text{ vpm}$$

The natural frequency of the system of two generators is

$$N_0 = \sqrt{\frac{P_{R1} \cdot N_2^2 + P_{R2} \cdot N_1^2}{P_{R1} + P_{R2}}} = \sqrt{\frac{1880 \cdot 117^2 + 2290 \cdot 86^2}{1880 + 2290}} = 101 \text{ vpm}$$

This practically gives resonance with the frequency of the disturbing impulses from unit No. 1, which is N_D = 100 per min. The resulting power pulsation must now be found.

$$I = 2.8 \cdot \text{hp} = 2.8 \cdot 520 = 1450 \text{ in-lb}$$

$$m_1 = \frac{WR^2 \cdot n^2}{34{,}700{,}000} = \frac{470{,}000 \cdot 200^2}{34{,}700{,}000} = 540 \text{ lb-in. sec}^2$$

$$\omega = \frac{\pi}{30} \cdot N_D = \frac{\pi}{30} \cdot 100 = 10.5 \qquad \omega^2 = 110$$

$$a_s = \frac{I}{m_1 \cdot \omega^2} = \frac{1450}{540 \cdot 110} = 0.0244 \text{ radian}$$

$$F = \frac{1}{\frac{\omega_0^2}{\omega^2} - 1} = \frac{1}{\frac{101^2}{100^2} - 1} = 50$$

or $$F = \frac{100}{\omega} = \frac{100}{10.5} = 9.5$$

The smaller value is used for F

$$a_r = a_s \cdot F = 0.0244 \cdot 9.5 = 0.232 \text{ radian}$$

$$\frac{1}{P_R} = \frac{1}{P_{R1}} + \frac{1}{P_{R2}} = \frac{1}{1880} + \frac{1}{2290} = \frac{1}{1030 \text{ kw}}$$

Power pulsation is

$$B = 2 \cdot a_r \cdot P_R = 2 \cdot 0.232 \cdot 1030 = 480 \text{ kw}$$

This is 137 per cent of the rated output of unit No. 1 and much more than can be tolerated. It must be added that this result is based upon the previous estimate made of the unbalanced torque of an internal-combustion engine. Whether this estimate corresponds to the actual condition when the two engines were operated in parallel is not known. It is known, however, that the lights on the network flickered.

Several Generators in Parallel

Fig. 9 illustrates the case of several generators running in parallel. The damping so far is not taken into consideration. This condition is actually the same as that shown in Fig. 1.

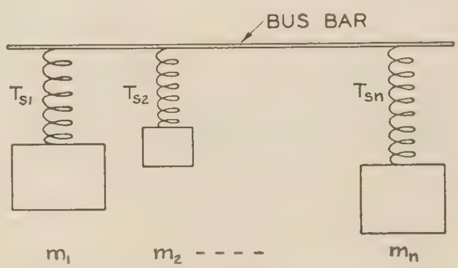


Fig. 9 Representation of Generators Running in Parallel

The natural frequency N_0 of this system is found as follows: Assume that the system is vibrating with a forced frequency N and that the bus bar has an amplitude r. The impulse I that

excites these vibrations acts directly on the bus bar. No outer forces act on any one of the generators so the vibration of each generator will depend upon N and r only. Each generator then will react on the bus bar with a certain force, and the total of these forces will determine the impulse I necessary to keep up the vibrations.

We will now keep the amplitude r of the bus bar constant and vary the forced frequency N. The necessary impulse I then will vary too and will be zero at some frequencies. These are the natural frequencies N_0 of the system.

Let the movement of the bus bar be $r \sin \omega t$ and let the movement of a generator be $v_n \sin \omega t$. At maximum amplitude, the forces on the generator mass m_n from spring and acceleration must total zero.

$$(r - v_n)\cdot T_{Sn} + m_n \cdot v_n \cdot \omega^2 = 0$$

But the natural frequency for an infinite system is found by

$$\omega_n^2 = \frac{T_{Sn}}{m_n}$$

so

$$v_n = \frac{r \cdot \omega_n^2}{\omega_n^2 - \omega^2}$$

The force reacting on the bus bar is

$$K_n = T_{Sn}(v_n - r) = r\omega^2 \frac{T_{Sn}}{\omega_n^2 - \omega^2}$$

All these forces must total zero if ω shall be the natural angular velocity ω_0, so

$$\Sigma \frac{T_{Sn}}{\omega_n^2 - \omega_0^2} = 0$$

or

$$\Sigma \frac{P_{Rn}}{N_n^2 - N_0^2} = 0$$

Either equation may be used for determining the natural frequency N_0 of the system. The second one is more convenient as the natural frequency for infinite system N_n usually is figured for all the generators on the network

$$\frac{P_{R1}}{N_1^2 - N_0^2} + \frac{P_{R2}}{N_2^2 - N_0^2} + \cdots\cdots \frac{P_{Rn}}{N_n^2 - N_0^2} = 0$$

This equation has $n - 1$ solutions. They may be found analytically but that is quite a problem if the number of generators is higher than three. A graphical solution, however, will make the problem quite simple. This is shown in Fig. 10 for the case with three generators, but it is evident that the same method may be employed for any number of generators.

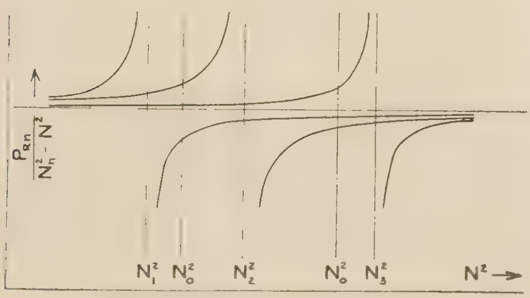


FIG. 10 GRAPHICAL SOLUTION FOR NATURAL FREQUENCY OF SYSTEM HAVING THREE GENERATORS

In Fig. 10 values are plotted for $\frac{P_{Rn}}{N_n^2 - N^2}$ for all three generators on the network with N^2 as abscissa. The total of the amplitudes will be zero for two values of N^2, which are easily found by a few trials, and these two values will correspond to the two natural frequencies N_0. It will be seen that there is always a solution for N_0 between consecutive values of the natural frequency for an infinite system for the various generators.

All these solutions, however, do not correspond to critical speeds, because the influence of the damping on a vibrating system of this kind is quite considerable. The damping not only limits the vibrations at resonance, but it changes the form of vibration to an extent that in some cases it will be possible to operate safely at what is believed to be a resonance point.

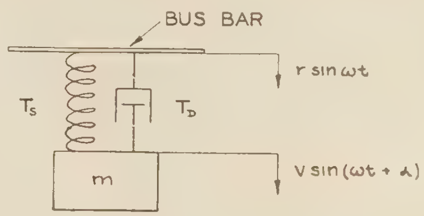


FIG. 11 VIBRATION OF A GENERATOR EXCITED FROM THE BUS BAR ONLY

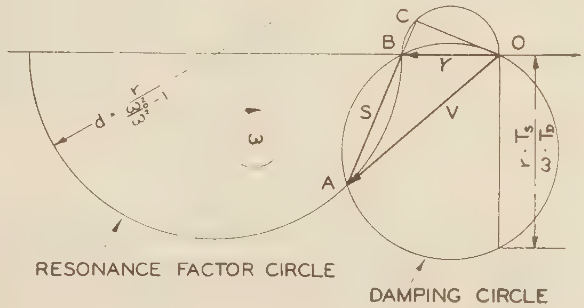


FIG. 12 VECTOR DIAGRAM FOR MOVEMENTS IN SYSTEM

A method has been developed to find the power pulsation in such a system with several generators. The basic idea is to substitute the system of n generators with a system of only two. One of these is the generator on which the disturbing impulse acts. The other one is equivalent to all the other generators. On this system the previously found formulas may be applied.

In order to use this method when there are several disturbing impulses in the system, it is necessary to take one impulse at a time. The resulting power pulsations must afterward be totaled in some way.

That the generator mentioned is equivalent to all the generators on which no outer forces act means this: The equivalent generator will react on the bus bar with the same force as all the other generators together. However, as will be seen later, the use of this equivalent generator is possible only as long as the forced frequency does not change. A different forced frequency will necessitate a different equivalent generator.

The vibration of the generators without impulse is excited by the movement of the bus bar only. Therefore, in order to find the reaction on the bus bar from these generators, we will first study the vibration $v \sin (\omega t + \alpha)$ of a mass m, connected to the bus bar through a spring constant T_S and a damping coefficient T_D, Fig. 11. The excitation for the vibration is a movement of the bus bar $r \sin \omega t$. Fig. 12 is a vector diagram for the movements. Value r is the vector for the bus bar, v is the vector for

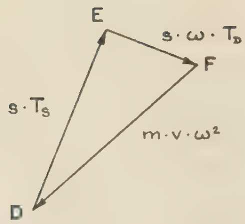


Fig. 13 Diagram of Forces

the generator mass m, and s is the vector for the relative movement.

A diagram of the forces on m is shown in Fig. 13. There are three forces. The spring force $s \cdot T_S$ proportional to the relative amplitude s, the damping force $s \cdot \omega \cdot T_D$ proportional to the relative velocity $s \cdot \omega$ and perpendicular to the spring force, and the acceleration force $m \cdot v \cdot \omega^2$ proportional to the absolute amplitude v. This force diagram is similar to the triangle ACO in Fig. 12, as the angle C is made 90 deg.

$$\frac{AC}{s \cdot T_S} = \frac{v}{m \cdot v \cdot \omega^2}$$

The natural frequency for an infinite system of this mass is determined by

$$\omega_0^2 = \frac{T_S}{m}$$

So

$$AC = s \cdot \frac{\omega_0^2}{\omega^2}$$

$$\frac{AB}{BC} = \frac{s}{s \cdot \dfrac{\omega_0^2}{\omega^2} - s} = \frac{1}{\dfrac{\omega_0^2}{\omega^2} - 1}$$

The angle C is 90 deg so the point C must be somewhere on a circle with diameter $BO = r$. As a consequence, point A must be somewhere on a large circle, called the resonance-factor circle, with diameter

$$d = \frac{r}{\dfrac{\omega_0^2}{\omega^2} - 1}$$

Another determination for the location of A is given by

$$\text{angle } A = \text{angle } D = \text{arc } tg \, \frac{\omega \cdot T_D}{T_S}$$

This means that point A also must lie somewhere on a circle, called the damping circle, in which arc $tg \, \dfrac{\omega \cdot T_D}{T_S}$ is inscribed. For the standard proportion $T_D = 0.01 \, T_S$ this will mean arc $tg \, \dfrac{\omega}{100}$ where ω is the angular velocity for the forced frequency.

The location of point A is now found by the crossing of the resonance-factor circle and the damping circle. The amplitudes for the system in Fig. 11, are thus determined by a simple graphical method.

When several generators without impulse are connected to the bus bar, all the vectors for their vibration may be determined in this way, Fig. 14.

It is here assumed that the proportion T_D to T_S is the same for all the generators. This assumption is not necessary for the solution of the problem, but it is thus made possible to use the same damping circle for the various generators.

The generators will react on the bus bar with vectors $m \cdot v \cdot \omega^2$ and the resultant R of these vectors will determine the combined force on the bus bar from all the generators without impulse, Fig. 15. It may be added that this diagram may be simplified by omitting ω^2; the result will be the same.

The equivalent generator m_E can now be determined. A line in Fig. 14, parallel to the resultant R, will give us $OA = v_E$, the amplitude of the equivalent generator. The mass is then determined by

$$m_E = \frac{R}{v_E \cdot \omega^2}$$

The natural frequency for infinite system N_E of this equivalent unit is found by taking into consideration that A is also a point of the resonance-factor circle with $d = \dfrac{r}{\dfrac{\omega_E^2}{\omega^2} - 1}$. This means that d may be determined, and then ω_E and N_E. The synchronizing torque for the unit then is $T_{SE} = m_E \cdot \omega_E^2$. The damping coefficient T_{DE} is determined by the same proportion T_D to T_S as for the other generators. If a different proportion is desirable, a special damping circle must be drawn for the equivalent unit.

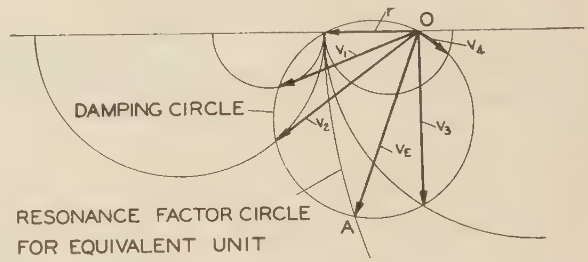


Fig. 14 Diagram for Determining Vectors for Vibrations of System Consisting of Several Generators

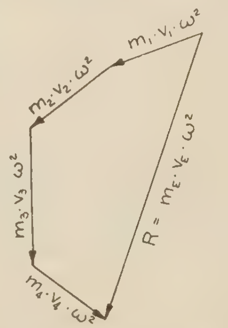


Fig. 15 Combined Force on Bus Bar From All Generators

A generator equivalent to all the generators without impulse is now determined with mass moment of inertia m_E, synchronizing torque T_{SE} and damping coefficient T_{DE}. From the way this equivalent generator was determined, it will be seen that it will depend upon the forced frequency. A different forced frequency will give a different equivalent generator. Therefore this equivalent generator can be used only for the same frequency for which it was computed.

The system of n generators has now been reduced to a system

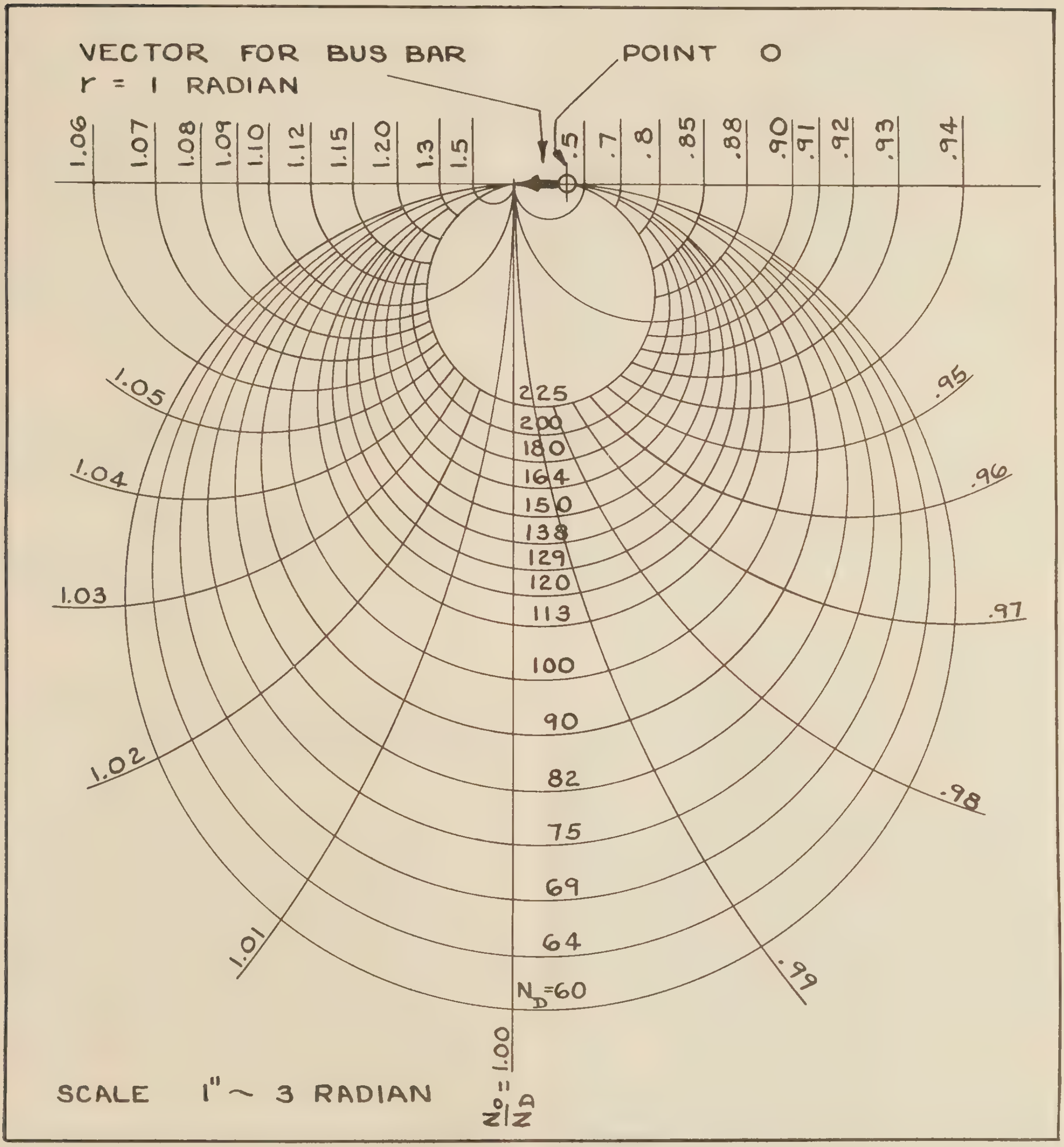


Fig. 16 Diagram for Equivalent Generator

Damping circles are based on $T_D = 0.01 \cdot T_s$. Center Distance from base line $= \frac{50}{\omega}$. Value $\omega = \frac{\pi}{30} \cdot N_D$

Resonance-factor circles have diameter $D = \dfrac{1}{\dfrac{N_0^2}{N_D^2} - 1}$

N_D = frequency of impulse
N_0 = natural frequency for infinite system

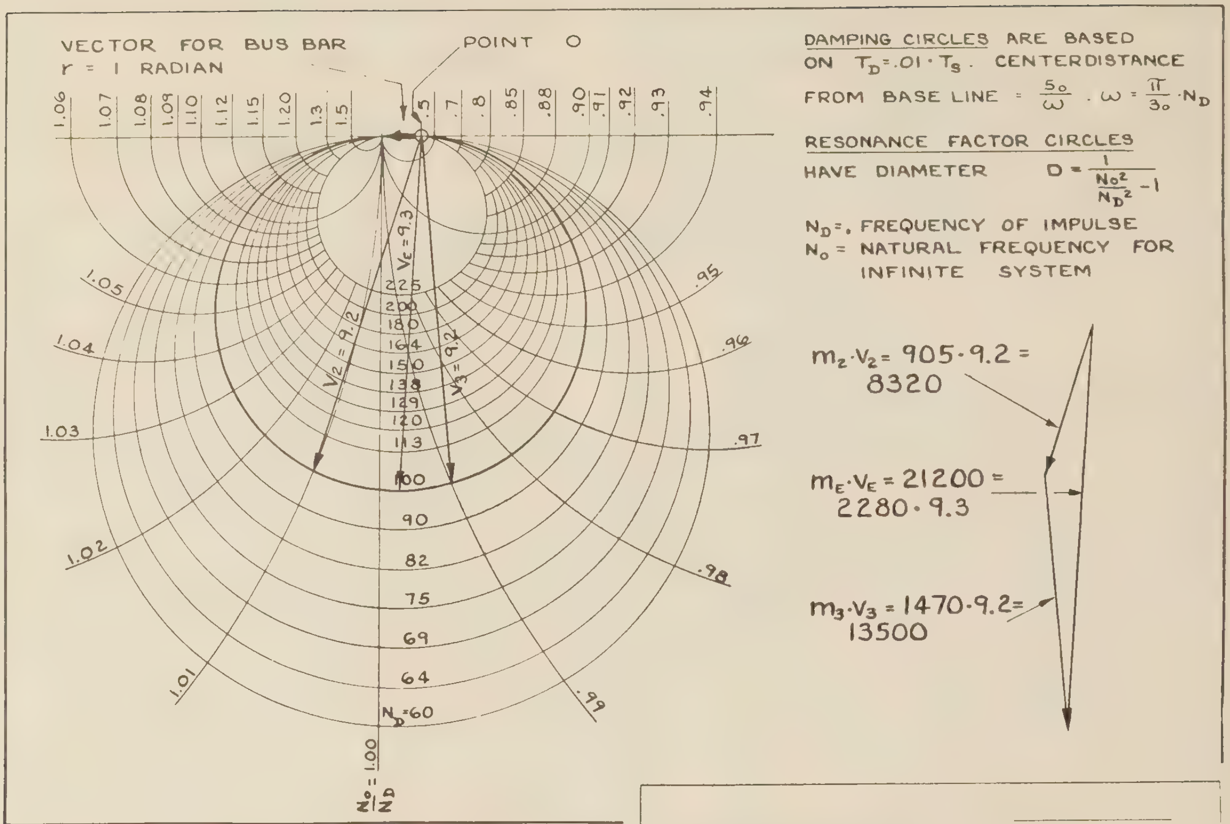


FIG. 18 RESULTANT EQUIVALENT GENERATOR DIAGRAM

of only two. One of these is the generator on which the disturbing impulse acts and the other one is the equivalent generator. By applying the previously found formulas, it is easy to find the power pulsation B_D kilowatts for this system. Term B_D will then be the pulsation for the unit with the disturbing impulse.

To find the pulsation for the rest of the generators, it must be remembered that the power pulsation for any generator is proportional to the torque with which it reacts on the bus bar. This is approximately also true when there is damping in the system. Therefore the pulsation B_n of any one of the generators without impulse may be found with help of the diagram Fig. 15.

$$B_n = B_D \frac{m_n \cdot v_n \cdot \omega^2}{m_E \cdot v_E \cdot \omega^2} = B_D \frac{m_n \cdot v_n}{m_E \cdot v_E}$$

The work of finding this equivalent unit is simplified greatly by using a copy of the diagram, Fig. 16. In this diagram is drawn a complete set of resonance-factor circles and damping circles, corresponding to an amplitude of 1 radian for the bus bar. The resonance-factor circles have diameters

$$d = \frac{1}{\frac{\omega_0^2}{\omega_D^2} - 1} = \frac{1}{\frac{N_0^2}{N_D^2} - 1}$$

where N_D is the forced frequency and N_0 is the natural frequency for an infinite system. The damping circles have their centers at a distance from the base line of $\frac{50}{\omega_D}$ corresponding to the average value of the damping coefficient $T_D = 0.01 \cdot T_S$. Term ω_D is the angular velocity of the forced frequency which, in most cases, is

POWER PULSATION

DATE ________
ORDER NO ________

UNIT NO.	$m = \frac{WR^2\, n^2}{34{,}700{,}000}$ LB IN SEC^2	$T_S = 23.5 \cdot P_R$ LB IN RAD^{-1}	$\omega_0^2 = \frac{T_s}{m}$	$N_0 = \frac{30}{\pi} \cdot \omega_0$ ~/MIN	POWER PULSATION $B = B_D \frac{m \cdot v}{m_E \cdot v_E}$ KW	POWER PULSATION %
1	1065	52500	49.3	67	230	29
2	905	100200	111	101	[illegible]	6
3	1470	157500	[illegible]	[illegible]	145	5
m_E	2260	250000	110	100	230	

m_E IS EQUIVALENT TO ALL THE UNITS WITHOUT IMPULSE
$m_E \cdot \overline{V}_E = \Sigma\, m \cdot \overline{V}$ (SEE DIAGRAM)

DISTURBANCE FROM UNIT NO. 1 ($= m_D$)
N_D = 100 ~/MIN $\omega_D = \frac{\pi}{30} \cdot N_D$ = 10.5 ω_D^2 = 110
BHP = 1200 IMPULSE I = 2.8 · BHP = 3360 LB IN

m_D AND m_E IN PARALLEL

$\frac{1}{T_S} = \frac{1}{T_{SD}} + \frac{1}{T_{SE}} = \frac{1}{52500} + \frac{1}{250000}$ T_S = 43400 LB IN RAD^{-1}

$\omega_0^2 = \frac{T_S (m_D + m_E)}{m_D \cdot m_E} = \frac{43400(1065 + 2260)}{1065 \times 2260}$ = 59.5 N_0 = 74 ~/MIN

STATIC MOVEMENT: $a_s = \frac{I}{m_D \cdot \omega_D^2} = \frac{3360}{1065 \times 110}$ = .0287 RADIAN

MAGNIFICATION: $F = \frac{1}{\frac{\omega_0^2}{\omega_D^2} - 1}$ = 2.18 NOT MORE THAN: $F = \frac{100}{\omega_D}$ = 9.5

POWER PULSATION FOR DISTURBING UNIT: $B_D = \frac{a_s\, F\, T_S}{11.75}$ = 230 KW

FIG. 17 CALCULATIONS FOR POWER PULSATION

one half of a synchronous speed. Consequently circles are drawn corresponding to one half of the usual synchronous speeds.

Using this diagram, it is very easy to determine the vectors in Fig. 14. All that is left then is to make the force diagram in Fig. 15.

The entire method will now be illustrated by an example with three generators in parallel. It is evident, however, that the same method can be used for any number of generators, with but little increase in the work involved.

Example. There are three engines on the network, one four-cycle and two two-cycle engines. The electric frequency is 60 cycles per sec so the corresponding formulas are used.

Unit No. 1 is a four-cycle engine, 1200 bhp, 800 kw, WR^2 = 925,000 lb-ft², 200 rpm, and P_R = 2230 kw. The natural frequency for an infinite system is

$$N_1 = \frac{272{,}000}{n}\sqrt{\frac{P_R}{WR^2}} = \frac{272{,}000}{200}\sqrt{\frac{2230}{925{,}000}} = 67 \text{ vpm}$$

Unit No. 2 is a two-cycle engine, 2250 bhp, 1585 kw, WR^2 = 618,000 lb-ft², 225 rpm, and P_R = 4260 kw. The natural frequency for an infinite system is

$$N_2 = \frac{272{,}000}{225}\sqrt{\frac{4260}{618{,}000}} = 101 \text{ vpm}$$

Unit No. 3 is a two-cycle engine, 3850 bhp, 2715 kw, WR^2 = 1,005,000 lb-ft², 225 rpm, and P_R = 6700 kw. The natural frequency for an infinite system is

$$N_3 = \frac{272{,}000}{225}\sqrt{\frac{6700}{1{,}005{,}000}} = 99 \text{ vpm}$$

To find the natural frequency N_0 for the system of three generators, curves could be drawn as shown in Fig. 10. In this case, however, it will not be necessary. We know there must be a solution for N_0 between 99 and 101 vibrations per min, and that must be rather close to 100 which is the frequency of the disturbing impulses on unit No. 1.

A computation, however, will show that the resulting power pulsation is not large. The calculations are arranged in Fig. 17 with the diagram for the equivalent generator in Fig. 18.

This example illustrates clearly the influence of damping on a vibrating system of this kind. Without damping in the system, there is a natural frequency of 100 vibrations per min. The masses of units Nos. 2 and 3 must then vibrate in opposite phase, 180 deg apart. The damping, however, changes this angle to a few degrees. The two masses no longer vibrate against each other but practically in the same phase. Their effect on the bus bar is almost the same as of one large generator with mass $m_E = m_2 + m_3$.

Discussion

E. J. Kates.[2] The author is to be congratulated on his simplification of a most complicated and often troublesome problem. The paper should be of great help to engineers faced with the problem of operating, in parallel, generating units having differing characteristics, which introduces difficulties not encountered when the units are alike.

V. L. Maleev.[3] The method presented in this paper seems to be original, logical, and to the point, and the author should be congratulated upon developing it.

In connection with the presentation, however, it is regrettable that the author did not number his numerous formulas and equations, as such numbering would aid their use and discussion.

The reading of the paper is also made more difficult by lack of explanation in respect to several symbols. Thus, there is no definition of ω ($\omega_0, \omega_1, \omega_2$). Seemingly it is the conventional designation for the number of free oscillations per sec. Other symbols, as n for rpm and f for cycles per sec are explained twice.

The symbol m is also not explained clearly. In one equation its dimension is given as lb-in. sec², later it is called a mass, but the dimension of a mass is lb/(ft/sec²).

The use of M_i for mean indicated torque is confusing, since torque is designated usually and in the same paper by T. The term "impulse" means the product of a force by the time interval during which it acts. Applying it to a torque is wrong and confusing.

If the author would refer to a standard text from which he took his symbols and formulas, and use them consistently, the reader would benefit considerably.

The example, based on two- and four-cycle engines in parallel, is very helpful for an understanding of the method and would be still more helpful for actual use of the method if the equations applied were numbered.

Also it seems regrettable that the author took for an example abnormal conditions in respect to P_R. If the generators had standard P_R = 2.7 kw, the natural frequency of the system would be N_0 = 85 vpm, the value of P_R for the system would be 697 kw, and the power pulsation B = 122.4 kw, or only 35 per cent of the rated output of unit No. 1.

Finally, the writer believes that, for the benefit of designers and builders of internal-combustion engines, the paper could be continued and an analysis made, whether it is possible by changing the WR^2 of the engines to eliminate an excessive power pulsation. If this is possible, it would be interesting to see in what direction and by how much WR^2 should be changed; for instance in the example cited, using the same not-standard generators, to eliminate objectionable pulsation.

There seems also to be an interesting possibility of applying the author's method for computing the flywheel effect (WR^2) necessary for parallel operation of generators driven by internal-combustion engines. So far the required coefficient of steadiness was given more or less arbitrarily.

Harte Cooke.[4] The writer has been interested in the author's analysis of the oscillation of various generators when running in parallel. One of the conditions, however, which causes trouble is something not easily susceptible of mathematical analysis.

When any oscillation of the generator takes place, be it from irregularities in the turning moment, such as the author outlined, or from any outside disturbance, the acceleration of the generator will affect the governor. This change in the governor will cause a slight displacement of the fuel racks, giving a slight increased increment of fuel.

This slight increase in the fuel increment will cause an acceleration of the engine, but this acceleration will be delayed by a certain time interval from the increase in velocity which caused it. Now, if this increased power increment, caused by the movement of the governor, happens to synchronize with the regular swing of the generator, it will tend to build up the swing of the generator and, as this swing builds up, the power increment caused by the action of the governor will be increased slightly each time. This has been known to be enough to cause the generator to go out of step.

There is another difficulty which might occur in the operation of generators in parallel, i.e., the generators might be unstable in

[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[3] Research Professor of Mechanical Engineering, Oklahoma A.&M. College, Stillwater, Okla. Mem. A.S.M.E.

[4] Engineer, Diesel Division, American Locomotive Company, Auburn, N. Y. Fellow, A.S.M.E. Deceased.

themselves. When a generator is displaced in phase from the system or from another generator, there is a difference in voltage. This difference causes a current to flow between the two machines. However, due to the resistance, inductance, etc., there is a certain time delay between the difference in voltage and the current flow which it causes. The flow of current causes a magnetizing effect in the fields which tends to bring the generators back into synchronism. However, due to hysteresis, etc., there is a certain time delay between the flow of the current and the magnetizing effect which it causes. The effect of these time delays is that, if we consider the displacement of the swing of the generator as a sine curve, the sine curve of the current will be displaced a little and the sine curve for the torque will be displaced still further.

Now with the sine curve, which is divided at the 90-deg points, the areas during each 90 deg will be equal. However, because of this displacement of the torque curve, the areas under it will no longer be equal, and the torque, which tends to bring the generators together, will be greater than the torque which tends to restrain them after they pass the synchronous point. If this difference in area gives a greater effect than the losses in the system due to this, the swing of the generators will become cumulative electrically without any assistance from the engine.

I. C. Benson.[5] This paper presents a clever mechanical analogy in which several generators operating in parallel are shown and mathematically analyzed as a mechanical system having masses, springs, and damping effect operating upon the same shaft, which shaft corresponds to the electrical bus bar. The method avoids the long, cumbersome calculations usually utilized and still includes the effect of electrical damping forces.

The paper presents a simple and concise theory and method for calculating and determining whether or not satisfactory parallel operation of generators will result, including operation near resonance. This represents a distinct advance in the art, even though for years synchronous motors have been operated near resonance when driving certain compressors and having certain damping effects. Also, in many cases, this results in a saving in flywheel effect, shaft size, bearing size, efficiency, over-all space, etc.

In both generator and motor applications connected to reciprocating forces, the damping force plays an important role along with the other system characteristics in determining the resulting operation. The damping force is due to the damper winding and field winding cutting the magnetic field, thereby producing torque which is in the opposite direction from that of the varying reciprocating forces of the engine or compressor, respectively. The damping effect varies widely with the design of the cage as illustrated in Mr. Linville's paper.[6]

The average generator operates at approximately 20 electrical deg at which displacement, the per unit damping torque on the 77,500-kva generator being approximately 0.01 due to the field winding alone, varies from 0.03 to 0.08, depending upon the percentage span of the bars in each pole for open damper windings, and is 0.35 for a closed damper winding (one having connections between end rings of adjacent poles). These values refer to an unusual generator having high speed and large capacity, but the values will serve to indicate the effects of the damper-winding design. Accurate calculations for determining the damping force is a laborious task, and therefore some assigned values should undoubtedly be set up for practical applications for use in the author's method. These values should be carefully set up by the generator manufacturers so as to be conservative but still provide satisfactory operating characteristics.

[5] Electric Machinery Manufacturing Company, Minneapolis, Minn.

[6] "Starting Performance of Salient Pole Synchronous Motors," by T. M. Linville, Trans. A.I.E.E., vol. 49, 1930, Fig. 11(a), pp. 531–547.

The graphical solution for the natural frequencies of a system of n generators for $(n - 1)$ frequencies provides a convenient and accurate method which arrives at the desired frequency by the use of a few calculations.

The circular diagram, showing the vectors of the bus bar along with those of the generators, illustrates the resulting operation, both above and below the forced frequency. It serves to clarify and explain the resulting operation of two or more generators operating in parallel, and therefore constitutes a welcome advancement in the predetermination of parallel operation of alternating-current generators.

F. K. Brainard.[7] When modern two-cycle, multicylindered Diesel-engine generating units are installed in old alternating generating stations having four-cycle units, the selection of suitable flywheels for the new units frequently becomes quite important. The two-cycle units, by themselves, generally require comparatively light flywheels, but the problem usually is to take care of the possible low-frequency impulses which may be introduced by irregular operation of the four-cycle engines.

In some cases, it is not feasible to keep all of the natural frequencies well away from the frequencies of the impulses which may be introduced by the four-cycle engines, and it becomes necessary to take the damping of the generators into consideration in computing power pulsation.

The author is to be congratulated for having developed a method for handling such cases, and for having shown that it is often feasible to operate with flywheels which would not be considered permissible if calculated by methods neglecting damping. This emphasizes the importance of damper design in such cases.

It is usually possible, of course, to design the dampers of a particular generator to be most effective for damping impulses of the frequency that is most likely to cause trouble, and for this reason close co-operation between the generator and engine builders is most desirable.

Has the author tried solving this problem by the method of mechanical impedance? This is the application of the algebra of complex quantities to the solution of problems involving mechanical vibration and has been used quite extensively for that purpose. This method can easily be applied to any number of generators without the necessity of substituting a single fictitious generator for those whose engines are not producing the unbalanced impulse, as the author has done, and seems to provide a simpler and more complete solution of the problem.

The case of two machines is analogous to a problem solved by A. R. Kimball.[8]

The case involving "n" machines has been solved by a member of our department, the final equations being as follows:

Equations for Calculating Power Pulsation Between n Synchronous Generators Resulting From an Unbalanced Impulse Applied to One of Them

$$q_1 = \frac{1 + j\,\dfrac{\omega T_{d1}}{P_{r1}}}{1 - \left(\dfrac{N_x}{N_1}\right)^2 + j\,\dfrac{\omega T_{d1}}{P_{r1}}}$$

$$q_n = \frac{1 + j\,\dfrac{\omega T_{dn}}{P_{rn}}}{1 - \left(\dfrac{N_x}{N_n}\right)^2 + j\,\dfrac{\omega T_{dn}}{P_{rn}}}$$

[7] Allis-Chalmers Manufacturing Company, Milwaukee, Wis.

[8] "Vibration Prevention in Engineering," by A. L. Kimball, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 40.

$$\delta_0 = \frac{2q_1F}{(P_{r1} + j\omega T_{d1})(1 - q_1) + (P_{r2} + j\omega T_{d2})(1 - q_2) + \ldots . (P_{rn} + j\omega T_{dn})(1 - q_n)}$$

$$\delta_1 = q_1\delta_0 + F\frac{2q_1}{P_{r1} + j\omega T_{d1}}$$

$\delta_n = q_n\delta_0$ for all excepting No. 1

$P_1 = (\delta_0 - \delta_1)(P_{r1} + j\omega T_{d1})$

$P_n = \delta_0(1 - q_n)(P_{rn} + j\omega T_{dn})$ for all excepting No. 1

where F = unbalanced impulse produced by engine of No. 1 unit
N_x = frequency of unbalanced impulses from No. 1, in oscillations per min
P_{r1} = synchronizing power of No. 1 generator
P_{r_n} = synchronizing power of nth generator
ω = 0.1045 N_x
ωT_{d1} = damping power of No. 1 generator
ωT_{dn} = damping power of nth generator
δ_0 = displacement of bus-bar voltage
δ_1 = displacement of No. 1 unit
δ_n = displacement of nth unit
q_1 = amplification factor of No. 1 unit on an infinite system
q_n = amplification factor of nth unit on an infinite system
P_1 = power pulsation of No. 1 unit
P_n = power pulsation of nth unit
N_1 = natural frequency of No. 1 unit on an infinite system, in oscillations per min
N_n = natural frequency of nth unit on an infinite system, in oscillations per min
T_{dn} can be taken as approximately $0.010P_{r_n}$

The units are numbered 1, 2, 3, etc., with the engine of No. 1 producing the unbalanced impulse under consideration. The subscripts of the various symbols indicate the numbers of the units to which the corresponding quantities apply.

Then, for the two problems in the paper, we computed the pulsation, based upon an unbalanced impulse of 12.7 per cent of the rating of the four-cycle engines, as he had done, and a damping factor T_d of 1 per cent of P_r in each generator. This is in accordance with the author's assumptions. In the first example, we got 330 kw which is somewhat different from the author's result. That is 94 per cent of the rating of the 350-kw unit, or 33 per cent of the rating of the 1000-kw unit. On the same basis, the second example showed 20.8 per cent pulsation for No. 1 unit, 4 per cent for No. 2, and 3.8 per cent for No. 3.

J. B. Sims.[9] The writer is superintendent of the plant referred to in the paper, hence some observations on the paralleling problem experienced in this plant may be of interest.

Three of the engine generators are 1200-hp, 800-kw, 2400-v, 200-rpm, four-cycle units. They were installed prior to the installation of a 2250-hp, 1600-kw, 2400-v, 225-rpm, two-cycle unit in 1937. There was a good deal more electrical disturbance, practically speaking, between the generators of the three four-cycle units when running together than when they were operating in parallel with the two-cycle unit which, incidentally, was equipped with a more sensitive engine governor. In the operation of the three four-cycle engines, there were two separate and distinct conditions observed: (1) A variation in speed and resultant frequency change with consequent power shifting from unit to unit due, apparently, to governor action; (2) a more or less constant pulsation between the three machines which apparently was a hunting, due we presume to inherent instability between the machines themselves. The older four-cycle engine governors are not as sensitive, and their frequency will often vary on the system from 1/2 to 1 cycle. This condition would appear to be almost entirely due to the lack of sensitivity of the governors, whereas, the pulsating effect might appear to be from other causes.

After the installation of the two-cycle unit in 1937, which was equipped with a more sensitive governor, the operation of this unit in parallel with the four-cycle units appeared to help stabilize the system. Since installation of the new 3850-hp 9-cylinder two-cycle unit, the system seems to be operating with fair stability in so far as we can tell. Although there remains some pulsation between the four-cycle units, it may be said that this is not as pronounced when operating with the larger engines. The hunting or pulsating between the machines seems to be particularly noticeable in the four-cycle engines. In fact, when these machines were installed, reverse-power relays were put in on each generator oil circuit breaker, apparently with the thought that if the fuel should fail the unit would not continue to operate as a synchronous motor. Later, it was found impossible to operate the generators with the relays due to their constantly kicking out the circuits, and the relays had to be disconnected. It is evident therefore, that there was a considerable transfer of power back and forth between the generators.

It may be of interest to know that the larger 9-cylinder-engine generator is connected through a bank of transformers to the outdoor distribution system. In other words, this 9-cylinder unit does not work directly on the bus with the other four engines. The other four are on a 2400-v bus, and the newer unit works through a bank of 2400/7200-v, 2000-kva transformers, which have approximately 6 per cent reactance.

[9] City of Grand Haven, Mich.

Author's Closure

Professor Maleev points out that some symbols are not explained and that the formulas are not numbered. That is admitted. Symbol ω is the circular frequency of a vibration, measured in radians per second. With frequency N vpm we have $\omega = N \cdot \pi/30$. Symbol m designates a mass moment of inertia with dimension lb in sec². In most of the figures in the paper mass moments of inertia are pictured as masses which is the reason why m is often called a mass. It is also admitted that the condition in the example cited is somewhat abnormal. However, it is an actual installation and it is the only case in our records where it is definitely known that there was trouble with the parallel operation. It was felt that this would be more interesting for the readers than an example made up at the author's desk.

Professor Maleev further suggests that an analysis be made to show whether it is possible by changing the WR^2 of the engines to eliminate excessive power pulsation. The answer is that this is possible of you can change the WR^2 for all the units in the plant. However, when a new unit is added to an existing plant, it is sometimes difficult to find a satisfactory solution.

If, for instance, two engines run in parallel, it is possible to lower the natural frequency within certain limits by adding WR^2 to any one of the units. However, when the power pulsation is considered, it will be found that WR^2 added to the disturbing unit can bring down the pulsation to any amount desired while even infinite WR^2 on the passive unit cannot bring the pulsation below a certain limit. In other words, it is very important to have considerable WR^2 at the source of the vibrations, which in most cases is a four-cycle engine in the plant.

This was demonstrated by a recent example. A power plant had a four-cycle engine running at 277 rpm, with a very light flywheel giving a natural frequency on an infinite system of 162 vpm. This is higher than the disturbing frequency of half engine speed or 138 vpm. The natural frequency of this unit in parallel with other units in the plant also was above 138 vpm.

This is not a satisfactory condition: (1) Because value P_R goes down with decreasing load, thus bringing thesystem into resonance, and (2) because the light WR^2 on the disturbing unit increases the amplitude of the vibrations. The mere fact that you are 10 per cent away from resonance does not guarantee perfect parallel operation in a case like this. It was actually found by computation that by increasing the WR^2 on the four-cycle engine, thus bringing the system closer and even into resonance, the power pulsation was hardly increased.

In a plant like this it is difficult to fit in new units. The cheapest solution might be to change the flywheel on the earlier installed four-cycle engine. It is gratifying to know that Mr. Edgar J. Kates considers the paper a simplification; many seem to find the treatment rather complicated.

Mr. Harte Cooke mentions two phenomena which are not dealt with in the paper. However, the first one should be taken care of with a correct governor and the second with a correct generator design and, as Mr. Cooke says, "these two peculiarities are apparently under control."

Mr. J. B. Sims gave some interesting information concerning the practical performance of the power plant used for the second example. The mathematics proves clearly that the resonance point where we happen to operate is an apparent resonance only, without any effects whatsoever. However, it is comforting to hear the superintendent of the plant testify that practice complies with theory.

Mr. Benson's remarks are very much appreciated. Further information about the damping in the generators certainly would be welcomed. However, knowledge about the exact damping is not always important. In the first example given in the paper, the effect of higher damping would be proportionally lower power pulsation, but in the second example, change in the damping would have very little, if any, effect, although we here run exactly at resonance. The effect of the damping here is to determine the phase angle, and a moderate change in this angle will make no difference. In the first example, we have an actual resonance point, in the second an apparent resonance only.

The method of "mechanical impedance," suggested by Mr. F. K. Brainard, is quite similar to the method described in the paper, the difference being an analytical instead of a graphical treatment. An earlier graphical method without the fictitious generator was practically identical to Mr. Brainard's method. However, it was felt that this fictitious generator would give a much clearer picture of the whole setup. Thus, using this method for the second example, it is evident without calculating more than the natural frequencies on an infinite system that the power pulsation will not be excessive although we run right at the resonance point.

Mr. Brainard's results for the two examples in the paper differ from the results given there. The reason for this is that Mr. Brainard uses a disturbance of 12.7 per cent of the kilowatt rating, while in the paper is used 12.7 per cent of the indicated horsepower. For the two cases, this will correspond to about 17.8 per cent of the kilowatt rating. By multiplying Mr. Brainard's results with this proportion, complete accordance will be obtained for the second example. In the first example there will remain a difference of about 4 per cent of the result, which might be explained by the approximations made in the paper.

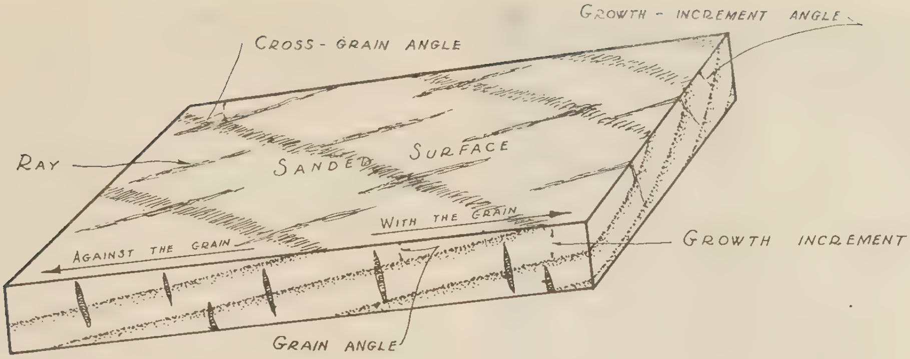


FIG. 1 SKETCH OF TEST BLOCK OF OAK WOOD, SHOWING LOCATION OF VARIOUS ANGLES OF GRAIN AND THEIR RELATION TO DIRECTION OF TRAVEL OF ABRASIVE

An Analysis of the Factors Responsible for Raised Grain on the Wood of Oak Following Sanding and Staining

BY G. G. MARRA,[1] SYRACUSE, N. Y.

Attempts to minimize the development of raised grain without fully understanding the causes of its development have led to costly processing techniques in industry. The cause or causes of this condition may originate from any one or a combination of several of the many factors involved in the sanding operation. What these factors are or how they function is by no means well understood. The research reported, therefore, was initiated for the purpose of determining the more important factors responsible for the raised-grain condition as they pertain particularly to the wood of oak.

NATURE AND DEVELOPMENT OF THE PROBLEM

THE raised-grain effect in wood is a most puzzling and aggravating phenomenon which often accompanies the sanding and staining of wood surfaces. It arises after a satisfactory surface has been obtained and is induced by the action of swelling agents such as water stains, spirit stains, or even by the humidity of the atmosphere. It is apparent that there is a direct relation between the presence of the swelling agent and the appearance of the raised-grain condition. It is also apparent that a latent reaction of the wood elements on the surface to the action of the abrasive is responsible for this condition. Consequently, the extent of raised grain may be used as an index to the severity of the treatment of the surface elements. Within limitations, it can also serve quantitatively as a measure of the effectiveness of a given set of conditions to produce a satisfactory surface.

In accepted sanding techniques, a satisfactory surface is secured by the removal of all visible blemishes including machine marks, dents, and scratches from the surface. This is accomplished by the use of a sequence of successively finer grits, each one in turn reducing the magnitude of the scratches produced by the preceding grit. The quality of the sanded surface is usually predetermined by the quality of the subsequent finish desired. After the required sanded surface has been attained by the sander, the finisher is still left with the problem of coping with whatever condition may have been created by the abrasive. One of the most common manifestations of the damaged condition of the surface is the raised-grain aftereffect of staining and, when present, it necessitates an extra hand-sanding operation before the filler can be applied.

It is obvious that this extra operation is costly in large-scale production and, in severe cases, may even be a bottleneck to production methods. Furthermore, it may be desirable in some cases to avoid expensive sanding and finishing techniques now employed to reduce raised grain in general such as "glue-sizing" the surface before final sanding, sanding with extremely fine abrasives, the use of special stains to avoid raised grain, and wash-coating after staining to facilitate hand-sanding the raised grain.

Koehler[2] has described the various types of raised grain occurring in the woodworking field in general and has pointed out that raised grain, caused by sanding and staining, is associated with the vessels. He showed that the abrasive action and the pressure applied are responsible for this condition by comparing a sanded surface with a planed surface before and after wetting. These are only two of the many factors involved in the sanding operation all of which may be divided into two groups, (1) those

[1] Inspector of Wood Aircraft Materials, U. S. Army Air Force.

This work was conducted under the supervision of the department of wood technology, New York State College of Forestry, Syracuse, N. Y., 1940–1942.

Contributed by the Wood Industries Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[2] "Some Observations on Raised Grain," by Arthur Koehler, Trans. A.S.M.E., vol. 54, 1932, paper WDI-54-9.

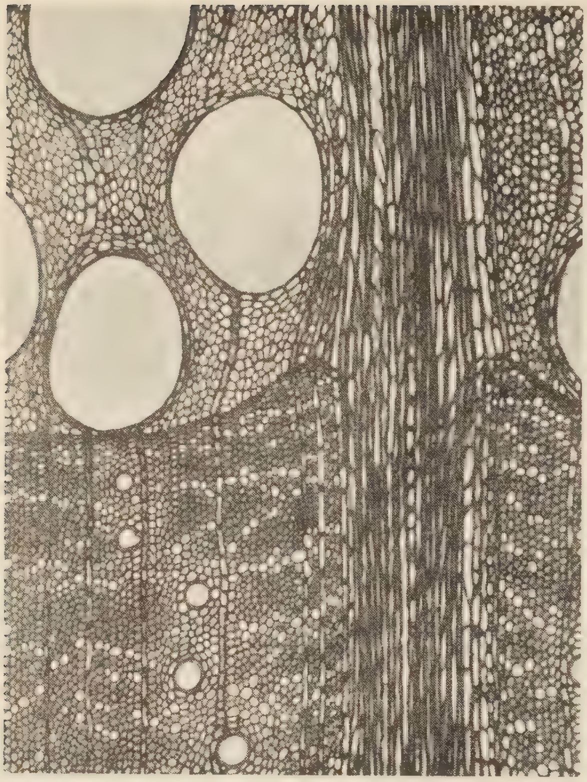


Fig. 2 Cross-Section Photomicrograph of Wood of a Red Oak Showing Springwood Zone Composed of Large, Thin-Walled Vessels Surrounded by a Relatively Soft Tissue and Dense Fibrous Tissue of the Preceding Summerwood; ×75
(Photomicrograph by S. Williams.)

pertaining primarily to the abrasives and their application, and (2) those pertaining to variations in the structure and grain of the wood.

Among the most important factors in the first group are:

(1) Abrasive minerals; (2) type of coating and deposition; (3) grit sizes; (4) condition of the abrasive; and (5) speed of abrasive and applied pressure.

The second group contains the most important factors from the standpoint of this paper and, in contrast to those of the first group, are not as easily controlled because, not infrequently, several factors are present which vary even within a small block. These factors include:

1 The alignment of the wood elements with respect to the surface being sanded.

2 The direction of travel of the abrasive with respect to the alignment of the wood elements.

3 The type and arrangement of the wood elements.

Fig. 1 illustrates the location of the various angles of the grain and their relation to the direction of travel of the abrasive; Fig. 2 is a photomicrograph of the wood of oak, showing the type and arrangement of the wood elements.

Some of the more easily controlled sanding variables, such as speed, pressure, condition of the abrasive, types of minerals used, and sizes of grits, were studied first. The wood used in this preliminary work included both flat- and edge-grain stock, but aside from this selection, no further attention was given to the wood than is the rule in actual production. Since hundreds of blocks, sanded under all possible combinations of these conditions, failed to show any definite correlation between raised grain and the sanding conditions, the problem was finally narrowed down to a study of the reactions with respect to the wood alone, as influenced by a constant set of sanding conditions.

This situation presented a continuous challenge because, in almost every experiment attempted, some blocks showed no raised grain whatever. Since the sanding conditions did not change, some characteristic of the particular block must have been responsible. Therefore, throughout the study, the opinion was held that if these characteristics could be ascertained in terms of wood structure it might theoretically be possible to eliminate raised grain by proper selection of material.

Of necessity, the method of attack was essentially "hit-or-miss." As soon as a pertinent wood variable was noted, a set of conditions isolating this variable was devised. This process of isolation was perhaps the most difficult part of the research. Because of the many variations in grain which may appear in a small block, it was frequently necessary to evaluate a given variable in combination with several others which could not be reduced to constancy.

Failing to obtain a good correlation was taken to indicate the presence of other unobserved factors, and the search proceeded until another was observed, and it in turn was subjected to further tests. This procedure finally led to an extensive use of the microscope and consideration of the anatomy of the wood.

Critical diagnosis of the conditions involved in a relatively few blocks, representing all degrees of raised grain, was the method of study finally employed. No means of measuring accurately the quantity of raised grain could be devised, but judgment conditioned by constant observation of sanded surfaces was brought to bear in analyzing various types of this phenomenon. In this manner, some of the more important factors were determined and the task of correlating these with various sanding conditions was attempted.

This study therefore does not represent the last word in the raised-grain problem but is rather a survey of the factors involved, with some definite correlations indicated.

Materials and Equipment Used

Wood. Most of the wood used in this experiment was supplied by a large furniture company in North Carolina, and part of the preliminary experimental work was carried out at its plant. The blocks consisted of 6-in. × 3-in. × $^3/_8$-in. kiln-dried samples of hard maple, oak, Honduras maghogany, and Philippine mahogany. These were kiln-dried as for ordinary production uses and, at the time of the preliminary tests, the following average moisture contents prevailed:

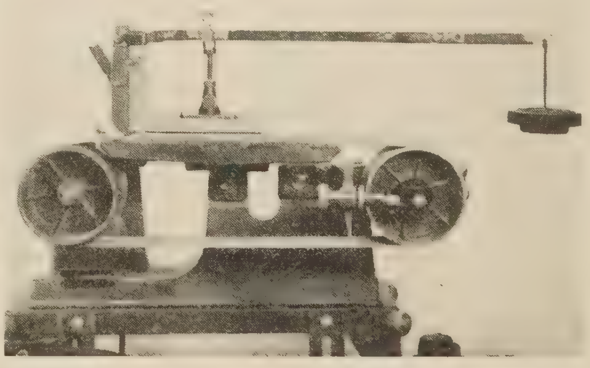


Fig. 3 Sanding Machine Showing Arrangement of Pressure-Control Mechanism

Wood	Average moisture content, per cent
Maple	9.5
Oak	8.0
Honduras mahogany	8.5
Philippine mahogany	4.5

Abrasives. The products of two leading abrasive-manufacturing concerns were used in this research. They included flint, garnet (open- and close-coated), aluminum-oxide (electrocoated), and silicon carbide (open- and close-coated). The grit sizes were 1/0 through 6/0 inclusive and were all mounted on E-weight paper.

Sanding Machine. The sanding machine used was a small variable-speed sander, employing a belt size of $52^1/_2$ in. $\times$ 4 in. It was capable of three no-load speeds of 1300, 2400, and 4400 abrasive fpm. These speeds are somewhat slower than those used in modern production, but faster speeds could not be employed safely.

A pressure-control lever system was installed above the machine which held the block securely while it was in contact with the belt. Also, the platen was covered with a $^3/_{16}$-in. felt pad shellacked in place and covered with graphited canvas. This arrangement reduced the shock on the grits and produced uniform abrasion over the surface of the block. The entire assembly is shown in Fig. 3.

Experimental

Effect of Size and Type of Abrasive, Angle of Grain, and Direction of Abrasive on Raised Grain. Preliminary investigations on various species of wood with various sanding conditions revealed that the orientation of the grain with respect to the surface was an important factor in relation to raised grain. Further studies under more controlled conditions were therefore undertaken to substantiate the relationship. At the same time, an attempt was made to determine if the various abrasives and the direction of sanding with respect to the angle of the grain had any effect on the amount of raised grain produced.

Although both oak and maple blocks were used in this particular experiment, only the oak, because of its greater variability, was considered in detail.

Procedure. Blocks were first carefully selected and divided into two groups, i.e., one group having a definite angle of grain, the other group being relatively straight-grained. This was done for both the oak and maple with special care being taken to select only flat-sawn stock. The blocks were originally kiln-dried to a moisture content of 8 per cent for the oak and 9.5 per cent for the maple. Previous to testing they remained in a warm dry room for 5 months, during which time they reached an equilibrium moisture content of 5.9 per cent for the oak and 6.1 per cent for the maple. The blocks were grouped so that equal numbers were sanded against the grain, with the grain, and on straight-grained surfaces, for each of the grits.

The abrasives used were 1/0 and 3/0 sizes of the following minerals:

1 Garnet, open-coated.
2 Garnet, close-coated.
3 Aluminum oxide, electrocoated.
4 Silicon carbide, close-coated.

Each belt was given a prerun on flat-grain maple before using on test blocks. The no-load speed of the belt was constant at 2400 fpm.

The pressure was maintained constant for the entire test, regardless of grit size or wood used. This was accomplished by suspending a weight pan from the pressure arm and placing a $1^2/_3$-lb weight in the pan. By application of the principles of the simple lever system, this weight was found to produce a static pressure of 0.76 psi on the bearing surface between the block and the abrasive.

Each block was subjected to a preliminary sanding with a 4/0 aluminum-oxide electrocoated belt, hand-manipulated until the surface was uniformly smooth.

In sanding, the actual length of time had to be extremely short because the belt was only $52^1/_2$ inches long and, at the speed used, individual grits were in contact with the wood 9.1 times per sec. Because of the rigidity of the pressure mechanism, each grit followed precisely the same path at each rotation, thus introducing a condition which never obtains on large production machines. Consequently, a sanding time of only 2 sec was allowed for each block to minimize this effect. This interval was sufficient to insure proper contact and to allow the abrasive adequate time to cut below any irregularities resulting from the previous sanding.

A sample of the sander dust was collected from each type of sanding for the maple blocks. This was accomplished by means of a vacuum cleaner, which was attached in such a position that it drew the dust directly from the belt. A small cloth bag fixed in the hose connection caught the dust before it reached the larger receptacle. It was anticipated that this material might reveal a difference in the abrasive action for the various conditions.

After sanding, the blocks were moistened with an aqueous-alcoholic solution of Bismarck brown stain[3] (0.1 per cent Bismarck brown, 7 per cent ethyl alcohol). This was applied gently with a small camel's-hair brush and allowed to dry under room conditions. The blocks were then examined with the aid of a binocular microscope.

Discussion of Results

Types of Raised Grain. Most of the raised grain on oak occurs in the springwood zone and is therefore associated directly with the vessels as indicated previously. Raised grain is also associated with the scratches and with the dense fibrous tissue, and on this basis it was classified into three main types as follows:

1 Type I represents that condition which results from injury to the vessel walls. The ruptured and collapsed vessel walls, under the influence of water or other swelling agents, tend to reassume their original shapes and positions, but, depending upon the character of their attachments, the fragments may assume any position. This type may be further subdivided into three forms, depending upon their location with respect to the vessel aperture and the angle of the grain:

Form A is represented by the condition occurring when the grain is raised at the point where a vessel dips below the surface, Fig. 4, and is therefore most prominent when the grain angle is pronounced. In degree, this form may vary from a slight protrusion of the thin vessel wall to a mass upheaval of the solid tissue surrounding the vessel. Consequently, the severest cases of raised grain are usually of this form.

Form B is represented by the condition in which the vessel wall has been caved in and lacerated for some distance and, consequently, is most likely to occur where the grain is straight or slightly angled. In such cases, the fragments are in part attached to the rest of the wall, and on wetting are forced outward, as shown in Fig. 5.

Form C is characterized by a washboard appearance resulting from a cave-in of the vessels, but without rupture or laceration, Fig. 6. In common with form B, it appears only on straight or slightly angled flat-sawn stock and, although of infrequent occurrence on oak, it is a common characteristic of raised grain in maple.

[3] A stain was used in the swelling agent to facilitate microscopic inspection.

(a)

(b)

FIG. 4 TYPE I RAISED GRAIN, FORM A; ×6.7
(a, Mild; grain angle 3 deg, growth-ring angle 0 to 2 deg, tyloses abundant, sanded against the grain. b, Severe; grain angle 7 deg, growth-increment angle 16 deg, tyloses sparse, sanded with the grain.)

FIG. 5 TYPE I RAISED GRAIN, FORM B, OCCURRING ON STRAIGHT OR SLIGHTLY ANGLED GRAIN; ×6.7

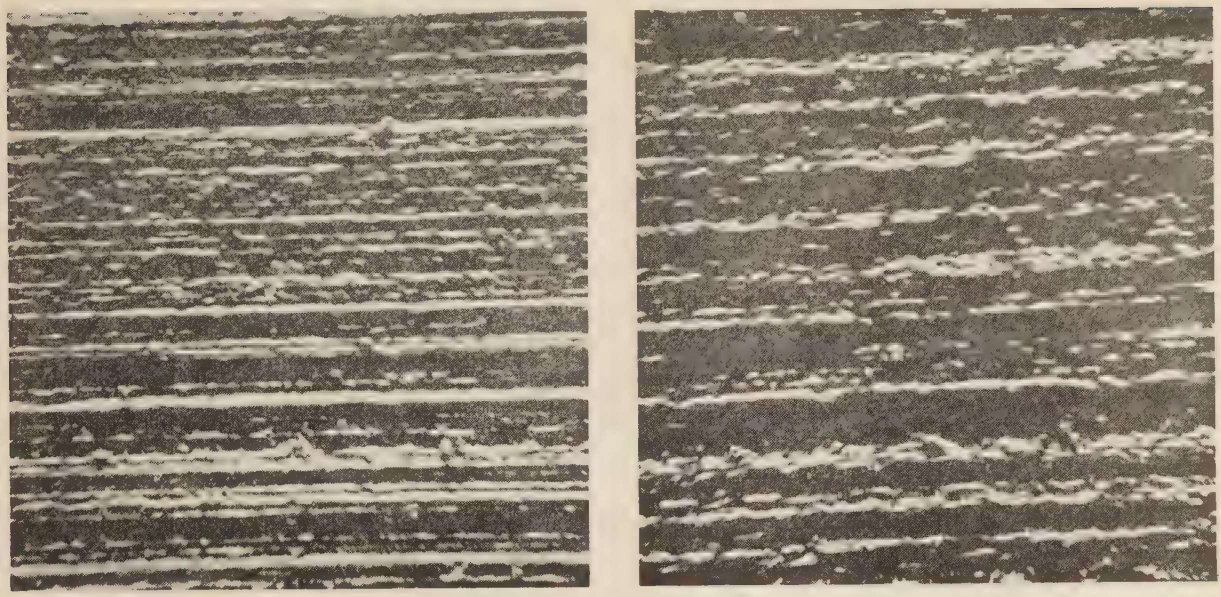
FIG. 6 TYPE I RAISED GRAIN, FORM C; ×12
(Washboard effect occurring on straight or slightly angled grain when plane of abrasive cut just grazes springwood zone.)

2 Type II is a type of raised grain which results from swelling of the elements constituting the ridges of the scratches and is, therefore, proportional to the scoring tendency of the abrasive, Fig. 7.

3 Type III is a type of raised grain resulting from fiber-fiber separations which may or may not be apparent after sanding. The fibers may be solitary or in groups and are attached at one end, Fig. 8. In oak, this type occurs for the most part in the fibrous summerwood zone, whereas in maple it appears only in the springwood zone.

These types, although in themselves clear-cut and well-defined, did not permit of a similar complete and accurate classification with respect to blocks as a whole. This was due to the fact that variations in grain and structure, even in these small samples, permitted a diversity of types and quantity of raised grain to appear on the same block. However, the foregoing classification was useful in segregating differences due to the various conditions in any particular block.

Effect of Size and Type of Abrasive. On the whole, the 1/0 grits



(a) (b)

Fig. 7 Type II Raised Grain Is Associated With the Scratches; ×21
(a, Before wetting; b, after wetting.)

were productive of more raised grain than the 3/0 grits, as might have been expected from their more severe abrasive action. Microscopic inspection revealed that raised grain of Types II and III was minimized with the 3/0 abrasive, but Type I was less consistent. In other words, Type I frequently appeared in quantity even with the 3/0 grits and occasionally was absent with the 1/0 grits. This indicated the presence of other factors in addition to abrasive action as noted later. However, in connection with the rule, a close examination revealed that the presence of more Type I raised grain in the case of the 1/0 grits was not due to greater caving-in of the vessel walls but to greater laceration of the walls which then were free to respond to the influence of the swelling agent. The action of the 3/0 grits resulted in just as much caving-in, but the walls were usually abraded free of contiguous elements and remained attached at the sides. Such collapsed walls were incapable of rising above the surface after wetting and, in appearance, had a striking resemblance to shallow open vessel lines. Actually, however, this was not the case, since the upper parts of the walls were merely collapsed in such a way that they formed "false bottoms" in the vessel cavities, Fig. 9. Such a condition might be detrimental to a satisfactory penetration of the filler.

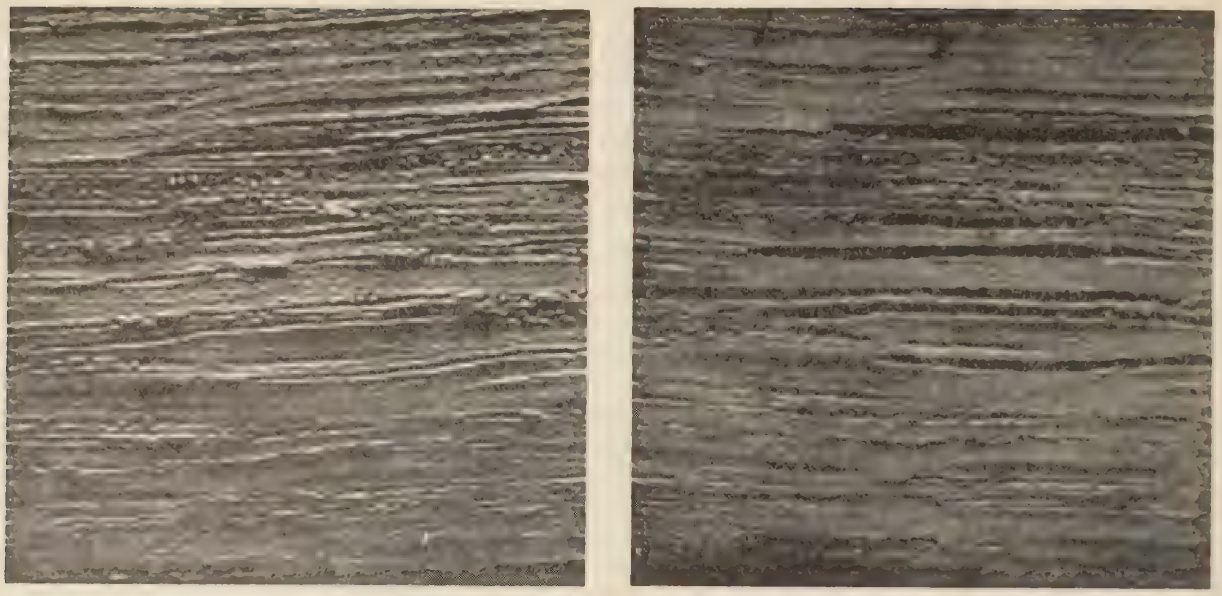
Under the conditions of this experiment, open-coated garnet seemed to be the most effective in reducing[4] Type III when the sanding was against the grain. Silicon carbide appeared to be most effective in reducing the permanent type of cave-in described in the previous paragraph and also in reducing Type I raised grain in general.

Effect of Angle of Grain and Direction of Abrasive. The experiment showed that the angle of the grain with respect to the surface being sanded was the most important variable in connection with the development of raised grain. The effect of the angle varied, within limits, not only with its magnitude but also with respect to the direction of travel of the abrasive.

The maximum effects were observed to occur for angles between 3 and 15 deg and depended largely upon the direction in which the belt was traveling. More raised grain resulted from sanding against the grain than in sanding in the reverse direction. In the lower portion of the range, sanding against the grain produced all three forms of Type I, as well as Type III, raised grain. In reversing the direction, Form A of Type I was usually reduced and Type III was entirely eliminated. In the upper portion of the range, Form A was dominant regardless of direction and was conspicuous even on the summerwood vessels. It was difficult to distinguish differences in the amount of raised grain when a large angle was present except for the consistent presence of more of Type III in every case of sanding against the grain.

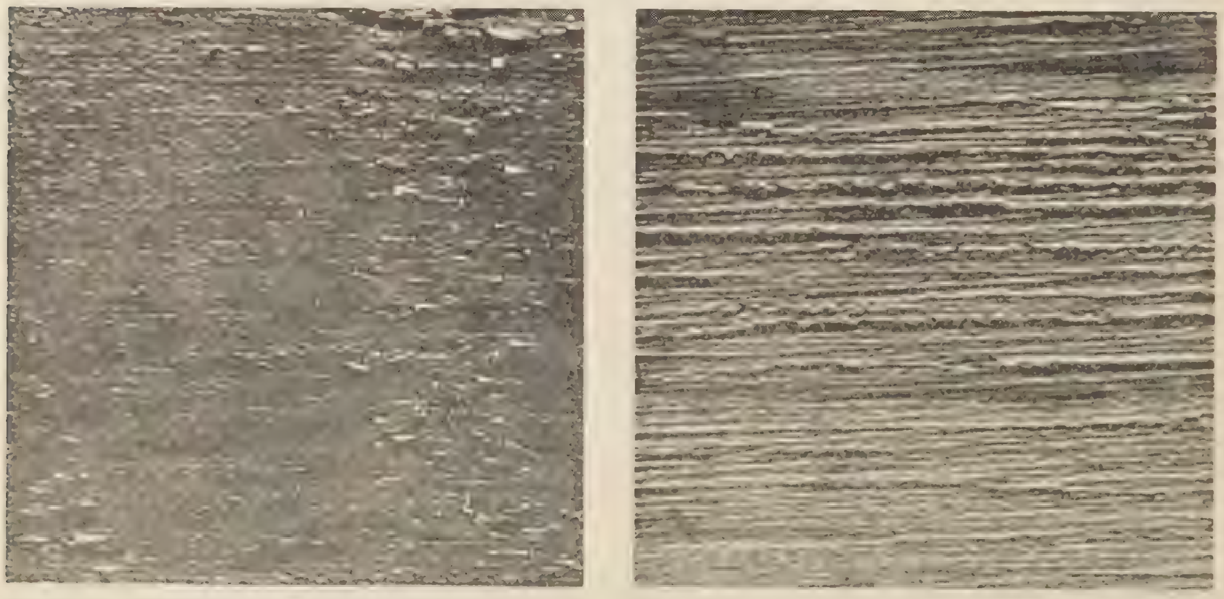
Fig. 8 Type III Raised Grain Due to Fiber-Fiber Separations in Summerwood of Oak; ×12

[4] The word "reducing," used in this sense, connotes a reduction not only in quantity but also in severity of raised grain, i.e., size of unit and height above the surface.



(a) (b)

Fig. 9 Straight-Grain Oak Lacking Raised Grain
(a, With typical "false bottoms" in vessel lines. b, Same spot with false bottoms removed, exposing vessel cavity below.)



(a) (b)

Fig. 10 Oak Block Showing Absence of Type I Raised Grain; ×6.7
(a, When grain angle is over 15 deg and sanding is against the grain. Note presence of Type III raised grain. b, When grain angle is less than 3 deg.)

When the angle was over 15 deg, very little Type I raised grain was produced regardless of direction of sanding. This was due to the fact that the vessel apertures were very short, and the supporting tissue above the vessels was sufficiently thick to protect the thin wall from injury except for a short distance below the surface. Consequently, very little cave-in resulted and hence raised grain from this source was at a minimum, Fig. 10 (a). However, Type III raised grain was still in evidence at this angle when the sanding was against the grain.

When the angle was below 3 deg the grain approached the straight-grain condition, and variations in type were not readily distinguishable. When the grain was relatively straight, the vessels extended for a distance of 4 to 5 cm on the surface, Fig. 10 (b). In such cases raised grain was usually at a minimum, and the direction of the abrasive had no apparent effect. When raised grain was evident, it was usually of Types I B and I C. Type III was occasionally present to a slight degree. As might have been anticipated, the permanent type of cave-in was more in evidence when the grain was straight because the condition extended for longer distances.

Additional Factors. Flat-sawn lumber was associated with more raised grain than quarter-sawn lumber because in this case the maximum area of the springwood was exposed. The extent of raised grain in both cases, however, was determined primarily by the angle of the grain and the direction of abrasion.

On the other hand, the relation was less predictable for lumber in which the growth rings were oblique to the surface, i.e., neither truly flat nor truly edge grain. These oblique-grained surfaces showed the greatest variation in the amount of raised grain. In some instances, pronounced raised grain resulted from sanding both against the grain and with the grain. Such cases were observed to occur when the slope of the grain was rather high and the growth-increment angle was between 15 and 20 deg, Fig. 1. Type I A raised grain was the most prominent type observed and was almost always characterized by the presence of solid fibrous tissue which was rigid to the touch, Fig. 4 (*b*). In other cases, no raised grain whatever resulted from sanding in either direction, when all other conditions were apparently favorable.

Blocks showing cross grain on the surface were so limited in number that the effect of this factor on the production of raised grain could not be determined under all conditions. However, observations revealed that it had the least effect on straight, flat-grained surface, and the most effect on angled grain when the direction of sanding was against the grain.

The bark and pith sides of the same block sanded in the same manner were also observed to develop varying amounts of raised grain. Blocks with pronounced differences were numerous but no consistency of variation with respect to the two sides was apparent. Flat-sawn straight-grain blocks, however, showed a greater tendency to produce raised grain on the pith side of the block than on the reverse side.

Tyloses (frothlike material common in the vessels of some oaks), when present in sufficient number, have a very marked effect in reducing Type I raised grain under all conditions. They are also effective in reducing the permanent type of cave-in, Fig. 11. This is obviously due to the support offered by the tyloses to the vessel wall in resisting collapse. However, both their presence and quantity are subject to great variation within a block, particularly for the species of the red-oak group.

Raised Grain on Maple. Although not studied as intensively as oak, maple was observed to be more predictable with respect to the production of raised grain. This was undoubtedly due to its finer and more uniform texture. Angled grain appeared to be the most active factor in relation to raised grain but the angle was subject to great variation even on small surfaces, not only in respect to degree but also to direction of slope. In addition, the cross-grain angle varied considerably due to localized fluctuations in the direction of the grain. Types I A and I C raised grain were the dominant types developed, and Type II was more prevalent than was true for oak. Type III, in contrast to the oak, appeared only in the springwood areas.

Maple Sander Dust. Not much was learned by a study of the sander dust from the maple blocks. In general, the dust from the straight-grain blocks appeared to be composed of longer and cleaner shavings than the dust from the angled-grain blocks. Recognizable structures were not common in either case, but the former appeared to display more fiber ends intact than the latter.

Fibers in groups of three or four were usually attached to a portion of a vessel wall. Fragments of vessel walls with no fibers attached were infrequent, which may indicate that the vessel walls probably always collapsed before the fibers above the wall had been removed.

Under the pressure used (0.76 psi), the 3/0 grits produced shavings of such width as to represent only one fiber. The 1/0 grits produced two-fiber shavings quite commonly and occasionally one of three-fiber width.

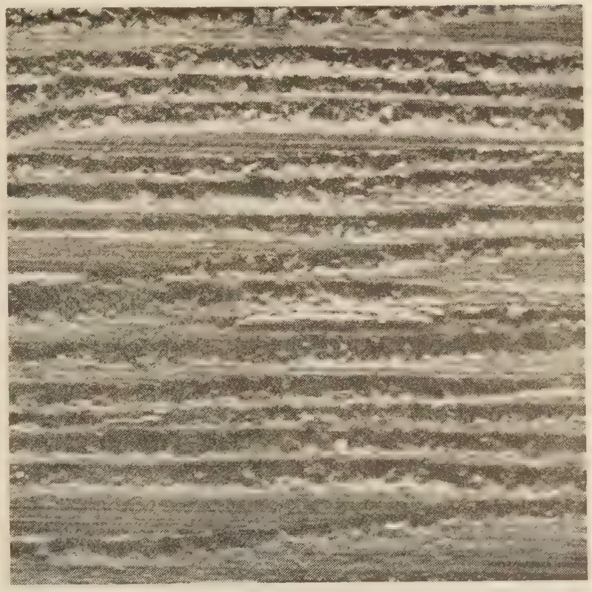


Fig. 11 Flat-Sawn Straight-Grain Oak Block Showing Abundant Tyloses and Lack of Collapsed Walls; ×12

Effect of Heat Generated During Sanding on Raised Grain

The effect of heat generated by the abrasive action on raised grain was determined by sanding the same blocks for alternating periods of 5 sec and 30 sec each, under the same conditions. The differences in temperatures were very marked, and with few exceptions, the 30-sec period resulted in more raised grain than the 5-sec period.

Effect of Applied Pressure on Raised Grain

An attempt was made to determine the exact effect of applied pressure on raised grain, but in spite of all possible refinements, no definite correlation could be obtained.

Some blocks which had previously shown abundant raised grain under other sanding conditions were sanded with a worn-out 6/0 aluminum-oxide belt under high pressure for 1 min. In every case, less raised grain resulted from this latter treatment than previously, indicating that pressure alone, in the absence of abrasive action, does not cause raised grain.

Mechanics Involved in Raising the Grain

Although raised grain of the type described in this paper has always been associated with swelling agents, no literature was found in which the true nature of the forces acting was explained. It has been assumed that the grain, i.e., loosened fibers, groups of fibers, or vessel walls, rise as a result of the twisting and turning incident to the drying of the swelling agent. This process has been likened to the defects resulting from rapid drying of boards during seasoning.

This view was held by the author until an attempt was made to reduce raised grain by reducing the drying rate of the swelling agent. Close observation of the entire wetting and drying operations under a binocular microscope revealed that the loose fibers appeared to rise as the swelling agent dried. This, however, was largely an optical illusion. As the surface of the liquid receded toward the surface of the wood, the fibers appeared to rise higher and higher. Actually, the fibers or groups of fibers,

which were more or less well attached at one end, exhibited very little motion during drying, and it must therefore be assumed that their position above the surface of the wood resulted from the swelling action and not from the drying action. Individual fibers or loosely attached groups of fibers may recede to the surface of the wood as the liquid dries, perhaps pulled down by the surface tension of the liquid, as pointed out by Campbell[5] in connection with the formation of a sheet of paper. Thin fragments of vessel walls with no fibers attached may remain in a raised position until after the surface of the wood has dried and then they may collapse completely.

The lack of motion of firmly attached groups of fibers during drying was further substantiated by the use of the compound microscope equipped with an eyepiece scale. As soon as a surface was moistened, the medium power of the microscope was brought to focus on the uppermost tip of a group of fibers which protruded above the surface of the liquid. When the eyepiece scale was aligned with the long axis of such a group, any vertical displacement of the fibers was instantly registered on the scale, as indicated in Fig. 12. Furthermore, any motion of the group

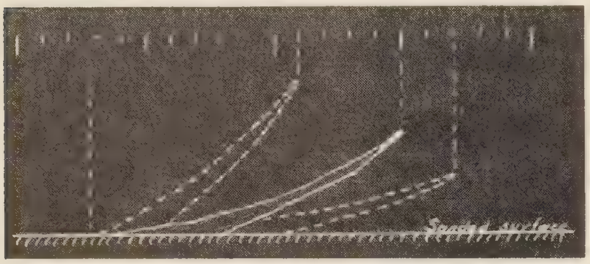


Fig. 12 Sketch Indicating Manner in Which Vertical Displacement During Drying of Loose Tip of a Group of Fibers Is Registered on Eyepiece Scale

resulted in moving the tips of the fibers under observation out of focus. By this technique, it was observed that very little motion resulted during drying and this mostly in a downward, not an upward, direction.

Conclusion

Over 2000 blocks were sanded and examined critically during the course of this research in an effort to establish the relationship between raised grain and the factors involved in the sanding operation. In placing the responsibility for the major portion of raised grain directly upon the wood variables, many sanding variables were first proved to be less directly responsible. These wood variables must be carefully controlled before a proper evaluation of the effect of various sanding conditions can be obtained.

The factors involved in a wood, such as oak, are not only numerous but are subject to variation from block to block and even within the same block. The resulting interrelationships make it difficult to evaluate the factors and establish their relative importance. For this reason, the broad term "raised grain" was found to be inadequate to use as a measuring stick with which to compare results, and it was therefore necessary to differentiate various types of raised grain. Even with this refinement, it was practicable to characterize only small areas in which the factor under consideration could be isolated. By this means some order was injected into the complicated cause-and-effect relations of the many factors which of necessity must operate simultaneously.

However, the elucidation of definite rules by which the extent of raised grain on a particular board may be predicted must be preceded by the further application of the techniques and refinements in selecting blocks as outlined in this paper. The most important factors have been ascertained, but the determination of the exact effect of some of them requires the careful selection of hundreds of blocks sanded under controlled conditions. The effect of oblique grain, for example, is not definite since it appears to be associated with both extremes of raised grain. Cross grain and the relation of the surface to the bark and pith are in a like category.

The effect of pressure also is not definite, and its relative importance, with respect to temperature and abrasive action, requires further study. If it can be definitely proved that the effect of pressure is not as significant as the effect of heat generated, then sanding techniques could be altered accordingly.

A more thorough study should also be made of the false-bottom condition so prevalent on straight-grain surfaces sanded with fine abrasives. Since this may inhibit the proper penetration of the filler, it might be advisable to take steps to avoid its occurrence.

As was previously stated, this study was not initiated to solve the raised-grain problem but merely to determine the most important factors involved. This has been accomplished and the opinion is still held that, although it may never be possible to devise sanding methods which will eliminate all raised grain, it may be possible with further study to select material which will produce a minimum of raised grain under predetermined sanding conditions.

Discussion

A. C. Fegel.[6] The types of abrasives and methods of application used in sanding wood have been subject to a number of investigations by the woodworking and abrasives industries. In making these studies, however, an attempt has usually been made to avoid any consideration of the influence of variations in the wood by confining the study to one species of apparently uniform structure. In the interpretation of results, the influence of variations in the anatomical structure of the wood has been given little consideration.

In this paper, the importance of wood structural characteristics is stressed and the effects of some of the possible variations are described. The results clearly show that variations in the wood structure and the alignment of the elements with reference to the path of the sanding belt are at least as important as the speed, pressure, abrasive, and other mechanical features of the sanding operation. It is to be hoped that investigators of wood sanding will give this more consideration in the future.

In the discussion of the effects of heat and pressure on raised grain, the author concludes that heat generated during sanding tends to increase raised grain, while pressure in the absence of abrasive action does not cause raised grain. Since the procedure used in studying the effect of applied pressure would probably generate more heat than that used to determine the effect of heat, it would seem that heat as well as pressure would have no tendency to cause raised grain in the absence of abrasive action.

Sern Madsen.[7] Usual methods of machining the surface of wood invariably crush down the walls of the wood cells, particularly when the cutting medium is blunt or dull. Ordinary sand-

[5] "The Cellulose-Water Relationship in Paper-Making," by W. B. Campbell, Forest Service Bulletin 84, Department of the Interior, F. A. Acland, Ottawa, Canada, 1933.

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[7] Mechanical Engineer, Mattison Machine Works, Rockford, Ill. Mem. A.S.M.E.

paper grits are not adapted to cut but rather push off the wood particles. The grit particles, no doubt, press downward proportionately as much as they push sidewise to remove stock.

Downward pressure tends to press down or crush the walls of wood cells, particularly where a cell is cut on an angle, and there is a thinned or feathered edge. The action is decidedly different from where a razor edge like a microtome is used in cutting the wood fibers.

It is also quite reasonable to suppose that considerable surface heat is present instantly at the time of the abrasive contact of the grit with the wood. Thus, heat and pressure may briefly deform and set the walls of the wood cells. Subsequently, moisture entering such deformed cell walls will tend to restore them to their original shape. This then shows up in the form of raised grain.

On this theory the raised grain would be in evidence to the greatest extent where the wood is most porous, a fact borne out in actual practice. Angle of grain will also cause a "spotted" effect in raised grain.

From this reasoning it seems that we may have to provide sharper and better cutting elements. We might also take a lesson from common practice in wood finishing. A patternmaker coats a completed wood pattern with shellac and then sands the surface. The effect is that of adding "starch" to the cell structure, thus helping resist cell crushing.

The analysis of raised-grain structure in the author's paper is timely and informative and offers much fact on which to base improved methods for its prevention.

Author's Closure

The abrasive action of the grits on wood surfaces is difficult to visualize. The author's conception is that it is roughly similar to the cutting action of a two-man crosscut saw in that all the grits act simultaneously as cutter teeth and raker teeth. Some plowing action is also operative as the abraded particles spiral past concave faces of the grits and into grit clearances. In this conception, the pressure required would be that necessary to maintain the grits at an optimum cutting depth. Additional pressure would result in further deformation of the fibers below the penetration of the grits with little increase in the cutting rate of the abrasive.

The character of the remaining sanded surface therefore depends upon the reaction of the fibers to whatever pressure is applied and upon the splitting or cleavage characteristics of fibers from one another, both of which are influenced by the many variations in grain and texture of the wood.

Behavior of Plywood Under Repeated Stresses

By ALBERT G. H. DIETZ[1] and HENRY GRINSFELDER[2]

Aircraft, with their rapidly increasing use of plywoods bonded with synthetic-resin adhesives, have made imperative a study of the behavior of both wood and adhesives under vibrating loads. It is desirable for aircraft engineers to know if the wood or the adhesive is likely to fail under repeated stress, what allowable stresses may be employed for various loading conditions, and what the endurance limit may be. Should the bond show a tendency to fail, it is desirable to know if certain adhesives behave better than others. In this paper are contained some of the results so far obtained in a continuing program of research into the behavior of plywood and laminated wood of both normal and high density. Results on only normal-density plywood and laminated wood, bonded with thermosetting phenol-formaldehyde resin and with cold-setting urea-formaldehyde resin, are presented.

Of the fatigue behavior of plain wood, comparatively little is known, and still less is known about that of plywood. Stanton (1)[3] in 1916 concluded that the endurance limit of spruce was about 27 per cent of the tensile strength which, in the material he employed, was low (6800 psi). Kommers (2) in 1927 reported tests made at the Forest Products Laboratory, on the basis of which values ranging from 26 to 30 per cent of the static bending strength were assigned Southern white oak, Sitka spruce, and Douglas fir. Schlyter (3) found that Sitka spruce, European spruce, and common European pine had endurance limits 22 to 25 per cent of the static bending strength. Kraemer (4) working with hardwoods (walnut and ash) and softwoods (pine and spruce) found endurance limits ranging from 25 to 33 per cent of the bending strength. He discovered, moreover, that the endurance limit of the heavier woods could in some cases be evaluated by as few as 20,000 cycles, whereas the lighter specimens required several million cycles. Kollmann (5), evaluating the foregoing results, points out that, although the endurance limits for wood are relatively low, the ratio of endurance limit to specific gravity may be relatively high compared to the metals used in aircraft. Kraemer (6) investigating plywood, bonded with synthetic phenolic-resin film and with casein, found that if the grain of face plies was parallel to the axis of the test piece, the endurance limit was 26 per cent of the tensile strength for resin-bonded and 25 per cent for casein-bonded plywood. If the grain of the face plies was placed at an angle of 45 degrees to the axis, the endurance limit for resin-bonded material rose to 52 per cent; for casein-bonded it remained practically the same, or 26 per cent. Failure of the casein-bonded plywood in this case was caused by exfoliation of the veneers.

Testing Procedure

Material. Aircraft-quality birch veneer, $^1/_{16}$ in. thick, was selected because of its strength and toughness and because it is moderately difficult to bond. The combination of strong wood and somewhat difficult bonding ability, consequently, was expected to reveal any tendency for the bond to fail under repeated stress. Adhesives were thermosetting phenol-formaldehyde film (Tego) and an aqueous solution of cold-setting urea-formaldehyde (Uformite CB551). These were representative of the two principal types of synthetic-resin adhesives commonly employed in aircraft.

Veneers were laid up and bonded as follows:

(*a*) Phenolic-resin film: Two-ply laminated, three-ply laminated, three-ply plywood.
(*b*) Urea-resin solution: Three-ply plywood.

Both laminated- and regular-plywood construction were investigated in the phenolic-bonded material, because differing stress conditions are set up in laminated wood and in plywood, as is indicated in Fig. 1. Laminated wood behaves much the same

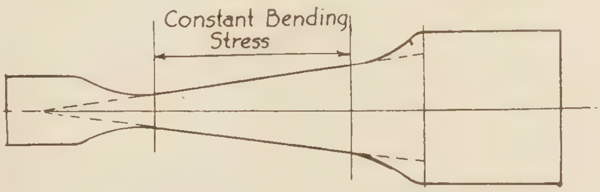

Fig. 1 Stress Distribution in Laminated Wood and Plywood

as solid wood; but in plywood, with the face grain parallel to the specimen axis, the bending stresses are almost entirely concentrated in the face plies, and the shear-stress intensity practically reaches its maximum at the glue line. If the bond were weakened by repeated stress, the behavior of laminated wood and plywood might be expected to differ. Only three-ply plywood was investigated in the urea-bonded material because experience with the phenolic-bonded materials indicated that glue-line failure was more apt to occur, if at all, in plywood than in laminated wood.

Machines. Cantilever-type machines were employed because these were more adaptable to flat plywood sheet material. Moreover, the loading conditions provided both bending and shearing stresses. Motors operated at 1750 rpm. Fig. 2 illustrates the method of testing; the specimen has failed.

Type of Specimen (Fatigue). Various types of specimens were tested to determine the shape which would most consistently yield fractures free of splits and would at the same time provide constant stress over a representative length of cantilever specimen. It was found that the general shape, illustrated in Fig. 3, gave good results for the varying lengths of specimens employed.

To provide standards of comparison, static bending and tension specimens were cut from each sheet of material adjacent to the points from which fatigue specimens were cut. Both bending

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[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Wood Industries Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

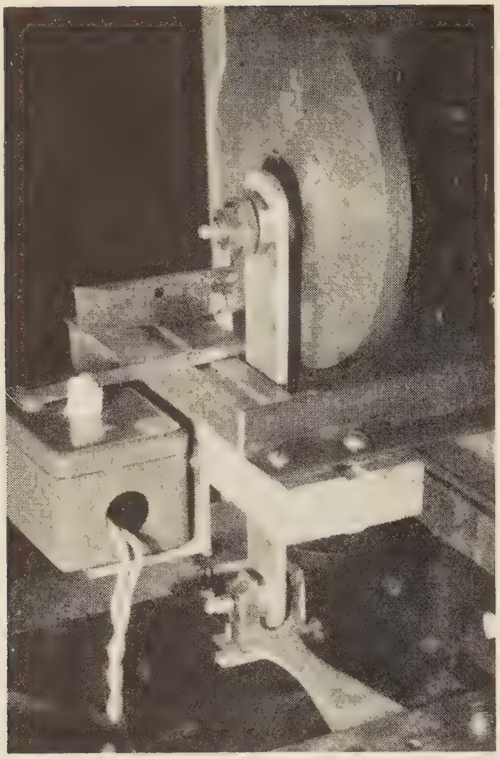

Fig. 2 Arrangement of Machine and Specimen for Fatigue Test

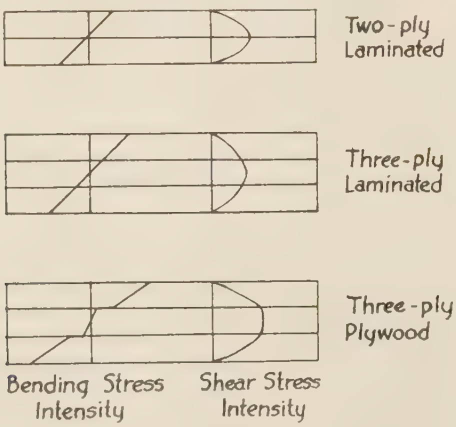

Fig. 3 Typical Fatigue Specimen

and tension specimens were cut from the phenolic-resin-bonded sheets but only bending specimens were employed for the urea-resin-bonded plywood, because experience seemed to indicate that the bending test provided a better basis of comparison, inasmuch as the fatigue tests were basically bending tests. Furthermore, reliable tensile tests are difficult to obtain with wood.

Static bending specimens were tested on a span 16 to 20 times the depth of the specimen, to avoid appreciable shear deformations, and loads were applied at the third-points through small, rounded blocks of compreg. Test specimens were 1 in. wide.

Glue-line shear tests were made on standard plywood shear-test specimens (7), cut from the fatigue specimens after failure, and from the static bending control specimens. In most instances specimens were the standard 1-in. width, but some were less.

Testing

Each fatigue specimen was first calibrated by clamping the fixed end and loading the free end with dead weights to obtain the load-deflection curve. From this the modulus of elasticity was obtained and the stress for a given deflection calculated. Deflections were read to the nearest thousandth inch with an Ames dial. Dimensions of the specimen were measured with a standard micrometer.

After calibration, the fatigue specimen was clamped in the machine and the motor started. In the preliminary runs the motor was stopped at rather frequent intervals, and the specimen was inspected for extent and type of incipient rupture. After a satisfactory shape and size of test piece had been found, specimens were in general permitted to remain in the machine until failure had occurred.

It was necessary to determine some criterion of failure because a piece which had cracked and was carrying no load might hang on for a considerable number of cycles before parting. Consequently, when a crack had formed completely across the width of the piece, so that the cantilever to all intents and purposes was merely hinged at the line of the crack, failure was considered to have occurred. In general, it was found that failure, so defined, followed soon after the first appearance of a fatigue crack.

Fatigue stresses ranged from 90 to 20 per cent of the static moduli of rupture of the controls. Because of the variability of the material, four to six pieces of each length were tested and six to eight lengths were employed. Two sets of runs from two ship-

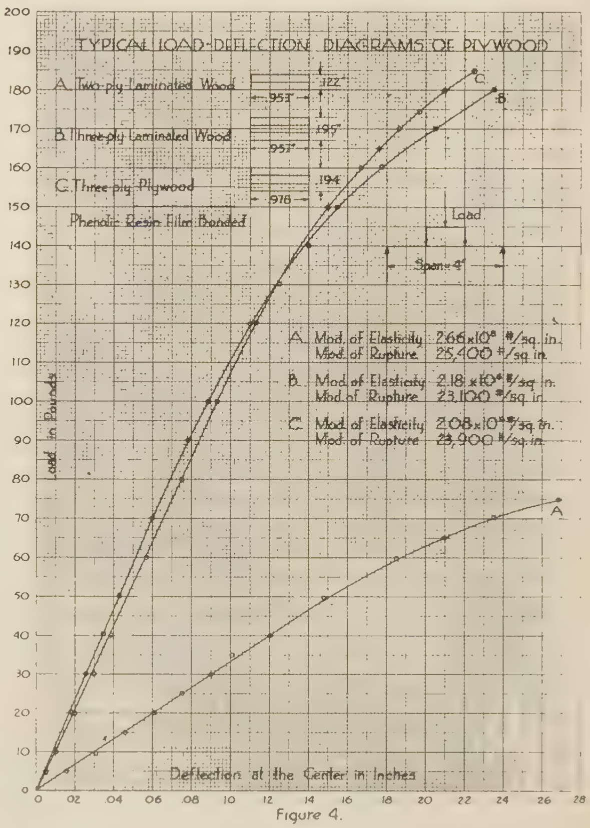

Fig. 4 Typical Load-Deflection Diagrams of Plywood

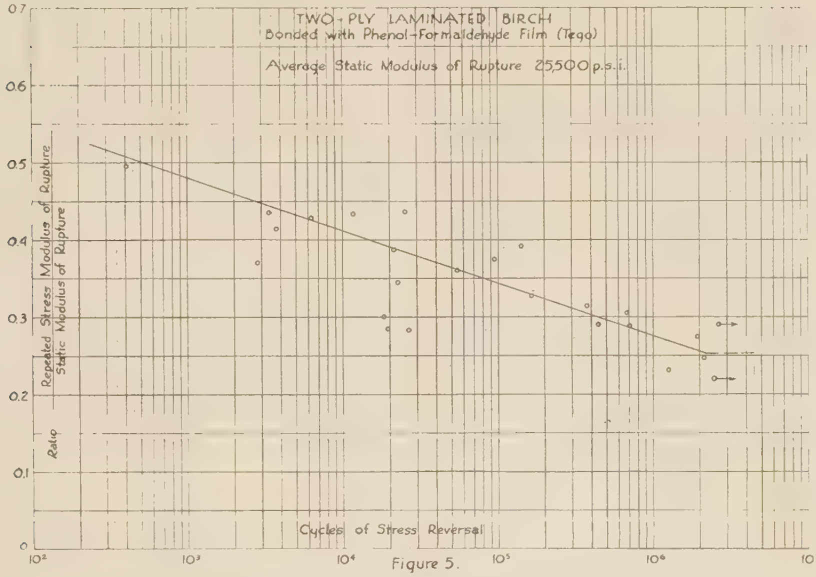

FIG. 5 RESULTS OF FATIGUE TESTS ON TWO-PLY LAMINATED BIRCH
(Bonded with phenol-formaldehyde film, Tego. Average static modulus of rupture, 25,500 psi.)

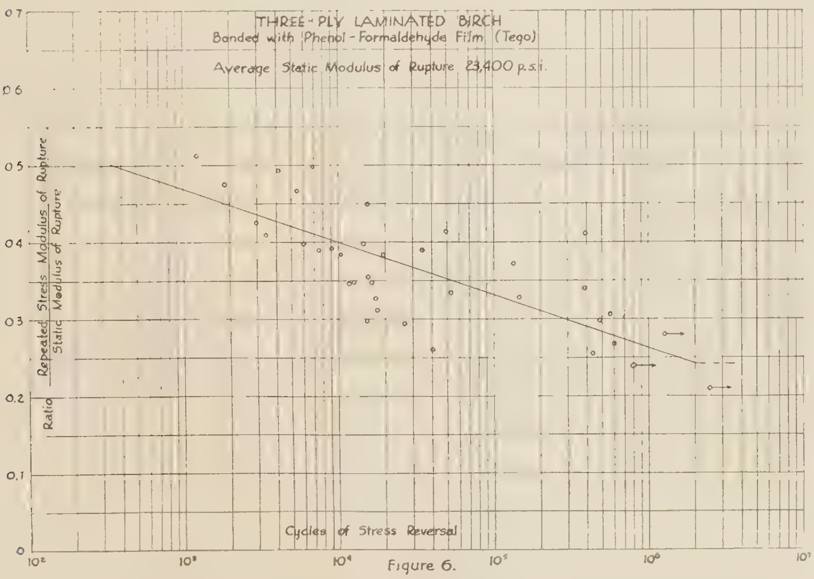

FIG. 6 RESULTS OF FATIGUE TESTS ON THREE-PLY LAMINATED BIRCH
(Bonded with phenol-formaldehyde film, Tego. Average static modulus of rupture, 23,400 psi.)

ments of panels were made on the three-ply phenolic-bonded plywood. Considerable scatter was found in both sets, but the averages fell fairly well along the same line.

TEST RESULTS

Results are presented in Table 1 and in Figs. 4 to 8, inclusive.

Static Tests. Typical static bending load-deflection curves for the phenolic-film-bonded materials are given in Fig. 4. They conform to the general shape of such curves for plain wood and the tangent proportional limit is found to be 65 to 70 per cent of the modulus of rupture. Moduli of rupture and of elasticity are representative of material of this species and quality.

In Table 1, average specific gravity, moisture content, proportional limit, modulus-of-rupture, and modulus-of-elasticity values are summarized for the four classes of test specimens.

Fatigue Tests. Fatigue results are summarized in Figs. 5 to

TABLE 1 RESULTS OF TESTS

Material	Specific gravity	Moisture content, per cent	Tangent proportional limit, psi	Modulus of rupture, psi	Apparent modulus of elasticity, 10^6 psi
Phenolic film:					
2-ply laminated	0.67	7–9	16500	25000	2.4
3-ply laminated	0.68	7–9	15000	23400	2.1
3-ply plywood	0.72	7–9	13500	20650	2.1
Urea formaldehyde:					
3-ply plywood	0.66	7–9	12000	17400	2.0

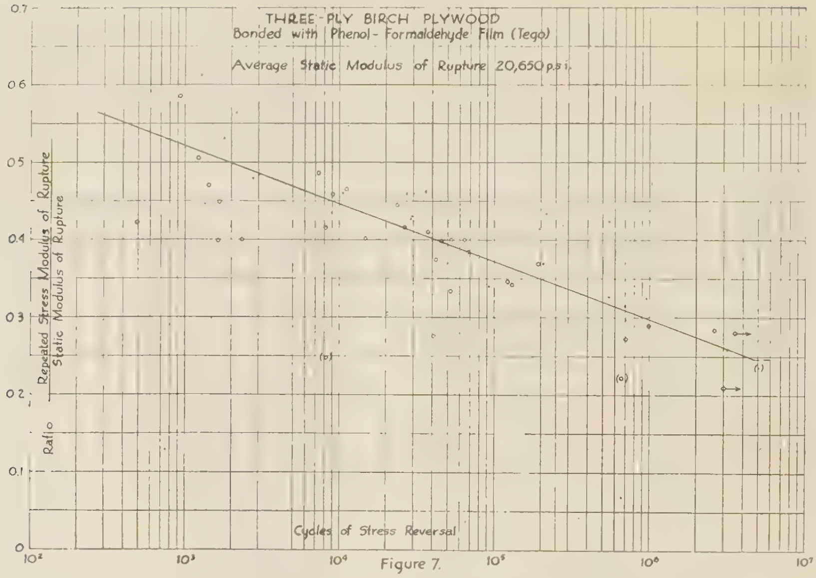

FIG. 7 RESULTS OF FATIGUE TESTS ON THREE-PLY PLYWOOD BIRCH
(Bonded with phenol-formaldehyde film, Tego. Average static modulus of rupture, 20,650 psi.)

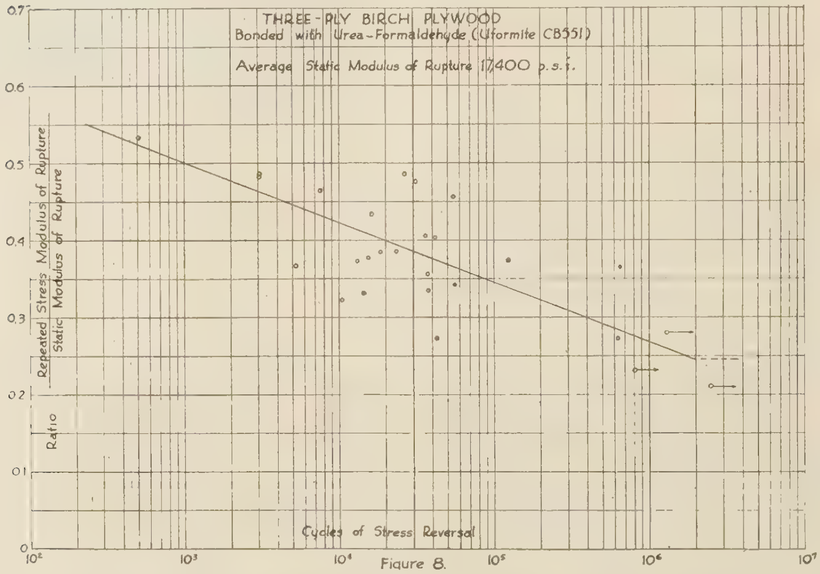

FIG. 8 RESULTS OF FATIGUE TESTS ON THREE-PLY PLYWOOD BIRCH
(Bonded with urea-formaldehyde "Uformite CB551." Average static modulus of rupture, 17,400 psi.)

8, inclusive. Fatigue stresses are plotted as ratios of the static moduli of rupture, and numbers of cycles are plotted on a logarithmic scale to bring out the trend. As is to be expected, a considerable amount of scatter is found in spite of the care with which the material was selected and cut. The general trend is apparent, however, and the averages of the plotted points fall fairly close to the indicated straight lines.

The horizontal dotted lines, at approximately 25 per cent of modulus of rupture, represent averages of specimens which had not failed at 1,000,000 to 3,000,000 cycles of stress reversal. Whether or not these represent true endurance limits cannot of course be stated categorically. Some materials appear to have no true endurance limit, in the sense that they can be repeatedly stressed for an indefinite period without failing.

At higher ratios of fatigue stress to static modulus of rupture, the number of cycles to failure drops off until at 90 per cent, 100 reversals may be sufficient.

Periodic examination of specimens during the test runs revealed little tendency for the bond to fail and the veneers to separate. After the outer plies had cracked through to the glue line, that is, after failure as previously defined had occurred, delamination did in some instances take place. This tendency was somewhat more prevalent in the plywood than in the laminated material.

CONCLUSIONS

The tests so far concluded in this program indicate that, in birch plywood and laminated wood, bonded with thermosetting

phenol-formaldehyde film (Tego) and in plywood bonded with an aqueous dispersion of cold-setting urea-formaldehyde resin (Uformite CB551):

(*a*) Fatigue failures are primarily wood failures.

(*b*) Delamination of the veneers occurs very seldom before the outer plies have given way.

(*c*) The material may be expected to withstand at least 2,000,000 stress reversals without failing, when stressed to 25 per cent or less of the static modulus of rupture.

Acknowledgments

The authors acknowledge their indebtedness to the individuals who assisted in this program, particularly to Mr. Harry Majors, Jr., assistant in the department of mechanical engineering at The Massachusetts Institute of Technology, who carried the brunt of the experimental work, and to Dr. Wm. M. Murray of the same department who was instrumental in devising the testing equipment. Messrs. S. N. Tu, M. Becker, and F. Carroll assisted in the testing.

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Discussion

M. Finlayson.[4] Were load-deformation curves taken after the material had been stressed through a large number of cycles, in order to determine whether or not the load as well as the deflection was constant throughout the test? It is suggested that either load-deformation curves at various stages of the test or a load indicator on the machine would show whether or not the load remained constant throughout the test, a point which might have some significance, especially at high loads or over a large number of cycles.

J. E. Gurvitch.[5] It may be well to add to the information in this paper that results obtained by A. R. Crocker,[6] on the fatigue strength of compreg (compressed laminated resin-impregnated wood), and the writer, on the fatigue strength of various glue lines on compreg, are in line with the work done by the authors. The fatigue strength of compreg was found to be 11,000 psi, approximately 25 per cent of the modulus of rupture of this material (40,000 to 50,000 psi). Compreg could be stressed with that load for several million cycles without failure. Fatigue failures were also similar to those described in the paper. After the initial crack developed in the outer ply of the compreg, the line of failure often followed the glue line between the outside and the adjacent ply.

The fatigue resistance of various glues, i.e., casein, urea formaldehyde, and phenol formaldehyde, were tested for fatigue strength by gluing thin compreg laminations ($^3/_{32}$ in.) together with each of the glues mentioned. These laminated beams were subjected to 11,000 psi for a minimum of 7,000,000 cycles. No failures occurred in the glue lines of any of these adhesives. Controls were given an accelerated aging test by heating at 65 C for 1 month and then fatigued for 7,000,000 cycles at the stated stress. No failures were observed.

All fatigue tests were made on compreg specimens $^1/_2$ in. $\times$ $^1/_2$ in. $\times$ 11 in., the direction of grain being parallel to the longer dimension. A cantilever system was used in which the beam was held stationary at one end and flexed by a cam mounted on an electric motor.

In all tests, it was noted that the specimen generated heat at the point of maximum stress. No quantitative measurements were taken, however, of the temperature rise.

P. D. Zottu.[7] Did the authors observe any differences between the specimens tested which were bonded with urea- and phenol-formaldehyde glues?

Authors' Closure

Load deformation curves were not taken on the test specimens after being stressed to failure, inasmuch as the type of failure generally precluded such measurements. In a subsequent series of tests, in which tests will not be carried to failure, it is intended to make load-deformation measurements both before and after repeated stress.

The tests on compreg reported by Mr. Gurvitch are very interesting in that they indicate the behavior of compressed impregnated wood under repeated stress to be similar to that of plain wood and plywood.

There was no significant difference in the behavior of phenol-formaldehyde bonded material as compared with urea-formaldehyde bonded types.

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High-Density Plywood

By MALCOLM FINLAYSON,[1] ARMSTRONG CORK COMPANY, LANCASTER, PA.

The physical properties of high-density plywood produced from Tego-bonded birch veneer, the grain direction of all plies parallel, are presented and compared with the corresponding properties of various metals. It is shown that the strength properties are directly related to the specific gravity. Stress-strain curves in tension indicate that this material not only has no normal yield point but actually shows a decrease in elongation per unit load at high loads. The moduli of elasticity in tension and compression are shown to be greatly different. The behavior in torsion is discussed, and the effect on the tensile strength of cross-laying the veneers is presented. The effect of immersion in water and the effect of various humidity conditions are given.

Introduction

The improvement of the properties of metal by heat-treatment, tempering, and alloying is well developed and well understood. The somewhat similar improvement of wood by subdivision, elimination of flaws, reassembly, and densification, while not new, has not been studied to any great extent and is not so familiar to engineers.

Subdivision and reassembly (cutting into thin veneers and reassembling these veneers with an adhesive) permits the elimination of local flaws and makes possible the preparation of a more homogeneous material by rearrangement of the grain direction of the various plies. The distribution of strength by the control of the grain direction of the various plies makes available a wide variety of materials, each having a combination of properties suited to a particular use. The engineer is in this way afforded an opportunity to design his work in a material particularly suited to the use which he has in mind.

In normal plywood, subdivision and reassembly add little to the aggregate strength of the resulting material. The use of many thin plies will show some improvement due to the elimination of local flaws and due to the fact that the grain direction in each sheet of veneer varies from a truly straight grain and from the grain direction of every other sheet. Because of this, the assembly tends to average out to a material having no weakness due to a crooked grain running throughout the thickness of the material. There may also be some slight increase due to the binder effect of excess resin adhesive which has penetrated the veneer at the surfaces during pressing, although there are grounds for the belief that the resin plays only a very minor part in the improvement of mechanical properties, the major part being due to densification only. There will also be some slight increase because of the slight densification occurring during reassembly.

A large increase in strength arises through densification of the veneers, which is obtained by using pressures much greater than normal during the reassembly process. It is well known that hot wood compresses more readily than cold wood and that wet wood compresses more readily than dry wood. It is therefore evident that by controlling the pressure, heat, and moisture content during reassembly, materials of any desired degree of densification can be obtained readily. The thermosetting adhesive used in the process cures while the wood is in the compressed condition and locks it in this condition, so that, providing sufficiently thin veneers have been used, little or no springback results after the material has been cooled and removed from the press. The compression referred to is, of course, only in the thickness dimension, end-grain or across-grain compression presenting entirely different problems.

In the unimpregnated type of high-density plywood, with which this paper deals, the adhesive can be either the film type, which is simple and clean to use and gives a uniform spread without adjustment, or the liquid type, which is spread by rollers and is apt to be somewhat messy to handle and relatively difficult to control to a uniform spread, but which is often cheaper. All the data presented here are on material made with the film-type adhesive. The accompanying charts refer to high-density plywood made from $^1/_{32}$-in. rotary-cut birch veneer, all local flaws excluded, with the grain direction of all the plies parallel, i.e., the so-called "laminated wood." There are a few exceptions which are specifically noted.

Testing Methods

Before testing, all samples were thoroughly conditioned at 70 F, and 50–55 per cent relative humidity. The samples used in the determination of the tensile strength were $13^1/_4$ in. long, having ends for the holder $4^1/_4$ in. long, and a constricted section $4^1/_4$ in. long, of which the center $2^1/_2$ in. was of constant cross section of $^1/_4$ in. The thickness of the sample was the thickness of the board as made, in all cases approximately 1 in. The width of the ends was $1^1/_4$ in. The long ends are required to insure that the shear strength at the shoulders shall be greater than the tensile strength at the constricted section. A Berry strain gage (5 magnification) was attached to the specimen at the constricted section in order to determine the modulus of elasticity. Elongation was measured with a dial gage reading in 0.001 in. Loading was by hand and slow, with 10-sec stops at the appropriate points for accurate reading of the elongation. Sample failures in this type of setup were clean tensile fractures.

The samples used in the determination of the compressive strength were 2 in. × $^1/_4$ in. × thickness; the thickness approximately 1 in. in all cases. Deflection was measured with a dial gage reading in 0.001 in. When the load is applied parallel to the grain, a clean compressive failure results, but when the load is applied perpendicular to the direction of the grain, failure is in tension perpendicular to the direction of compression and perpendicular to the plies.

The samples used in the determination of the modulus of rupture were 9 in. × 1 in. × 1 in. The span was 7 in., and the load was applied at the center of the span. In the stress-strain curves, corrections were made for the deflection due to the indentation of the beam by the loading bar.

The water-immersion and humidity tests were taken on 1-in. cubes. The samples were first conditioned for at least 1 week at 70 F, and 50 per cent relative humidity, and then exposed to the test conditions. The materials were kept at the test conditions until the dimensions were stable over a period of 3 days, except in the case of the boiling-water test, which extended over a period of 8 hr.

All points plotted in the curves are the average of at least three

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Contributed by the Wood Industries Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers.

tests, except for Fig. 9, in which the points are for one test only. No results have been omitted in the computation of the averages.

Characteristics in Tension and Compression

Fig. 1 shows the relationship between ultimate tensile strength and specific gravity, and between ultimate compressive strength and specific gravity. Included are several results on materials made from $^1/_{40}$- and from $^1/_{48}$-in. veneer of the same type. Apparently, the use of thinner veneers, at least in this range of thickness, does not affect significantly the relationship of ultimate tensile and ultimate compressive strengths to specific gravity. This supports the assumption that significant improvement in mechanical properties is a function only of the degree of densification and is not related to the resin content, unless the resin content is increased to such an extent that the wood fibers act as a filler. It is true, of course, that increasing the number of plies by decreasing their individual thickness will give better averaging of grain direction and better averaging of imperceptible local flaws, but it is quite probable that the resulting improvement falls within the experimental error of measurement. This slight general improvement, at an increased cost, naturally, may often be worth while in applications where rigid demands are made on the material and where all precautions to prevent local very slight weaknesses are essential.

It is interesting to note that the straight lines showing the relation of tensile strength and compressive strength to specific gravity, when extended, pass through the origin. This means that, on a strength-for-weight basis, material of relatively high density is no stronger than material of relatively low density. The strength-for-weight ratio is constant at about 24,700 psi. A certain minimum specific gravity, probably about 0.80, is required to insure adequate bonding of the veneers, i.e., that the bond shall be stronger than the wood. On a volume basis, however, the higher-density material is superior and should have a wide scope of use where neat dense design is desired. The same thing is true of compressive strength, with the strength-for-weight ratio about 15,000 psi. The range of strength in tension, 20,000 to 35,000 psi, and the range of strength in compression, 12,000 to 21,000 psi, make this material suitable for a wide variety of applications.

Fig. 2 shows stress-strain curves in tension for various materials in comparison with a typical curve for high-density plywood. The curve for high-density plywood is for tension parallel to the grain direction. The high-density plywood shows a decided increase in modulus of elasticity at high loads in place of the abrupt yield point so common in metals. This is shown some-

TABLE 1 COMPARISON OF HIGH-DENSITY PLYWOOD AND VARIOUS METALS; TENSILE STRENGTH FOR WEIGHT; ULTIMATE-STRENGTH VERSUS YIELD-POINT STRENGTH

Material	Specific gravity	Yield point in tension, psi	Strength, psi / Specific gravity
High-density plywood.........	0.8–1.3	None	24700
Low-carbon steel..............	7.75	36000	4600
Alclad aluminum.............	2.89	47000	16200
High-carbon steel............	7.83	75000	9500
Magnesium alloy.............	1.78	23000	12900

TABLE 2 COMPARISON OF HIGH-DENSITY PLYWOOD AND VARIOUS METALS; TENSILE STRENGTH FOR WEIGHT; ULTIMATE STRENGTH VERSUS ULTIMATE STRENGTH

Material	Specific gravity	Strength in tension, psi	Strength, psi / Specific gravity
High-density plywood...	0.8–1.3	Varies with specific gravity	24700
Steel, heat-treated.......	7.75	100000	12900
	7.75	125000	16100
	7.75	150000	19300
	7.75	175000	22600
Aluminum, various alloys	2.81	40000	14200
	2.81	50000	17800
	2.81	60000	21400
Magnesium alloy........	1.78	43000	24200

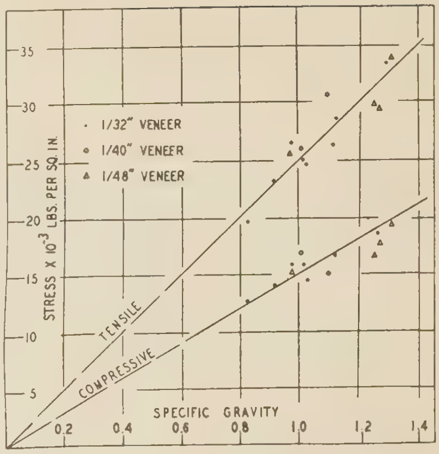

Fig. 1 Ultimate Tensile and Compressive Strengths Versus Specific Gravity

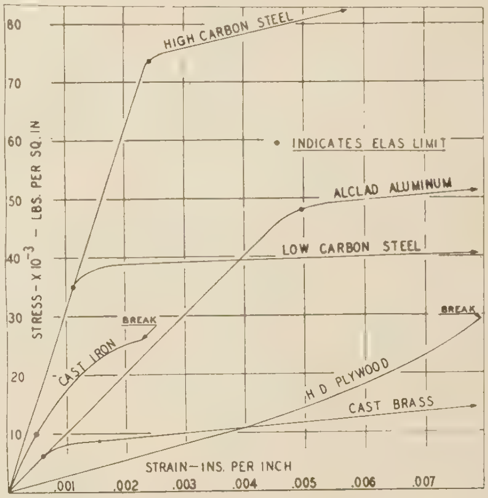

Fig. 2 Stress-Strain Curves of High-Density Plywood and Various Metals in Tension

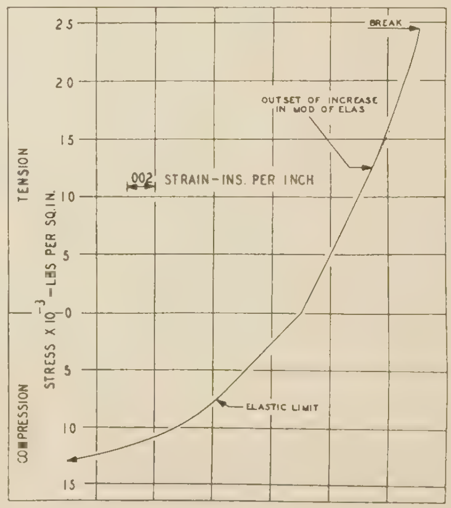

Fig. 3 Typical Stress-Strain Curve of High-Density Plywood in Tension and Compression

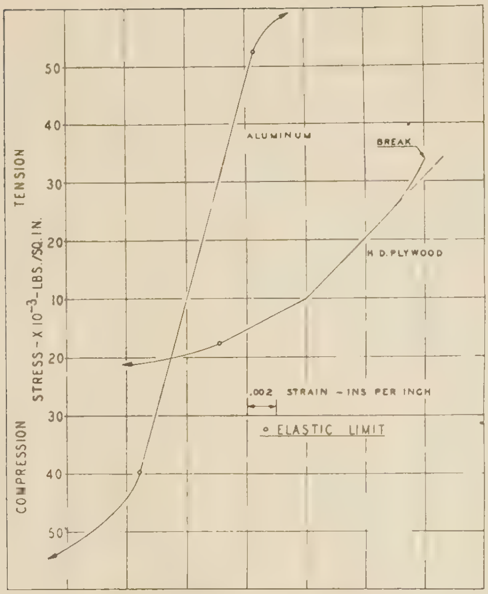

Fig. 4 Comparative Stress-Strain Curves

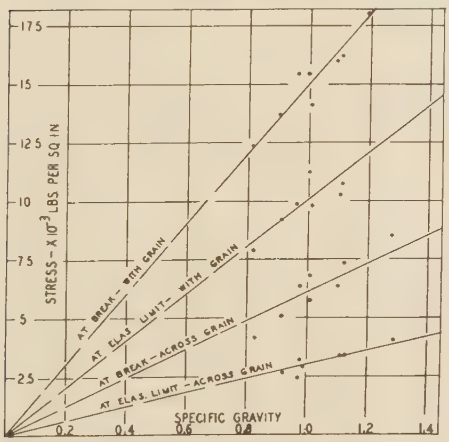

Fig. 5 Compressive Strength Versus Specific Gravity

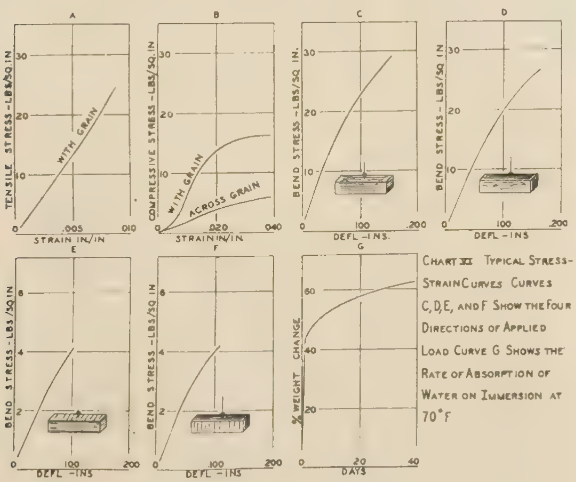

Fig. 6 Typical Stress-Strain Curves

what more clearly in Fig. 3. In every tension test, this decrease in elongation per unit load was quite apparent and, unless some unknown constant error is present in the method, is characteristic of the material. This anomalous behavior in tension indicates that high-density plywood, of the laminated type at least, can be loaded to the breaking point without the appearance of permanent distortion and suggests that, in any comparison on a strength-for-weight basis, the ultimate strength of the high-density plywood can be compared with the yield point of the metal, since in many applications the point at which permanent distortion occurs is the limiting point. Regardless of whether the comparison is on the basis of ultimate strength versus yield point or ultimate strength versus ultimate strength, high-density plywood yields its place at the top of the list to none of the commonly used metals. Table 1 gives a comparison of the strength for weight of high-density plywood and several metals on the ultimate-strength versus yield-point basis, while Table 2 gives a comparison on the ultimate-strength versus ultimate-strength basis.

Comparison With Metals in Compression

In compression parallel to the grain high-density plywood does not show the same superiority but is still well up on the list. If compressive strengths at the yield point are compared on a strength-for-weight basis, high-density plywood is exceeded only by the heat-treated grades of aluminum alloy. If the comparison is made on the basis of ultimate strength for weight, the untreated aluminum alloys also exceed high-density plywood, and cast iron moves well up to the same range as the high-density plywood. These comparisons are shown in Table 3. This table indicates that in compression as well as in tension, although not to the same degree, high-density plywood is a very promising material. It should be stated, however, that the modulus of elasticity in compression parallel to the grain is not comparable, even on a weight basis, to the modulus of elasticity in compression of various metals. This is shown in Table 4.

TABLE 3 COMPARISON OF HIGH-DENSITY PLYWOOD AND VARIOUS METALS; COMPRESSIVE STRENGTH FOR WEIGHT; YIELD STRESS AND AT ULTIMATE STRESS

Material	Specific gravity	Compressive strength Yield, psi	Compressive strength Ultimate, psi	Yield strength / Specific gravity psi	Ultimate strength / Specific gravity psi
Aluminum alloy					
25SW	2.79	25000	48000	9000	17200
25ST	2.79	35000	58000	12500	20700
Cast iron	7.23	20000	90000	2270	12400
Wrought iron	7.70	36000	60000	4670	7800
Structural steel	7.80	36000	60000	4600	7650
High-density plywood	0.8–1.3	...	...	10000	15000

TABLE 4 COMPARISON OF HIGH-DENSITY PLYWOOD AND VARIOUS METALS; MODULUS OF ELASTICITY IN COMPRESSION ON A WEIGHT BASIS

Material	Specific gravity	Modulus of elasticity in compression, psi	Modulus of elasticity / Specific gravity psi
Aluminum	2.79	10×10^6	3.58×10^6
Cast iron	7.23	12×10^6	1.65×10^7
Steel, various types	7.80	29×10^6	3.72×10^6
High-density plywood	0.8–1.3	....	1.00×10^6

Fig. 4 gives a typical stress-strain curve in both tension and compression for a sample of high-density plywood (this particular sample having a specific gravity of 1.03), together with a similar stress-strain curve for aluminum. It is evident that the moduli of elasticity in tension and compression are appreciably different for high-density plywood but are the same for aluminum. This lower modulus of elasticity in compression in high-density plywood is apparently not due to bending during the test, since samples 2 in. × 1 in. × $^1/_2$ in., 2 in. × 1 in, × 1 in., and 1 in. × 1 in. × 1 in. all give the same slope of the stress-strain curve and the same elastic limit. It seems reasonable to

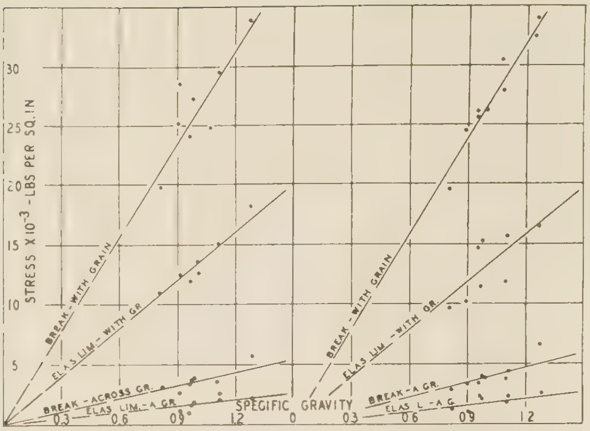

Fig. 7 Bend Strength Versus Specific Gravity; Load Perpendicular to Plies

Fig. 8 Bend Strength Versus Specific Gravity; Load Parallel to Plies

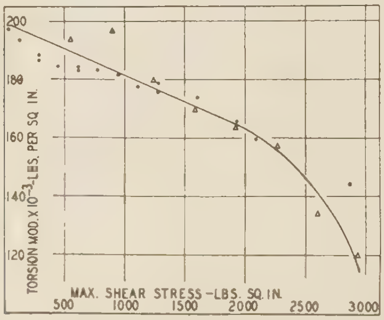

Fig. 9 Torsion Modulus Versus Shear Stress

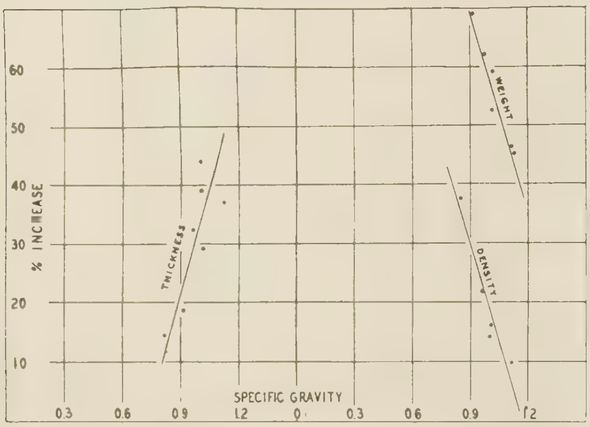

Fig. 10 Change in Thickness, Weight, and Density on Immersion in Water at 70 F

suppose that, since voids are still present, the material, under a compressive load, is squeezed into these voids, rather than against itself as in a metal. The fact that distortion under a compressive load is different from distortion under a tensile load may have to be taken into consideration in certain applications.

Fig. 5 gives the relationship to specific gravity of the compressive strength both with grain and across grain, at ultimate strength, and at the elastic limit.

Elastic Moduli

Fig. 6, curves *A-F*, gives typical stress-strain curves of high-density plywood. The range of the modulus of elasticity in tension of this material is from 2.2×10^6 psi to 4.1×10^6 psi, increasing with increasing specific gravity, as do all the moduli reported. The range of the modulus of elasticity in compression, when the load is applied parallel to the grain, is from 0.8×10^6 to 1.3×10^6 psi. The range when the load is applied perpendicular to the grain and perpendicular to the direction of compression in making is from 0.12×10^6 to 0.28×10^6 psi. The modulus of elasticity in bending, when the load is applied perpendicular to the grain, ranges from 1.8×10^6 to 4.0×10^6 psi. When the load is applied parallel to the grain in bending, the range is from 0.25×10^6 to 0.55×10^6 psi.

Characteristics in Bending

Figs. 7 and 8 show the relationship to specific gravity of bending stresses, both ultimate and at the elastic limit, and both with and across grain. The four directions are here shown to resolve into two directions, since the two directions in which the load is applied perpendicular to the grain have the same strength, and the two directions in which the load is applied parallel to the grain have the same strength, indicating that high-density plywood is anisotropic in only two directions, as is to be expected. Thus, Figs. 7 and 8, although derived from data obtained from tests in different directions, are identical.

Characteristics in Torsion[2]

A cylindrical sample of high-density plywood made from $^1/_{40}$-in. rotary-cut northern-yellow-birch veneer, having a specific gravity of 0.95, was tested in torsion in order to determine the shear strength and the torsion modulus. The test was made with the laminations parallel to the long dimension of the sample. Three tests were made on the same specimen. In the first test, increments of 165 psi of shear stress were used to a maximum of 1323 psi. The rod was then unloaded and recovered to an angle of 0.035 deg per in. of specimen length in 3 min. In the second test, increments of 332 psi of shear stress were used to a maximum load of 2935 psi. The rod was then unloaded and recovered to an angle of 0.116 deg per in. of specimen length after 5 min. In the third test, increments of approximately 345 psi of shear stress were used to the point where failure occurred. The results of these tests are shown in Tables 5, 6, and 7, respectively. The ultimate shear stress in torsion of this relatively low-density material of 3426 psi is a decided improvement over normal woods, and in consequence the transmission of forces from wood into metal is greatly facilitated.

TABLE 5 TEST NO. 1: SHEAR STRESS AND TORSION MODULUS ON HIGH-DENSITY PLYWOOD

Torque, in-lb	Shear stress, psi	Angle of deflection in deg per in. of specimen length	Torsion modulus, psi
0	0	0	
12	82.7	0.053	198000
24	165.4	0.108	194000
48	331	0.220	189000
72	496	0.340	185000
96	662	0.456	184000
120	827	0.570	184000
144	993	0.692	182000
168	1158	0.823	178000
192	1323	0.953	176000

Fig. 9 shows the torsion modulus plotted against the shear stress. Apparently, the deflection is not truly proportional at any point on the curve and the torsion modulus varies continuously. This information may be of value in certain instances in which the material may be subjected to severe torsional stresses.[3]

[2] Data by Pittsburgh Testing Laboratory.

[3] End of excerpts from Pittsburgh Testing Laboratory Report.

TABLE 6 TEST NO. 2: SHEAR STRESS AND TORSION MODULUS ON HIGH-DENSITY PLYWOOD

Torque, in-lb	Shear stress, psi	Angle of deflection in deg per in. of specimen length	Torsion modulus, psi
0	0	0	
48	331	0.224	187000
96	662	0.452	185000
144	993	0.692	182000
192	1323	0.937	179000
240	1654	1.20	174000
288	1983	1.51	166000
312	2150	1.71	160000
426	2935	2.58	144000

TABLE 7 TEST NO. 3: SHEAR STRESS AND TORSION MODULUS ON HIGH-DENSITY PLYWOOD

Torque, in-lb	Shear stress, psi	Angle of deflection in deg per in. of specimen length	Torsion modulus, psi
0	0	0	
87	600	0.391	194000
137	944	0.608	197000
187	1289	0.904	180000
237	1634	1.22	170000
287	1979	1.53	164000
337	2323	1.88	157000
387	2668	2.52	134000
437	3013	3.21	119000
447	3082	4.05	96400
487	3358	4.73	89900
497[a]	3427	4.82	90200

[a] Failure.

A comparison of the ultimate shear strength of high-density plywood and aluminum seems, at first glance, to show high-density plywood to be greatly inferior, since its 3426 psi is only about $^1/_9$ that of aluminum and its specific gravity is $^1/_3$ that of aluminum. However, as shown by De Bruyne (17),[4] if high-density plywood is considered as a metal, and the effect of energy absorption is ignored for a moment, a comparison of their resistance to torsional shock shows them to be about on a par. The maximum shear stress f developed, if a circular shaft of radius r is suddenly given an angular velocity w, is given by

$$f = rw(Ep)^{1/2}$$

where E is the modulus of rigidity and p is the specific gravity. Thus, the ratio of shear stresses developed in similar shafts of aluminum and high-density plywood will be

$$\left(\frac{E_A p_A}{E_P p_P}\right)^{1/2}$$

Putting

$$E_A = 4.2 \times 10^6 \qquad p_A = 2.85$$
$$E_P = 0.18 \times 10^6 \qquad p_P = 0.95$$

the ratio of shear stresses is found to be about 8.4 to 1. The ratio of ultimate shear strengths is 8.8 to 1. Thus, even without considering the effect of energy absorption on the reduction of shear stress, high-density plywood is shown to have a high resistance to torsional shock.

It is known that high-density plywood has a high energy absorption compared to metals. The beneficial effects of this high energy absorption can be illustrated by considering a member such as a propeller blade which is free at one end and given a sudden twist at the other (17). The effect of the sudden twist will be to send a torsional wave down the member; this wave will be reflected at the free end (the tip of the blade) and will return to the fixed end with practically undiminished energy if the material has small energy absorption. As the free end and the fixed end form an antinode and a node, respectively, the original and reflected waves will interact to produce a maximum stress at the fixed end of twice the value which would obtain in the absence of reflection. If, however, the material has large energy absorption, the reflected wave will return with diminished energy and the maximum stress will be correspondingly reduced. If we take into account the difference in energy absorption between high-density plywood and aluminum alloy, it is quite possible that high-density plywood has superior resistance to torsional stresses of this type.

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

Variable-Density Materials

As an example of the variations possible in the manufacture of high-density plywood, two types of variable-density boards were made. The first consisted of 31 plies of $^1/_{28}$-in. mahogany veneer, alternated with 30 plies of $^1/_{30}$-in. poplar veneer, with the grain direction of the poplar veneer at right angles to the grain direction of the mahogany veneer. This construction at the high-density end was stepped down gradually to a total of 35 plies at the low-density end. The entire assembly was pressed to a uniform thickness of 1 in. The second consisted of a total of 69 plies of which only 5 were poplar and only the poplar veneers were laid cross-grain. This construction at the high-density end was stepped down gradually to a total of 41 plies at the low-density end. Table 8 gives the tensile strength, specific gravity, and the number of plies at various points along the first board. Table 9 gives the tensile strength at the high- and low-density ends of four boards, of the second type, all of exactly the same construction, together with the corresponding specific gravities and number of plies.

TABLE 8 TENSILE STRENGTH OF VARIABLE-DENSITY BOARDS AT VARIOUS POINTS ALONG THE BOARD; PLIES CROSS-LAID

Number of plies	Tensile strength, psi	Specific gravity
61	16700	1.23
55	15600	1.10
49	14900	1.01
39	14300	0.84
35	13900	0.75

TABLE 9 TENSILE STRENGTH OF VARIABLE-DENSITY BOARDS AT HIGH- AND LOW-DENSITY ENDS; PLIES LAID PARALLEL

Number of plies	Tensile strength, psi	Specific gravity
69	35000	1.40
41	22000	0.86
69	32000	1.37
41	21300	0.84
69	34300	1.39
41	21200	0.88
69	35200	1.39
41	22200	0.87

The figures given in Tables 8 and 9 are the result of a single test only, and can therefore be relied upon only to indicate the general order of magnitude. In Table 9, the checks from board to board are very good and the results are probably quite reliable. It is obvious that the cross-laid structure has resulted in a material showing only a slight difference in tensile strength for a large difference in specific gravity. The material having a very large preponderance of parallel-grain plies gives a very much greater change in tensile strength with specific gravity. This increased change is, of course, at the expense of tensile strength in the opposite direction, the predominantly long-grain material having very low strength across the grain, and the cross-grain material having just as high strength in both directions.

Effects of Moisture

Fig. 10 shows the change in thickness, weight, and density of materials of various specific gravities after immersion in water at 70 F until apparent equilibrium is reached.

Curve G, Fig. 5, shows the rate of absorption by weight of a typical sample. The change in thickness is the change from the original dimension to the final dimension while in equilibrium with water. No account is taken of the regain of compression, which would be the difference between the original thickness and the thickness after swelling and redrying. In this respect, it

might be mentioned that it has proved feasible to hold this regain to essentially zero per cent by proper control of moisture content and temperature during the manufacturing process. However, when a material is subjected to immersion in water, it is usually the total change in dimension which is of interest to the prospective user, rather than the change measured after redrying. It is interesting to note that, while the change in thickness is considerable, there is no tendency toward delamination in properly processed material. Since it is to be expected that the water will tend to release the strains holding the material in the compressed condition, as well as swelling the released fibers, the high-density plywood should show an increase in thickness larger for the higher-density materials, since the higher-density materials have been compressed to a greater degree. This is borne out by the curve showing increase in thickness with increase in specific gravity. The curve showing increase in weight indicates that the lower-density samples take up a greater percentage of water than the higher-density samples. Since the increase in weight is greater than the corresponding increase in thickness (changes in the other dimensions are slight) there is an increase in density, greater for the initially low-density material.

Exposure to various moisture conditions results in relatively large changes in thickness, small changes in the cross-grain dimension, and insignificant changes in the with-grain dimension. Table 10 gives the maximum and minimum dimensional and weight changes, over the entire range of specific gravity, for the indicated conditions.

TABLE 10 CHANGES IN DIMENSION AND WEIGHT OF HIGH-DENSITY PLYWOOD UNDER VARIOUS CONDITIONS

Change in Dimension, per cent

Dimension	Over water —30 F[a]	At temperature noted 70 F	At temperature noted 120 F
Thickness	—2.4 to —4.6	9.0 to 22.4	9.0 to 24.8
Parallel to grain	—0.2 to —0.1	0 to 0.2	0 to 0.3
Across grain	—2.5 to —1.3	0 to 1.6	0.6 to 1.6

[a] Over water vapor at this temperature.

Change in Dimension, per cent

Dimension	Bone-dry at temperature noted —30 F	70 F	212 F
Thickness	—7.4 to —3.2	—2.4 to —6.4	—6.4 to —1.8
Parallel to grain	—0.2 to —0.1	—0.4 to 0	—0.2 to —0.1
Across grain	—2.4 to —1.0	—2.8 to —1.2	—2.3 to —1.0

Change in Dimension, per cent

Immersed in water at 70 F, to equilibrium: Thickness	Parallel to grain	Across grain
14 to 44	0.1 to 0.6	1.3 to 3.2

Change in Dimension, per cent

Immersed in boiling water for 8 hr: Thickness	Parallel to grain	Across grain
15 to 54	0 to 0.7	1.0 to 3.5

Increase in Weight, per cent

Immersed in water at 70 F to equilibrium	Immersed in boiling water 8 hr
45 to 70	50 to 63

It is obvious that high-density plywood of this type is unsuited to exposure to water or high-humidity conditions unless very thoroughly protected by an impervious coating. Since the penetration of moisture is relatively quite slow through the faces, and quite rapid on the end grain, material having a large area could perhaps be adequately protected from large changes in size by coating the ends and edges only.

Uses for High-Density Plywood

The possible uses of high-density plywood are almost without limit. It is evident from the preceding data that this material is not the long-sought, universal, "cure-all," but it is believed that it does have a definite place in applications where high stresses are encountered, providing that it is used intelligently and that a sufficient background of information concerning its properties can be assembled. In actual practice, this new type of material has been widely used in only a few applications. This lack of wide utilization of such an adaptable material is largely because stress data are not available, and because relatively few industrial concerns have pushed the development of this type of product. There are few presses of sufficient capacity to produce this type of material, in the large sizes desired, which do not have a large backlog of other essential work.

The greatest progress in the utilization of high-density plywood has been made in England in the development of this material for aircraft propellers, peculiarly enough, one of the most highly stressed units in an aircraft. These several types of high-density plywood in use on fighter and fighter-bomber planes. It is claimed that, for propellers for high-horsepower engines, high-density plywood is much superior to metal. Some of this superiority may be attributed to the ability to get much lighter propellers which satisfactorily withstand the conditions of use. As an example, it is reported that a three-bladed propeller for a 1750-hp engine weighs 300 lb less in high-density plywood than in metal. This saving is of great value not only because of over-all weight reduction but also because of the decreased inertia of the blade. This weight saving is possible because metal blades for engines of high horsepower require a high percentage of steel and large hubs to absorb the high stresses, both causing large weight increases. Conversely, on engines of low horsepower, low-steel-content alloys and small hubs can be used, and high-density plywood shows very little or no weight advantage.

In addition to uniform-density materials, there has also been developed and used a variable-density material having a high density at the hub end and low density at the tip end. By using variable-density blanks, it is possible to step down the density gradually, thus avoiding any area of sharp transition from high to low density and, at the same time, obtaining the strength of high-density material at the hub end where the stresses are concentrated, and the lightness of the low-density material at the tip end where the stresses are smaller. The Jablo propeller blade is of this type.

In addition to the weight saving, the high-density plywood shows a superior resistance to the effect of notches and dents and is more easily repaired. It is reported, on the basis of considerable combat experience, that repairability of high-density-plywood blades is 80 per cent as compared with 60 per cent for metal blades. The fatigue resistance of high-density plywood is excellent and its energy absorption much greater than that of metal.

In the United States, the emphasis on high-density plywood for propeller blades has been in its use as a cheap, noncritical material, substituting for metal in propellers for engines of low horsepower. High-density plywood is, however, being used as the hub material in the Schwarz blade which is being produced in America, although of European origin. This type blade utilizes a high-density-plywood hub scarfed to spruce tips and is well protected by suitable coatings. While there is a sudden transition from high- to low-density material at the scarfed joint, it is understood that this blade has proved quite successful.

High-density plywood should be of great value in solving spar-design problems, since it gives a definite economy in weight and space over spruce spars in applications where high bending moments are present. An additional weight saving is made possible because the bearing strength is so much greater than that of normal wood. At mechanical joints and at all points where high bearing strength is required, the number of bolts and the size of the fittings can be greatly reduced when the spar is designed in high-density plywood. As an example, it is reported that, in spars of spruce and high-density plywood designed to carry the same bending moment, there is an over-all weight-saving of 35 per cent in favor of the high-density plywood (10).

Outside the aircraft field, one application has been developed and is now in use. This is the flare base for the M26 parachute flare. This piece was formerly produced from die-cast aluminum and is required to take the full shock, across a very small area, of the sudden stress occasioned by the parachute stopping the rapid descent of the heavy flare. High-density plywood withstands this stress satisfactorily and has proved successful in this application, releasing many thousands of pounds of aluminum for vital aircraft parts. In this instance, the direct substitution of high-density plywood for aluminum without redesign of the part was possible. In many cases, redesign to take advantage of the properties of high-density plywood would be required. Undoubtedly, there are many other somewhat similar applications where high-density plywood could replace metals of which there is a critical shortage. However, in many cases, the necessity of redesign and its attendant problems act as a bar to the development of such applications.

Several concerns are investigating this material for many wartime uses, and it will undoubtedly have a wide peacetime application. Among the many uses for which this material shows promise, in addition to those discussed, are reinforcing plates, plywood clips, boat parts, and aircraft parts, of many types, and as dies for forming sheet metals.

Acknowledgment

The author is indebted to H. A. Robinson, S. W. Eby, and the laboratory staff for their help in the preparation of this material.

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Heating Wood With Radio-Frequency Power

By J. P. Taylor,[1] Camden, N. J.

Radio-frequency power can advantageously be used to provide the heat required in various types of wood processing such as seasoning, drying, curing of impregnating materials, gluing, and bonding. This method is of considerable importance to the production of plastic plywood planes, boats, and housings of various types. This paper is intended to form an introduction to the subject. The theory of radio-frequency heating is considered briefly, the factors which determine its applicability and usefulness are discussed, and some data for the calculation of power required, time cycle, and operating cost are presented. Means of handling some specific problems are reviewed and several typical installations briefly described.

RADIO-FREQUENCY power has been used to overcome what seemed an insurmountable problem in the production of airplane propellers made of compregnated wood. Radio frequency is being used to expand the production of laminated and box spars, truss-type rib constructions, bomber floors, bomb-bay doors, and other aircraft parts. It is being tested for use in making various molded plywood forms such as wing elements and fuselages. Experimentally it is being used for seasoning wood, drying aircraft-quality plywood, and curing various types of impregnating resins. In all of these applications the desired effect is obtained as a result of the heating developed in the wood by the use of radio-frequency power.

Obviously, these applications are important, and this lends importance to the questions which naturally arise: How is radio-frequency power applied? What makes the wood heat? Why is radio-frequency current used? What advantages does the process offer? What limitations does it have? What kind of equipment is required? How much does it cost to operate? How do results compare with those of other methods?

Actually the method is too new to give final answers to all of these questions; however, enough work has been done to make it worth while to review the information which is available.

How Electric Power Is Applied to Wood

(Heating wood with electric power is aside from the strictly application problems which arise) a relatively simple operation. Theoretically, at least, it is only necessary to have a generator of suitable characteristics and to connect it by means of wires or other conductors to the wood which it is desired to heat.

Such an arrangement is illustrated in Fig. 1. A generator ⓔ is connected to two metal plates (usually called "electrodes"). The wood which it is desired to heat is placed between these metal plates. The reason for the plates can be understood by inspection of Fig. 2, in which the dotted lines represent current paths and the number of lines in any particular part of the wood block is an indication of the current density in that section.

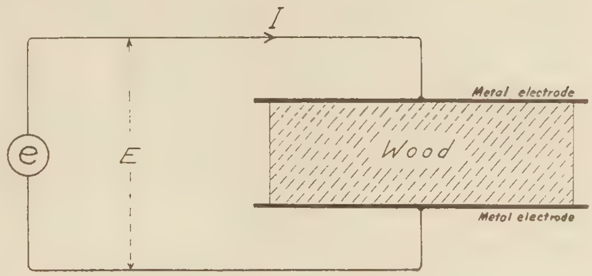

Fig. 1 Fundamental Circuit Used in Heating Wood With Radio-Frequency Power

(The wood is placed between metal electrodes which are connected to a generator ⓔ. Resistance to passage of current I causes wood to heat.)

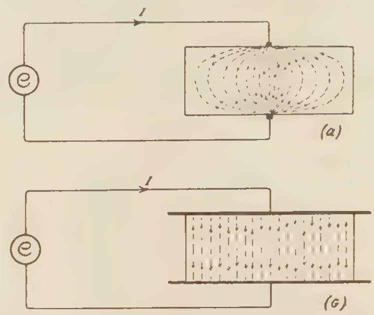

Fig. 2 Distribution of Current Paths Through the Wood

(a, When only small contact points are used; b, when large metal plates called "electrodes" are used.)

If connections to the wood are made at single points, or by means of small contact plates, the current distribution will be approximately as shown in Fig. 2(a). In this case the corners of the block, since there is less current through them, will be heated very little, and the parts of the block which are close to the contacts will be overheated. In some cases such a distribution might be desirable, but ordinarily a more even distribution of heating is desired, and this can most easily be obtained by using large electrodes as in Fig. 2(b). With such an arrangement the voltage generated at ⓔ causes a current I to flow around the circuit, through the wood, and back to the generator as shown. The magnitude of this current will be determined by the voltage available from the generator and the resistance (to the passage of the current) presented by the wood.

What Makes the Wood Heat

There is nothing very mysterious about the fact that wood can be heated by radio frequency. Almost any material we know of will heat up if an electric current of sufficient intensity is forced through it. Obvious examples are the resistance wires in electric stoves, heaters and irons, the filaments of lamp globes, the windings of motors. All of these, of course, are metals, which means that they are relatively good conductors. Their resistance to the passage of direct or low-frequency current is low (compared to that of nonconductors), and hence the force (i.e., voltage) which is required to cause a sufficient current to pass

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through them is of reasonable value, usually 110 or 220 v

In the case of the poorer conductors (sometimes called insulators) such as wood, the resistance to the passage of direct or low-frequency current is very high. The force (i.e., voltage) which would be required to cause an appreciable 60-cycle current to pass through these poor conductors would be out of practical reach (of the order of millions of volts). However, as the frequency of the current is increased, the equivalent resistance of these materials drops almost inversely. At frequencies in the range of what we normally call radio frequencies it becomes low enough so that it is practical to force through these poor conductors enough current to heat them as we desire.

Although some writers on the subject have taken the opposite view, there is no essential difference between high-frequency (hereafter called radio frequency) heating and low-frequency heating. For a purely practical reason, viz., that the required voltages are lower, it is preferable to use radio frequency for the heating of poor conductors. The mechanism of heating is exactly the same.

Physicists picture the heating which occurs as being due to "molecular friction" caused by the passage of current through the materials. They visualize this current not as a stream of electrons each of which flows all the way across the material but rather as the net effect of the motion of all the electrons. Normally these electrons have random orbits. When a voltage is impressed across a section, those electrons which are not too tightly bound change their paths somewhat so as to produce an over-all effect of a charge moving from one side to the other. This displacement of the paths of the electrons represents work done and this work appears as heat.

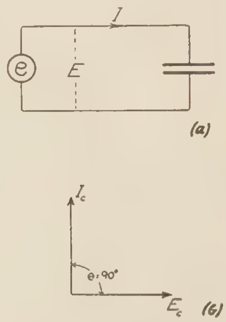


Fig. 3 Schematic (a) and Vector (b) Diagrams for a Circuit in Which a Generator is Connected to a "Perfect" Condenser
(The current and voltage are 90 deg out of phase; hence, no power is consumed.)

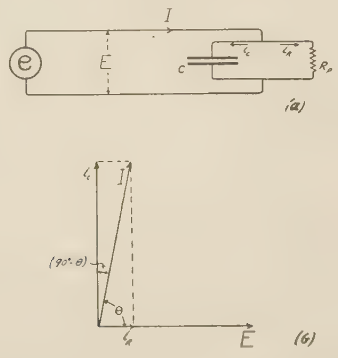


Fig. 4 Schematic (a) and Vector (b) Diagrams for Circuit, Similar to Fig. 3, in Which Condenser Has a Poor Dielectric Such as Wood
(The current i_R flowing through the equivalent resistance R_p represents power which is expended in heating the wood.)

The difference between good conductors and poor conductors is represented by the degree of freedom of the so-called "orbit electrons." There is no fundamental difference in the heating effect. In either case it is due purely to the "conduction loss" which occurs with actual passage of current through the material.

The Effect of Frequency

Where the dimensions of the "package" of wood to be heated are known, it is relatively easy to calculate the voltage which will be required to obtain a certain heating effect. Electrically the metal electrodes between which the wood is placed form a condenser. If these electrodes were separated by air, they would form a so-called "perfect condenser." Such a condenser would be represented schematically, as in Fig. 3(a), and the voltage across and the current through this condenser, by the vectors of Fig. 3(b). In this case the voltage and current are 90 deg out of phase, and the average power dissipated in the condenser is zero.

When, however, the wood is placed between the electrodes, we no longer have a perfect condenser since the wood presents a leakage or "conduction" path. The imperfect condenser thus formed can be represented schematically by a perfect condenser paralleled by a resistance R_p, as shown in Fig. 4(a). The capacity of C is the same as the capacity of the imperfect condenser and hence is a constant. Term R_p, however, is purely an "equivalent" resistance and is not independent of frequency. It is therefore necessary to calculate R after the frequency of operation has been chosen.

The relations of voltage, current, power, and frequency of the circuit, shown in Fig. 4(a), can be obtained directly. Fig. 4(b) is the vector diagram. In this case, the total current I leads the voltage across the load by something less than 90 deg. The total current I is made up of two components, viz., the capacity or "out-of-phase" current i_c and the resistive or "in-phase" current i_R. Since the first does not represent any power furnished to load, we are not interested in it, but only in i_R and the resistance R_p through which it flows. In order to calculate R_p we note that

$$\text{Power factor (PF)} = \cos\theta = \sin(90° - \theta)$$

for small angles

$$\sin(90° - \theta) \cong \tan(90° - \theta)$$

hence

$$\text{PF} \cong \tan(90° - \theta) \cong \frac{i_R}{i_c}$$

$$i_R = \frac{E}{R_p} \quad i_c = \frac{E}{X_c}$$

$$\text{PF} = \frac{X_c}{R_p}$$

$$R_p = \frac{X_c}{\text{PF}} \qquad [1]$$

The value of X_c can be calculated from

$$X_c = \frac{1}{2\pi fc} \qquad [2]$$

where

$$C \text{ (in farads)} = \frac{8.85 \times 10^{-14} KA}{d}$$

where

K = dielectric constant
A = area of plates, cm²
d = separation of plates, cm

The value of the power factor PF can easily be measured by means of a radio-frequency bridge or Q-meter. The power factor of a material is usually thought of as a constant. (However, recent measurements, see Figs. 5 and 6, show that PF varies considerably with frequency, moisture, and impregnation.) For the calculation as here, an approximate value will suffice.

Having determined R_p, it now remains to note that the power delivered to the wood and which appears as heat is given by

$$P = i_R^2 R_p \qquad [3]$$

If we have calculated (as described later) the amount of power required to heat the material, we will know P and from Equation [3] we can calculate i_R. Then, since

$$E = i_R R_p \qquad [4]$$

we also obtain the value of E which is the voltage which will be required to force through the wood the current necessary to heat it in the desired amount.

If we now substitute some actual values in the foregoing calculations, we will immediately perceive the reason for using high frequencies.

Example 1. It is desired to use 60-cycle current to heat the

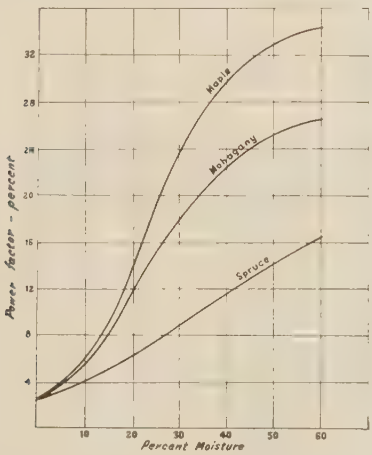


FIG. 5 VARIATION OF POWER FACTOR WITH PER CENT MOISTURE FOR THREE SPECIES
(Measured values for one species vary considerably: hence, these curves should be used only as a first approximation.)

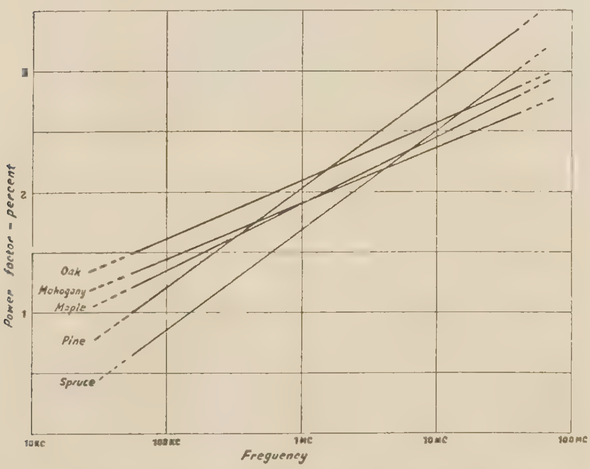


FIG. 6 VARIATION OF POWER FACTOR WITH FREQUENCY
(At lower frequencies these curves tend to flatten out.)

test propeller, shown in Fig. 21, to 240 F. By the following calculation it has been determined that a power of 6000 w is required to do this in a time of 8 min. By calculation, C was found to be 150 $\mu\mu$f; hence from Equation [2]

$$X_c = \frac{1}{2\pi fc} = \frac{1}{6.28 \times 60 \times 150 \times 10^{-12}}$$

and from Equation [1]

$$R_p = \frac{17{,}700{,}000}{0.05} = 350{,}400{,}000 \text{ ohms}$$

taking PF = 0.05

From Equation [3]

$$P = i_R^2 R_p$$
$$P = 6000, \quad R_p = 354{,}000{,}000$$
$$i_R^2 = \frac{6000}{354{,}000{,}000}$$
$$i_R = 4.13 \times 10^{-3} \text{ amp}$$

and from Equation [4]

$$E = i_R R_p = 4.13 \times 10^{-3} \times 354 \times 10^6$$
$$= 1{,}460{,}000 \text{ v}$$

In other words, the desired heating could be accomplished with 60-cycle current only by the use of an entirely impractical voltage.

Example 2. Now assume that the same block is to be heated with current at a frequency of 1 megacycle:

From Equation [2]

$$X_c = \frac{1}{2\pi fc} = \frac{1}{6.28 \times 10^6 \times 150 \times 10^{-12}} = \frac{10^6}{943} =$$

$$X_c = 1010 \text{ ohms}$$

From Equation [1]

$$R_p = \frac{1010}{0.05} = 20{,}200 \text{ ohms}$$

From Equation [3]

$$P = i_R^2 R_p = 6000$$
$$i_R^2 = \frac{6000}{20{,}200} = 0.297$$
$$i_R = 0.545 \text{ amp}$$

From Equation [4]

$$E = i_R R_p = 0.545 \times 20{,}200$$
$$E = 11{,}000 \text{ v}$$

In other words, if we use 1-megacycle current we can operate with 11,000 v across the load. We might go still higher, say, to 10 megacycles, in which case the required voltage would drop to 3480 v. In order to show this effect of frequency graphically, the values of X_c, R_p, and E for a large range of frequencies have been plotted in Fig. 7.

As can be seen in Fig. 7 the voltage required for a given power input (i.e., a given heating effect) is inversely proportional to the square root of the frequency. This means that, generally speaking, the higher the frequency the better; although a practical limitation is encountered due to the fact that the efficiencies of some types of tubes fall off at the higher frequencies. There may also be difficulties due to current distribution at the higher frequencies, as will be pointed out later. The actual maximum voltage that can be tolerated will depend chiefly upon the thickness of the load. For very thin materials, not more than a few

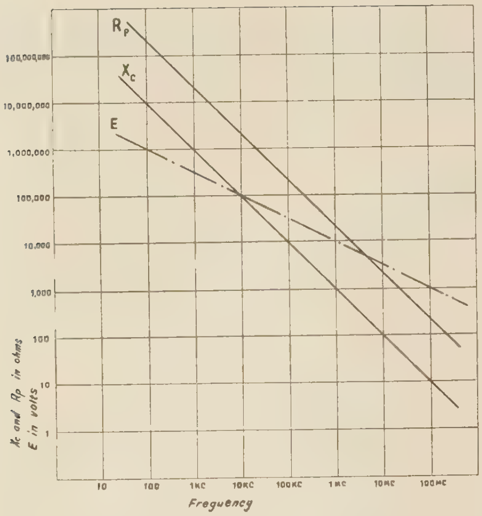


Fig. 7 Variation With Frequency of Reactance X_c, the Equivalent Resistance R_p and Voltage E Across the Load for a Typical Radio-Frequency Heating Setup

(Note that voltage E varies inversely as the frequency; hence, higher frequencies mean less danger of voltage flashover.)

hundred volts can be used before arc-over occurs. In thicker sections, as much as 15,000 v can be used. Generally, voltages much above 15,000 cannot be used, no matter what the thickness, due to corona effects which become evident at higher voltages and which are only partially dependent upon electrical spacing.

Averaging these various factors together, it has been found that the range of 1 megacycle to 10 megacycles presents the best immediate possibilities. Some very thin sections, however, will require higher frequencies. Looking to the future (when a wider choice of high-frequency tubes will presumably be available) it seems very probable that a higher range of frequencies will come into use.

Calculation of Power Required

The amount of heat (in gram calories) required to raise the temperature of a certain quantity of wood (or any material) a certain number of degrees can be calculated from the relation

$$H = pc\,\Delta t\cdot\text{volume}$$

where

p = specific heat, cal per g per deg C
c = density, g per cc
Δt = change in temperature, deg C
v = volume, cc

The power required to produce this amount of heat in a given time is

$$P\ (\text{watts}) = \frac{4.187 \times H}{\text{time (sec)}} = \frac{4.18\cdot pc\cdot \Delta t\cdot\text{volume}}{\text{time (sec)}}$$

If we substitute cu in. for cc, and deg F for deg C, and time in minutes

$$P\ (\text{watts}) = \frac{0.637\cdot pc\cdot \Delta T\cdot \text{volume (cu in.)}}{\text{time (min)}}$$

where

$$\Delta T = \text{deg F}$$

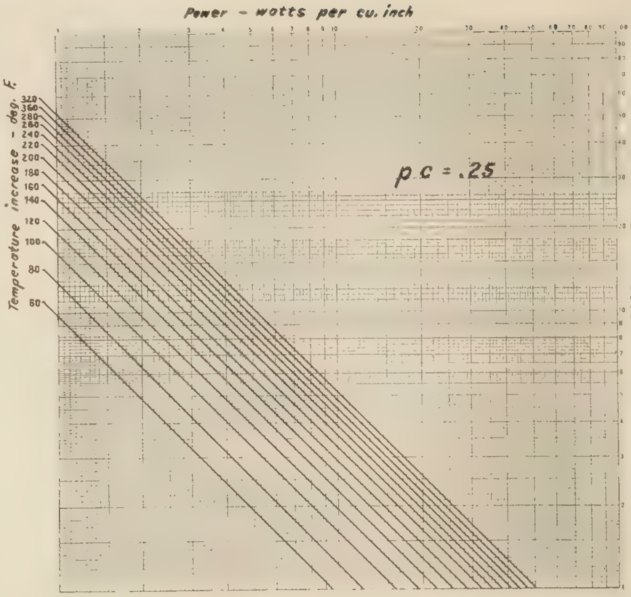


Fig. 8(a) Power, in Watts Per Cubic Inch, Required to Raise the Temperature of a Wood, or Other Material, Having a Value of pc = 0.25 to a Given Temperature in a Given Time

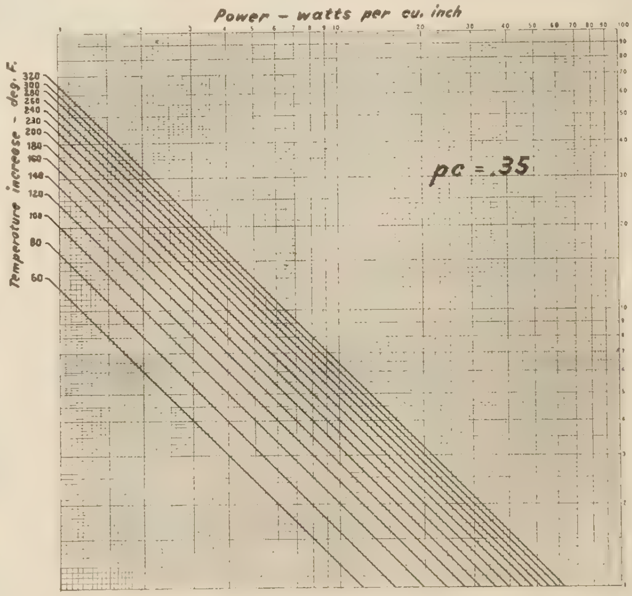


Fig. 8(b) Power, in Watts Per Cubic Inch, Required to Raise the Temperature of a Wood, or Other Material, Having a Value of pc = 0.35 to a Given Temperature in a Given Time

In some cases it is more convenient to express this in terms of required power concentration, viz.

$$\text{Power concentration (per cu in.)} = \frac{0.637\ pc\,\Delta T}{\text{time (min)}}$$

In other cases where a limited amount of power is available, it may be desired to know the time required to raise the material so many degrees. This is simply

$$\text{Time (min)} = \frac{0.637\ pc\,\Delta T}{\text{watts per cu in.}}$$

In order to give an idea of the powers required in typical instances, two sets of curves have been made up, Fig. 8, showing the relation of power concentration to time interval for several different temperature increments. These curves are based on value of $pc = 0.25$ and $pc = 0.35$. Most woods give values which lie between these two curves. Where accurate data are available, the curves may be used for other values of pc simply by noting that the time will be increased or decreased in proportion to the value of pc.

It should be noted that the power requirements, as indicated in Fig. 8, are the power which must be used up in the wood itself. In other words, if there is any loss of power, it will have to be supplied in addition to the foregoing. Fortunately, the losses (through conduction to the press and radiation to the surrounding air) are small if the cycle is relatively short. For purposes of calculating the power to be supplied to the press, they can be assumed to be of the order of 10 to 20 per cent. This is in marked contrast to most hot-plate presses where a large part of the power furnished the press is used to heat the mass of metal and a very considerable proportion is dissipated in heating the surrounding air. The lack of heat about a radio-frequency press is in marked contrast to the vicinity of a hot-plate press in operation.

The required power as estimated from the foregoing is a good indication of the radio-frequency power to be furnished by the generator. In cases where the generator is rated in output power, that is, in terms of the radio-frequency power it actually puts out, the indication is direct. Where the generator is rated in input power, that is, power it takes from the line, an efficiency factor of 50 per cent should be used. In other words, a generator with an input rating of 10 kw will actually put out about 5 kw of radio-frequency power. The same factor should be used in calculating power consumption, viz., a press that requires 15 kw of radio-frequency power for a certain heating process will, with associated equipment, draw about 30 kw from the line.

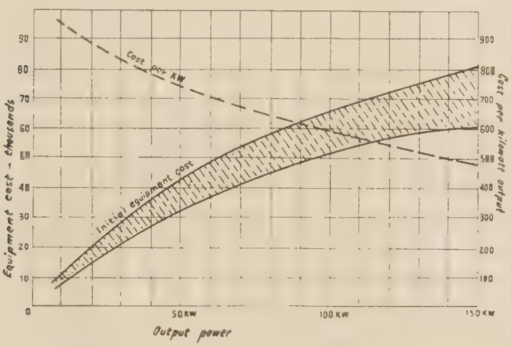


Fig. 9 Initial Cost of Radio-Frequency Equipment and Cost Per Kilowatt
(Based on a number of installations made by different manufacturers.)

Initial and Operating Costs

The cost of radio-frequency power per kilowatt is not uniform but rather decreases gradually as the power of the installation increases. This is illustrated by the curve in Fig. 9, which has been drawn on the basis of such information as is available on installations made to date. Unfortunately, varying applications and engineering costs were necessarily involved in these. Also, there was some difference in cost of equipments of different manufacture. For this reason all available information was plotted and a broad curve drawn so as to include all points. The result at least gives a quick indication of what installations to date have cost. The dotted line on this graph indicates the average "cost per kilowatt" at various powers.

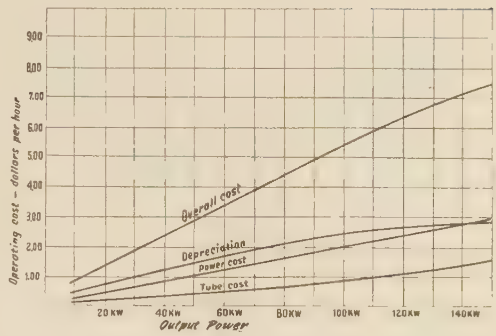


Fig. 10 Approximate Tube Cost, Power Cost, and Over-All Operating Cost of Radio-Frequency Equipment Versus Power Delivered to Load
(Basis of calculations given in text.)

Operating costs in equipment of this type are made up mainly of tube-replacement and power costs. Since there are no moving parts, other maintenance costs are negligible. Contrary to some statements, the useful life of the equipment is long and will most likely be terminated by obsolescence rather than wear. Depreciation, therefore, can be based only on some arbitrary figure such as that used for tax purposes.

In order to gain some idea of operating costs, a number of equipments of various powers were analyzed and the results shown as curves in Fig. 10. In this calculation, average tube life was assumed as 5000 hr (which is a reasonable estimate of the life of present-day tubes in this service). Power was assumed to cost 1 cent per kwhr. Depreciation was figured on the basis of 25,000 hr of operating life (approximately 3 years at 24 hr a day, or 9 years at 8 hr a day). The over-all cost, of course, is simply the total of depreciation, tube, and power costs.

It is worth noting that these costs can only be assumed to hold for the present period. Simplifications in design, improvements in tube construction, and other factors which can be quite clearly foreseen will undoubtedly lead to lower first cost and lower operating cost. However, it is apparent that even on the basis of the costs, indicated in Fig. 10, there are many present-day jobs on which this type of equipment can easily be justified from the cost standpoint.

Advantages of Radio-Frequency Method

The more obvious advantages of the radio-frequency method follow from the fact that, in this method, heat is caused to be generated simultaneously and uniformly throughout the whole body of the wood. This means that (neglecting losses) the whole block of wood comes up to temperature evenly. It also means that the time required for a given increase in temperature is independent of the thickness of the wood. These effects are in marked contrast to those which occur with other methods of heating. In all other methods, heat which originates outside the wood (as, for instance, in steam platens) must travel into the wood by conduction. As a result, the outer layers of the wood come up to temperature much more quickly than the interior. Moreover, the time required to heat a section of wood depends entirely upon the thickness. In the case of thick sections this becomes very long.

In order to obtain a more detailed picture of the temperature gradients in the two methods the time-temperature curves have been computed for several typical cases. These are shown in Figs. 11 and 12. The values from which the steam-plate curves of Fig. 11 were plotted were calculated from the formula

$$\theta = \theta_s - (\theta_s - \theta_0)\, f\left(\frac{\alpha t}{d^2} \cdot \frac{x}{d}\right) \qquad [5]$$

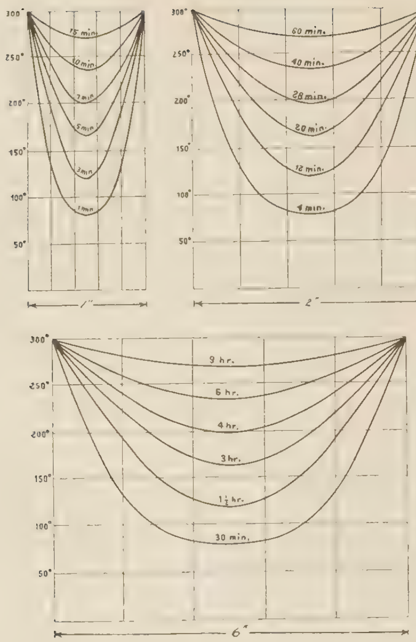


Fig. 11 Time-Temperature Distribution Curves for 1-In., 2-In., and 6-In. Thicknesses of Wood Heated by External Means

(External means refers to steam platens, heat tunnel, etc. Curves are calculated for thermal diffusivity of 0.0063 for spruce at 8 to 10 per cent moisture content.)

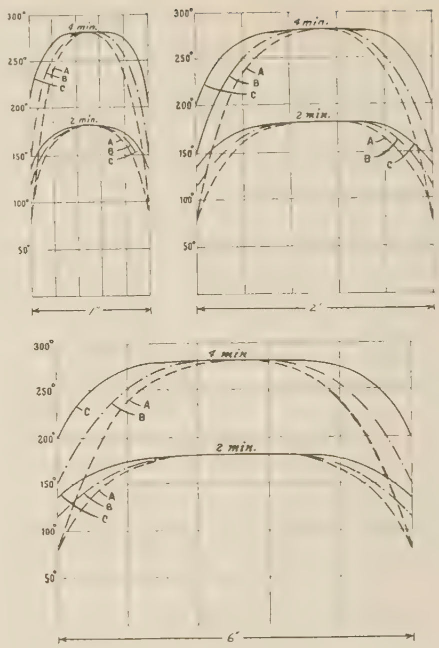


Fig. 12 Time-Temperature Distribution Curves for 1-In., 2-In., and 6-In. Thicknesses of Spruce Heated by Radio Frequency

(Power concentration, 5 w per cu in. Curve A is for the wood directly against cold plates; curve B for wood against thin metal electrodes; curve C for a thin insulating material between wood and press platens.)

where θ is the temperature at point x and time t; θ_s is plate temperature; θ_0 is original temperature of wood; α is the thermal diffusivity, and d the thickness of the wood. Values of the function $\left(\frac{\alpha t}{d^2} \cdot \frac{x}{d}\right)$ are given by Brown and Marco. They are also given by McAdams and other sources, but the first reference is the most usable in this case. A value of 0.0063 for the thermal diffusivity of spruce at 8 to 10 per cent moisture content was used.

The curves, shown in Fig. 12, are based mainly on experimental measurements made on various sections of wood heated by the radio-frequency method. For each time-thickness relation, a set of three curves is given. This is necessary because in this method of heating the temperature gradient depends upon the conduction losses; and these losses depend to some extent upon the setup used. The three typical setups illustrated by these curves are (a) for the case where the material is directly against the faces of a large cold press; (b) where the material is clamped between relatively thin metal plates which are exposed to air; and (c) for the case where a thin layer of heat-insulating material (such as pressboard) is placed between the material and the press or between the electrodes and the press.

The curves for the cold-press setup are shown dotted. The large mass of metal acts as a "sink" and the outer surface of the wood never rises above ambient temperature. The curves for the thin electrodes are shown broken. In this case, the surface temperature rises to some extent and the gradient is less sharp. The curves for the setup where heat insulation is used are shown solid. These are purely indicative since in this case the gradient depends upon the amount of heat-insulating material. The curves shown are for measurements made with insulating sheets having $^1/_8$ the thickness of the wood itself.

Comparing the curves for the two methods, the most striking feature is the fact that for a 1-in. thickness, the radio-frequency process gives a time cycle of 4 min (to 280 F), where the steam-plate method requires some 15 min to bring the center to the same temperature. Further increase in the thickness of the wood makes the discrepancy even greater since (as can be seen from Equation [5]) the time increases as the square of the thickness. Moreover, the steam-plate time cycles are fixed and there is no way of shortening them appreciably. On the other hand, the time cycles required for radio-frequency heating depend entirely upon the power used. The curves of Fig. 12 were calculated on the basis of a power to give 5 w per cu in. of material. Increasing this to 10 w per cu in. would cut the time in half. The temperature would then be raised to 280 F in 2 min. Whether or not this quick cycle would sufficiently set the glues is a moot point. Some authorities on glue believe that the glue must be held at fairly high temperature for at least 2 or 3 min.

When the comparative time cycles for thicknesses greater than 1 in. are considered it is immediately evident that the advantages of radio frequency are enormous. As indicated in Fig. 12, the time to heat thicker sections by radio frequency is the same as the time for the 1-in. section. This statement is true, of course, only if the power per cubic inch of wood is kept constant; in other words, the total power increased as the thickness. If the total power is held constant, then the time varies directly as the thickness. In the steam-plate method, on the other hand, the time varies as the square of thickness (as can be seen by reference to Equation [5] and Fig. 11). This leads directly to the second important advantage, viz., thick sections which by older methods had of necessity to be cold-glued can now be hot-glued. This means they can be produced more quickly, and, since higher-temperature-setting glues can be used, they can be made better.

Still another advantage is evident when the shapes of the curves in Figs. 11 and 12 are considered. In the hot-plate method the outer layers of wood are at a high temperature for a very considerable length of time. As a result these tend to dry out and a degree of "case hardening" sets in. In critical sections such as aircraft parts, this is very objectionable. Sometimes, the wood

must be "conditioned" after the gluing by being wet on the outside to increase the moisture content of the outer layers. With radio frequency this drying out does not occur, even where the cycle is fairly long, because the temperature gradient has a slope the reverse of that which would cause such an effect.

An important advantage where new press installations are considered is the fact that the presses themselves can be of much cheaper design, since the massiveness associated with steam platens and multiple openings is done away with. Moreover, the use of radio frequency makes it much easier to use hot-gluing in conjunction with very large presses and presses of unwieldy dimensions. Presses designed originally for cold-gluing are readily adapted for hot-gluing.

There are also numerous minor advantages which vary in accordance with the job to be done. These include the convenience of not having to work in close proximity to hot plates; the fact that the heat can be closely controlled and can be turned off instantly; the possibility of making hot-glued joints on jigs designed for cold-gluing; and others of a similar nature. The importance of these will become more evident as this process comes into more widespread use.

When making compregwood parts, radio frequency has another very important advantage, this is, the fact that since the whole mass of wood is heated uniformly the compression takes place uniformly. The contrast with what takes place when using a steam die only is very marked. In the latter instance the outer plies, of course, heat up very quickly and the resin in them begins to set up. Compression must, therefore, be started immediately. However, at this point the interior of the wood has not even started to heat. As a result the compression is very nonuniform and terrific internal stresses result. Such stresses can be partially relieved by very long cooking periods, but even then results are often unsatisfactory. Direct comparisons made between similar sections done with and without radio frequency show that in the latter these stresses have been 90 per cent or more eliminated.

All of these advantages are cited on the basis of work which has actually been done. In considering the use of radio frequency for making curved surfaces, such as fully stressed wing or fuselage elements, it should be noted that this is something which apparently has not yet been done in production. Should the radio-frequency process be found feasible, there would, of course, be marked advantages. While the autoclave method appears to be the most satisfactory found to date, it is unquestionably cumbersome, time-consuming, and expensive. Use of radio frequency might conceivably eliminate the cost of the autoclave, greatly reduce the time cycle, the quantity of jigs required, the number of operations, and the inconveniences attendant on the present process. There are numerous problems to be worked out before this will be possible, but already the answers to some of these have been indicated.

To reduce the advantages mentioned, which are more or less general, to specific cases, the following tabulation has been made:

Advantages of radio frequency as compared to hot-plate gluing:

(*a*) Temperature is uniformly distributed.
(*b*) Time cycles usually greatly reduced (for all but thin sections).
(*c*) Thicker sections can be glued.
(*d*) Higher-temperature-setting glues can be used.
(*e*) Temperature can be more closely controlled.
(*f*) Surface of wood is not dried out.
(*g*) Cheaper presses can be used.

Advantages of radio frequency as compared to cold-gluing:

(*a*) Very large reduction in time cycles.
(*b*) Allows use of phenol glues.
(*c*) Makes for better, more consistent joints.
(*d*) Requires fewer presses or jigs.
(*e*) Results in a large saving in floor space.

Advantages of radio frequency in making compregwood:

(*a*) Considerable reduction in over-all time cycle.
(*b*) Reduction in number of dies required.
(*c*) Large reduction in internal stresses.
(*d*) Close control of temperature.
(*e*) Less critical operation.
(*f*) Less likelihood of exothermic reaction.
(*g*) Greater leeway in design.

Possible advantages as compared to autoclave method:

(*a*) No autoclave required.
(*b*) Dimensions not limited by size of autoclave.
(*c*) Time cycles greatly reduced.
(*d*) Fewer forms required.
(*e*) Fewer operations involved.
(*f*) Less floor space needed.
(*g*) Higher-temperature-setting glues used.
(*h*) Over-all inconvenience much reduced.

Factors Limiting Applicability of Radio Frequency

With all the advantages of radio frequency, one wonders why it has not been more widely adopted. The answer seems to lie in the fact that there are definite problems in applying it to other than simple jobs. These problems have been too much for wood engineers to solve by themselves and, at least until very recently, radio engineers have not been interested. The present plywood-plane and glider program has changed this situation overnight. The importance of this work, plus the necessity for quantity production, plus the desirability of using phenol glues have caused a sudden demand for radio-frequency equipment. This has revived the interest of radio engineers, and they are now tackling these application problems on a wide front. A detailed consideration of these problems is obviously beyond the scope of this paper. However, a brief review of them will be worth while as a means of indicating the factors which should be considered in making a decision on the use of radio frequency.

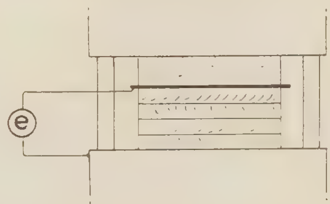


Fig. 13 Typical Setup of Gluing One Assembly at a Time
(The insulator *I* must have good loss characteristics if a large waste of power in heat in the insulator is to be avoided. If wood is used as an insulator it should be several times the thickness of the wood to be heated.)

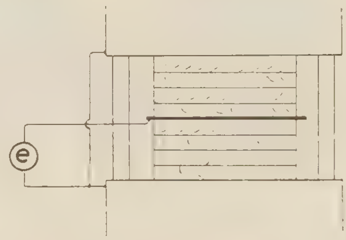


Fig. 14 The "Sandwich" Method Whereby Two Assemblies Are Made at Once
(This setup requires no insulating material and insures high efficiency.)

Necessity of Insulating Electrodes. Referring to Fig. 1, the fundamental method of applying radio-frequency power is by means of two metal electrodes connected to the radio-frequency generator. If these electrodes could be the regular plates of the press, everything would be easy. Unfortunately, this is not the case since in such an arrangement the frame of the press would form a "short" across the electrodes. If desired, one plate of the press can be one of the electrodes, but the other electrode will have to be insulated. Such an arrangement is indicated by Fig. 13. If the thickness of this insulator is to be kept to reasonable proportions, it must be made of a material having a low loss, such as glass, hard rubber, or a ceramic. In the sizes required these materials are hard to obtain and difficult to use. For this reason wood is ordinarily used. However, it is obvious that if the wood insulator is of the same characteristics and same thickness as the wood to be heated, then the insulator will heat equally and this represents a 50 per cent power loss. Therefore the wood insulator must ordinarily be several times thicker than the piece to be heated.

One easy and efficient means of solving the insulator problem

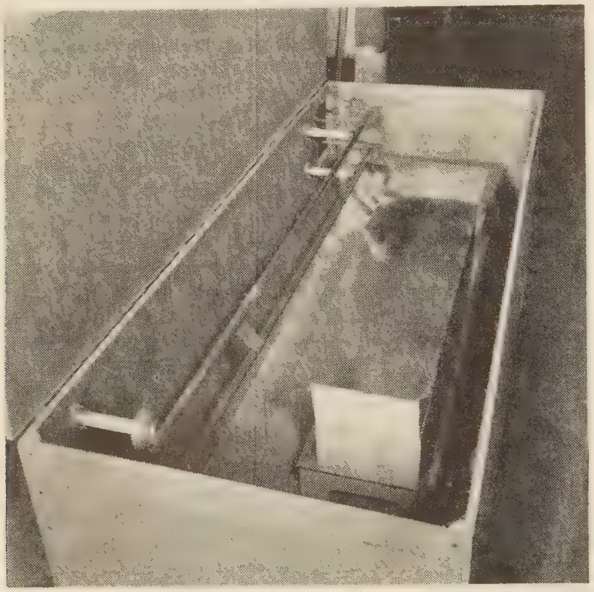


Fig. 15 Screened Box for "Preheating" Large Block Previous to Time It Is Placed in Press
(Note copper-pipe tuning stubs mounted on side of box.)

is indicated in Fig. 14. Two of the assemblies are glued at a time, being placed one above and one below a copper caul as illustrated. This copper caul is connected to one terminal of the generator and the press to the other terminal. Thus the two assemblies are effectively connected in parallel and heated together. There is no need for any insulating material and hence no insulator heat loss.

There are numerous other ways of getting around the insulator problem. One of these is by the device of "preheating." This is particularly applicable to the making of compregwood and similar procedures wherein a steam-heated die of more or less complicated shape is ordinarily used. In such cases an analysis indicates that the problem is really one of bringing a thick material up to temperature; a process which often requires several hours. Once it has reached the necessary temperature there is no difficulty in maintaining it. Therefore, the radio frequency is used only to bring the material up somewhere near the final temperature. This may be done entirely outside the press—merely by placing the material between the electrodes as shown in Fig. 15. Or it may be done with the material in the press but with the press left open so that a temporary electrode can be placed on top of the material. As soon as the material is brought up to temperature, it is placed in the press (or the temporary electrode removed), the press closed, and the material left to "cure" under steam alone. This process can also be used with paper-base and canvas-base laminates, with preforms for automatic molding, and in similar operations.

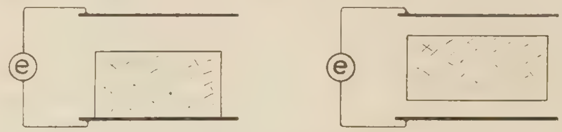


Fig. 16 Heating by Radio Frequency Does not Require Contact
(Either arrangement can be used. However, voltage is increased appreciably, which may be a disadvantage.)

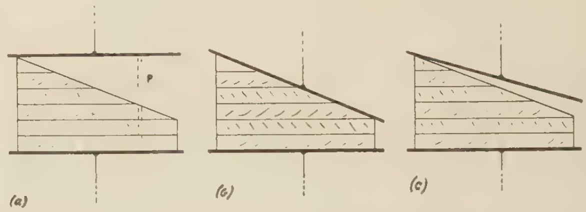


Fig. 17 Incorrect Setups (*a*) and (*b*) for Heating an Assembly of Uneven Cross Section; Correct Setup Shown at (*c*)
(Since path through air offers higher resistance, an arrangement such as (*c*) is necessary to obtain even heating.)

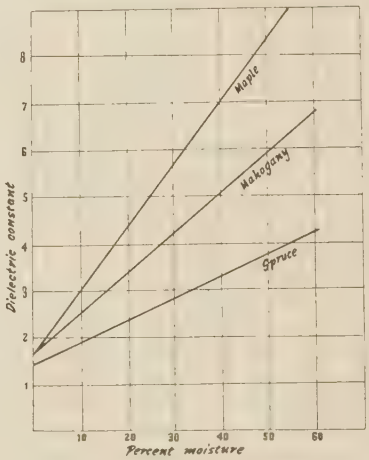


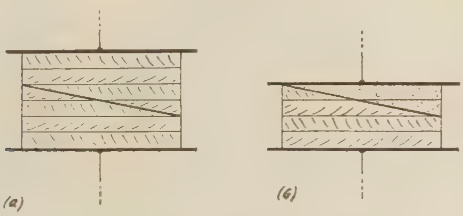
Fig. 18 Variation of Dielectric Constant With Per Cent Moisture Content for Three Species

Shape and Spacing of Electrodes. The shape of the electrodes used and the spacing between them must also be taken into account. If the material to be heated is flat or square and is homogeneous in characteristic, the plates should be flat. They may be spaced or adjacent to the material, but spacing, as in Fig. 16, increases the voltage across the system and is undesirable from that viewpoint. If the material does not have a parallel surface and is placed between parallel plates, as in Fig. 17(*a*), the thick part of the wood will heat more than the thin part. This is because the air space at the thin end presents a much higher impedance to the passage of the current, and therefore less current flows through. If the arrangement shown in

Fig. 17(*b*) is tried, the thin end of course heats up more since the shorter path has the lower impedance.

In order to insure even distribution of heating, each small elemental path *p* from plate to plate must have the same impedance as all other such paths. But the sections of these paths which are in the air offer much more resistance per unit length than the parts which lie in the wood; hence to equalize the currents, the air space at the thin end must be much less than the difference of wood thickness.

As an approach to calculation of the proper spacing, we can use the dielectric constant. The dielectric constant of wood varies with species and moisture content, somewhat as shown in Fig. 18. We can estimate the spacing of the electrode by assuming that the ratio of the resistance of the paths in air and in wood will vary inversely as the ratio of the dielectric constants of air and wood. Thus, where the wood has a dielectric constant of, say, 4, then the path in air has a resistance 4 times as great; hence $^1/_4$ in. of air along path *p* is the same as 1 in. of wood. If the wood is 3 in. thick at one end and 1 in. thick at the other, then the electrode spacing at the thin end should be $1^1/_2$ in.



(*a*) (*b*)

FIG. 19 METHOD OF OBTAINING EVEN HEATING OF UNEVEN CROSS SECTIONS

(*a*, By placing two assemblies together; *b*, by using a shaped spacer.)

Other simple ways of taking care of the spacing are to do two blocks together, as in Fig. 19(*a*), or to use a spacer of similar material, Fig. 19(*b*).

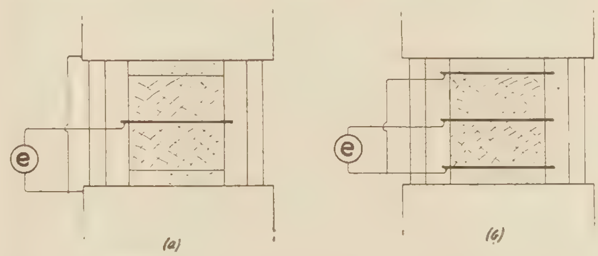
Avoiding Voltage Flashover. A prime difficulty with early radio-frequency installations was the occurrence of voltage flashover. This will always, of course, be something to be considered in the design of the equipment. However, it is felt that it is a problem which can be handled. The tendency to arc across is, of course, a function of the spacing, the material, and the voltage. The voltage which will occur across a given load can be calculated by methods which have been indicated. Furthermore as shown in Fig. 7, it can be reduced by increasing frequency.

How much voltage can be tolerated is hard to calculate since the wood is usually not dry and moreover the glue, if a liquid, tends to squeeze out so that voltage breakdown is far less than would be indicated by the figures for the dielectric strength of the wood. As a result, the value of the voltage to be used must be based largely upon experience. Work done to date is not conclusive, but indications are that for thicknesses of the order of 1 in., several thousand volts can be used. For increasing thicknesses, somewhat greater voltages can be used, but the value does not increase proportionally and there is a limit between 10,000 and 15,000 v (depending upon curvature, etc.) beyond which it is undesirable to go since in this range corona effects become rather troublesome.

Effect of Moisture. One of the most common questions regarding the radio-frequency method is the effect of moisture. The best answer is that the effect is approximately the same as with any other method of heating. Chiefly, this is that an increase in the average moisture content (which means an increase in density) results in a longer time cycle, since more power is required to give the same increment in temperature. The radio-frequency method has some advantage in that it allows the power to be increased enough to hold the time cycle to the same value if this is desirable.

If the moisture content is uneven, there is of course a tendency for the current to flow more through the moist sections. The effect, however, becomes important only for considerable differences. The currents through various sections will vary approximately as the dielectric constant. Fig. 18 shows the variation of dielectric constant for three species. For spruce, for instance, the variation is only 15 per cent between moisture contents of 5 and 10 per cent. This would in all likelihood produce a smaller difference in heating than this same difference in moisture content would when steam platens are used. Generally speaking, we can say that the range of moisture differences met with in gluing operations will cause no difficulties. With regard to drying or curing, where the differences may be great, there is obviously no objection to concentration of power at the more moist points, since this tends to bring the material to an equalized condition.

Minimum and Maximum Power Ratio. There is one effect of moisture which occurs when the heating is done very rapidly and the section is fairly thick. This is the formation of vapor pressure. (The same effect would, of course, occur with steam if the wood were heated equally fast.) It has been claimed that this pressure forces moisture out of the wood and thereby reduces the power required in drying wood by the radio-frequency method. This may be true for small pieces or thin sections, but for large pieces, it is questionable. If the section is large, the pressure produced is considerable and it cannot easily relieve itself. As a result it is quite likely to "blow" out the end, thereby ruining the piece.



(*a*) (*b*)

FIG. 20 METHOD OF REDUCING TEMPERATURE GRADIENT WHICH OCCURS BECAUSE OF CONDUCTION TO THE PRESS

(In *a*, sheets of heat-insulating material are placed between the wood and press; in *b*, they are placed between outer electrodes and press.)

It appears, therefore, that there is a definite rate at which power can be forced into a given package of material. As a result, the possibility of drying large pieces in a matter of minutes is definitely out. It may be that at some lower rate the process may still be of advantage. For instance, even drying which took 2 or 3 days might well be an advantage in an emergency, as compared to a 60-day kiln-drying period. The economics remains to be proved.

It is perhaps worth noting here that there is also a minimum power-input rate for a given job. This follows from the fact that the conduction, convection, and radiation-loss rates increase with temperature. At some point, these equal the power input. Thus, it is necessary to employ a power-input rate which will insure that the equilibrium point is well above the temperature it is desired to reach.

Heat Losses. In considering heat losses, the shape of the curves of temperature gradient cannot be disregarded. If the wood is placed in a press adjacent to large cold plates the gradient is as indicated in Fig. 12. If the laminations are thick so that no glue line is near the outer edge, no difficulty results; but if the laminations are thin, some provision must be made. One simple

expedient is to place extra layers of material between the package to be glued and the electrodes, as in Fig. 20(*a*). This, of course, requires more power. A method which avoids this power loss is shown in Fig. 20(*b*). The value of this arrangement depends of course upon the insulating properties of the buffer.

Of course if hot plates are already available, they can be used to keep the plates up to temperature thereby completely avoiding conduction losses. In some cases it may be desirable to consider the use of heating the plates electrically. These expedients will be required, of course, only for thick sections and since only one set of plates need be heated, the power for electric heating is relatively small.

In most cases the losses by convection and radiation will be much smaller than the conduction losses and probably will not need to be considered. In the case of a thick piece of relatively small area the radiation loss might become fairly large. This loss can be computed from the Stephan-Boltzmann equation

$$\text{Watts per square inch of surface} = 36.8 \times 10^{-12} \times K(T^4 - T_0^4)$$

$K = 0.9$ (for wood)
T_0 = original temperature (absolute)
T = final temperature (absolute)

If the piece is large and the time long, the convection losses can become large, if no precautions are taken. Use of a heat-insulated chamber would overcome this to a degree.

Coupling the Generator to the Load. Failure to "get the power into the load" explains the poor results many experimenters with radio frequency have had. In order to do this properly the electrodes must be correctly shaped and placed (as previously explained), the load must be "tuned," and the impedance presented by the tuned load must be coupled to the generator in such a manner that there is no mismatch of impedances.

Since the electrodes used have a considerable capacity, tuning the load necessitates the provision of an inductive reactance equal to the capacitive reactance of the plates. At the higher frequency (8 megacycles or above) this can be a hairpin of copper pipe with a shorting bar to provide variable electrical length. Such an arrangement is shown in Fig. 15. The tuned circuit provided by the capacity of the electrodes with this loop of pipe in parallel presents a rather high impedance. If the load is close to the generator this circuit can be coupled to the tank circuit of the generator without too much difficulty and usually without resort to a "matching" circuit.

If the load is more than a few feet from the generator, some sort of transmission line is required. For purposes of shielding, this will usually be a concentric line (that is, one conductor inside another) of some type. Such a line will have a rather low impedance of the order of 70 to 200 ohms. If it were coupled directly to the tuned load a mismatch would occur with the result that likelihood of line loss and arc-over would be increased. In these cases it is therefore necessary to employ a "coupling" or "matching" circuit. Where a single type of load is to be fed, as in production, the elements of this circuit can be fixed. When various loads are to be fed, it is preferable to have these elements easily variable. Once set up, such a circuit adds no great complexity to the operation. However, the original calculation and design of a suitable network is a problem for radio engineers.

Shielding Considerations. Shielding must be considered from two angles. The first of these is safety of personnel. Since the generator employs high alternating-current and direct-current voltages it must be completely enclosed and interlocked. The voltages on the press, being of radio-frequency, are less dangerous in that radio-frequency does not produce shock. However, contact with radio frequency at high potential is likely to result in severe burns and for that reason it is well to consider at least a guard rail around the press.

Fig. 21 Carved "Preform" (Right) and Finished Blade of Compreg Test Propeller Made by Radio-Frequency Method
(Reproduced by courtesy of Camfield Manufacturing Company, Grand Haven, Mich.)

The second reason for shielding is to prevent radiation which might cause interference with radio-communications channels. To keep radiation at a minimum, the generator should be completely shielded, the transmission line should be concentric (or shielded), and the press or electrode setup should be shielded as well as possible. In addition, the whole system should be well grounded, preferably through a ground connection not connected to other grounds or grounded pipes. When the heating is done between electrodes outside the press, it is usually possible to enclose the whole assembly in a box lined with copper screen, as shown in Fig. 15. Enclosing a press is, of course, more difficult and usually inconvenient. Though circumstances will vary, it is felt that a metal screen, which comes down around the platens, will give sufficient shielding in most cases. Obviously, either this screen or the lid of the box can be furnished with electrical interlock devices, thereby providing completely automatic protection for the operator.

Typical Installations

(*a*) *Compregwood.* The first installation for the use of radio-frequency power in the production of compreg parts was made by the Camfield Manufacturing Company, Grand Haven, Mich. This equipment has been employed with particularly gratifying

results in the production of compreg propellers. Not only has the time cycle been markedly reduced, but also a much better propeller is obtained.

In the process developed at Camfield, a carved "preform," Fig. 21, is made up under carefully controlled conditions. This preform is cured under heat and high pressure to obtain a homogeneous block having high tensile and shear values. High density at the hub end with decreasing density toward the tip is automatically achieved, and close dimensional tolerances are maintained. The difficulty with this procedure was that, because of the time required for the heat to penetrate the thick block, the curing cycle was very long (of the order of 7 or 8 hr).

The radio-frequency equipment shown in Fig. 23 was installed several months ago and is now used to expedite production by preheating the preform to a temperature of 230 to 260 F. This is achieved by placing it between formed electrodes, as shown in Fig. 22, for approximately 8 min. The preform is then placed in the die and pressure applied. Since the material is relatively soft at

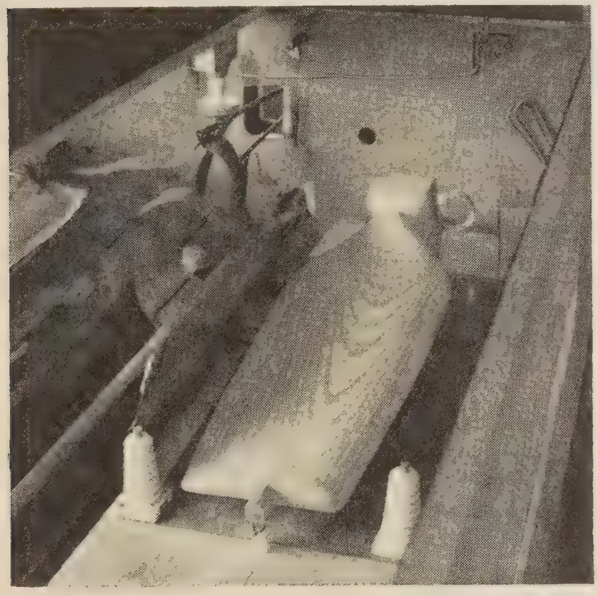


FIG. 22 SCREENED BOX FOR "PREHEATING" PREFORM SHOWN IN FIG. 23

(Note that upper electrode has been removed to show preform in place for heating. Reproduced by courtesy of Camfield Manufacturing Company, Grand Haven, Mich.)

these temperatures, the die closes in a matter of 3 or 4 min. This compares with a time of $2^1/_2$ hr to close the die (and 4 hr for the center to reach 285 F) when the preform is put in cold. Moreover, since the compressing is now all done with the block uniformly soft, very little warping and distortion occur. This contrasts sharply with the uneven compression effect which occurred with steam alone due to the fact that the heat was unevenly distributed in the early stages. As a result, the use of radio frequency almost completely eliminates the internal residual stresses which were previously a major problem. In overall time the reduction attained is 50 to 65 per cent.

A diagrammatic comparison of the time cycles for the procedures with and without radio frequency is shown in Fig. 24. This application of radio-frequency power is obviously one of the most promising. In addition to cutting down the time cycle, improving the quality of the product, and saving considerable in the cost of dies, it opens up many interesting possibilities in the way of sizes and shapes heretofore considered too difficult to attempt. Apparently, thickness and curvature will no longer be limitations on the use of compreg.

The Rudolph Wurlitzer Company at DeKalb, Ill., is another company which is using radio-frequency power in the manufacture and assembly of compreg sections. The equipment setup is generally similar to that shown in Fig. 23. It is arranged to feed power to either of several presses or to a "preheating" setup as shown in Fig. 15. Among the interesting products turned out at Wurlitzer is a compreg propeller block.

(*b*) *Spars and Ribs.* The manufacture of spars and ribs is

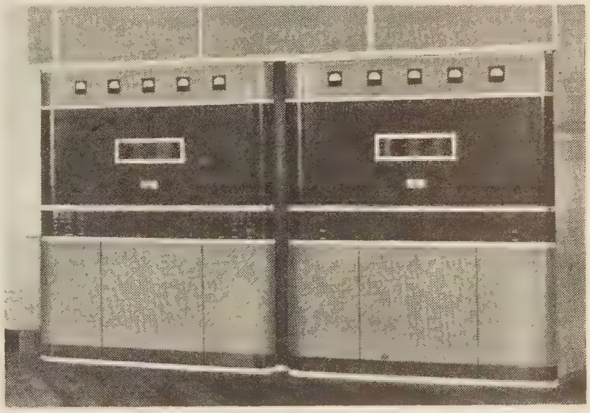


FIG. 23 RADIO-FREQUENCY EQUIPMENT; 15-KW OUTPUT SETUP FOR WOOD HEATING

(Radio-frequency equipment proper is in cabinet at left; power and control equipment in cabinet at right. Reproduced by courtesy of Camfield Manufacturing Company, Grand Haven, Mich.)

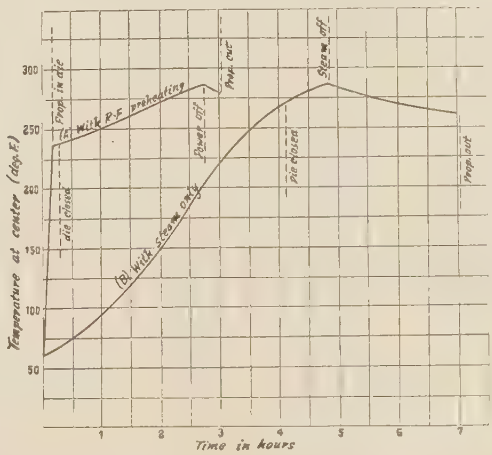


FIG. 24 TEMPERATURE-VERSUS-TIME CURVES FOR PRODUCTION OF COMPREG PROPELLERS (*A*) USING RADIO-FREQUENCY PREHEATING, AND (*B*) WITH STEAM ONLY

obviously one of the most promising fields for the use of radio frequency. Because of their thickness, these structural parts are hard to heat in a steam press, and because of their dimensions they are inconvenient to handle by the bag method. Neither the thickness nor the size is an obstacle when radio frequency is used. In fact, these are among the easiest shapes to heat by the radio-frequency method. Where a press of suitable dimensions is available, they can be heated in the press, preferably using the "sandwich" arrangement, i.e., two pieces with an electrode between. If a press is not available, or those available are not big

Fig. 25 Twenty-Foot Press Setup for Use With Radio-Frequency Equipment in Production of Aircraft Spars
(Two $3^1/_4$-in. × $6^1/_2$-in. × 16-ft spars are shown in process of being heated. Reproduced by courtesy Tolerton Lumber Company, Alliance, Ohio.)

enough, the laminations of the spar or rib can be clamped between metal electrodes arranged for the application of radio-frequency power. It has been suggested that it should be possible to make up fairly simple jigs of the "fire-hose" type for use with radio-frequency power. One such jig would be capable of producing as many pieces as several dozen jigs made for cold gluing only.

A large press used by the Tolerton Lumber Company, Alliance, Ohio, in making spars by the radio-frequency method is shown in Fig. 25. Laminated spars having a cross-sectional area of 1 in. × 6 in. and 17 ft 6 in. long can be produced at a rate of two every 5 or 6 minutes. Larger spars having a total cross section $3^1/_4$ in. × $6^1/_2$ in. can be turned out at a rate of two every 20 to 25 min. In this latter operation a glue containing some catalyst is used. The radio-frequency power applied to the wood for these spar-gluing operations is 10 kw to 12 kw; a frequency of 8 megacycles is used. With operation at this frequency, no flashover trouble has been encountered, even when the squeezed-out glue has run down the side of the spars.

Acknowledgment

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"Wood Aircraft Fabrication Manual," by J. R. Dalzell and J. McKinney, Forest Products Laboratory, July, 1941. Available from CAA on a "restricted" basis.

Ten Years' Progress in Management[1]

FOREWORD

Following a time-honored custom, and in commemoration of the one to whose foresight and leadership in the field of management this custom owes its origin—Leon Pratt Alford, able engineer, dean of management editors and historians, developer of men—this Ten Years' Progress in Management Report is presented by the Management Division, of which Dr. Alford was one of the founders. It is the fourth presentation of its kind, the three previous Reports having been written by Dr. Alford and delivered by him to The American Society of Mechanical Engineers at Annual Meetings, respectively, ten, twenty, and thirty years ago.

Frederick Winslow Taylor, Father of Scientific Management, and a Past-President and Honorary member of the Society, began his pioneering achievements in 1881, and in 1895, 1903, 1906, and 1911 presented the results of certain of his important discoveries and developments at its meetings. The Society was thus the first organized association to recognize the vital significance of this new field of industrial and economic endeavor. The 1911 paper, "Principles of Scientific Management," caused considerable discussion within the Society, so much so that a Subcommittee on Administration was appointed to make a thorough investigation of Scientific Management, presumably to justify it, or recommend its rejection, as a part of the Society's activities.

THE 1912 REPORT ON INDUSTRIAL MANAGEMENT

At the Annual Meeting in 1912, just thirty years ago, this Subcommittee presented its report on "The Present State of the Art of Industrial Management."[2] The chairman of the Committee was James Mapes Dodge, the secretary—who developed the report—was Leon Pratt Alford, then editor of *The American Machinist*, and other members were D. M. Bates, H. A. Evans, Wilfred Lewis, W. L. Lyall, W. B. Tardy, and Henry R. Towne. This report was a "Majority Report," a "Minority Report" also being submitted by those who disagreed with some of the majority findings.

A reading of this first Report reveals, to those familiar in subsequent years with Dr. Alford's progressiveness in engineering and editorial endeavors, his clear conception and prophetic foresight, at this early date, of the vast opportunities ahead in applying scientific-management principles intelligently to bring about engineering, industrial, economic, and social progress.

TEN YEARS' PROGRESS REPORT OF 1922

In 1922, after the first World War and the subsequent 1920–1921 depression, Dr. Alford, whose career meanwhile had been marked by engineering and editorial leadership in the field of management, conceived the idea of preparing for the Annual Meeting of the Society a "Ten Years' Progress in Management" Report to summarize the widespread developments made since the first Report was presented. He was particularly interested in doing so because, in 1920, the Management Division had been formed, with his active participation, and he had been its first chairman.

THE 1932 REPORT ON TEN YEARS' PROGRESS IN MANAGEMENT

After the publication of the 1922 Report there followed a period during which more widespread progress was made in the field of management than had occurred in all of the years before, since Taylor had first conceived his scientific approach to this branch of human endeavor. National progress and prosperity continued until after 1929, when the long depression of the 1930's suddenly broke. In 1932, Dr. Alford, continuing his custom of reviewing the course of events in management, presented at the Society's Annual Meeting the second of his "Ten Years' Progress in Management" Reports, again summarizing, with his keen discernment, the high spots of advancement, assaying their value, and indicating their portent and potency for further industrial and economic progress.

THE TEN YEARS' PROGRESS REPORT OF 1942

To those associated with Dr. Alford it was known that he contemplated a continuation of his custom by offering a third "Ten Years' Progress in Management" Report at the 1942 Annual Meeting. Unfortunately, his untimely passing from the scene of his labors, on January 2, 1942, while actively engaged in many Society undertakings, and in educational work as head of the department of administrative engineering in the college of engineering, New York University, prevented the carrying out of his desire in person. At least four others prominent in the management field and in the Society also passed away during the last ten years: C. B. Auel, Robert I. Rees, W. L. Conrad, and Harold B. Bergen.

With his customary careful preplanning and his invariable foresight in discerning the outstanding developments, Dr. Alford had already begun to collect pamphlets, papers, articles, and data on subjects which he considered significant indicators of management progress. Close friends of Dr. Alford in the Management Division committees felt that no more important step could be taken, and nothing more pleasing to Dr. Alford could be done than to continue the practice of presenting a Ten Years' Progress in Management Report, at the 1942 Annual Meeting, along the sound lines for which he had already set an outstanding precedent. The plan, in fact, became the guiding consideration in selecting topics and speakers for all of the Management sessions at the Annual Meeting, thus unifying the program around the most significant management thinking of the day. Obviously, after the most severe depression ever experienced, which had not ended when the most destructive and widespread war in all history engulfed practically the entire globe, the preparation of a progress report on management became a problem of major difficulty.

DEEP DEBT OWED TO CONTRIBUTORS

A committee appointed for the purpose began in January. While all members of the Management Division Executive Committee, and of its General Management Committee, were

[1] Contributed by the Management Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

[2] Previous Reports appear in the Transactions of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS for the respective years.

affiliated with the work, special acknowledgment is due those who untiringly selected authors for the various sections of the extensive Report planned, and then as untiringly co-operated in securing the delivery of manuscripts from men who unselfishly added to their pressing burdens that of writing parts of the Report. These committee members are: Prof. John R. Bangs, James M. Talbot, Gideon M. Varga, Joseph M. Juran, Lawrence A. Appley, Andrew I. Peterson, Dr. E. H. Hempel, Professor Carlos deZafra, and John A. Willard. To the contributors, whose names and professional connections appear at the head of the respective sections of this Report which they wrote, we are forever grateful. To the Report Committee Chairman, Dr. Lillian M. Gilbreth, is due full credit for envisioning the possibilities of the Report, directing the work of planning the sections and the selection of authors, arranging the method of presentation and steering the activity through its many vicissitudes during a time when demands upon managers and engineers heavily loaded them with unprecedented difficulties.

Through presentation of the Report by Prof. John R. Bangs, chairman of the Management Division, to Harold V. Coes, incoming President of The American Society of Mechanical Engineers, and by him to James W. Parker, President of the Society during the year 1942, as part of The Society's archives, we trust that the custom set by Dr. Alford may now become a precedent for future Management Division Committees to follow. Several of the sections were lengthy papers which had to be condensed for presentation in the Report, because of limited space. The originals are available, however, and may be read in full by anyone applying to the editorial department of the Society, where they will be on file.

The Management Division and its Committee preparing this Report submit it in commemoration of Dr. Alford's unflagging interest in Management, his outstanding contributions to the fund of management information, his active support in many new developments in advance of their general acceptance, his unswerving loyalty to the engineering profession and to The American Society of Mechanical Engineers through which he found a ready outlet for his varied creative energies, his devotion to his friends and associates in whose progress he many times exerted a helping hand, and his sincere interest in engineering education where his leadership and inspiration were constructive influences in shaping for many students broader and more useful careers. He has taken his place among the leaders who have placed the Society in a position of leadership in the Management field.

George E. Hagemann
Vice-Chairman, Ten Years' Progress in Management Committee, and Editor of Report

Management Division

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James M. Talbot, Vice-Chairman
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Administrative Organization

By LOUNSBURY S. FISH,[3] WASHINGTON, D. C.

PERHAPS the most significant trend in the field of administrative organization during recent years has been an increasing recognition of the need and importance of a well-designed plan of organization in facilitating and expediting the direction, co-ordination, and control of an enterprise. Organization is the chassis upon which management is mounted—if adequately designed, the management job is made easier and more effective, if poorly designed, management is difficult and results often unsatisfactory.

Organization Often the Result of Mere Evolution

Most large organizations have just "evolved"—have "grown up like Topsy." Additions and changes have been made to meet specific problems on a basis of expediency with little consideration for over-all design or rationality. The difficulties of management under these conditions may be likened to those which would be involved in living in a house built one room at a time over the years by different tenants without benefit of architecture.

In some cases the rationalization of organization plan has been prompted by enlightened recognition of the advantages to be gained in increased management effectiveness; in others it has been forced by the problems of size, unwieldiness, lack of flexibility, and overburdening of top executives. Regardless of cause, however, an appreciable number of major concerns have overhauled and clarified their general organization plans within the last 10 years, to their great subsequent advantage. Others recognize the need but are reluctant to move because of the complexity of the problem, lack of specialized assistance in this field, or hesitancy about changing the assignments of key personnel.

Trends During the Past Ten Years

In general, the trend and emphasis in organizational planning during recent years have been along the following lines:

1 *Freeing top executives of administrative detail to concentrate on policy determination, long-range planning, and over-all control.* In the case of large enterprises, this result is often accomplished by relieving a number of well-qualified top executives of operating and administrative responsibility to assist the president in the general management and direction of the business as a whole, defining over-all policies and objectives, deciding matters of major concern, co-ordinating the various aspects of the business into a successful whole, and appraising over-all progress and results. This arrangement proceeds from the recognition that in any very large concern it is virtually impossible for any one man, such as the president, to do full justice to the top executive job single-handed. While in a smaller business the operating executives are normally in a position to render such assistance to the chief executive, in very large organizations the operating heads are almost invariably so heavily burdened with day to day administrative problems that they seldom have the time and objectivity to assume a major role in the general or over-all management of the enterprise, and it is necessary to set up a separate

[3] On loan from Standard Oil Company of California as Director of Organizational Planning for War Production Board.

group of carefully chosen men for this special purpose. These men function virtually as "assistant presidents."

2 *Decentralization of the burden of management by dividing and subdividing the enterprise into its logical, separable components, each of which can be held fully responsible and accountable on a proprietary basis.* Due to increasing size and the growing complexities of management there has been a distinct trend toward the subdivision of large enterprises into smaller components which are reasonably complete entities in themselves and which, therefore, afford the basis for effective decentralization. Examples of this type are the many companies which in recent years have subdivided on the basis of product divisions or regional divisions, each having, as a rule, its own product engineering, manufacturing, and marketing organization, and thus constituting, in effect, a separate enterprise which can be held accountable on a profit and loss basis. This type of organization requires special provision for co-ordination of basic functions (product engineering, manufacturing, marketing, etc.) between divisions, usually through the medium of functional vice-presidents who serve in a staff capacity.

3 *Delegation or placement of the power of decision at the lowest practicable organization level, resulting in the elimination of unnecessary layers and levels of management, multiple handling, and red tape.* The importance of this fundamental concept in simplifying organization structure and relationships is just beginning to be recognized. When supervisors in immediate charge of operations are well selected and trained, given a maximum degree of responsibility and authority for making their own decisions within general policy and budgetary limitations, and finally held accountable for results, it is remarkable how well they usually perform, relieving their superiors of unnecessary burden and often eliminating the need for intermediate "layers" of management.

4 *Better co-ordination and integration of staff functions.* Another promising trend is the clarification and standardization of staff relationships. For many years organization of the comptroller's functions has served as a model of staff effectiveness in many concerns. The general plan has been for the comptroller to place a "subcomptroller" trained in the over-all company system, in each department or subsidiary, with the acquiescence of the local manager. The subcomptroller is directly responsible to his manager for supplying necessary information and service in support of operations; at the same time, he is functionally responsible to the comptroller for compliance with the general company system and methods within his field. In recent years this well-proved plan has been increasingly used as a model for the organization of other staff functions which find application throughout the enterprise, such as personnel relations, engineering, purchasing, etc.

5 *Clear-cut definition and understanding of the basic functions, objectives, relationships, and extent of authority for each principal position or agency.* A well-designed organization structure or chart goes only part way in defining a sound organization plan. It must be supplemented by a thorough understanding on the part of each responsible executive as to just what is expected of him and his organization in relation to the rest of the enterprise. As a basis for such understanding there is an increasing tendency in well-run enterprises clearly to define the primary purposes, functions, relationships, and limits of authority of each principal position or agency within the organization—the board of directors, the president, vice-presidents, managers, committees, etc. These written statements can best be developed by staff representatives thoroughly familiar with the existing and projected plans for the organization as a whole, in collaboration with the executives concerned, thus assuring full understanding of the proper relationship of each part with respect to the whole.

In view of the increasing complexities of management and the difficult problems and adjustments which lie ahead, it seems probable that the next decade will witness further major progress toward rationalizing and clarifying industrial organization plans in order that management may be free to concentrate major attention upon broad needs and objectives and have effective means of their fulfillment.

Purchasing

By STUART F. HEINRITZ,[4] NEW YORK, N. Y.

THE industrial purchasing executive during the decade 1932–1942 has operated within business and economic conditions ranging from the depths of depression and low levels of industrial activity to the new peaks of demand and production generated by the need for materials and equipment in the present World War. It has been a period of extremes. In popular terms, these two extremes represent the conditions characterized as a buyer's market and a seller's market.

During the stage of depression, supplies of materials and potential producing capacity are substantially in excess of demand, and tactical advantages are on the purchaser's side. In the ensuing prosperous period, when supplies of materials and production capacity are short of meeting the demand, the situation is exactly reversed. The tactical advantages are on the side of the seller.

Influence of Government Regulation

Within the past decade, this theoretical position at both extremes of the business cycle has been greatly modified by economic controls exercised by the government. The codes of fair practice under the National Recovery Administration during the business depression, in effect, condoned price-fixing activities that served to set minimum levels; curbed expansion which would have thrown the supply/demand ratio still further out of balance; and tended to maintain the existing competitive position of various units within an industry. In the present wartime regulations, at the other end of the scale, we have the direct imposition of maximum prices; priority ratings, limitation orders, and allocation of materials combine to channel industrial output, not according to the manufacturer's preference in selecting his customers, but according to an over-all national plan; inventories are no longer a matter of company policy, but of government regulation. Meanwhile there has been a succession of legislative enactments against commercial discrimination, and a solicitude on the part of the government, well intentioned if not always effective, to protect the smaller business establishments which have not the economic resources or resourcefulness to protect themselves in these times of stress.

Another important consideration is the fact that in the War Production Program, the government itself—directly or indirectly—is the dominant customer for by far the largest sector of producing and manufacturing industry. This automatically places the greater part of business under the control of government contracts and contract conditions, on a mandatory basis. In addition to the regulations already mentioned, extraordinary powers in respect to procurement have been established, including limitation of profits and salaries, permissible items of cost, and the privilege of re-negotiation. Further regulations of a similar or more drastic nature can and will be made whenever the national interest in an emergency demands them. It has been truly pointed out that under circumstances like these, a seller's market in the ordinary sense is impossible; when the government is the buyer, we have a buyer's market.

[4] Editor of *Purchasing* magazine.

Purchasing Policies

Purchasing policies, like any other phase of management policy, are flexible and capable of adjustment to changing conditions of business. In relation to the normal business cycle, that adjustment follows a well-defined pattern. During the period of declining activity and declining prices, purchasing is on a "hand-to-mouth" basis, and inventories are held to the lowest practicable working quantities. The reasons for this are obvious. Under such conditions the delivery cycle is relatively short, so that there is little danger of interrupted production for lack of materials. Storage and carrying costs of inventory are curtailed along with all other costs. By covering for current requirements only, and "following the market down" with small-quantity purchases, the potential loss in declining inventory values is held to a minimum. This was characteristic of purchasing policy in 1932.

In the depression of this past decade, another policy affecting purchasing was highly developed, and came to play an important part in many buying and selling transactions. This was the policy of reciprocity, by which a company's desirability as a supplier was rated largely on its value as a customer, thus injecting an additional factor into the purchasing executive's selection of his sources of supply. Reciprocity is nothing new in business, but its intensive development as a major policy and its widespread application in purchasing is distinctly a product of the pressure for business during the lean years of the early 1930's. It can be readily rationalized: All other things being equal, we prefer to buy from those who buy from us, and so we help each other. The trouble with this oversimplified statement is that other things are rarely exactly equal, and the problem becomes one of balancing the advantages and disadvantages involved in a reciprocal relationship.

Purchasing men have been traditionally opposed to reciprocity as an infringement of their prerogative of free selection of suppliers and as a practice which is easily subject to abuse. They have resented the sales approach which is based on a reciprocal claim for preference. Reciprocity is primarily a sales argument, as frequently originating in the sales department of the buyer's own company as in that of his suppliers. When reciprocal buying was forced upon the purchasing department, it was not uncommon for the purchasing executive to take great pains in establishing the fact that it was accepted only as a sales policy and not a purchasing policy, even to charging any forfeited price advantages to the account of sales expense.

Under the stress of the depression, when all departments became more sales-minded, this traditional attitude underwent a very general change. Reciprocity was accepted as a management policy, applicable to both sales and purchasing. It came to be regarded as a business factor to be considered in making purchase decisions along with other economic and market factors; and many purchasing departments organized their records and procedure with a view to using this factor most intelligently, just as their records and policies were adjusted to take account of commodity fluctuations, seasonal markets, the business cycle, and similar considerations. This development kept pace with the development of reciprocity itself, even when the latter progressed from the simple two-way relationship to three-way and up to elaborate cases of seven-way reciprocity involving not only customers but distributors and customers' customers. In some companies, large and small alike, purchase records are analyzed according to vendors and volume and correlated with sales records so that purchases of common supplies such as lubricating oil, for example, can be distributed proportionately to sales to the respective suppliers; to a lesser extent, purchase records are regularly furnished to sales departments as a part of their sales information; in a few large companies, individuals in the purchasing department were given the specific responsibility of handling reciprocal relationships.

When the business cycle turns upward, purchasing policies are adjusted for longer coverage, longer credits, and higher inventories. The reasons for this are as obvious as the reasons for hand-to-mouth buying during the preceding stage. This predictable change in buying policy became apparent as early as 1937; the new phase was in full effect by 1939 and was being intensified month by month under the impetus of the European war. A true sellers' market was in the making.

Unfortunately, these perfectly normal purchasing policies operate to exaggerate the swings of business fluctuations, whether on the upward or the downward movement. With wartime inflation and material shortages clearly imminent, every logical move of the purchasing executive to protect his company against these hazards could only tend to hasten the day of shortages and inflationary prices—a fact which purchasing men genuinely regretted but which they were powerless to avert except through concerted action which was both impracticable and illegal. The policies of resistance to price advances and moderation in buying for inventory, on a voluntary basis, were strongly advocated both from within purchasing circles and from the outside, but they were effective only to a limited extent in stemming the tide.

Before the process had advanced very far, however, governmental controls were developed, mildly at first and then with increasing scope and severity, to cope with the situation. These controls were prompted by a dual purpose: to prevent a runaway inflation, and to assure that the needs of the country's armed forces would have first call on the materials available to industry. It is not necessary to detail these regulations here. A large proportion of them directly affect the purchasing operation in the procurement and application of materials, permissible inventories, and prices.

Despite the fact that the burden of detailing requirements, making application for materials, extending priority ratings, and submitting a great variety of reports to the government control agencies, has fallen particularly heavily upon purchasing departments just at a time when the volume and difficulty of their work were greater than ever before, purchasing men have welcomed these regulations and co-operated willingly in carrying them out, accepting them not only as a necessary measure in the national interest but as a means of maintaining an orderly flow of materials under conditions which might otherwise have developed into ruthless and destructive competition for the limited supplies available. Criticisms of the regulations have been directed chiefly toward simplifying the control procedures and toward permitting the exercise of more discrimination in applying the regulations to excessively small quantities and to certain types of "shelf products"—both being everyday problems of purchasing procedure and both constructive, calculated to make the system work more smoothly and effectively without attacking the basic plan.

Particular Problems

The modern conception of the industrial purchasing function had its origin about 1915, under conditions of wartime procurement comparable to those of today though not on such a vast scale. Since that time, stimulated by collective thought and effort, and by the newly awakened functional consciousness on the part of buyers, progress has been sound and steady. The mechanics of purchasing procedure are not complicated. The underlying principles which make for a more effective procurement program, correlated and co-ordinated with the other departments of industry, take time to develop.

In 1932, particular attention was being given to simplification and standardization, purchase specifications, and purchase

budgets. Substantial progress has been made in respect to all of these matters. A significant trend at that time was toward the insistence on more technical or engineering training and experience for men entering the purchasing field, and the provision of more adequate educational facilities for purchasing work. Both of these programs have been carried forward consistently and successfully during the decade. There are now upward of sixty specific courses in purchasing offered by schools and colleges of business administration throughout the country, ranging from the vocational-type course led by men from the ranks of purchasing to required courses integrated with the management curriculum in graduate schools such as the Harvard Business School and the Wharton School of Commerce at the University of Pennsylvania.

The major new and current topics to which purchasing men are directing particular attention are largely prompted by the changes wrought by the war emergency. Typical of these problems and activities are the following:

Substitution of Materials. Common practice since the more general use of specifications in buying has been to compile "approved lists" of products and suppliers for each regular requirement. Products and firms represented on these lists were capable of meeting specifications and were considered satisfactory sources from both the purchasing and the technical or production viewpoint; the purchasing executive selected the supplier from these lists. Under present conditions of scarcity, conversion, etc., they no longer represent assured sources of supply. The purchasing agent therefore seeks new sources or substitute materials which will serve the purpose and which are still available. With the approval of technical executives, such alternative materials or sources are added to the approved list, broadening the potential field from which materials may be drawn, even though this sometimes involves basic changes in product or operations. The search for materials covers the whole range of industries and producing centers.

Plant Conversion. With manufacturing industry converting more and more completely to the production of war materials, the character of many plants has radically changed. The purchasing executive is now dealing with materials previously unfamiliar to him, and suppliers with whom no previous contact or experience has been established. This too involves an enormous amount of exploration and research.

Conservation of Materials. The disposition of surplus and obsolete materials and equipment has long been considered an incidental responsibility of the purchasing department. It is logical, therefore, that purchasing men have had a prominent part in salvage and conservation activities which are now assuming greater proportions and greater importance in every industrial project.

Government Regulations. It has been previously pointed out that a large part of the wartime regulations concern materials and procurement. Starting early in 1941, many purchasing departments found it advisable to set up a Priorities Section within the department to handle the records involved in these controls and to keep abreast of changing regulations. This is now a major responsibility in every company, and in many cases the Priorities Division has been developed as a clearing house for all information of this sort.

Expediting. Follow-up of orders to secure delivery is another normal purchasing function that has assumed new importance, and it is not uncommon for an expediting staff in 1942 to outnumber the actual purchasing staff in a buying department. Outstandingly successful examples make use of men recruited from sales departments, whose broad industrial contacts, temperament, and training are particularly adapted to such work.

Administration. The broadening scope of purchasing responsibilities previously suggested, as well as increased volume of operations, have resulted in greatly expanded personnel. The typical purchasing department in an industry serving the war effort has doubled or tripled in size over the past two years. This has brought about a change in the function of the department head. Whereas most purchasing directors a decade ago were actively engaged in buying, along with an assistant or a staff of buyers depending on the size of the company, today's department head is necessarily an executive concerned with policies, planning, and administration of the procurement function, dealing only with major contracts, if indeed he is actually engaged at all in the buying operation.

Development of the Procurement Function

The position of the purchasing executive in his company organization, and in relation to the various other departments, has been fundamentally changed as a result of these recent developments. In the accepted management formula of ten years ago, purchasing was regarded as a purely service function, carrying out plans and decisions of design and production departments. A new relationship has now emerged, which is indicated by the more general use of the term "procurement" in place of "purchasing." Purchasing is an act. Procurement is a function—prerequisite to production, and often affecting design.

With materials, supply occupying the key strategic position in our present wartime economy, this distinction has particular significance and the purchasing or procurement officer is placed in a similarly strategic position in the planning councils of his company. The procurement of materials is no longer a service operation, but a determining factor in policies and operations. The purchasing man who does not measure up to these responsibilities cannot survive in that position. Fortunately, the emphasis on technical competence and functional training previously mentioned have gone far toward preparing purchasers to meet this situation.

The requirements of the purchasing job have been enormously raised. It has always been true that a successful purchasing man must be a little of an economist, a little of a factory man, a little of an engineer, a little of an accountant, and a little of a lawyer. That whimsical description is now a very serious matter. Writing a purchase order no longer gives assurance of delivery. The formal type of specification has given way to a consideration of the function or service to be performed. As one leading buyer has phrased it, it is no longer sufficient to make the best possible purchase, but the best legally defensible purchase.

The economic and philosophical approach to purchasing has also been altered by the times. Prior to the wartime economy in which we are now working, the purchasing man, however broad-minded and progressive he might be, was to a large extent committed to the *laissez-faire* theory of business; the particular responsibility assigned to him by management was the protection of the company's interests in respect to material costs—always remembering that cost and price are not synonymous. In his purchasing, he was necessarily the instrument of the competitive system.

Now, as the instrument through which a national planned economy is directing the flow of materials, with national welfare rather than individual operations as the controlling factor, and in his new position of executive responsibility, he has an important part in implementing new commercial policies which will go far in determining the economy, philosophy, and sociological aspects of the whole postwar world, for materials and their distribution—among industries, among nations, and among classes—are recognized as one of the basic problems which must be solved in the establishment of a stable civilization.

Looking Ahead

Looking ahead, it is certain that the future of purchasing will be a process of further adjustments to change. Many of the specific problems of 1942 are developments of wartime conditions and are temporary in nature. But there have been permanent developments as well. In particular, the emergence of procurement as a basic management function, co-ordinate with marketing, production, engineering, and finance seems to be fundamental. There is significance in the action of the Harvard Business School in correlating its marketing and purchasing curriculum under the inclusive title of "Distribution."

Historically, the first great advance of purchasing, in technique and influence came with the first World War. Permanent values were revealed and developed in that experience, which became the basis of the past quarter century of progress in buying. The values and opportunities now apparent in the present emergency are immeasurably greater. It is reasonable to assume that the experience of that earlier emergency will be repeated, to whatever extent purchasing men display the capacity and willingness to accept the new responsibilities of distribution and procurement.

Defunctionalization of Industry

By HENRY H. FARQUHAR,[5] ALEXANDRIA, VA.

Summary

In the 1920's there was a trend toward "functionalization." Thus, in addition to the usual major departments (of production, of selling, of finance), various companies went much further in separating out certain over-all functions or activities common to each of these major operating departments—functions such as personnel work, public relations, cost and accounting methods, the setting of work standards, and so on—and giving a man in charge of each of these authority either within one of these departments or over all departments according to his location in the organization. But such a high degree of subdivision of labor and the setting up of these "functional" officials cross-cut and interfered with "line" operations of producing and selling, and the past decade has witnessed a decided straight-lining of industry back to one-man control at any one point in the organization.

Perhaps the most notable trend in organization during the past decade has been the widespread setting up of "staff" assistants to various officials, to advise and instruct operating men in handling these problems of personnel, public relations, finance, and the rest, but without authority to issue orders to operating men. In clear-cut organizations it is made plain to everyone that such staff men have no authority (except as may be specifically and publicly delegated on special occasions). A sharp distinction has been made between "line," "staff," and "service" or housekeeping functions. The natural tendency for staff men gradually to assume line authority has been reduced through definition of the staff's place in the organization, as well as through increasingly sympathetic understanding and use of the staff by line executives.

The other notable organization trends during the past decade connected with the movement away from undue functionalization, are the development of more adequate control over total operations; the integration of research and production through definite organization machinery; and the raising to the top levels and the building up of high-grade men and methods for collective bargaining and other personnel matters.

[5] Organization and Management Counsel.

Organization

The outstanding over-all development during the 1930's is the great progress our leading corporations have made in revamping organization structure to meet the imperative demands of the times.

Anyone who has not followed closely the recent trends in decentralization, and in the simplification of lines of authority supplemented by technical staff and service assistance at various points, will be impressed with the modifications which many companies have made. There is still, to be sure, considerable variation in the degree of formality of organization and control. In general, as might be expected, the close corporations usually have an informal setup, whereas those with wide stock ownership tend to develop a more formal structure and to minimize the danger of repercussions and drastic changes upon the withdrawal of any one individual.

Practices Among Progressive Companies

A few major points of emphasis, indicative of modern practices and trends among the more prominent and progressive business and industrial firms, are these:

1 In the larger companies, it is now customary to have one official, usually the chairman of the board, who devotes his undisturbed time to major matters of policy, contact, public relations, and long-time plans and programs, free from the many demands of the day-to-day running of the business. The actual operating is left to a chief executive—the president or an executive vice-president—the chairman concerning himself with current administration only in very exceptional cases. In some companies the chairman has no direct authority over operations.

2 Particularly noteworthy is the extent to which the staff has grown in prevalence and importance during the past few years. (By "staff" in this discussion is meant a group of specialists composing a part of an executive's own office, but outside of the line of authority, with no administrative or routine duties, whose principal job is to enable the executive more fully to exercise co-ordinating and other functions pertaining to his own office, and who also carry on broad constructive work, keep in touch, advise, inspect, and supervise with respect to their particular specialties, and act as delegated from time to time by their executives in definite fields, functions or projects.) These staff officers have charge of no departments, have no regularly recurrent or other duties except as delegated by the president or by the subordinate official to whom they are advisers. In some cases such staff officials are found at headquarters only; in other cases corresponding staff men are found at divisional or regional headquarters, and also at each individual subsidiary within each region. In some cases these distant staff men report directly to the corresponding headquarters staff men; in other cases, they are under the jurisdiction of the local manager. In either case, however, it is the practice for staff men wherever located to correspond and deal direct with each other in the discussion of programs, plans, and technical matters.

Among the firms which have taken the lead in modern organization, it is the universal practice that staff men have no regular authority except by virtue of their knowledge and position. When their recommendations are to be translated into action, they must persuade the line official concerned that such action is desirable, and all orders putting such action into effect are transmitted by and through line officials. This keeps the lines of authority and responsibility clear, and enables the chief executive to hold one man responsible for results along specific lines.

An individual normally assigned to line work may be and often

is transferred for a period to a staff position, the staff man meanwhile being assigned perhaps to the line. There is ordinarily much interchangeability between line and staff personnel. But the positions remain distinct; during the time the line or the service man is occupying a staff position, he is an advisory assistant to his executive and no longer exercises authority. He is not expected adequately to carry on both line or service and technical staff duties simultaneously, or switch indiscriminately from one to the other.

The staff thus neither exercises authority over the line nor performs line work for an operating official. [A staff assistant for personnel management, for example, does not do personnel work for the line, nor lessen any line official's responsibility for maintaining good industrial relations. This staff assistant helps his executive to see that all line and all service officials conduct personnel work in their own units in accordance with announced policy and procedure.]

In some notable cases this general staff principle embodies a definite group of advisers to the chief executive, each of whom is assigned as adviser also to one or more specific departments. It is the duty of such advisers to relieve the chief executive of many details in running the business, to keep informed of what is going on in the division to which he is assigned, to keep the chief executive informed regarding such activities, and to serve as a correlating and reference agency to which the particular department head is morally obligated to refer. In such cases many more officials report direct to the chief executive than has heretofore been considered practicable—it leaves the avenues of approach open, but removes from the chief executive innumerable conferences on minor matters. In fact the "span of control" has been immensely extended through such means as these.

A most important element in the full use of the staff, is the judicious use of advisory group discussions at various levels. Groups are becoming indispensable in big concerns for purposes of co-ordination, for developing high morale, for training younger executives, and for the simultaneous bringing out of all points of view. For these purposes advisory boards in a few cases extend throughout the personnel. They should not be confined to the top levels, although unfortunately this is where too many of them are found today. Their use has been materially increased during the last ten years; they are today of major service in helping determine company policy as a whole.

It is significant that such use of the staff, supplemented by advisers and advisory boards, has come about on a large scale almost entirely since the depression, and in the face of the necessity of economizing in every practicable way.

3 In large corporations with many subsidiary plants it is of course customary to have in the headquarter's office a vice-president of production, of sales, of engineering, and so on. It is the general practice, however, to allow utmost freedom of action to the president or local manager of each subsidiary and, in effect, these vice-presidents in the headquarter's office are advisory rather than line officials.

4 All necessary authority for the accomplishment of specific work is delegated to one individual as directly and with as few limitations as possible; such authority being delegated in turn by the particular individual to his subordinates with a view of transmitting it as close to the point of action as possible. Certain exceptions and reservations, of course, are made in various cases, either in written form or in well-understood procedures evolved over long periods of time and controlled by headquarters in various ways as will be discussed under the next topic.

Control

As has been indicated, it is becoming increasingly the practice to delegate authority with as few limitations, overlappings, or indefinite areas as possible, coupled with the development of various means by which the exercise of such authority may be checked. Among the outstanding measures by which central control with decentralized operation is secured are the following:

1 The use of advisers as just explained. With an understanding on the part of each line official that he is not only to keep his adviser informed but also to secure his concurrence where possible in contemplated action, the chief executive has provided a series of spokesmen for himself in transmitting his desires downward, and a spokesman for each operating official whereby the latter's activities and wishes are transmitted upward. There is an understanding by line officials that such advisers have freedom to look into any activity at any time. There is thus provided a means by which the executive can secure sifted summaries of what is going on below, with the safeguard that lower officials can come direct to him or he to them; he thereby has multiple sources of information regarding each activity.

2 The creation of staff specialists at various levels throughout the organization with much the same effect as in the case of the advisers previously described. Such staff specialists have the additional function of specializing on current research or other problems referred by operating officials—problems which would otherwise occupy the time and attention of such officials and their subordinates at the expense of getting the work done. It might seem that such control through the advisers and the staff would result in a slowness of action, but as a matter of fact it has the opposite effect in that each line official is given full and unmixed authority under close inspection and advisory guidance, with the opportunity to reach his adviser or a staff man much more readily than he can reach the chief executive.

3 The increasing use of carefully prepared operating plans and budgets, whereby individual responsibility for expenditures and results is secured, with monthly or other short-term comparisons and measurements. Matters of budgetary control and of adjustments of funds are in but few cases entrusted to anyone other than the chief executive, or to his immediate personal or staff assistant delegated by him to make adjustments in all possible cases without reference to him.

4 The much more extensive use of advisory committees in the past few years. These committees consist of both permanent and special groups, selected with the definite purpose not only of getting a breadth of viewpoint, but also of serving as correlating agencies. In a few cases they are used definitely for the purpose of taking details of administration off the chief executive's shoulders by his delegating to the committee authority to take action where general agreement can be arrived at. Reports of the meetings of such groups, and reports covering subsequent action, are submitted to the chief executive.

5 In various companies the careful detailed definition of duties and authority of each individual; in most cases, however, such written instructions are couched in broad general terms only, and in some cases the attempt to draw detailed specific lines is not favored. The latter is more often true, however, with old established organizations than where recent changes in functions and relationships have been made.

6 The interchange of headquarters and outlying personnel in so far as practicable, and the detail of outlying personnel to the headquarter's office.

7 Inspection by specialized officials from headquarters, and particularly by staff men; in some cases through the medium of traveling inspectors who do nothing else; and in all cases by occasional field trips of top officials.

8 Encouragement of all important officials to go to the top for advice and guidance if they feel so impelled, without necessarily clearing through their immediate superior, and encouragement of the practice of the chief executive's calling directly on

anyone down the line for information and advice. In such cases, the chief executive is careful to confine his contact to information and advice, and not give even *implied* instructions to any subordinate except through that subordinate's superior officer.

Research

Research plays and will play a vital part in the leadership, and in fact in the very existence of practically every large company today. It is particularly important to observe how operating officials and activities, and research officials and activities, are correlated and made of maximum benefit to one another and to the company as a whole. The following has become pretty much standard practice during the recent past:

1 With the exception of projects which might be termed "pure" research looking to long-term product or process developments which may or may not have an eventual commercial value, it is almost universally customary that all research projects are passed on either by the president, by the executive committee, or by a special committee delegated authority for this purpose. In all of these cases each proposed project is "shot at" from all major angles—sales, production, finance, engineering. In some cases formal recommendations from each one of these officials are turned in to the central approving agency, in other cases the president or agency calls in such affected officials as seems necessary for proper consideration of the special case.

2 Actual conduct of research projects is turned over by the central approving agency either to the operating or to the research end of the business, each of which thereafter has exclusive jurisdiction over the project under the established procedure that the other is to assist if necessary in an advisory capacity. In some cases technical men are borrowed from operating departments for temporary work under research, and vice versa.

3 It is quite generally customary for operating officials to conduct specified minor research projects through utilizing their own technical personnel, but with prior approval of the chief executive or of an appropriate official or group reporting to him. Such limitations, furthermore, are usually expressed in terms of maximum estimated cost, or left to the operating official to be carried on within his budget, or to be referred to the research division at his option.

4 It is customary that progress reports be required at short intervals covering each continuing or new project assigned. The progress reports go to the central approving agency, and to other interested or affected personnel. In various cases the extent to which research is to be prosecuted is defined in advance—for instance, in pharmaceutical manufacture, research is in one company limited to determining the medical practicability of the project, after which the president determines commercial practicability through conferences with various officials.

5 When a project is completed to a defined point the results go back to the approving agency, which through additional conferences determines the department (research, production, or sales) which is to have future responsibility for putting results of research into practice.

6 Some interesting variations of paragraph 5 above is where specific departments, removed from the jurisdiction of either research or operating, have the responsibility of converting the results of research into factory or other operating standards and instructions. Such departments have no authority to put standards and instructions into effect, these being finally approved by and executed through the line official affected.

7 In many cases much of the work of research dealing with current or short-term projects originates from operating officials. In such cases where the project is to be turned over to research to conduct, plans are drawn up by research and approved by operating, the work being under the supervision of research but with constant touch and advice from operating officials.

8 Where it becomes advisable to assign a research technician to conduct his work within an operating department (such as chemical control of manufacture, of rubber compounding, of synthetic products, or of engineering work involving machinery for processing), the dividing line does not seem to be clear in all cases. In some few instances a research man is definitely delegated to supervise the production processes involved; in more numerous instances, the research man temporarily reports to the line official for general supervision but with an understanding that research is to specify and control technical methods and procedures for the conduct of the experiments.

Personnel—Labor Relations

It has been only during the second half of the past decade that noteworthy changes as respects the organization status of those handling personnel management and labor relations have become widespread. The reasons are obvious. Such changes have taken two principal directions:

First, the need for uniform dealing with the national labor unions has required that the personnel department or some similar office be given added functions and added status in respect to collective bargaining. In some cases this has involved the actual bargaining and the writing of contracts at the top level; in other cases it has involved participation by the personnel officer as an observer and counselor; in still other cases the personnel officer has acted as management's adviser and as a co-ordinator of data and principles to keep uniform the bargaining covering the various divisions of the company to avoid the error of giving one local more than another.

Second, along with the raising of collective bargaining to the top level, has come a decided emphasis on employee participation; on the institution of more carefully planned and conducted suggestion systems; on the improvement of personnel techniques, job-evaluation methods, merit rating, standardization in wage and salary determination, and so on. Perhaps most significantly, there is just now beginning to be put into general practice a conscious development of more systematic and rational interviewing and counseling programs in order to help the individual worker adjust himself to the total situation—both within and outside the plant—which confronts him.

It is increasingly customary to find that the man who handles such work as is indicated in the two preceding paragraphs now reports directly to the chief executive—a major shift in organization in tune with the times.

Miscellaneous Developments

A few miscellaneous organizational developments merit mention:

1 Quite general decentralization of clerical and service functions; thus each major division of the business has its own office management (with the occasional exception of a central clerical pool, the use of which is optional), its own personnel officer (who must keep in close touch with the company personnel officer), its own mail and files, its own maintenance, and so on.

2 Where a central service unit exists, any other unit of the company is free to utilize the services or not as they choose. In no case does such service unit have control or authority over the activities of any other department, the necessary co-ordination being performed by the chief executive or his "staff" (as defined).

3 The practice of having accounting personnel in outlying subsidiaries report directly to the accounting personnel in the headquarter's office (functional), is followed in some cases, but this practice is decreasing. In most companies the outlying accounting personnel reports to the local manager; the head-

quarter's officials have authority to regulate over-all company procedures as approved by general management.

4 Training activities are outlined by headquarter's specialists, but in most cases actual conduct of training is under the jurisdiction of the department head, or of the resident manager of the subsidiary, whose personnel is being trained. In at least one company the personnel manager reports to any one of three co-ordinate manufacturing vice-presidents, according to which group of employees is being dealt with; this personnel manager, furthermore, has a personnel man in each plant, who, under the functional principle, reports direct to him.

5 Quality inspection, of course, is now quite generally removed from the supervision or influence of the man who is immediately responsible for quantity production.

There have thus been outstanding contributions during the past decade to the organization problems of structure, of control, of research, and of personnel relations. Under the fire of the depression, of defense, and then of war, industrial managers have forged a direct and flexible, yet essentially simple structure which combines maximum opportunity for direct action with minimum necessary central control. They have refined and made more responsive that control. They have extended and integrated research—both for war and post-war use. And the field of human relations has been raised in status, and refined in technique.

Real progress has been made toward finding the answers to some of the most perplexing problems which modern conditions have forced upon top administration—the problems of determining sound policy, of minimizing bigness, of the burden on the chief executive, of the two-way flow of information, of permanence and continuity of management, and of striking the balance between decentralized operation and central control.

Gaging and Inspection in Interchangeable Manufacture

By CARLOS deZAFRA,[6] NEW YORK N. Y.

AT the close of World War I industry recognized the difficulties caused by lack of international accord in the field of precision measurement. There was no accepted factor for converting metric into English units of measurement and vice versa. Nor was there any standard temperature at which precision measurement should be made. These points were subsequently cleared up so that now 25.4 mm equal one inch and the standard temperature at which precision measurements should be made is 68 F or its equivalent of 20 C.

This international agreement has tremendously facilitated co-operation between allies in war efforts—a co-operation that was sadly needed and practically impossible of accomplishment in the first World War.

Advances in Inspection Techniques and Instruments

In recent years great advances have been made in the technique of inspection and in the measurement and inspection instruments. Contributing to these advances are two outstanding accomplishments. First, the adoption of the American Gage Design Standards by the gage manufacturers. These standards apply to gages of commercial design and establish measurements and tolerances for gages and components thereof, affording interchangeability between components produced by different gage makers. Second, the National Screw Thread Commission has issued a revised edition of "Screw Thread Standards for Federal Services" (National Bureau of Standards Handbook H28) bringing up to date and clarifying the various modifications in screw-thread elements, design and classification, with standards in tolerances for each type of fit for each size range of screw threads. Thus manufacturers of screw cutting or forming tools, threaded products, and thread gages all have a common reference standard which now eliminates the confusion which existed over these many years of standards development.

Formerly, the human equation played an important part in the use of micrometers and in setting snap gages with precision gage blocks. Tolerances seldom were closer than three decimal places. "Go" and "Not Go" gages were individually applied to check each element of a component part and often the acceptance or rejection hinged upon the technique employed by and the "feel" of the inspector. Today this human factor is being rapidly eliminated by the use of fixtures to hold gaging points making simultaneous contact at the various locations on the inserted component where checking for size is necessary. The proper pressure of contact of the gage element against the component is uniformly maintained so that the same results may now be obtained by novice or skilled inspector. The advance attained by automatic inspection is illustrated by the practice of one automobile-engine manufacturer who employs a device whereby twenty-seven elements of a camshaft are simultaneously inspected, the accepted shafts rolling out of the inspection machine on one level and the rejected shafts rolling off at another level. An inspection is accomplished in from four to six seconds. A few years ago such an inspection would have required 27 applications of "Go" gages and 27 applications of "Not Go" gages, or 54 gage applications in all, requiring many minutes to perform, depending upon the skill and dexterity of the inspector, who had to be experienced.

The Air Gage

The air gage has become quite popular for certain types of work. It is based upon the principle of the escape of air, under pressure, through an orifice. If the orifice is so obstructed as to restrict the escape of air, a back pressure is built up to actuate the pointer on a dial and thereby indicate the error in dimension of, say, the inside diameter of a ring slipped over a mandrel of proper size through which the air passes to the orifice outlet on its side. If the ring is too small it will block the free passage of air; if too large the clearance between ring and mandrel will permit too much air to escape so that the pointer will indicate correspondingly the oversize.

Electrolimit Gages

Likewise so-called "electrolimit" gages are fast making their appearance and contributing at this time to the rapid inspection of munitions. In this type of electric gage very great rapidity of inspection can be maintained. It is applicable to tolerance ranges from 0.00005 in. to 0.012 in. with an accuracy of measurement of 0.000005 in. The instrument is set for any given dimension by the use of precision gage blocks or other standard, and several gaging heads may be combined in the design of a fixture to receive the work, such as shells, fuses, cartridge cases, etc., upon which eight or more dimensions can be simultaneously checked.

Each gaging point is individually connected to a light on the indicator panel, in addition to being integrated to a master signal light usually set at the head of the panel. The inspector needs only to insert the work and glance at the master signal which flashes red if any gage dimension is in error while the individual

[6] Director, Gage Laboratory, College of Engineering, New York University. Mem. A.S.M.E

light for that gaging point informs him which is the erring dimension. Theoretically, at least, the capacity of the instrument is unlimited, pinions $^1/_8$ in. in length and 0.010 in. in diameter being checked as easily as large-caliber shells and large automotive parts. Nor is there any limit to the number of gaging points, for interior or exterior dimensions, that may be used at one time.

Women Inspectors

Although women have been employed to a limited extent on peacetime production inspection, the shortage of qualified men for this work during the current national emergency has greatly stimulated the training of women inspectors. In such training courses they are given the same instruction as the men. The subjects include review of mathematics, drawing and blueprint reading, materials testing, metallography, manufacturing methods, machine-shop work, jigs, fixtures, gages and inspection methods, and actual inspection practice with the use of a wide variety of precision-measurement instruments.

The magnitude of the present war production has prompted tremendous improvements in inspection instruments and technique so that we may look forward to the ultimate resumption of peacetime activities on a far more advanced level than prior to the war, necessity being still the mother of invention and progress.

Statistical Control in Applied Science

By W. A. SHEWHART,[7] NEW YORK, N. Y.

Foreword

Statistical quality control was born eighteen years ago. It was Dr. Alford, at that time editor of *Manufacturing Industries*, who two years later announced the birth, as it were, by the publication of a short article in that journal.[8] Thereafter he watched with an active, critical interest the development of statistical-control techniques and when, in 1932, he wrote his report, "Ten Years' Progress in Management, 1923–1932," he called attention to statistical control as a tool of far-reaching significance to management. It is therefore particularly gratifying to be able to present here a brief survey of developments in statistical control that are of interest from the viewpoint of management in the hope that the comparatively rapid development of the theory and application of statistical control both here and abroad during the past ten years will bear additional testimony to Dr. Alford's ability to sense the importance of new developments in scientific method to the solution of everyday problems of management.

THIRTEEN years ago The American Society of Mechanical Engineers, in co-operation with the American Society of Testing Materials, called a round-table conference on the application of statistics in engineering and manufacturing, out of which came the organization of a committee which is now sponsored jointly by five societies.[9] Progress in the application of statistical quality control during the past ten years is largely attributable to this co-operative attempt on the part of representatives from several engineering and scientific organizations to get people from different groups to merge their common knowledge of statistical techniques and to discuss their problems in order to get a broader view of the usefulness of statistics. Co-operation of this character did not stop in America but, through the efforts of the Joint Committee and its sponsor groups, engineering societies in other countries, particularly in Great Britain, were asked to join in surveying the potential contributions of statistics.

Increase in Applications of Statistical Control

A report of the Joint Committee describing these early activities was published in Mechanical Engineering for November, 1932, and should be consulted for a brief review of the early steps in organizing this co-operative effort to further the applications of statistics in engineering. Since then, members of this committee and its sponsor organizations have taken an active part in the development of the application of statistics. A large share of the credit for progress in this direction should go to The American Society of Mechanical Engineers which, through its journal and its sectional and annual meetings, has done much to promote the work of the Joint Committee.

The few applications of statistical quality control in industry in 1932 have grown to many in 1942; too many to list here. It may be interesting, however, to review some events, both here and abroad, which, viewed from their aftereffects, seem to have been most influential in helping to spread the knowledge of the value of statistical methods.

Through the co-operation of the American Standards Association and the engineering societies represented on the committee, in 1932, the British Standards Institution became interested and appointed a committee to look into the subject with the result that an excellent monograph[10] by Prof. E. S. Pearson, was published by them in 1935. Next in line chronologically was the awakening of the interest of the United States War Department in the value of statistical methods in the production of ordnance[11] which finally resulted in the request of the War Department to the American Standards Association to standardize the quality-control-chart techniques to make possible their more general use throughout the country. In accord with this request, the American Standards Association has recently issued three standards on this subject.[12]

At the present time, the Office of the Chief of Ordnance has undertaken a program of training conferences on the principles of the quality-control technique in the various ordnance districts throughout the country. The American War Standards prepared by the American Standards Association were reissued by the British Standards Institution in England. Last April a joint meeting of the Institutions of Civil, Mechanical, and Electrical Engineers was held in London, which, according to reports in several scientific journals,[13] showed by the large

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[7] Bell Telephone Laboratories. Chairman of the Joint Committee for the Development of Statistical Applications in Engineering and Manufacturing.

[8] "Finding Causes of Quality Variations," by W. A. Shewhart, *Manufacturing Industries*, vol. 11, no. 2, February, 1926, pp. 125–128.

[9] Joint Committee for the Development of Statistical Applications in Engineering and Manufacturing sponsored by The American Society of Mechanical Engineers, the American Society for Testing Materials, The American Mathematical Society, the American Statistical Association, and the Institute of Mathematical Statistics.

[10] "The Application of Statistical Methods to Industrial Standardization and Quality Control." In 1942, the part of this devoted to quality-control charts was reissued in a revised edition prepared by B. P. Dudding and W. J. Jennett, BS 600R: 1942. Prior to this the A.S.T.M. had issued an important monograph "Manual on Presentation of Data," 1932; supplement, 1935.

[11] See "An Engineer's Manual of Statistical Methods," by Col. L. E. Simon, John Wiley and Sons, 1941, and "Quality Control and the War," by the same author, *Electrical Engineering*, September, 1942.

[12] Z1.1, Guide for Quality Control; Z1.2, Control Chart Method for Analyzing Data; Z1.3, Control Chart Method of Controlling Quality During Production.

[13] See, for example, "The Statistical Method in Quality Control—A Review of Progress in a New Industrial Technique," by H. Rissik, *BEAMA Journal*, May, 1942, pp. 130–133.

number (over 800) in attendance the interest already aroused and at the same time did much to promote new applications.

Contributions to Mass Production

Statistical control in mass production may be thought of as an attempt to maximize the advantages to be attained through interchangeability, a commonplace of production today but most revolutionary when Eli Whitney made his muskets in 1798. Four specific ways in which statistical control makes this contribution may be mentioned:

1 *Minimizes cost of inspection.* At each stage in the process of attaining a state of statistical control of a production operation, the application of statistical theory makes possible the establishment of sampling plans,[14] that will screen at minimum cost the output of such an operation so as to meet previously specified tolerance requirements and previously specified producer and consumer risks.

2 *Minimizes number of rejections.* By helping the engineer to detect the presence of assignable causes of variation so that these causes may be discovered and removed, statistical control techniques help to reduce variability of quality and hence the number of rejections.

3 *Maximizes quality assurance.* As assignable causes of variation are detected and removed, the quality of a given product approaches a state of statistical control for which the assurance that the quality of a piece of product will meet its tolerance requirements is a maximum. This fact is of particular importance for goods that cannot be given 100 per cent inspection because of the destructive nature of a test.

4 *Minimizes tolerance range.* The operation of statistical control provides an experimental technique for minimizing tolerance ranges. Such an operation makes possible the most efficient use of limited quantities of raw materials and provides the maximum degree of refinement attainable by any production process. Preliminary studies indicate that the operation of statistical control also provides a useful technique for eliminating assignable causes of variability in certain kinds of human effort as, for example, typing and other forms of transcription. Both strategically and commercially, industrial groups and even nations often need every increment of efficiency in the use of limited quantities of raw materials and human effort that can be provided through the application of the operation of statistical control. Likewise, they often need maximum refinement in quality through elimination of assignable causes, not only in pursuit of the arts of peace but also in time of war. As one example, the attainment of maximum homogeneity and hence minimum tolerance ranges in the properties of raw and fabricated materials may extend the potential carrying capacities of ships both in the air and on the sea.

Need for an Adequate Science of Control

Out of the successful effort to apply statistical techniques in the control of quality has grown a general theory and technique of statistical control in applied science that is applicable in the whole field of the *science* of management defined[15] by the Management Division of the A.S.M.E. as follows:

> "Management is the art and science of preparing, organizing, and directing human effort applied to control the forces and to utilize the materials of nature for the benefit of man."

Management of today is interested not only in a science of control helpful in "preparing, organizing, and directing human effort" to win this war but also in one helpful in utilizing to a maximum "the materials of nature for the benefit of man" when peace comes, because transition to peacetime production will present again many of the problems encountered in going from peace to war production. But that is not all. After the war, there may come proposals from many quarters, industrial, social,[16] political, and the like, about the art and science of organizing and directing human effort in producing goods to satisfy in the most adequate, dependable, and economic manner the wants of all. As pointed out in a recent editorial on "Science and Politics" in one of the journals of the American Institute of Physics: ". . . it behooves scientists to give their serious consideration to the *role of science*[17] in a state which is becoming increasingly centralized. . . . Important problems exist between these fields[18] which can be solved to the great benefit of each if, firstly, the will to co-operate exists, secondly, the problems are fairly and properly formulated, and, thirdly, their solutions are determinedly sought under wise and resolute leadership."[19]

One example of a field of universal interest wherein exist many important technical problems that overlap the fields of natural and social sciences is that of price-quality control. Perhaps there are few fields where there is greater need that the problems be fairly and properly formulated. To do this, means that we must discard many popular methods of control based upon the concept of an exact or deterministic science and replace them by scientific methods that take into account, as does the theory of statistical control, the fact that the quality of goods cannot be specified with exactness and that even though they could be specified with exactness, they could not be inspected with certainty because of the inherent variability between measurements.[20] Then there are those many instances where the qualities cannot be measured at all without destroying that which is measured, as in the case of many quality characteristics of foods, drugs, clothes, ammunition, and so on indefinitely. Hence it is that the science of control cannot be exact but only probable. In order that we may judge wisely in these days to come, we shall likely need as never before to distinguish clearly what is, from what is not, *an adequate science of control.*

Requirements of an Adequate Science of Control

To make clear what I have in mind, let us first consider a field familiar to mechanical engineers, namely, that of mechanics. Many observed phenomena can be described satisfactorily by the laws of Newtonian mechanics. These are *deterministic* in the sense that they assume that if such and such an operation is performed, such and such measurable events will surely happen. However, some mechanical phenomena, such as those treated in statistical mechanics, cannot be explained in terms of deterministic laws: instead *statistical* laws must be introduced. Moreover, there is not *one* statistical theory of mechanics but several.

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[14] For tables to assist in establishing sampling plans see "Single Sampling and Double Sampling Inspection Tables," by H. F. Dodge and H. G. Romig, *Bell System Technical Journal*, vol. 44, January, 1941, pp. 1–61.

[15] Trans. A.S.M.E., vol. 35, 1913, p. 1272.

[16] While writing this paragraph, an announcement of a book, "Readings in the Social Control of Industry," to be published by the Blakiston Company, Philadelphia, came to my desk. In addition to carrying fifteen signed articles, it is to give an indexed bibliography of more than 250 journal articles published mainly in the last twenty years.

[17] Italics author's.

[18] Science and politics.

[19] "The Review of Scientific Instruments," vol. 13, August, 1942, p. 313.

[20] For example, Dean-Emeritus Roscoe Pound of the Harvard Law School has discussed "The Relation of Statistical Quality Standards to Law and Legislation," in the volume, "Fluid Mechanics—Statistical Methods in Engineering," published by the University of Pennsylvania Press, 1941, page 137–146. As a background for this discussion he makes use of the article, "Some Aspects of Quality Control," published in *Mechanical Engineering*, December, 1934.

The theories differ fundamentally in the physical assumptions made the basis of the assumed statistical laws. The choice between them as an interpretation of physical phenomena must be based upon their comparative abilities to fit observed facts.

The fact that observable phenomena are not explainable in terms of deterministic laws does not necessarily mean that they are explainable in terms of statistical laws. In general, as has been shown elsewhere, there are what have been called *assignable causes* that must be found and either eliminated or taken into account before valid scientific predictions can be made in terms of the tolerance limits on observable values. An adequate science of control must provide practical techniques for discovering such causes. Such a theory is provided by the theory of statistical control.

Statistical Control Not Mere Application of Statistics

Most of us have a certain curiosity to know something about what we are getting into before we begin to learn a new discipline. Unless we have some ideas about it and its relation to things otherwise familiar, we may not wish to study it at all. In the remaining paragraphs, I shall go a little way toward satisfying this curiosity, by showing how a statistical theory of control differs from simply the application of statistics to certain problems in control.

Common to any statistical theory either of mechanics or of statistical control is the use of mathematical probability or distribution theory. But the theories differ in the underlying physical postulates. In this sense, there may be more than one statistical theory of control in much the same sense that there may be, and is, more than one statistical theory of matter, and any such theory is more than classical distribution theory of mathematical statistics.

Four of the specific ways that statistical control theory in the present sense differs from classical statistics are:

1 Classical statistics start with the assumption that a statistical universe exists, whereas control theory starts with the assumption that a statistical universe does *not* exist.

Even in the statistical theory of mechanics and of radiation phenomena, it is assumed that if a deterministic theory is not adequate, then a theory based upon the assumption of the existence of laws of chance will be adequate. In the field of quality control, as already noted, it is now generally accepted, however, that measurable phenomena do not obey laws of chance until what is known as assignable causes have been discovered and taken into account.[21]

2 Statistical control theory assumes that assignable causes can be found and either eliminated or taken into account in making valid predictions.

Statistical quality control has developed and provided a proving ground for two techniques for discovering such causes. These are: (1) the control chart technique for control of quality in production, now standardized by the American Standards Association and the British Standards Institution, and (2) the statistical run chart technique for finding assignable causes of variation in research and development.

3 Classical statistics ignores completely the ultimate goal of an applied scientist to make valid predictions in terms of tolerance limits as contrasted with the confidence limits of classical statistics.

Control engineers pointed out this fact in 1928 but it was not until 1941 that an academic statistician took note of the problem of making valid predictions in terms of tolerance limits.[22] Incidentally, knowledge of the theory of tolerance-range type prediction gives promise of contributing much to the theory of estimation and the theory of testing hypotheses of classical statistics in much the same manner that consumer and producer risks introduced into statistical quality-control theory in 1925 were the forerunner of the very important developments by J. Neyman and E. S. Pearson, in the theory of errors of the first and second kinds in modern mathematical statistics. That is to say, control theory has had to consider some problems really belonging in the realm of classical statistics before they were recognized there.

4 Classical theory is based upon the concept of inference from a single sample from a statistical universe, the ordering within the sample being ignored, while control theory must be based upon evidence provided by a succession of samples, ordering within the sample, and other pertinent information.

Hence the three scientific steps, hypothesis, design of experiment, and test of hypothesis in approaching a state of statistical control differ from those discussed in classical statistics.[23] Moreover, the problems, before a state of statistical control has been reached, as viewed by the control statistician are essentially different. For example, at the round-table conference thirteen years ago, statistical science was implicitly defined by the chairman, Colonel Rorty, as: (1) The assembly of broad masses of data, (2) the reduction of such data graphically or mathematically to a more compact and useful form, and (3) the analysis of such data to determine useful conclusions and general laws.

But the control statistician is not concerned with assembling and reducing data in this sense until he has data worth assembling and reducing. Likewise his experience in the field of quality control has shown the uselessness of inferring "statistical laws" until the effects of assignable causes have been taken into account, or, in other words, until such laws exist to be inferred. Moreover, the help of the control statistician is usually needed in designing the experiment that will give the data necessary for tracking down assignable causes; in fact his greatest contribution is most likely that of helping to design such experiments rather than in analyzing the data.

After a state of statistical control has been attained, the principles of statistical inference provided by modern mathematical statistics may be taken over and applied directly for the purpose of inference in terms of confidence limits and tests of statistical hypotheses. However, even under these conditions, the control statistician needs to go beyond the mathematical theory discussed in texts on mathematical statistics as noted previously, if he is to be able to make valid predictions in terms of tolerance ranges as is desirable in applied science.

The Future Problem

An adequate science of control for management should take into account the fact that measurements of phenomena in both social and natural science for the most part obey neither deterministic nor statistical laws, until assignable causes of variability have been found and removed. Statistical control provides practical control-chart and run-chart techniques for discovering such causes so that they can be removed, or taken into account, and it provides statistical hypotheses, experiments, and tests of

[21] For a fuller explanation of this see Chapter 1 of "Statistical Method from the Viewpoint of Quality Control," by W. A. Shewhart, published by the Graduate School of the Department of Agriculture, 1939.

[22] "Determination of Sample Sizes for Setting Tolerance Limits," by S. S. Wilks, "Annals of Mathematical Statistics," vol. 12, no. 1, March, 1941, pp. 91–96. This is a very important paper from the viewpoint of control.

[23] For a specific illustration of how these differ, see Shewhart, *op. cit.*, pp. 39 and 40.

hypotheses for discovering and using statistical laws resulting after the assignable causes have been removed.

The steps involved in attaining and making the most efficient use of a given degree of control often involve the co-ordinated effort of literally thousands of employees, including physicists, chemists, engineers, sales agents, purchasing agents, lawyers, and economists. Very few of these people have ever had training even in classical statistics and probability and yet many of them must be sold on the use of statistical control techniques if the control statistician is to have an opportunity of making his full contribution to management in the solution of its problems. This situation constitutes a problem, not only for those now in industry, but also for those responsible for training the industrial leaders of tomorrow so that they will have sufficient knowledge to help them recognize the potential contributions of statistical control theory and technique.

In the future, the control statistician must do more than simply study, and measure the effects of, existing cause systems; he must help his colleagues devise means for modifying these cause systems in the best way to satisfy human wants. The control statistician must not be satisfied with simply measuring the demand for goods; he must help change that demand by showing, among other things, how to improve the quality of these goods to the consumer. He must not be content with measuring production costs; he must help decrease them.

The future contribution of the statistical control statistician lies not so much in analyzing data put to him as in helping to get data in which assignable causes have been segregated so that analysis will lead to valid conclusions not otherwise possible. Not only may each industry expect to profit by having on its consulting staff a highly trained control statistician with a broad background of training in the physical and social sciences and with a flair for co-operation with his colleagues, but there is also great need for creating, through college training,[24] a statistically minded new generation of those natural and social scientists who will have charge of preparing, organizing, and directing the effort of those who are "to control the forces and to utilize the materials of nature for the benefit of man."

Job Standardization and Work Simplification

By HAROLD B. MAYNARD,[25] PITTSBURGH, PA.

Summary

The author traces the growth of interest in job standardization and work simplification, contrasting the decade 1932–1942 with the preceding decades, calling attention to the increase in literature devoted to methods-improvement subjects and the researches being conducted in the field.

The development of the techniques of applications and the practices employed to introduce them into industry are outlined. Special emphasis is placed on the fact that up to the present time the proportion of jobs which have been studied and simplified to those capable of being improved is disappointingly small.

The tremendous expansion of methods-improvement activities during 1932–1942 is then described. The author concludes by predicting the development of a new procedure which will shift the emphasis from the improving of existing methods to the engineering of highly efficient methods before the jobs are put into production, thus avoiding the difficulties inherent in technological change and increasing the operating effectiveness of industry.

In the report which follows, the terms "Job Standardization" and "Work Simplification" will be used largely as they relate to methods of performing work in industry. Because of the rapid growth of interest in finding better and more economical ways of doing work, there has been a correspondingly rapid development of procedures for accomplishing this betterment. The procedures have been given different names by different investigators, but they all have in common the objective of developing and standardizing upon the best way of performing each given task.

Growth of Interest in Methods

Although the importance of effective working methods was stressed by the pioneers in the time and motion-study field and by some of their followers, during the years immediately following World War I, managements tended to focus more interest on the introduction of financial wage-incentive plans as a means of securing economies in plant operation. The more enlightened managements recognized the importance of accurate standards in this connection and organized time-study departments to handle the function of task measurement. Interest was centered on accurate measurement procedures often to the exclusion of matters of greater importance. At a meeting of the Society of Industrial Engineers held at Detroit, Michigan, in 1928, for example, such subjects as the proper type of stop watch to use, the development of time formulas for rate-setting purposes, and the like, were the subjects in which the chief interest was shown.

By the time the decade 1932–1942 was beginning, two important considerations had developed which caused a rapidly growing shift in emphasis from the setting of accurate standards to the establishing of better methods. In the first place, it had become apparent to the time-study technicians themselves that with all their strivings for accuracy, the standards which they established did not prove satisfactory over a period of time. Carefully measured standards which were accurate when established were sometimes difficult for operators to meet later on or in other cases proved to be so liberal that the soundness of the original time-study work was questioned. In investigating the reasons for this looseness of standards, it at length became clear that the method of doing the work was of greater importance than had hitherto been clearly realized. Minor changes in method which were formerly considered unimportant, were seen to have sufficient influence to make a standard appear liberal or unduly low depending upon whether or not the change was an improvement upon the method being followed at the time the standard was set. True, the time-study technicians had never overlooked the importance of major methods improvements, and they almost always sought to improve methods before establishing standards. Their knowledge of methods was imperfect, however, because adequate techniques had not been developed up to that time.

At the beginning of the past decade, the country was headed for the depths of the worst depression in its history. Business was almost at a standstill. A buyers' market existed, and competition was on a strictly price basis. Anything which would result in cost reductions was eagerly sought. Hence when the various methods-improvement techniques began to be introduced, they found management in a receptive mood. When in addition they appeared at a time when the more advanced time-study technicians were becoming aware of the incompleteness of

[24] This fact is beginning to be appreciated not only in the industrial field but also in academic halls where many of the managerial leaders of tomorrow are to receive their training. For example, the announcement of the Graduate School of Business Administration at Harvard for 1940 carries the description of a course on "Statistical Controls for Company Management." This course has been developed by Prof. Theodore H. Brown, a pioneer leader in the application of statistics in the field of management.

[25] President, Methods Engineering Council.

their own procedures, it was only to be expected that methods-improvement activities would be accepted as desirable and necessary.

The procedures used for methods-improvement work go under several different names, but they all have in common the bettering of methods as was pointed out above. Much methods-improvement work still goes under the name of time study. The older time-study procedure has merely been expanded to include methods-study activities. Motion study as developed by Frank B. Gilbreth has always been used to study and improve methods. It is part of, or forms the basis of, several other procedures known variously as operation analysis, work simplification, motion-time analysis, and methods engineering. The procedures differ to some extent in detail, but there is sufficient similarity of purpose in all of them to justify treating them together in this 10-year report of progress.

Methods-Improvement Literature

During the past decade, the literature devoted to the discussion of methods-improvement techniques has expanded several fold. Prior to that time, the literature covering the subject was limited to the writings of Taylor, Gilbreth, and their early associates. A book "Time and Motion Study and Formulas for Wage Incentives" was published by Lowry, Maynard, and Stegemerten in 1927, but although it did not overlook the factor of method, the major emphasis was on time-study practices.

In 1932, the book "Common Sense Applied to Motion and Time Study" by A. H. Mogensen was published. This combined several articles by various men interested in methods-study work with some of Mogensen's own experiences and gave one of the first co-ordinated presentations of recently developed methods-study practices. Following this book came an increasing number of others, all stressing the importance of methods study. Among the more widely distributed of these were "Time and Motion Study and Formulas for Wage Incentives," by Lowry, Maynard, and Stegemerten, second edition 1932, third edition 1940; "Motion and Time Study," by Ralph M. Barnes, 1937, second edition 1940, "Time Study for Cost Control," by Phil Carroll, 1938; "Applied Time and Motion Study," by Walter G. Holmes, 1938; and "Operation Analysis," by Maynard and Stegemerten, 1939.

In addition to these books, magazine articles discussing methods-improvement techniques increased from an occasional article to a standard feature in several industrial magazines. Recently articles appeared in such nationally distributed magazines as *Reader's Digest* and *Fortune*.

As a further indication of the growth of interest in methods-improvement work, it may be pointed out that many of the more recent books on management and foremanship subjects include one or more chapters on this subject.

Research Past and Present

During the past decade, research in the field of work simplification has been carried on by several investigators. A. B. Segur has continued to develop his motion-time analysis procedure, although published reports of his work have been meager. Professor Ralph M. Barnes has conducted at the University of Iowa a number of investigations of specific motion-study problems, using students as operators. Although the practical application of the results of these investigations has probably been rather limited up to the present time, the published reports of findings have served to stimulate an interest in this type of research and to make clear the vast amount of work which must be done before all of the facts about motions and motion times are known.

Undoubtedly, other research has been and is being conducted in this field, the results of which either have not been published or have not come to the attention of the author. In the ranks of industry, for example, the Westinghouse Electric and Manufacturing Company has given G. J. Stegemerten and the author the broad assignment of conducting motion-study research to the end of developing a practical procedure which will lead to the establishing of better methods more surely than do the existing procedures. Although this research is progressing satisfactorily, no findings have yet been published.

Development of Techniques of Application

The development of techniques of application related to job standardization and work simplification has progressed rapidly during the past decade. At the beginning of this period, the value of process charts, the questioning attitude, micro-motion study, and the like as tools of methods-improvement were fairly well recognized, but their application during a given methods study was likely to be hit or miss. Too many technicians tended to conduct their studies by concentrating on the correction of obvious inefficiencies and thus overlooked greater possibilities. In effect, they often established efficient methods for doing unnecessary work. Recognition of this caused the leaders in the field to develop a more systematic approach in the application of methods-improvement techniques.

For example, empirical procedures were developed to determine at the outset of any study the amount of study justified by the economics of the situation. From that point, procedures for conducting studies in accordance with the results which could be expected were outlined. The operation-analysis procedure organized upon the principle of considering first things first was evolved. The questioning-attitude tool used since the time of Taylor and Gilbreth was expanded to include lists of practical questions to be applied to every step of operation analysis and even to every therblig.

During this period, an understanding of what constitutes effective methods was developing on the part of various technicians. Through the medium of motion pictures, particularly of the "before-and-after" type in which ineffective and effective methods were compared, this understanding was spread throughout large sections of industry. Although evidences of this understanding in the form of practical applications were somewhat slow in appearing, the technicians could at least begin to talk of two-handed operation, normal and maximum working areas, drop delivery, and the like outside of their own immediate ranks with an increasing chance of being understood.

One of the stumbling blocks in the road of application of work simplification proved to be in getting the operators to use the improved methods which the technicians devised. This led to the development of better methods of operator training which are now an integral part of many work-simplification programs. The methods evolved include improved techniques for imparting verbal instructions, more complete and understandable written instructions, the use of motion-picture-film loops, or a combination of all three. The importance of operator training was recognized by the War Production Board during the early stages of the defense program. The program of job-instruction training which it encouraged throughout industry undoubtedly included many points which were first developed for teaching improved methods derived from motion study.

The practices employed in introducing work simplification into industry have followed a fairly uniform pattern during the past decade. Interest in the possibilities for improvement appears first, usually in the ranks of management. This interest may be inspired by a pressing necessity for cost reductions, a progressive desire to improve the competitive position at all times, or a desire to follow the example set by others. The next step is to educate the key members of organizations as thoroughly as possible with

regard to the principles upon which methods improvement is based. Groups of technicians are taught how to apply the principles in every detail. The foremen and other supervisors are usually taught only enough about the principles to enable them to co-operate with the technicians intelligently and to arouse in them a desire to receive the benefits which work simplification brings. In some cases, the education is extended to certain key workmen or labor representatives. It is not expected that these men will actively undertake methods-improvement applications, but rather the purpose of the training is to inform them of what methods-improvement work is and to show them what they themselves may expect to get out of it. The training is usually done at first by a consultant or by a member of the organization trained by a consultant. Later as more knowledge of the work is gained, the training may be undertaken wholly within the organization.

The final phase is that in which the methods-improvement procedures are applied. It is expected to be a phase which continues indefinitely. During the past decade, a great deal of interest in methods improvement has been created in industry, and much excellent training has been done. Many applications of the principles of methods improvement have been made, and yet it has been fairly evident to competent observers that the proportion of improved methods to the number of jobs capable of being improved has been decidedly small. A number of factors have perhaps been responsible for this. The procedures themselves are new and in the development stage. In some cases, they have been competently applied; in other cases they have been overapplied. Managements in certain instances have allowed interest in other matters to prevent them from giving work simplification the aggressive backing which it must have if the greatest results are to be realized.

Finally, during the past decade with the growing strength of organized labor, there has been an increasing resistance to technological change. This has seldom proved to be an insurmountable barrier when work simplification is properly presented, but it has made it necessary to do a much more careful job of introducing it and to proceed more slowly than was the case in the preceding decade. An excellent report on this problem has been published by Professor John W. Riegel entitled "Management, Labor, and Technological Change" (1942).

Expansion of Methods-Improvement Activities

The past decade has witnessed a tremendous growth of interest in and application of methods-improvement techniques. Following the pioneering period during which Taylor, Gilbreth, and their associates worked in industry, there was a slow but steady growth in the use of time and motion studies. The major emphasis tended to be on time-study rather than on motion study, however, and although many methods were improved as the result of time study, it was more due to the fact that it was impossible to overlook certain inefficiencies while making the close and detailed observations required by time study than because methods improvements were systematically sought through the application of methods-improvement techniques. There were, of course, exceptions to this, but the fact remains that time study received the chief attention.

The period during and following World War I witnessed the mushrooming into prominence of a number of efficiency experts. Among them were unqualified, untrained, unethical individuals who by misapplying a technique which they did not understand did much to blacken the name of time and motion study and to retard its application.

During the first part of the decade, 1922–1932, the procedure had to win back the ground which had been lost. This was accomplished by sound relatively unspectacular work on the part of a number of technicians who, convinced of the beneficial results which the procedure, properly applied, would bring, pressed for its application at every opportunity. Then followed a brief era of fairly smooth sailing. An increasing number of managements accepted time and motion study—although they still meant mainly time-study—as being a necessary function and gave it a permanent place in their organizations. In the growing prosperity of the late twenties, few people questioned the wisdom of technological change. Unorganized labor in fact accepted it as a part of the normal development which was carrying our country to a greater material prosperity than had ever been dreamed of. It was an era during which a golden opportunity existed for undertaking methods-improvement work unresisted and unquestioned with plenty of funds available for carrying on the work. The opportunity, however, was missed, for the urgent necessity for doing the work did not exist.

Industry Searches for Cost Reduction

When after 1929 prosperity vanished, industry began to look for every help it could find. Always before when sales fell off, if prices were reduced, sales were regained. For this and other reasons falling prices became the order of the day. As prices dropped, the necessity for reducing costs increased in proportion. Wage reductions were put into effect, but the more far-sighted realized that this method of reducing costs also reduced the purchasing power of potential customers. They searched for another answer and found it in cost-reduction activities of all types. Among these, emerging at a time when the demand for it was acute, was work simplification. During the decade of 1932–1942, work simplification came into its own, at least as far as recognition is concerned.

Although the interest in methods-improvement techniques has been great during the past decade, the actual results accomplished have been disappointing to those who appreciate the potentialities. The results which have been obtained have been good, of sound and lasting benefit, but quantitatively they have been too few, at least from the technician's viewpoint. Some of the reasons for this have already been discussed.

In spite of this, it may be said that the development of job standardization and work simplification up to the present state is of marked significance to the management field. In fact, the situation is in the making wherein it will be necessary for management to have an understanding of methods-improvement techniques and related problems in order to be successful. During the past few years, many firms whose organizations have grown old or fallen into a rut have turned to work simplification as a means of revitalization. Besides the cost reductions which the procedure brings, there is a stimulating effect on the thinking of an organization when it begins to grasp the almost limitless possibilities for improvement which lie dormant in the familiar operations within the plant. Work simplification has proved to be a developer of men as well as a developer of methods.

Managements were realizing these things in increasing numbers as the decade 1932–1942 drew to a close. At the same time the need for handling technological changes more skillfully was also becoming more apparent. In turning to a consideration of future trends in the development of methods-improvement work, it appears that these factors furnish the background which will shape the next advances.

Probable Future Developments

Up to the present time, work simplification has been largely a procedure of methods improvement. The term "methods improvement" has purposely been repeated frequently throughout this report to give emphasis to the underlying philosophy at present in vogue. The term presupposes that an ineffective method

is already in existence at the time the study is begun. In probably the majority of cases, the method has been in effect long enough for those following it to have become thoroughly accustomed to it. They know what constitutes normal production, how much employment the job may be expected to give, and what the earnings will be. At length, the work simplification specialist appears on the scene. He studies the job with all of the enthusiasm of creative endeavor. Suddenly the familiar aspects of the job disappear. The future looms vague and uncertain through the mists of the unknown. Technological change has appeared.

It is only natural to expect that this process will generate resistance on the part of those who want the certain security of things as they are rather than the promised benefits of things that may be. Up to the present, the answer to tbe problem has been to try to sell the benefits so thoroughly that the uncertainties will be overlooked. To the author, this solution appears to be merely a stop gap.

In the past, work-simplification technicians have proceeded under the principle that it is always possible although not always economically practical to improve a method by subjecting it to detailed, intensive study. Even after a job has been motion-studied, it is usually possible to improve it further by more intensive study during which the problem is tackled afresh. Experience has borne out the correctness of this principle, for it has been possible, in case after case, to do this very thing. This may be an unavoidable situation, for certainly most progress results from a series of developments. At the same time, in the case of work simplification, it does not seem inevitable that as many steps have to be taken to approach the best method for doing the work as is the usual practice.

Improving Methods Before Adoption

In addition, it should not be necessary to wait until a method is already in effect before beginning to study and improve it. This results in technological change which puts a strain on industrial relations. Obviously, the best time to work out the improved method for performing an operation is before the operation is set up in the shop. It may not be possible to achieve perfection from the start, but it should be possible to set up methods which approach perfection much more closely than is now the case.

By learning to work out good methods of doing work before new operations are introduced into the shop, by introducing these methods through proper operator training, and then by leaving them alone for a while at least, good methods can be established without strain on industrial relations. The disturbing effect of technological change from this source will be minimized, and a more efficient and a more smoothly functioning industry will result.

This appears to be the next logical development in the field of job standardization and work simplification. The present procedures must be improved and perhaps new ones developed which will make it possible to pre-determine effective methods for doing all classes of work. The design engineer and the tool designer will have to be brought into a much closer relationship with the work-simplification man; in fact, these functions may tend to merge into a new function.

In any event, it will be necessary to take the information which has been obtained from past researches, and adding to it by new investigations, develop a procedure which can be applied by one trained in it to devise efficient methods while the work is still in the design stage. Under this new procedure, the emphasis will shift from methods improvement to true methods engineering.

Foundation Already Laid by Research

As a matter of fact, these predictions do not belong wholly to the future, for the foundation for these developments has already been laid by the researches previously referred to which are being conducted at the Westinghouse Electric & Manufacturing Company by G. J. Stegemerten and the author. These researches are leading to a more fundamental approach than is now in use which will result not only in the predetermining of effective working methods, but also in the predetermining of scientifically accurate production standards. In connection with the latter point, one of the controversial points in industry today is the determination of the task which constitutes a fair day's work. Time-study procedures to be sure have been developed which in practice do this reasonably satisfactorily, yet nevertheless the subjective judgment which must be used in determining just how well an operator is working while being studied is something which technicians, management, and labor alike would be glad to eliminate. The development of methods formulas based upon careful research which will establish methods and time standards from basic data which can be proved to be correct will accomplish this end.

The researches thus far conducted indicate that the development and application of methods formulas will require the services of much more highly trained technicians than the present-day motion- and time-study or work-simplification man. In all probability, as complete and comprehensive a course of training will be required as is now considered necessary to turn out a mechanical engineer or an electrical engineer. A full four-year college course devoted entirely to methods-engineering training appears likely enough to justify consideration by universities and colleges teaching engineering.

It will require considerable research and practical experimentation to develop the complete methods-engineering procedure of the future, but the results will make the effort worth while. The new procedure will cause a better job of cost engineering, will markedly improve industrial relations, and will increase the efficiency of American industry to the point where it can expect to maintain its world leadership in the uncertain years ahead.

Cost Accounting and Budgetary Control

By JOHN A. WILLARD,[26] NEW YORK, N. Y.

It is generally considered that standard cost accounting "came of age" in the period from 1922 to 1932. In the decade now under review, it has been subjected to two major forces.

The first of these influences is concerned with the gradual development of improved methods which would normally occur with the lapse of time in a stable economy. The second is the result of the pressure of unusually disturbed economic conditions followed by an intensified amount of governmental regulation of business in an attempt to offset the effect of the economic collapse of the early 1930's.

Cost Accounting a Management Force

In connection with the problems concerned with the normal growth or development of accounting techniques, it is especially noteworthy that cost accounting has become a management force rather than a check or audit. There has been developed in this period a far better understanding of the meaning and scope of control. In fact, control has become an inherent part of management rather than a tool. In addition, there has arisen an in-

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creasing recognition that no single formula or routine can meet the needs arising from all the problems of all businesses; that purely accounting records must be supported by an adequate amount of statistical reports arising from the accounting routines, but not necessarily tied in with the books.

Naturally, techniques have been further polished and developed. Sound applications to a wide variety of businesses and manufacturing processes have been evolved and in turn subjected to greater refinement.

Interpretations have been formulated of the true significance of material, labor, and burden variances. The disposition of these variances among asset or liability and operating accounts at the close of the fiscal period, has been thoroughly developed until procedures adapted to a wide variety of operating conditions have become fairly well standardized.

The beginning of the decade brought the application of costing and statistical techniques to market studies, sales analysis, and selling and distribution expense. This field will still reward extended research. At present it is pretty much limited to a relatively few companies that present averaged data over a period of several months instead of setting it up for current monthly cost control. The few exceptions serve principally to emphasize the need of a better knowledge of distribution.

Fifty years hence, these fundamental gains in managerial control through improved cost knowledge will more than likely overshadow the more temporary but likewise more voluminous changes caused by the impact of governmental control in an effort to bring about economic recovery.

Effects of Attempts at Economic Planning

The decade was still young when the National Recovery Act was passed in an attempt to alleviate the effect of presumed technological unemployment, overproduction and underconsumption. Economic planning and the development of mass purchasing power were resorted to in an attempt to block the descending spiral of deflation with its attendant human suffering.

Trade associations and "industry codes" grew with rapidity, and with them businesses needed new accounting techniques. It became illegal to sell below cost; statistical reports on selling prices were required. And these needed "uniform cost-accounting methods" to show actual violations. The leak due to slack methods of calculation of depreciation was closed by governmental order and new plant-equipment accounting methods. Price control and allotment of business caused several changes in accounting—even if very few changes in human nature and avarice.

Discouragement over continuous losses caused a wave of fixed asset revaluations to try to offset them. Under the umbrella of the N.R.A. normals were recomputed to boost costs to higher levels.

The N.R.A. was a prodigious experiment in industrial self-government. While opinions may differ as to its ultimate value, it did bring forth a fundamental definition of interstate commerce in the Schechter case; both accountants and industry gained valuable experience with uniform accounting systems (industry) and simplified accounting, and acquired a better understanding of flexible budgetary procedure.

The Social Security Act made a lasting impression on both employment policies and the accounting procedures identified with plant pay roll.

Again, accounting for capital assets and depreciation began to assume new importance in connection with excess-profits-tax formulas.

Like the N.R.A, the Robinson-Patman Act also made it obligatory to adopt accounting techniques sufficiently detailed to enable the manufacturer to show that he was not selling below costs. New activity in selling and delivery cost accounting became vital, as well as more highly refined distributions of expense.

Development of Pricing Policies

It is significant that these new accounting techniques were extended to form a base for establishing pricing policies. In a world struggling to free itself from the deepest economic depression ever experienced, the fundamentals of sound pricing policies were developed. They ought to prove of lasting benefit.

One of the most important effects has been the impact of new policies of federal taxation on cost accounting, especially on the valuation of inventories.

For years, industry had used either the first-in first-out method of pricing inventories, or an average method. The rising tide of material prices in the earlier stages of the present war centered attention on the use of a method known as the last-in first-out method of pricing material inventories.

It is important to note that the thing which influenced the technique was what would save the most taxes rather than what would produce the most accurate figures. This fact is mentioned principally to illustrate the point that the greatest shifts in accounting procedure in the last decade have come about as a result of economic and social changes and the legislation devised to meet them.

At the moment new techniques for accelerated depreciation and amortization on account of the high rates of plant usage, when running 24 hours a day for seven days a week, are receiving a good deal of attention.

Again, those executives responsible for industry stability are gravely concerned over whether the high rates of profits indicated by the books (and on which taxes are being paid) are real profits. They question whether the overabsorbed burden now being thrown into profit should not be set up as a reserve to cushion the shock of postwar operations. The long-term stability of the present capitalistic structure is vitally involved in their ultimate decision.

Improvements in Budgetary Control

The technique of budgetary control has developed very much more slowly and over a considerably longer period of time than cost accounting. On the other hand, there are probably fewer people who can really do a good job of establishing a modern flexible budget than can turn out a creditable job of cost accounting. Budgetary procedure is based far too much upon unsupported estimates rather than upon a careful analysis of the "pattern" of expense variation under varying operating conditions.

Here again economic forces of the depression made it necessary for budgeteers to draw up budgets at varying levels of business. Many of our largest corporations did an extremely able job of controlling expense in proportion to available income through the use of this variable budgetary technique.

If the cost standards in the accounting plan are properly prepared and correctly broken down into their fixed and variable elements, the cost control can be made to embody many of the better controls inherent in budgetary procedure. In all probability, the greatest gains for the future in flexible budgeting technique will come from studying departmental expense to determine what causes the upward and downward surges so characteristic of many expense items.

In addition to making a detailed study of departmental expenses for one or two years, it is necessary to chart them to see more clearly the cause of the expense variations which accompany changes in departmental volume. This step permits the drawing of characteristic expense curves from which the equation of variable expense may be obtained as well as the fixed departmental costs.

It is in improving this portion of the technique of budgetary control that the greatest advances in the future are to be expected. However, it should be definitely borne in mind that sound costing technique and budgetary procedure are closely interrelated. Continuing gains in the one should result in improvement in the other.

Industrial Marketing

By HARRY J. LOBERG,[27] ITHACA, N. Y.

THE depression of the 1930's was beneficial to genuine progress in the industrial-marketing field. The roots for this progress go back many years in some cases, but their origin is not of prime importance here. The essential point is that a buyers' market, coupled with broad economic readjustments, forced management to reconsider the importance of sales to their well-being. A concept, new to many, that sales too had to be manufactured, was forced on industry. To make a product was not enough—markets were glutted with products. To just make sales was not enough—sales had to be made at a profit. Manufacturing products and manufacturing sales had to be in balance. A great productive capacity was being strangled by the inability to move those goods in the market place. Management had to reappraise their techniques and the driving force was survival.

These generalities may seem to be too all-inclusive, too sweeping to be specific, but as one breaks down the whole into its parts, one becomes aware of a serious attempt to be more scientific, at least factual, in the handling of the distribution of goods. To some individual concerns, it was a case of carrying on work previously started. To others, it meant a complete change in attitude and outlook—they were living in an era that was fundamentally different.

In speaking of progress, one must consider the over-all effect. At any one time some are in the vanguard, others are lagging far behind either because they are unaware of change, through ignorance, or are too stubborn or unwilling for a multitude of individual reasons to recognize that the old order changeth. Table 1, taken from the Biennial Census of Manufacturers, shows at a glance that survival was an acute problem.

TABLE 1 CHANGES IN NUMBER OF PLANTS AND VALUE OF PRODUCTS

Year	Number of establishments	Per cent 1929 = 100	Value of products (1000 dollars)	Per cent 1929 = 100
1929	206,663	100	$67,994,041	100
1931	171,450	83	39,829,888	59
1933	139,325	67	30,557,328	45
1935	167,916	81	44,993,699	66
1937	166,794	80	60,712,872	89
1939	184,230	89	56,843,025	84

The impact of the depression varied from industry to industry, but few escaped the pangs of hunger and undernourishment of few sales.

Market Research

To overcome these difficulties, marketing became somewhat more factual and less intuitive. Market research in various forms and guises became a better understood mechanism. Management began to apply research techniques to their marketing problems analogous to what they had been doing for years in reference to production.

The field of marketing research has been defined in the following statement:

> Marketing research is the study of all problems relating to the transfer and sale of goods and services from producer to consumer, involving relationships and adjustments between the production and consumption preparation of commodities for sale and physical distribution, wholesale and retail merchandising, and the financial problems concerned. Such research may be undertaken by impartial agencies, or by specific concerns or their agencies, for the solution of their marketing problems.[28]

"The Technique of Marketing Research,"[29] published under the sponsorship of the American Marketing Association, lists four main categories of research:[30]

1 *Policy research* embraces all marketing research affecting the policy of a company, such as studies concerning advertising allowances, cancellations, compensation, credit granting, pricing, public relations, and returned goods.

2 *Product research* includes studies of such subjects as the addition of other products; change in design, color, texture, taste, etc., to increase sales; comparison with competitive products as to performance, style, quality, size, price, etc.; customer requirements and ideas; new uses of products; packaging; and pretesting of new models and design.

3 *Market research*, as its name implies, covers such studies as analysis of consumer markets by sales territories; analyses of wholesale markets by sales territories; analysis and interpretation of current market statistics; business forecasting; classes of consumers purchasing product; from point of view of amount and nature of income, buying habits, seasonality of interest, location and number of individual plants represented; the potential market for a product or line; sales quota construction; and sales territory delineation.

4 *Methods and means research* covers questions concerned with advertising, selling, and service. The subjects of studies under this heading are competitors' practices in advertising and selling; distribution costs; effectiveness of advertising program; brand preference and related topics; effectiveness of various advertising mediums; selection of channels of distribution; and the training of salesmen.

No clear-cut picture can be given of the exact amount of marketing research being carried on, nor is it possible to state in absolute values the increase that has taken place in the 1930's. The increased interest in the subject is reflected in the increase of pertinent published material during the period, however. For example, the publication in 1939 of the "Industrial Market Data Handbook," Department of Commerce Series No. 107 was the result of the need by industrial marketers for more data. The increased interest in marketing as such by societies, such as the A.S.M.E. and the N.I.A.A., can be accepted as further evidence. While the success of the Check Sheet—Introduction of New Industrial Products first issued in June, 1935, by the Department of Commerce and revised in November, 1937—again shows the aliveness of this subject, without laboring the point, there is enough evidence on hand to show that this interest was not superficial but dictated by a desire and need for applying factual analysis to marketing problems.

The National Industrial Advertising Budget Surveys indicate an increase of expenditure as shown in Table 2.

The amount spent is relatively small, but it has been increasing. It is also fair to assume that the principles and techniques are applied by some individual concerns without making a specific appropriation or allocating their time and effort specifically to marketing research.

The "Marketing Research Activities of Manufacturers" made

[27] Associate Professor of Administrative Engineering, Cornell University.

[28] Report of the Committee on Definitions, American Marketing Association.

[29] "The Technique of Marketing Research," prepared by the Committee on Marketing Research Technique of the American Marketing Society, McGraw-Hill Book Company, 1937.

[30] Adapted from "Marketing Research Activities of Manufacturers" (Market Research Series No. 21), United States Department of Commerce, April, 1939.

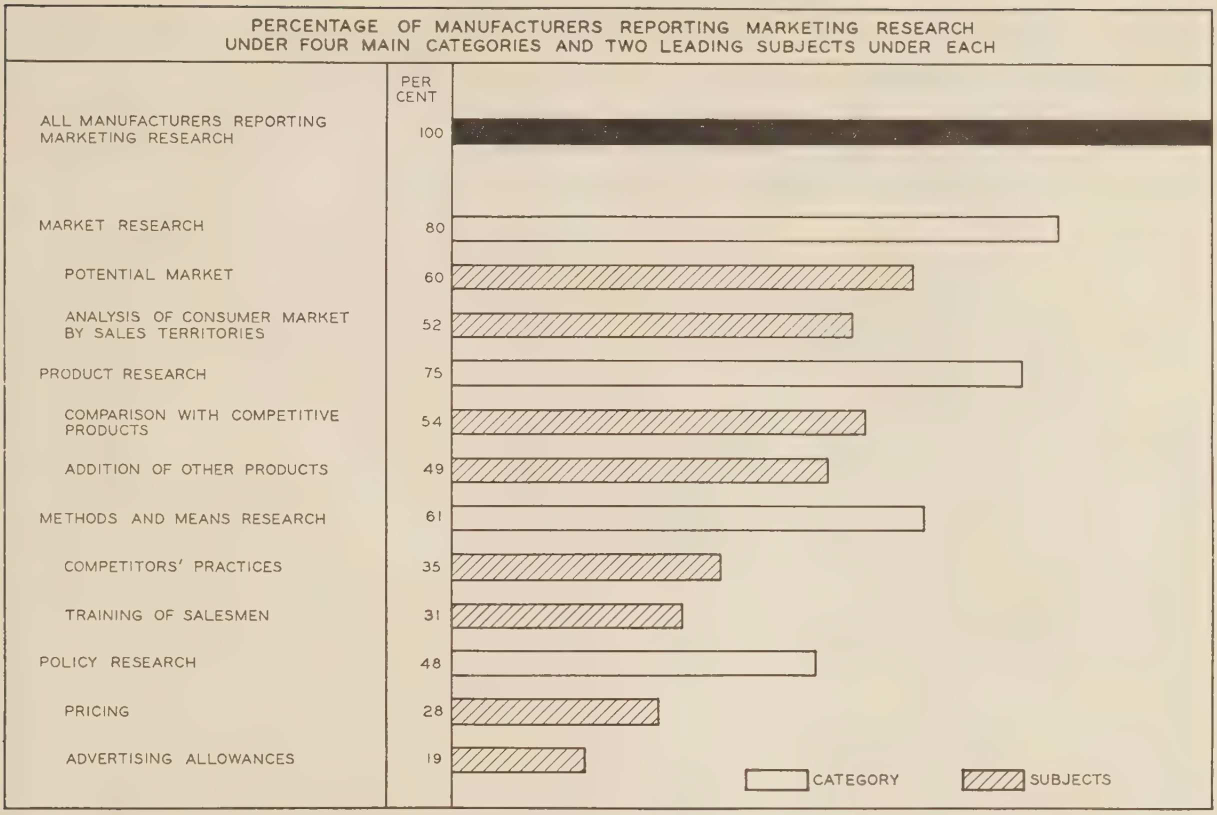

FIG. 1 ANALYSIS OF REPLIES TO QUESTIONNAIRE ON MARKETING RESEARCH

TABLE 2 PERCENTAGE OF TOTAL ADVERTISING BUDGET SPENT ON MARKET RESEARCH

Year	Total number of questionnaires	Average for total number of questionnaires	Average for actual number reporting: Number	Per cent
1937	134	0.1	..	..
1938	164	0.1	..	..
1939	416	0.28	36	3.13
1940	345	0.20	36	1.87
1941	305	0.18	28	1.91

TABLE 3 SUMMARY OF REPLIES OF MANUFACTURERS REGARDING MARKETING RESEARCH

Classification	Total manufacturers replying: Number	Per cent	Manufacturers conducting marketing research, Number
Total	556	100	188
1 Consumer goods............	133	24	54
2 Industrial goods............	341	61	105
3 Consumer and industrial goods	82	15	29

in 1938 gives further evidence, Table 3. This is biased somewhat since the questionnaires were mailed to many who were known to be carrying on research—it was not a complete census.

From the same study the following chart indicates the fields of greatest interest.

Market research in its various forms was steadily being used more intelligently and extensively. With its broader application came improvements in techniques. The use of small samples and their statistical reliability became more firmly entrenched. Field work and questionnaire development was improved. The knowledge of how to ask questions associated with the psychological needs became greater. Difficulty is still encountered in the field of semantics—wording of questions to get true answers is still a real problem. In the industrial field, the necessity for properly qualified interviewers capable of obtaining maximum information on a more informal basis has been realized. The distinctions between consumer household investigations and industrial management investigations are more clearly understood, resulting in studies that are far more informative than those based on just counting heads.

The effect of increased use of market research is evident in all phases of distribution. It is discussed here first and in somewhat more detail to set the background for its application in other major branches of marketing.

PRODUCTS AND PRODUCT DEVELOPMENT

In 1932, *Business Week* wrote: "An outstanding characteristic of business today is the hard driving search for new products, something to keep idle machines busy, something to give a fillip to the market." In back of this drive came the realization that products must be geared to the needs of the user. The buyers' market brought on a better understanding of the value of designing products, not only from a utility point of view but also for eye appeal. The so-called industrial designer came into the picture. Streamlining became a by-word. In this process the designer was brought closer to the user—products were redesigned—simplic-

[31] United States Census.

TABLE 4 MANUFACTURERS' SALES—DISTRIBUTION BY PRIMARY CHANNELS[31]

Per cent of distributed sales made to

Own wholesale branch offices 1929	1935	1939	Own retail stores 1929	1935	1939	Wholesalers and jobbers 1929	1935	1939	Retailers for resale 1929	1935	1939	Industrial, etc., users 1929	1935	1939	Consumers at retail 1929	1935	1939
17.4	21.7	23.0	2.4	2.3	2.7	32.8	26.2	26.9	17.9	20.9	19.8	27.7	26.6	25.8	1.8	2.3	1.8

ity, functional analysis, and inherent beauty became necessary. The packaging of many industrial goods was part and parcel of this newer attitude toward product design.

The effect of this trend toward product improvement was to bring production and selling closer together. It has resulted in more definite plans and procedures for locating and appraising product ideas.[32] The effect to the average individual has been more noticeable in consumer goods, but it definitely has had its counterpart in industrial goods.

Distribution Channels

Since the bottleneck was the distribution of goods, it seems obvious that the ways and means of getting goods into the hands of the ultimate user would be more carefully scrutinized. The over-all picture is shown in Table 4.

The over-all picture indicates a trend toward more direct control of the selling process on the part of the manufacturer. Naturally these shifts and changes have not been uniform or of the same magnitude and direction in various industries. This trend was due to many causes, but probably the most important ones were: (1) The need for more aggressive selling and merchandising in a highly competitive era than was being supplied by the average middleman; and (2) the better transportation and local storage facilities made it easier to accomplish.

Selective distribution became more widely used and important. Having measured markets more carefully, it was logical to select such methods and organizations that would and could cover the sales territories more effectively.

The tendency toward voluntary chains and group buying in the consumer's field has had its repercussions in the industrial market. For example, in the hardware field the jobbers, who formerly refrained from soliciting business in the industrial field because they did not want to compete with their own customers for local industrial business, have now discarded that policy and are energetically developing the industrial market. Thus a new group has entered the industrial distribution picture because of actions taken by those primarily concerned with consumer distribution. This has resulted in a tendency toward larger and financially stronger units taking over most of the distribution.

The typical industrial distributor has not been completely idle. He does perform definite functions and services. Many of them have also changed their selling techniques to meet the demands of users and the effect of intensified competition. The present war may reverse any trends established in the 1930's. Certainly distribution problems will be intensified when peace comes. Perhaps the industrial market will see the counterpart of the supermarket in some fields, especially those bordering closely to consumers' goods in use and application.

Sales Analysis and Sales Control

The importance of more careful sales analysis, not only by territories but by products or lines, was evident. To stop the profit leaks, it was necessary to make more detailed studies. The over-all picture was frequently misleading. Shifts in emphasis on profitable items and profitable customers were necessary, leading again to more selective distribution.

Concomitant with sales analysis came more definite control of the salesman's time and effort. A study made by the writer in 1938 on controlled routing of salesmen did not indicate that it was universally applied, but it did indicate that more effort was being made to direct salesmen's efforts in the channels that would produce greater sales.

The extent of control exercised varied considerably with the type of product and the character of the selling job needed. However, more and more concerns realized that sales just did not grow. Careful planning and direction were needed. This was achieved either by training salesmen to operate more efficiently or, on the other extreme, routing him on practically a fixed schedule. The success of routing hinges largely on the judicious use of free time to cover emergencies as determined from the analysis of the selling job to be done.

From sales analysis and control came reduced distribution costs. Also more information from a sales-accounting point of view for more reliable budgeting and sales forecasting.

Salesmen's Training and Compensation

The new problems of selling necessitated new selling methods. High-pressure methods had to give way to more service or at least a better understanding of the customer's needs. Sales training programs were revised. The trend was toward doing a more thorough job and replacing textbook and classwork with more training directly in the field.

The cost of training tended to increase. More companies began to retrain old salesmen as well as new recruits. But with the realization that the possession of a catalog and price list did not make a man a salesman, companies devoted more time to training either on an informal of formal basis. Closely coupled with this is the problem of hiring new men. Psychological testing is showing an increase of use. This trend may easily become more pronounced based on the results being obtained by some firms at the present time in relation to production workers.

Compensation plans have been adjusted to meet the requirements of doing a balanced sales job. "The past ten years have seen two major trends in compensating salesmen. At the beginning of the period, there was a stampede to straight commission basis of remunerating salesmen. Then, as business conditions improved, Dartnell surveys began to disclose a gradual but definite trend away from commission bases and toward salary plans."[33]

A study by the writer in 1938 indicated that in the industrial field a combination of straight salary and commission or bonus was most popular, although straight salary was a close second. During the period under consideration, it is evident that more thought was given to sales compensation. Many companies tried to get the advantages from straight salary and commission forms of payment by combining the two, thus eliminating the disadvantage of each. No single compensation plan is best under all circumstances and for all products, but a sincere attempt was made to make salesmen's earnings more equitable.

Advertising and Sales Promotion

Advertising has been in the limelight and critically appraised. The net result has been beneficial. More of top management have realized that it is one form of selling tool that can be used effectively if properly administered and co-ordinated with the rest of the selling program.

Much more interest has been shown in attempts to find out the value of advertising. The most monumental study in this respect has been "The Economic Effects of Advertising," conducted by the Harvard Business School under the direction of Professor Neil H. Borden, published in 1942. Other studies, such as the "Proof of the Effectiveness of Industrial Advertising" by the Eastern Industrial Advertisers made in 1940, indicate the trend of thinking of both advertising men and management. To quote from this study: ". . . for too many years too many of us have been working in the dark, in so far as knowing the actual value, the definite proof of advertising."

This critical attitude has been helpful. Industrial advertising

[32] "Locating and Appraising Product Ideas," by Ross M. Cunningham, *Journal of Marketing*, July, 1942.

[33] "Salesmen's Compensation and Salary Incentive Plans," Dartnell Report No. 506, 1940. Dartnell Corporation, Chicago, Ill.

has improved in quality. A better realization of what it can do and what readers of it expect in the way of information has come about. Under the suggestion of many leaders in the field and summarized by the Associated Business Papers campaign of "Tell All," industrial advertising has not only become more factual but more informative. More companies are finding out what readers want to know and then supplying this information through their advertising media. At the same time the advertising has taken a more human approach and is not quite so dry as dust. This improvement has not been one based on a radically new approach, but rather a better understanding of what, fundamentally, industrial advertising can do. The institutional type of advertising being carried on when no products are available for general sale, as exists at the present time, indicates some belief at least in the role advertising plays in long-range planning.

Coupled with this altered attitude toward advertising has been the increased use of copy testing in its various forms. Here again one finds the basic ideas started much earlier but real tangible progress made during the period of the 1930's.

The co-ordination of advertising with other forms of selling has been noticeable if not capable of actual measurement.

The sales tools available for salesmen have also undergone considerable revamping. Visual presentation kits, better models and samples, the increased use of slides and movies all give emphasis to the necessity of properly equipping a representative to do the kind of selling job needed.

The objective behind these moves has been to increase selling effectiveness and to reduce sales costs. The constant drive for increased efficiency has resulted in improvements—slow in some cases, much more rapid in others.

Miscellaneous Trends

The impact of government regulations and laws on industrial goods has not been so marked as with consumers goods. Nevertheless, the important element of pricing has not been overlooked. With more study being given to sales costs and analysis, pricing as an element in marketing strategy is slowly taking on more significance.

The use of market-research principles in the field of public relations—both at home and away from home—has been cropping out here and there. The development of better internal relations in view of recent labor legislation may become an extremely important outgrowth of the seeds planted largely in the period under consideration.

Likewise, the humanizing of corporation annual reports so that stockholders and employees knew what was really being done is important enough to bear further watching.

Summary and Conclusions

The attempt here has been to highlight the more important trends—others might be of more importance to a particular industry, while still others may blossom out in full vigor after the war when they in themselves seem minor at present. The biggest step forward in the industrial-marketing field has been the realization that distribution is essential. The application of more scientific analysis to the various problems involved is evident. The result has not been Utopia but at least more companies are carrying on their activities with a better understanding of the needs of the users. Realizing that sales are made in men's minds, they are becoming more human in their approach.

The effect of government regulations has been purposely omitted. The impact of war may have more far-reaching effects than anything indicated or suggested by previous legislation.

Practically all other trends will probably continue. Surely with our tremendous production capacity being developed for war, the conversion to peacetime needs will again make distribution rather than production of prime importance. The problems of product development and adaptation, the actual selling and advertising of goods, the training of men, and the unremitting research for changes in both location and preferences of markets will become vital again.

The foundation and trends established in the decade of the '30's will carry on, for most of them have been based on a more reasonable and logical approach to the problem.

Job Evaluation and Merit Rating

By ASA S. KNOWLES,[34] KINGSTON, R. I.

THE past decade has witnessed both growth and refinement in many of the tools of industrial management. Those which have received most attention, however, are the tools which are related to the management of manpower. Because wages and salaries have been subjects of controversy, managers and labor leaders alike have sought for tools and ideas to assist in ironing out many of the difficulties which arise when matters of pay determination are under discussion. Job evaluation and merit rating have proved helpful in providing both useful and accurate information about jobs and men. Consequently, a great deal of attention has been devoted to their development and refinement.

Job Evaluation

Significant among the steps of progress which have been made in job evaluation during the past decade are those presented in the following paragraphs.

Job evaluation has been misunderstood by many in both management and labor circles. Some have thought of it as a system for deciding the amounts to be paid for various jobs and consequently to determine the total pay roll which a firm must be prepared to meet. Others have thought of job evaluation as a system for justifying existing pay scales through paper work. Still others, and particularly many in the labor groups, have regarded job evaluation as "just another pay system," and have so branded it.

An Understanding of Purpose

Intelligent discussion of job evaluation has done much to remedy these false concepts. In practice, job evaluation results in compensating for work on the basis of its requirements to make rates of pay *equitable* for all jobs in an organization. This result is accomplished as follows:

1 The work requirements of all jobs are expressed in common factors—skills, responsibilities, etc.

2 These common factors are used as a measuring stick by which all jobs are appraised, i.e., the relative requirements of the jobs, one in relation to the other, are decided.

3 The relative values decided upon are converted into job ratings.

4 The ratings become the basis for deciding what amount of the total pay roll each job is entitled to command—the amount that each job justifies for anyone who can meet its minimum requirements.

Any tool of management which deals with wages and salaries must be clearly understood by both management and labor if it is to be used constructively. It is significant to record, therefore,

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that the past decade has witnessed a greater understanding of job evaluation on the part of management and labor.

Improvements in Terminology and Techniques

At the outset, most instruments and tools are crude. They are perfected and refined with use and application. In this respect remarkable strides have been made in job evaluation during the past ten years. Both the terminology and techniques which are requisite to having a job-evaluation program have been refined and perfected. This is a result of the merging of information on the part of many firms now having job-evaluation plans in operation and individuals who have devoted particular attention to research on this subject.

In the early twenties, the initial job-evaluation programs used a number of factors to measure jobs which are now discarded. It was discovered through use that such factors as skill, training, and experience overlap considerably. Fatigue, prevailing wages, cost of living, profit of the company, and similar factors which were deemed to be desirable by some to use in appraising jobs have been discarded.

Four Major Classifications of Factors

Experience reveals that there are essentially four major classifications of factors which can be used to compare jobs. These are requirements of various jobs with respect to (1) skills, (2) responsibilities, (3) effort, and (4) working conditions. Each of these lends itself to subdivision for purposes of definitions which measure relative requirements of individual jobs.

It is increasingly recognized by all who operate job-evaluation plans that definitions must be broad and yet discriminating if comparisons are to be made among jobs which are similar, as well as different, in requirements. This demands that those in charge of developing evaluation systems must develop terminology which measures types and degrees of skill, responsibilities, etc., rather than specific kinds of these. For example, it is possible to compare the skill of a switchboard operator to that of a business-machine operator as to type. The particular things which they do are not important, but rather the degree of types of skill—manipulative ability which is paid for.

Improvements have been made also in the development of techniques used to collect job information which is translated into descriptions and specifications and in this same connection great strides have been made in the preparation of descriptions and specifications which are useful not only for job-evaluation purposes, but also other functions which are normally performed by the personnel department—selection, training, guidance, etc.

The past decade has seen a growing tendency to adopt the use of weighted-point systems to evaluate jobs as compared to the job-ranking or classification system and so-called factor comparison system. It is recognized that the weighted-point system permits use of numerical definitions as well as word definitions to measure the relative degrees of difficulty of various jobs.

Another refinement which has been helpful in the improvement of techniques has been a careful definition of what is meant by *key* jobs as used in the job-evaluation process. Key jobs are now selected in the light of these requirements:

1 Cover a large portion of the personnel of the company.

2 Have counterparts in other companies and therefore permit cross-comparisons.

3 Are stabilized as to duties and not in a state of flux.

4 Are familiar to the entire organization.

5 Are distributed over the entire range of pay of the company.

Adoption as Tool of Collective Bargaining

Firms having job-evaluation plans in operation find that the information needed to evaluate jobs is useful also in matters pertaining to collective bargaining—particularly when rates of pay are under consideration. Job descriptions and specifications provide data upon which to base discussions and conclusions regarding what jobs are worth.

Significant in this respect is the insistence of some labor unions that job evaluation be the basis of discussion between management and labor regarding jobs and the pay they are to receive. Some recent labor-union contracts contain clauses which require a job-evaluation system for working out base rates of pay.

Usefulness in Personnel Administration

The information collected for job descriptions and specifications has been found to have real usefulness also in making selections and placing employees in an organization. When specific information is available about jobs it is possible to fit applicants more carefully into the types of work for which they are best qualified. Job information is useful also for training purposes. Good job descriptions become the basis of desk or bench manuals. In this same connection much progress has been made in the development of standard titles for jobs. Instead of using widely varying names for the same or similar kinds of work, titles are being made more uniform from department to department as well as among various companies. Job evaluation has helped also in the promotion of better supervision. The rating of jobs forces supervisors to study more carefully the jobs for which they are responsible and they know what to expect of their employees in performing their work.

Application of the Principles to Salaried Jobs

The early work in the field of job evaluation was developed in connection with jobs compensated for on an hourly basis, i.e., jobs ordinarily related to the shop and factory productive work. Within the past ten years an increasing number of commercial organizations as well as manufacturing establishments have made application of the basic principles of job evaluation to the determination of base rates of pay for jobs compensated on a salary basis—clerical, supervisory, and executive jobs. The basic principles of evaluation are no different for salaried jobs than those employed to evaluate the work done by the worker paid on an hourly basis. The major and sub factors used and their definitions do vary, however. The development and refinement of the definitions of these factors has been a singular contribution to the job-evaluation field during the past ten years. Particular mention should be made too of the application of job evaluation in appraising the worth of occupations in the public service, i.e., jobs of federal, state, and municipal governments. Worth-while contributions to the literature of job evaluation have been made by those who are applying basic principles of job evaluation to salaried governmental jobs.

Development of Wide-Scale Programs

A further expansion of job-evaluation work which had its major development almost wholly during the past ten years is the participation of trade associations in the promotion of job-evaluation as a desirable system for determining wages and salaries. Typical of these is the National Metal Trades Association which has a job-evaluation plan designed for the metal-trades industries which are its members. The work of this association and others has been a real contribution. It has promoted interest among firms in the same or related businesses to do the following: (1) Adoption of uniform titles for the same work, (2) elimination of discrepancies in pay for the same or similar work, and (3) provision of accurate job information upon which to base pay decisions.

Development of Professional Status of Job Evaluation

Job evaluation has reached a point in growth where there are today a number of persons who devote their major attention to

work in this field. They are primarily engaged in installing systems for various firms seeking to adopt job-evaluation plans. These persons are specialists in job evaluation and their work has given this particular field a professional status.

As an increased number of persons prove their competence in this field, there should result (1) a more widespread adoption of job evaluation as a program of wage and salary determination and (2) further refinement of techniques used. Conceivably, these specialists will help to bring about wider use of standard job-titles, standard definitions of jobs, and standard terminology which will be used in making job comparisons. The net result will be to iron out unfair and inequitable pay discrepancies throughout all industry and to promote better labor relations.

Merit Rating

The developments which have taken place in the rating of men which have proved significant during the past ten years are outlined in the succeeding paragraphs.

Closer Tie-In Between Job Rating and Man Rating

At one time men were rated without the raters having available any particular information about the jobs filled by the persons being rated. Today it is standard practice to have job specifications available for raters to guide their appraisal of performance. Since job specifications are an essential part of a job-evaluation program, a closer tie between merit rating and job evaluation is a natural outcome.

Job evaluation and merit rating are being used in conjunction by a growing number of organizations. Jobs are rated to determine base rates and men are rated to determine the pay differentials which they are entitled to receive as rewards for performance and contributions to their work, etc. When pay is decided solely on the basis of job evaluation and merit rating, pay inequalities are minimized. Information about what jobs require is combined with knowledge of what men do in the performance of them to establish fair and equitable rates of pay. Moreover, when controversies arise about pay matters, information is readily available for discussion and settlement of disputes.

Development of Man-Rating Techniques

The rating of men is not new. For a long time there have been rating systems based on the use of letters, numbers, and groupings. Until recently, however, little distinction has been made between *objective* and *subjective* measures. The objective measures are those which lend themselves to specific calculation—measures of quality, output, safety, performance, etc. The subjective measures are those which are based wholly on judgment. Modern rating sheets are divided into two parts—objective and subjective—and wherever possible, objective measures are used so as to minimize prejudice and bias.

Another step forward in techniques which has been perfected during the past decade is in the use of weights. Raters are now told not only to appraise particular characteristics which men possess, but also to decide what values these characteristics have in the performance of the jobs they fill. Characteristics are given numerical weights which indicate whether they are essential, desirable, or unimportant in meeting minimum job requirements.

Improvements have been made within the past ten years also in the thinking about the frequencies of ratings, i.e., how often they are to be made. In addition, considerable attention has been focused on the training of raters, i.e., having those who are to do rating properly instructed as to what they are doing and its importance to the individuals rated.

Broadening of Applications of Merit Rating

During the past ten years merit rating has been extended to include not only those who work in the factory but also supervisors and top executives. It is not uncommon for rating systems to be applied to an entire organization, those at the top being rated for their capacities in the performance of their tasks just as ratings are made of those who are classified as employees. Supervisory rating techniques have been developed and applied successfully for purposes of both selecting supervisors and upgrading personnel.

Use of Rating in Connection With Selection

Merit rating has been found to be valuable also in selecting and placing employees. This application involves the use of merit rating to select from among present employees those who are most competent in doing the type of work for which new employees are sought. The persons selected as having the requisites sought in new workers are given various tests and the scores they achieve are calculated. The tests on which the scores are highest are used for testing new applicants. The results reveal which applicants have similar qualities to the employees found already to be satisfactory and, therefore, are most likely to succeed when hired.

More Intelligent Use of Merit-Rating Information

In many organizations merit rating has been regarded since its inception as a tool of dismissal and any information uncovered about various employees has been kept secret. As merit rating has come into wider use, its real applications have been recognized and encouraged; these are:

1 The uncovering of individual weaknesses and use of rating results to help individuals to correct them.

2 The use of the results to select persons for upgrading and transfer.

3 To guide training programs. When persons are rated and their weaknesses known, they can be told just what steps should be taken to improve their competence and work status.

4 Forcing of supervisors to study men. Those who do the rating must study the men they rate carefully. This in itself is excellent because it requires supervisors to study their men carefully. In this way they come to know their employees better and they themselves become better supervisors.

Trade Association Activity in Developing Rating Systems

Just as the development and promotion of job evaluation has become a major function of some trade associations, merit-rating systems have been developed to be used in conjunction with the job-evaluation plans. In this respect, rating systems which have application over a wide number of companies have come into existence. The work of trade associations in this connection is making a worth-while contribution to the field of man-rating literature and practice.

Development of Merit-Rating Specialists

Just as in the case of job evaluation, a number of persons are now devoting their full attention to the development and installation of merit-rating systems. Some of these persons are professional consultants and others are individual staff members of organizations where merit-rating plans are in operation. Their research and experience cannot fail to enhance the uses to which merit rating can be put during years to come. Many have made already marked contributions to techniques and it is to be expected that their work in this direction will continue.

Wage Plans

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Summary

During the past decade, one development, that of participation by labor unions, has towered over all other considerations in the development of wage plans. While there have been local instances in which union pressures or management resistances have yielded distorted wage plans, the broad effect of labor-union participation has been to accelerate the adoption of improved and more sound methods of wage and salary determination.

There is reason to believe that this wave of adoption of improved methods will continue for some time to come. There is, however, a strong possibility that continued and unbalanced union pressure may result in the forced adoption, on a broad scale, of wage plans whose scientific aspects are twisted out of shape to meet the pattern of compromise.

The last decade will long be remembered by managers, personnel men, and wage-practice engineers as the decade in which the earlier pattern of techniques of wage determination had to open up wide to receive a new and potent variable—that of collective bargaining. This new variable was productive of considerable confusion in the ranks of management. A new variable suddenly injected into the laws of gravity or into the laws of friction would require some nimble footwork on the part of engineers and designers to adjust to the new variable. Union participation in wage plans was no less a problem for the wage-determination engineers.

The attack took place along all fronts—time studies, piece rates, job evaluation, merit rating, etc. As the National Labor Relations Board and the courts clarified the National Labor Relations Act, it became apparent that this entire subject matter fell within the collective-bargaining area. This legal status of the wage question automatically brought out of the confidential files of the various companies the details of their wage-determination techniques. Those companies which operated with rudimentary plans or with no plans at all were caught flat-footed. Even those companies which had earnestly studied the problem found a necessity for drastic adjustment because of the new variable.

The greatest problem was presented by the fact that these two forces, the union and management, operated on unlike principles without knowing that their principles were unlike. "It is this basic difference which has left managers and labor leaders groping about like two players on the same board, one using checkers and the other using chess pieces, and each astonished at the unorthodox moves being made by his opponent."[36]

To the union, which operated toward the objective that employees should get a greater share of the fruits of industry, it was perfectly consistent to seize on any device which approached that objective. For example, lean piece rates were dragged out into the open to be raised, but the cutting of fat piece rates was resisted vigorously. This was consistent with the prime union objective, but decidedly inconsistent with the management objective of making the earning power of piece rates uniform. True, the unions likewise wanted uniform piece rates, but this and all other objectives were subordinated by them to the prime objective of getting for their members a larger share of the fruits of industry.

Extent of Piecework

The early thirties found a tendency in some industries, notably the automobile industry, to abandon piecework and to go to some form of measured daywork. It would be an over simplification to find some one reason for this. There were many factors, some of which were as follows:

1 The desire to cut overhead caused managers to re-examine the man-hours being spent in rate setting, in computing piecework earnings, and in auditing the piecework system.

2 The dangers that output would fall after piecework was abandoned wás minimized by the prevailing low level of employment.

3 The drastic layoff had made managements strain to minimize further layoff, even if this meant tolerating lower levels of output.

4 The volume of production dropped in many cases below the point where establishment and maintenance of piece rates was profitable.

While the evidence is conflicting, it would seem that a trend toward more piecework was resumed as schedules went up. There appears to be no basis for assuming that piecework had been found lacking, on technical grounds, as a good means for incentives to employees.

Adequacy of Piece Rates

The whole problem of low earning and unequal piece rates stood out in unfortunate relief when collective bargaining turned a spotlight on one of the corners management had traditionally kept in the deepest darkness. Managers were suddenly called on to defend piece rates of very long standing, as well as piece rates established by methods which had for years been only as good as the conscience and fortitude of neglected piece-rates departments. The question of "how accurate can rates be set" came up for vigorous examination.

The problems of adequacy of piece rates resolved themselves generally into:

1 Minimum earnings.
2 Accuracy of piece rates.
3 Uniformity of piece rates.
4 Guarantee against cutting piece rates.

Minimum Earnings

This problem may be broadly stated as one involving the principle that whatever may be the figured earnings of the employee, he should in no case receive less than a stated minimum rate, this generally being known as the base rate. Many companies for years had such a principle in effect. Through the combined pressure of the Wage and Hour Laws and the unions, there was a universal adoption of the principle.

Accuracy of Piece Rates

The normal process of establishing a piece rate involves:

1 Setting the job up as it ought to run.
2 Timing one or more appropriately selected operators.
3 Adjusting the time study to normal operating conditions and to the pace of an "average experienced operator" (or whatever words are used to describe the standard operator).

This last step always involves estimating the extent to which the operator under observation deviates from the standard, and this estimate is the most important single variable in the piece rate—more important than all the others put together.[37] The

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[36] Review of Book, "The Dynamics of Industrial Democracy," *Mechanical Engineering*, vol. 64, 1942, p. 624.

[37] See Progress Report of Committee on Rating of Time Studies, *Advanced Management*, vol. 6, no. 3, July–September, 1941, p. 110.

estimate is a very complicated one and, under existing methods, involves at best a "standard error" of about 7 per cent.[38]

The idea that a third of the piece rates can deviate from the "standard" by more than 7 per cent appears to concern the time study engineers more than it does the unions. The engineers are concerned because there have been many claims about piece rates being set within 3 per cent, 5 per cent, etc., of each other. There are no published data to substantiate any such claims—in fact there are precious little data of any kind published on the subject.

In the over-all view there appears to be no real reason for alarm. In F. W. Taylor's day, the going rate of output of operators was very low, and the piece rates established required levels of output twice, or several times the going daywork output.[39] If in one generation we have developed methods which have cut this error of 100 per cent, 200 per cent, etc., to one tenth of its former value, it is not a poor performance. What *is* unfortunate is the absence of published data on what is the accuracy of the prevailing methods. There is great need for publication of data on this very point. If the engineers do not supply them, the unions will, and will thereby put the engineers on the defensive. F. W. Taylor himself overstated the attainable accuracy on this point,[40] and union leaders have for years made his comments a target.

Uniformity of Piece Rates

Uniformity of piece rates is closely allied to the problem of accuracy, but has many practical offshoots. One difficulty is that there are several perfectly sound principles which can nevertheless conflict with each other in this area. For example, an old and active piece rate for machining part A may be too fat. A new piece rate for machining part B (a new part) may be admittedly a fair rate. Yet the machining operations on these parts may be identical—in fact the same operator may work on both jobs. The union will urge that the piece rate for part B be fattened up to be equal to that part for A. In the management's view, this will accomplish uniformity by making the rates uniformly wrong.

The attainment of uniform piece rates depends on:

1 The precision of the rate-setting techniques; the elements can be no more uniform than the precision of the mechanism which produces them.

2 The maintenance of the piece rates; constant changes in operations will make the rates obsolete unless the changes are incorporated into the piece rates.

3 The establishment of new rates to the standard, flexible though the present methods of setting standards may be, rather than by comparison with rates not at standard. This procedure avoids pyramiding errors but runs into the problem of guarantee of piece rates.

On the whole, it must be said that the problem of uniformity of piece rates has not yet been fully aired. The nub of the problem is the precision of the present rate-setting techniques. Until this factor is given a thorough going over in the engineering laboratories and in the literature, there is little hope for any improvement on a broad front.

Guarantee of Piece Rates

The phrase "We guarantee our piece rates" is about as meaningless as the phrase "These shoes are guaranteed." To be meaningful, there must be defined:

1 What is it that is being guaranteed.
2 Who is guaranteeing it to whom.
3 What terminates the guarantee.

What Is It That Is Being Guaranteed?

The possible differences in the subject matter of the guarantee can best be explained by an example (much simplified):

Element no	Standard time
1	0.05 Min
2	0.40 Min
3	0.15 Min
4	0.10 Min
5	0.30 Min
Total	1.00 Min

Time rate = 60 pieces per hour, for which 25 per cent incentive is paid.

Base rate of pay = 80 cents per hour, so that for doing 60 pieces per hour, the employee will be paid 1.25 × $0.80 = $1.00 per hour.

Now, does the guarantee apply to the $1.00 per hour, to the 1.00 minutes per piece, or to each and every standard time for each element? Unless this and related questions can be answered, the parties do not understand the guarantee.

Who Are the Parties to the Guarantee?

To follow out the foregoing example, employee no. 1 starts to work with the understanding that so long as the method of assembling product A remains unchanged, so long will he be credited with one hour of work for every 60 assemblies. Now clearly this means that tomorrow, and the next day, etc., employee no. 1 must continue to be so credited.

However, tomorrow, employee No. 2 (long an employee in the same department) starts on this job. Does the guarantee hold as to him if the piece rate has meanwhile been found to be in error? How about employee no. 3, a brand-new employee who never heard of the company at the time the piece rate is established—was he a party to the guarantee? Is the guarantee between the company and the union? Again, unless questions such as these can be clearly answered, the understanding is confused.

Duration of the Guarantee

Finally, there is the question of the duration of the guarantee. Is it for a limited time, or is it in the nature of a truce, or is it "as long as the method does not change." This last is much more complex than appears at the first sight. Going back to the example of the 5 elements, if one of these elements and no other is changed, then is the guarantee terminated?

Summary on Guarantee of Piece Rates

The foregoing considerations must make it abundantly clear that understanding of fundamentals is today quite essential to administration of a piecework system. Questions such as just stated have always been present, but until the unions were able to ask their questions with their new-found articulation, the managements were never called upon to answer them with directness. In being called upon to make these answers, the managements themselves learned much about the problem.

Job Evaluation and Merit Rating

These techniques are fully described by Dean Asa S. Knowles in another section of this ten-year report. It remains in this connection only to point out that these techniques are within the

[38] "Time and Motion Study Cross-Examines Itself," by J. M. Juran, Unpublished paper delivered at meeting of New York Chapter of Society for Advancement of Management, Feb. 15, 1940.

[39] "The Principles of Scientific Management," by F. W. Taylor, pp. 47, 71, 81, and 95.

[40] "Life of Frederick W. Taylor," by Frank B. Copley, vol. 2, p. 420, quoting the *Bulletin of the Taylor Society*, vol. 1, no. 1, Dec., 1914, p. 3.

collective-bargaining area, and that accordingly managements must use plans which are on their face reasonable. Many of the estimates in these plans are necessarily quite arbitrary, and in such zones the union can claim as much voice as does the management. Until means are found for making these estimates more precise, there seems little to be gained by adopting a doctrinaire attitude which says in effect that only management is qualified to make these estimates.

Wage and Salary Surveys

The growing usage of "prevailing wages" in collective-bargaining contracts, in legislation, and in translating the scale of job evaluation into dollars, has led to much development of the technique of conducting market surveys of wages and salaries. Fortunately there have been valuable contributions to the literature in this field.

The flexibility present because of sampling errors and because of foggy techniques has permitted unions and managements to arrive at different conclusions in the same labor market, or even in the same wage survey. To date there are still a number of practical obstacles to joint union-management wage surveys, though there have been instances of such collaboration. In some instances wage and salary surveys are made by trade associations, such as the National Electrical Manufacturers Association, or the National Metal Trades Association. In cases where the union is very large and powerful, while the employers are small individually, the prevailing wages may in a large measure be surveyed by the union's engineers. International Ladies' Garment Workers Union is a case in point.

Cost-of-Living Adjustment

The fact that wages generally all rise together or all fall together provides a means for managements to secure a relatively constant proportion of labor in the cost dollar, while at the same time offering to the unions a means for keeping real wages constant. During the last decade managements grew quite willing to adopt this principle, but the unions took the view that real wages were not high enough, and that they (the unions) did not want to freeze real wages at so low a level. The coming of the war with the possibility of a reduced standard of living for all, has caused the unions to re-examine their position and to argue for increases in pay on the ground that cost of living has gone up.

Profit Sharing and Other Bonus Plans

The last decade has brought no outstanding development in the field of profit sharing and bonus plans. Some old plans have been abandoned, and some new ones have been begun, but there is no evidence of a trend here. However, the federal wartime income-tax policy has made it inexpensive for companies to pay bonuses. If 90 cents of the excess-profit dollar goes to the government anyway, why not stimulate performance by liberal bonuses? This practice has gone on apace subject only to keeping within reasonable bounds lest the U. S. Treasury Department declare the system out of bounds.

Guaranteed Annual Income

The depression of the early thirties and the "recession" of 1937 stimulated the consideration of means for guaranteeing income to employees. The problem of guaranteeing income is closely allied with the problem of guaranteeing employment, and in turn guaranteeing sales volume. Much progress was made in determining how seasonal fluctuations in sales could, through good planning, through diversification, and through use of variable stock piles, be ironed out into a relatively constant manufacturing load.

These adjustments, while distinctly easing the burden on specific industries, nevertheless did not go to the root of the problem—national unemployment. Furthermore, the feature of guaranteed employment for those already employed tended to raise problems of guaranteed unemployment for those already unemployed.

It would be unrealistic to say that these plans of guaranteed income or guaranteed employment have to date any more than dented the basic problem of national unemployment.

Tendency to Overstress Importance of Techniques

It is well to close this discussion of "Wage Plans During 1932–1942" by calling specific attention to the inadequacy of techniques *per se* as a means of solving differences in fundamentals. The big change of this decade was the broad-scale adoption of collective bargaining as a force in industrial relations. Such being the case, no amount of twisting and adjusting of techniques designed for other systems of industrial relations could be of any avail. Perhaps of greater consequence was the lack of realism—the failure to see or to believe that the National Labor Relations Act meant what its words said.

In any event there is now an opportunity, during the truce forced by the war, for unions and managements to endeavor to agree on fundamentals. To the extent that they do so, the strains within the industrial system will be relieved. To the extent that they fail, the unions and managements of tomorrow will continue to go through needless strife to develop the fundamentals which must precede any true agreement.

Wage Plans During the Decade to Come

The probability that labor unionism has not yet attained its full growth would seem offhand to suggest that we will have "more of the same" during the next decade. Clearly, there is great likelihood that the unions will gain rather than lose ground, and that they will accordingly increase their stature at the bargaining table. Yet even if this view is borne out by the events of the next decade, it must be noted that the unions themselves will be confronted with new variables which will have a great bearing on wage plans.

Because of favorable circumstances, and through vigorous leadership, labor has substantially increased its proportion of the national income. This increase in proportion must in time come to a halt, even if this means getting it all. When the halt is achieved, two problems already confronting the labor leaders will become problems of the first order of magnitude:

1 Members of the unions will press the union leaders for redistribution of labor's fixed share among themselves.

2 Under conditions of a fixed proportion of the national income, any increase in standard of living will be possible only by increasing the national income itself.

In this way the union leaders, bearing the direct brunt of employee pressures, will have a foremost concern in the problem of equal pay for equal work, and in increasing productivity. It remains to be seen how they will rise to the occasion.

A History of the Man Situation

By C. G. MARCY[41] and M. M. BORING,[42]
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FOR many years industry operated under conditions that made the recruiting of help relatively simple. Labor of all kinds was plentiful. Employment offices had on file

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[42] Supervisor of Personnel, Schenectady Works, General Electric Company.

large reserves of applications of skilled, semiskilled, and unskilled men and women seeking work. Employment was a matter of selection rather than recruiting. If an employer needed a toolmaker he either called back to work a former employee whose abilities were well known to him or he perused his file of applications, selected a number of the most promising ones, sent for them, and then hired the one he considered best qualified.

Technical graduates were plentiful and colleges were contacted in an orderly, systematic way. Seniors were interviewed and employed with great care—each company representative carefully selecting men who seemed to have characteristics which fitted them for particular jobs available.

Unprecedented Need for Workers

It is obvious that when the nation is engaged in the most gigantic production program of all time—when it is estimated that 20 million war workers will be needed next year as compared with 7 million last year and perhaps only a million in 1940—recruiting and selection procedures and techniques must undergo substantial changes from the simple process described.

This tremendous force of workers must be recruited and trained in the face of the withdrawal from civilian pursuits of from 10 to 13 million fighting men. The manpower shortage will be further accentuated by the need for greater agricultural production in order that the fighting forces and the civilian population of the United Nations may be adequately fed.

In 1939 when the nation was coming out of the recession of 1938, further impetus was given the expanding industrial activity by the onset of the European War. A defense program was launched in America and it was not long until the reserves of skilled labor were exhausted and industrial and economic prophets foretold impending shortages of manpower.

Many employers, anticipating their needs for help, advertised for it widely—sent labor scouts into industrial areas seeking men who might be encouraged to leave their present employers for better opportunities elsewhere. For a time these efforts were conducted on a dignified and reasonable basis, and avoided approaching employers of other defense industries. Later, as the need for men became more urgent, many enthusiastic employers, in their desperation, resorted to unethical practices and enticed workers away from their jobs with little regard for the importance of the work they were on and without consideration of the detriment that their removal might be to the war-production program. This practice of unethical employment procedure is referred to as pirating or scamping.

The elimination of labor pirating is considered essential to the war-production program by the War Manpower Commission, the United States Employment Service, and the War Production Board, all of whom are working in co-operation with industrial leaders to curb the practice voluntarily.

Rapid Growth of Training Programs[6]

As manpower shortages became more and more evident, manufacturers and educational authorities alike began to accelerate their training programs by setting up short training courses designed to train individuals in special skills that might be utilized to advantage in well-planned manufacturing processes. High schools, vocational and trade schools encouraged people to take courses in machine-shop practice, blueprint reading, precision measurements, etc. Employers established vestibule training courses to teach machine-tool operation to many of their employees capable of upgrading. Highly skilled all-round mechanics were used as instructors, leaders, or set-up men to utilize these new, superficially trained men to the best advantage.

The results obtained by the use of these quickly trained men on close-tolerance work has been amazing. This development could not have been brought about without good planning by competent engineers in breaking down jobs into simple elements, or without the wholehearted co-operation of a nucleus of skilled men who have helped impart some of their skill to the newcomers in the shop.

The United States Employment Service has assisted greatly in the selection of recruits for training in vocational and trade schools and in special defense schools set up for the purpose in connection with colleges and secondary schools.

The establishment of the E.S.M.W.T. program in the technical schools provided a large number of new semitechnically trained workers, which greatly augmented the dwindling supply of draftsmen and engineers. The upgrading of such people was greatly stimulated as the need for this group expanded.

Various government agencies were established to find and bring to the war industries large numbers of technical people who had left engineering and science jobs during the depression years, and the refresher courses in the colleges were of tremendous aid in again making these people effective.

Requirement for Proof of Citizenship

Until a few years ago employers paid little attention to citizenship, except to provide a space on their application blanks for recording it. Apparently the only use made of the information was for statistical purposes. Legislation was passed in 1926 requiring that only U. S. citizens be permitted to work upon or have access to certain classes of contracts for the armed forces. Provision was made, however, for special permission to be granted for the employment of aliens in particularly meritorious cases. Many employers referred to their records and if these records indicated U. S. citizenship for employees engaged on these contracts they assumed that they were in full compliance with the regulations and were not greatly concerned further with the matter.

As the requirements of the government for war goods became more urgent and as manufacturers accepted more and more defense and war contracts, personnel men began to require proof of citizenship for new employees. When it developed that such evidence in many cases was obtainable only with considerable difficulty and that many who thought or represented themselves to be citizens were found to be aliens upon examination of the evidence, these personnel men in conjunction with government authorities went a step farther and required proof of citizenship of present employees.

These new requirements caused a tremendous demand for birth certificates. Bureaus of vital statistics had to enlarge their forces, and even then delays were experienced. Thousands of prospective employees were born before vital statistics were kept with the care that they are now, which meant that these people had to find other evidence to prove their citizenship. Infant baptismal certificates, statements from physicians who attended at birth, affidavits of parents or older relatives, old census records, family Bibles and other family records were relied upon to convince employers that those who presented them were U. S. citizens. Bureaus of vital statistics set up procedures whereby citizens for whom no record of birth had been made could obtain a delayed birth certificate following the presentation of sufficient corroborating evidence indicating the place and date of birth. Almost invariably it is easier for a foreign-born individual who has been naturalized to demonstrate his citizenship than it is for a native-born citizen, particularly if beyond middle age.

More recently, because many capable men and women found it impossible to obtain employment in war industries through lack of adequate proof of citizenship, the government set up a procedure whereby such people could declare themselves citizens

in the presence of Army or Navy procurement officers. Obviously this procedure must be surrounded by safeguards to prevent its abuse and good presumptive evidence should be available before resorting to its use. It does provide a way, however, for many competent workmen to obtain employment on war projects who may be qualified in every respect and who are in fact citizens but are unable to present documentary evidence to prove it.

Need for Engineers

As the need for engineers in the armed forces increased, various branches of the Army, Navy, civil service, and other governmental agencies began to compete with war industries for men in the colleges. The various reserve plans were established, and pressure was brought to bear to bring to the armed forces a large portion of the young undergraduate engineers in the colleges. It became apparent early in 1942 that the normal supply of engineering graduates was reduced to an insignificant number. A few physically handicapped young men and a small number of older men, who by their vocations had not used their engineering for many years, comprised the only supply.

Many older men were so out of date that it would have taken much training and considerable time before their engineering could be very effective. Many of these people who held lucrative positions in the past had difficulty in adjusting themselves to the point where they could accept pay commensurate to the jobs they were fitted to do in the war effort.

Naturally with the increasing acuteness of the manpower situation, employers abandon some of their traditional standards for prospective employees. They find that physically handicapped people can be used to advantage. By exercising some ingenuity they can modify some of their jobs in order that individuals with physical handicaps can perform them acceptably. Examining physicians are guided by the necessity of determining what kind of job a man can do. In wartime rejections are made only for physical limitations that make it unsafe or inadvisable for the individual having them to be employed. Today many employers look upon moderate physical handicaps as assets in prospective employees for they feel that they will not be so liable to be called for military duty.

Employment of Older Persons

There is a trend toward the employment of older people as the labor shortage becomes more stringent. Some latent skills and aptitudes have been rediscovered by this process and many employers are finding that these people have a seriousness of purpose and a degree of dependability that offsets to some extent their reduced physical vigor.

During periods of labor surplus, employers may set up standards for education and training that may not be fully justified by the requirements of the job, but because better trained people are available, they get the jobs. Now that labor is scarce, many employers are not so particular about requiring high-school graduates for messengers, clerks, typists, and similar jobs.

It must be acknowledged that industry—and by industry we mean the employee group as much if not more than management has been reluctant to accept certain minority racial groups into the industrial family. Considerable progress has been made toward modification of this attitude and substantial sources of manpower are thereby becoming available.

Women the Greatest Source of Potential Workers

The greatest source from which new workers in industry may be recruited is the women of the nation. It has been amply demonstrated in England that women can do many things American industrialists have thought it impossible for them to do. It is inevitable that as more men are drawn into military service, their places must be filled to a great extent by women. To make the adjustment successfully, women must be trained in short training courses and by providing competent instructors for on-the-job training. Many employers have already made considerable progress in substituting women for men on many types of shop work such as machine-tool operation, assembly , welding, wiring, riveting, crane operation, etc. In general, on these jobs the principle of equal pay for equal work is recognized. More employers are getting under way with such programs now, and it is anticipated that as the military branches of the government call the young manhood of America into the Service, the womanhood of America will rise to the situation and play a most vital part in supplying the mechanized equipment that is so necessary in the successful prosecution of the war.

There has never been a supply of technically trained women, however, and the adjustment necessary to replace fully trained engineers by women is extremely difficult. The nearest approach to such a reservoir of womanpower has been that small group of college women trained in abstract mathematics and science. Educationally, these people are at best only at the sophomore level and also they completely lack the mechanical sense found in boys and young men. They cannot attain in a short time the full engineering understanding so needed under war conditions.

However, the breaking down of jobs and the continuous shifting of trained engineers has made a real place for these women. Their eagerness of purpose and their willingness to study give them ample opportunity to develop technical skill to a remarkable degree.

The long period of depressed conditions immediately preceding the war, during which time all types of training were retarded, has greatly reduced the supply of all skilled people. The progressive steps from depression and national defense to war has caused great difficulty. These steps naturally led to elaborate expenditures of time and money in training young men who were called to the armed forces about the time they became effective in production. This training was lost to industry and now the job must be done over again by the training of women whose usefulness we did not before recognize and still must cause much readjustment. The nation must use this womanpower, the older men, and the physically handicapped, however. It is imperative to keep the production machine rolling to supply our armed forces and the armed forces of our Allies to the extent that by all pulling together the war may be brought to a successful and more rapid conclusion.

Labor Relations in Evolution

By W. R. BURROWS,[43] SCHENECTADY, N. Y.

THE war has brought new problems for both management and labor and, although the top representatives of the two major unions have agreed that there shall be no strikes during the war, this agreement is not always taken seriously by the officers of militant locals. As much care has to be exercised in handling labor relations in the industrial plant as before Pearl Harbor.

Destructive Results of Industrial Strife

A strike represents a collision on the road of industrial progress, a collision caused by two groups going in the same direction, where the effort of one to beat the other forces both off the road, to their mutual disaster. Usually both are at fault, the one for

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recklessly trying to get ahead and the other for not veering off a little in compromise, even though he knew he was within his rights.

No car drivers deliberately court accidents, and neither do labor and management purposely set the stage for an industrial battle. But at the time when the trouble occurs at least one party has gone out of bounds, and the other, in spite of the danger, often holds to his course as a matter of maintaining his rights, or of forcing the aggressor back into place. Neither cares particularly at the time what happens to the other and neither counts the losses or the effect on the general public.

Strikes can best be prevented by methods like those used to reduce traffic accidents. The two factors are to observe normal safety precautions and to become "safety conscious." If each party carefully refrains from the acts most likely to aggravate the other, and both are on the alert to prevent occurrences which may injure one another, there is far less danger that some unfortunate event will happen to run them off into the ditch.

No Reason for Management and Workers to Collide

Generally speaking, under democracy in industry there is no reason why the interests of management and workers should collide. Management does not go out deliberately to exploit the worker, and labor does not maliciously plan to scuttle management. It is usually some misunderstood situation or some unpremeditated act which plunges both into conflict. Neither side is intentionally selfish but both often guard their rights so jealously that neither can see the disputed point in the light of the other.

The past history of labor relations is filled with such misunderstandings. When the factory owner had full power to determine hours of work, wage rates, working conditions, and other terms of employment the worker had no guarantee of fair dealing if the employer chose to be arbitrary and exacting. If the bargain was one-sided in favor of the employer, the worker knew he had been exploited and the injustice was an affront to his personal liberty. This is why he was aroused to fight for his rights, even to the extent of violence, bloodshed, and destruction of property.

Economic progress and technical improvements in time brought about better hours, wages, and working conditions for workers but in many cases not any improvements in employer-employee relations. It is true that forthright managements took a broad view of their responsibilities and realized that, besides their obligations to stockholders, they had a trust to discharge toward their employees and the public. The more discerning and better-balanced labor leaders, likewise, saw that nothing was to be gained by constant agitation against management, and found that public support fell away when labor demands were unreasonable or in conflict with national interests. But there were still those in both camps who stood obstinately on their rights, or used their existing powers to force through measures which were to their own distinct advantage, even though unfair to the other.

Why Strikes Were not Alarming in the 1920's

During the prosperous years between 1922 and 1929, both labor and capital benefited by the good times; jobs were plentiful, wages were reasonably high, dividends were generous, and stock values pyramided. Aside from the efforts of some managements to circumvent the labor movement, and the arbitrary attempts by some organized groups of workers to boost wages, restrict output, and exert union pressure to obtain concessions there were no really violent disturbances. Neither side could be too discontented with current conditions.

Basis for Labor Unrest in the Past Decade

When the severe depression broke in the 1930's, however, and the national income was cut to less than half, 17,000,000 workers, or more than one third of the employable persons in the United States, were out of jobs. Stocks dropped to unheard-of lows and dividends turned to deficits. A struggle for self-preservation broke out between capital and labor. National legislation was passed mainly with the idea of protecting those victims of the crash who had the lesser resources upon which to draw. At the same time that economic pools were established by the government to alleviate misfortunes, social legislation was enacted partly to provide measures for greater security of individuals and partly to bring about certain reforms according to the more radical ideas of some of the proponents of the legislation.

Labor Legislation of the 1930's

There were several labor Acts put into effect. The first was the National Labor Relations Act, which guaranteed to workers the right to bargain collectively without restriction by the employer, and prevented the employer from functioning in any way in the setting up or operation of whatever kinds of labor organizations his employees chose to establish for the purpose of dealing with him on questions of wages, hours, working conditions, and all other matters concerned with employee relations.

A second measure was the Walsh-Healey Act which guaranteed to workers in companies holding government contracts of $10,000 or over the maintenance of fair labor standards as practiced in the industry at large in matters of pay and hours of work.

A third enactment was the Fair Labor Standards Act (Wage-Hour Act) which established for all but certain excepted businesses and occupations minimum hourly rates of pay (now 30 cents) and maximum normal working hours per week (now 40), requiring time-and-a-half rates for all hours above the established normal number.

This legislation, in spite of certain defects which have arisen largely from somewhat extreme regulations and interpretations in applying various provisions of the Acts, represented only what companies leading in the development and practice of good labor relations had already set as equitable standards. Just as factory laws previously passed in the various states to govern fire protection, safety, sanitation, and similar matters involving public welfare had compelled lagging companies to provide facilities of this kind on a par with those already existing in the better-managed plants, so the enactment of labor laws, in effect, raised the level of fair standards for all companies to those of progressive organizations which had clearly seen that whatever benefits labor, by and large, also reacts to the advantage of employers. In other words, factory legislation in the states had set a "floor" or minimum below which no manufacturer was allowed to go, thereby protecting the humanitarian employer from the cut-throat competition of the ruthless employer who lowered his costs by abuse of his workers. National labor legislation, in the same manner, set standards or floors above which all employers were required to measure, so that managements which were maintaining fair con itions for their workers would not suffer from the inroads of competitors whose profits resulted from the low wages and long hours of their workers.

Management's Mistakes Prior to 1932

Where many managements made a serious mistake in the ten years prior to 1932 was in the failure to deal with workers on an honest and equitable basis under the less coercive and more friendly atmosphere of the smaller local labor groups. In many industrial areas, during those years, progressive managers became well acquainted with union organizers and were able to

help them with their own managerial problems in the formative years of the union. Through this contact these plants gained experience in analyzing employee-relations problems, so that they were able to train members of their staffs in the fundamentals and mechanisms of handling grievances, planning for labor co-operation, and all the other constructive features of up-to-date personnel practices.

On the contrary, plants in which the managers took a highly individualistic attitude and set up an entirely wrong conception of responsible unionism, fell into the error of prejudiced antagonism, and gave opposition instead of offering co-operation to a movement which could not be put down. Other managers were unwilling to go through the long period required to educate members of their organization in the new attitudes and practices required in dealing with unions of a national extent.

Dealing With Unions During the Past Ten Years

Unionization of plants during the past ten years has been highly educational to management. By meeting together periodically to discuss mutual relationships, the representatives of management and of labor—the latter mainly workers in the respective plants—have learned much from one another. Out of such contacts has come a broader understanding of what is required to establish and maintain a satisfactory and mutually beneficial relationship.

One of the primary fundamentals for successful labor relations is the realization by management that it is responsible for making contractual relations successful. A written contract should be set up specifying working conditions, machinery for handling grievances, methods of making decisions, and provisions for modifying contracts. The procedure for adjusting grievances is especially important. In the General Electric Company, 95 per cent of all grievances are handled by the foreman. All questions, however, can go to the top. One of the General Electric vice-presidents, for example, settled a request for a two-cent an hour increase.

It is essential that management establish friendly relations with union executives and representatives. All dealings, moreover, should be conducted in a straightforward manner and without loss of temper. In addition, it is vital to have a factual basis upon which to act. Hence the management should be willing to furnish data to union representatives. Otherwise unreasonable demands may be filed, and these are sometimes most difficult to adjust.

During the past ten years, also, as one of the results of unionization, management has been aroused to the need of training foremen and supervisors. The foreman has grasped a new conception of his job—that of not merely getting out production, but also of representing management and taking care of labor relations within his department. Some instruction in industrial psychology and special training in methods of handling labor problems and grievances are most useful adjuncts to his preparation for such responsibilities. In addition, since he is concerned with the training of workers under his direction, the foreman is greatly helped by intensive courses in methods of teaching workers how to do their jobs.

As an example of what may be done to aid good labor relations, one of the plant studies in the General Electric Company covered an investigation as to why some workers failed to earn good wages under piece rates. It was found that one man who had fallen steadily below standard had defective vision. Proper glasses corrected his poor vision, increased his earnings, and improved his whole attitude toward his job and himself.

Principle of Collective Bargaining

The main principle brought out by experience with collective bargaining and relationships with labor unions during the past ten years is as follows:

In the evident absence of any real conflict between the aims of management and workers, the best interests of both are best served by intelligent co-operation to work out agreements and establish bases of dealing which will eliminate serious clashes and bring to both the greatest economic and social benefits. The public is sincerely interested in this outcome of the labor question in industry.

Federal Administrative Management 1932-1942

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THE most conspicuous and widely noticed characteristic of the federal government in the past decade has undoubtedly been the vast expansion of governmental functions first to meet a depression crisis and now the war—an expansion that has carried the influence and activity of federal agencies into every community and virtually every farm and home. But to those who have given attention to the capacity of the federal government to manage large enterprises effectively, the decade is equally significant as the period during which there was the first concerted and continuous effort to improve the processes of management.

These two developments are in a sense related. Better administrative methods grew in part out of the sheer necessity of bringing a vast and rapidly expanding organization under some degree of control. Methods for improving management had been developed both inside and outside the government, and had been applied in limited fashion by various governmental agencies. The depression-born growth of government functions provided the setting for widespread application of these methods.

Previous Improvement in Management Was Slow

Important improvements in government management had been instituted in previous decades but progress had been slow and piecemeal—the civil-service system in 1883, the beginnings of improved purchasing methods in 1909, the budget system in 1921, a classification and compensation plan in 1923, and the work of the Bureau of Efficiency between 1913 and 1933. Between 1932 and 1942 all of the tools of management were developed more fully, and for the first time the President was provided with immediately accessible management facilities through the organization of the Executive Office of the President. This was perhaps the most significant development of the decade since it represented the crystallization of the concept of the President as the general manager of the federal government. Between 1932 and 1942 there were also notable improvements in the quality of government personnel, in the internal management of departments and agencies, and in the structure of the federal government.

There was not only outstanding progress in federal management in the past decade; there was likewise a change in the approach to government management problems. Whereas previously the motivation had been variously "get rid of the spoilsman," "reduce government costs," or "run the government like a business," the new touchstone was "how can the goals established

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Author's note: This paper is the concensus of ideas of the staff of the Bureau of the Budget and certain outside authorities as to what have been the most significant aspects of management in the federal government during the past ten years.

by Congress best be attained?" This did not mean disregard of economy and efficiency, but rather relating these concepts to the purposes of government activity. It meant a greater recognition that the purpose of government is to perform whatever functions or services the citizens agree upon through their chosen representatives and to do these things with a minimum of expense and effort.

But progress in management cannot be measured solely against the standard of previous decades. It must also be measured against the almost limitless wartime demands for strong and effective government. Granted that in the absence of recent advances, chaotic management might well have been the outstanding feature of the war program, the question remains whether our management capacity has reached the point where it will assure well-planned and co-ordinated war administration. The federal government is notably better equipped today than ten years ago to organize and guide the nation through an emergency, but there are still serious weaknesses. Some of the successes and failures of war administration that are now evident will be noted.

Management Tools Put on a Firmer Basis

Efficient management depends upon the use of certain administrative tools. No administrator, however competent, can manage his organization effectively or give assurance that the public's money has been spent with care, in the absence of staff officers and management units to assist him. Such management aids—primarily budgeting, program planning, management planning, and personnel administration—little developed in previous decades, have become well accepted and have been put on a much firmer basis in the past few years. This is one result of an accelerated interest throughout the government in the improvement of administrative management and of organized leadership and facilities at the center of the government.

The Executive Office of the President grew out of the recommendations in the 1937 Report of the President's Committee on Administrative Management. It was established in 1939 by bringing together into one organization the White House staff, augmented by six administrative assistants to the President; the Bureau of the Budget; the National Resources Planning Board; and the Office of Government Reports. Since that time the latter has been transferred to the Office of War Information and the Office for Emergency Management has been developed. The O.E.M. is the means by which emergency organizations to administer designated Presidential powers can be set up and co-ordinated either in time of war or in peace.

Although the Executive Office has been able to make important contributions to the management of the federal government and to take the first steps in integrating the federal government into a more cohesive structure, its potentialities as a management arm of the President are very much greater than the accomplishments of the first three years of its existence.

Development of the Budget System

A budget system for the federal government was launched in 1921. During the first eighteen years of its existence, the function of Presidential review and co-ordination of departmental requests for funds was established, review of legislative proposals involving the expenditure of money was undertaken, and some attention was given to organization problems of agencies. But with a small staff—65 in 1939—it was out of the question for the Bureau of the Budget to do an intensive piece of work on all budget functions. In the past three years, however, with expansion of the staff to almost 500, it has been possible to organize for a comprehensive budget job. The work of the Bureau is now carried on through five major divisions—Estimates, Legislative Reference, Fiscal, Statistical Standards, and Administrative Management, and a special War Projects Unit which inspects construction work.

Review and co-ordination of departmental requests for funds and follow-through on expenditures has been intensified. The larger staff makes possible much more field work and the building up of detailed knowledge on the activities and problems of government agencies. For example, five staff members devote full time to the Department of Agriculture. Formerly six persons were responsible not only for that agency but also for the War and Navy Departments.

Techniques for executing the budget program have been strengthened. In 1933, the power to apportion appropriations and establish budgetary reserves was transferred from the departments to the Bureau of the Budget. This authority has become an important method of keeping agency expenditures within appropriation limits and of achieving savings. One obstacle to using this device effectively has been a lack of information on a comparable basis as to the progress of expenditures. Often the reports submitted have been little better than good guesses.

In 1940 the foundations were laid for a uniform system of fiscal and work reporting by government agencies. This will not only give the departments and the Bureau a better basis for judging the pace at which expenditures are being made, but eventually will produce the first comprehensive information on expenditures in relation to units of work accomplished, and on the financial condition of the government and the operations of government corporations.

The work of reviewing legislative proposals advocated by the departments has been extended. This activity now covers all types of bills whether or not the expenditure of money is involved and includes executive orders. Proposals are reviewed to determine whether they are mutually consistent and in accord with the President's policies.

Departmental budget systems have likewise taken a spurt forward during the decade. Although every department was required by the Budget and Accounting Act to appoint a budget officer, little was done at first to develop offices that did much more than total up the estimates of the several bureaus and divisions. In recent years, however, appreciable progress has been made in a number of agencies, for example, Agriculture and Post Office Departments and the Veterans Administration.

These improvements in the work of the Bureau of the Budget and the department budget offices, which reflect the acquisition of more intimate understanding by budget officers of the operations behind the figures, have marked an important change in the approach to the budget function in the federal government. The federal budget is considered increasingly as a work program expressed in financial terms rather than merely the arithmetical result of sharp bargaining. Another development which is still in its beginning stages is closer integration of the budget with over-all fiscal and program policy decisions.

Growth of Departmental Program Planning

Program planning as a tool of federal management implies the use by administrators of research and planning assistants who analyze the problems on which policy decisions may be required and prepare recommendations and programs of action for consideration by administrators and, where appropriate, by Congress.

Program planning units, not often labeled as such, have existed for many years for specific government activities, for instance the Bureau of Public Roads as the central planning agency for the national highway system. In the past decade as the government has entered many new fields of activity where programs were not

always carefully defined in advance, the number of agencies having program planning units has increased; also the first central planning agency for the federal government—the National Resources Planning Board—was established.

Illustrations of some of these newer units are: the Division of Tax Research of the Treasury Department, the Division of Research Statistics of the Social Security Board, the Directive Council of the office of the Co-ordinator of Inter-American Affairs, the Planning Committee of the War Production Board, and the Bureau of Program Planning and Review of the War Manpower Commission.

In some agencies the effort during the past decade has been to put program planning on a broader basis. In the Department of Agriculture, for example, the Bureau of Agricultural Economics was stripped of its operating functions and designated as the central planning agency for the department in 1938. It was made responsible for planning a national farm program, including co-ordinated programs of agricultural production and distribution, land use, and conservation, and for co-operating with state and local planning bodies concerned with similar problems. More recently the Department's War Board, which is advisory to the Secretary on policy questions, has also participated in the development of departmental plans and programs.

The National Resources Planning Board

The National Resources Planning Board had its beginnings in the effort to plan the public-works program authorized under the National Industrial Recovery Act in 1933. Since then it has grown into a central planning agency to advise the President on development, use, and conservation of the nation's resources, to encourage the establishment of planning units in agencies, regions, states, and municipalities, and to co-ordinate the activities of such agencies.

Regional problems such as the development of the water resources of drainage basins have received particular attention from the Board. Handling problems of this type involves co-ordinating not only agencies of the federal government but also the agencies of different governmental jurisdictions. The Board has prepared numerous reports on long-range problems such as land use, urbanism, housing, transportation, industrial development, public-works programming, relief, and several pamphlets to stimulate thinking on postwar problems. The Board also makes a quarterly report on employment to the President. Recently the President has requested this agency to bring together and correlate the postwar planning efforts of both public and private agencies.

Although there has been an increase in planning activities, they have not been integrated with actual operations to the extent necessary if planning is to bear fruit in policy decisions. One illustration of the kind of integration that should be more frequent is the work of the National Resources Planning Board and the Bureau of the Budget in programming public works. Federal agencies responsible for construction programs are required by law to submit six-year advance plans to the Board and the Bureau jointly and the two work together in appraising the proposals. In this fashion, budget decisions on construction estimates are made in the light of long-range considerations brought forward by the Resources Board.

Management Planning Takes Hold in the Government

During the past few years great strides have been made in the development and installation of improved organization, procedures, and business practices both for the government as a whole and within departments. Prior to the present decade the primary work in this field had been that of the Bureau of Efficiency. Although that agency worked on administrative and operating problems from 1913 to 1933, it did not give major attention to the over-all structure of the government. Furthermore, the fact that it was never a part of the central administrative stream and did not have a close relationship with the Chief Executive detracted from its influence in over-all governmental matters. After the Bureau of Efficiency was abolished in 1933, there was no central administrative planning agency for the federal government until the Division of Administrative Management in the Bureau of the Budget was built up in 1939 and 1940.

The Division now has approximately 75 staff members with experience and background in both government and business who by virtue of their position in the Executive Office of the President are able to facilitate numerous fundamental improvements in federal management. Beginning with the comprehensive reorganizations of the government structure put into effect in 1939 and 1940, the Division has worked continuously on the formulation and effectuation of reorganization measures. Improvements in such government-wide business practices as travel regulations, standardization of charges for living quarters and meals allowed for federal employees, control of government-owned vehicles, and a wide range of similar problems have received the Division's attention.

The major part of the work of the Division of Administrative Management, however, has been assistance to individual departments and agencies in helping them to work out internal organization and management problems. In a sense the Division supplies a consulting service at the immediate call of every department head who wishes outside sympathetic help in eliminating organizational and procedural difficulties. More recently, attention has been devoted almost exclusively to ways and means of improving war organization and administration.

Administrative Management in Agencies

This increased attention to over-all federal organization and administration has been accompanied by a rapid increase in the number of management planning units within departments and agencies. There are now more than fifty such units throughout the government and most of them have been organized within the past few years. The Office of Organizational Planning in the War Production Board, the Office of Budget and Administrative Planning in the Post Office Department, the Planning and Budget Section in the General Accounting Office, the Division of Fiscal Management in the Department of Agriculture are among those that operate on an agency-wide basis. Others, such as the Planning and Review Section of the Bureau of Old Age and Survivors Insurance, and the Departmental Planning Section in the Farm Credit Administration function within Bureaus or other agency subdivisions.

The job of the management planning units is to assist operating officials in streamlining agency organization, to solve the tough problem of controlling field operations, to improve management practices, to expedite such central services as filing, mails, and communications, and generally to survey, develop solutions, and install new systems covering any administrative bottleneck or sore spot. The work of management planning units in the military agencies has been especially significant. The Control Division of the Army Services of Supply, the Directorate of Management Control of the Army Air Forces, the Administrative Management Section of the Coast Guard, and the Management Engineers Office of the Navy, have worked on rationalization of organization and other basic improvements and have achieved spectacular results in eliminating "red tape." More than 1000 useless report and record forms have been scrapped recently in the two military departments.

Co-Ordination of Statistics

Steps to co-ordinate the collection of statistics by approximately one hundred units of the government illustrate another aspect of progress in administrative planning. The Division of Statistical Standards in the Budget Bureau at the present time reviews all proposals for the collection of statistics by the government and affixes an identifying number to each report form approved, in order to eliminate duplications, co-ordinate the statistical activities of federal agencies, and control the quantity and quality of statistical inquiries.

The Division also stimulates the establishment of similar central review units within agencies collecting a diversity of statistical data, for example, the Office of Price Administration, the War Production Board, and War and Navy Departments. Other functions are to encourage agencies needing similar data to obtain it from a single source, and in co-operation with other agencies, to develop statistical programs that will bring about co-ordination of governmental activity in specified fields and collection of data that will be useful to the largest number of agencies. There had been little activity in this field before 1933.

New Approach to Civil Service

An "about face" has taken place in personnel administration in the past ten years. The Civil Service Commission, the oldest of the federal agencies concerned with central management, has shifted from the time-honored philosophy of policing personnel transactions to that of service to the operating agencies. Maximum flexibility within the framework of the merit system is the guiding principle of the present wartime procedures of the Commission. Placing eligibles on lists according to rank in examinations and submission of only the top three names to appointing officers has been abandoned. Qualifications of applicants are investigated, but less frequently by written competitive examinations, and appointing officers have relative freedom of determining which applicant best fits the particular requirements of a job after Civil Service Commission standards have been met.

Several factors have had a bearing on this development. Recent Commissioners have understood the management role of a personnel agency. Much of the new personnel that has come to the government service in the past decade has been impatient with the rigidities of the traditional civil-service system. The success of the Tennessee Valley Authority in making the personnel function an integral part of management has been a challenging example. The Report of the President's Committee on Administrative Management presented a strong case for reorganizing the Commission to make it an integral part of the President's management machinery. But perhaps of greatest importance has been the effect of the war. The change from surplus to shortage in the labor market, and the acute and immediate need of government departments for personnel put on the Commission a pressure to give service that could not be resisted.

Although the Civil Service Commission has become in effect an agency for facilitating overhead management rather than for control, a statutory prohibition has prevented integration of personnel administration with the other management functions in the Executive Office of the President. However, through the designation of one of the President's administrative assistants as liaison officer for personnel management, it has been possible to make the link between the Civil Service Commission and the Chief Executive a more vital one. This officer meets with the Commission each week and advises with them. He also watches personnel developments throughout the government, and proposed Presidential actions, such as amendment of the Civil Service Rules, are first cleared with him.

Equal gains in the federal personnel system have been made in departmental personnel management. Ten years ago the Department of Agriculture was the only department that had developed personnel management in any comprehensive fashion Today almost all agencies have substantial personnel offices and many have broad-gage programs. This is an outgrowth of a Presidential directive in 1938 requiring all departments and the large independent agencies to establish personnel divisions.

At the same time the Council of Personnel Administration was established to give further stimulation to the improvement of personnel practices. Although previous sporadic efforts have been made to bring the departments and agencies together for the solution of common personnel problems, the present Council is the first one to become a continuing influence for improved personnel administration. Only in the past four or five years have there been experienced personnel officers to serve as members of such a body. The Council aided by a technical staff has been a focal point for organizing a unified attack on such problems as recruitment, placement, classification, compensation, training, service ratings, employee relations, leave, separations, personnel forms, and health and safety measures.

Extension of Merit System

The development of improved management techniques in the federal government would be of little significance in the absence of able and imaginative administrators devoted to the public welfare. The tradition of political patronage in American government has meant that the first years of civil service had to be spent in establishing the merit principle of appointment. At the opening of the present decade this had been very largely accomplished. At that time approximately 80 per cent of all federal positions were subject to the Civil Service Act and Rules.

Subsequently, during the early depression years there was a recession in the coverage of the civil-service system. This was due, in part, to the temporary basis on which many of the emergency programs were established, as well as to the difficulties of setting up programs rapidly under the rigidities of the old civil-service system. Many competent administrators were convinced that the Civil Service Commission, under its traditional methods, could not supply the quality of talent needed. However, the exemption of positions from civil service was no solution, as the pressure for patronage appointments proved an equal obstacle to meeting administrative necessities.

In consequence, in 1938, the President placed under civil service all positions over which he had control and asked Congress for similar authority over all other non-policy-determining positions. Such extension of the civil-service system had likewise been advocated by the President's Committee on Administrative Management and by such citizen groups as the National League of Women Voters and the National Civil Service Reform League. These efforts to extend the merit system culminated in 1940 in the passage of the Ramspeck Act which made all but the policy-determining positions subject to the Civil Service Act and Rules. This law is a landmark; for the first time it became the policy of the government to include all the top administrative and professional positions within the merit system. Another significant step was taken when this policy was applied to some 4500 legal positions.

Recruiting Administrators

At the same time that this movement was taking place, the public service was becoming more attractive as a career for persons of top ability. In past decades a tradition of high-quality permanent-career officials in many scientific and technical branches of the public service had developed, but no corresponding custom had been built up with regard to positions requiring general administrative competence. In recent years,

however, there have been numerous educational efforts calling attention to the necessity of improvement on this front, beginning in 1935 with the report of the Commission of Inquiry on Public Service Personnel, "Better Government Personnel." Also depression conditions and the inauguration of important new government programs in 1933 and 1934 attracted large numbers of able persons to the government service who normally would not have been available.

At about the same time the Civil Service Commission began to depart from its normal recruitment procedure. Traditionally, applicants had been examined for proficiency in each type of clerical or technical position with little regard to capacity for growth and ability to discharge higher responsibilities. In 1934 a program was launched to bring into the government service the most intelligent and talented graduates of the colleges and universities. As a result more than 5000 junior appointments of this type, in some thirty professional fields, had been made by June 30, 1941. In 1940, civil-service examinations were given for the first time for persons trained and experienced in the general field of management and in recent months have been held almost continuously.

Training programs have been another constructive approach to the improvement of the quality of government employees. The public-service training programs of the universities were greatly accelerated throughout the decade. Government internships for training outstanding college graduates for the public service were established by a private agency—the National Institute of Public Affairs, in 1936. In-service training within the government also began to receive attention, and again the Tennessee Valley Authority set a notable example.

Unfortunately, all of these trends have not produced enough administrative officers for the war effort, in spite of the fact that increased numbers of competent administrators have come to the government from private enterprise since the start of the war program. Most breakdowns in administration are the result of inadequate leadership by the head of an agency or his principal subordinates. Heroic steps are needed to solve this problem. The Civil Service Commission, through its Committee on Administrative Personnel, is now engaged in an intensive effort to identify and secure administrative talent both from within and without the government service, but the need continues to outstrip supply.

Organization for Management Within Departments

One evidence of the improved quality of government administrators has been the great increase in attention given within departments and agencies to the co-ordination of programs, conduct of field operations, introduction of new technical devices, development of effective staff services, and methods of reporting on work done. This interest has been fostered by professional societies such as the American Society for Public Administration, the Society for the Advancement of Management, and organizations of public officials within specific fields of government. The conferences, research programs, and publications of such organizations as the American Public Welfare Association, the Civil Service Assembly, the National Association of Housing Officials, and the American Society of Planning Officials have been great stimulants of better administrative management. Employee organizations are also increasingly focusing their attention on administrative and procedural improvements.

Establishment of departmental personnel, budget, management planning, and program-planning facilities on a firmer foundation has already been noted. Mere organization does not suffice, however. These units will not contribute to better management unless top agency executives use them. There needs to be a common focus for management facilities either in an administratively minded department head or in a general administrator working in close association with the policy leadership of the agency.

A few years ago observers thought they saw in a few departments the beginnings of general managership positions which could meet this need, but the development has not continued. Recently there has been some experience with trying to solve the management problem by appointing career administrators to Assistant Secretaryships or Undersecretaryships of Departments, positions traditionally occupied by political appointees. There is not yet a consensus on the best solution.

Neither is there complete agreement on the proper role of individual management units, although the trend is clearer. The temptation for staff officers to engage in actual operations often has not been resisted. There has been, therefore, considerable following for the view that departmental policies and responsibility can be effectively established only if there is a vigorous review of bureau transactions. The contrary view, rapidly gaining in adherents, is that exercise of leadership, establishment of departmental standards, and general periodic review of results is a more effective method of operation.

In the departmental personnel offices, for example, there has been a tendency to think in terms of centralized operations and paper work, although there is growing appreciation of the fact that most operations must be decentralized and that there are better ways of securing a high level of performance than by routine review every step of the way. In the legal offices the issue has been posed in a different form. In some departments, the auxiliary legal aids to the bureaus have been centralized to one unit for the whole department, instead of being kept at the operating level.

Need for Greater Decentralization to the Field

Corollary to the need of organizing departments more effectively has been the need for greater decentralization of responsibility to the field. It is out of the question to attempt to carry on nation-wide programs when every decision has to be referred to Washington, and in general, the last ten years has seen progress in this respect. There is a trend toward greater devolution of operations to regional offices.

Changes in Civil Service Commission procedures are an example. Traditionally a highly centralized agency, the Commission has recently decentralized operations along two lines. Commission employees attached to departments and agencies have been given increased authority to approve position classifications, appointments, and promotions without prior clearance with the Commission. District offices of the Commission have also been given greater authority in such matters as recruiting.

Some decentralization likewise is taking place as a result of wartime efforts to co-ordinate field operations. The regional directors of the Social Security Board, for instance, have been designated co-ordinators for wartime programs in health, welfare, and related fields. One consequence of this effort to correlate operations in the field has been that field officers have sought greater freedom of action and decision.

Improvement of the Government Structure

Reorganization of the basic structure of the executive branch of the federal government has been urged periodically in many quarters ever since the Taft Commission on Economy and Efficiency made the first comprehensive report on the subject in 1912. But prior to the past decade, the only action came as a result of delegation of reorganization authority to President Wilson in 1918 as a war measure. In the present decade there have been three delegations of power to reorganize—in 1932, 1939, and 1941.

President Hoover issued the first orders under the 1932 authority, but they were disapproved by Congress shortly before the end of his term. Subsequently, President Roosevelt in 1933 and 1934 issued several orders which went into effect. The Procurement Division of the Treasury Department was established to facilitate standardized and economical government purchasing and warehousing; the various agricultural credit agencies were brought together in the Farm Credit Administration; the Shipping Board was abolished; the disbursement function was centralized in the Treasury Department; and the power to apportion appropriations was transferred from the departments to the Bureau of the Budget.

Under the powers granted in 1939 and 1941, more spectacular results have been achieved, partly because in the meantime the government structure had expanded greatly and the need for structural reorganization was much more obvious. Leaving out of account the growth of temporary agencies to carry out the war program, the more recent reorganizations have simplified the general outlines of the permanent structure of the government and reduced drastically the number of agencies theoretically reporting to the President but in fact getting limited supervision from him. In 1937, a classification of agencies in the Executive branch showed in addition to the ten departments, some fifty major agencies and numerous minor organizations. Of these fifty agencies, twenty-eight have now been abolished, placed in one of the established departments, gathered into one of the three new agencies which are virtually departments, or placed in the Executive Office of the President.

Of particular interest in these reorganization moves is the creation of the Federal Security Agency, the Federal Works Agency, and the National Housing Agency. A Federal Loan Agency was similarly organized but has since been liquidated by transfer of its constituent agencies to the Department of Commerce. Although the President may not create departments, through his reorganization authorities he has been able to bring associated activities together under one common head, thereby centralizing administrative responsibility for related functions at a point short of the White House. The heads of these agencies, through the power to hire and fire, have virtually as much power to achieve co-ordination as the head of any department, and in effect, have been given cabinet rank since they attend cabinet meetings. The main obstacle to securing administrative responsibility in these agencies is the fact that numerous subordinate posts are subject to appointment by the President and confirmation by the Senate. This defect could not be corrected under the reorganization authority.

New Technique for Reorganizing

The main problem in effecting administrative reorganization has been to determine how the Congress could give the Chief Executive authority to take action without, on the one hand, giving up the power of the legislature to reject proposals or, on the other hand, putting such proposals in jeopardy because of minority opposition. Plans for improving administration almost always attract vigorous opposition from special interests affected —employees or pressure groups—and in the absence of a procedure to protect the majority, run the risk of defeat if the matter is not of sufficient interest to attract general congressional attention.

The technique embodied in the 1939 Reorganization Act solved the dilemma by providing that Presidential plans should go into effect unless a majority of the Congress was opposed. The President was in effect made an agent of the legislature for the purpose of preparing reorganization plans. Such plans had to lie before the Congress for sixty days while it was in session. At the end of that time, in the absence of a concurrent resolution of disapproval accepted by both Houses, the plans went into effect. Such resolutions were not subject to Presidential veto, although in fact none was passed during the time the act was in operation.

Although this two-year experiment was a success, there has not yet been a willingness in Congress to give the President such reorganization authority on any continuing basis. Perfection of organization in government as in private business is a continuous process, although periodically a large-scale revamping is needed to catch up with a new era or with basic changes in program. To deny the Chief Executive, or the head of an agency within the executive branch, considerable latitude in readjusting organizational arrangements is to deny him an essential tool of management.

Plus and Minus Side of Wartime Management

The past decade has seen unprecedented progress in federal management. The magnitude of the management problem brought forth by the war is also unprecedented. With acute shortages of vital raw materials, with the necessity of producing not only for our own armed forces but also for our allies, of relating production to strategy decisions that affect every corner of the globe, and of working out effective administrative relationships with other nations, there is no leeway for incompetent personnel, inadequate planning, or poor management. All operations must be thought out, arranged, and co-ordinated with a precision that is rarely achieved.

In general, there has been recognition of the necessity for such precision, but the administrative means for achieving it are still distressingly inadequate. Effective production planning is in its very earliest stages, the relationships between the agencies that govern production and those concerned with strategy are ineffective, and there still remain difficult problems of co-ordination of such interrelated matters as military requirements, production possibilities, manpower needs, shipping, and foreign policy. Some of these weaknesses have their origin in the fact that there has been no comprehensive scheme for bringing the best talent of the country into the government service. They also grow out of the great difficulty of finding and identifying administrators adequate for these vast management tasks even when concerted efforts are made to find them.

Given these serious shortcomings, what is the positive side of the war organization picture? For one thing the availability of the Executive Office of the President to assist in the establishment of new programs and in ironing out jurisdictional disputes has been an important asset. In this connection, the device of the Office for Emergency Management within the Executive Office has been especially helpful. The President cannot organize new operating agencies, but he can organize his own office and assign duties over which he has discretion to units within it. In this fashion and through use of emergency funds made available by Congress, it has been possible to put the war program into operation quickly and to make adjustments when needed.

To a greater degree than is generally realized the regular government agencies have been harnessed to the war effort. This has been particularly true of the agencies whose functions have a clear-cut wartime applicability, such as the Department of Agriculture, the Civil Aeronautics Authority, or the Treasury Department. There have also been notable efforts to co-ordinate government activities, as in the organization of the National Housing Agency, the Office of War Information, the War Manpower Commission, and the Board of Economic Warfare. The error made in the last war of using boards for important administrative purposes has been avoided; in the case of current boards the chairman is given full authority to act.

Political patronage considerations have played almost no part

in the selection of key personnel. Furthermore, the introduction into government of many capable persons from the business and industrial worlds is not only of importance now but is bound to leave a permanent impress. Some of these newcomers undoubtedly will remain in government service as a permanent contribution to the improvement of the quality of federal personnel. Those who return to private jobs will have left behind them some of the management ideas and practices of business and will take back a better understanding of the problems of government.

Another hopeful aspect of the present governmental scene is the fact that in some agencies, management is improving at an accelerated rate. When progress in management in the federal government in the past decade is considered in the abstract, the results seem spectacular. Considered in the light of what is needed—better administrators and better use of personnel throughout the government, less selfishness and personal aggrandizement by officials, more effective planning, further development of management facilities, improved supervisory practices, greater decentralization, improved procedures, etc.—the picture is discouraging. However, under the pressure of wartime necessity, new gains are being made that should become permanent.

Public Management and the Future

Looking toward the future and what it will require of management, two problem areas stand out. One is the aftermath of converting the productive machinery of the country to meet war requirements. The mobilization of the country's resources, with the attendant necessity of substituting government directives for ordinary economic controls, is one of the most complicated management jobs ever undertaken in this country, but the task of releasing war-time controls promises, if possible, to be even more complex. Our ability to solve this problem and to reorient the nation's economy to meet postwar needs, will depend in great measure on the imagination and resourcefulness of our administrators.

The other great management challenge lies in the field of international administration. Various combined boards representing Great Britain and the United States have been established for co-ordinating United Nations operations. These must be implemented with effective administrative machinery not only for prosecuting the war, but also to lay the foundations for the future. Without the means for carrying on co-operative administration at the close of hostilities, it will be impossible to build the kind of world order envisioned by the Atlantic Charter.

Increase in Adaptability of Workers to Job Requirements

By EDGAR W. LANCASTER,[45] WASHINGTON, D. C.

During the last war there occurred a large influx of women into industry, which required the extensive development of training methods and dilution, or job-breakdown, techniques. This same influx of women into industry has occurred in the present war but on a much greater scale.

The introduction of women in the current war, however, does not require so extensive a readjustment of training methods and processes, nor the development of as many new mechanical techniques as was necessary in 1914–1918, because industrial mechanization and subdivision of work have developed constantly since that time, particularly so from the depression years, the 1930's, to the present. It is also true that in the past few years more and more women have continued to be employed on processes suited to them, due primarily to the development of research departments which studied and adjusted such situations, and also due to the recent development of mechanical aids for women in industry. These developments created a situation for women in industry in September, 1939, greatly different from that of 1914.[46]

Increase in Mechanized Equipment

There has been, in the past ten years, a tremendous increase in mechanized equipment in the factories, on the farms, and in other fields throughout Great Britain and America. Basically, it stands to reason that the women of America and England through coming in contact and by working with these thousands of mechanical devices, which are becoming a part of their home activities and outside life, have developed knowledges and skills which have given them greater adaptability for acquiring mechanical skills in war industries today.[47]

An article published by the British Press Service states, "Explaining factory work in terms of simple household apparatus is proving a valuable method in introducing inexperienced women to wartime factory work in England."

The article goes further to explain that the egg beater, the can opener, the washing machine, and all the other highly developed present-day electrical and hand-operated mechanical devices are among the well-known appliances that are compared to workshop machines in employing women in munitions plants. These women are taught that in manipulating one drill which bores through hard metal they practice the same technique as when using a rotary egg beater and in working with another drill that bores out the middle of shells, they use the same movement employed in still another household appliance.

Women are shown the analogy between present kitchen scales and the apparatus that tests the diameter of primers for shells to see if they have been made the exact size for the gun from which they will be fired. They see that the finished primer is merely placed on something resembling a balance and that their job is to watch an indicator that registers if the size is not true.[48]

Need for Millions of Women

In the United States there are 13,200,000 working women, with 3,000,000 in various kinds of manufacturing and mechanical industries. It is estimated that if war production goals are to be reached and maintained, there must be an increase in the number of women in war industry from the 1,000,000 in May, 1942,[49] and approximately 3,000,000 at the end of 1942, to 6,000,000 by the end of 1943. Some of these women will be recruited from among young girls finishing school and college and housewives not now in the labor market.[50]

In 1930 the working women of the United States totaled nearly 11,000,000; the number at work in manufacturing and mechanical industries was 1,386,307.[51]

[45] Research Specialist, Civilian Personnel Division, War Department. Paper prepared in collaboration with Lawrence A. Appley, consultant for War Department Director of Training and Personnel, and vice-president, Vick Chemical Company, New York.

[46] "Women in War Industries in Great Britain," U. S. Department of Labor, Women's Bureau, October, 1941. See also "American Labor in the World War Period 1914 to April, 1917," *Monthly Labor Review*, October, 1939. Industry is spending 17 times as much for research annually as in the first year of World War I.

[47] "Women in Industry," *Independent Woman*, May, 1937.

[48] "British Housewives Introduced to Machinery Via Egg Beaters and Mangles," *British Press Service*, June, 1942.

[49] "Woman's Place," *Business Week*, May 16, 1942. "Today out of an estimated total of 8,500,000 war workers, 1,000,000 are women. By the end of 1943 there will be 20,000,000 or more war workers, and 4,000,000, or $^1/_5$, will be women."

[50] "Arms and the Woman," *Survey Graphic*, May, 1942. See also "Women in War Work," *New Republic*, May, 1942. "Women at Work," *Newsweek*, January, 1942. "Women in War Work," *New York Times*, March 15, 1942.

[51] "Occupational Progress of Women 1910–1930," Bulletin of the Women's Bureau, U. S. Department of Labor, No. 104.

In July, 1939, there were over 4,000,000 women workers in Britain. Since then even approximate totals have not been given out. During the past winter, the Minister of Labor, acting on the principle that "nothing that a woman can do or can learn to do should be allowed to absorb a man of military age," has ordered the compulsory registration of all women between the ages of 16 and 41. In some munitions plants they make up 80 to 90 per cent of the total labor force. Some aircraft plants are 50 per cent manned by women with the expectation that the proportion will rise to 80 per cent in the months ahead.[52] In small-arms munitions plants in Wales, 80 per cent of the workers are women, as against 40 per cent in the United States.

At an early date in World War II it became clear that England's war as well as our own would be fought primarily in the factories. Consider for the moment the following facts: In Caesar's time the value of the life of one soldier who died defending his country was estimated at 75 cents; in the Thirty Years' War, the estimate rose to $50; and in the American Civil War to $5,000. The First World War raised that figure to $25,000, and now roughly the estimated cost of the Second World War in terms of individual military fatalities is already over $125,000. Of greater significance is the fact that most of this money is spent not on the battlefield, but in the machine shop; not in the trench, but on the assembly line.[53]

Jobs for Women

From 1932 until the outbreak of the present war there were obstacles to the employment of women in industry. Women thus did not have too great an opportunity for developing versatility in specialized mechanical work. Economic changes in the way of living did, however, tend to bring women into factories—changes such as: the increased mobility of workers, their migration in order to obtain employment, the need for housewives to assist in supporting the family, the increase in apartments as away-from-home dwellings, and the government work program.[54]

In Germany and in Great Britain present reports indicate that 40 to 50 per cent of the employees are women, and certain trends indicate that women will get a chance to show their skills in America.

In April, 1941, an airplane assembly plant hired 16 girls for the covering, paint, and electrical-assembly departments. The experiment was so successful that by the end of the year the company was employing 500 women, utilizing their services on nearly all production processes, even in the machine shop and on the final assembly.[55]

Plants making ammunition for artillery and small arms already employ large numbers of women. Of 28 operations in the small-arms ammunitions plants, women were found to be the only workers on seven; 12 others, it was considered, could be turned over to women entirely or, at least, the number of women working on them could be greatly increased. The remaining nine operations were adjudged unsuitable for women, chiefly because they required physical strength.

In the plants making heavier artillery ammunition, of the 46 operations observed, six were entirely in the hands of women; on 29 the employment of women could be greatly increased; 11 were considered unsuitable. In the mechanical time-fuse department of a government arsenal women held only 2 per cent of the jobs three years ago; now 96 per cent of the workers are women, and more fuses are being produced per employee than ever before. There has been a complete change in the breakdown of operations.[56]

In one General Electric plant the ratio was 60 to 40 in March, 1942, in favor of the men. "But by the end of the year," said the general manager, "it will be reversed, 60 per cent women to 40 per cent men." The Ford Motor Company has pushed the civilian estimate of women workers it would need in 1942 until the figure stood at 20,000. The Sperry Gyroscope Company, in one of its new plants, will want 6,000 to 8,000 women employees. A machine company where only 20 per cent of the working force were women in the fall of 1941, had increased the proportion to 80 per cent by February, 1942. These are random indications of a nation-wide trend.[57]

In an effort to indicate how many and how varied are the new openings suitable for women, the United States Employment Service has been analyzing all jobs occurring in war industries. At present 623 essential occupations have been studied. Women are employed in only 27 of these. A study of the duties performed by workers in the remaining occupations indicates that 251 are wholly suitable for women, with another group calling only for some rearrangements of equipment and process. A study by the United States Women's Bureau showed that women were able to step immediately into about 600 out of 1900 major war occupations, and could be quickly trained for another 600 such occupations.

Percentage of Women Workers Growing Fast

The U. S. Department of Labor in a survey of munitions plants found that more than 30 per cent of the workers in small arms manufacture were women;[58] in shell and bomb loading plants the figures ran from 33 to 42 per cent; in bag loading plants the figures ran from 30 to 48 per cent. In aircraft manufacture fewer than 2 per cent of the total labor force were women, though in West Coast plants the proportion of women on the assembly lines ran much higher.[59]

The following are reports of the Women's Bureau from their visits to other aircraft plants in more recent months:

One plant having none but male employees expected to hire 6000 women within a few months after the survey.

An aircraft assembly plant was making plans to take on more women; at the time it had 27 women on the production line but this number was expected to increase very soon to 2,000. Women were to be tried first in the electrical department and on small precision assembly.

In another aircraft assembly plant women are being seriously considered for light woodwork operations, electrical assemblies, taping and doping, and tube assemblies. (There is one woman engineer in this plant, but only nine on the production line.)

Kinds of Jobs at Which Women Can Work

Now what are the prospects that women will be readily absorbed into the rapidly expanding production program?[60] New

[52] "Arms and the Woman," *Survey Graphic*, May, 1942.

[53] Leo M. Cherne, "Your Business Goes to War," p. 5.

[54] "Withdrawal of Young Women From Certain Pursuits," *Monthly Labor Review*, February, 1942. See also, "Women in Federal Defense Activities," *Monthly Labor Review*, March, 1942. "Arms and the Woman" (replacing millions of men in war production), *Survey Graphic*, May, 1942; "25,000 Women Will Be Employed by Armed Forces in Radio," *Science*, March, 1942.

[55] "Women's Jobs in War Production" (training workers and supervisors for war production), American Management Association Personnel Series no. 56, February, 1942. See also "Women in Wartime," *Monthly Labor Review*, May, 1940.

[56] *Ibid.*

[57] "Arms and the Woman," *Survey Graphic*, May, 1942.

[58] See "She Works in an Arms Plant," *New York Times*, M, April 12, 1942.

[59] "Arms and the Woman," *Survey Graphic*, May, 1942.

[60] See "Employment of Women in U. S. Defense Industries," *Monthly Labor Review*, January, 1941. See also, "Woman's Place" (Surveys show that they can not only do the job but they can do it better), *Business Week*, May, 1942. "Women in War Work," *New Republic*, May, 1942.

jobs have been created through the breakdown of "big" processes into simpler and less-skilled operations. Some jobs can be done equally well by men and women—for example:

Milling-machine work
Light-punch and forming-press work
Work on bench and watchmakers' lathes
Burring, polishing, lapping, buffing (on lathes)
Packing, labeling.

Women can do some jobs even better than men. They are more painstaking; they mind tedious work less; they have greater finger dexterity. Women adapt themselves readily to repetitive jobs requiring constant alertness, if not skill, strong fingers, and tireless wrists, and they can keep at these jobs without flagging. Examples are:

Drill-press work
Assembly—all types
Winding coils and armatures
Soldering
Taping
Painting—all kinds (spray, stencil, radium, touch-up)
Visual inspection.

But women can also do certain notably skilled work after training. They have the ability to work to precise tolerances; they can detect variations of a ten-thousandth of an inch. And they can make careful adjustments at high speed with great accuracy. Such skills include:

Welding
Sheet-metal forming and riveting
Work on light turret lathes
Work on light-duty hand and automatic screw machines
Work on setting up machines
Production, planning, routing
Tracing, drafting.[61]

Changes Affecting Employment of Women

There are two developments in the change of attitude toward the women workers which have simplified the infiltration of women into the skilled jobs. The first is the improvement in the tools themselves, which has made the work easier.[62] The second is that the very exclusive unions, which never before accepted women as members, have undertaken to negotiate for the women workers and in some cases have even supported equal pay for equal work. This step is in contrast to the situation during the last war when large numbers of men opposed the entrance of women into industry.

An important point is that, in contrast to what happened in the last war, we have vastly improved working conditions in the war factories. Clinics are conducted, and trained nurses are employed; we have tried to prevent eye strain by providing proper lighting; rest rooms have been installed in all the vital factories and rest pauses instituted. In addition, the tea wagon is sent around, and extra rations are provided in the middle of each shift.

In manufactures that were first to undertake defense production an average of about 275,000 women wage earners were employed in 1939, distributed in the following broad industrial groups:

Electrical machinery	87,000
Iron and steel and their products	60,000
Chemicals and allied products	42,000
Nonferrous metals and their products	31,000
Machinery (other than electrical)	27,000
Automobiles and their equipment	26,000
Other transportation equipment	1,570

The total number of women in these groups had declined by about 20,000 since 1932, the number of men by about 313,000. Exact comparisons usually cannot be made in specific groups because of many shifts in classification. However, in one—automobiles and their equipment—women had increased by about 4000 while employment of men declined by about 52,500.

Most of these industries had to be converted from production for consumers to production for defense. A few already were producing war materials, and in most of these women's employment had declined, though in aircraft making it had increased by 45 per cent. Chief among those producing war materials were the following:

Industry	Women, average number	Per cent of total
Aircraft and parts	535	1.1
Aluminum products (ingots, castings, plates, sheets, etc.)	862	5.8
Ammunition	1360	31.9
Explosives	1224	16.9
Firearms	240	4.8

Response of Women to Training

Nineteen-year old Evelyn Duncan went to a Birmingham factory and asked for war work. She said she wanted to operate a capstan lathe. The production manager looked at her, saw a slight, frail girl and told her that she wasn't strong enough. He would give her some other job. But she was so disappointed that he relented. All right, she could have a lathe. So she was given training . . . That was two years ago. On Saturday night the frail girl stopped her lathe after a six-day week's work. She had just set up a world's record by turning out in the week 6130 A. A. shell components, 1130 more than the previous best.[63]

The training programs for women in Great Britain are considerably in advance of our own training programs for women here in America. Originally all training in technical processes was done on the job itself. As specialization grew, however, specialized technical training became necessary, and training centers, specially equipped, grew up outside the workshops. Thousands of women owe their ready absorption into British war industry to the intensive training they received from the skilled instructors in Government training centers.

It had been the policy of the Ministry of Labor and National Service to encourage trainees to develop to the full their capacities as working men and women. To this end a course of training extending over a period of four months or even longer, in the postwar years, was the normal practice. Today a more rigorous standard of selection is being undertaken and initial courses of from four to eight weeks are given to all entrants to training and only those showing particular aptitude for engineering work are retained for the long course. The department aims to avoid overtraining in those instances where a short course of a few weeks is sufficient.[64]

Industry of Great Britain is likewise reducing the time of its training programs in order to place workers on the job as quickly as possible. According to one plant manager, once the fear of the machine is overcome women learn quickly due to their usage of mechanical equipment in past years. They are keenly conscious of the value of the metal they machine and make few errors. Of four hundred to five hundred women trained in a comparatively short time, only six failed to make good on production.[65]

In some plants, women were working satisfactorily on machine and assembly operations for which, in certain other plants,

[61] "Women's Jobs in War Production" (Training Workers and Supervisors for War Production), American Management Association Personnel Series no. 56, February, 1942.

[62] See "Increased Efficiency and High Performance Standards Definitely Linked to Fitting Workman to Job," *Coal Age*, November, 1941.

[63] "New Chronicle," Engineering Bulletin No. 6, November, 1941.

[64] Engineering Bulletin No. 10, March, 1942.

[65] "The Employment of Women in Canadian Gun & Rifle Factories." January, 1942.

women have not even been considered. Though most of the plants have not offered any special training to women for job progression and upgrading, have not tried to give them training or responsibility for set up of machines, and have limited their work to unskilled repetitive tasks, the generalization often is made that women have no mechanical ability, interest, or aptitude. In a few plants where they are allowed to do so, some women set up their machines, a responsibility of many women in British factories.

Training Develops Increased Adaptability

In England there is abundant evidence that the contribution to the war effort made by Government Training Centers has been immense. Innumerable unsolicited testimonials bear witness to the work of these Centers. Here is a sample:

No doubt you will be interested, and gratified, to learn that two of the women who passed through your Training Center as trainee toolmakers have now, after a period of only a few months, proved extremely capable and efficient at press toolmaking. We have left them to make blanking tools throughout, and work which they have recently done reflects great credit on them and the initial training. It is very pleasant for us to record these details, as it shows that in our particular instance the dilution of skilled labor in toolmaking is being carried out with great success.[66]

Those who doubt the possibilities of women trained in a couple of weeks to run the machine to which they have been allocated should be interested in the experience of a machine-tool works at Halifax. Here women are engaged in bench and machine work, and all on lathe parts. Under supervision, they operate—Herbert & Ward capstan and turret lathes; Small surfacing and boring lathes; Sunderland gear planers; Fellows gear shaper; Churchill internal grinder; Churchill external grinder; Archdale miller; Archdale sensitive drill; Hey tooth rounder; Defires keyseater; Pratt & Whitney duplex spline miller.[67]

Two thirds of the workers on rifle production are women. The first group were trained in the plant. Today girls go to the technical high schools for a three weeks' course under the Dominion-Provincial War Emergency Training Program. Two weeks are given over to learning the principal machine tools and one week to bench work. Thereafter, the women are assigned production jobs under section men and foremen. They begin with less exact machine operations and are advanced to more difficult work with experience.[68]

Women comprise a much larger proportion of the munition makers abroad than in the United States. In the *London Times* of May 11, 1941, a representative describing a visit to the new Royal Ordnance Plant in Wales reported that 80 per cent of the workers were women. None had more than 7 months' experience, and the majority had less. A supervisor expressed the opinion that a typically intelligent girl trained in the factory was as efficient at her work after 6 months as the average boy who had worked longer.[69]

So far as capstan operating is concerned, experience in the North Western Division is particularly enlightening. Many firms are concerned, and in relatively few of them are women restricted to work for .001-in. limits. Hobson's limits for women are .0002-in. while Rolls Royce have women working to .0001-in. limits.[70]

At Newton-le-Willows women are trained and are now operating ordinary 8-in. lathes, that have been speeded up for brass work. They turn out the parts to fine limits, bored, turned, and screw cut. This is not ordinary repetition work either, for the jobs are commonly put through in small numbers—say only a dozen at a time. The charge-hand shows the woman how to produce the first one, and she then finishes off the remainder, exactly as a male operator would. Working with calipers and micrometers, the women have become "very efficient." One of them is getting a man's rate, and earning up to 75 per cent bonus weekly.[71]

Six-weeks' basic training at a technical college has enabled women trainees to operate 16-ft-long gun-barrel boring lathes at a Royal Ordnance Factory.[72]

After three months' experience of general machine-shop work in a Scottish optical factory, women work to drawings and set up their own jobs, even for such work as cutting double square threads.[73]

After a two months' course at an A.I.D. Training school, ex-trainees from one Center took the first five places in the recent final examination.[74]

A woman trainee, subsequently placed as an assistant examiner with the A.I.D., came first of a class of nine, (seven men and two women) in a recent A.I.D. written examination.[75]

A woman trainee at a Government Training Center in the south of England was recently placed with the Mechanical Engineering Department of one of H. M. Dockyards. At the Center the woman had turned out work as good as that produced by any male trainee in the Center's history. After a little time in her job at the Dockyard she was put in charge of all the women employed in the Mechanical Engineering Department.[76]

Machining tank bogie wheels (steel) which is usually considered a highly skilled man's job, is being done after only 7 weeks' training by a woman at a factory in the North East Region. She operates a War No. 9 machine, and the wheels (which are machined all over) have a 13-in-diam flange. They are bored to standard with an allowance of .003 in. for finishing on mandrel. The girl, formerly a textile operative, works to a micrometer for sizes. The management is proud of her.[77]

A government trainee engaged by one firm as an instrument maker proved particularly efficient on lathe work. The firm which had taken her on therefore employed her as a tool turner. She showed such aptitude that she did this work as well as men who had been for some years with the firm.[78]

Who Is Being Trained?

The following division into age-groups of the women employees, and the approximate percentage of each group in relation to the total number of women employed at one of the larger factories in Great Britain is of special interest. The 30–40 and the 40–50 age groups are remarkable for their size; and the quite considerable proportion of "over 50's" is striking testimony to the way in which the older women of the neighborhood have rallied to the firm:

Age group	Percentage
18–20	15.0
20–30	36.0
30–40	24.1
40–50	18.1
50–60	6.0
60–70	0.8

Before the war women were found mainly in the press shop and on inspection work; and a visit to the press shop today is particularly interesting. The press-shop foreman was very enthusiastic over the work now being done by the women. For his work he definitely prefers a woman over 30 years old; and while he considers the best age is between 30 and 40, there is no age limit, and his staff includes one woman of 64 years of age (she operates a press) and many others well over 40.

As a matter of special interest, the press shop claims a record of five grandmothers on its staff! In this and other departments women are used on all the usual run of machines, such as capstan lathes, facing lathes, drilling machines, thread-milling machines, centerless grinders, automatics, tapping machines, and power presses. They also work on welding, assembly, and inspection.[82]

Fitting and assembly are other spheres where women seem particularly suitable after a short training period. Complete wing assembly, as well as cabin details, bomb sights, wireless, and landing gear are among the aeronautical examples, while the complete assembly of 2000 typewriter parts affords an instance from an entirely different sphere. Press operating also is com-

[66] Engineering Bulletin No. 4, September, 1941. (Manchester Metal Works, Ltd.)

[67] Engineering Bulletin No. 1, June, 1941.

[68] "The Employment of Women in Canadian Gun & Rifle Factories," January, 1942.

[69] Bulletin of the Women's Bureau, No. 189-2.

[70] Engineering Bulletin No. 1, June, 1941.

[71] Engineering Bulletin No. 1, June, 1941.

[72] Engineering Bulletin No. 1, June, 1941.

[73] Engineering Bulletin No. 1, June, 1941.

[74] Ibid., No. 6, November, 1941.

[75] Ibid.

[76] Ibid.

[77] Engineering Bulletin No. 10, March, 1942.

[78] Engineering Bulletin No. 7, December, 1941.

[79] Engineering Bulletin No. 11, April, 1942.

monly done with success by women working under widely varying conditions.

Mechanical Aids for Women Operatives

During the war of 1914–1918 women undertook a good deal of heavy work formerly performed by men, and this led to a government-sponsored inquiry into the whole question of weight lifting. A pamphlet describing the study states: "From a consideration of the physique of women in the heavier industries it would seem as if women workers are naturally attracted by and retain work for which they are physically fitted. The weight-carrying capacity of these selected—perhaps 'self-selected'—women is remarkable. Women seem to know their capacity to a nicety, and in spite of the undoubtedly heavy work they undertake in certain industries they rarely figure in accidents due to weight lifting and carrying."

Fortunately today's problem has been simplified by the introduction of numerous mechanical aids. Most firms have installed cranes and hoists, telphers and conveyers, and other such devices, which have made possible the large-scale employment and training of women who, with the aid of these devices are today handling 180-pound shells with ease.[80]

The increase in the adaptability of women to the skilled trades has been amazing, even in the heavier war industries. When all the causes are summed up, the following results are revealed:

1 Today's war does not require the extensive readjustment of training methods and processes that was so necessary in the first World War.

2 Women excel in work requiring care and constant alertness, use of light instruments, such as gages, micrometers, vernier calipers, and can be trained for this work in an amazingly short time.

3 Women also excel at work requiring manipulative dexterity and speed, which work skill has been brought about by the use of the many high-speed mechanical devices of today.

4 Women can operate with skill large machines on heavy work because such work has been made easy by the use of improved lifting and handling devices.

5 Industrial gates have been opened to women employees during the past few years, and thousands of women are equipped, for the first time in history, with the mechanical knowledges and skills necessary to produce needed materials for today's war.

Management Research

By EDWARD H. HEMPEL,[81] NEW YORK, N. Y.

"As a function of industrial operation the art of management is now firmly established because of the general acceptance of the fundamentals. The promise of the future is to lead to a higher professional standing through the development of intellectual, societal, and moral attitudes and values."

L. P. Alford, 1932.

The Scope of This Report

In mechanical, chemical, or any other branch of engineering it is relatively easy to discern and segregate research from actual work performance or established practice. In management matters this is more difficult since often enough research is made merely a process of thinking up a new method of doing or organizing, which then is immediately applied without much experimental work or scientific description. The good old habit of doing, describing, and publishing research work on management subjects has become much less frequent than it was during previous decades. It seems that the last ten years have been perhaps too hectic and disturbed to induce the composition of an extensive "research" literature on management, and that the progress which has been made expressed itself more in actions and practical work than in treatises on new ideas.

Many who actually do management work do not identify part of it as "management research," although it might fall under this heading. Even definitely new and novel ideas, policies, methods, and applications of management are still merely considered as part of the daily job, as improvements, or routine. They are not especially rated or reported by their authors as scientific achievements, even if they would merit it, and are evolved by quite the same methods of recording, fact finding, analyzing, modifying, and perfecting, exactly as novel ideas are developed in technical research.

Since there is no patent literature on management subjects, and the various contributions to management research are scattered widely and not always discernible or marked as new ideas, this report has been prepared for the very purpose of bringing out the new thoughts which have been added to the science of management. Efforts have been made to consider actually applied but not reported developments as well as those which have been reported in the literature.

In view of the complexity of actual developments, and considering the great number of books and articles published on management subjects during the last ten years, the tracing of the progress in management research has been a sizable task, and, in spite of all efforts, the findings may not be complete. But it is hoped that the following will bring into focus at least the most pertinent developments. Most difficult has it been to draw the line where "economics" ends and "management" begins, or vice versa, since the two fields have become much more closely interwoven than ever before.

Government Research

The government, having at its disposal not only millions but billions of dollars year after year, undoubtedly has taken over the lead in economic and management research. As it extended its influences, and later on its activities, from welfare and unemployment-relief programs into labor legislation, then into centrally directed defense production, and now into outright control of most war and many civilian activities, it extended its research facilities far beyond any precedent. Vested with legal powers of inquiry and investigation it could obtain data and information as no other researcher or research institute ever had at his disposal.

The Departments of Labor, Interior, Commerce, Justice, Treasury, Agriculture, alone or together with special Committees (Temporary National Economic Committee, for instance), and quite a few new Administrations (N.R.A., W.P.B., O.P.A., R.F.C., Economic Defense Board, etc.), all have researched extensively in subjects pertaining to the field of management, and to some extent they even have developed management and administrative policies and techniques of their own which should not be overlooked. The War Department, for instance, employing close to one million people in technical or administrative work certainly became the biggest single employer and producer practicing administrative as well as technological management.

A great many of the more recent government activities in the field of management have not been described as yet, and of the research studies made during the depression and recovery years many are predominantly legal or economic in character. But those which clearly deal with management subjects are listed in this section under the proper subject titles.

[80] Engineering Bulletin No. 7, December, 1941.

[81] Assistant Professor of Industrial Engineering, Columbia University. Mem. A.S.M.E.

Industry and Trade Association Research

While even in "normal" times quite a few management problems are being investigated by the associations for the benefit of their members, the subjects added during the last 10-year period were unusually numerous. The main efforts were directed on the economic and legal aspects of management and operation. The occasions to make such studies were caused either by extraordinary economic developments or by new government policies or administrative regulations.

A fairly good survey of what were the main industrial problems and how American Industry proposed to solve them, can be obtained from a study of the "Annual Declaration of Principles" proposed at the Annual Congresses of the National Association of Manufacturers, which are well complemented by the speeches and papers presented at the meetings. They also show how the traditional concepts and practices of industry were gradually assimilated to the new thoughts.

Besides, nearly every one of the many trade associations covered those specific difficulties and problems which were of particular interest to their industries. The Food, Drug, and Cosmetics Act, the N.R.A., O.P.A., Wagner Act, and quite a few other regulations were considered by nearly every association, which has caused a considerable literature of its own. As interesting as some of these presentations are, no effort has been made to incorporate them into this report.

Scientific and Professional Association Research

The research efforts of most of the scientific and professional institutions have been extended considerably into the economic aspects of management and in general more thorough work and thought has been applied in the various subjects covered than in past decades.

Outstanding in this respect are the publications of the American Academy of Political and Social Science, which has sponsored the investigation of the most important problems of the period and has published the findings in book form. Arranged in chronological sequence they show the following list:

1931 Insecurity of Industry.
The Coming of the Industry to the South.
1933 Essentials for Prosperity.
Social Insurance.
1934 Social Welfare in the National Recovery Program.
Banking and Transportation Problems.
The Ultimate Consumer; A Study in Economic Illiteracy.
1935 Pressure Groups and Propaganda.
Increasing Government Control in Economic Life.
Economics of Planning. Principles and Practice.
Education for Social Control.
1936 Railroads and the Government.
Government Finance in the Modern Economy.
Progress of Organized Labor.
1937 Revival of Depressed Industries.
Consumers' Co-operation.
1938 Present International Tensions.
1939 Government Expansion in the Economic Sphere.
Ownership and Regulation of Public Utilities.
Refugees.
1940 When War Ends.
Marketing in Our American Economy.
1941 Billions for Defense.

The nearest to it in systematic completeness are the publications of the Chamber of Commerce of the United States whose pamphlets and bulletins support and extend the list:

1931 Distribution in the United States, Trends in Organization and Methods.
1932 Banking Legislation.
1933 Federal Expenditures (annually).
Federal Bankruptcy Legislation, Municipal Insolvencies and Corporate Organization.
Working Periods in Industry.
Discriminatory Legislation Affecting Retailers.
1934 New Opportunities for City Planning.
Local Code Problems.
Standardization of Consumers' Goods.
Federal Budget and Recovery.
1935 Government's Relation to the Power Industry.
Quality Standards and Grade Labeling.
Federal Taxation (3 Parts).
1936 Report of Committee on Employment.
Co-operative Business Enterprises Operated by Consumers.
1937 Surtax on Undistributed Corporate Earnings.
Farm Income in the United States.
Restrictions on Price-Making Methods.
1938 Special Sales Events.
1939 Distribution Services and Costs.

This is by no means a complete list of publications, and only titles related to management have been given. If tax and financial topics would be considered in this field, the list would be appreciably longer. The publications of the National Industrial Conference Board and similar institutions are given under the specific fields of management to which they refer.

The contributions to management research published by the professional societies are more restricted to the traditional management subjects, as can be seen from the articles contained in the bulletins of the Taylor Society, the journal of the Society for the Advancement of Management, and in the publications of The American Society of Mechanical Engineers. While there are hundreds of articles they do not reveal any predominant trends. Attention is fairly equally distributed among all phases of enterprise management. Quite a few of the presentations are merely reiterative, old principles are stated differently or applied in a new way; but there is sufficient material which shows the spark of real innovation and progress. To segregate these articles into those which might be called real advancements or progress in management, and into reiterations, has been the most time-consuming part of this study. It might be well for the editors of management publications to either honor the "progress" articles by a star, or by printing them in a special "Progress in Management" Section, of their respective magazines.

Scholastic and Academic Research

Professors and educational institutions established a greater number of courses on Management, and especially time-motion- and method-study courses have come into greater demand than ever. It seems that also Industrial Engineering, which is really a combination of engineering and production management, is coming more into its own since many companies have begun to attempt real savings in the costs of their production.

The many textbooks on scientific industrial organization which were brought out during the period were definitely more voluminous than in past decades, but not all of them represent progress. Some are definitely more academic and lifeless than they ought to be and they fail to inspire a student to think up new ideas beyond those which he needs to solve the "problems" offered in the book.

More recently the tendency to bring out more specialized texts for the various fields of management has been a useful move, e.g.:

Maynard, H. B.: Operation Analysis, 1939.
Rautenstrauch, W.: The Design of Manufacturing Enterprises, 1941.

More of such texts on specific management subjects suited for graduate training, and closer alignment of the subject with actual practice, instead of adherence to generalization and assumed "cases," would be more desirable.

Research in Administrative Organization

In this field relatively little new thought was proposed, because it was very well developed during the previous decade. There was obvious, however, a clear tendency to consolidate the knowl-

edge of management and to extend it, as is proved by many books and articles on this subject. Only a few are listed:

Alford, L. P.: Cost and Production Handbook. Ronald Press. 1934.
Whitehead, A. C.: Planning, Estimating, and Rate-Fixing for Productive Engineers and Students. Pitman, 1933.
Person, H. S.: Technique of Planning. Industry's Contribution to Social-Economic Planning. *Taylor Society Bulletin*, No. 1, pp. 29–34. November, 1934.
Geschelin, Joseph, and Younger, John: Work Routing-Scheduling and Dispatching in Production. Revised edition, Ronald Press, 1942.
Thompson, S. E.: Optimum Productivity in the Workshop. *S.A.M. Journal*, 4, pp. 39–44. March, 1939.
Landy, T. M.: Production Planning and Control of the General Electric Company. *S.A.M. Journal*, 1, pp. 3–8. January, 1936.

In actual practice a more definite placing of responsibilities and decentralization of executive functions could be noted and a general desire to co-ordinate the various departments of a company or the sections of a shop into better functioning units. The number of companies using planning systems was increased and planning was adjusted to numerous specialized fields, for instance:

Stone, N. I.: Systems of Shop Management in the Cotton Garment Industry. *Monthly Labor Review*, vol. 46, pp. 1299–1320. June, 1938.
Thompson, H. L.: Going Straight Line. Advice out of the Clothing Industry. *Factory Management and Maintenance*, vol. 97, No. 1, January, 1939.

Research on Procurement and Material Control

Up to 1937 neither the procurement nor the control of materials caused any difficulties. Nor was industry caught with great excess stocks and losses after 1929 as it had been after 1919. The various methods and systems of material control developed by scientific management were well established and successfully applied in most plants.

Only as the various attempts to stimulate recovery failed to succeed convincingly, did the need for better co-ordination of purchasing with actual production requirements and for very close control of this phase become obvious:

Zink, W. C.: Purchases Are Tied to Production. *Factory Management and Maintenance*, vol. 98, no. 3, March, 1940.
Lewis, H. T.: Standards of Purchasing Performance. Analysis of Purchasing Expenses. *Harvard Business Review*, no. 4, pp. 480–493, July, 1936.
Brechin, C. H.: Standards for Handling Incoming Materials at Westinghouse Electric and Manufacturing Company. *Factory Management and Maintenance*, vol. 95, pp. 67–68. September, 1937.

When material scarcities became pronounced with the intensified defense, lend-lease, and war programs, and aggravated by the loss of the Far Eastern material supplies, quite a few new features were introduced into the procurement of materials:

War Production Board, see Priorities Division, Materials Division, and Contract Division publications on Priorities, Subcontracting District Offices, etc. For Scrap Salvage see: W. P. B., Bureau of Industrial Conservation.
A.S.M.E., Committee on Conservation and Reclamation of Materials. A.S.M.E. Annual Meeting, 1941. *Mechanical Engineering*, January, 1942, p. 25.
See also:
Engineering Aspects of Industrial Scrap Salvage. *Mechanical Engineering*, June, 1942.
Spooner, W. B.: Preference Ratings Are Your Job! *Food Industries*, January, 1942.
Editorial: Let's Look at Substitutes. *Factory Management and Maintenance*, vol. 100, February, 1942.
Odlum, F. B.: How I Plan to Spread Defense Contracts (Division of Contract Distribution, Office of Production Management). *Factory Management and Maintenance*, vol. 99, no. 11, 1941.
Van Vlissingen, Arthur: Farm-Out Plan Adds 67% Capacity. *Factory Management and Maintenance*, vol. 99, no. 2, February, 1941.

The more recent economic, military, and political aspects of material procurement will be found covered in the following references:

U. S. Senate, Committee on Military Affairs: Strategic and Critical Materials and Minerals. Hearings, 1941. Superintendent of Documents. (U. S. 77th Congress, 1st Session.)
U. S. Tariff Commission: Latin America as a Source of Strategic and Other Essential Materials. Report No. 144, 2nd Series, 1941.

On synthetic rubber and other chemical substitutes *Chemical and Metallurgical Engineering* and *Chemical Industries* have published excellent information.

Research in Job Standardization and Work Simplification

Job standardization and work simplification are also definitely among those subjects which have been done in large scale without having been described much in the literature. Only occasionally do companies or their employees take out time to report on such work for publication.

Also the methodical presentation of the scientific principles involved in this kind of work is still only tentative in character, partly because the field is a very wide one, partly because those developing the subject have not access to sufficient actual shop information, and in school laboratories only some kinds of work simplification can be experimentally developed. Nevertheless, progress has been made through the following contributions:

Maynard, Harold B., and Stegemerten, G. J.: Operation Analysis. McGraw-Hill Book Co., New York, N. Y., 1939.
Mogensen, H.: Work Simplification Conferences, Lake Placid, N. Y., and various publications (annual).
Wansky, S. L.: Foremen Eat Up This Work-Simplification Course. *Factory Management and Maintenance*, vol. 99, no. 9, September, 1941.
Koch, B. C.: Motion Economy for All. International Business Machines Corporation, *Factory Management and Maintenance*, vol. 96, pp. 56–57, June, 1938.
Klopsch, O. Z.: Sat Down and Wrote Up the to Do and the Why. Wolverine Job Standards. *Factory Management and Maintenance*, vol. 96, pp. 59–60, May, 1938.
Carroll, Jr., P.: Time Study for Cost Control. McGraw-Hill Book Co., New York, N. Y., 1939.

The techniques necessary for work simplification also have been advanced through new ideas:

Barnes, Ralph M.: Photocell Motion-Study Research. Various articles in *Factory Management and Maintenance*, vol. 97, no. 1, January, 1939, and subsequent issues.
Kosma, A. R.: Motion Analysis in Three Dimensions. Stereoscopic Method. *Factory Management and Maintenance*, vol. 99, April, 1941.

Research on Cost Control and Budgetary Control

When sales declined during the depression, and especially when they declined to the dwindling point in 1932 and 1933, costs, cost-savings, and cost controls gained in importance. Therefore, cost accounting and budgetary controls which had been, so far, helpful in reducing costs were improved by many organizations. Mostly, however, the traditional methods of analyzing standard setting and budgeting of costs were found to be inadequate. The seriousness of the situation required more direct and truly effective measures.

The first step taken was usually a careful *check of the "book costs"* of operation, processes, parts, or products as established by cost-accounting methods. It was found over and over again that even careful allocation or distribution of burden items did not give true actual costs. Furthermore, even if costs appeared to be fairly accurate, they still had to be brought down.

This led to the second step: A careful analysis and *study* of each operation or process *from a technical angle combined with the purpose of reducing costs.* When this approach was found to be truly effective, quite a number of companies which previously had relied on accounting and budgeting controls began to build up industrial-engineering departments and by now the larger companies employ sizable technical staffs which control and try to reduce time costs by time and motion studies and revisions; material costs by trying out substitutes or cheaper materials; machine and tool costs by careful checks, redesign, and control of pertaining charges, etc.

Thus the real cost control has been in many instances passed on from the cost accountants to industrial engineers, and cost engineering has become much more real than it has ever been, at least in the best managed and controlled companies. In the majority of plants, however, accounting cost control still prevails, with or without the use of standard costs and budgets to serve as measuring sticks.

The literature in this field was ample and diversified. The following merely are samples:

Bigelow, C. M.: Gone Are the Good Old Days; Suggestions for Reducing Costs. *Factory and Industrial Management*, vol. 83, pp. 17–19, January, 1932.
Dutton, H. P.: How to Install and Operate a Cost Production Department. *Factory and Industrial Management*, vol. 83, pp. 413–415, November, 1932.
Sittig, L. P.: Methods Engineering Will Get the Most Out of Your Plant. *Factory and Industrial Management*, vol. 81, pp. 966–969, June, 1931.
Knoeppel, C. E.: Profit Engineering. McGraw-Hill Book Co., New York, N. Y., 1933.
Gorofski, I.: New Developments in Controlling Labor Costs. N.A.C.A. Bulletin No. 20, pp. 1–11, September, 1938.
Gillespie, G. M.: Accounting Procedure for Standard Costs. Ronald Press, 1935.
Peck, S. A.: Managerial Aspects of Controls. N.A.C.A. Bulletin No. 20, pp. 471–490. December 15, 1939.
Carter, W. L.: Industrial Management and Accounting. *Journal of Accounting*, vol. 60, pp. 345–356. November, 1935.
Ward-Benson, D.: Cost Control of the Belt Conveyor. The Department Recovery System. *Mechanical Handling*, vol. 25, pp. 205–206. July, 1938.
Mallet, L. C.: Problems in Costing Airplanes. N.A.C.A. Bulletin 18, pp. 1152–1158. June 15, 1937.
Ebert, G. M.: Cost Accounting for Airplane Production. N.A.C.A. Bulletin 19, pp. 1333–1351. August 1, 1938.
Krafft, E. H., and King, H. F.: Budgeting for Varying Volume. *Factory Management and Maintenance*, vol. 99, June, 1941.
Merrill, A. A.: The ABC of the Variable Budget. *Factory Management and Maintenance*, vol. 98, April, 1940.

Research on Industrial Marketing

The most satisfactory features in the scientific progress made in this field are two: (1) A great deal of the superficialities and easy success philosophies which prevailed before the depression were abandoned under the convincing weight of actualities, and (2) More careful and detailed research was given to all those phases of industrial marketing, which were found worthy and useful of development under the more severe sales conditions.

While at first the difficulties encountered in selling were exclusively economic in character, gradually also government regulations were introduced prohibiting below cost selling (N.R.A.), later on came price supervision, and finally O.P.A.'s War Price and Rationing Boards. The simultaneous co-ordination of all sales efforts to war demands and an ever-increasing amount of restrictions on the manufacture of certain civilian goods created quite difficult sales conditions in some industries.

Since also the consumer, consumer organization, the co-operatives, the development of supermarkets, product research, forecasting, and the dependence of sales and selling on economic conditions were taken up as special research fields, a multitude of new thoughts were contributed in this field. The literature is so numerous that only samples can be offered:

Garland, C. M.: Depressions and Their Solutions. Guilford Press, 1936.
Dulles, E. L.: Depression and Reconstruction. University of Pennsylvania Press, 1936.
Hart, A. G.: Failure and Fulfillment of Expectations in Business Fluctuations. Record of Economic Statistics, vol. 19, pp. 69–78, May, 1937.
Frey, A. W.: Manufacturers' Product, Package and Price Policies, Modern Merchandise Management. Ronald Press, 1940.
Stidstone, G. W.: Field of the Industrial Engineer in Merchandising. *S.A.M. Journal* vol. 1, pp. 78–81, May, 1938.
Cowan, D. R. G., How Sales Effort Can Be Audited. *Advertising and Selling*, vol. 32, pp. 41–43, June, 1939.
Fox, W. M.: Profitable Control of Salesmen's Activities. McGraw-Hill Book Co., New York, N. Y., 1937.
Cowan, D. R. G.: Improved Standards of Sales Performance. *American Statistical Society Journal*, vol. 32, pp. 75–82, March, 1937.
Holleran, O. C.: Check Sheet, Introduction of New Consumer Products, With List of Government Sources of Market Research Material. U. S. Dept. of Commerce, Superintendent of Documents, June, 1935.
Henderson, Leon: Consumer and Competition. Annals. P. 363.
Mann, W.: Do You Favor Consumer Research? The Sales Executives' Forum. *Sales Management*, vol. 36, pp. 166–167, February 1, 1935.

Agricultural Adjustment Administration, Consumers Counsel Divison:
Consumers' Guide, published semimonthly.
Consumers' Market Service, published semimonthly.
Consumer Services of Government Agencies, 1936, 1937, 1938.
U. S. National Recovery Administration, Division of Review: Legal Aspects of Price Control. February, 1936.
Ibid: The Control of Geographic Price Relations Under Codes of Fair Competition. March, 1936.
Ibid: Minimum Price Regulations Under Codes of Fair Competition. March, 1936.
Ibid: Price Filing Under NRA Codes. March, 1936.
U. S. Congress, Temporary National Economic Committee: Hearings. During these hearings the sales and price policies and methods of quite a few industries were investigated and a great number of other management subjects were probed. Among the references frequently referred to were:
Means, Gardiner C.: Industrial Prices and Their Relative Inflexibility. 74th Congress, 1st Session, Document No. 13.
Nourse, E. G., and Drury, H.: Industrial Price Policies and Economic Progress. Brookings Institute. No. 76, 1940.
Hamilton, W. H.: Price and Price Policies. McGraw-Hill, Book Co., New York, N. Y., 1938.
Dennison, H. S., and Galbraith, J. K.: Modern Competition and Business Policy. Oxford University Press, New York, N. Y., 1938.

Many new thoughts were expressed in these texts and they were used to some extent as a basis for argumentation on the government's side.

The hearings were held for the "Investigation of Concentration of Economic Power" and they were published in 31 parts. Of special interest are the following:

Part	1	Economic Prologue
"	2, 3	Patents
"	5	Monopolistic Practices (5 and 5A)
"	6	Liquor Industry
"	7	Milk and Poultry Industries
"	8	Problems of the Consumer
"	11	Construction Industry
"	14–17A	Petroleum Industry
"	18–20	Iron and Steel Industry
"	21	War and Prices
"	25	Cartels
"	26, 27	Iron and Steel Industry
"	30	Technology and Concentration of Economic Power
"	31	Investments, Profits, and Rates of Return for Selected Industries
"	31 A	Supplemental Data

Recovery Plans. TNEC Monograph No. 25, 1940.

Research in Job Analysis, Job Rating, Employee Merit Rating and Wages

While in past years these subjects were still in their development stages, mostly theory, and considered as separate fields, during the period under review a considerable number of companies introduced all of these methods into their activities and daily practice. Tying them together and adjusting them to their specific needs, they began to investigate in fine detail the relationships between the characteristics of all jobs, then they established definite ratings for the jobs as well as for those doing them, and finally they began to establish wage rates in line with the findings. In this manner they arrived at truly scientific evaluation practices and solidly founded wage and reward policies.

The main reason for this intensified scientific approach to job rating and wage problems was the ever-increasing pressure of the unions to obtain higher wages for any kind of job. These attempts could not be met in any other way but by a better coordination of job analysis with all the other problems. Thus the companies are now in a position to prove or check how a wage rate was arrived at and how it is justified by the character of the job. Once a wage structure can be "proved," the unions have little left to attack it, and a most troublesome problem in labor relationship is solved or at least made more satisfactory for all concerned.

While a truly impressive amount of work has been done by the companies, the unions, and also by federal agencies (Federal Security Agency—Employment Service Division), the literature contains little evidence of this research. The nearest to it are:

U. S. Employment Service: Directory of Occupational Titles, Part 1: Definitions of Titles, Part 2: Job Families. Washington, D. C.: Superintendent of Documents.
Kress, A. L.: How to Rate Jobs and Men. *Factory Management and Maintenance*, vol. 97, October, 1939. (Discusses National Metal Trades Association Rating Plan.)
Mills, N. G.: A Point Plan for Rating Jobs. *Factory Management and Maintenance*, vol. 98, December, 1940.
Clarke, W. V.: Rating Employees. *Personnel Journal*, vol. 15, pp. 100–104. September, 1936.
National Industrial Conference Board: Plans for Rating Employees. Studies No. 8, 1938.
Riegel, J. W.: Wage Determination. University of Michigan, Bureau of Industrial Relations, No. 138, 1937.

Research on Personnel and Employment Practices
(A) White Collar Personnel

The executive, his responsibilities and functional duties, came up for special study, and during the decade it was his performance which counted rather than his "superpersonality." New light also was cast on the role which the foreman ought to play in a good organization.

Sacksteder, O.: Putting It Up to Department Heads to Swim or Sink. *Printers Ink*, vol. 156, pp. 98 ff, July 31, 1931.
Barnard, C. J.: Functions of an Executive, 1935.
Metropolitan Life Insurance Co.:
Functions of the Production Executives, 1935.
Functions of the Controller, 1935.
Eddy, H. P.: What an Employer Looks for in a Young Engineer. *Civil Engineering*, vol. 6, p. 832, December, 1936.
Lenz, A.: Foreman Has Keyrole in Maintaining Harmonious Relations With Employees. Chevrolet Foundries. *Automotive Industries*. January 12, 1935.
Tinsley, J. F.: Foremen Invested With Responsibility Are Keymen to Success of Industry. *Steel*, vol. 95, p. 38, December 10, 1934.
Kress, A. L.: Foremanship Fundamentals. *Factory Management and Maintenance*, vol. 99, No. 10, October 1941, and
Same: The Foreman on His Job. *Factory Management and Maintenance*, vol. 99, No. 10, October, 1941.

(B) Workers

Hiring. While during the depression dismissals prevailed, the requirements and standards for those to be hired were raised and actual skill was desired and tested for by the companies. They were not any longer satisfied with the mere fact that a man had "experience." The unskilled or less skilled men were gradually absorbed by Works Provision Administration projects, or had to remain on relief.

On this aspect of employment practices quite some literature of a social-economic nature was produced, but little was published treating this phase from the management angle.

Aptitude Tests. With the defense program and its final evolution into full-fledged war production the necessity arose to select a great number of workers from the as yet untrained or only semitrained applicants. In order to avoid too great mistakes the "aptitude tests" previously developed were revived and new tests were developed to suit specific company or job needs. In most instances tests like those used in Army aptitude testing were applied. More specific testing methods were described by:

Gillette, Robert W.: Help for the Hirer. *Factory Management and Maintenance*, vol. 98, no. 3, March, 1940.
Gillette, Robert W.: Tests Help You Hire Right. *Factory Management and Maintenance*, vol. 99, no. 10, October, 1941.
Editorial: General Electric Tests and Trains Meter Makers. *Factory Management and Maintenance*, vol. 98, no. 7, July, 1940.
Taylor, Harold C.: The Selection of Subordinate Personnel. *Mechanical Engineering*, November, 1941, pp. 807–810.

Women in Industry. When the draft took many men from industry and prevented the availability of sufficient men as required in the rapidly expanding war industries, women had to be hired, at first only for light work in the aircraft industries and later on also for heavier work in mechanical shops. When women were paid the same wages as would have been paid to men, the results were considered as satisfactory.

Editorial: What Women are Doing in Industry. *Factory Management and Maintenance*, vol. 100, March, 1942.
Editorial: Enter Women—to Do Men's Work. *Factory Management and Maintenance*, vol. 100, April, 1942.

Prevention of Seasonal Unemployment. Among the most noteworthy innovations in employment practices is the as yet limited tendency among employers to plan and provide for an exchange or shifting of employees from one company to others, so as to provide employment for the workers in off-seasons or seasonal slow-downs. More research and wider application of this practice with a view to postwar conditions are highly desirable. References so far known are:

Editorial: Big firms interchange workers in Chicago to fit each other's busy season. *Business Week*, January 28, 1939.
Lum, M.: Swapping Workers in Seasonal Slacks. *Nations Business*, vol. 27, p. 44, September, 1939.

U. S. Government Agencies for Employment. Another very important feature in personnel administration is the establishment of various government agencies for the purpose of recording and using the manpower of the country to best advantage. The most important of them are:

National Resources Planning Board, National Roster of Scientific and Specialized Personnel, Washington, D. C.
U. S. Employment Offices, established throughout the country to assist in finding and placing workers, and
U. S. Manpower Commission, Washington, D. C.

Research on Labor Practices

This field was considerably reformed by government action. New laws and regulations were put into effect, discussed, and management had to be adjusted to them. A complete review is impossible but the following references should help in revealing the new developments during the period.

U. S. Dept. of Labor: Annual Reports.
U. S. Nat. Recovery Administration, Division of Review, Labor Studies Section: The Labor Program under the N. I. R. A., 5 parts, March, 1936.
U. S. Works Projects Administration: National Research Project on Re-employment Opportunities and Recent Change in Industrial Techniques (Productivity of Labor, etc.). Various volumes 1939, 1940.
Appraisal of Results of W. P. A. December, 1938 (Works Progress Administration).
U. S. National Labor Relations Board: Functions and Relations to other Boards. Activities, etc. July 9, 1934, October, 1934, and later.
Decisions and Orders (Annually since 1936).
Annual Reports (since 1936).
U. S. Department of Labor, Labor Standards Division: Digests of Labor Bills, Investigations, Health and Safety Regulations (Since 1935).
U. S. Department of Labor, Labor Statistics Bureau. Studies on Unemployment Insurance, Employment, Pay rolls, Employment Office Service, Labor Standards in Government Contracts, Co-operative Movement, Union Progress. 1934 and later.
See especially:
Labor Unions, Characteristics of Company Unions, 1938.
Fair Labor Standards Act. Interpretive Bulletins on Methods of Payment, Maximum Hours, etc., 1938.
Labor Research Association: Labor Fact Book. International Publishers. 1941.
McNaughton, W. L.: The Development of the Labor Relations Law. Complete Bibliography. pp. 182–193, 1941.
Slichter, S. H.: Union Policies and Industrial Management. Brookings Institute, 1941.
Twentieth Century Fund: Labor and National Defense. 1941.
Balderston, C. C.: Industrial Relations. (Misnomer) Executive Guidance of Industrial Relations. University of Pennsylvania. 1935.
Westbrook Pegler was the most outspoken critic of labor practices and especially of union policies. His editorials were published by the *World Telegram* and many other newspapers.

Increase of Adaptability of People to Industry (Training)

A great deal of effort and research was extended to the training of workers, foremen, and junior personnel, and excellent methods were developed by official as well as by private initiative. The Division for Training Within Industry (War Production Board) established a nationwide organization to assist employers in their job-training efforts by providing training-course outlines, instructors, and material. While this work was exceptionally successful, hardly anything has been published as yet about it or the methods used.

During the recovery period, and more recently, quite a few companies, and in some industries (aircraft, steel) nearly all of them, developed systematic training, ranging from fundamental job operations to complete training programs on technical and management subjects. The following references are helpful in tracing at least the main developments:

Cheney, A. S.: Workers' Education in the U. S. A. International Labor Office Review, vol. 32, pp. 39–59, July, 1935.
Dildine, P. L.: They All Go to School. Goodrich offers every worker an opportunity to get more education. *Factory Management and Maintenance*, vol. 93, pp. 159–160, April, 1935.
Editorial: Four out of Five Industrial Companies Training Employees. *Steel*, vol. 101, p. 30, September 6, 1937.
Editorial: American Trains Her Industrial Army. *Factory Management and Maintenance*, vol. 98, November, 1940.
Irwin, R. R.: Quick Training Gives Lockheed the Men It Needs. *Factory Management and Maintenance*, vol. 98, December, 1940.
Wood, T.: Training Operatives for Conveyor Production. *Mechanical Handling*, Vol. 25, p. 268 ff, September, 1938.
Stephenson, A.: Training of Unskilled Labor. *Engineering*, vol. 145, pp. 347 ff, April 1, 1938.
Vanden Bosch, J. W.: Training People for Factory Work. *Factory Management and Maintenance*, vol. 98, September, 1940.

The training of college and subcollege type employees by companies, too, was intensified considerably and quite novel features were developed as may be seen from the following articles:

A.S.M.E., Survey of Training for National Defense.—Government, Civil Service Commission, Navy Training, Aeronautics Industries, etc. (Excellent). *Mechanical Engineering*, March, 1941, pp. 183–194.
Stevenson, Jr., A. R., and Howard, A.: General Electric Company's Advanced Course in Engineering. *Electrical Engineering*, vol. 54, pp. 265–268. March, 1935.
Kirby, F. B.: Study Clubs for Employees. Abbott Laboratories. *Nations Business*, vol. 26, pp. 108–109. April, 1938.
Editorial: 50 Per Cent of Tuition Fee Paid. *System*, vol. 64, p. 20. March, 1935.
Davis, E.: Educational Refunds in Industry. Princeton University, Industrial Relations Section, 1935.
Freund, C. J.: College Crop. Fitting Engineering Graduates to Their First Jobs. *American Machinist*, vol. 79, pp. 520–521. July 17, 1935.
Teichroew, H. W.: Trade School Crop Vs. College Crop. *American Machinist*, vol. 79, pp. 796 ff, October, 1935.

Research on Managements' Attitudes and Industry's Relations With Federal Government

In view of the ever-growing importance of government policies giving a new background to industrial management, quite some literature evolved dealing with the new kind of relationship. As indicative presentations on this subject may be considered:

Clark, J. M.: Social Control of Business. McGraw-Hill Book Co., New York, N. Y., 1939. Second edition.
Dykstra, G. O.: Textbook on Government and Business. Callaghan, 1939.
Fortune Magazine: Business and Government. March to October, 1938.
Lee, M. G.: Government's Hand in Business. Baker Voorhis & Co., 1937.
Burnham, James: Managerial Revolution. John Day, 1941.
U. S. National Recovery Administration: 557 Codes of Fair Competition, Supplements, Amendments, Executive and Administrative Orders. For detailed information see U. S. Government Catalog of the Public Documents 1933–1934, pp. 488 to 575.
Phillips, C. F. and Garland, J. V.: Government Spending and Economic Recovery. Wilson 1939.
Kimmel, L. H.: Cost of Government in the U. S., National Industrial Conference Board, 1938.

The Management Problems of the Future

Which problems will be in need of special investigation and research in the years to come is as yet difficult to state. Management's problems and character will undoubtedly be finally formed by the present war, and following it by the prevailing government attitudes, but there are certain trends and indications, which allow to discern a few pertinent facts:

The high taxation on corporate and personal incomes is bound to remain in effect for so many years to come that it will require lowest cost production and further research toward that end in all management branches pertaining to it.

Organized labor and legislation on the rights of labor have become such strong factors in the industrial pattern that research will have to find better than present methods of reconciling labor's and managements' interest. Scientific management should prove itself superior to the policies and attitudes now in vogue.

Considerable changes in materials and material supply will become permanent features and will involve not only technical problems, material and product research but will evolve also new methods of supply planning, expressing themselves in novel methods of purchasing and more improved stockroom management.

The systematic co-ordination of production of entire industries achieved by now on an unprecedented and never-expected cen-

tralized scale will probably be modified, but its advantageous features will be maintained and novel schemes of industrial co-ordination may be expected.

The art of selling and methods of distribution, deprived by now of much of their previous expensiveness, will have to be simplified further, and greater effectiveness and directness will be favorite subjects of research.

Price controls and price ceilings are not popular in peacetime and will probably be abandoned; but determination of true costs, price setting, and pricing policies will be restudied and the voluntary achievement of price stability will become more general policy as soon as it becomes better recognized as one basic pre-requisite for better planning.

Military service, and especially the air-force training now given to millions of young men, specializing them in many new technical subjects and in their application under most difficult conditions, training them in teamwork, discipline and human co-ordination—all of this is bound to make a deep imprint on the management of tomorrow.

Only time can and will point out more clearly the specific problems and detail tasks which will have to be investigated for new solutions.

Management Attitudes

By ERWIN HASKELL SCHELL,[82] CAMBRIDGE, MASS.

SOME psychologists maintain that attitudes are much more the product of our surrounding culture than of our personal experience. I believe the attitudes of the manager are an exception to this rule. It is not what people say to him, but what the world does to him that orders his temper of mind. And when we examine his environment during the past ten years, what an extraordinary kaleidoscope of change lies before us.

We first find our industrial economy in company with that of almost every other nation spiraling close to the bottom of a deep pit of international depression. And today, at the end of these ten years, we are well on the way to the greatest industrial output we have ever known. With such a spectrum of change before us, it is to be expected that management attitudes should reflect important adjustments. To say that these trends have, on the whole, marked progress is to make a bolder statement, for there are those who question if our standards of life have increased during this decade, even though standards of living in the United States have shown clear advance.

There has been one fundamental attitude which has revealed little variance during these years. It is management's ever-present temper of extreme concern over the health and continuance of the individual business. In its highest manifestation, this is an attitude of responsibility; in its primitive form, it is an expression of self-preservation. In either instance it is the product of the precarious state in which business and industry have found themselves. Even in normal times, as everyone knows, it is the exceptional establishment which weathers the pressures of competition. During this decade, the casualties have been unusually high, and the number of enterprises in this country, whose corporate life has for months or years literally hung by a thread, are numbered in the thousands. This basic fact has had little advertising value, and therefore has not been granted the weight that it deserves in explaining the attitude of industrial managers during these difficult years. The psychologists again tell us that people have public attitudes and private attitudes. When management appears at times to strike out unreasoningly at whatever obstacle confronts it, we may be sure that the irritability displayed is due to a deeper concern which the manager bears for those whose welfare is in his charge.

Yet we must admit that with the increasing waves of change which are sweeping across our country, we are witnessing two different managerial attitudes emerge. On the one hand, there are those reaching forward boldly and even eagerly to ride upon these wave-crests and to travel with them to the new. Others are turning and seeking the sand beneath their feet so that they may run for the shore. As this report is a component part of a larger survey of progress in management, we feel that we may properly deal only with the attitudes of those fighting folk who see in change an opportunity rather than a difficulty.

Needless to say, managerial attitudes have been infinite in number and variety, for they spring into being with every individual response to every managerial situation. To detail any fragment of these attitudes, therefore, is to open oneself to the inevitable criticism of omission or under-emphasis. Conversely, a commentator has the advantage of presenting his own viewpoint without fear of serious challenge, inasmuch as no objective measurement via attitude polls has, during this period, acquired sufficient reliability or validity to be scientifically dependable. As the advent of war during the last year has introduced the most radical changes in managerial attitudes occurring during the decade, it will be reserved for the final topic in this presentation.

Methods

The beginning of the ten-year period saw the end of the rapidly waning opposition to the advent of science in management. Again, almost the last barriers of secrecy in methods had disappeared, the final stand being in the areas of new product development. Managers increasingly turned with favor to methods or procedures which could be mechanized and almost automatically operated. An extraordinary growth in clerical machinery and in control mechanisms reveals the readiness with which management has accepted the benefit of these devices. There has also been a change of attitude toward methods of maintaining quality. Originally, quality was viewed largely as a disciplinary problem. More recently, emphasis is being laid upon technical improvements in fabrication and in quality measurement as roads to improvement. As the period advanced, the presence of standards was no longer viewed as an innovation but as a necessity in the same category as machinery or cash. In the Midwest, a whole new methodology has developed around the refinements of flow production, and a technique has been perfected which permits adjustment to demands for variations in product as well as rates of output. Perhaps the most dramatic change in managerial attitudes toward methods has been the accelerated acceptance on the part of industrialists of work simplification applied through the medium of foreman and employee training.

Facilities

Here, the greatest fundamental change in attitude has been the growing acceptance of process as the dominant element around which facilities are to be arranged. In many plants at the beginning of the decade, to move a machine required almost the same mental wrench as to move a tombstone. Today, flexibility of equipment and versatility in arrangement is sought by many aggressive managers. In the last analysis, the pivot upon which the enterprise turns in capitalizing upon the winds of change is now the resource of managerial and technical skill. Managers hold that facilities, materials, and products should be readily shifted and altered in order that these fundamental resources may be applied most directly and efficiently.

In no previous decade has there been greater change and over-

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turn of orthodox methods of fabrication than in this. Flame-cutting, welding, die-casting, powder metallurgy, and the advent of the synthetics have revolutionized many processes which, for over a hundred years, have held their position in our industries. These developments have called for a much more open-minded attitude on the part of managers than hitherto, a point of view easiest accepted by those who are familiar with the humilities of the scientific spirit.

Personnel

In no field has the manager found it necessary so radically to vary his attitudes as in that of personnel. The economic holocaust which swept across the world in the early '30's revealed to everyone that neither the individual establishment, nor the industry, nor the combined abilities of a nation were sufficient to prevent unemployment. With perhaps 15,000,000 men without a place to work as our era begins, it was clear that marked changes in employee as well as employer attitudes must inevitably result. Out of the maelstrom of conflicting opinion, confusion, and dismay, there have arisen certain constructive attitudes which merit mention. The first is the growth in awareness of the dignity and importance of simple human relationships. At the height of the turmoil, one manager remarked, "We have no idea how things are coming out. But one resolution we have made. It is that when the dust has settled, we shall have retained our personal friendships with our employees, which we have built up over so many years."

A second change of attitude is the increase in managerial interest in relation to the individual. As one executive put it, "Ten years ago, my foremen knew their employees by numbers; five years ago they began to call their men by their last names; then we swung to a first-name basis, and today (prewar) my supervisors are trying to know the members of the family as well." Managerial attitudes have given new emphasis to the maintenance of employee goodwill. Complaints are viewed with greater concern; inequalities of basic wage have been the cause of extensive analyses in many plants, and the techniques of evolving mutually satisfactory working relationships between management and labor have been given far greater weight than in previous years. To a growing extent, management is looking at personnel as co-managers, rather than as employees, inasmuch as the manager of a machine is in many regards an executive in his own right, although his control is not over human stuff.

Supervision

At the beginning of our period the importance of the supervisor was rapidly growing in the eyes of management. Yet the depression years forced many companies to skeletonize their organizations, and improvement in this sector resulted as much from the removal of mediocrity in supervisory personnel as from foreman training. With the rapid acceleration in the growth of labor organizations, new problems of such severity developed that in many establishments management short-circuited the supervisor in an attempt to allay difficulties. The ineffectiveness of this procedure was soon recognized, and as our period draws to a close it is the growing attitude of management that to the greatest extent possible the supervisor should be trained to fulfill his new role of company representative in dealing with labor's appointed departmental spokesman.

A second curious error developed during this interval. In management's anxiety fully to prepare the supervisor for his new duties, his loyalties to the company were so strengthened that his relationships with the men were somewhat weakened and the principle that a good foreman believes in his company but stands for his men was overlooked. This condition was quickly corrected, and today in our progressive establishments we find management hoping that their first-line executives will be men whose heads are with the company and whose hearts are with the employees.

Vendors

An interesting change of attitude is found here. During the ten years, vendors have been viewed less as adversaries in negotiation and more as partners in production. Indeed, one organization has required that its sources of supply be known as its resources. The mutuality of interests between vendor and customer has been more widely explored, and agreements upon inspection methods and other technical details effected. Back of this change of attitude is the realization by management that the extreme fluctuations in the business world when shifts from a buyers' to a sellers' market occur almost overnight, make it necessary that beneath commercial relationships there should be constructed a stratum of fundamental goodwill and friendship. To maintain a lively competitive spirit has been found entirely possible without the corrosion of mutual personal regard and respect. These trends are reflected in the modern purchasing installations which give increased attention to the convenience of sales representatives and economy in their expenditure of time.

Stockholders

It is a truism that in times of peril all animate life seeks protection through closer relationships with its kind. The long depression followed by growing international uncertainties has doubtless been a contributory factor in the change of attitude on the part of management with respect to stockholders. At the beginning of our period the stockholder was a name in a big book. At the close of the decade he is a person who management sincerely hopes may become interested in the welfare of the business of which he is a part. Early in the decade when it became clear that management had lost the vote there were sporadic efforts made by some executives to awaken sudden interest on the part of stockholders. This was but a surface phenomena however, and a deeper and more hopeful change of attitude has been the tendency throughout the country to provide the stockholder with such facts about the business as will reflect the situation truly and yet simply. The problem has not been an easy one, inasmuch as the growing tendency of the American buyer has been to measure his purchases solely by results with less interest in the methods by which these results are obtained.

The Public

Despite sporadic evidences to the contrary, management's attitude toward the public has rarely been antagonistic. The physical nature of manufacturing processes with their screen of buildings and property have naturally insulated production from the community and the public. That there was any reason for closer mutual understanding and familiarity was little realized. But when it became clear that without such relationships industry might find itself in a weak strategic position, a wave of concern developed in these areas. Little by little it became clear that it was not only the privilege but the responsibility of management to make its activities an acceptable and understood part of the community. Applying the common principles of hospitality, management found it sound practice to "put the house in order before the guests are invited." Frequently these preparatory activities took longer than was anticipated. As our period drew to a close many managers throughout the country were reflecting attitudes of satisfaction that such constructive steps had been taken and that such responsibilities had been shouldered. In some instances definite resources of immediate practical benefit to the company had been disclosed; in others potential gains are in the offing. Chief among the changes in attitude, however,

was the realization that the industrial establishment may wisely view its market as only a fragment of the public with whom it should be on close and friendly terms.

Government

When it became clear that the depression suffered in the United States was common to practically every other industrial nation, it became equally patent that our country as a unit must inevitably assume new responsibilities, undertake new activities, and develop new relationships with its people. Later when the fire of war began sweeping over the planet, it was doubly obvious that our unity as a nation must be given new significance, dignity, and implementation. Management's attitude toward government during the past decade has been colored by the complete spectrum of human emotions. At one end of the scale we find intense antagonism and now at the other we find the height of loyalty and support. Over the period two simple facts have come to light widely affecting management attitudes. The first is that government has become the largest business in the country and that it is carried on by human beings who live and breathe as do managers. Indeed, many of these individuals are themselves managers in but a slightly different field of endeavor.

The second fact is that management increasingly has had need of government in view of the heightening world conflagration. These two certainties have gone far to lessen the pain of limitation which governmental action has inevitably found it necessary to throw about American business and administration. Managers and government officials are getting to know each other better, and attitudes of uncertainty and suspicion are changing to those of familiarity and mutual understanding. As the nation entered the war there was no question but that the trend of management attitude toward government was definitely in the direction of increased co-ordination, co-operation, and harmony.

Wartime Attitudes

While our so-called defense period served to usher the nation into a war-time economy under somewhat more gradual auspices than would otherwise have been the case, yet it is true that with Pearl Harbor there came a sudden, and in some instances, drastic change of attitude on the part of management. Still in a state of flux, these points of view are yet plastic in terms of current changes, and may be only incompletely discerned. Their apparent contour as revealed by recent contacts with several hundred wartime establishments may, however, be highlighted. Wartime attitudes toward method have fortunately responded promptly to the revolutionary point of view which the emergency made essential. During the initial stages, all thoughts of cost or efficiency had to become secondary to the objectives of sheer quantitative output. Where conversions were most severe, and where great expansion was called for, intelligent management sought effective production first and qualitative cost or economy values second. Already many firms are rapidly swinging to the intensive cultivation of qualitative manufacturing methods which will markedly enhance production without commensurate increases in expenditures.

Wartime facilities proved an initial point of constriction. Once more management attitudes swung from costs to output as the dominant objective, with the ensuing refurbishment of old machines, conversion of existing facilities and building by company establishments of their own new equipment. The wartime attitude is "any machine is better than none."

A new note has been sounded in personnel relations which has been expressed by an able commentator as follows:

> In France, the nation collapsed in important part, because labor's spirit had been broken; because workers didn't see much for which to fight; in England, the nation rallied in important part, because labor had a real stake in government; because workers felt that they did have a reason to fight; in Russia, the resistance exceeded all expectations because the people felt that they had a stake in winning; because they had a reason to fight.
>
> Here nothing should be done to discourage workers, to give them the idea that there is little to defend. Everything should be done to give labor a big stake for which to show an interest and for which to fight.[83]

Here is the underlying philosophy which is basing all decisions of personnel policy and which dominantly colors the attitude of constructive management.

In the field of supervision, a new spirit has arisen. Supervisors increasingly take the attitude that they are working *with* their men *for* their country. A new quality of morale, not to say devotion, results from this feeling and we may anticipate an even closer bond of camaraderie between the rank and file of our industry, as labor-management committees continue to grow in numbers.

Never before have vendors attained so high a position of eminence in the eyes of the industrial buyer. The attitude of management is one of prayerful thankfulness they have been able to build any residue of goodwill which now may be capitalized in these relationships. Unquestionably, this new mutuality of interests the war has brought will go far toward effecting stronger and more creative opportunities for closer productive co-ordination between buyer and seller when the hostilities come to a close.

As Army and Navy pennants are awarded and as trade literature reveals the efforts companies are making in the interest of the nation, an unusual surge of support and enthusiasm has come from stockholders who now feel themselves, in part at least, related to the war effort of the industries whose securities they hold. There is little question but that here the war is bringing stronger bases for close affiliation than have been enjoyed in many years.

Never has industry been so inextricably involved in the concerns of the community as in this wartime period. The recruiting of our vast army has involved mutual sacrifices on the part of both to which each has cheerfully responded. Community and company have stood shoulder to shoulder facing the inevitable losses of industrial manpower which mobilization has demanded. At this writing, management is in the process of turning to the sisters and mothers of these young men for their assistance in manning machines and work places. New sociological relationships between industry and community are an inevitable outcome of these activities and closer contacts will continue to strengthen mutual understanding and regard.

With respect to management's attitudes toward government, the wartime temper can best be reflected by a distinguished manufacturer who recently said, "Yesterday the man in the White House was my president; today he is my commander in chief." To offer unselfishly, unfalteringly, and unreservedly the services of management to those in whose hands the conduct of the war rests is the kind of attitude upon which ultimate victory for the nation depends. Its wide prevalence throughout our land holds high promise for victory.

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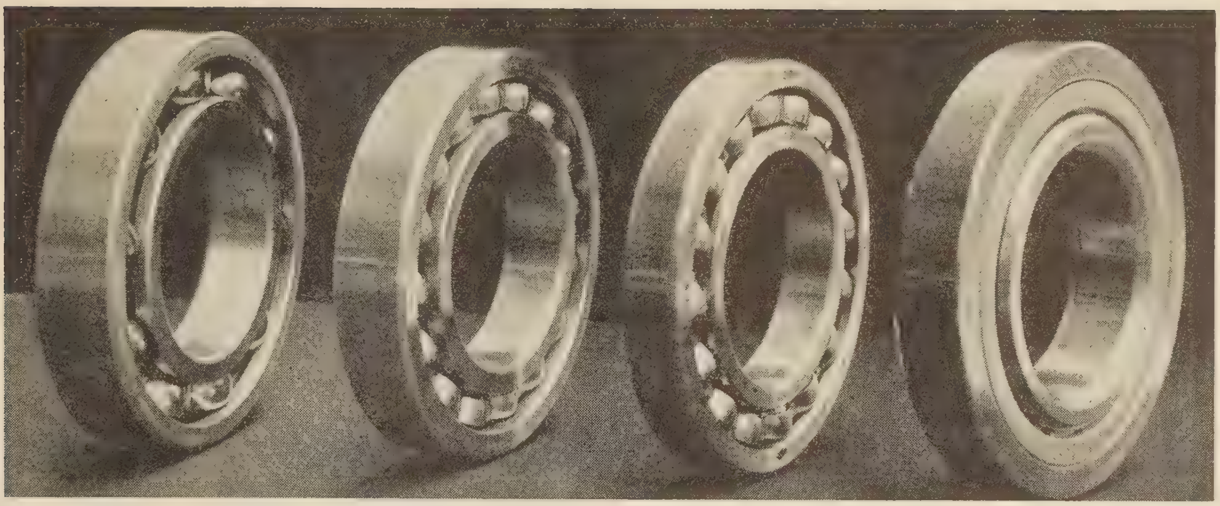
[83] *United States News*, vol 2 Aug. 2 1941.



(a) (b) (c) (d)

Fig. 1 Changes in Type of Bearing Used Progressively in the Same Engine as Power Was Increased, and Also Used Progressively From Small to Large Engines in Present-Day Production
(a, Conrad or "deep-groove" type. b, Notch-filled or maximum-capacity type. c, Same as b, but narrow or "aviation" width. d, Roller bearing.)

Antifriction-Bearing Developments for Aviation Engines

By THOMAS BARISH,[1] HYATTSVILLE, MD.

Rapid advances in aircraft-engine design for higher powers have required corresponding improvements in antifriction bearings. From an intimate knowledge of the requirements the author presents a comprehensive outline of the progress made in bearing developments with particular reference to current practice. The discussion includes details of bearings for the following services: crankshaft, propeller thrust, rocker arm, supercharger, and controllable propeller.

Crankshaft Bearings

Special ball and roller bearings have been developed in the past decade for nearly every location on aircraft power plants. The departures from the industrial standard bearings were kept to a minimum, especially at first; mainly they were reductions in width for weight saving, or large corner radii and smooth finishes for reducing stress concentrations. In some places completely special designs proved necessary with increased requirements.

The crankshaft bearings are by far the most difficult problem. They must take care of the following:

1 Very heavy loading: for example, 10,000 lb mean and 14,000 lb maximum on the front bearing of a 1000-hp, single-row, radial engine on a bearing with $3^7/_8$-in. bore. These loads are doubled momentarily due to gyroscopic forces which reach a peak in a tight spin.

2 Large shaft deflections due to these heavy loads, combined with minimum shaft thicknesses, the worst location being at the rear where crank-cheek yield and torsional vibration have their greatest effect.

3 High speeds: The present peak is about 2800 rpm on a $7^1/_2$-in. bore for the center bearing of a two-row radial engine, giving 6000 fpm and requiring far better performance than standard industrial bearings.

4 Relatively high temperatures, especially in this same center location close to both rows of cylinders.

Ball and roller bearings are employed on crankshafts only on the radial airplane engine, primarily because of their compactness and ease of replacement. They also afford less dependence upon a continuous oil supply, less susceptibility to deflections and, in addition, are valuable for the relatively slow progress of failure when it does occur. Usually the bearing will carry through to a landing and frequently remain unnoticed until major overhauls.

For in-line engines, it would be very difficult to install roller bearings. German engines employ them on the large end of the rod and occasionally on the main bearings. Notable is the Daimler Benz rod-end rollers, which ride directly on a hardened crank. There is a probable saving in power and heat, perhaps 50 hp per engine, but this is questioned in the United States and may be checked soon.

Radial engines were at first put into production with ball bearings on the crankshafts. The small and medium-sized engines still retain them as in Table 1.

Notch-filled-type ball bearings, Fig. 1 (b), are used except for a

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Contributed by the Aviation Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

few of the smallest lightly loaded jobs where Conrad or non-filling slot-type, Fig. 1 (*a*), are accepted as alternates.[2]

The only common departure from standards is a reduction to "aviation widths" in the light or 200 series, Fig. 1(*c*), retaining the standard ball size and cages, bores, and outside diameters.

Sizes were first selected by arbitrary rules, necessarily so since the calculated loads far exceeded the ratings. Actual experience on crankshaft ball bearings used loads up to 2.5 times the catalogue ratings for mean loads and 5 times the ratings for take-off loads and speeds.

The gathering of this experience followed the usual pattern for aviation work. Nine years ago, a No. 218 size (3.54-in. bore light series) was selected for the rear end of a 550-hp single-row radial engine. This same engine has been improved step by step to over 1100 hp with no change in bearing size. The first power increases eliminated the Conrad type (11 $^{13}/_{16}$-in. balls) and specified only the notch-filled type (17$^{13}/_{16}$-in. balls). Even that was found unsatisfactory above 850 hp for air-line service with its extreme demands for reliability and long life (1000 hr are guaranteed and 2000 are under consideration). A square-section roller bearing, Fig. 1 (*d*), of the same dimensions was then introduced, and finally the roller bearing needed improvement; all of this in the same over-all dimensions.

These same steps are reflected in present production radial engines as size increases: Conrad-type ball bearings in the smallest group, maximum type in the medium-size group (Table 1), roller bearings in these same engines at their higher ratings (Table 2), and roller bearings of the revised designs for the latest and highest ratings.

TABLE 1 RADIAL ENGINES USING BALL BEARINGS ON CRANKSHAFTS

Engine type	Horsepower	Front	Rear
Group (1)			
Le Blond	110	211V	210M
Warner	150	211CA	211CA
Rover	75	210M	
Group (2)			
Lycoming	240	214M	214M
Jacobs	225	214V	214V
Continental	210	213M	212M
Continental	300	214V	214V
Kinner	370	Bush	217M

TABLE 2 CRANKSHAFT BEARINGS

Engine type	Horsepower	Front	Center	Rear
Lycoming	380	214 Roller	...	214 Roller
Jacobs	325	214V Roller	...	214V Roller
Guiberson	240	216 Roller	...	216 Roller
Whirlwind	450	216V Roller	...	Bushing
Wasp Jr.	450	215 Roller	...	216 Roller
Cyclone	900	219 Roller	...	218 Roller
Hornet	875	121 Roller	...	121 Roller
Two-row Wasp	...	120 Roller	037 Roller	216 Roller
Two-row Cyclone	...	220 Roller	136 Roller	220 Roller

Further increases in power required improvements in these roller bearings; investigations made in 1938 produced relatively minute but extremely important internal changes that brought

[2] The Conrad type, Fig. 1 (*a*) (formerly called "deep-groove type"), has continuous raceways and is assembled by raising the inner ring and filling the crescent-shaped opening with balls, then distributing the balls and adding the separator or cage. Hence, the bearings are only slightly more than half-full of balls. Note the spaces between balls. The notch-filled or maximum-capacity type starts to assemble the same way, but about 40 per cent more balls are added through notches in the side of the rings. These races are sprung or wedged apart on each side of the notch during this assembly operation so the notch need not start at the very bottom of the ball path. The increased number of balls produces more radial-load capacity. The notches do limit the thrust capacity of these bearings, but not so noticeably in the lighter series bearings used in aviation because the light rings can be sprung more without permanent harm and, therefore, have greater distances from the bottom of the groove to the beginning of the notch.

Fig. 2 Typical Roller-Bearing Failure, Showing Stress Concentration

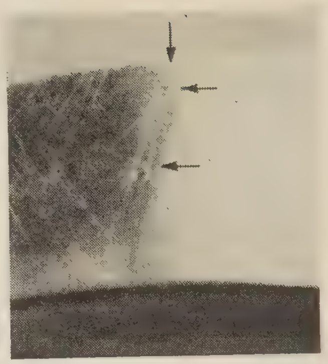


Fig. 3 Brinell Mark of Straight Roller on Flat Race

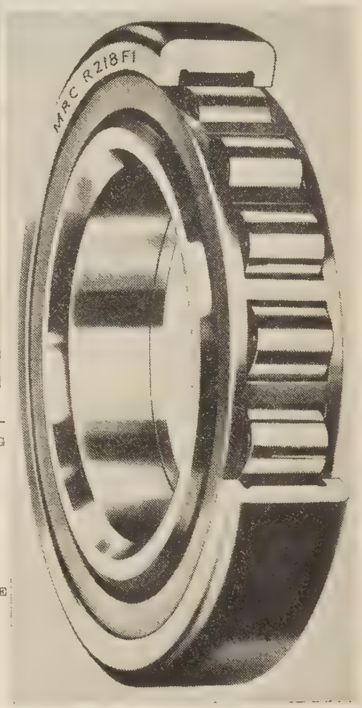


Fig. 4 (*Right*) Roller Gage With Square Broached Holes

to use some of the unrealized possibilities of this type.[3] These new features have become standard for all heavily loaded locations:

(*a*) Edge relief of rollers: required because the capacity is limited by what happens at the edge of the long, thin rectangular contact area. The very high "Hertz" pressure (200,000 to 300,000 psi) along the center of the rectangle tapers off sideways but changes abruptly at the end having no side support. Without edge relief failure occurs as a shear under this edge (typical failure Fig. 2). This stress concentration is strikingly indicated when a straight roller without relief is pressed into a flat plate for checking brinelling capacity (Fig. 3). A small crescent-shaped permanent deflection forms under the end of each roll. A small edge relief, of about 0.0002 in. (approximating deflections) produces startling improvement in life under heavy operating loads.

(*b*) Crowning of the outer race: The very small deflection of

[3] "A Theoretical Comparison of Ball and Roller Bearings," by Thomas Barish, paper presented at the American Gear Manufacturers Association Meeting, April, 1938.

roller bearings makes them extremely sensitive to misalignment or shaft deflections. The latter are considerable in airplane engines because shaft sizes are minimized. The total yield in a straight 1-in. roller is only about 0.0002 in., and a tilt of only 0.0002 in. in 1 in. or $^1/_{10}$ deg would shift the contact entirely away from one end. Even more serious, the loaded end tends to drag and swerves the roller to one side. Crowning the outer race track largely eliminates these effects and without loss of capacity because the surface pressure of the concave outer race can still be kept below that of the convex inner. Crowning the outer race introduces serious production problems that have been satisfactorily solved.

A similar effect may be produced by crowning the rollers and leaving both race surfaces flat, but this may sacrifice part of the inner-race contact area, and hence part of the bearing capacity.

Incidentally, the crowned race improves the load distribution among all the rollers as well as along the roller length because of increased deflections.

(*c*) A new one-piece retainer design, with square broached holes (Fig. 4) permitting 10 to 20 per cent more rollers primarily designed to eliminate the rivets which have always been a hazard; the design permits complete disassembly of the bearing for overhaul inspection. The last feature is especially valued by the air lines.

(*d*) A control of the oil flow to the critical points in the bearing.

These design features produced (1) two to three times the life, (2) a marked reduction in operating temperaures and also (3) in noise in testing machines. Higher permissible operating speeds are indicated: In recent tests, a No. 219 size bearing (3.9 in. bore light series) ran 960 hr at 9000 lb load and 6000 rpm. The load-carrying possibilities of these improved rollers have not been exhausted yet, especially at higher speeds. Probably much of the unusually high load capacity at top speeds results from the very low coefficient of friction and low heat generation.[4]

Frequently all the inner rings of any one size are made interchangeable for replacement of separate parts. This involves extremely close production control and even then requires wider tolerances on internal looseness.

Much effort has been expended on the subject of internal looseness and fits. Shaft galling, due to very high unit pressures with relatively narrow bearings, has been largely cured by increasing press fits to 0.0005 to 0.0007 in. minimum. Additional internal looseness is then needed, as the maximim press fits often reach 0.0025 in. and inner races expand correspondingly. Research still continues on internal fits since theory indicates a marked drop in roller-bearing capacity for looser bearings. Also certain types of engine vibration can be helped by reducing looseness both internally and around the outside diameter. Experimental checking on this is still in progress.

High operating temperatures in bearings near the cylinders may produce slight softening of the steel and a small permanent growth (standard bearing steel S.A.E. 52100). Judging from this growth, the temperatures often reach 350 F. A new steel-drawing technique, "stabilizing," corrected this condition by the use of very much longer soaking. New alloys are on test to permit higher operating temperatures without much loss of hardness or growth.

Some discussion continues as to the need of highly polished finishes on all exterior surfaces including corner radii. This common practice in the United States is not so general in Europe.

These same roller bearings serve usefully in gearboxes with in-line engines. Several of the late United States engines follow English practice of eliminating inner races and having the rollers ride directly on the pinion extensions or gear shaft, thus eliminating fits, galling, and nuts for clamping.

[4] "Friction Torque in Ball and Roller Bearings," by Haakon Styri, *Mechanical Engineering*, vol. 62, 1940, pp. 886–890.

Propeller Thrust Bearings

All but the smallest airplane engines regularly employ the single-row radial ball bearing for the propeller thrust and radial loads. The major departure from standards is in reduced "aviation widths."

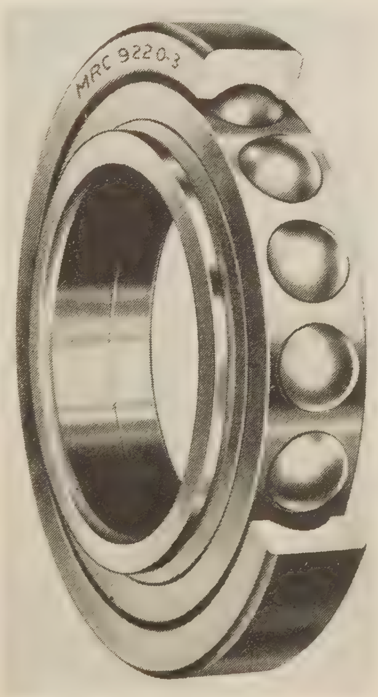
The notch-filled and Conrad-type ball bearings remain standard in all of the medium-sized group (Table 3).[5] With the advent of constant-speed controllable propellers, the larger engines found it necessary to use only the Conrad type because of a great increase in take-off thrust. Take-off loads jumped from about 2000 lb up to 5000 to 6000 lb, and these momentarily encroached too closely on the beginning of the notch.

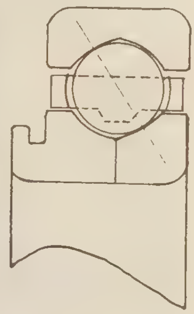
TABLE 3 PROPELLER THRUST BEARINGS[5]

Engine type	Horsepower	Bearing type
Le Blond	85	210M
	110	210M
Warner	90	212M
	150	212M
Kinner	100	211M
	160	212V
	210	214M
Guiberson	185	215M
	240	216M
Lycoming	300	213M
Jacobs	170	212M
	300	213M
Continental	300	214V
Whirlwind	420	214S
Wasp	550	216S
Hornet	700	217SV
	800	220S
Cyclone	900	220SV

Further increases in power (again without change in bearing size) brought forth a revival of an old (1912) Gurney bearing type with split inner race, comprising two angular contact-type bearings in a single row (Fig. 5)—what appears to be a 4-point bearing but is really two 2-point bearings—thus permitting a

[5] Letter M in table means notch-filled "maximum" type, and S means Conrad type.





Figs. 5 and 6 Single-Row Two-Direction Steep-Angle Propeller Thrust Bearing

nearly full complement of balls, and a steeper angle of contact, with 2 to 3 times the thrust capacity. Again the bearing is designed to separate completely for thorough overhaul inspection, Fig. 6.

English engines have been using a very similar design with two-piece outer race and one-piece inner.

The propeller bearing must carry a very large momentary radial load because of gyroscopic forces when the airplane goes through violent gyrations. Coming out of a dive does not represent the peak; other conditions in the airplane limit the dive loads to about 10 G. Far worse loads occur in a tight radial spin taken at 2 radians per sec for bombers, and a possible 3 radians per sec for highly maneuverable pursuit ships. Theoretical moments of 200,000 in-lb produce forces taxing the bearings, especially when the two front bearings are relatively close together.

On all but the largest engines, the front ball bearing carries this large momentary radial load as well as the thrust load. The largest engines frequently introduce a separate roller bearing for radial support, but recent tests indicate that the new split-inner-race thrust bearing may make the roller unnecessary.

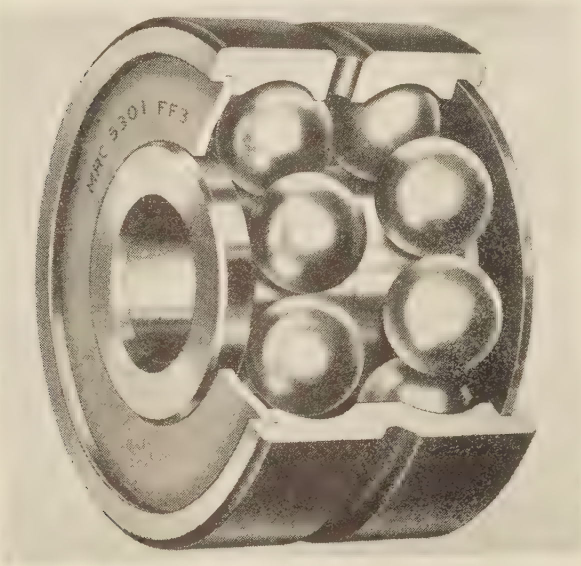


Fig. 7 Rocker-Arm Ball Bearing

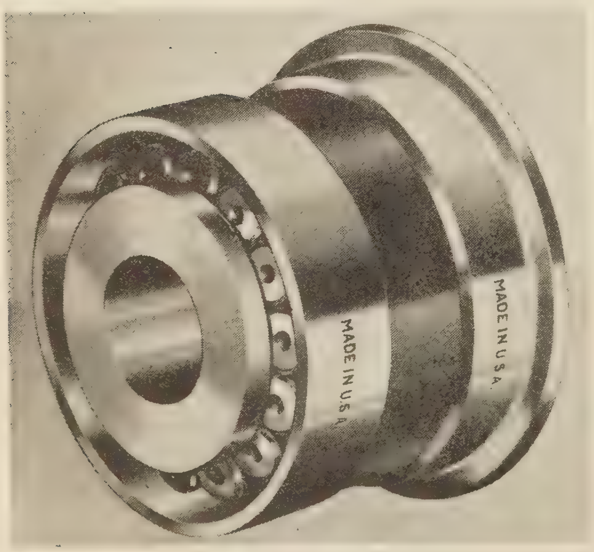


Fig. 8 Tapered Roller Bearing for Rocker Arms

Rocker-Arm Bearings

Fig. 7 shows a current large engine rocker-arm bearing. This is a one-piece double-row ball bearing, with the maximum possible number and size of balls and no cage or separator. Tapered roller bearings of roughly similar design are used, Fig. 8.

Very fine race finish and unusually accurate bearings help ward off "false brinelling,"[6] a race-pocketing condition which results from the very short movements (about $^1/_{32}$ in. on the race path).

Antifriction bearings of this back-to-back construction are common on radial-engine rockers, because the push rods angle considerably and produce severe sideways tilting effects, which this bearing type resists. One engine accidentally used the wrong type for resisting tilt; the resultant side movement of the valve roller produced elliptical wear of the valve guides and in one case broke off the valve head, with unfortunate results.

The smaller radial engines still retain two single-row ball bearings opposed, thus obtaining a similar effect with standard parts.

Besides relatively high loads, the rocker-arm location also involves high temperatures, high enough to definitely soften the common ball- and roller-bearing steels. Checks by the softening indicate 450 F frequently and higher in rare cases. Fortunately, these occur only during periods of maximum engine output and do not last long. The indicated research is for steels that will maintain high hardness at these temperatures.

Noteworthy is the fact that inner races suffer more than outer races. The heat evidently travels along the cylinder walls, and slight improvements might be obtained by reduced heat paths and localized greater cooling, where the strength requirements for rocker-box support permit.

The same heat problem accounts for the relatively poor performance of the early, grease-lubricated, rocker bearings on high-output radial engines. The temperature undoubtedly baked the grease out rapidly leaving the bearings unprotected and adding a hard carbon residue that fostered false brinelling. High-output engines have now been converted to oil lubrication for rocker arms giving:

1 Maintained lubrication.

2 Covering of surfaces to help keep the air away and reducing the "fretting-corrosion" action of false brinelling.

3 Better heat transfer to the cooling surfaces.

Oil lubrication has not been found necessary on some of the smaller engines with lower head temperatures.

On in-line engines where push rods are perpendicular to the rocker axis, no side tilt occurs; ball bearings have not been found necessary; bushings prevail with occasional use of needles for their lesser oil demands. Bushings are also on test for the radial engine.

Supercharger Bearings

Extreme speeds and large power concentrations have been successfully met with the same superprecision ball bearings originally developed for high-speed grinder spindles. A typical mounting, Fig. 9, delivers about 100 hp maximum at 36,000 rpm on a $^3/_4$-in. shaft. The counterbore or one shallow-shoulder type (Gurney "radial") is commonly used for its greater number of balls and for the one-piece retainer, made for this service out of tightly laminated bakelite tubing with cotton base.

[6] "Lubrication and False Brinelling of Ball and Roller Bearings," by J. O. Almen, *Mechanical Engineering*, vol. 59, 1937, pp. 415–422; also discussion by Thomas Barish, *Mechanical Engineering*, vol. 59. 1937, pp. 703–704.

Note one of the three bearings carrying only the thrust load resulting from the air pressure on the back side of the rotor. The occasional reverse loads are taken by one of the radial supports. This isolation of radial and thrust loads yields more capacity per space and theoretically allows higher top speeds.

One very difficult problem is sealing against blower pressure, alternating with high suction when throttled. Oil leakage at this point may dangerously foul the valve system. In very similar designs for cabin superchargers, the seals must be so tight as to prevent a leakage of even a "smell" of oil. The most common method provides two seals with a very liberal drain to the atmosphere between them to carry off any engine-oil leakage and to permit only air leaking into the blower system.

The design of Fig. 9 was at first frequently used with two cylindrical roller bearings and one Conrad ball bearing for thrust only but in the United States was superseded by the three interchangeable radial-type ball bearings shown. However, it is now questionable whether the change in bearing type contributed as much to the great increase in reliability as did the increased accuracy and certain radical changes in mounting principles:

(*a*) Liberal oiling instead of the scant lubrication previously thought necessary for top speeds. However, the lubricant is not passed through the bearing but is permitted to wash off the sides, serving mainly for heat dissipation and temperature equalization. In one test on an engine with thermocouples on the bearing outer race, the temperature started at 295 F with very slight oil feed; gradually dropped to 225 F as the oiling increased, and then started to rise again slowly. These temperatures may seem high, but the entering oil starts at 170 to 190 F.

(*b*) Relatively loose bearings and looser fits in the housings. An extensive German test[7] indicated that the internal looseness served mainly for oil films which otherwise developed excessive heating at high speeds. Allowances must also be made for appreciable temperature differentials, especially when an aluminum housing may be considerably chilled by a large volume of very cold stratosphere air. In one extreme case, the bearing outer race needed 0.0025 in. radial looseness in the housing.

(*c*) Present bearing-capacity tables are practically ignored. Apparently the lower loads that go with extreme speeds are all within the "fatigue limit." The major problem is removing the heat generated within a small volume of metal. Even with very low coefficients, there is an appreciable friction energy to be dissipated because of extreme speeds and high power. Based on present ratings, factors down to 0.35 (load/rating) are in use here and 0.20 on one foreign military engine.

In spite of these low factors, astonishing reliability has been achieved at high speeds; as low as one or two field failures in 10,000 sets. Of course, this is based on conservative air-line practice of replacing supercharger bearings at every major overhaul (500 to 600 hr) even though it is rarely indicated by the condition of the bearings.

Antifriction bearings prove highly desirable for the overhung rotors, because the bearings are so close together, and small radial and axial clearances must be maintained between the blower and casing. Some superchargers "straddle" mount the blower between bearings, and bushings become more practical, but they involve high oil flow and careful design, especially for thrust load. One European design employs a bushing at one end and a ball bearing for thrust and radial loads at the other end.

Controllable-Propeller Bearings

The blades of a controllable propeller require rigid support and fairly free rotation for blade-angle changes. The bearings must be designed for the following:

1 Extremely large centrifugal forces (200,000 lb maximum).
2 Aerodynamic forces at large overhangs.
3 Severe tendency to vibrate and false-brinell.

The late German and English designs use mostly tapered roller bearings or ball thrust bearings.

One of the most popular makes of American controllable propellers uses a flat-plate thrust bearing with short cylindrical rollers of square section, Fig. 10, and two separate bushings inside the blade for overhung air forces. Most of the other United States designs and one English propeller employ what is now called "stacked bearings" (Figs. 11 and 12), a series of angular

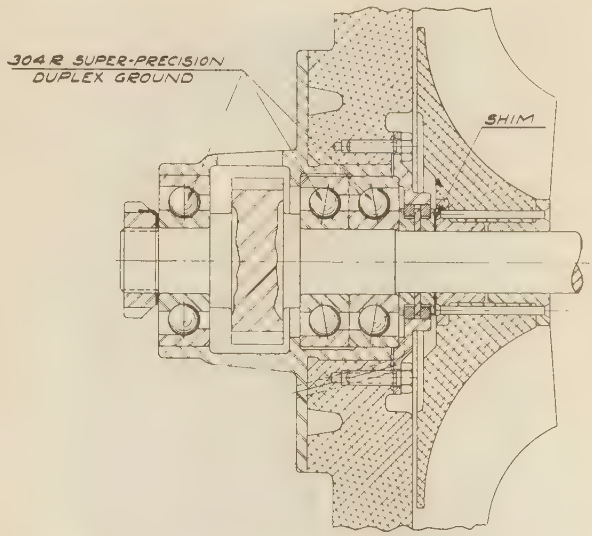


Fig. 9 Typical Supercharger-Bearing Layout

Fig. 10 Flat-Plate Thrust Bearing for Controllable Propeller

[7] "Experiments on Ball and Roller Bearings Under Conditions of High Speed and Small Oil Supply," by Gunter Getzloff, *Luftwaffe Forschung* Jahrbuch, 1938. Translated in N.A.C.A. Technical Memorandum No. 945, 1940.

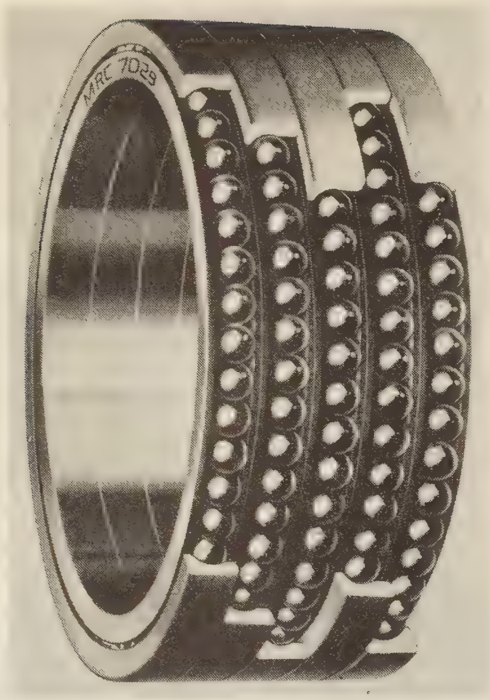


Fig. 11 Stacked Bearings for Holding Blades on Controllable Propellers

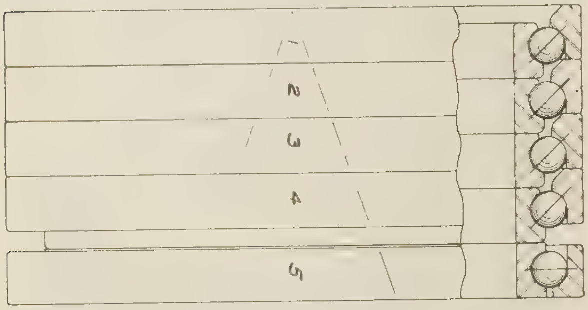


Fig. 12 Offset Stacked Bearings With Preload Bearing

contact-type ball bearings, ground to divide load.[8] Several rows of small balls dividing the load provide larger thrust capacities per pound weight and for the space than can be obtained with one row of large balls. The usual "stack" weighs about 6 lb for 250,000-lb-thrust brinell capacity, a very much higher intensity of loading than has ever been used before.

To obtain such high capacities, the race curvatures are made to extremely close conformity with the balls. No retainer is now used, but may yet be introduced because of ball-to-ball scoring which, however, has not caused any failures.

The race sections are cut to a minimum for space and weight saving but introduce serious production problems in maintaining tolerances and roundness. Also, large race distortions occur under the heavy loads. Although both inner and outer rings are loosely fitted for assembly, thrust load expands the outer and contracts the inner rings to provide very tight radial support. Furthermore, this distortion is not uniform. The rings tend to become conical and polygonal. Race sections have been recently redesigned with an "offset," Fig. 12, to keep the bores and outside diameters more nearly cylindrical under the load.

Fig. 12 also shows a preload bearing. By pulling the stacked bearings very tightly initially against the preload bearing, all looseness is eliminated providing smoother throttling conditions and also less deflections under load for better location of gears on the propeller blade. The preload bearing also provides inner radial support even when the operating thrust of the blade removes all the preload.

Load variations and vibration produce considerable working or "fidgeting" between the bearings and the blade and also on the outside diameters. Severe "galling" results, sometimes reaching the stage where it might be called "gouging." This has been alleviated by slightly looser fits, and even more by special plating, both on the blade and on the bearings; usually a hard material on the blade and a softer thin plating on the bearing bores. So-called antigalling lubricants or compounds have proved of little help as might be expected under very high unit pressures. Likewise rustproofing or treating the surfaces has not so far been useful as the galling is not a true "false brinelling" or "fretting corrosion."

False brinelling refers more exactly to what happens at the ball-race contact under the continuous slight oscillations of an automatic propeller. The most effective antidotes are, first, adequate bearing size to keep the unit pressures down and, second, complete submersion of the bearings in oil or special greases. In any case, the large centrifugal force packs the lubricant into the bearings quite tightly and a very effective seal is required.

A new concept of "brinelling capacity" or "nonbrinell" capacity has been developed in selecting the bearings for the occasional large momentary loads, i.e., the maximum load that can be applied without permanent surface deformation. A surprisingly large step exists between the first permanent surface impression visible under a small magnification and the first measurable depression.

However, this brinelling capacity is not the only control on bearing size, since such loads cannot be employed often, and since the automatic propeller has eliminated large loads from momentary overspeeds as in a dive. The other control of size is resistance to false brinelling under normal loads and the brinelling capacity serves only as a method of comparison.

Also worthy of mention is the development of smaller controllable propellers where single-row ball thrust bearings are likely to be standard for simplicity and low cost.

Normally the friction of a ball or roller bearing is small compared with the torque needed to hold the blade position against centrifugal "self-righting" forces. In certain cases, however, friction becomes critical, especially with counterweighted blades. Then flatter race curvatures may be employed even at some sacrifice of capacity and, in the case of stacked bearings, the use of ball retainers or cages helps very appreciably.

Very thin bearings similar to the stacked bearings are also employed for gears, cams, etc., in the blade-actuating mechanism.

[8] The method of obtaining load division is described in an article, "Preloaded Antifriction Bearings," by Thomas Barish, *Machine Design*, vol. 3, Oct., 1931, pp. 36–40.

Bevel Gears in Aircraft

By ALLAN H. CANDEE,[1] ROCHESTER, N. Y.

This paper illustrates and describes some important applications of bevel gears in aircraft, and explains some of their advantageous features. The selection of bevel gears instead of parallel-shaft gears is dictated in many instances by the locations and arrangements of the driven equipment and the source of power. In cases where the related parts can be arranged to suit either general type of gear, the designer will make his selection according to experience and preference.

Propeller Drives

One of the most important applications of gears in aircraft is in the main drive from the engine to the propeller, in which a medium reduction of speed is usually required. The planetary arrangement of bevel gears, Fig. 1, has been very successful. In the earlier drives of this type, the gears had straight teeth which were hardened but not ground. In newer applications, the gears have curved teeth of zero spiral angle (Zerol teeth), which are ground after hardening. Grinding the teeth is beneficial, because the improved accuracy so obtained increases the assurance that each planet pinion will carry its proper share of the load.

In the application of this planetary arrangement to an aircraft propeller, one of the side gears is held stationary; and the other is the driving gear. The spider which carries the several planet pinions drives the propeller. When right-angle bevel gears are used, Fig. 2, the reduction ratio is 2/1, irrespective of the numbers of teeth.

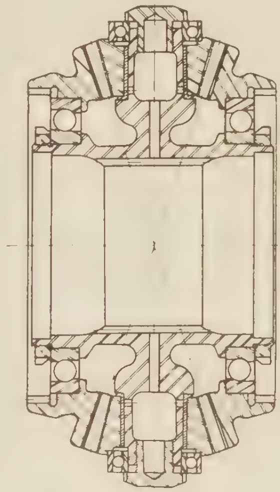


Fig. 2 Planetary Bevel Gears, 90 Deg Shaft Angle
(With one gear fixed, other gear drives spider at speed reduction of 2/1.)

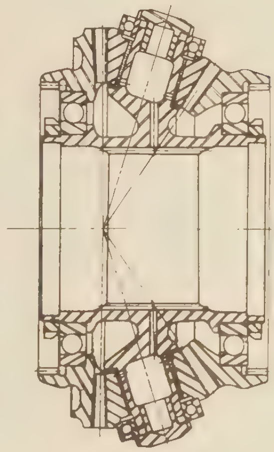


Fig. 3 Planetary Bevel Gears, Two-Speed
(Angular bevel gears. Higher or lower reduction ratio to spider, according to side gear used as driver.)

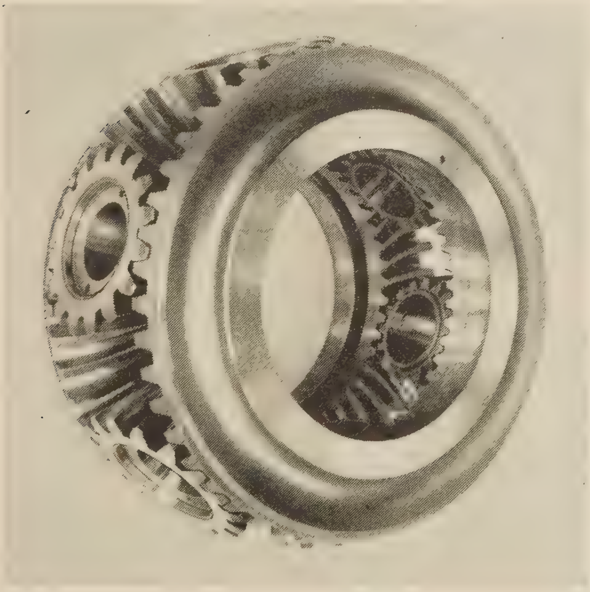


Fig. 1 Planetary Bevel Gears for Propeller Drive
(90-deg shaft angle, 2/1 reduction, Zerol teeth which are hardened and ground.)

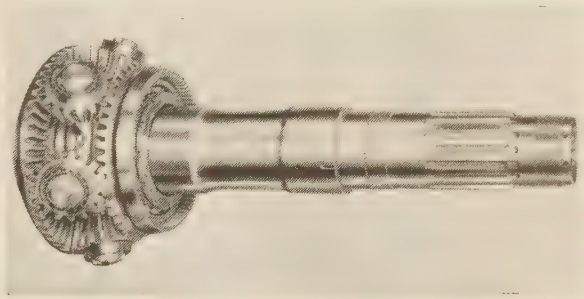


Fig. 4 Planetary Angular Bevel-Gear Unit Assembled With Shaft
(Continental Aviation & Engineering Corp.)

By changing to angular bevel gears, the two side gears become unequal, as in Fig. 3, and ratios different from 2/1 can be obtained. Fig. 4 shows such a unit. In this case, the internal spur teeth, which act as splines for fixing one gear and driving the other, are designed to be exactly similar. It is thus possible to turn the unit end for end, and to select either of the two reduction ratios thus available (as 1.82/1 and 2.22/1).

The reason for using these planetary arrangements is the decrease in size and weight over ordinary single-reduction gearing, due to carrying the load at several tooth contacts instead of at only one. In order to obtain a rough idea about how much this reduction in size may amount to, we may consider geometrically similar bevel gears in planetary and single-reduction arrangements. These would have the same numbers of teeth and the

[1] Mechanical Engineer, Gleason Works. Mem. A.S.M.E.

Contributed by the Aviation Division and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers.

same ratio of face width to cone distance. Then the formulas generally in use, both for surface durability and for strength of gear teeth, show that gear diameter should vary directly with the square root of the torque. With six planet pinions having tooth contact on two sides of each, the individual tooth load is decreased to $^1/_{12}$ of the amount in a corresponding single-reduction pair of equal gear diameter. Accordingly, for equal stresses, the diameter in the planetary arrangement can be decreased approximately in the ratio of $\sqrt{12/1} = 3.46/1$. This is a deciding factor in aircraft design, even though the cost of the eight gears and pinions is obviously more than of a larger single pair.

Efficiency of Planetary Drive

The question of comparative efficiency in these planetary drives naturally arises. It is well known that some planetary arrangements and applications of gears are seriously inefficient, when some of the gears are not only heavily loaded but also rotate at comparatively high speed. In the case of the bevel-gear arrangement shown for propeller drives, it is therefore interesting to find that the efficiency is practically the same as that of single-reduction gears. This will be explained by reference to Fig. 5.

In order to make a simple and direct comparison of the planetary and single-reduction arrangements, it is assumed at first that there is only one planet pinion in the assembly, and as many features as possible are made the same in both arrangements, as follows:

1 Same mean radius R in the gears and $R/2$ in the pinions;

2 Same number of teeth N in the gears and $N/2$ in the pinions;

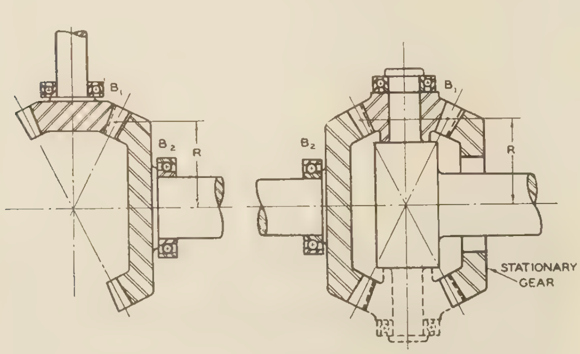


Fig. 5 Comparison of Single-Reduction Bevel Gears and Planetary Bevel Gears

(Multiple-tooth contacts in planetary arrangement allow decrease in size.)

3 Same torque T and tangential load $W = T/R$ on the driven gear of the single reduction, and on the spider of the planetary reduction;

4 Same assumed amount of rotation of the driven gear and of the spider.

It is seen then that the amount of tooth sliding is the same at each place of engagement, and that the load is W on the single-reduction pinion and $W/2$ on each side of the planet pinion. The friction loss in the two contacts at load $W/2$ on the planet pinion will obviously be practically the same as in the one contact at load W on the single-reduction pinion. In this way we arrive at the interesting fact that, with gears of similar proportions, the friction losses and therefore the efficiency of the gear teeth are the same, at least approximately, in the 2/1 planetary reduction and in a simple 2/1 single reduction.

This analysis has been made with the assumption of a single planet pinion. A change to multiple pinions of course does not alter the result.

When values of gear efficiency are mentioned, they usually include the bearings. The loss in ordinary plain bearings, with a coefficient of bearing friction of 0.05, is less than one half the loss in the gear teeth. In ball bearings, the coefficient of friction is much lower, and the loss in bearings is only from 0.1 to 0.2 times the gear loss, thus being relatively unimportant. The comparative conditions in the supporting bearings in Fig. 5, however, will be pointed out. In the ball bearings B_1 of the pinions, the thrust load is the same for the planet pinion and single-reduction pinion. The friction loss due to radial load in the plain bearing of the planet pinion, of course, is greater than in the ball bearing of the single-reduction pinion.

The thrust load on bearing B_2 of the driving gear in the planetary arrangement is one half the value on the corresponding bearing in the simple reduction, because the tooth loads are, respectively, $W/2$ and W. On the other hand, the speed of rotation of the driving planetary gear is twice that of the driven gear. Therefore, the bearing losses due to thrust are practically the same. In the single reduction, bearing B_2 is subjected to a radial load, whereas with multiple-planet pinions, the corresponding bearing B_2 in the other arrangement has no radial load. These general considerations show that differences in bearing losses in the two arrangements are comparatively unimportant.

The builders of the 2/1 planetary bevel-gear propeller drives have made tests and report efficiencies of 98.5 to 99 per cent. An Appendix to this paper gives an analysis of friction losses and a method of calculating the efficiency in gear teeth; and in an example in which the coefficient of sliding friction is assumed as 0.1, the result obtained is 98.7 per cent (Table 4). These gears would be used in any case, because of the saving in space and weight; but it is very satisfactory to find that they also compare so favorably in respect to efficiency.

Drive for Coaxial Propellers

Another type of bevel-gear drive is for two coaxial propellers rotating in opposite directions, Fig. 6. Bevel gears are used in the familiar differential or reversing arrangement. One propeller is driven directly by a shaft extending through the gearbox. The second is driven in the opposite direction by the gears and a sleeve surrounding the inner shaft.

Coaxial propellers are not yet in wide use, but they may be adopted more frequently in the future. They increase the power at a single propeller location, with smaller diameters. They also eliminate the torque effect of a single propeller.

Driving Arrangement of the Future

In present-day aircraft, the engines for multiple propellers are housed in enlargements of the wings, or nacelles, which increase drag. An arrangement for aircraft propulsion which has long been considered is to locate one or more engines in the fuselage and to connect them to propellers out on the wings by means of gears and shafting, as indicated in the diagram, Fig. 7. Such an arrangement will not only decrease the drag and make a larger portion of the engine power available for useful work, but will also give accessibility to engines for making adjustments and minor repairs during flight. Transport planes of the future, both commercial and military, almost certainly will use some such arrangement. Fig. 7 shows one engine driving two propellers. Of course a separate engine can be used for each propeller. Bevel gears are naturally required for this plan.

Variable-Pitch Propellers

Variable-pitch propellers, which operate at constant speed, naturally make use of bevel gears. The angular direction of the blades is varied, in order to control speed and power, Fig. 8. Several methods of operation and control are in use. In Fig. 9, straight bevel-gear teeth on the inner ends of the blade shafts

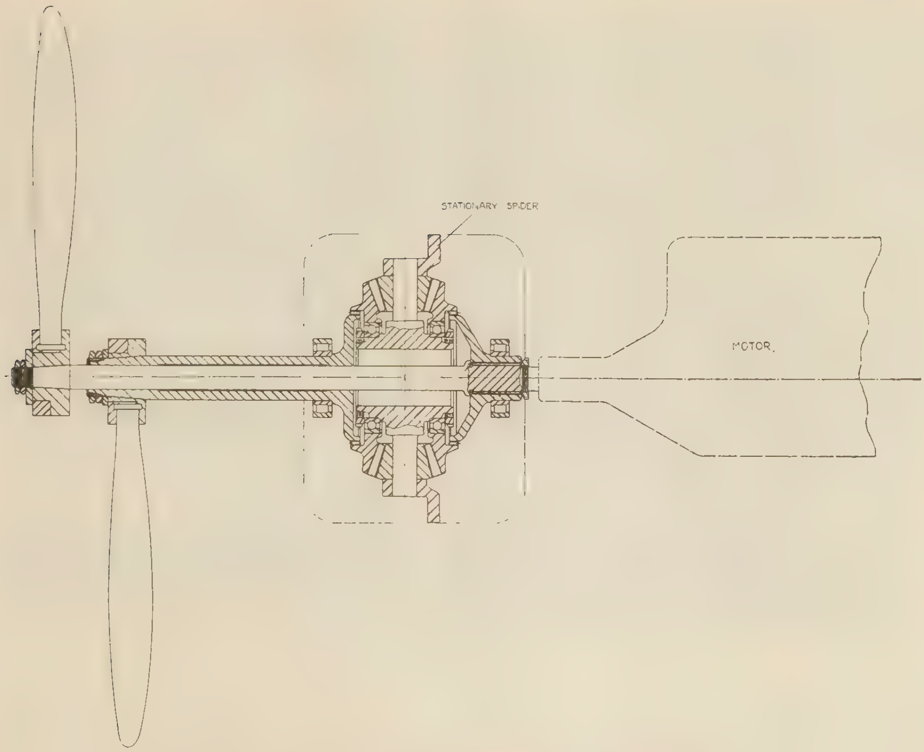


Fig. 6 Bevel-Gear Drive for Coaxial Propellers
(Counter-rotating propellers at a single location exert more power and eliminate torque on plane.)

are engaged with a central bevel gear. This gear is given a slight relative rotation by means of hydraulically operated rollers moving in cam slots in order to change the pitch of the blades. In this design, the twist of the blade is resisted through the gears.

In a second form of propeller control, Figs. 10 and 11, the bevel gears have spiral teeth which are hardened and very accurately ground, so that the parts can be assembled with practically no backlash. This decreases likelihood of vibration. Here again, the gears carry the twist load of the blades.

In a third propeller arrangement, the position of each blade is controlled by an individual hydraulic piston and cylinder. Bevel gears are used, however, to make certain of the same angular position of all three blades.

Advantageous Features of Bevel Gears

Bevel gears have at least three important features which are different from other types of gears as usually made and which are very valuable in aircraft as well as in other fields:

1 Their conical form makes it a comparatively simple matter to control the amount of backlash when assembling the gears, by small adjustments of the relative axial positions. This feature is of particular value in obtaining proper load distribution, in the planetary arrangements which have been shown, by adjusting each pinion for the same amount of backlash.

2 Another feature in bevel gears is unusual flexibility in tooth design, which comes about from the methods of cutting and the types of tools used. In all methods of generating gear teeth, it is just as easy to cut long-and-short addendum teeth as standard-addendum teeth; but with bevel gears it is also just as convenient to cut teeth of one thickness as another. This being so, when maximum strength is important, it is customary to proportion tooth thicknesses in pinion and gear in the most favorable way. To do this with spur gears or helical gears is not usual and would necessitate the making of nonstandard cutters or hobs.

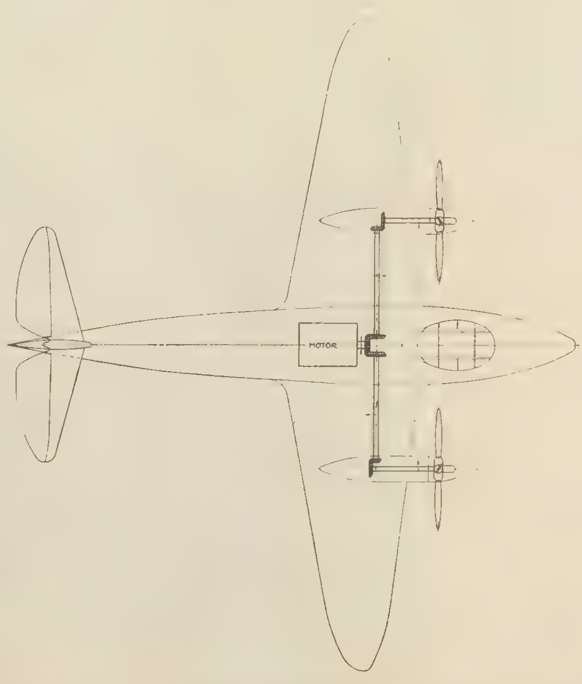


Fig. 7 Probable Driving Arrangement for Aircraft of the Future
(Location of engines in fuselage decreases drag and gives accessibility for adjustments and minor repairs.)

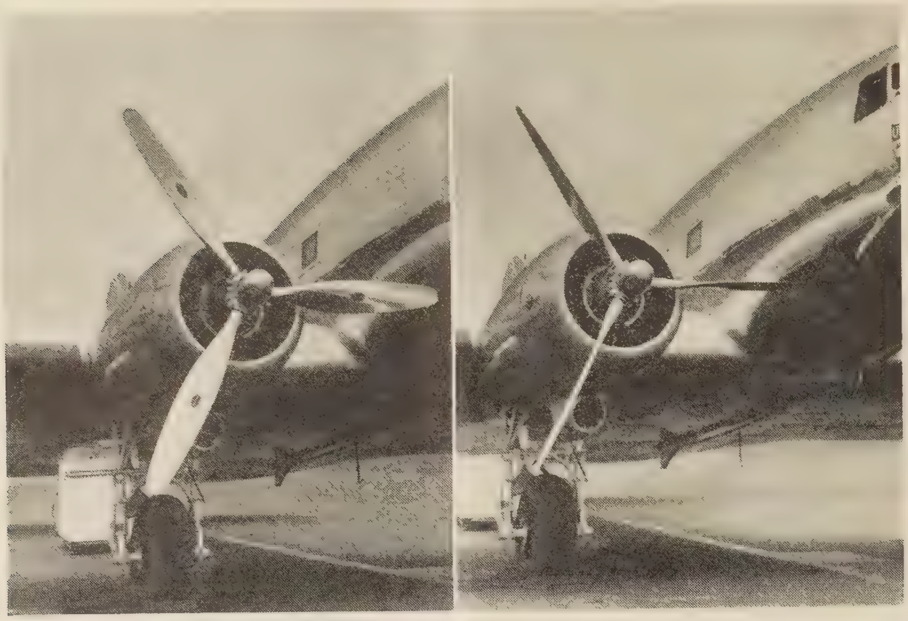


Fig. 8 Variable-Pitch Propellers
(Hamilton Standard Propellers. Allow operation at constant speed.)

3 An extremely valuable feature of curved-tooth bevel gears is the characteristic tooth bearing clearly shown in Fig. 12. The tooth surfaces are so generated that contact is kept away from the ends of the teeth. Then, when deflections occur under variations of load, or when gears are not assembled with exact accuracy, the load does not shift to one end of a tooth so as to cause early breakage and impaired operation. The degree of this localization of tooth bearing is completely controllable in the generating operation.

Requirements for Gears in Aircraft

There are two distinctly different sets of requirements for gears in aircraft. One set applies to gears which must transmit power at high speeds like those driving propellers, or which must turn very smoothly and accurately. In order to operate safely and smoothly they must be manufactured accurately and also assembled and maintained in proper relative position as accurately as possible. Such gears must be of precision grade with hardened and ground teeth for maximum results.

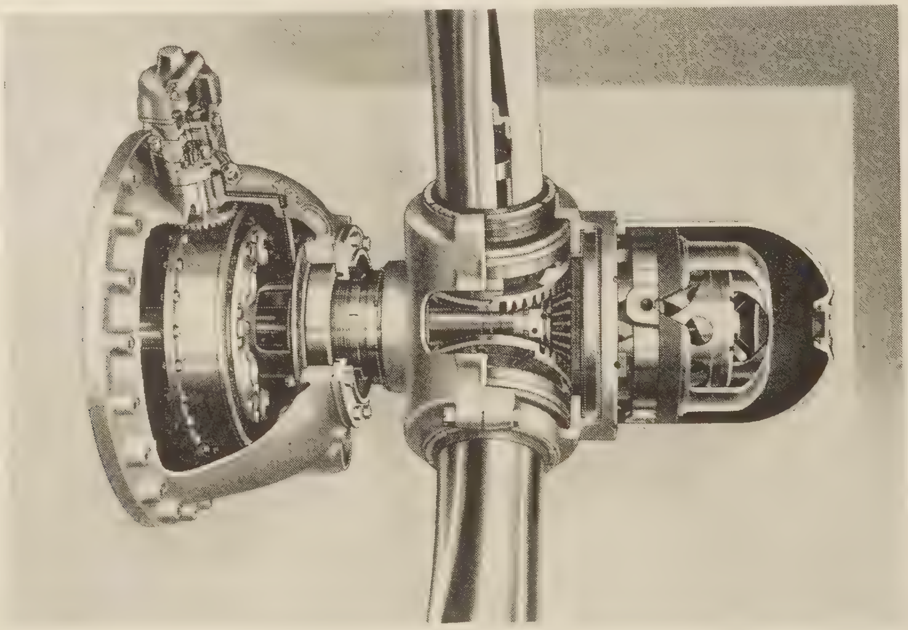


Fig. 9 Hydraulically Controlled Variable-Pitch-Propeller Hub
(Hamilton Standard Propellers. Bevel gears are used to rotate blades for change of pitch.)

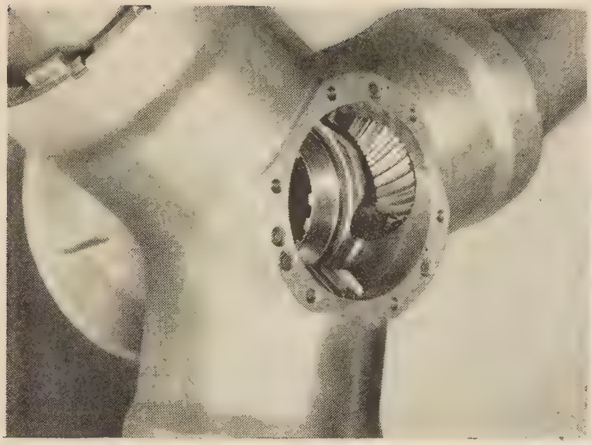


Fig. 10*a*

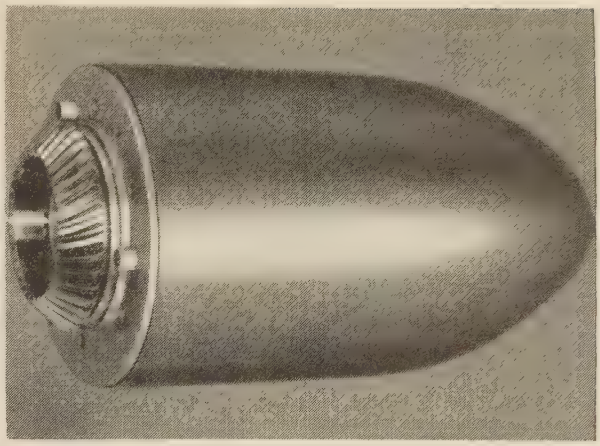


Fig. 10*b*

Fig. 10 Bevel Gears in Hub of Electrically Controlled Variable-Pitch Propeller
(Curtiss Propeller Division, Curtiss-Wright Corporation. Hardened and ground teeth in spiral bevel gears to adjust blade pitch are extremely accurate and uniform and allow assembling with practically no backlash.)

A different set of requirements applies to the gears which drive engine accessories and other equipment. These may have to carry only light loads, but frequently very wide assembling tolerances may have to be allowed. It often happens in aircraft construction that two engaging gears are mounted in separate assemblies; and when the assemblies are put together, considerable variations in the relative positions of the gears may occur. Furthermore, these gears are likely to be designed with thin sections which save weight but which lead to hardening distortions. Under such conditions, it is very beneficial to adopt curved teeth in bevel gears, which can be ground after hardening. This insures uniformity in the gears themselves, thus narrowing the range of variations and decreasing the time required for proper assembling. When sufficient quantities are to be manufactured, gears with ground teeth can be produced at no increase in cost. Zerol bevel gears with a generous amount of tooth-bearing modification of the kind shown in Fig. 12 compensate at least partly for inaccuracies in mounting and are proving remarkably satisfactory.

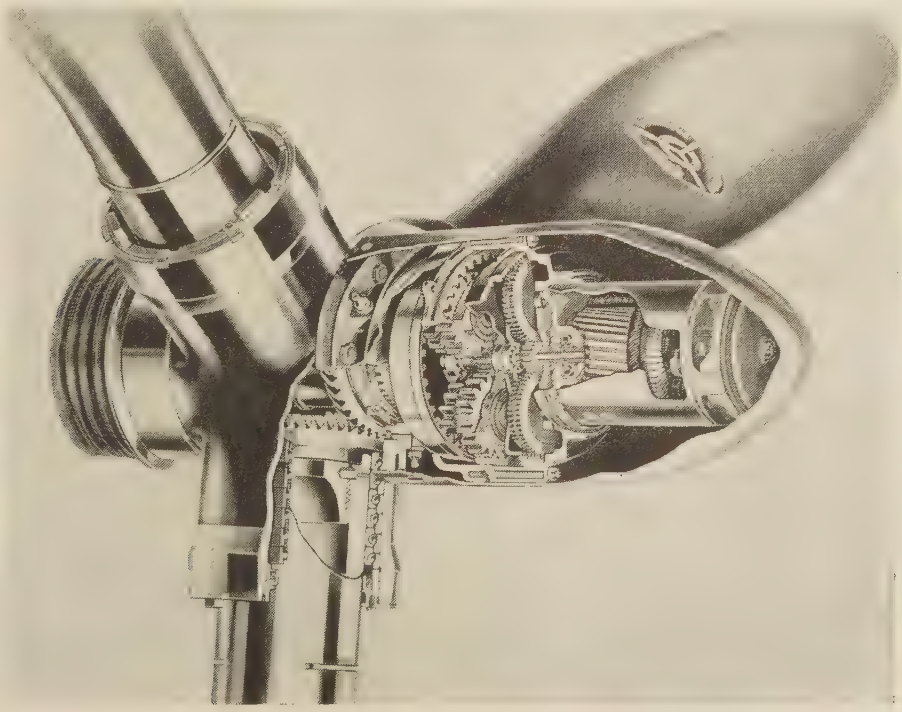


Fig. 11 Sectional View Showing Control Mechanisms of Electrical Variable-Pitch Propeller
(Curtiss Propeller Division, Curtiss-Wright Corporation. Electric motor controls pitch of blades through planetary spur gears and bevel gears.)

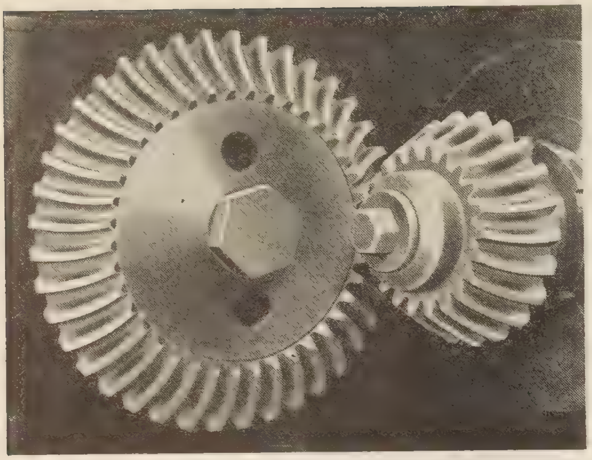


Fig. 12 Localized Tooth Bearing in Curved-Tooth Bevel Gears

(Amount of contact is readily controlled by varying diameters of cutters when generating teeth. Experience has demonstrated that such teeth operate more smoothly and have longer fatigue life.)

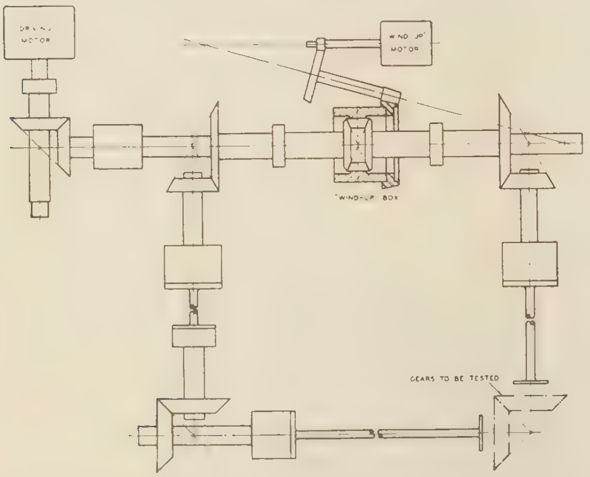


Fig. 13 Drive Diagram of Bevel-Gear Testing Equipment

(Gears are tested at full load with power circulating in a closed circuit.)

New Testing Equipment

One of the natural developments in the present expansion of aircraft construction is the designing and building of machines to grind the teeth of hardened bevel gears of larger diameter than in the past. It is obviously desirable to know the performance to be expected from such larger hardened and ground gears.

The Gleason Works was commissioned by the U. S. Army Air Corps a few months ago to install some new testing equipment of larger capacity than had previously been available. This installation is almost completed; and various tests of the load-carrying capacities of bevel gears are to be run in co-operation with the design personnel at Wright Field. The general arrangement for running the gears under load is diagrammed in Fig. 13. This is another application of the well-known method of circulating power in a closed circuit and having to supply only the power to overcome friction. A "wind-up" box is provided in combination with torsion shafts, by means of which the tooth load can be applied and varied while the gears are running. The data which will be obtained with this new equipment will be a valuable extension of the knowledge of bevel-gear performance previously gained from numerous tests of gears in automobile rear axles. At some later time, results will undoubtedly be made available through the authorities at Wright Field.

Miscellaneous Applications

It is not practical in the space of this paper to illustrate all the various applications of bevel gears in aircraft. Table 1, however, lists most of the kinds of equipment for which bevel gears have been used.

TABLE 1 BEVEL GEARS IN AIRCRAFT

Engine	Variable-pitch propellers
Main drive to propellers	Airplane, miscellaneous controls:
Accessories:	Retractable landing gear
Starter	Wing-tip floats
Fuel pump	Steering mechanism
Oil pump	Trimming tabs
Coolant pump	Radio
Camshaft	Bomb-door operation
Magneto	Gun turrets
Supercharger	
Governor	
Gun synchronizer	

When the designer is free to choose between bevel gears and parallel-shaft gears, the special characteristics of bevels should be considered. Whether the bevel-gear teeth should be straight,

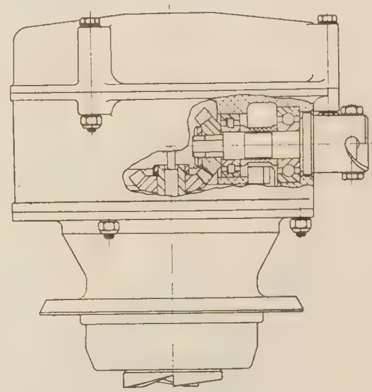


Fig. 14 Bevel Gears in Engine Starter

(Straight bevel gears give good service under relatively heavy load and at fairly high speed.)

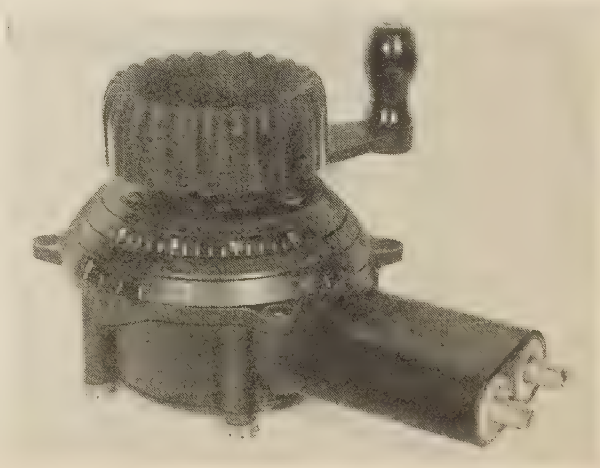


Fig. 15 Trimming-Tab Control

(Compact design built around two parallel symmetrically positioned shafts.)

spiral, or Zerol (zero spiral angle) must be decided according to the conditions and performance required. Two further examples of actual applications will be illustrated.

In the engine starter, Fig. 14, straight-tooth bevel gears have been entirely satisfactory. This is a case of intermittent service at relatively heavy load and fairly high speed.

An interesting application of small hypoid gears is shown in the control for trimming tabs, Figs. 15 and 16. In this unit, it was required to have two parallel and symmetrically arranged shafts independently adjustable by small intervals and turning in opposite directions. Hypoid gears with pinions in offset positions resulted in a compact and satisfactory design.

Bevel and hypoid gears are capable of greatly differing arrangements and variations to suit almost any application or requirement. On the basis of demonstrated performance, many designers prefer them whenever the general arrangement of related parts permits them to be used.

Appendix

Analysis of Friction Losses in Gear Teeth and Comparative Gear Efficiencies

Friction losses and efficiency of gears received more attention from engineers years ago than in recent times. This is natural because designers now know from numerous reported tests that the efficiency of a simple pair of gears will almost certainly be between 97.5 and 99 per cent; and exact determinations are not important. It has come about, however, that no publication, as far as is known to the author, gives a satisfactory analysis of friction losses for application to modern designs of gear teeth. It is believed that the analysis presented in this Appendix is of sufficient interest to justify having it on record.

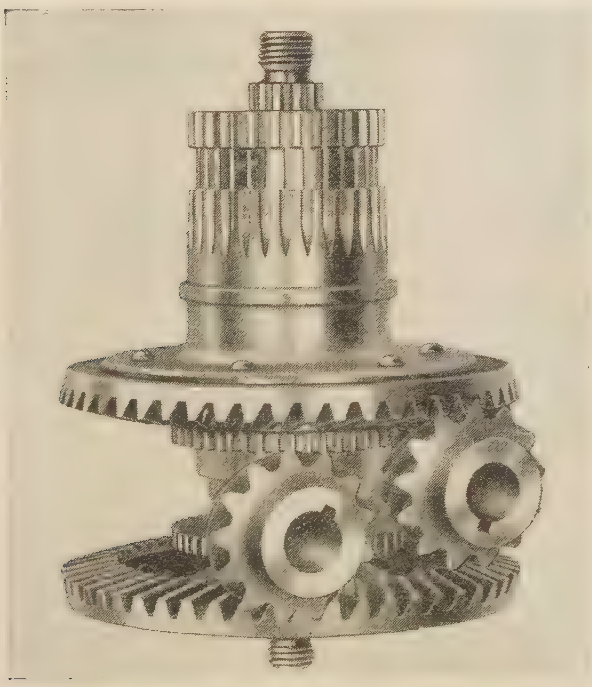


Fig. 16 Unusual Combination of Hypoid Gears

(Offset pinions become basis of compact design in Fig. 15. Relative angular positions of the two pinions can be adjusted, after which control operates as a unit.)

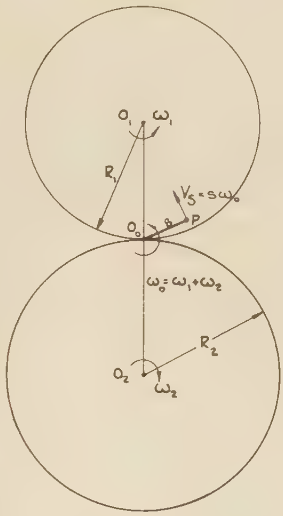


Fig. 17 Velocity of Sliding Between Gear Profiles

Sliding of Gear Profiles

Velocity of Sliding (Fig. 17):

R_1, R_2 = pitch radii
ω_1, ω_2 = angular velocities
$\omega_0 = \omega_1 \pm \omega_2$ = relative angular velocity. Negative sign applies when one gear is internal
s = distance from pitch point to point of contact
V_S = sliding velocity of profiles

Since the pitch point is the instantaneous center of relative motion

$$V_S = s\omega_0 \ldots\ldots [1]$$

This equation of course is given in textbooks.

Amount of Sliding:

dx = infinitesimal arc of rotation on pitch circles
$d\theta_1 = dx/R_1$ = angular displacement of first gear
$d\theta_2 = dx/R_2$ = angular displacement of second gear
$d\theta_0 = d\theta_1 \pm d\theta_2 = (1/R_1 \pm 1/R_2)\, dx$
= relative angular displacement

Then the infinitesimal increment of sliding is

$$dS = s\, d\theta_0 = (1/R_1 \pm 1/R_2)\, s\, dx \ldots\ldots\ldots\ldots [2]$$

The total amount of sliding which occurs during the displacement s from the pitch point is

$$S = \int s\, d\theta_0 = (1/R_1 \pm 1/R_2) \int s\, dx \ldots\ldots\ldots [3]$$

The value of the integral $\int s\, dx$ depends upon the form of the path of the point of contact, that is, on the kind of tooth profiles.

Amount of Sliding in Involute Teeth (Fig. 18):

φ = pressure angle, which is constant
$s = x \cos \varphi$
$dx = ds/\cos \varphi$

in which ds is the infinitesimal displacement along the line of action corresponding to dx.

Then $$S = \sec \varphi\, (1/R_1 \pm 1/R_2) \int s\, ds$$

or $$S = {}^1/_2 \sec \varphi\, (1/R_1 \pm 1/R_2)\, s^2 \ldots\ldots\ldots\ldots [4]$$

The general mathematical relationships of the sliding velocity and the amount of sliding, to displacement are shown in the graphs, Fig. 19. The upper graph is particularly convenient for the gear engineer, because the amount of sliding which occurs between two displaced positions of a tooth can be calculated as a corresponding area under a sloping line.

Length of Line of Action in Involute Teeth (Fig. 20):

Given R = pitch radius
a = addendum
φ = pressure angle
s = distance along line of action from pitch point to addendum circle of teeth

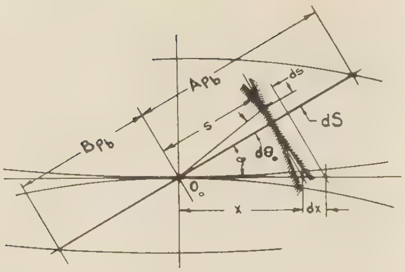


Fig. 18 Displacements and Sliding in Involute Teeth

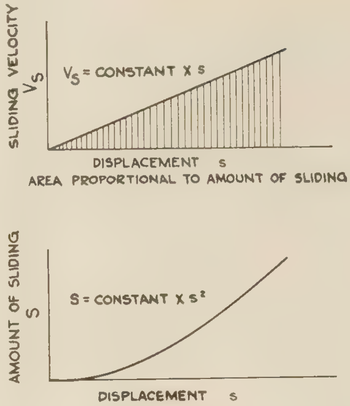


Fig. 19 General Mathematical Relationships for Sliding in Involute Gear Teeth

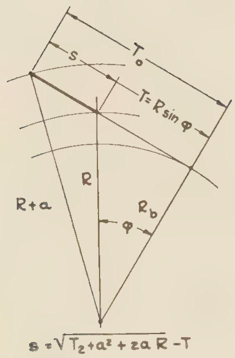


Fig. 20 Length of Line of Action for Addendum of Involute Gear

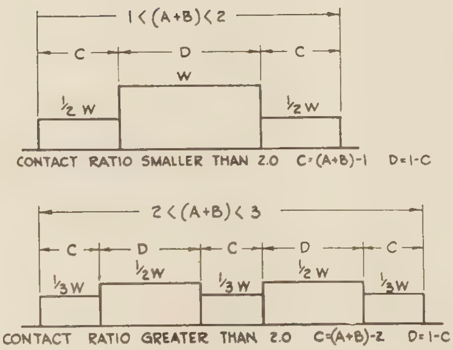


Fig. 21 Variations of Load on Tooth

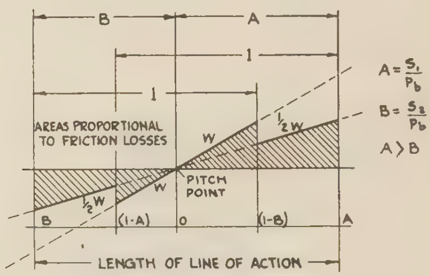


Fig. 22 Diagram for Summation of Friction Losses

A convenient method to calculate s is

$$T = R \sin \varphi \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

$$s = \sqrt{(T^2 + 2aR + a^2)} - T \quad\ldots\ldots\ldots\ldots [6]$$

For a pair of engaging gears, of course, the whole length of the line of action is made up of the two individual distances s on opposite sides of the pitch point.

Friction Losses and Efficiency

Effect of Contact Ratio on Load Distribution

"Contact ratio" is defined as the ratio of the arc of action to the circular pitch, or more conveniently in involute gears as the ratio of the length of the line of action to the base pitch. The base pitch $p_b = p \cos \varphi$. The contact ratio can never be less than 1 for continuous gear action. It sometimes exceeds 2. When the contact ratio lies between 1 and 2, there are two teeth in contact during part of the action. When the contact ratio lies between 2 and 3, there are three teeth in contact during part of the action and two teeth during the rest of the action.

In this analysis, it is assumed that when two teeth are in contact each carries one half of the load, and that when three teeth are in contact each carries one third of the load. This is a reasonable assumption to make, although it is not fulfilled exactly in actual gear contact.

Let the two portions of the line of action into which it is divided by the pitch point be given in terms of the base pitch, as Ap_b and Bp_b, as in Fig. 18. Also for greater simplicity let $p_b = 1$ Then the length of the line of action and the contact ratio are both represented by the sum $(A + B)$. Changes of the load on one tooth, as it moves along the line of action, are found to be of a cyclic nature and are illustrated in Fig. 21.

Lost Work of Friction on One Tooth:

W = total load tangential to pitch circles
W_N = load normal to tooth surface = $W/\cos \varphi$
μ = coefficient of sliding friction
μW_N = friction load tangential to profiles

Assuming first that the contact ratio is smaller than 2

S_1 = amount of sliding while one tooth is in contact
S_2 = amount of sliding while two teeth are in contact
E_f = friction work

Then
$$E_f = \mu W_N \left(S_1 + \frac{1}{2} S_2 \right) \quad\ldots\ldots\ldots\ldots\ldots [7]$$

In arriving at the value of $\left(S_1 + \frac{1}{2} S_2 \right)$ it is found that there are three possible cases, depending on the position of the pitch point in the line of action; that is, on the values of A as follows:

Case 1	Case 2	Case 3
$A < 1$	$A = 1$	$A > 1$

Case 1 applies, for instance, when the addendums of the two gears are equal or nearly so and is illustrated by the diagram in Fig. 22. The shaded areas are proportional to the friction losses. Positions in the horizontal direction correspond to displacements along the line of action, that is, to values of s. The two slopes used in the construction of the diagram correspond to the tooth loads W and $W/2$ for different portions of the action. By referring to Equations [4] and [7] and by inspection of Fig. 22, the total friction loss on the tooth is written as

$$E_f = \frac{1}{4}\mu\, W_N \sec\varphi\,(1/R_1 \pm 1/R_2)\, p_b^2\,[A^2 + B^2 + (1 - A)^2 + (1 - B)^2]$$

The expression in the brackets will be different for cases 2 and 3, and for additional cases with contact ratios of 2 and greater than 2. For convenience let the value in the brackets be denoted by K. Then the equation for friction work becomes

$$E_f = \frac{1}{4} W_N \sec\varphi\,(1/R_1 \pm 1/R_2)\; p_b^2\; K\; \mu$$

Useful Work by One Tooth. The useful work E_U performed by one tooth during its movement along the line of action, can be determined as the summation of the work done by single-tooth and double-tooth contact, in a way exactly similar to the method used to determine E_f. The same result is arrived at more directly, however, by recognizing that E_U must be the useful work during one revolution of a gear, divided by its number of teeth. It is then seen that $E_U = Wp = W_N\, p_b$.

Proportional Friction Loss. It is next possible to write the proportional friction loss as

$$f = \frac{\text{Lost work of friction}}{\text{Useful work}} = \frac{E}{E_U}$$

or

$$f = \frac{1}{4}\sec\varphi\,(1/R_1 \pm 1/R_2)\; p_b\; K\; \mu$$

which is the same as

$$f = \frac{1}{4}(1/R_1 \pm 1/R_2)\; p\; K\mu$$

A further simplification is obtained by introducing numbers of teeth instead of pitch radii, thus

$$N_1, N_2 = \text{numbers of teeth}$$

$$R_1 = N_1\, p/(2\pi) \text{ and } R_2 = N_2\, p/(2\pi)$$

Substitution of these values results in a final form of the equation for proportional friction loss as

$$f = (\pi/2)\,(1/N_1 \pm 1/N_2)\, K\mu \ldots\ldots\ldots\ldots\ldots [8]$$

Values of K. It has been noted that various cases occur with different values of A and B. Those most likely to occur for contact ratios not only smaller than 2, but also equal to 2 and greater, are indicated by the equations for K in Table 2. Some numerical values of K are given in Table 3. Attention is called to the fact that the minimum value of $K = 1$ occurs with the minimum contact ratio of 1 and with $A = B = 0.5$. For any different values of A and B and for any greater contact ratio, the value of K increases, because of increased sliding. Table 3 is directly interesting because it shows how friction losses vary with different amounts of tooth contact, in a pair of gears with given numbers of teeth.

Gear-Tooth Efficiency. Of course when the friction loss is known, the efficiency is obtained as

$$e = 100/(1 + f) \text{ per cent}$$

Tooth Friction in Bevel Gears

It is customary to use Tredgold's approximation for the cross-sections of the teeth in straight bevel gears. The foregoing analysis of friction in spur gears can be applied to the bevel-gear profiles so obtained. This is done most simply by the idea of the equivalent numbers of teeth in the bevel gears, Fig. 23.

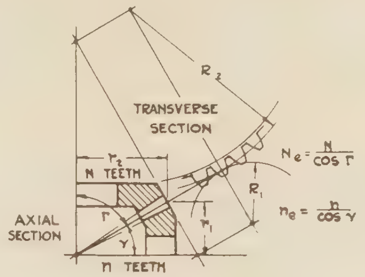


Fig. 23 Equivalent Numbers of Teeth in Bevel Gears (Tredgold's approximation.)

TABLE 2 EQUATIONS FOR K

Case No.1	Contact Ratio	Conditions (A) B)		K =
1	Smaller than 2.0	A < 1		$[A^2 + B^2 + (1 - A)^2 + (1 - B)^2]$
2		A = 1		$2[B^2 - B + 1]$
3		A > 1		$[A^2 + B^2 - (A - 1)^2 + (1 - B)^2]$
4	Equals 2.0			$[A^2 + B^2]$
5	Greater than 2.0	A < 2	B > 1	$\frac{2}{3}[A^2 + B^2 - A - B + 3]$
6		A < 2	B = 1	$\frac{2}{3}[A^2 - A + 3]$
7		A < 2	B < 1	$\frac{2}{3}[A^2 + 2B^2 - A - 3B + 4]$
8		A = 2	B < 1	$\frac{2}{3}[2B^2 - 3B + 6]$
9		A > 2	B < 1	$\frac{2}{3}[2B^2 + 3(A - B)]$
For Case 1, K also equals $2[A^2 + B^2 - A - B + 1]$				

TABLE 3 NUMERICAL VALUES OF K

A	B 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5					1.00							
0.6				1.04	1.02	1.04						
0.7			1.16	1.10	1.08	1.10	1.16					
0.8		1.36	1.26	1.20	1.18	1.20	1.26	1.36				
0.9	1.64	1.50	1.40	1.34	1.32	1.34	1.40	1.50	1.64			
1.0	1.82	1.68	1.58	1.52	1.50	1.52	1.58	1.68	1.82	2.00		
1.1	2.02	1.88	1.78	1.72	1.70	1.72	1.78	1.88	2.02	2.07	2.15	
1.2	2.22	2.08	1.98	1.92	1.90	1.92	1.98	2.08	2.11	2.16	2.23	2.32
1.3	2.42	2.28	2.18	2.12	2.10	2.12	2.18	2.18	2.21	2.26	2.33	2.42
1.4	2.62	2.48	2.38	2.32	2.30	2.32	2.29	2.29	2.32	2.37	2.45	
1.5	2.82	2.68	2.58	2.52	2.50	2.45	2.42	2.42	2.45	2.50		
1.6	3.02	2.88	2.78	2.72	2.64	2.59	2.56	2.56	2.59			
1.7	3.22	3.08	2.98	2.87	2.79	2.74	2.71	2.71				
1.8	3.42	3.28	3.15	3.04	2.96	2.91	2.88					
1.9	3.62	3.46	3.33	3.22	3.14	3.09						
2.0	3.81	3.65	3.52	3.41	3.33							

When N and n are the numbers of teeth, and Γ and γ are the respective pitch angles, the equivalent numbers of teeth are

$$N_e = N/\cos\Gamma \text{ and } n_e = n/\cos\gamma$$

Then the value $(1/N \pm 1/n)$ for spur gears becomes

$$\left(\frac{\cos\Gamma}{N} \pm \frac{\cos\gamma}{n}\right)$$

for bevel gears; that is, for bevel gears Equation [8] becomes

$$f = \frac{\pi}{2}\left(\frac{\cos\Gamma}{N} \pm \frac{\cos\gamma}{n}\right)K\mu \ldots\ldots\ldots\ldots\ldots [9]$$

This equation is recommended for practical use. Straight bevel gears, however, are analyzed most accurately by considering tooth profiles on a surface of constant cone distance, that is, on a spherical surface. Spherical involute profiles for bevel gears, corresponding to plane involute profiles of spur gears, are referred to in textbooks, but have probably never been made. Octoid profiles are only slightly different and are approximated in actual gears.

The general method of plane geometry applying to spur gears can be worked out in spherical geometry. This determination is too long to be included here, but comparative results will be given. Taking bevel gears with 14.14/14.14 teeth, equivalent to spur gears of 20/20 teeth, with 20-deg pressure angle and standard addendums, the comparative results are as follows:

	Contact ratio	Proportional friction loss
By plane geometry................	1.556	0.20552
By spherical geometry:		
Spherical involute profiles........	1.559	0.20555
Octoid profiles	1.527	0.20021

The differences here shown are obviously so small as not to justify the increased work of using the spherical method. They also show that there is no reason practically not to use the spur-gear method for straight bevel gears.

A sample calculation of the efficiency of bevel gears is given in Table 4.

Comparative Friction Losses in Gear Teeth

By applying the method derived in the foregoing analysis,[2] some interesting comparisons of the amount of friction in various designs of gear teeth have been obtained.

Value of Coefficient of Sliding Friction (μ)

Textbooks and handbooks usually state that this coefficient may vary from 0.1 to 0.25 or 0.3, and that under some conditions it may be less than 0.1, depending upon materials, surface smoothness, lubrication, and sliding velocity. It is assumed in this analysis that μ has a constant value throughout the gear-tooth action. Allowance for variation of μ with sliding velocity could be made, but would mean added complication. In the comparisons to follow, the convenient value $\mu = 0.1$ is used.

[2] The method of calculating friction losses in involute gear teeth here presented has been known previously, at least for the value of K in Case 1. Marks' "Mechanical Engineers' Handbook" gave the same equation except with different notation, in the first and second editions, but omits it in the third edition. The equation is also given in "Zahnräder," Schiebel, Julius Springer, Berlin, 1922 (first edition, 1911), p. 56, with a reference to "Zahnreibung," Kohn, *Zeitschrift des Vereines deutscher Ingenieure*, 1895, p. 459.

Comparison of Tooth Numbers

Spur gears, 20 deg pressure angle, equal addendums of $1/(DP)$			
Numbers of teeth....................	N/N	20/20	30/30
Friction losses.....................	f	0.0205	0.0149

It is of course apparent that, with greater numbers of teeth, there is less curvature of the pitch circles and therefore less sliding and less friction.

Comparison of Pressure Angles

Spur gears, N/N = 30/30, equal addendums of $1.0/(DP)$			
Pressure angle..................	φ	14½ Deg	20 Deg
Friction losses..................	f	0.0199	0.0149

All gear designers are familiar with the fact that, with standard working depth, the length of the line of action becomes shorter with an increase of pressure angle. This decreases the amount of sliding more rapidly than the increase in the load W_N normal to the tooth surfaces.

Of course, if it should be desired to decrease the amount of friction in teeth of 14½ deg pressure angle, it could be done by shortening the addendums. This, however, would decrease the contact ratio; and most designers are more concerned with durability as dependent upon contact ratio, than with slight variations in friction and efficiency.

Comparison of Addendum Proportions

Spur gears, n/N = 20/40, 20 deg pressure angle			
Addendums (1 DP)........	a_P/a_G	1.0/1.0	1.3/0.7
Friction losses............	f	0.0166	0.0173

As is to be expected, a departure from equal addendums means a slight increase in sliding and in friction loss. The change is so slight, however, as to be negligible in practical work.

Comparison of Bevel Gears and Spur Gears

20 deg pressure angle, equal addendums of $1.0/(DP)$				
Gears		Spur	Bevel	
Numbers of teeth..........	N/N	20/20	20/20	14.14/14.14
Friction losses............	f	0.0205	0.0156	0.0205

As has already been explained, the determination for bevel gears can be made on the basis of the equivalent numbers of

TABLE 4 CALCULATIONS FOR EFFICIENCY OF BEVEL GEARS

		Pinion	Gear
Number of teeth	N	16	48
Diametral pitch	P	1	
Pressure angle	φ	22°30'	
Addendum	a	1.220	0.620
Pitch angle	γ	18°26'	71°34'
Equiv. number of teeth	$N_e = N/\cos\gamma$	16.866	151.800
Equiv. pitch radius	$R = N/(2P)$	8.433	75.900
	$T = R\sin\varphi$	3.227	29.045
Line of action outside of pitch circle	$s = \sqrt{T^2 + 2aR + a^2} - T$	2.472	1.582
Circular pitch	$p = 3.1416/P$	3.142	
Base pitch	$p_b = p\cos\varphi$	2.903	
	$A = s/p_b$ (pinion)	0.852	
	$B = s/p_b$ (gear)	0.545	
Contact ratio	$(A + B)$	1.397	
Case No. 1)	$K = [A^2 + B^2 + (1-A)^2 + (1-B)^2]$	1.252	
Coef. of sliding friction	μ (assumed)	0.1	
Proportional friction loss	$f = 1.571\,(1/n_e + 1/N_e)\,K\mu$	0.013	
Efficiency	$e = 100\,(1 - f)$ per cent	98.7	

teeth. Bevel gears with 20 teeth, being equivalent to spur gears with 28.28 teeth, have a smaller loss and are more efficient than spur gears with 20 teeth. This fact, of course, has been indicated in textbooks and handbooks for many years.

Conclusion

The friction loss calculated by the foregoing method for a given set of gears naturally will differ somewhat from the true value because of differences between assumed and actual conditions. Such differences may be the following:

1 The assumed tooth profiles may be different from the actual ones. The determination of the amount of sliding for given profiles is merely a geometrical problem.

2 The actual distribution of load with two or three teeth in contact will not be uniform as assumed, but will be affected by deflections even in teeth of ideally correct form, and is almost certain to vary in the manufacture and assembling of the gears.

3 The value of the coefficient of sliding friction may not be correctly assumed, unless determined by experiment.

For any given conditions, however, the comparisons of various designs of teeth are considered to be reliable.

For gears with oblique teeth, such as helical and herringbone gears, and spiral bevel gears, further analysis must be made, to take account of the different conditions of load distribution.

Development and Performance of a Coal-Fired Unit Heater

By R. M. Rush,[1] Pittsburgh, Pa.

The paper describes an advanced design of coal-fired unit heater; explains the means by which high rates of heat transfer comparable to those of heating boilers are attained in an air heater, and by which the steel shell of the combustion chamber is kept at a safe temperature level; and gives results of tests of a large heater of this type. Various modifications of the installation arrangements are described, and the savings of critical materials and of man-hours, important under the present war conditions, attainable by the use of this type of heater are pointed out. A design of heater having all-refractory heat-transmitting surfaces, is also described. The reasons why the over-all saving of metal effected by the last-mentioned design must be disappointingly small, are explained.

Direct-fired unit heaters are being widely used in war production, largely because of their savings of critical material, amounting in many cases to over 50 per cent of the metal required to install a comparable steam-heating system. Most of these installations have been made for oil or gas firing, where it was comparatively simple to pipe the fuel to each unit. The introduction and continued improvement, over a period of 10 years, of such units, resulted in hundreds of installations, particularly in the steel industry. The recent restrictions on oil and gas made the coal-burning unit heater a logical development. The purpose of this paper is to outline one approach to the solution of the problems involved.

Development of Coal-Fired Unit Heater

In 1933, there was introduced for steel-plate air heaters a type of corrugated heating surface, with fins and deflectors of original form, and this combination for the first time made it practical, from the economic or commercial as well as the engineering standpoint, to transfer heat from the wall of a combustion chamber to a moving air stream at about the same rate per square foot of heating surface as that at which the surface of a heating boiler transmits heat to water. This may fairly be said to have revolutionized direct-fired unit-heater design. Being self-fired and compact, taking up no more space and requiring about the same power to drive as steam unit heaters, and giving efficiencies of 80 to more than 85 per cent, the units attained widespread use.

Sectional views of gas- and oil-fired units are shown in Fig. 1, and a unit with the outer cover removed in Fig. 2.

The recent restrictions on these fuels emphasized the need for a coal-fired unit heater which would be compact in size, dependable, and efficient. Coal-firing, because of the more intense radiation from the fuel bed, transmits more heat per square foot to some parts of the combustion-chamber shell, than in the gas- or oil-fired heaters, and heat abstraction from the outer surface must be adequately effective in order to avoid overheating the metal. A number of coal-fired heaters, having this corrugated heating surface with fins and deflectors, had been in operation since 1935,

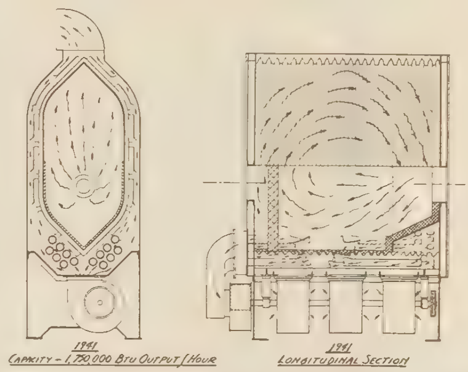


Fig. 1 Cross Section and Longitudinal Section of Redesigned Heater as Built in 1941

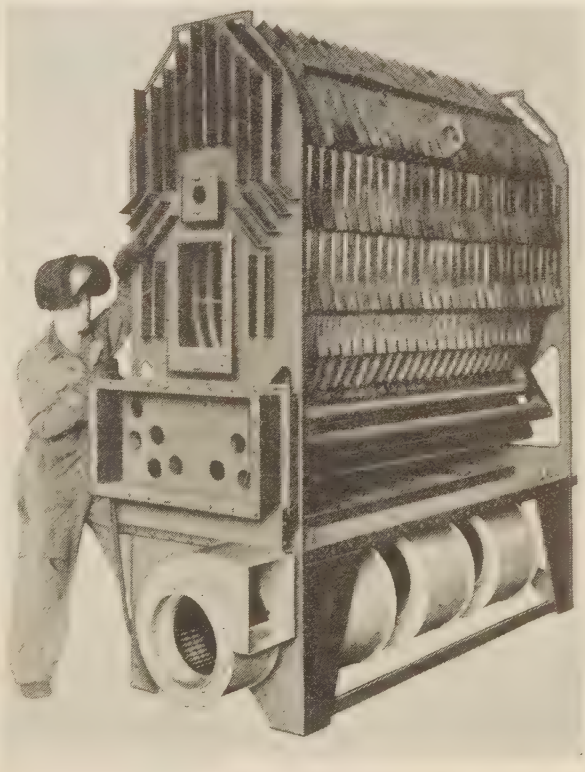


Fig. 2 Oil- or Gas-Fired Heater With Outer Cover Removed
(View shows fins and deflectors on corrugated surface, also tube bank, fans, and exhauster.)

[1] Manager, Industrial Department, Dravo Corporation. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

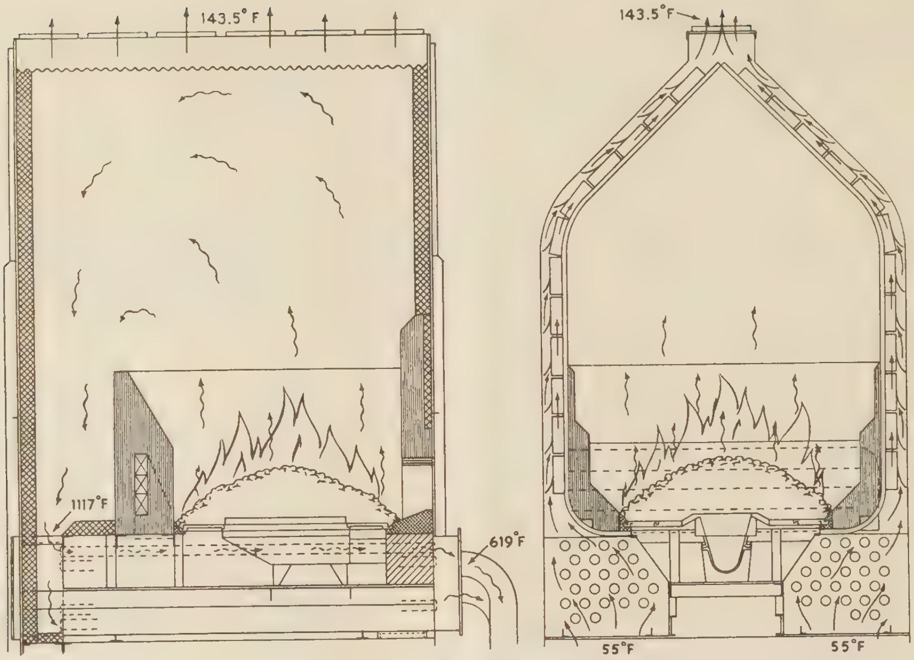

Fig. 3 Cross Sections of 4,000,000-Btu per Hr Coal-Fired Heater Showing Temperatures of Air and Gases During Test

and had given satisfactory results. The experience with these had shown this type of heating surface to be as successful with coal as with any other fuel, and indicated that, with an air velocity over the heating surface of 35 to 45 fps, heat abstraction was amply rapid to avoid overheating or burning the metal.

A development program was undertaken in the winter of 1941–1942, to bring the coal-fired heater to the same degree of perfection as the most recent oil- and gas-fired units. A coal-fired unit of 4,000,000 Btu per hr output was designed and built, and complete tests were conducted. The results of these tests indicated that the coal-fired unit had about the same heat-transfer rate per square foot of surface as the oil-fired heater, and that comparable efficiencies were easily maintained.

Cross sections of the coal-fired heater are shown in Fig. 3. The fuel is burned on a simple type of underfeed stoker, in a combustion chamber having a monolithic refractory lining, which is not unlike a modern air-cooled boiler setting. The plastic refractory is molded against the inner side of the corrugated surface extending about one quarter of the height of the combustion chamber and fastened to it with steel studs welded to the metal and embedded in the refractory. The flexibility of the corrugated steel shell as regards thermal expansion makes it a nearly ideal container and support for the refractory. Heat is conducted through these comparatively thin refractory walls so rapidly that the refractory is saved from deterioration. Since the temperature of the inside surface of the lining is below the fusion temperature of most kinds of ash, slag does not form nor clinkers fuse to the walls. From a practical operating standpoint, this makes an underfeed stoker very easy to handle, even when burning coal having a high content of ash of low fusion temperature. The ash takes the form of a light porous clinker which is easily removed.

The combustion conditions approach the ideal, in that there is plenty of furnace volume and sufficiently long flame path for the gases distilled from the coal to burn completely without smoke, before the combustion is arrested by their striking cold surfaces; the tightly welded combustion chamber prevents the infiltration of excess air, enabling high CO_2 to be maintained. Draft in the furnace, suitable to the rate of operation, is maintained by the self-contained induced-draft fan, independently of the variables of natural draft.

The fuel-bed temperature is comparatively low, due to rapid radiation to the large area of exposed heating surface above the refractory and in sight of the fire. The greater part of the total heat transmission occurs through this "black surface" of the upper part of the combustion chamber. The bridge wall is also cooled by means of a duct through its center through which a stream of air, induced by vanes in the air passages at the sides, flows horizontally from one side of the heater through the bridge wall to the opposite side. The bridge-wall refractories are thereby kept so cool that clinkers do not adhere, while the cooling air effects additional heat recovery.

Details of Experimental Unit

A long series of tests of this 4,000,000-Btu per hr unit established its ability to withstand even such conditions as sudden power failure while operating at 20 per cent overload. From a commercial standpoint, however, there were certain objections to the design, e.g., the location of the fans alongside the combustion chamber required excessive floor space, and the plenum chamber, with tubes beneath, added to the already excessive height. Accordingly a new design based on the established heat-transfer rates was developed which overcame these objections, as shown in Fig. 4. A unit of 1,000,000 Btu per hr output was built to this design and found to give results comparable to those obtained with the larger heater of the previous design, while requiring about one half the floor space and about one third less weight per 1,000,000 Btu. In this design the tubes are placed, to save height, above the combustion chamber, and the outlets are at the rear, but near the peak of the combustion-chamber shell. Means therefore had to be provided to prevent the gases from short-circuiting directly to the outlet. At first this was effected by a refractory-covered metal baffle. Later, in order to prevent overheating of the metal, the baffle was air-cooled. The cooling proved so effective that the refractory could be eliminated except

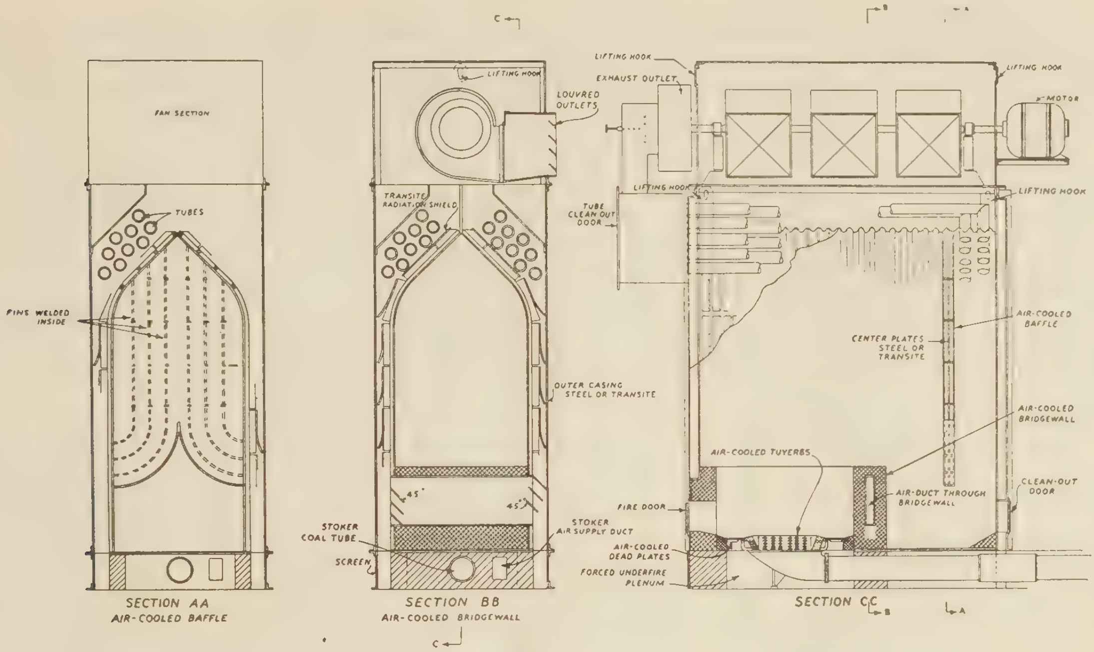

Fig. 4 Air-Cooled Setting

(The coal-fired heater has a completely air-cooled setting. Some of the more outstanding features of this principle are illustrated. Sectional views show air-cooled baffle; air-cooled bridge wall; air-cooled stoker dead plates and tuyères; nonmetallic radiation shield.)

Fig. 5 Detail of Air-Cooled Baffle

for a small amount at its lower edge, Fig. 5. The all-metal baffle with streamlined air passages, and a center partition, which acts as a radiation shield for the rear of the baffle, is shown in Figs. 4 and 5. The pressure difference in the air stream at the heater sides, between points at the inlet and at the outlet of the hollow baffle, forces air through the baffle at a velocity sufficient to keep the temperature of the metal wall below the danger point. The cooled baffle adds effectively to the "black surface" of the interior, and to the heat recovery of the unit. The front wall of the combustion chamber also is cooled by a rapidly moving stream of air passing over finned surfaces. The furnace bottom consists of air-cooled stoker dead plates.

Thus, all sides of the refractory lining are cooled, not only by rapidly moving air on the outside, but by radiation from their inside face to large areas of cool, metal, heat-absorbing surface. With such an air-cooled furnace, the refractory repair costs naturally are low.

The gases, as previously stated, leave the combustion chamber at the upper rear, and for the final stage of heat extraction pass to a bank of steel tubes extending across the air stream. The design of the outlets through which the gases leave the combustion chamber was first developed for the gas- and oil-fired units. In the sloping top at the rear of the combustion chamber are a number of $3^1/_2$-in. holes evenly spaced and of total area suitable to the quantity of gases to be handled. Standard-gage boiler tubes, enter these openings, at right angles, turn through 90 deg and run horizontally the length of the heater to a tube header at the front end, Fig. 4. The flue gases are drawn through the tubes to the exhauster. The advantages of this arrangement are that the gases are taken from the combustion chamber into a multiplicity of small, distributed openings, whereby they are well spread over the air-cooled surface, and both localized overheating and short-circuiting are prevented. The tube bank is thus located out of the way, in a space for which there is no other use. The location is also convenient for leading the gases to the exhauster and for finally disposing of them.

In a heater of this type, the overfire draft is normally about 0.1 in. of water, and is maintained with an adjustable damper. With this controlled draft, and with the top rear location of the flue outlets behind a vertical baffle, there is little tendency to pull fly ash and soot into the tubes; but hinged flue doors are provided to give easy access to the tubes, and any soot which does collect in them can be brushed to the rear where it falls to the floor of the combustion chamber which has a door at the rear to give access for cleaning.

The effectiveness of a tube bank as a heat-abstraction means is well known. The gases split up into a number of small streams, are so effectively cooled by the rapidly moving air stream from the fans passing over the tubes that average flue-gas temperatures through the variable load of a heating season, of 500 F, or lower, can be expected.

In buildings where heaters of this type are used, floor space is usually at a premium. The addition of the stoker makes it im-

practical to locate the fans directly beneath the combustion chamber. For this reason, in the arrangement which is standard for all except the largest sizes, the fans are mounted on top of the heater. They can, however, be located alongside the heater if space is available. The fan assembly consists of three air fans and the flue-gas exhauster, the fan wheels all being mounted on a common shaft and driven by a motor through V-belts. The stoker has its own separate fan for supplying combustion air and, in case of main-air-supply failure, the stoker including the forced draft is shut down by a sail-switch in the heater air stream. The induced-draft fan mounted on a common shaft with the main-air-supply fans provides a positive induced-draft cutoff in case of failure of the main air supply, and therefore is a built-in safety feature to protect the heater. In other words, there is always cooling air passing over the heating surface while there is draft in the furnace.

Depending upon where it is desired to locate the ducts or to discharge the warm air, the air from the fans can either be blown down across the tube bank and over the sides of the combustion chamber into underground ducts or, if overhead ducts are to be used, the fans can be reversed and used as suction fans, and cold air taken in near the floor all around the heater and discharged at the top either vertically or horizontally. In the smaller units rated at 750,000 to 2,500,000 Btu per hr output, the warm air can be discharged at the top horizontally through louvered outlets or stub horizontal ducts at velocities of 2000 fpm. In this manner, heated air can be effectively distributed at distances of 100 to 200 ft, displacing cold air drawn from near the floor.

Experience has shown that, with the latter method of distribution, the temperature rise of the air as it passes through the heater should be about 80 F in order to provide greatest comfort for the occupants of the building. Accordingly, the quantity of air passed through the heater is about 11,600 cfm for each 1,000,000-Btu per hr output.

Arrangement for Testing

Careful tests of the 4,000,000-Btu per hr heater were made in May, 1942.

In the heater tested the fans and the tubes, instead of being on top of the unit, were located at the sides. Fig. 6 shows the test setup. Instead of the louvered outlets for horizontal discharge of the heated air, a rectangular box was provided at the top, through which the air was discharged vertically upward.

The opening was cross-hatched so that 55 temperature readings covering the entire outlet area were obtained, using mercury thermometers. The temperature of the incoming cold air was measured by mercury thermometers located in each of the two openings common to adjacent forced-draft fans. The temperature of the air leaving the tube bank was likewise measured by mercury thermometers.

The higher temperatures were measured by means of thermocouples and a potentiometer. These include the temperature of the gases at the entrance to the tubes on each side, and the temperature leaving the tubes. The temperature of the gases leaving the tubes and before entering the exhauster was considered as the final gas temperature for the unit.

The coal was weighed on scales on the shop floor, and was then raised and deposited in the hopper of the stoker. For accuracy, the upper flared part of the hopper was removed and the coal was leveled with a straightedge at the beginning and after each hour of the test run.

For sampling, the tentative method of the A.S.T.M. was used. Both proximate and ultimate analyses of the coal sample were made. The calorific value was determined in a Parr adiabatic bomb calorimeter.

The ash in the form of clinker was removed at suitable times, and subsequently reduced and subjected to laboratory analysis.

The quantity of air heated was found from the air-temperature rise, and the heat imparted to the air determined as the difference between the heat in the fuel and the sum of all measured losses. The gaseous products of combustion were sampled and analyzed with the standard Orsat apparatus.

Pressures and drafts were measured at a number of significant points on both the combustion and air sides.

Test Results

The measured test data, calculated values, and heat balance are given in Tables 1 to 3.

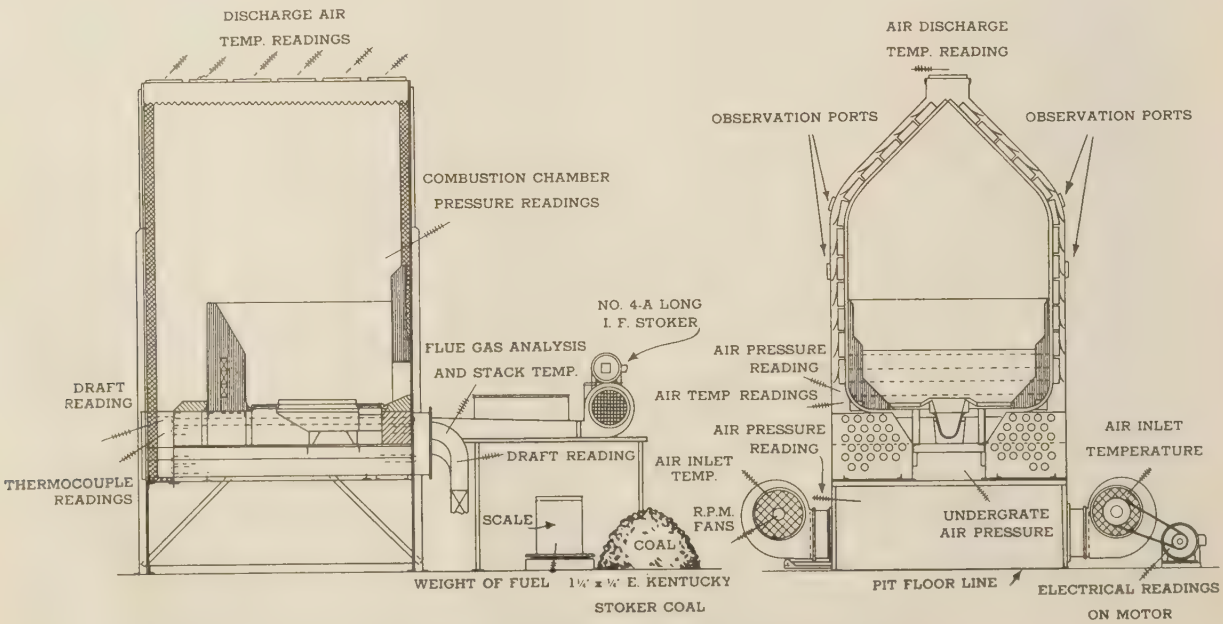

Fig 6 Test Setup of 4,000,000-Btu per Hr Coal-Fired Unit Heater

TABLE 1 TEST OF COAL-BURNING-TYPE DIRECT-FIRED HEATER

Date of test	May 15 and 16, 1942
Location	Waukegan, Ill.
Duration of test	8 hr
Stoker	Iron Fireman No. 4-A, Long
Grate area	26.25 sq ft
Kind of coal used	1¼-in. × ¼-in. Eastern Kentucky stoker coal

MEASURED TEST DATA

1	Coal consumed, as fired, lb	3403
2	Higher or gross calorific value of coal, as fired, Btu per lb	13799
3	Weight of solid refuse, lb	178.2
4	Temperature of air entering fans, F	55
5	Arithmetical average of temperatures of air leaving heater, F	143.5
6	Temperature of air leaving tube bank, F	78
7	Average temperature of gases entering tubes, F	1117
8	Average temperature of gases leaving tubes, or final gas temperature, F	619
9	Temperature of air outside building, F	38
10	Static pressure in air plenum chamber of heater, in. of water	3
11	Power input, average kw	28.92

PROXIMATE ANALYSIS OF COAL

		As received	Dry
12	Moisture, per cent	3.33	...
13	Ash, per cent	5.03	5.20
14	Volatile matter, per cent	39.07	40.42
15	Fixed carbon, per cent	52.57	54.38
		100.00	100.00
16	Sulphur, per cent	2.20	2.28

ULTIMATE ANALYSIS OF COAL

		As received	Dry
17	Moisture, per cent	3.33	...
18	Carbon, per cent	75.65	78.26
19	Hydrogen, per cent	5.40	5.59
20	Oxygen, per cent	7.07	7.30
21	Nitrogen, per cent	1.32	1.37
22	Sulphur, per cent	2.20	2.28
23	Ash, per cent	5.03	5.20
		100.00	100.00

PROXIMATE ANALYSIS OF ASH

24	Combustible, per cent	0.72
25	Ash, by difference, per cent	99.28
		100.00

ANALYSIS OF DRY FLUE GASES

26	Carbon dioxide (CO_2), per cent	14.22
27	Oxygen (O_2), per cent	4.32
28	Carbon monoxide (CO), per cent	0.08
29	Nitrogen (N_2) by difference, per cent	81.38
		100.00

TABLE 2 CALCULATED VALUES

30	Weight of coal as fired, lb per hr	425.37
31	Weight of dry coal, lb per hr	411.21
32	Heat input in coal, Btu	5,869,681
33	Btu release per cu ft hr	11,500
34	Excess air, per cent	21
35	CO escaping unburned, lb per lb of coal	0.00423
36	Weight of dry flue gases, lb per lb of coal	13.415
37	Weight of carbon unburned, in refuse, lb per hr	0.1604
38	Carbon burned, lb per lb of coal	0.7561
39	Relative humidity of air at fan inlet, per cent	51.0
40	Moisture entering in air, lb per lb of coal	0.0600
41	Moisture in coal, lb per lb of coal	0.0333
42	Power input to fans, shp	31.79
43	Mean temperature rise of air, F	88.5
44	Heat imparted to air and radiated from casing, calculated by difference (input minus sum of losses), Btu per hr	4,777,931
45	Rate of firing, lb of coal per hr per sq ft of grate area	16.2
46	Power factor, average, per cent	76.2
47	Motor efficiency, estimated from known efficiency of similar motors, per cent	82.0
48	Air delivered, as calculated from items 46 and 47 (55 F, 51 per cent relative humidity), cfm	48,119

TABLE 3 HEAT BALANCE

		Coal as fired, Btu per lb	Per cent[a]
49	Heat input in coal	13,799	100
50	Heat loss due to moisture in coal	44	0.319
51	Heat loss due to water from combustion of hydrogen in coal	642	4.651
52	Heat loss due to moisture in combustion air	16	0.114
53	Heat loss due to sensible heat in flue gases	1,815	13.158
54	Heat loss due to unburned gaseous combustible	44	0.319
55	Heat loss due to unconsumed combustible in refuse	6	0.040
56	Total losses accounted for	2,567	18.601
57	Heat imparted to air and radiated (by difference) Item 2 minus Item 57	11,232	81.4

[a] It is realized that figures to the third decimal place are not warranted by the degree of accuracy of engineering measurements, but they are given for the sake of arithmetical accuracy.

No trouble from clinkering developed during the test. The output of 4,777,000 Btu per hr was almost 20 per cent above the rated capacity of the heater. The efficiency, based on the heat-balance-by-difference method was 81.4 per cent.

The accuracy of heat-balance tests for steam-generating equipment has been ably advocated in papers presented by B. J. Cross,[2] and E. L. McDonald and R. Winters.[3] In the Cross paper, it was pointed out that the total maximum error in obtaining the efficiency of a boiler and furnace by this method was approximately 1.4 per cent. It might be well to mention that the variable which is due to radiation loss is hard to measure in a large boiler, while direct-fired heaters are either located in the heated space or in a plenum chamber through which the heated air is passed and, in consequence, any radiation loss goes into useful heat. For all practical purposes, the heat-balance method is probably the most accurate way to test a large air heater.

The figure for the combustible content of the ash, namely, 0.72 per cent, would appear to be low. In this heater, fired with a simple underfeed stoker, the carbon loss in the ash is low because the glowing clinker having formed on the dead plates (not shaker grates) alongside the stoker retort is, due to the air-cooled setting, easily turned up with a bar to the top of the fire and burns there for a considerable length of time before removal so that the carbon is rather completely burned out.

A discrepancy of greater importance seemed to be indicated by the figure 0.08 per cent of CO in the flue gases, which also would appear low in view of the very high carbon-dioxide (14.22 per cent) and low oxygen content of the gases (4.32 per cent). The slowness of action of the Orsat solution used for absorbing CO, and the rapidity with which it becomes exhausted, are well known, and the true CO content of the flue gases therefore may well have been higher than the 0.08 per cent indicated by the Orsat readings. There may also have been fractional percentages of hydrogen and methane escaping unburned in the flue gases, as the Orsat does not show these components.

Even after making a tentative reduction, by assuming more probable values for these questioned items, and by allowing for the possible presence of H_2 and CH_4, it still appears that the efficiency is around 80 per cent, which, for a coal-fired heater, operating considerably beyond its rated capacity, is decidedly good.

HEAT TRANSMISSION

An interesting question is the rate of heat transfer in the combustion chamber. Because of the number of interacting factors such as the continuation of combustion in the gases rising from the fuel bed, this is not amenable to calculation or prediction by the use of rationally derived equations, and empirical formulas such as that of Orrok[4] are ordinarily used. Figuring back from the measured temperature of about 1120 F, for the gases entering the tubes, calculating with known values of heat transfer by convection plus radiation from gases in the section back of the bridge wall, it is found that the gases just beyond the top of the bridge wall must have had a temperature of about 1250 F, which means that about 63 per cent[5] of the heat in the fuel has been abstracted up to that point. According to the Orrok formula, only 57 per cent[5] of the heat would have been abstracted to this point, and the gases there would have had a temperature of about 1500 F.

[2] "The Heat Balance Tests as a Means of Obtaining Evaporative Efficiencies of Steam-Generating Equipment," by B. J. Cross, Trans. A.S.M.E., vol. 52, 1930, paper FSP-52-33.

[3] "Heat Balance Versus Weighed Boiler Tests," by E. L. McDonald and R. Winters, *Mechanical Engineering*, vol. 59, 1937, pp. 93–96.

[4] G. A. Orrok discussion of paper "Radiation in Boiler Furnaces," by B. N. Broido, Trans. A.S.M.E., vol. 47, 1925, pp. 1123–1147; discussion, pp. 1148–1155.

[5] These figures refer to the total heat in the coal. When referred to the total heat transmitted to the air or water, the percentage of heat transmitted in the combustion chamber would, in both cases, be much higher.

The heat abstraction, therefore, appears to be considerably more effective in this heater than in steam boilers from tests of which the Orrok formula was derived. During the tests, the heater was carefully watched for "hot spots" which might develop, the corrugated shell being observed through peep holes located at various points in the outer casing. No such localized overheating was observed at any time.

That it is possible in an air heater to transmit heat through the combustion-chamber shell at a rate which is of the same order of magnitude as that attained in low-pressure steam-heating boilers or about 6000 Btu per sq ft per hr, is due to the corrugation-fin-deflector combination shown in Figs. 2 and 7. The value 6000 Btu per sq ft refers to the combustion-chamber walls and not to the total heating surface, comprising prime, secondary, and tube surface. A maximum of 3000 Btu per sq ft per hr for the total surface has been found satisfactory, with this heater.

Each fin is welded throughout its length to the outer peak or ridge of the corrugation, and carries a deflector bent back toward the outer casing. This construction acts in a number of ways to increase the heat abstraction from the corrugated shell. It might be thought that the only effect of the deflectors is to increase the velocity of the air, but such is not the case; their chief function is to hold the flowing air in contact, or throw it back into contact with the corrugated shell and to force it to flow in the corrugations, especially when passing over the rounded parts where the air tends to hug the outer casing. The deflectors also increase the turbulence of the air stream at the place where this is most effective for wiping off the stagnant film of air from the corrugated shell. It is thought that vortex sheets are formed in the air stream at the trailing edge of each deflector, and are forced into contact with the corrugated surface by the air stream constrained by the succeeding deflector. The fins, being welded to form a metallic joint with the corrugated sheet, conduct heat outward and dissipate it to the air stream. Calculations, based on the theory of heat conduction, show that the fin surface would be about 55 to 60 per cent as effective per unit of area as the corrugated surface. The deflectors also act as secondary heating surfaces, receiving heat by radiation from the shell and dissipating it to the flowing air; in doing so they act as radiation shields with respect to the outer casing, which accounts for the very low percentage of heat loss from the latter.

That a high velocity of the air was not of itself sufficiently effective was shown by a trial in the gas-fired unit in which the same fins were used but the deflectors were omitted, and the outer casing was moved closer to the corrugated shell. Although the true air velocity was as high or even higher than with the deflectors, overheating of the shell persistently occurred at some places, notably in the curved parts at the top.

Sizes and Arrangement of Heaters

Coal-fired heaters are manufactured in nine standard sizes of from 750,000 to 4,000,000 Btu per hr output. Larger units up to 8,000,000 or 10,000,000 Btu per hr output are available to suit special conditions, but it is usually more economical to use several smaller units to make up the required capacity. Many variations in the arrangement of the coal-fired heaters are possible. They can be fired with either anthracite or bituminous coal. The stoker may be provided with a hopper or may be arranged to feed coal automatically from the coalbin by an extended feed worm. The stoker can be located at the front of the heater or at the rear, whichever is the more convenient (but not at the side of the heater). The stokers are equipped with controls for safety, automatic operation and refueling. This latter term refers to the functions of a control which automatically turns on the stoker every few hours, to keep the fire lighted in mild weather, even though the thermostat does not call for heat. For burning coal,

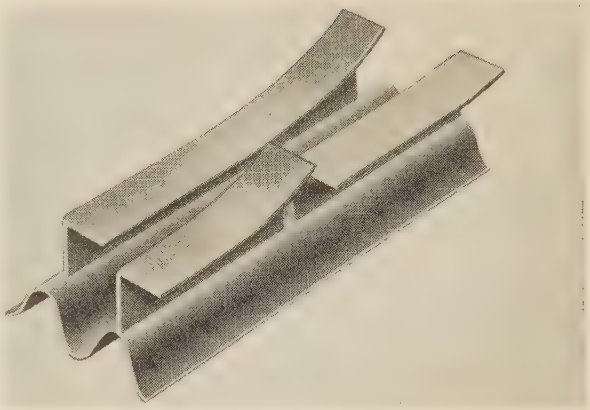

Fig. 7 Detail Showing Fins and Deflectors Welded to Corrugated Surface

hand-firing is possible in this heater, but not usually desirable because of the smoke nuisance and the difficulty of controlling the fire. The capacity and efficiency of a stoker-fired heater is so much greater than that of a hand-fired one that it is usually advisable to use the stoker, except when critical material restrictions make this impossible. An exception to this occurs when burning coke, which on the one hand, burns smokelessly, and on the other hand, cannot be fired satisfactorily with an underfeed stoker. When hand-fired grates are used, either for coke or coal, an undergrate forced-draft blower is furnished to synchronize automatically with heat demands. Coal-fired heaters can be fired with oil or gas if changing fuel conditions make this desirable. This is important in view of probable future removal of restrictions.

These heaters require no expensive foundation, nor do they require insulation, as the outer steel casing is kept cool by the rapidly moving air stream inside it. What heat does escape is radiated as useful heat into the room, or plenum chamber, so that the only losses are those in the flue gases and carbon in the ash. The saving of fuel, as compared to a steam-heating plant with its possible radiation, condensate, and stand-by losses, may amount to 25 per cent or more.

Some objections which have always been raised to the use of a number of coal-fired heaters or boilers distributed throughout the plant, are the troubles of distributing coal to and collecting the ashes from a number of scattered points, and the presence of dust and ash dust floating about and settling on everything in the plant or building. Some years ago, these were very pertinent objections, but with recent improvements not only in heaters but in fuels they have lost most of their point.

The increased use of domestic stokers and the competition of oil and natural gas in recent years for house heating, have resulted in the production of "stoker coal," which is sized, washed or air-cleaned, and treated by one of several processes to make what dust remains nonfloating. Such stoker coal is now on the market almost everywhere, and when using it there need be no fear of dirt from the coal. Those who have used it for heating their homes know the advantage over the old-style untreated coal and find their furnace rooms practically as clean as when burning gas. The stoker coal costs more than the untreated coal, but the 20 to 25 per cent reduction of fuel consumption in the unit heaters, as compared to a steam-heating plant, usually makes up for the additional cost per ton. As to ashes, as previously stated, the stoker does not produce fine ashes, but porous clinkers, and if the latter are removed with reasonable care into covered cans and transported in these, there will be practically no ash dust in the building.

With such an installation of unit heaters, large-scale and costly coal-handling equipment including conveyer, bunkers, etc., is eliminated. Usually, small individual bins can be suitably spaced around the outside of the building, and from each bin the coal is fed to its stoker by an extended feed screw. Trucks with lift bodies are today available almost everywhere, and from them the clean stoker coal may slide directly into the coalbin. In general, it costs less to handle coal in this way than with a large central coal-distributing plant. In many localities, the dealer who supplies the coal will also take away the clinkered ash.

Where it is necessary or preferable to have the coalbins inside an industrial building, the coal and ashes can be handled by the traveling crane available in the building. A central coal-storage bin is provided, accessible to the crane and filled by a simple coal hoist. Containers filled at this bin are carried by the crane to each stoker bin or hopper, and discharged into the latter through a spout from the bottom of the container.

In those plants where it is not desirable, or convenient, to heat with small units arranged throughout the heated space, central systems are advantageous. The heaters, in large sizes, are placed in a heater room similar to a boiler room, and the heated air is taken by overhead or underground ducts to the various parts of the building to be heated. This provides a central location for handling coal and ashes. This type of system, as a whole, is ideal for ventilation as well as heating, but is considerably more expensive than smaller units without ducts.

Particular Advantages of Direct-Fired Unit Heaters

Two advantages of direct-fired unit heaters are of particular importance at the present time in relation to the man-power situation. Specialized or skilled labor is not necessary, as janitors and maintenance men can operate them. Few man-hours are consumed in assembling the heaters in the field, since the lining of the combustion chamber with refractory is done at the factory; the fan assembly and other component parts are completely factory-fabricated, and only the installation of the stoker and the bolting together of the various components need be done in the field.

The saving of metal by the use of these heaters is of even greater importance in view of the present serious shortage of steel. For example, a comparison of the total weight of steel necessary for heating a plant which required 183,000,000 Btu per hr, showed that a steam plant, using high-pressure steel boilers, would have required 880 tons of steel, of which the steam and return piping accounted for more than the boilers themselves; whereas the complete unit-heater installation required a total of 530 tons of steel, resulting in a saving of about 40 per cent in the critical-material requirement. An even greater saving in metal was reported in the case of some naval supply depots requiring 40,000,000 Btu output. The steel-plate heaters, actually installed, including stokers, fans, and motors, required less than 250,000 lb of critical materials, whereas the steam plants considered, using low-pressure cast-iron boilers, were reported to require over 600,000 lb of steel and cast iron.

Design of a Refractory Heater

Regardless of these large material savings, the metal shortage developing with war demands, made it imperative to eliminate yet further the now-precious material, if practical, by the design of an all-refractory heater. This meant a radical change in the entire setup, because of the entirely different physical and thermal properties of the refractory materials available, from those of steel. Refractories have much lower thermal conductivity and tensile strength, and usually a greater thermal expansion than steel. They are brittle rather than tough, and they have considerable permeability to gases. Furthermore, because of the limitations of the processes by which refractory shapes are manufactured, if made with the thin walls required for fairly rapid conduction of heat, they cannot economically be made of any large dimensions in length or breadth; and even if they were so made, would of themselves crack into smaller pieces because of the stresses produced in them under temperature differences. The maximum linear dimension found to be satisfactory in thin sections of fireclay refractory is about 2 ft or at most 3 ft, and the minimum wall thickness at present is about $^1/_2$ in.

These limitations mean that, in order to obtain sufficient heat-transmitting surface, the heater must be built up of a large number of refractory units, through the joints of which leakage of air or of gases tends to occur. The joints cannot be cemented so that the entire assembly is monolithic, as cracking would then be sure to result. Furthermore, no packing material is known which retains its elasticity at the temperatures prevailing. The solution of the difficulty lies in having the joints horizontal, so that the weight of the tiles above will tend to keep them closed; in making the ends of the tiles smooth, setting the joints with suitable non-fusing cement, and making sure that the temperature conditions are as uniform as possible throughout each horizontal plane of the heater; and most important, in balancing the pressures or drafts so that the difference of gas pressure between the two sides of the heat-conducting wall is negligibly small.

The requirement of temperature uniformity in each horizontal plane means that the tiles must not be exposed to direct radiation from the fuel bed in coal-fired heaters, as the side of the tile nearest the fuel bed would become much hotter than the other side. This means that a Dutch oven must be provided, in which combustion will be completed and from which the hot gases will pass through an opening made rather narrow to reduce radiation, into the space above the top of the hollow tiles.

In a Dutch oven, at best, only 20 to 25 per cent of the heat can be recovered by circulating air about its walls, leaving 75 to 80 per cent to be handled by the heat exchanger. In a corrugated-steel-plate heater, approximately 75 per cent of the heat recovered is in a combustion chamber, leaving only 25 per cent in the tube bank.

The limitations of the refractories at present available set a rather definite limit to the rate of heat transmission and the difference of temperature which can be allowed between the gas side and the air side of the tiles in any one plane, if cracking is to be avoided. With present fireclay tiles, this difference is about 1000 to 1200 F for walls $^1/_2$ in. thick. Evidently then it is necessary (1) to design for counterflow (direction of motion of hot gases opposite to direction of air flow) in order to have the hottest air opposite the hottest gases; (2) to cool the combustion gases as much as possible before they enter the hollow tiles. The latter object could be attained by making the firebrick walls of the combustion chamber or Dutch oven as thin as is consistent with strength, providing an outer casing around the oven, and forcing a part of the air to be heated through the space between the oven wall and the casing. This has the additional advantages of helping to save the highly heated walls of the combustion chamber from rapid deterioration, by reducing their temperature somewhat, and of providing heat-transmitting area in addition to that comprised in the tubes or hollow tiles.

Even these two expedients, however, would be insufficient, and, in order to reduce the temperature differential at the hot end of the tile flues to a safe value, the amount of air passed around the tubes or hollow flues must be restricted to a fraction of the total volume of warm air required, so that it will attain a final temperature of at least 1000 F. This highly heated portion of the air must then be blended with the warm air passed over the walls of the combustion chamber and with the greater part of the total air quantity drawn directly from the atmosphere, so that the mix-

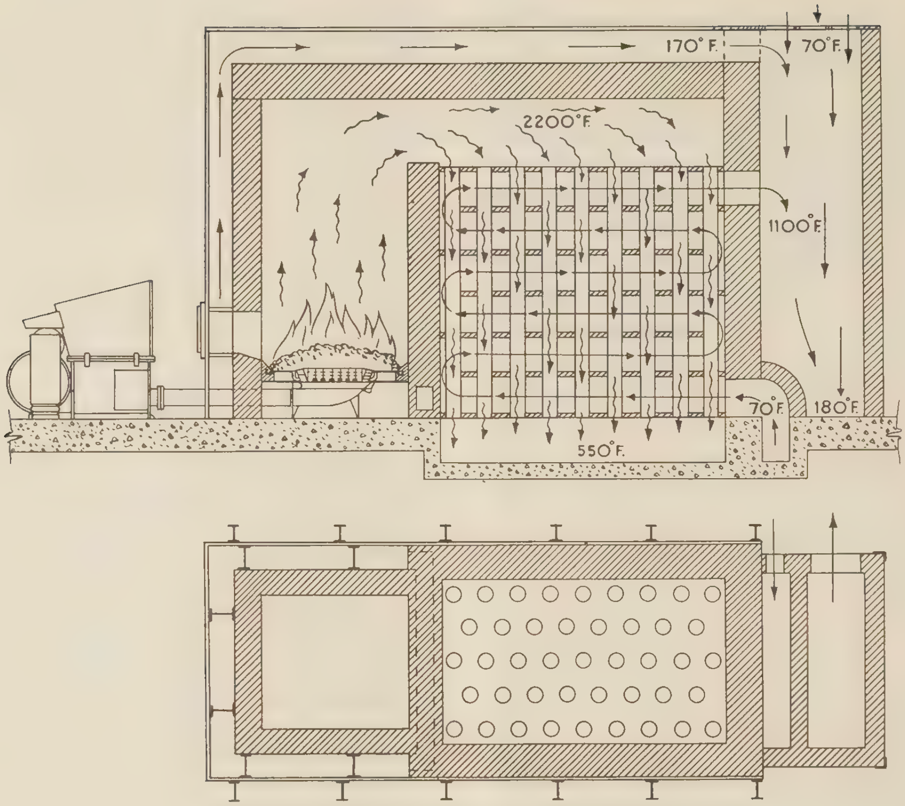

Fig. 8 All-Refractory Coal-Fired Unit Heater

ture finally leaves the distributing fan at about 180 F. This temperature is suitable in a duct-distributing system, and, of course, by varying the amount of tempering air can be adjusted to suit any reasonable demand.

In order to balance the pressures, two air fans are provided. The first forces cold air over the combustion-chamber walls and also supplies that part of the air, dampered down to a slight positive pressure, which is to pass around the tile flues. The second fan supplies the suction required for drawing in the additional cold air from the room and blending it with the warm-air stream; drawing the mixture into its intake, further blending it in its vanes and then raising it to the positive pressure required, either for discharging it in the form of directed free streams of air into the room, or for passing it through distributing ducts.

Such an all-refractory heater, as designed, is shown in Fig. 8. The refractory units making up a heater of this type have been in use in the steel industry[6] as recuperators for many years, and are suitable for use in this application if the steel savings are justified. Refractory material for their manufacture is said to be available for prompt delivery. As to cost, such heaters would be considerably more expensive than steel units, and necessarily are more bulky, because the combination of low velocity on the air side as limited by the small pressure difference available without causing leakage, low thermal conductivity of the refractory, and smaller average temperature difference as limited by danger of cracking the tiles, necessitates a much larger area of heat-transmitting surface than in the steel-plate heaters.

It is interesting and astonishing to note that the weight of the structural steel required for the binding (buckstays and tie rods), for the Dutch oven, suspended arch, and heat-exchanger walls, amounts to 35 or 40 per cent of the entire weight of steel in a comparable steel-plate unit heater. This small saving in steel does not offset the other disadvantages of a refractory heater, and would seem to indicate its use for this application as not being justified. It should be borne in mind, however, that with such a refractory heater, extremely high-temperature air is available for certain process or drying applications, where heretofore heat-resisting alloy steel or even silicon-carbide tubes were thought essential.

Summary

From the foregoing, it will be evident that a coal-fired heater of the steel-plate type has been developed which is compact, self-contained, and adapted not only to the present fuel situation but also to any changes which may occur later; and that in this heater the requirements of critical materials, of man-hours of labor, and of fuel have been reduced to the minimum, so that it is especially suited to the exigencies of the present war conditions.

[6] "Studies of Regenerative and Recuperative Furnaces," by W. A. Morton, *Iron and Steel Engineer*, vol. 15, Jan., 1938, pp. 24–34.

Improved Hydraulic Presses for Wartime Requirements

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This paper describes a new rapid-action self-contained hydraulic press developed for wartime industries in Canada. Since this mass-production press employs oil as the pressure medium, the power unit is of special interest. This the author deals with at some length. The operations for which the press is particularly adapted, namely, drawing, indenting, and loading cartridge cases, are described in some detail, as are also the methods of press control. A suitable hydraulagraph was developed for measuring pressure changes, and this instrument provides a permanent graphic record of tests on modern presses. The "one-shot" press and a 300-ton gun-straightening press are also discussed.

Introduction

WHEREAS certain improvements have been indicated in the economy of operation of a hydraulic press, quite a number still employ water as the pressure medium,

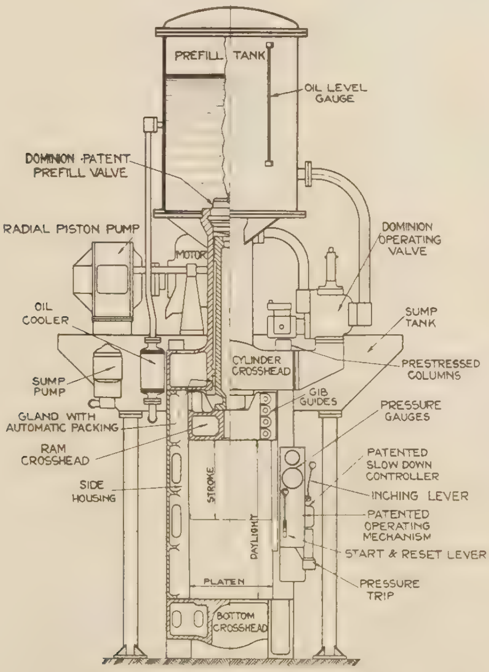

Fig. 1 Principal Features of Dominion Speed-Hy-Matic Press

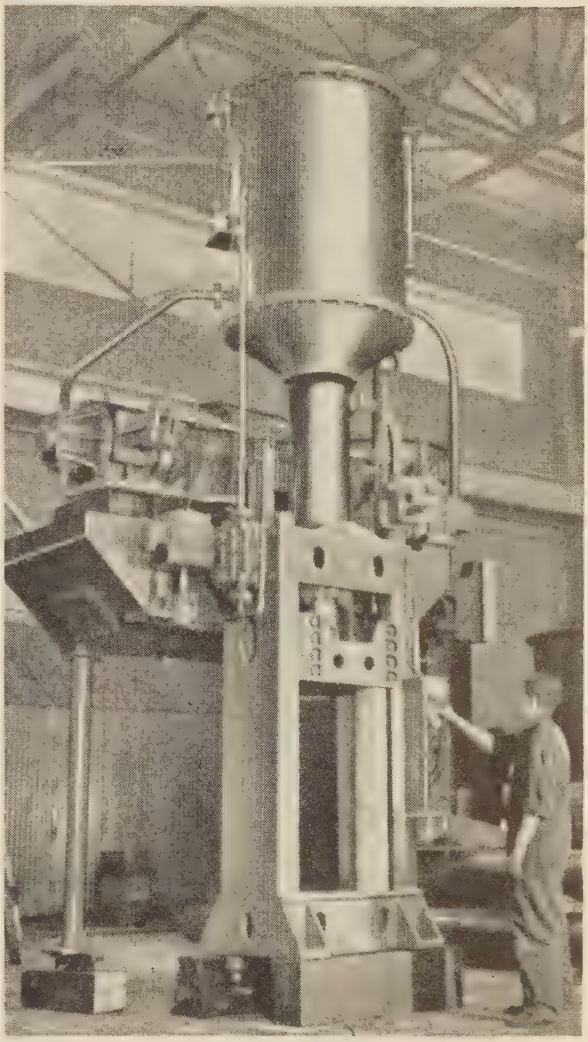

Fig. 2 Typical Dominion Speed-Hy-Matic Press

and the original idea of a separate press with its power units strategically grouped around it is still very prominent.

In modern times the hydraulic press has been improved in accordance with requirements of mass production, which demands have grown continually more exacting. An entirely new type of press has been developed.

The New Oil-Hydraulic Press

The principal features of a Dominion "Speed-Hy-Matic" press are indicated in Fig. 1, and the machine itself is illustrated in Fig. 2. The schematic circuit diagram is indicated in Fig. 3, which shows a press employing oil as the pressure medium, and

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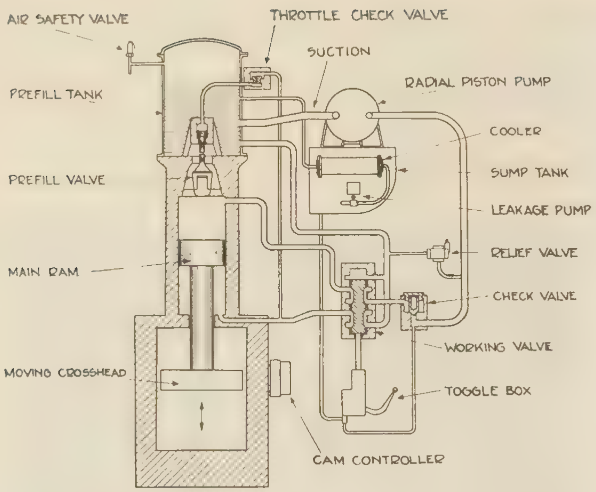

Fig. 3 Schematic Diagram of Typical Circuit

which is entirely self-contained with a circulatory system. The power unit comprises one or more high-pressure radial piston pumps, direct-coupled to the motor, and these with suitable electrical starting equipment and oil tanks are all carried on the press proper. The overhead prefill tank carries the oil supply and a filling check is located in the top head, interposed between this tank and the cylinder. This prefill valve is of special design, streamlined for flow in both directions, as indicated in Fig. 4, in order to permit the main cylinder to fill without turbulence, thus preventing cavitation and eliminating time lag prior to pressure development. With this design, we obtain approach or prefill speeds up to 20 ips without cavitation. As soon as the work is contacted, the prefill valve closes automatically so that the pumps which were previously only augmenting the flow now develop pressure immediately against work resistance. This work stroke occurs at absolutely uniform speed. The press is tripped to return automatically to its upper position, by certain embodied devices, either on the attainment of a predetermined adjustable pressure, or at any adjustable distance in any portion of the work stroke. The upper position limit is also adjustable at the will of the operator, which means that the daylight is variable as well as the pressure stroke. This permits the most efficient operation, since the press is adjustable in all respects to suit the peculiar nature of the work. When the slide attains the desired top of stroke, the continuously running pumps are stroke-neutralized in order to pump just enough pressure fluid to overcome system leakage and to develop no more pressure than is necessary to support the moving mass.

Another feature of the modern press is the use of the recently developed automatic vee packing in split-ring form, all externally applied to facilitate repacking, with bronze-lined, steel, gland rings, as indicated in Fig. 5.

The rigidity and permanence of press alignment is prescribed by the heavy side housings, which are keyed both ways, top and bottom, to the press heads and are provided with accurate guiding faces for the adjustable gib guides shown in Fig. 6. The forged-steel columns are preshrunk through the whole assembly by heating and then tightening beyond their normal working stress so that there is no working between adjacent contact surfaces.

Power Unit

The radial piston pump, Fig. 7, which is of the positive-displacement type, supplies the pressure oil to the various cylinders by suitable valve control.

The pump consists of a cylinder assembly mounted in ball

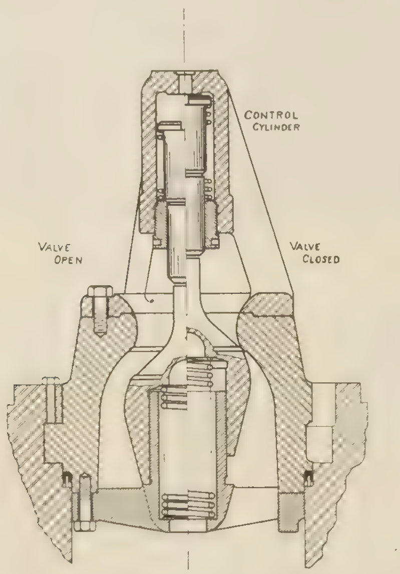

Fig. 4 Dominion Streamlined Prefill Valve

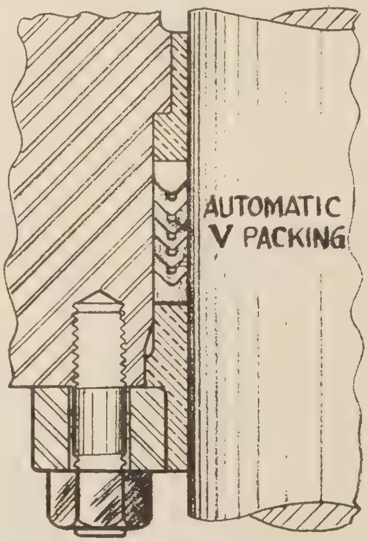

Fig. 5 Automatic Vee Packing

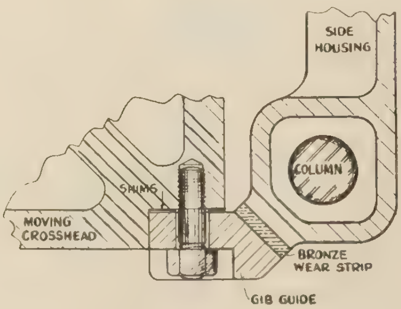

Fig. 6 Adjustable Gib Guides

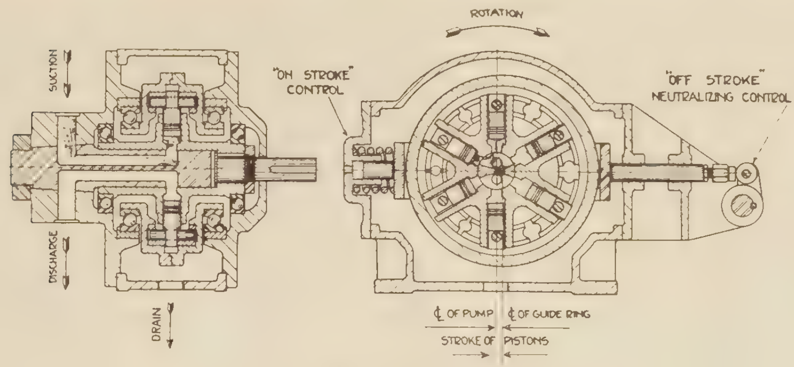

Fig. 7 Dominion High-Pressure Radial Piston Pump

bearings, fitted with lapped radial plungers, and driven by a spindle, so that as the cylinder rotates, the plungers are carried with it. The outer ends of these plungers are pivoted on bronze slippers, and their thrust is taken in the floating-ring assembly. The floating ring, which gives stroke to the pistons, is moved in a lateral direction by one or more devices. The "fluid-end" cover carries the stationary ported-valve pintle in the center of the assembly. Rotation of the drive shaft and cylinder body causes rotation of the pistons also, so that when the floating ring is moved off center, piston displacement occurs, and the pump discharge is proportional to the shift of the ring. A 7-cylinder pump rotating at 720 rpm will develop 84 small pressure crests per second, so that the overlapping discharge is, for all practical purposes, without pulsation.

The dual automatic pump control is also shown in Fig. 7, one side being an "on-stroke" and the other an "off-stroke" control. The former is a "stroke-holding cylinder assembly" to hold the pump on full stroke and maximum discharge, obtained by means of a spring at the starting low pressure, which is augmented by an assisting hydraulic cylinder at the higher developed pressures. The off-stroke control is a mechanical leverage operated by the press return at the upper position limit of travel to neutralize the pumps for leakage discharge and press sustenance.

The lubricating oil is employed particularly for pump requirements, being approximately S.A.E. 45 and highly temperature-stable, with a viscosity approximately 920 SUV at 100 F and better than 340 SUV at 132 F. The internals of the pump derive lubrication from a 3 to 5 per cent prescribed leakage past the lapped pistons and valve pintle to absorb the heat of the internal friction, which oil drains to the sump tank and is pumped back to the prefill tank through an oil cooling system.

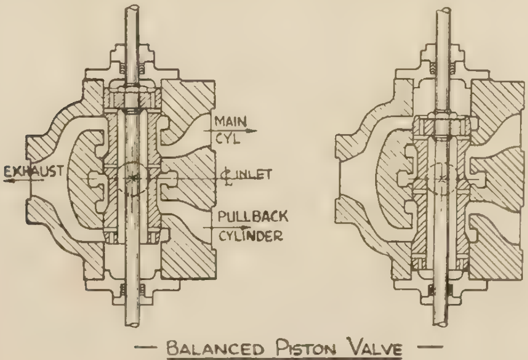

Fig. 8 Balanced Piston Valve

Simple balanced piston valves as shown in Fig. 8 are satisfactory in operation with this oil, and wear and maintenance are negligible.

Drawing, Indenting, and Heading Cartridge Cases

In the selection of a press required to perform a prescribed function in cartridge-case manufacture, one should choose a machine which is designed to suit the nature of the work, and if possible it should be capable of handling another allied operation just as efficiently when another machine in line is temporarily shut down for adjustment or repair. At first sight this requirement of double duty in emergency, with maximum efficiency for both cycles, appears to be almost beyond attainment. An examination of the leading characteristics of available machines indicates that this is not the case, and that they may be fitted into either of two important classifications:

1 The fixed-stroke type with fixed daylight.
2 The variable-stroke machine with variable daylight.

The two groups are so fundamentally different that a comparison of the leading characteristics shows the former to lack certain desirable features. A fixed-stroke press may be described as a machine whose mechanism, such as slider crank or crank toggle, will employ an effort over a long distance to produce a higher effort through a shorter distance and thus develop mechanical advantage. In this kind of machine, the opening gap, or daylight, is a fixed dimension except for a minor essential adjustment, and the stroke of the machine is definitely fixed by the throw of the crank. Once a mechanical press of this kind is set up for a certain operation, and the crank speed decided upon,

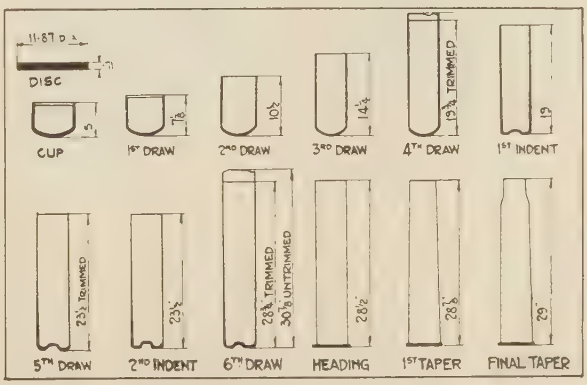

Fig. 9 Diagrams of Stages in Manufacture of 4-In. Cartridge Case

the maximum drawing speed becomes fixed, and this is not a desirable feature.

Drawing. Experiments in a certain arsenal, where special tools and Carboloy dies were employed, indicate a limit draw speed for cartridge-case brass as follows, in connection with 75-mm cases:

First and second draw	35 fpm
Third draw	30 fpm
Fourth draw	25 fpm

and this under favorable conditions with a laboratory atmosphere, accurate annealing control, with trained workmen and first-class technical supervision.

In Canadian factories, with the new oil-hydraulic press, raw labor, and standard commercial high-carbon-steel tools, we regularly draw at a speed of 27 fpm, and this on the fourth-draw operation of the 25-pounder case.

A 60-ton Speed-Hy-Matic press with an adept operator has regularly produced 1450 first-draw pieces per hr for the 25-pounder cartridge case.

Fig. 9 shows various stages in the manufacture of 4-in. cases.

Fig. 10 shows the velocity-stroke diagram of a crank press

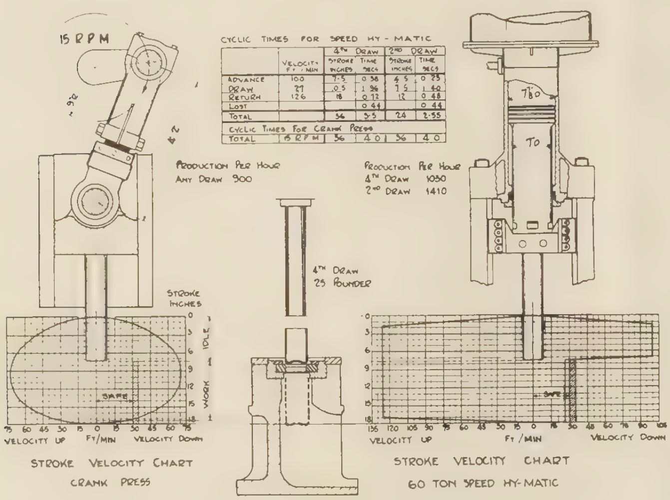

Fig. 10 Comparison Between Action of Speed-Hy-Matic and Crank Press

compared with the modern oil-hydraulic press, with the tooling employed in the fourth draw of a 25-pounder cartridge case. Cyclic times for the operations are also given.

It is axiomatic that the velocity diagram of the crank press is a sine curve, and it is seen that the tool must strike the work with a very considerable impact, for at the instant of contact the speed is 70 fpm which diminishes to 27 fpm throughout the stroke according to the law of simple harmonic motion. This means that as the draw punch proceeds, the metal is constrained to move at a speed which varies greatly; this is not the best practice for drawn work. Present evidence, borne out in practice, indicates that this rapidly changing velocity prevents full advantage being taken of the capacity of the metal to suffer plastic deformation, and rupture may occur.

Indenting and Heading Cartridge Cases. "Indenting" is the operation of making the pressed depression in the head of the case, subsequently machined to take the primer.

"Heading" is the term used to describe the pressing operation to increase the hardness of the head and form the retaining flange. Automatic variable-tonnage heading and indenting are now accepted as standard practice. In both of these operations, the press must necessarily operate against solid resistance in the form of a tool post. With mechanical presses, it is essential that the tool post be set accurately as regards height, but a gradual reduction of tool-post height occurs by repeated operation, because of the ironing and work-hardening of the member which must necessarily occur; the effect is expressed in reduced tonnage on the work so that adjustments become necessary. Whenever a tool post is replaced because of breakage or undue wear, the tonnage adjustment on a mechanical press must be repeated, for without this there is risk of mechanical failure due to excess tonnage developed at the end of the stroke.

In the new oil-hydraulic press it is stressed that the stroke

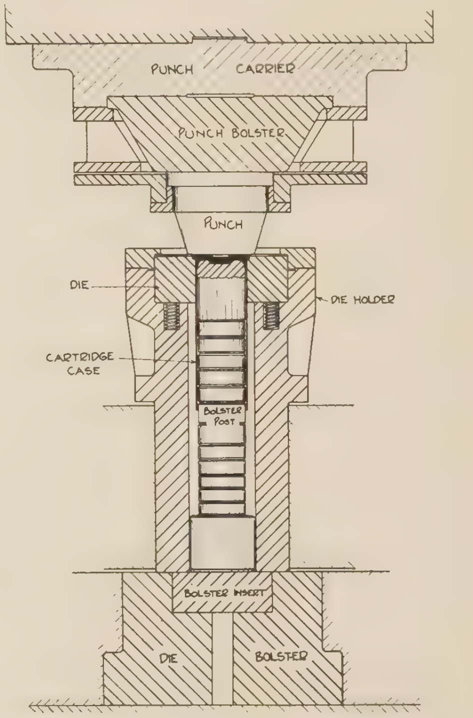

Fig. 11 Tooling for Heading Operation

and daylight may be varied so that variation in tool-post height does not affect either the safety or the life of the press or of the dies themselves.

Fig. 11 shows the typical tooling employed in either of these operations. In an oil-hydraulic press, overloading is impossible, first because of a pressure trip which is adjustable and active up to the maximum tonnage of the press, and also because of relief-valve protection.

Heading and indenting sometimes have to be done in two stages, and it is possible to employ two alternate separate selected tonnages in one press, each as best suited for either stage of the work. In a toggle-operated press, there is no slowdown at work contact, whereas, in an oil-hydraulic press, this feature is normal design so as to eliminate the impact on the workpiece. Furthermore, on work contact, the pressure is gradually built up hydraulically against resistance until the desired maximum is exerted. This gradual pressure build-up means that, in heading some cases, such as in 75-mm and 6-pounder cases, double-stage heading has been found unnecessary.

Comparison of the Two Methods of Press Control

The unidirectional pump discharge with shockless valve control has already been explained in connection with the Dominion Speed-Hy-Matic self-contained press. At a later stage in this paper a description is given of a hydraulagraph which was developed to permit an analysis of the valve action, thus providing a shockless tuning-up of the press in operation.

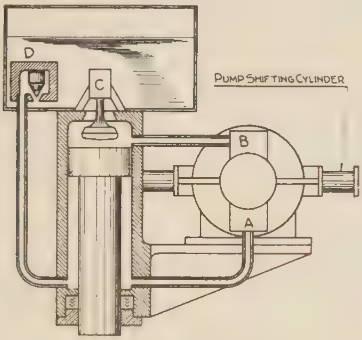

Fig. 12 Circuit With Two-Way Pump Discharge

A brief description of a press circuit having a two-way pump is necessary at this point. Fig. 12 indicates such a typical circuit.

During prefill, the pump withdraws oil from the pull-back annulus so that the pump port A is the suction, and pump port B discharges into the main ram area. The pumping rate governs the prefilling speed so that high approach speeds are limited. The prefilling valve C in the cylinder head provides oil from the overhead tank to augment the pump discharge into the main-cylinder volume. At work contact, the prefill valve closes, and the pump pressure completes the pressing stroke.

During this part of the cycle, the quantity of oil from the ram annulus is insufficient for pump suction, so that the check D operates to supply make-up oil from the overhead tank.

During the whole of this portion of the cycle, the pump has remained "on stroke" in the one direction, but, for the press-ram return, the pump is forced over through neutral in order to reverse the flow. It seems obvious that this action cannot be effected as rapidly as the movement of a simple shockless balanced piston valve. Hence, for a really fast operating cycle of 1500 per hr and over, the valve-control design is a natural selection.

Development of a Satisfactory Hydraulagraph

This instrument was designed to make possible the accurate measurement of the rapidly changing pressures encountered in modern high-speed hydraulic presses and to provide a permanent graphic record of tests.

Recording pressure gages of various types were tried and rejected because of their high inertia errors and delicate actions which made them unsuitable for recording rapid pressure fluctuations. Further investigation revealed that the indicators used for testing Diesel-engine-cylinder pressures were both rugged and accurate and had a very low inertia error. An instrument of this type was obtained with a view to converting it to record in the higher range of pressures under study. The instrument, Fig. 13, was of Lehmann manufacture, with a high-pressure spring, designed to give a scale reading of 500 psi. The pressure cylinder was removed from the body of the indicator, and a steel block, having at its upper end a cylinder, one quarter of the area of the original, was screwed into the indicator body from below. The projecting lower part of the block formed the body of a three-way plug cock for isolating and zeroing the instrument. A hard-steel loose piston was lapped into the cylinder; this had a mushroom head to bear on the original piston rod.

Following calibration by means of a standard gage and dead-weight testing machine, the instrument was found to give excellent stroke records, registering experimentally produced pressure surges up to 4000 psi intensity. However, it was found that in the stroke ordinate card, the important phenomena accompanying press reversal were crowded at the point where the card was moving most slowly. In order to improve the picture and at the same time to enable the experimenter to analyze the time cycle of the press, it was decided to add an electric-motor drive to the indicator drum.

A Warren Telechron clock motor, giving an output torque of 0.1 in-lb at 4 rpm, was bracketed to the instrument and connected by a rubber-belt drive to the drum from which the return spring had previously been removed. The ratio of drive selected gave a horizontal ordinate of 15 sec duration, generally enabling two or three complete cycles of the press to be recorded on one card. A large number of successful cards were taken with this instrument on hydraulic presses employed in the manufacture of brass cartridge cases. It was found of great value in the final adjustment of the valves used in the more complex type of press, in which several cylinders were required to operate in sequence.

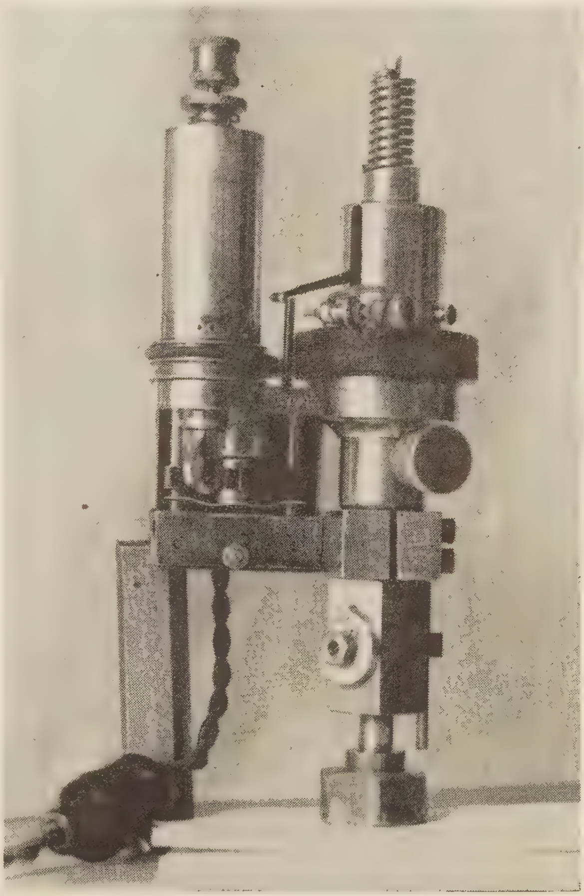

Fig. 13 Lehmann Diesel-Engine Indicator Changed to Hydraulagraph

Analysis of Press Indicator Cards, and Time-Cycle Studies

In tuning up the 23 presses operating in a modern fully hy-

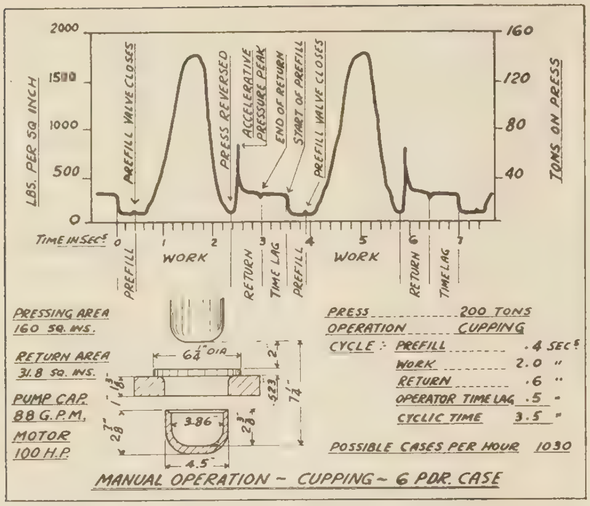

Fig. 14 Indicator Card for 200-Ton Press

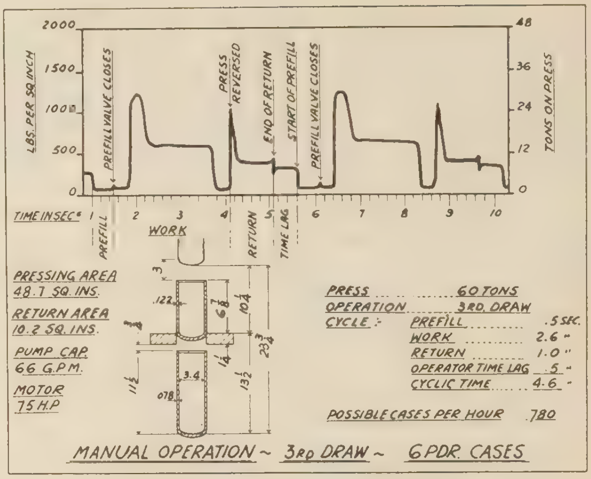

Fig. 16 Indicator Card for 60-Ton Press

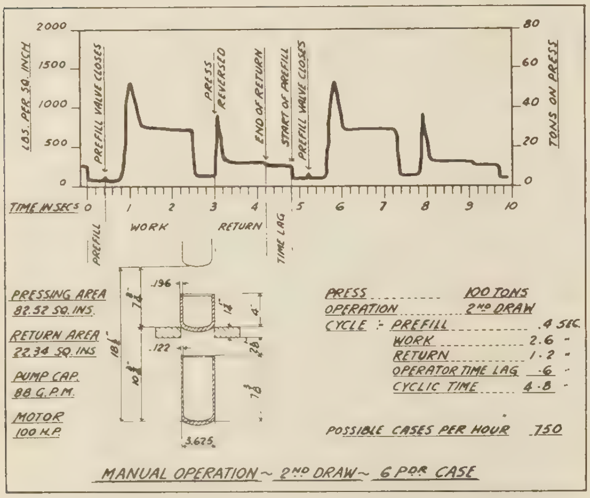

Fig. 15 Indicator Card for 100-Ton Press

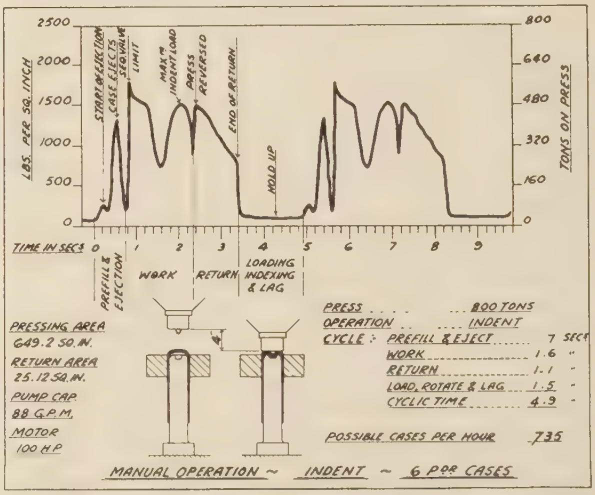

Fig. 17 Indicator Card for 800-Ton Press

draulic plant, some very interesting diagrams were obtained; Fig. 14 shows one obtained from a 200-ton Dominion Speed-Hy-Matic press engaged in the cupping operation. Female labor is employed for press operation. The record was taken during a normal run in the early stages of production without the operator's knowledge that the check was being made. Here it is seen that the necessary anticipatory feeding action on the part of the novice operator was lacking for a varying time lag of 0.5 to 0.6 sec. This should be eliminated as the operator becomes more adept. The cyclic time of 3.5 sec should reduce to 3 sec to increase output from 1030 to 1200 workpieces per hr. The diagram, being smooth in character during the entire cycle, indicates shockless action because of correctly designed and well-adjusted valving.

The diagram, Fig. 15, shows the card obtained from a 100-ton Dominion press engaged in the second draw operation of a 6-pounder case. Here again, there is evidence of an operator time lag of 0.6 sec, which when eliminated will reduce the cyclic time from 4.8 to 4.2 sec and increase output from 750 to 860 per hr. Some remarkable authenticated outputs are being obtained in another plant from a similar machine on 2-pounder cases. A regular run in 7 hr 40 min gives a production of 11,000 second-draw pieces $4^1/_4$ in. long from $3^1/_4$ in. long, and the shift record is 12,500. The maximum, obtained with manual operation, shows an average rate of 1640 per hr and an average cyclic time of 2.18 sec, sustained over an 8-hr shift.

Fig. 16 shows one of the longer draws performed in a rapid 60-ton Dominion hydraulic press. In this instance the load required at any instant of the cycle is accurately portrayed. The approach speed is very fast, giving 20 ips average. The pressing speed is nearly $5^1/_4$ ips, while the return speed is exceedingly high, namely, nearly 25 ips. The effect of the latter may be observed at the end of return as the pumps are over-neutralized a little, as evidenced by the small local pressure drop, the subsequent pressure build-up, and the settlement to sustain the mass during the operator's time lag of 0.5 sec. The cycle time with a trained operator can be reduced from 4.6 sec to 4.1 sec, with an increase in output from 780 to 880 per hr.

In the fourth section of this paper a general description of indenting and heading has been given. A diagram obtained from a modern 800-ton indenting press is given in Fig. 17. This machine is equipped with a three-station automatically operated turret, viz., load, press, and eject. In the work cycle

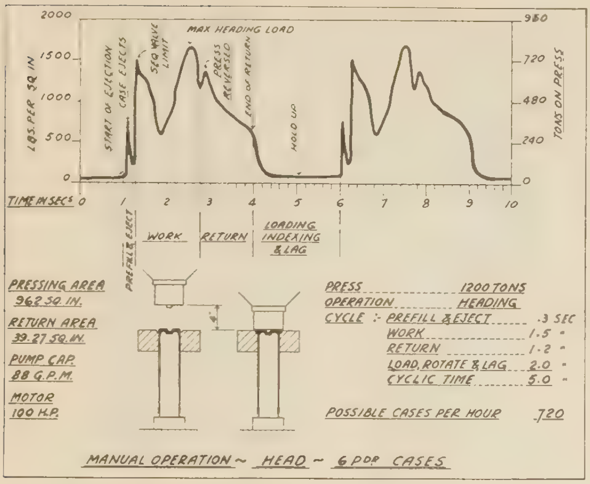

Fig. 18 Indicator Card for 1200-Ton Press

prefill or run down of the press ram occurs simultaneously with the hydraulic ejection of the pressed case, automatic pick-off of the case occurs at the same time as the work stroke, while the ejecting ram returns simultaneously with press return, the whole being a valve-controlled sequential cycle of events. The press is manually loaded prior to turret rotation and index, after which the operator initiates the press cycle by depressing the start button. The chart indicates that loading, indexing, and lag take up 1.5 sec which can be reduced. The indicated output, however, is 735 workpieces per hr.

Fig. 18 shows the chart obtained from a similar press but of 1200 tons capacity employed for heading. Here the indicated output is 720 per hr; the chart also shows a possible reduction in handling time.

Shell-Forging Presses

In the last war the cavity of the shell forging was rough-punched and subsequently finish-machined. Today, shell cavities are finish-forged to fine finish and tolerance but, despite this radical change in manufacturing procedure, the method of shell forging in general use today shows but little advance over that employed previously. In a number of present-day plants two machines are used, one to bump and punch and one to draw. While the old draw rings are often replaced by draw rollers, the essentials are the same. Another process, developed in England, extrudes the shell forward through a free opening in an extrusion die. The billet is about twice the outside diameter of the finished forging, and hence tremendous pressure is required to effect the work.

Another method which employs a single machine is of the multiple-progressive-punch type, and as many as five or six separate operations must occur successively on the billet, with consequential scale inclusions in the cavity.

In the majority of methods in use, the metal must flow in a direction opposed to punch travel with consequent rapid wear of punch and die liners. The Dominion Engineering Works, Limited, made extensive studies of shell-forging methods, and the analysis indicated the following facts:

To limit eccentricity when piercing, three improvements were necessary, namely, a very stiff press, an accurate punch guide, and the billet located centrally in the die. To locate the billet as required, a split die becomes necessary to develop a pinching grip. The best cross section of billet was decided as square, mosaic, or polygonal, with corner dimension slightly in excess of the die bore so as to provide the grip. The understood limit in piercing speed was 25 to 30 fpm, and the draw limit speed 22 fpm. However, if a punch tip is to retain its diametral dimension during piercing to develop an accurate bore, and also to show any wear life, it must be removed from the hot metal before absorbing excessive heat. In consequence, it was decided that a higher speed was desirable, so that 85 fpm and over has been attained.

When a punch enters a square billet in a die, the metal from the cavity has two possible paths in which to flow: The segments between the billet sides and the round-bore die are filled and the remainder increases the length. Lengthening can take place only in one of two directions, either against or with the direction of punch travel. Since all previous methods have a fixed base in the die pot, great punch pressure must be applied to pierce and cause metal flow past the punch-tip taper, up the die liner, and to oppose punch travel, especially since the pierced metal cools rapidly. The idea was developed to employ a hydraulic cylinder, whose ram filled the die-pot base, with suitable valves, so that the retraction of this ram would provide controlled resistance. It was apparent that, if the billet were positioned longitudinally in the die pot by sufficient pinching grip, the excess metal would be constrained to move ahead of the punch tip. Thus, an entirely new "one-operation" process for forging shells with finish cavities was evolved.

A semiautomatic press employing but a single stroke of the punch into the hot billet, and having a self-opening-and-closing die, hydraulically cushioned, was designed to utilize the new process.

It proved to be highly successful. Four machines have been put into operation, making 3.45-in. high-explosive, 3.7-in. anti-aircraft, 4.5-in. howitzer, 4-in. naval, and 5.5-in. high-explosive,

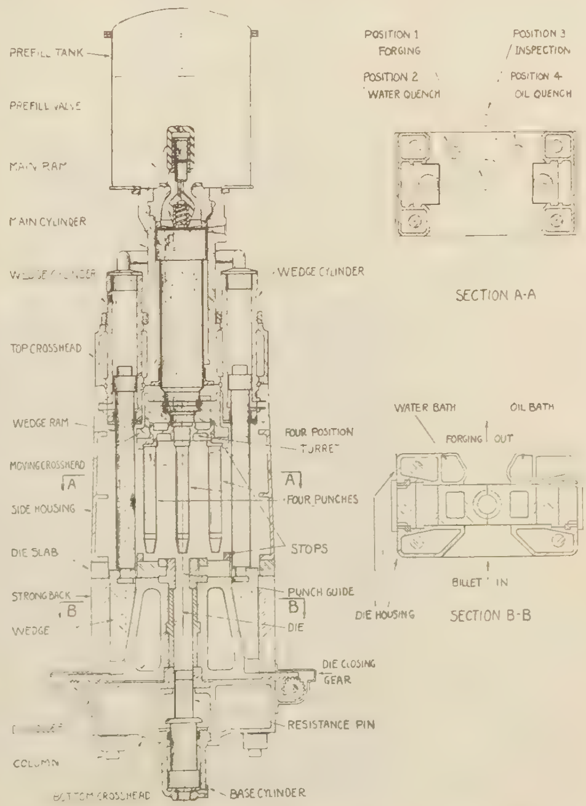

Fig. 19 Diagram of Dominion Shell-Forging Press

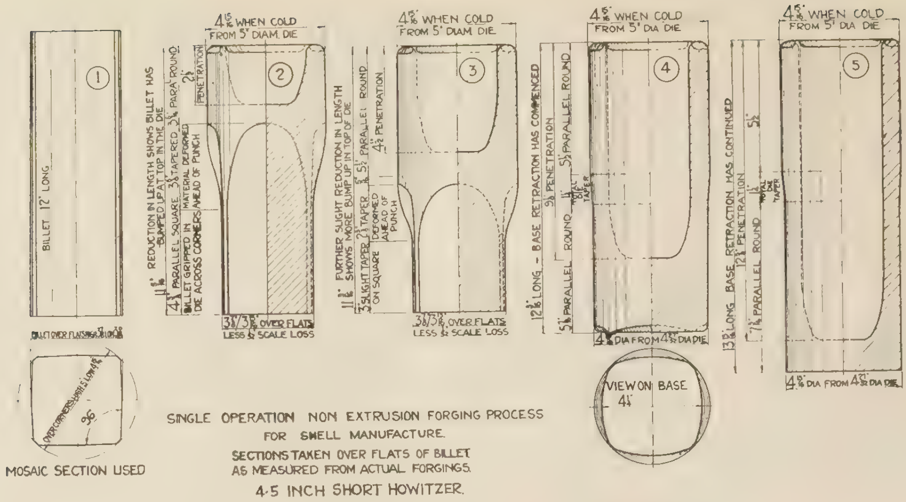

Fig. 20 Diagrams of Stages in Shell Forging

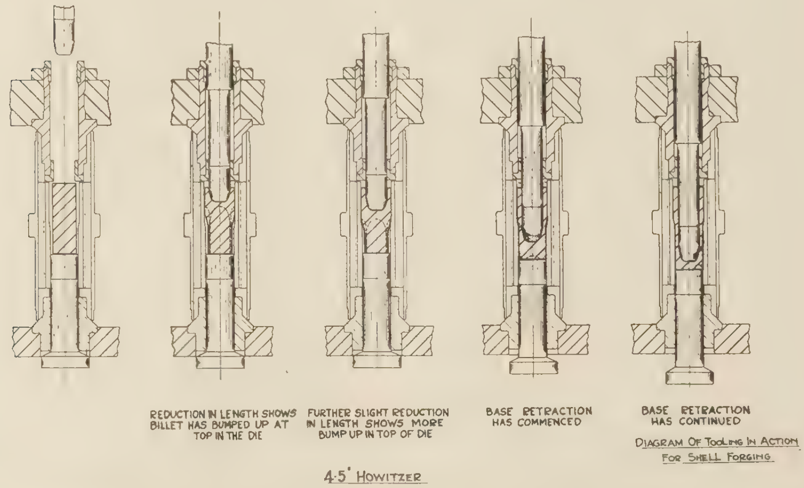

Fig. 21 Diagrams of Tooling-in Action for Shell Forging

Fig. 22 Sections Through Six Special Forgings With Plug Inserts

Fig. 23 Enlarged Detail of Cavity in Special Forging With Plug Inserts

shell forgings. The forging pressures have proved to be so low (about 60 per cent of that for normal piercing) that an alloy cast-iron tip is employed with cast-iron die liners, and the overall tooling cost is less than 10 cents per shell forging. Rejected forgings are exceedingly few with these machines. For one contract 15,707 forgings were made with only 121 rejections. Production rates are high, and results to date show that man fatigue and not the press imposes the restriction on output. As an example, two hundred 4-in. naval shell forgings are easily handled in an hour.

Fig. 19 is a diagram of the forging machine; the forging action is shown in Fig. 20. These diagrams are taken from a series of partially forged billets. The tooling in action when piercing a billet is seen in Fig. 21.

In punch and draw methods of shell forging, there is great difficulty in obtaining a forging without a "button" at the base of the cavity. This is due to faulty forging action and also the necessary technique to make a forging. After extrusion punching, the forging is often placed over a steam jet to reduce partially the temperature at the base of the cavity before the forging is pushed through the draw dies or rollers. This is often necessary to hold the base thickness and prevent the draw punch from going through the base. The "button" is thus formed by faulty forging action, because the metal remains static, and also by steam-quenching and cooling by the punch tip.

In order to test the plastic flow of the metal during shell forging in the Dominion process, and to demonstrate that the metal actually moved in the same direction as punch travel but ahead of it, certain experiments were performed.

A series of 9 holes was drilled through several shell billets, some across flats and some across the diagonal, all on center. Threaded stainless-steel rods were inserted in these holes and the ends welded over and dressed. The various billets were forged $^1/_3$, $^2/_3$, and full depth. Subsequently, the forgings were parted on the same center and ground. Figs. 22 and 23 illustrate the perfect forging action and the complete absence of the "button," for every inserted plug can be traced around the tip diminished to a hair line in some instances.

Gun-Barrel Forging Presses

Fig. 24 illustrates a 2000-ton hydraulic forging press, designed and built in Canada by the Dominion Engineering Works, Limited, and used for the forging of gun barrels. It is equipped with front and rear pits which carry the hydraulic racking gears for mandrel forging. The press was supplied complete with all valves and piping pumps and motors to form a self-contained circulating system.

There are three fluid systems, i.e., automatic low-pressure prefill to the work at 80 psi, for economy of high-pressure fluid; auxiliary medium pressure for the racking slides and pull-back cylinders at 2500 psi; and high pressure for forging only at 5675 psi, up to a maximum of 2000 tons total pressure.

The medium-pressure circuit includes a vertical triplex pump having a capacity of 37 gpm and driven by a 60-hp motor and a hydraulic accumulator of 10 in. diam and 12 ft stroke.

The high-pressure circuit includes two horizontal duplex pumps, each having a capacity of 160 gpm and each driven by a 600-hp motor. The press has a dual control for either forging or planishing and has an automatic pump unloader gear, with single-lever control.

Gun-Barrel Straightening Presses

Fig. 25 shows a 300-ton Dominion hydraulic press of the horizontal gap type, complete with the necessary tank, valves and piping, pump, motor, and controls to form a self-contained circulatory system.

While this press was designed by Dominion Engineering Works, Limited, primarily for the straightening of gun-barrel forgings, it will be readily seen that the press could be used for straightening bars, pipes, or structural shapes, etc., as well as for bending.

The pressing head is on the ram and there are two adjustable resistance blocks which can be varied in span from either side of the press. The forging to be straightened would be carried on stands located on either side of the press, the stands being equipped with roller rotating mechanisms to rotate the work, and adjustable wedge-operated supports to take the weight of the forging and bring it onto the pressing center line of the press. The press being of the horizontal gap type, the work is easily handled in and out of the press. Several presses of this type

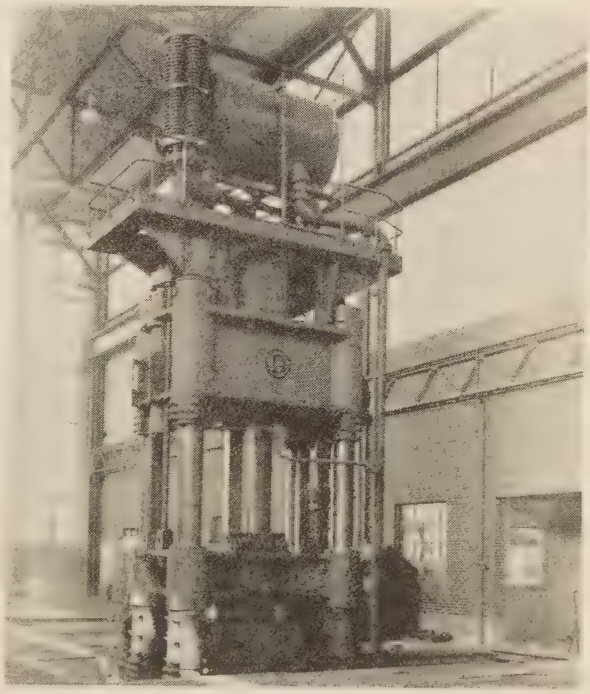

Fig. 24 Dominion Forging Press of 2000 Tons

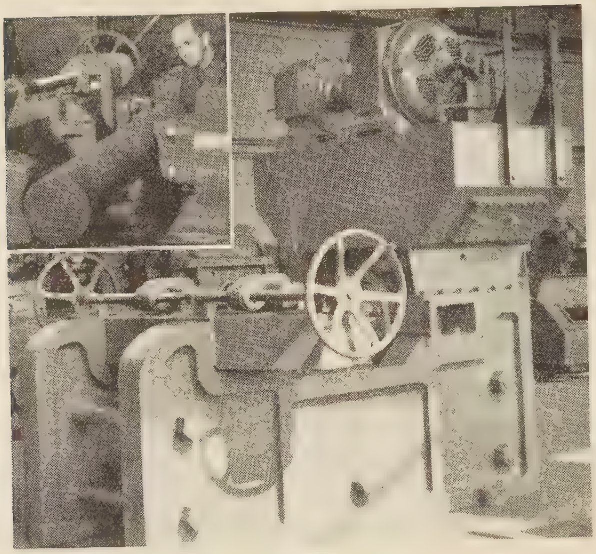

Fig. 25 Dominion Straightening Press of 300 Tons

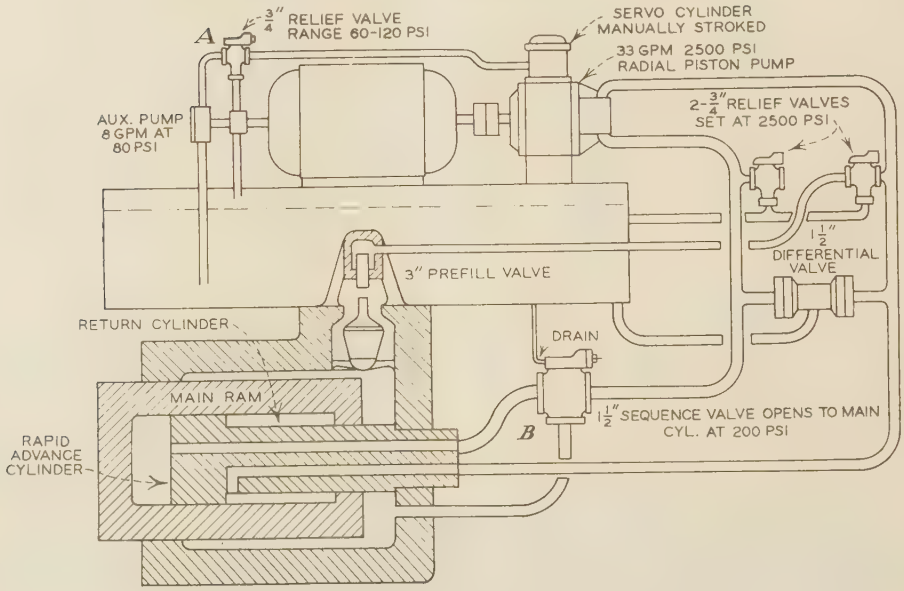

Fig. 26 Straightening-Press Circuit Diagram

have been built and are now in operation in gun-manufacturing plants in Canada.

This circuit is most interesting, as two fluid systems are employed, as will be seen from the circuit diagram, Fig. 26. The pump is stroked by servomotor control, operated from the low-pressure pump at 80 psi. A differential valve is also employed to compensate for the volumetric difference between the rapid-advance cylinder and the press-return cylinder, so that oil is pumped from one to the other as the press moves. Thus, when the press is advancing, the pump pressure closes the valve on the rapid-advance-cylinder side and opens between the return-cylinder connection and the oil tank.

As the volume of the return cylinder is less than that of the rapid-advance cylinder, the additional oil required by the pump flows from the tank into the system. When the press is returning, the pump draws oil from the rapid-advance cylinder and exerts pressure in the return-cylinder line, moving the valve over. As the volume of the system is now decreasing, the excess oil flows from the rapid-advance cylinder back to the tank.

The servomotor for the pump-stroke control is a device permitting of infinitely variable discharge from either flange of the pump, by the application of a very light effort.

The control operates through the admission of oil from an auxiliary pressure source to either end of the control cylinder through a special central valve. This valve is so ported that any movement in one direction causes oil to be admitted to the rear of the piston and to be exhausted from the front of the piston so that the latter moves in the same direction until it becomes again centered with respect to the valve.

The oil exhausted from the control passes through the pump casing and returns to the sump tank along with the leakage oil.

A spring is provided on the control to bias it in one direction, thus eliminating the effects of backlash in the control mechanism.

Stress Analysis of Passenger-Car Trucks

By J. C. TRAVILIA, JR.,[1] and W. E. BURDICK[2]

This paper discusses the Reuleaux loadings as applied to railroad passenger-car trucks, together with the effect of modern high-speed brakes. The importance of laboratory verification of stresses is emphasized and test methods described.

Modern high-speed railroad passenger traffic has dictated an ever-increasing vigilance on the part of passenger-car designers to insure that the high standards of safety shall be maintained. This is particularly true of car trucks where both vertical- and lateral-impact loads increase with an increase in the speed. Higher brake-shoe pressures have also added to the burden of the trucks.

It is the purpose of this paper to outline briefly the design loads producing critical stresses and to offer supporting test data.

A standardization of stress analysis for this type of equipment is not practical because of the wide variety of designs. For instance, a few of the variables involved are as follows:

(a) Four- and six-wheel trucks.
(b) Equalized and nonequalized trucks.
(c) Single- and double-equalizer trucks.
(d) Different spring suspensions.
(e) Various hanger suspensions.
(f) Various brake arrangements.
(g) Tread brakes and disk brakes.

Comparisons Between Trucks of Similar Design

The resiliency of springs and hanger systems bears on the problem of load transmission so that judgment on stress limitations must be weighted by the type and manner of these suspensions. For this reason it is often wise to proceed with simple formulas on trucks of similar designs, comparing the stresses thus obtained with trucks in similar service which have proved satisfactory.

If the absolute stresses are desired, maximum loadings should be considered with the stresses kept well within the fatigue limit of the material. However, the designer is cautioned that truck frames with their several integral members are statically indeterminate under combined loadings, besides being subjected to secondary stresses as the result of twist and nonaxial bending, re-entrant corners, etc., so that absolute stresses are difficult to calculate and must often be determined by laboratory analysis.

Wheel, axle, and box designs have been well established by the Association of American Railroads, mechanical division, and will not be discussed. For ready reference, however, the standard axle designations and their capacities are given in Table 1.

It will be noted that recognition of the increased impact loads occasioned by the higher speeds is reflected in the lower capacities in passenger service, being roughly 85 per cent at 85 mph, and 80 per cent at 100 mph of the freight capacities.

If a basis for calculating the absolute stresses is desired, the authors offer the Reuleaux loadings which were accepted in 1896 by the Master Car Builders[3] as a basis of axle design which has persisted until the present day. However, the brake reactions must also be considered and are treated later.

TABLE 1 CAPACITY PER AXLE

Axle classification	Journal size, in.	Freight service, lb	Passenger service 85 mph max, lb	Passenger service 100 mph, lb
A	3 3/4 × 7	15000	12500	12000
B	4 1/4 × 8	24000	20500	19000
C	5 × 9	32000	27000	25500
D	5 1/2 × 10	40000	34000	32000
E	6 × 11	50000	42500	40000
F	6 1/2 × 12	60000	51000	48000

The lateral-load value which was chosen by the Master Car Builders committee was 40 per cent of the vertical load which is approximately sufficient to overturn a car whose center of gravity is 6 ft from the rail. The vertical-impact-load values were obtained by calibrating springs in a car set of Fox-type trucks and by assuming that the maximum lateral force was acting at the time of the maximum vertical impact. The vertical load was increased 26 per cent to cover impact. The combination of 40 per cent lateral force and 26 per cent vertical overload acting simultaneously is quite severe. Although present-day speeds are in excess of those at the time of the test, the spring suspensions are much more resilient so that it may be assumed that the spring-suspended parts are safely designed if calculated with the Reuleaux system of loading. It will be shown later wherein tests have indicated that the actual lateral loadings may be less than under the Reuleaux system, even in high-speed main-line service.

Fig. 1 represents the generation of the Reuleaux system through a typical truck. The original forces are as follows:

P = vertical load carried plus 26 per cent
H = $0.40\,P$
w_1 = weight of bolster, bolster springs, hangers, and spring plank
w_2 = weight of truck frame, brake etc.
w_3 = weight of equalizer springs, equalizers, etc.

h_1, h_2, h_3 are 40 per cent of w_1, w_2, and w_3, respectively.

The reactions, vertical R and horizontal T, with illustrative subscripts can be readily obtained by considering the free-body equilibrium of the particular part in question. These reactions then become the loads for the adjacent part. In Fig. 1 the double arrows indicate the action-reaction forces.

Swing Hangers

Fig. 2 illustrates the forces transmitted by the swing hangers. For any initial angularity, $\frac{d}{s}$

$$R_1 = \frac{P}{2} - \frac{Hh}{L} \qquad R_2 = \frac{P}{2} + \frac{Hh}{L}$$

$$T_1 = R_1 \frac{d - y}{\sqrt{[s^2 - (d - y)^2]}} \qquad T_2 = R_2 \frac{d + y}{\sqrt{[s^2 - (d + y)^2]}}$$

A first approximation will permit neglecting $(d - y)^2$ and $(d + y)^2$ as small quantities of the higher order. Solving the foregoing equations for H gives

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Contributed by the Railroad Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[3] Proceedings Master Car Builders, 1896, p. 150.

$$H = \frac{PLy}{Ls - 2hd}$$

Ordinary hangers will transmit about 0.15 P, laterally, at maximum swing. Any excess will be transmitted by direct contact between the bolster and frame.

In high-speed operation high brake forces are necessary for economical stops. There is a marked reduction of brake-shoe

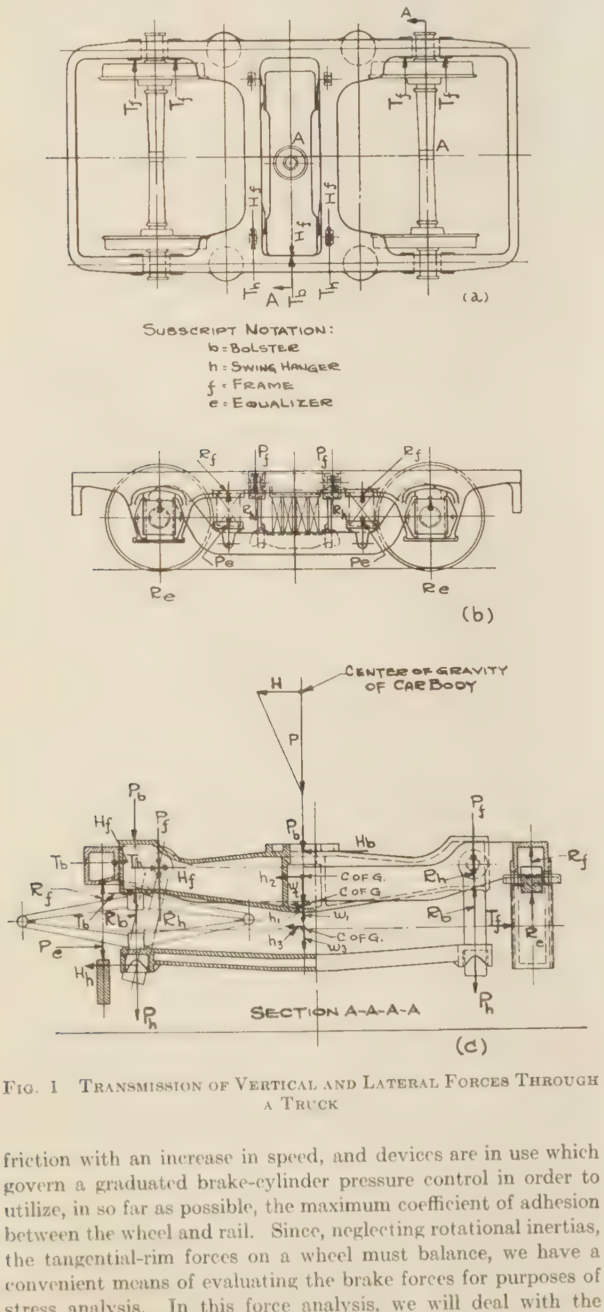

Fig. 1 Transmission of Vertical and Lateral Forces Through a Truck

friction with an increase in speed, and devices are in use which govern a graduated brake-cylinder pressure control in order to utilize, in so far as possible, the maximum coefficient of adhesion between the wheel and rail. Since, neglecting rotational inertias, the tangential-rim forces on a wheel must balance, we have a convenient means of evaluating the brake forces for purposes of stress analysis. In this force analysis, we will deal with the frictional coefficient μ acting upon the normal force N, producing the retarding effort on the rim of the wheel. This method is more exact than the method of setting the rim friction as a function of the force applied to the brake shoe by the brake rigging.

Fig. 3 illustrates the forces acting on the brake heads during a brake application.

The conditions of equilibrium which must be satisfied are as follows:

1 The forces acting on the brake-head pin, assuming a free pin connection must be coincident.

2 Hence the normal wheel reaction N and the friction component μN have a resultant T which must pass through this pin. This force, together with the hanger reaction Q and the brake-lever or beam force F, must form the closed-vector diagrams shown in Fig. 3(b) and (c).

The maximum retardation which can be exerted will cause im-

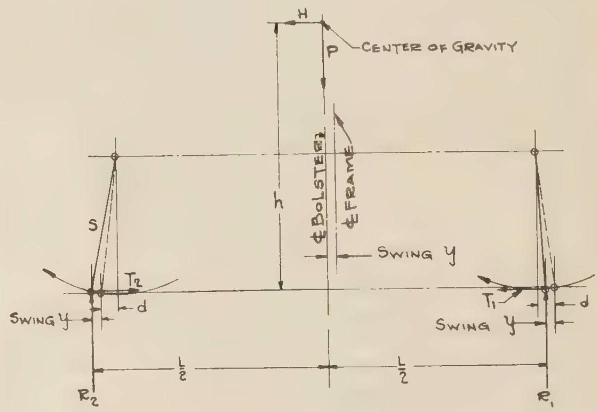

Fig. 2 Resistance of Swing Hangers to Lateral Forces

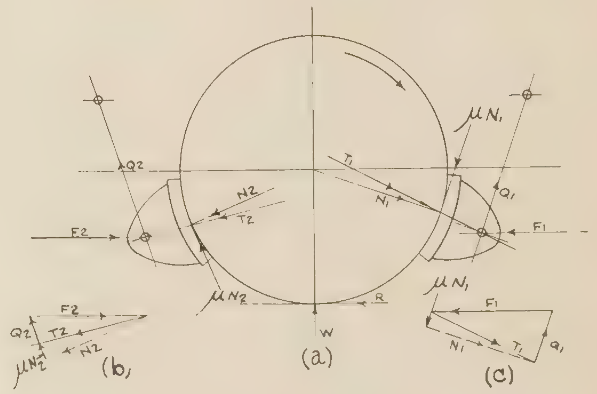

Fig. 3 Vector Evaluation of Forces Acting on a Brake Head During Brake Application

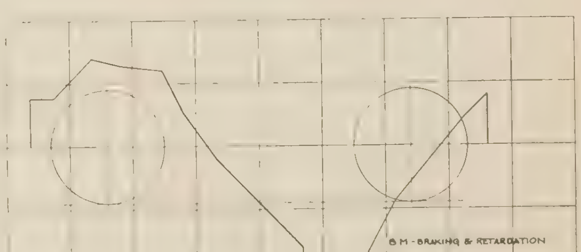

Fig. 4 Bending-Moment Diagram of Forces Due to Braking and Retardation

minent wheel slipping. Extensive tests[4] have shown that the coefficient of adhesion R/W averages 0.25 for all speeds and best surface conditions of wheel and rail so that μ can be determined from the relationship, neglecting the rotational inertia of the wheel and axle

$$\mu N_1 + \mu N_2 = R$$

From the foregoing, the reactions on the truck frame can be determined which are due to the combined action of the brake rigging and retardation forces.

A typical bending-moment diagram of the forces due to brake and retardation reactions is shown in Fig. 4.

It should be noted that the action of the brake causes a weight transfer on the center plates and within the trucks so that the forward truck receives an overload and the front axle a still higher overload. Assume, for example, a car body weighing 100, 000 lb, center of gravity 62 in. above rail, truck weight 18,000 lb, with center of gravity 20 in. above rail, truck centers 56 ft, truck wheel base 9 ft, and center-plate height 26 in., and consider an average coefficient of 0.25 for retardation. It will be found that the transference to the front truck will be 1340 lb and to the rail at the front wheels about 3900 lb, an increase of $11^1/_2$ per cent over normal.

From the loadings in Figs. 1 and 3 the stresses in the truck frame may be calculated. The bolster presents a relatively simple problem in that it is of the form of a simple beam. Likewise, the maximum reactions on the hangers can be determined readily. The equalizers may also be calculated, but proper concentration factors should be applied at the upper and lower curved portions as the inner forming radii cause a rise in stress. Fig. 5 illustrates the concentration of stress in the upper radius of a typical equalizer in a photoelastic model.

Fig. 5 Stress Bands in a Photoelastic Model of an Equalizer

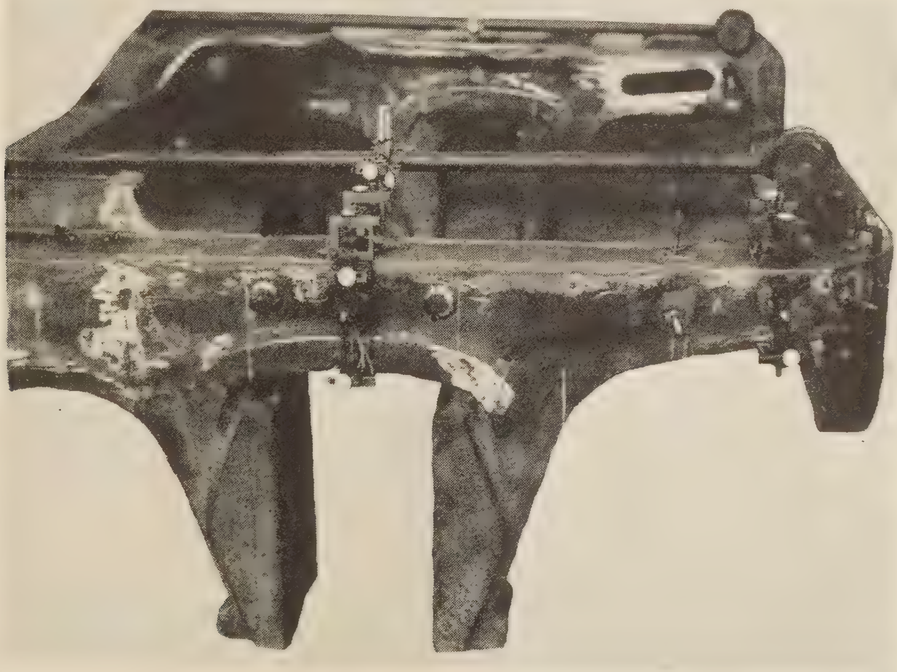

Fig. 6 Truck Frame Set Up for a Static Strain-Gage Test

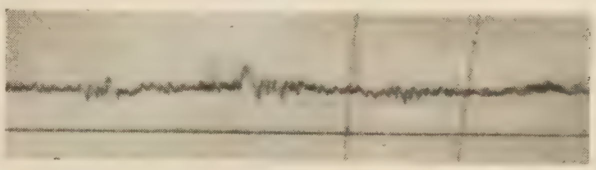

Fig. 7 Deformations Recorded on Target of a de Forest Scratch Strain Gage

(Four-wheel passenger-truck test; ×300.)

[4] "Mastering Momentum," by L. K. Sillcox, Simmons-Boardman Publishing Corp., New York, N. Y., 1941, p. 47.

Strain-Gage Tests of Truck Frames

As previously mentioned, the redundant nature of the frame structure makes it difficult to appraise accurately the true stresses, and analysis should be supplemented by laboratory experiments.

Fig. 6 illustrates a truck frame under one such strain-gage test. All questionable points were investigated under different methods of loadings taken to simulate the actual loading conditions. A sufficient number of points were investigated to evaluate the redundancies.

Following the static strain-gage test mentioned, a test using the de Forest scratch type of strain gage was run with the truck in actual operation in high-speed main-line service. This type of gage consists of a fixed arm with abrasive material embedded on its point which rubs on a plated target so that tensile or compressive deformations are recorded, which may be enlarged by a microscope and then photographed. Fig. 7 shows such a strain record. Thus, actual operating stresses were determined for the same points which were investigated in the static test, which gave a comparison of the assumed and actual loadings. From the comparison it was determined, for this particular test run, that the lateral forces were about 25 per cent of the vertical, and the vertical load impacts on both the bolster and frame were of quite a low order. The impact values on the unsprung equalizers were, however, of a higher order. These values will vary depending upon operating conditions.

Conclusions

1 The Reuleaux method of loading still serves as a basis of design, and properly calculated stresses based upon this loading should yield safe values.

2 Modern high-speed braking requires a careful investigation of the resultant stresses.

3 In the design of a complex structure such as a truck frame, calculations should be supplemented by strain-gage tests in order to investigate points of high stress concentration and to evaluate redundancies.

4 In designing trucks, experience is always the best guide and radical departures from conventional practices should not be attempted without careful supplementary laboratory investigation.

Corrosion of Unstressed Specimens of Alloy Steel by Steam at Temperatures up to 1800 F

By G. A. HAWKINS,[1] H. L. SOLBERG,[2] J. T. AGNEW,[3] AND A. A. POTTER[4]

This paper presents the results of tests made at the Engineering Experiment Station of Purdue University to determine relative resistance to corrosion by steam of unstressed specimens of various alloy steels at temperatures of 1500 F and 1800 F. The data include results from 500-hr tests at 1500 F and 1800 F for a representative selection of steels containing up to 18 per cent chromium, and for a 500-hr and a 1300-hr test on 25-20 and 25-15-2-W steels at 1800 F. The results permit extension of previously published data[5] to cover the temperature range from 1000 to 1800 F. All of the steels tested except the 25-20 and 25-15-2-W specimens show rapid corrosion beyond a limiting temperature which increases with chromium content. The 25-20 and 25-15-2-W steels were extremely resistant to steam corrosion for exposures up to 1300 hr at 1800 F.

Description of Apparatus

STEAM from the laboratory main was passed through a counterflow gas-fired steel-tube superheater, after which it flowed through an electrically heated superheater which consisted of 50 ft of 25-20 stainless-steel pipe $^5/_8$ in. OD by $^3/_8$ in. ID. The 25-20 material was selected for the high-temperature superheater element after serious difficulties were encountered during preliminary tests when using a superheater made of 18-8-Cb. This superheater was wound in the form of a cylindrical helix having a diameter of approximately $1^1/_2$ ft and a space between successive turns of 3 in. The electrically heated superheater was placed in a firebrick enclosure which was in turn surrounded by 6 in. of loose insulation. The heating was accomplished by connecting the secondary terminals of a current transformer directly to lugs welded on the ends of the superheater piping. After leaving the electric superheater the steam flowed through a stainless-steel connection housing a thermocouple to a 10-ft reaction chamber made from 2-in-OD double-extra-heavy 25-20 pipe. A plate was welded over one end of the reaction chamber and was drilled and tapped for the superheater-inlet pipe. The other end of the chamber was threaded and closed with a pipe cap made of 7-Cr steel to facilitate insertion and removal of the test specimens.

The reaction chamber or test section was heated externally to the same temperature as the entering steam by means of four electric heaters formed by winding No. 8 chromel wire around grooved refractory tiles placed around the reaction chamber. The inlet-guard heater was 1 ft long, each of the two main heaters was 3 ft in length, and the outlet-guard heater was 2 ft long. The reaction chamber together with the heaters was mounted horizontally in a tunnel of firebrick around which was placed a covering of 6 in. of loose high-temperature insulation. Pyod-type chromel-alumel thermocouples were placed in the reaction chamber 1, $2^1/_2$, $4^1/_2$, and 7 ft from the inlet end.

The steam leaving the test section was condensed in a coiled-copper-tube condenser. The quantity of steam flowing was regulated by means of a needle valve on the discharge side of the condenser. The valve was regulated to maintain a flow of 37.5 lb per hr.

Temperature-Control Circuit

The temperature of the steam leaving the electric superheater was maintained at any desired value by means of a complex thermocouple relay system which has been described in another paper.[5] Each of the four heaters around the test section was equipped with a variable resistance so that the temperature of the heater could be maintained at a fixed value over long periods of time. The specimens were protected from steam at any temperature other than the test temperature by means of the control system which would automatically interrupt the electric load and steam flow in case of high or low steam temperature. Lead wires from all couples were connected to a jack-board cold junction. The temperature readings were taken on a portable potentiometer.

Test Specimens

All of the specimens were machined to a length of 6 in. and a diameter of $^1/_2$ in. The machined surfaces were sandblasted in order to obtain identical surfaces on all specimens.[5] Each specimen was stamped with a classification number and a specimen number.

The general heat-treating procedure for the various steels is given as follows: Low-carbon steel was annealed at 1600 F, slow-cooled to 1200 F at a rate of 40 F per hr, and then air-cooled from 1200 F to room temperature. Steels having a chromium content between 1 and 12 per cent were annealed at 1580 F, slow-cooled to 1200 F at a rate of 40 F per hr, and air-cooled from 1200 F to room temperature. The 18-8 steel was annealed at 1950 F and water-quenched. The 25-20 alloys were annealed and water-quenched from 1900 F.

Outline of Testing Procedure

After the surfaces had been sandblasted the samples were weighed on a sensitive balance and then stored in a container fitted with a calcium-chloride drier. After the test chamber had been brought up to the test temperature and steam was flowing through a by-pass at test temperature, the various samples were placed in small cages made of alloy-steel welding wire and then inserted into the reaction pipe. The location of each specimen

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[5] "Corrosion of Unstressed Steel Specimens and Various Alloys by High-Temperature Steam," by H. L. Solberg, G. A. Hawkins, and A. A. Potter, Trans. A.S.M.E., vol. 64, May, 1942, pp. 303–313.

Contributed by Special Research Committee on Critical Pressure Steam Boilers and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

TABLE 1 CORROSION OF GROUP 1 STEEL BARS IN CONTACT WITH STEAM FOR 500 HR

Steel	Chemical analysis, ladle, per cent C	Mn	P	S	Si	Cr	Ni	Mo	Cb	W	Steam temp, F	Number of samples	Loss in weight, per cent of original weight
S.A.E. 1010	0.08	0.30	0.017	0.034	..	...	...	..	..	..	1736	3	100.0[a]
3 Cr-Moly	0.11	0.51	0.014	0.016	0.36	2.95	...	0.98	..	..	1736	2	74.8
4-6 Cr-Moly	0.11	0.33	0.020	0.027	0.28	5.66	0.22	0.50	..	..	1736	2	70.0
9 Cr-Moly	0.11	0.38	0.010	0.016	0.27	9.00	...	1.22	..	..	1728	2	60.1
18-8-Cb (stabilized)	0.07	0.36	0.015	0.012	0.39	18.62	9.90	..	1.11	..	1728	2	12.3
25-20	0.07	1.62	...	...	0.34	24.45	20.30	..	..	..	1751	4	0.06
25-15-2-W	0.10	1.75	...	...	0.56	24.18	14.34	..	..	2.06	1751	4	0.07

[a] No remaining steel (100 per cent Fe_3O_4).

TABLE 2 CORROSION OF GROUP 2 STEEL BARS IN CONTACT WITH STEAM FOR 500 HR

Steel	Chemical analysis, ladle, per cent C	Mn	P	S	Si	Cr	Ni	Mo	Cb	Steam temp, F	Number of samples	Loss in weight, per cent of original weight
4-6 Cr-Moly	0.10	0.33	0.020	0.027	0.28	5.66	0.22	0.50	..	1772	3	77.7
7 Cr-Moly	0.11	0.43	0.012	0.011	0.92	7.33	..	0.59	..	1772	3	57.4
9 Cr-Moly	0.11	0.38	0.010	0.016	0.27	9.00	..	1.22	..	1772	3	62.7
12 Cr	0.10	0.51	0.022	0.290	0.40	12.70	..	..	..	1765	4	100.0[a]
18-8-Cb (stabilized)	0.07	0.36	0.015	0.012	0.39	18.62	9.90	..	1.11	1765	2	11.5

[a] No remaining steel.

TABLE 3 CORROSION OF GROUP 3 STEEL BARS IN CONTACT WITH STEAM FOR 1300 HR

Steel	Chemical analysis, ladle, per cent C	Mn	P	S	Si	Cr	Ni	W	Steam temp, F	Number of samples	Loss in weight, per cent of original weight
25-20	0.07	1.62	...	...	0.34	24.45	20.30	..	1776	14	0.11
25-15-2W	0.10	1.75	...	...	0.56	24.18	14.34	2.06	1776	15	0.20

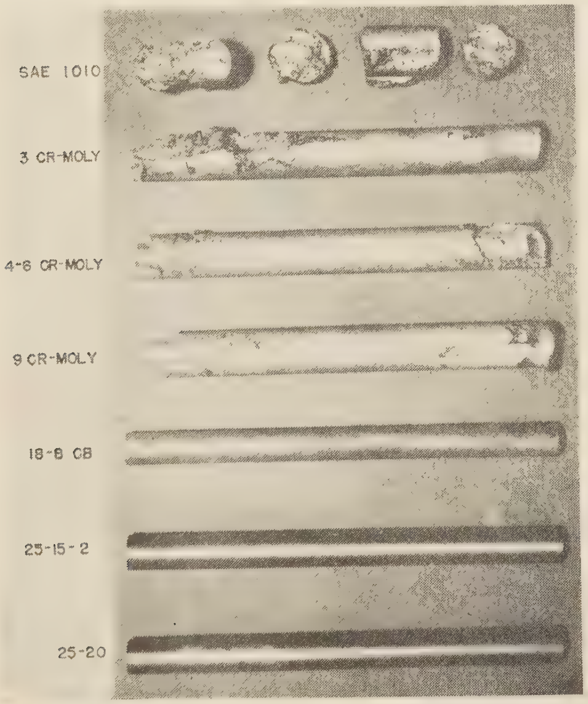

Fig. 1 Group 1 Steel Samples After Completion of Test

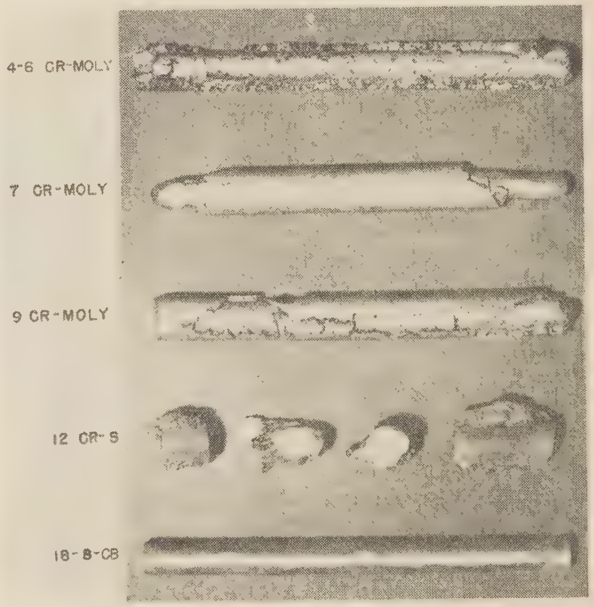

Fig. 2 Group 2 Steel Samples After Completion of Test

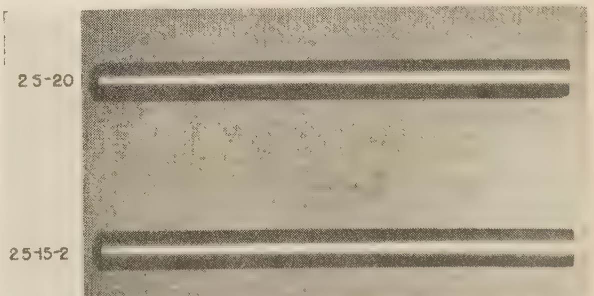

Fig. 3 Group 3 Steel Samples After Completion of Test

and cage was recorded at the start of each test. After the specimens had been in contact with the steam for the desired length of exposure, the specimens were withdrawn and the scale removed initially by mechanical means where a thick scale had been formed. The remaining scale from the heavily corroded specimens and the scale on the high-chromium alloys was removed by making the specimen the cathode in an electrolytic cell containing a 10 per cent solution of sulphuric acid with 1 g per l of quinoline ethiodide as an inhibitor and with a current density of 1 ampere per sq in.[5]

TABLE 4 CORROSION OF GROUP 4 STEEL BARS IN CONTACT WITH STEAM FOR 500 HR

Steel	Chemical analysis, ladle, per cent C	Mn	P	S	Si	Cr	Ni	Mo	Cb	Steam temp, F	Number of samples	Loss in weight, based on original weight, per cent
S.A.E. 1010	0.08	0.30	0.017	0.034	..	...	..	..	..	1498	3	62.4
3 Cr-Moly	0.11	0.51	0.014	0.016	0.36	2.95	..	0.98	..	1501	3	44.3
4-6 Cr-Moly	0.10	0.33	0.020	0.027	0.28	5.66	0.22	0.50	..	1501	3	28.8
7 Cr-Moly	0.11	0.43	0.012	0.011	0.92	7.33	..	0.59	..	1506	2	12.2
9 Cr-Moly	0.11	0.38	0.010	0.016	0.27	9.00	..	1.22	..	1506	3	14.6
12 Cr-S	0.10	0.51	0.022	0.290	0.40	12.70	..	..	..	1506	3	0.03
12 Cr	0.10	0.52	0.014	0.015	0.32	12.92	0.12	..	..	1508	3	0.05
18-8 Cb (stabilized)	0.07	0.36	0.015	0.012	0.39	18.62	9.90	..	1.11	1508	2	0.03

Test Program

One test at approximately 1800 F was conducted for 1300 hr and was divided into two 500-hr tests and one 300-hr test operated in series. Also, one test was conducted at 1500 F for 500 hr. The 1300-hr test was started using a number of samples of S.A.E. 1010, 3 Cr-Moly, 4-6 Cr-Moly, 9 Cr-Moly, 18-8-Cb, together with 18 specimens of 25-20, and 19 specimens of 25-15-2W steels. At the end of 500 hr, all samples of steels containing up to 18 per cent chromium were removed together with four samples each of the 25-20 and 25-15-2-W alloys. The samples removed at the end of 500 hr are classified as Group 1. Immediately after removal of the Group 1 samples a new group (Group 2) consisting of 4-6 Cr-Moly, 7 Cr-Moly, 9 Cr-Moly, 12 Cr, and 18-8-Cb steels was placed in the reaction chamber. At the end of the second 500 hr or a total elapsed test time of 1000 hr, the samples of Group 2 were removed and the test continued on the remaining samples of 25-20 and 25-15-2-W stainless steels which were placed in the reaction chamber at the start of the test. At the end of 1300 hr, the samples of 25-20 and 25-15-2-W were removed; these constitute Group 3. A new 500-hr test was then started using a steam temperature of 1500 F on S.A.E. 1010, 3 Cr-Moly, 4-6 Cr-Moly, 7 Cr-Moly, 9 Cr-Moly, 12 Cr-high-sulphur, 12 Cr, and 18-8 Cb steels. These steels are known as test Group 4.

Operating Experiences

At the end of 500, 1000, and 1300 hr for the 1800 F tests, it was found impossible to remove the 7-Cr steel caps from the ends of the reaction chamber by unscrewing. All of the caps were burned off by use of a torch without damaging the threads on the reaction chamber. This turned out to be a rather difficult task since a very thick layer of scale was formed on the cap.

The temperature controls and heating elements operated without a failure throughout the entire test program. At the end of 317 hr, during the 1300-hr test, a power-plant failure interrupted the power supply for 45 min. During this period the temperature of the reaction chamber dropped 150 deg. No other interruptions occurred during the test program.

Results

The results for the various groups of steels are tabulated in Tables 1, 2, 3, and 4. In a previous paper[5] the corrosion was reported in terms of surface penetration in inches. Due to the large change in diameter and the end effects, this procedure was not followed in reporting the results of these tests. Instead, the results are expressed in terms of the loss in weight of the specimen expressed as a percentage of the original weight of the specimen.

The influence of chromium as a corrosion-preventing agent is clearly shown in Table 1. The 25-20 and 25-15-2-W steels were extremely resistant to the corrosive action of the steam. Fig. 1 shows various unstripped samples taken from Group 1 steels.

Data dealing with steels of Group 2 are presented in Table 2. Here again, the marked influence of chromium content is clearly shown except for the 12-Cr steel. The correlation between the data shown in Tables 1 and 2 is extremely good. Fig. 2 shows samples of the steels in Group 2. The difference between 7 Cr-

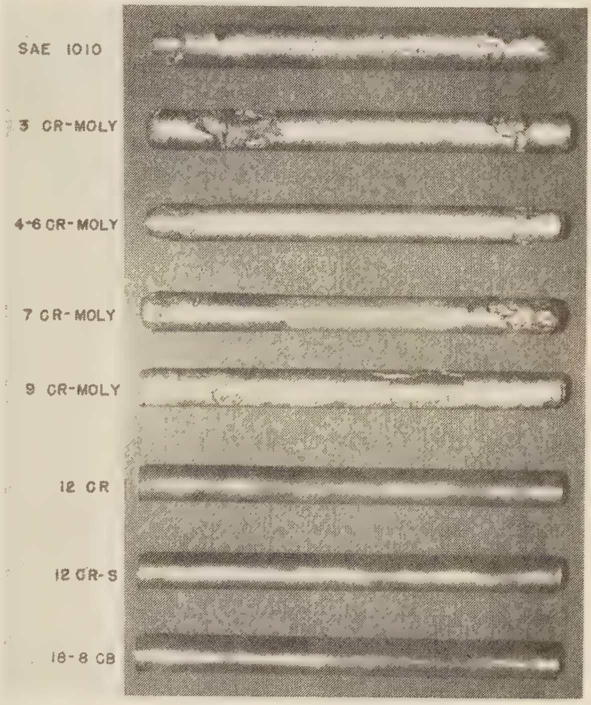

Fig. 4 Group 4 Steel Samples After Completion of Test

Moly and 9 Cr-Moly steel is probably not significant in view of the large amount of corrosion products in both cases. The 12-Cr steel, listed in Table 2, was corroded to such an extent that no parent metal remained. The inner core was solid in texture but extremely brittle. The core had a greenish gray color. The external layer appeared similar to the scale formed on the low-carbon-steel samples. The chemical analysis for the core and external layer of scale for the 12-Cr steel follows:

	Core, per cent	External layer, per cent
S	0.04	Trace
Cr_2O_3	24.8	0.5
Fe_3O_4	72.6	99.5

Apparently the iron ions have diffused outward at a faster rate than the chromium ions. In view of this condition and the decrease in corrosion with increased chromium content in the case of all other alloys which have been tested, the complete destruction of these particular specimens can probably be attributed to the high sulphur content of the steel. During the test, 93 per cent of the sulphur disappeared, as determined by an analysis of the corrosion products.

Results for the 1300-hr test are shown in Table 3. The results indicate that the 25-15-2-W steel lost 0.20 per cent while the 25-20 steel lost 0.11 per cent. In view of the extremely small

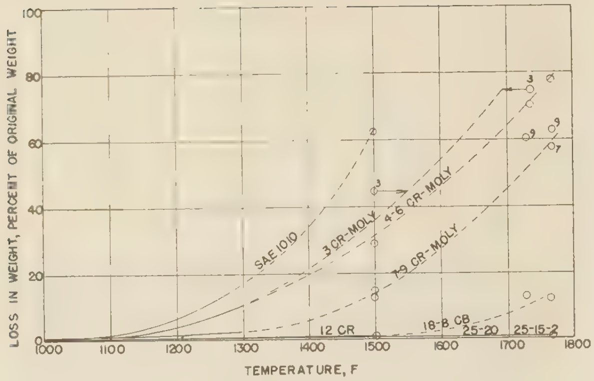

Fig. 5 Corrosion of Steel Bars in Contact With Steam for 500 Hr at Various Temperatures

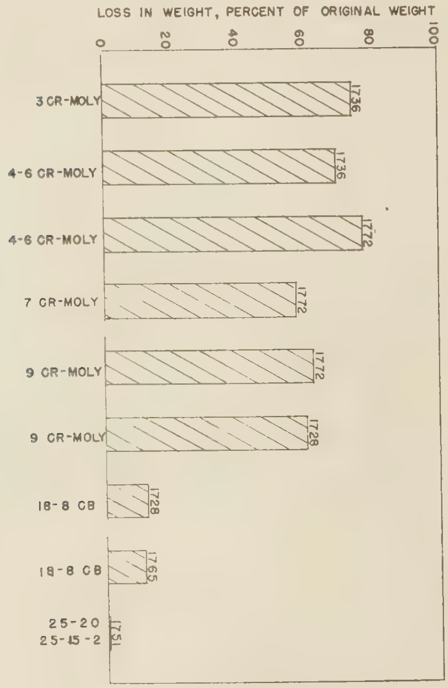

Fig. 6 Corrosion of Steels in Contact With Steam From 1728 F to 1772 F for 500 Hr

amount of scale produced on either specimen, and the difficulty of determining accurately whether or not all of the scale had been removed during stripping, the difference indicated is not significant. The important conclusion which may be drawn from the data is that both steels are extemely resistant to attack by steam at 1800 F for 1300 hr.

Greenish deposits were found on the 25-20 samples after 1300 hr. An analysis of the greenish material revealed no sulphide or sulphate. It was felt that the deposit was chromic oxide, with possibly a minute quantity of metallic chromate. Fig. 3 shows two samples of these steels.

The data presented in Table 4 were obtained during a 500-hr test at 1500 F on the steels in Group 4. Fig. 4 shows samples of these steels taken immediately after the conclusion of the test. In this test, two types of 12-Cr steel were used to determine the effect of the sulphur content on the corrosion resistance at 1500 F. From the results, no difference could be detected at the temperature of 1500 F. Apparently the complete destruction of the 12-Cr high-sulphur steel which occurred at 1800 F is associated with the higher temperature.

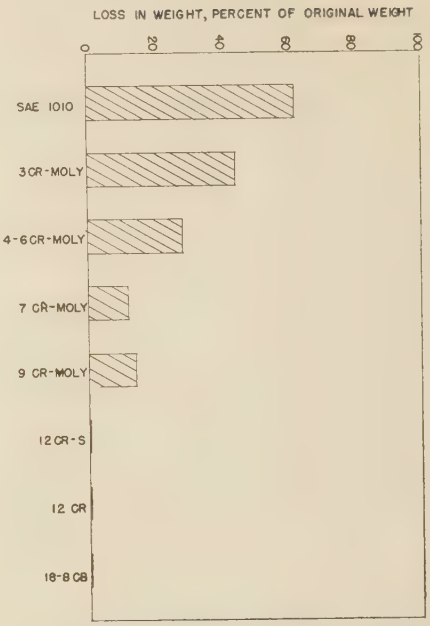

Fig. 7 Corrosion of Steels in Contact With Steam at 1500 F for 500 Hr

Conclusions

The results obtained on the steels in Groups 1, 2, and 4 have been plotted together with previously reported data[5] in Fig. 5 to show the effect of temperature. All of the steels tested except the 25-20 and 25-15-2-W specimens start to corrode rapidly at some temperature less than 1800 F. The temperature at which rapid corrosion begins increases with chromium content. The 18-8-Cb steel shows the same tendency toward rapid corrosion above some limiting temperature that the low-carbon steel shows at a much lower temperature. A single line has been used in Fig. 5 to represent the 7 Cr-Moly and 9 Cr-Moly steels, since the data are not comprehensive enough to distinguish between them.

Figs. 6 and 7 have been constructed using the results obtained on Groups 1, 2 and 4, and show the marked influence of the chromium content as a corrosion-retarding alloy. The free-machining 12-Cr high-sulphur steel is not shown in Fig. 6 because it is not intended to be used for applications where high corrosion resistance is a factor.

The 25-20 and 25-15-2-W steels are extremely resistant to steam corrosion up to temperatures around 1800 F for an exposure time of 1300 hr.

Acknowledgment

This investigation was made possible through the financial support of The Babcock & Wilcox Company, the results of the 1800 F test being released for publication through its courtesy and its suggestion. The authors are deeply indebted to Mr. H. J. Kerr and Mr. J. B. Romer of The Babcock & Wilcox Company and to Mr. H. D. Newell of The Babcock & Wilcox Tube Company for valuable assistance and constructive criticisms.

Appendix

The following statements, dealing with the mechanical properties and structural characteristics of the 25-20 and 25-15-2-W steels be-

Fig. 8 Structure of Material Prior to Steam-Oxidation Treatment; Heat No. 60494; ×100
(All Specimens illustrated etched with aqua regia.)

Fig. 9 View of Specimen in Greater Detail Than Shown at Left; Heat No. 60494; ×1000

Fig. 10 Structure of Material After Steam-Oxidation Test; Heat No. 60494; ×100

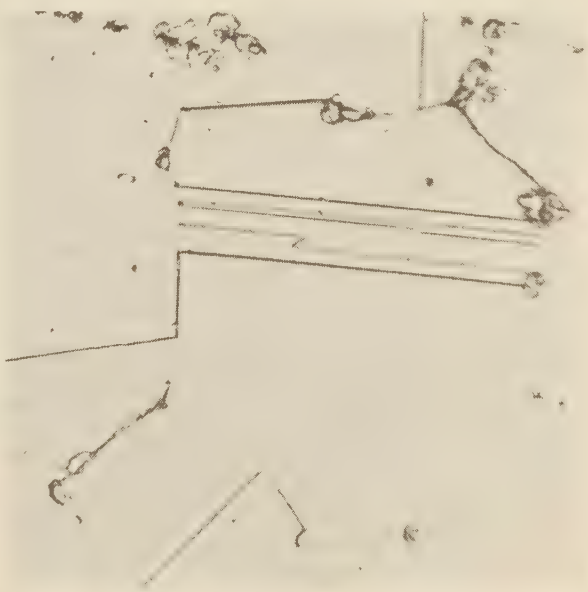

Fig. 11 View of Specimen in Greater Detail Than Shown at Left; Heat No. 60494; ×1000

fore and after exposure to steam at approximately 1800 F for 1300 hr, have been prepared by the research laboratory of The Babcock & Wilcox Tube Company:

"The exceedingly low rate of scale formation on the two steels tested by Hawkins, Solberg, Agnew, and Potter is of great interest from the metallurgical point of view. Specimens of the two steels have subsequently been examined and compared with the same materials prior to testing for the purpose of determining whether significant changes in mechanical properties or structural characteristics occurred during the tests.

"Figs. 8 to 17, inclusive, show representative microstructures from the same rods before and after testing. The filmlike character of the scale is shown in Figs. 16 and 17. These latter two specimens were chromium-plated over the scale and cut in a longitudinal direction along a plane normal to the circumference of the rod.

"Tensile specimens 0.438 in. diam were machined from the 1/2-in. rounds; the amount of metal removed being more than sufficient to clean up all surface imperfections. Table 5 shows both tensile properties and hardness values for the two steels before and after subjection to the steam-corrosion test.

"The photomicrographs indicate that no significant increase in

Fig. 12 Structure of Material Prior to Steam-Oxidation Treatment; Heat No. 1013; ×100

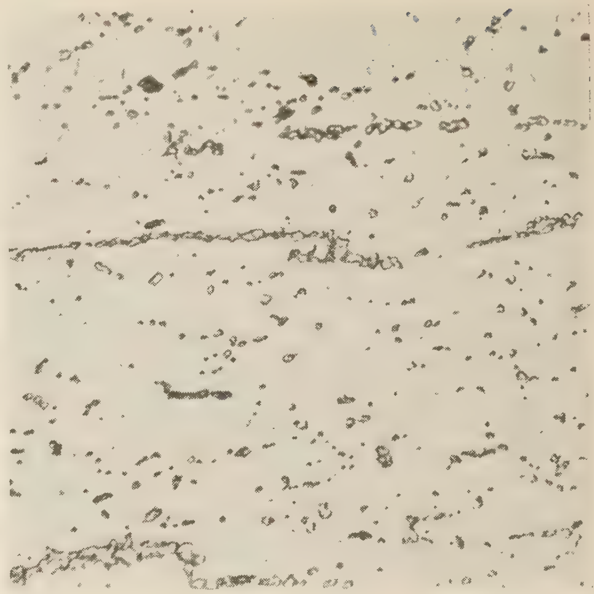

Fig. 13 View of Specimen in Greater Detail Than Shown at Left; Heat No. 1013; ×1000

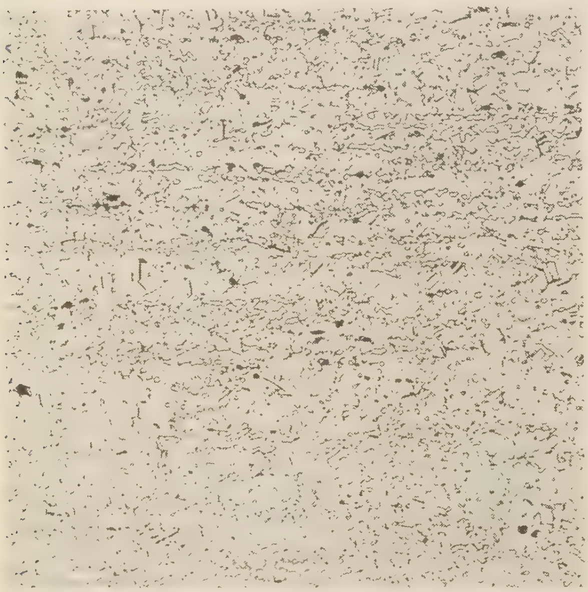

Fig. 14 Structure of Material After Steam-Oxidation Test; Heat No. 1013; ×100

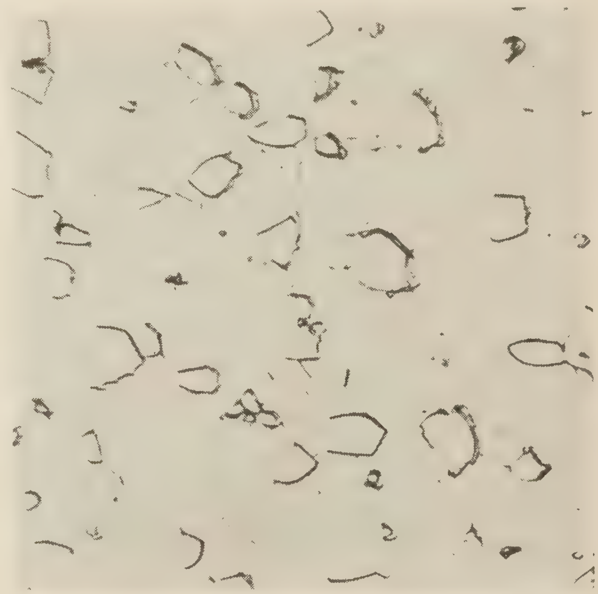

Fig. 15 View of Specimen in Greater Detail Than Shown at Left; Heat No. 1013; ×1000

TABLE 5 EFFECT OF STEAM-CORROSION TREATMENT ON MECHANICAL PROPERTIES

Heat no.	Treatment	Hardness Rockwell B scale	Hardness Equivalent Brinell	Yield strength, psi	Ultimate strength, psi	Reduction in area, per cent	Elongation,[a] per cent
60494	Original material (25-Cr, 20-Ni)	92.5	190	47000	91900	74.2	53.1
60494	Steam oxidation	80.0	146	42400	85500	73.7	49.7
1013	Original material (25-Cr, 15-Ni, 2-W)	96.5	215	66650	104850	57.6	43.4
1013	Steam oxidation	88.0	174	49580	95000	38.3	33.7

[a] Elongation measured on specimen $1^3/_4$ in. gage length and 0.438 in. diam.

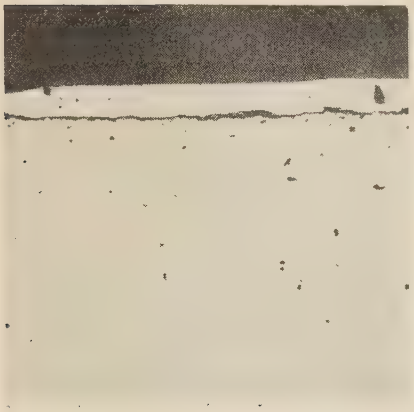

Fig. 16 Section Through Scale Film After Depositing Chromium Layer to Prevent Excessive Beveling and Pitting; Unetched; Heat No. 60494; ×100

Fig. 17 Section Through Scale Film After Depositing Chromium Layer to Prevent Excessive Beveling and Pitting; Unetched; Heat No. 1013; ×100

grain size occurred in the specimen from heat No. 60494, the straight 25-Cr, 20-Ni steel. In view of the fact that the grain size of the original steel was established by heating at 1900 F, prior to the steam-corrosion treatment, grain growth would hardly be expected at the testing temperature employed in the absence of phase changes.

"Heat No. 1013 (25-Cr, 15-Ni, 2-W) apparently was not quite stable at the testing temperature. Figs. 12 and 13 show ferrite stringers parallel to the direction of rolling, whereas the same material after the steam-corrosion treatment shows a more random distribution of equiaxed ferrite grains which are also coarser than in the original material, Figs. 14 and 15. The structural changes also are reflected in the mechanical properties as can be noted in Table 5."

Discussion

T. S. Fuller.[6] Of particular interest to the writer are the losses in weight values for 12 per cent chromium steels with 0.290 sulphur, and with 0.015 sulphur, in contact with steam for a period of 500 hr at a temperature of 1500 F.

No corresponding data are to be found in the paper for the low-sulphur alloy at 1800 F. Should this work be continued by the authors, the writer suggests including the low-sulphur alloy in subsequent 1800 F tests.

R. F. Miller[7] and G. V. Smith.[7] We have recently published[8] data on the relative resistance to oxidation by air of several alloy steels at 1700 F for a week, which show an order of resistance much the same as that in Fig. 6 of this paper for oxidation by steam.

The determination of resistance to oxidation or corrosion merely from the gain or loss in weight after a definite time at temperature fails to give desirable information on the earlier progress of the process. This information is readily obtained by a method of substantially continuous observation which has been used recently at the research laboratory of the United States Steel Corporation to follow the progress of air oxidation of a wide variety of steels at several temperatures; these results, with a description of the apparatus, will be published shortly.

The marked deterioration of corrosion resistance of the 12-chromium alloy at 1800 F brought about by the presence of sulphur is rather interesting. Might this be attributed to a combination of sulphur with some of the chromium so that the local content of chromium, though sufficient to keep the alloy resistant at 1500 F, is not large enough to prevent oxidation at 1800 F?

J. B. Romer.[9] The data presented in this paper are of great interest from the high-temperature-corrosion viewpoint.

Oxidation and corrosion losses are frequently calculated in terms of penetration, inches per unit of time, and heretofore these authors have so reported their data. In this instance, they encountered severe damage to certain low-alloy samples and, as depicted in their illustrations, corrosion or oxidation proceeded at the ends at a faster rate than along the central portion of their specimen. Therefore they have reported their data in terms of loss in weight, percentage of original weight. This method is not entirely free of end effects.

Our interest was more with highly alloyed materials, and therefore their data on loss can be recalculated in terms of penetration. Table 6 of this discussion shows the penetration in inches calculated to a 10,000-hr interval.

TABLE 6 PENETRATION, INCHES PER 10,000 HR[a]

Material	Duration of test: 500 hr	1300 hr
9 Cr-Mo	1.4	...
18-8 Cb	2.8×10^{-1}	...
25-20	14×10^{-4}	9.6×10^{-4}
25-15-2-W	17×10^{-4}	17.4×10^{-4}

[a] Temperature of test, 1750–1800 F.

For all the alloys tested, there is good agreement between the two 500-hr periods (see authors' Tables 1 and 2).

A study of the penetration at the end of 500 hr and at the end of 1300 hr, in conjunction with the calculated penetration for a time period of 10,000 hr, as shown in Table 6 of this discussion, indicates that the 25-20 alloy has a rate which decreases with time. We could therefore expect that a prolonged test would show a lower rate. The 25-15-2-W alloy shows the same rate for 10,000 hr in both the 500- and the 1300-hr tests, or in other words,

[6] Engineer of Materials, Schenectady Works Laboratory, General Electric Company, Schenectady, N. Y.

[7] Research Laboratory, United States Steel Corporation, Kearny, N. J.

[8] "Alloy Steels for High Temperature Service," by R. F. Miller, G. V. Smith, and P. A. Jennings, *Metals & Alloys*, vol. 16, 1942, pp. 438–441, 881–885.

[9] Chief Chemist, The Babcock & Wilcox Company, Barberton, Ohio.

we have a constant rate, and hence a prolonged test would not be expected to show a lower rate.

Fig. 5 of the paper shows the effect of temperature on the corrosion or oxidation rates. For a plain steel (S.A.E. 1010) there was a pronounced increase in rate at about 1100 F; and at 1500 F the rate was so high that the specimen was entirely destroyed in less than 500 hr. The 7-9 chromium-molybdenum types of alloys do not begin to show a pronounced increase in rate until the temperature is at least 1300 F. They do show an extremely high rate at 1800 F, even in a 500-hr test.

Apparently the 12 chromium and the 18-8 columbium stabilized do not show a break until somewhere in the 1500 to 1600 F range.

The 25-20 and the 25-15-2-W do not show a break at the maximum temperature of the test, namely, 1750 to 1800 F.

A very interesting condition has to do with the 12-chromium high-sulphur steel in the 1500 to 1750 F temperature range. This steel showed a very low loss, namely, 0.03 per cent of its original weight when tested at 1500 F, whereas it was completely destroyed when tested at 1750 F. The eutectic for the binary alloy of iron and sulphur is a little above 1800 F. For the complex alloys, it is not known. However, the change in rate just referred to suggests that the test temperature is near enough to the eutectic temperature of the complex alloy to lead to the belief that, for high-temperature service, a low-sulphur content in the steel is highly advantageous. This belief is well supported by the experiments reported by Whiteley[10] wherein he describes his observations on scale formation. The analysis of the core and the external layer of the 12-chromium high-sulphur steel, as reported by the authors, indicates that the chromium and iron diffused and that the sulphur was completely converted to some form which was removed from the test apparatus by the steam.

Authors' Closure

The high-sulphur 12 per cent chromium steel referred to by the discussers was a free-machining steel and was placed in the test group by accident. However, it was felt that the results obtained on it might be of considerable interest and were therefore included.

The authors are looking forward with interest to the forthcoming publication by Messrs. Miller and Smith of the results of air-corrosion tests involving substantially continuous observation of results. A correlation of the corrosion rates in air and steam and further data on the effect of time should be valuable contributions to existing knowledge.

[10] "An Effect of Oxygen and Sulphur on Iron in Scaling," by J. H. Whiteley, *Journal of the Iron and Steel Institute*, vol. 131, 1935, pp. 181–190.

Effect of Deoxidation Practice on Creep Strength of Carbon-Molybdenum Steel at 850 and 1000 F

By R. F. MILLER,[1] KEARNY, N. J.

A co-operative investigation with the General Electric Company, Westinghouse Electric & Manufacturing Company, Climax Molybdenum Company, and Crane Company, of the effect of deoxidation practice on creep strength. One heat of C-1/2-Mo steel was deoxidized with silicon and 1.5 lb of aluminum per ton, another with silicon and 0.5 lb of aluminum per ton. Creep tests at 1000 F showed that, with similar microstructure, the low-aluminum steel has superior creep behavior. At 1000 F coarse ferrite-pearlite has higher creep strength than fine ferrite-pearlite but at 850 F the reverse is true.

Introduction

Several previous investigations (1, 2, 3, 4)[2] have indicated that, at temperatures above the recrystallization or "equicohesive" range, the creep strength of a metal is increased by a moderate increase of grain size. While this may be clearly shown in a pure metal or a single-phase alloy, the effect of grain size on the creep strength of steel is complicated by the fact that in its usual condition steel contains at least two phases (ferrite and iron carbide), change in the distribution of which may have a more profound effect on the creep strength (5) than change in the grain size itself.[3]

Another complication, as yet not always appreciated, is that the grain size of a steel of given nominal composition is not always the same even for a specific heat-treatment but varies with the deoxidation practice used in manufacture of the material (7, 8, 9). The austenite grain size of a steel deoxidized with silicon, or silicon plus a small amount of aluminum, begins to coarsen immediately above the A_3 temperature, while that of a steel deoxidized with silicon and a larger amount of aluminum may remain fine until the material is heated 100 F or more above the A_3 temperature. Since steel used for high-temperature applications is usually normalized or annealed from the vicinity of its A_3 temperature, a steel deoxidized with little or no aluminum may show larger grain size, and hence higher creep strength (at least above its "equicohesive" temperature range), than will the same type of steel deoxidized with a larger amount of aluminum. There has been some doubt, however, as to whether the improvement in creep strength is due solely to the coarser grain size, or in part to the difference in aluminum content.

Investigation Conducted on Two Heats of Carbon-Molybdenum Steel

In order to answer this question and to determine the effect on creep strength of variation in the grain size and microstructure a co-operative investigation has been made of two heats of carbon-molybdenum steel. One heat was deoxidized with silicon and 1.5 lb of aluminum per ton of steel, resulting in a high austenite-grain-coarsening temperature; the second was deoxidized with silicon and 0.5 lb of aluminum per ton, resulting in a low austenite-grain-coarsening temperature. Heat-treatments were evolved which produced three similar microstructures in each steel, i.e., coarse ferrite-pearlite, ferrite Widmanstätten, and fine ferrite-pearlite. Comparisons of creep strength were made at 1000 F to determine the degree of difference between the pairs of similar structures. Even under such comparable conditions, the steel deoxidized with the smaller amount of aluminum (having the lower austenite-grain-coarsening temperature) had the higher creep strength.

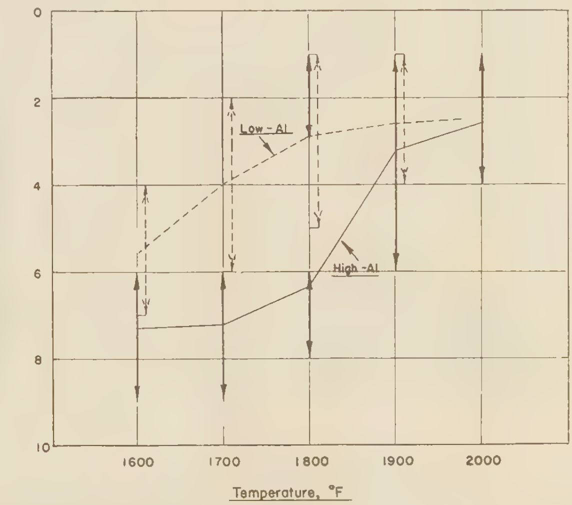

Fig. 1 Austenite-Grain-Coarsening Characteristics of C-Mo Steels

(Vertical arrows show range, and intermediate connecting lines show average austenite grain size, determined from tempered martensite.)

Steel deoxidized with silicon and 1.5 lb of aluminum per ton ————
Steel deoxidized with silicon and 0.5 lb of aluminum per ton — — — —

In order to show the effect of grain size on creep strength as a function of temperature, the coarse ferrite-pearlite and fine ferrite-pearlite structures of one of the steels were tested in creep at 850 F as well as at 1000 F. It was found that the material with the coarse ferrite grain size was the stronger at 1000 F, but at 850 F that with the fine ferrite grain size was stronger.

The open-hearth steels[4] selected for this work had the chemical composition given in Table 1.

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[1] Research Laboratory, United States Steel Corporation.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

[3] The grain size of steel has been referred to as austenitic, inherent, McQuaid-Ehn, actual, structural, ferritic, etc. The significance of these qualifying phrases has been discussed by Vilella (6) in a recent article.

Contributed by Committee on Effect of Temperature on Properties of Metals and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[4] The steels were supplied through the courtesy of the National Tube Company.

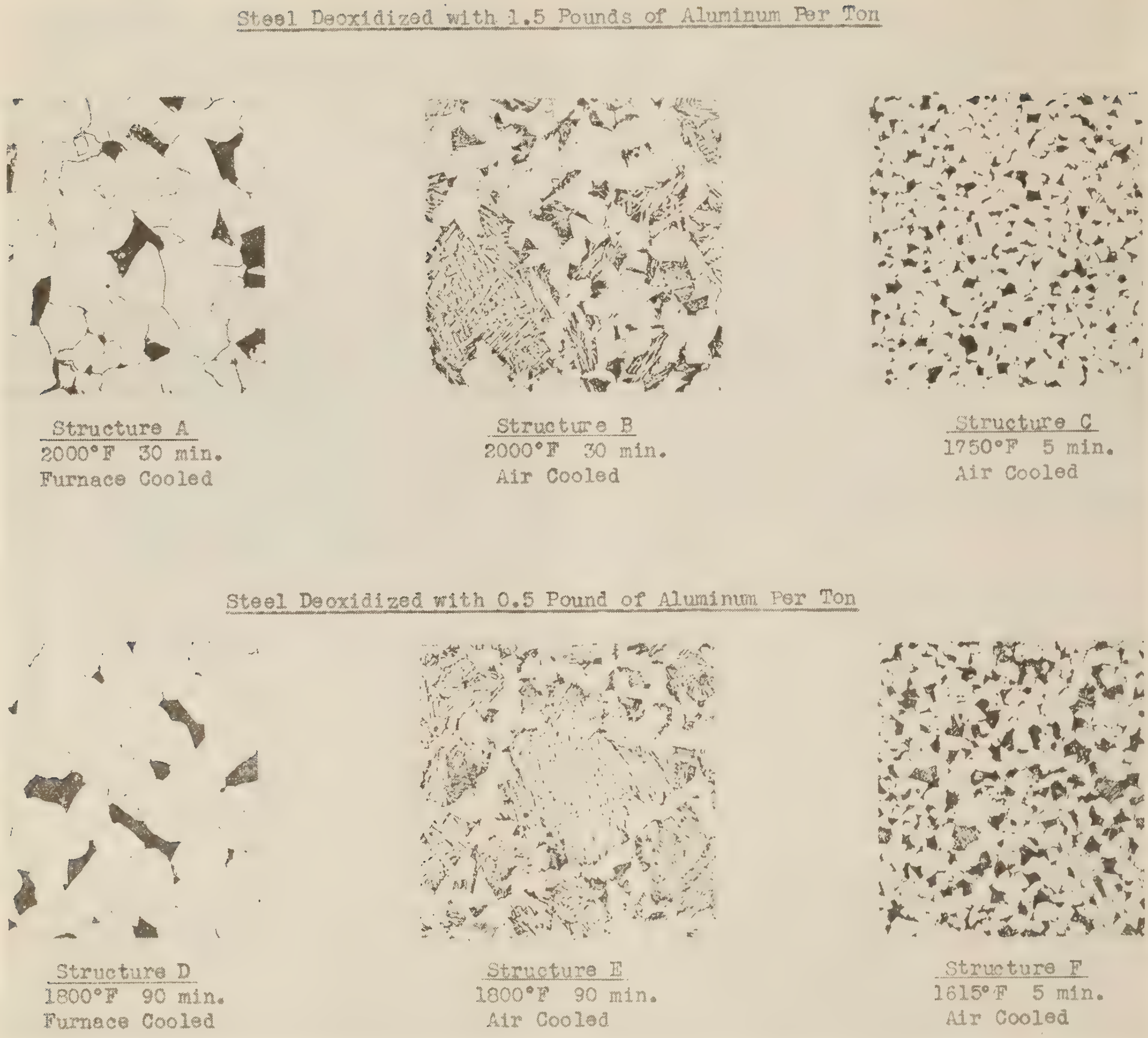

Fig. 2 Microstructures of Steels Tested in Creep; ×100

TABLE 1 COMPOSITION OF OPEN-HEARTH STEEL USED IN TESTS

Steel	C	Mn	P	S	Si	Mo	Al	Al_2O_3	Al added, lb per ton
High aluminum	0.11	0.41	0.028	0.010	0.14	0.49	0.016	0.002	1.5
Low aluminum	0.13	0.43	0.020	0.010	0.12	0.50	0.010	0.001	0.5

Heat-Treatment

While these two steels are of the same nominal composition, they have different austenite-grain-coarsening characteristics due to the difference in amount of aluminum added. The austenite grain size was determined at temperatures from 1600 to 2000 F by the tempered-martensite method, described by Vilella and Bain (10); the results are shown in Fig. 1. The high-aluminum steel remains fine-grained up to about 1800 F, at which temperature it begins to coarsen; from 1800 to 1900 F the grains are partly coarse, "duplex;" above 1900 F the grains are all coarse and increase in size with increase of temperature. In contrast, the low-aluminum steel begins to coarsen immediately above its A_3 temperature, shows no "duplexing," and the austenite grain size increases with increase of temperature more slowly than in the high-aluminum steel.

With a knowledge of the austenite-grain-coarsening characteristics of these steels, it was not difficult to determine experimentally a schedule of heat-treatments, producing in each steel three comparable microstructures. The heat-treatments carried out by the General Electric Company on 1-in-diam bars were as follows:

High-al steel → 2000 F, 30 min → Furnace cool (at 200 F per hr) → Structure A
High-al steel → 2000 F, 30 min → Air cool → Structure B
High-al steel → 1750 F, 5 min → Air cool → Structure C

Low-al steel → 1800 F, $1^1/_2$ hr → Furnace cool (at 200 F per hr) → Structure D
Low-al steel → 1800 F, $1^1/_2$ hr → Air cool → Structure E
Low-al steel → 1615 F, 5 min → Air cool → Structure F

TABLE 2 GRAIN SIZE, HARDNESS, IMPACT STRENGTH, AND CREEP DATA OF SPECIMENS

	Specimen	Structure*	Grain Size: Austenite	Grain Size: Ferrite	Hardness, VPN: Before Test	Hardness, VPN: After Test	Impact Strength, ft lbs**: Before Test	Impact Strength, ft lbs**: After Test	Stress 1000 psi	Duration, Hrs.	Total Ext. %	Intercept (%) and Creep Rate (Millionths (in/in/hr) During Time Intervals: 500 to 1000 hr. Int. %	500 to 1000 hr. Rate	1000 to 2000 hr. Int. %	1000 to 2000 hr. Rate	2000 to 3000 hr. Int. %	2000 to 3000 hr. Rate
Tests at 1000°F																	
1.5 lbs Al added per ton	1A1	Coarse F-P	1-4	2-5	102	136	12.5	6	8	3000	0.032	0.012	0.10	0.014	0.07	0.014	0.07
	2A1					113		3.5	12	3000	0.116	0.118	0.30	0.129	0.17	0.139	0.10
	3A2					105		3	15	3000	0.156	0.109	0.48	0.127	0.30	0.168	0.19
	1B1	F-W	1-4	4-7	146	106	37	30.5	8	3000	0.052	0.004	0.16	0.004	0.16	0.004	0.16
	2B1					138		31.5	12	3000	0.068	0.074	0.13	0.065	0.23	0.058	0.25
	5Bx					137		34.5	15	3000	0.081	0.105	0.14	0.036	0.24	0.060	0.32
	1C1	Fine F-P	5-7	7-8	118	120	43	38.5	8	3000	0.175	0.025	0.38	0.020	0.42	neg.	0.72
	2C1					121		38	12	3000	0.256	0.106	0.44	0.082	0.53	neg.	1.10
	3C2					119		39	15	3000	0.747	0.220	1.20	0.150	1.83	neg.	3.04
0.5 lb Al added per ton	3D1	Coarse F-P	1-4	3-5	109	112	11.5	4	10	2000	0.151	0.121	0.32	0.136	0.14	-	-
	3D2					-		-	15	3000	-	0.255	0.95	0.341	0.45	0.399	0.25
	4D2					107		2	17	3000	0.727	0.494	1.66	0.568	0.68	0.705	0.39
	3D2					-		-	10	1500	-	-	-	-	-	-	0.07
	1E1	F-W	1-4	5-7	151	149	34.5	30.5	12	3000	0.087	0.028	0.24	0.034	0.18	0.042	0.14
	2E1					148		29.5	12	3000	0.072	0.087	0.20	0.087	0.20	0.095	0.16
	3E1					141		26	12	2000	0.152	0.112	0.38	0.126	0.24	-	-
	4E2					150		28	12	2000	0.071	0.067	0.28	0.078	0.24	-	-
	5E1					141		28	12	3000	0.085	0.097	0.24	0.113	0.20	0.081	0.19
	4E4					-		-	15	3000	-	0.090	0.40	0.103	0.28	0.097	0.29
	3E2					-		-	15	2000	-	-	0.39	-	0.28	-	-
	5E2					153		33	17	3000	0.180	0.134	0.50	0.146	0.41	0.146	0.40
	4E4					-		-	10	1500	-	-	-	-	-	-	0.11
	3F1	Fine F-P	5-7	7-8	131	129	38	35	10	2000	0.189	0.122	0.52	0.146	0.42	-	-
	4F2					-		-	15	3000	-	0.181	0.76	0.192	0.69	0.184	0.72
	3F2					-		-	15	2000	-	-	0.95	-	0.73	-	-
	4F2					-		-	10	1500	-	-	-	-	-	-	0.24
Tests at [illegible]°F																	
1.5 lbs Al/ton	3A1	Coarse F-P	1-4	2-5	102	147	12.5	2	32	2000	4.077	3.826	1.90	3.970	0.75	-	-
	5A1					119		3	25	3000	1.390	1.350	1.00	1.380	0.52	1.640	0.24
	3C1	Fine F-P	5-7	7-8	118	130	43	37	32	2000	2.029	1.910	0.90	1.960	0.60	-	-
	5C1					130		40	26	3000	0.126	0.152	0.42	0.173	0.24	0.187	0.17

*F = ferrite, *P = pearlite, *W = widmanstatten ** modified (2/3 size) Charpy impact strength

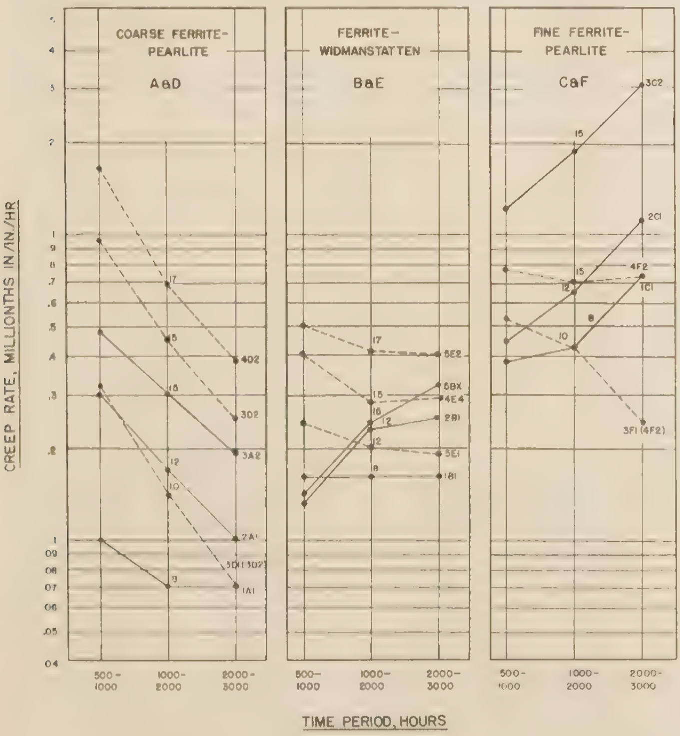

FIG. 3 CHANGE OF CREEP RATE DURING CREEP TESTS AT 1000 F; STRUCTURES A, B, C, D, E, F

(Time period plotted against average graphical creep rate during period. Numbers above curves show applied stress in 1000 psi. Symbols to right of curves are specimen numbers.)

Steel deoxidized with 1.5 lb of aluminum per ton ————

Steel deoxidized with 0.5 lb of aluminum per ton — — — —

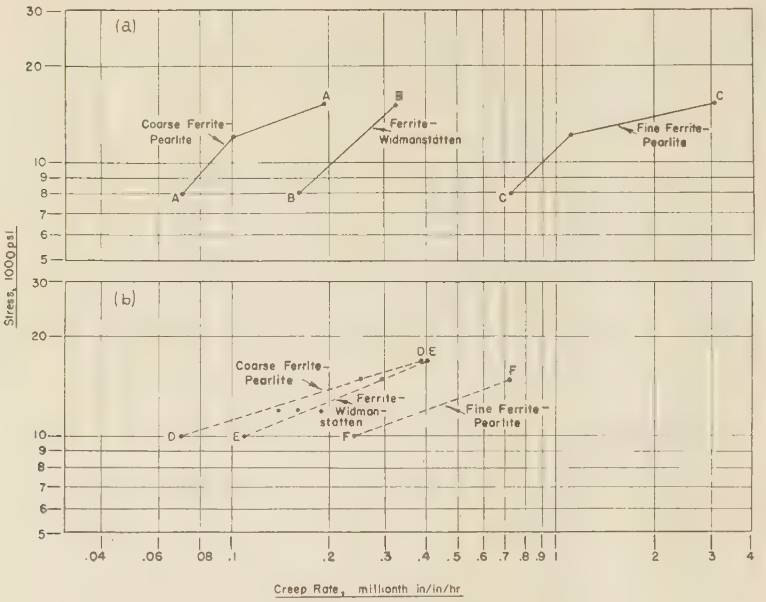

Fig. 4 Creep Tests on C-Mo Steel at 1000 F

a, Deoxidized with 1.5 lb of aluminum per ton of steel. *b*, Deoxidized with 0.5 lb of aluminum per ton of steel.)

High-aluminum steel ————
Low-aluminum steel — — — —

Specimens 3D1, 3E1, 3E2, 3F1, and 3F2 were run only 2000 hr, and, in order to obtain additional information beyond 2000 hr on D, E, and F material tested under 10,000 psi at 1000 F, the stress on specimens 3D2, 4E4, and 4F2 was reduced from 15,000 to 10,000 psi at the end of 3000 hr, and the test continued for an additional 1500 hr. The creep rate was determined for the last 500 hr of the additional 1500-hr test period, and the data recorded in Table 2.

Since the creep tests were made in five different laboratories, each using a somewhat different technique, check tests were run to determine the variation in their results. Structure *E*, chosen as the check material, was tested by all five laboratories under 12,000 psi at 1000 F. The check tests showed good agreement (Table 2), giving values from 0.18 to 0.24 millionth in. per in. per hr for the time period 1000 to 2000 hr and 0.14 to 0.19 millionth in. per in. per hr for the time period 2000 to 3000 hr.

The creep rate of all of the specimens was found to be changing over the entire 3000-hr test period, and considerable useful information was obtained by study of the change of creep rate during the tests. The average creep rate and intercept[5] were estimated graphically from the time-elongation creep curve for the three time periods 500 to 1000, 1000 to 2000, and 2000 to 3000 hr. These data are listed in Table 2, and the creep data replotted against the time period in Fig. 3. For the same stress and microstructure, the low-aluminum steel (dotted lines) shows a decreasing creep rate more consistently than the high-aluminum steel (solid lines). Comparison of the creep behavior of different structures of the same steel shows that, under the same stress, the coarse ferrite-pearlite structure exhibits a more pro-

The microstructures resulting from these heat-treatments are shown in Fig. 2. It will be noted that the coarse ferrite-pearlite structures *A* and *D* are almost identical, as are the ferrite Widmanstätten structures *B* and *E*. However, comparison of the fine ferrite-pearlite structures *C* and *F* shows that the Widmanstätten pattern is a little more pronounced in structure *F* of the low-aluminum steel, presumably due to its low austenite-grain-coarsening temperature.

Results of Creep Tests

The duration of the creep tests was 2000 or 3000 hr. The creep data are listed in Table 2 and plotted in Figs. 3 to 6.

[5] For each time period, the average creep rate was extended to the left to intersect the extension ordinate of the creep curve. This intersection or "intercept" gives an indication of the amount of deformation accompanying any particular creep rate.

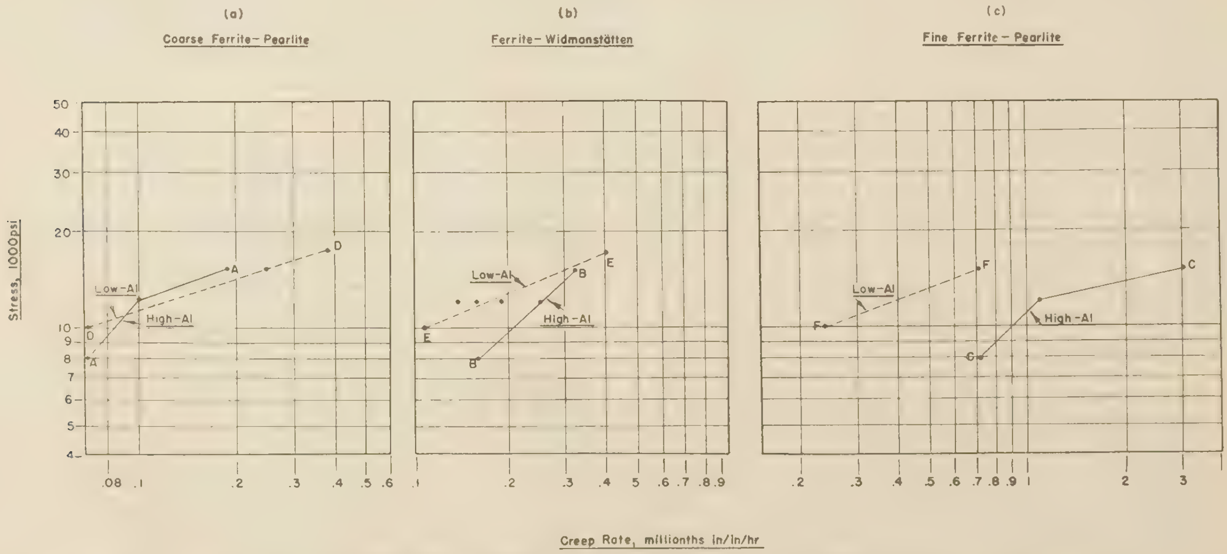

Fig. 5 Creep Tests on Similar Microstructures of C-Mo Steel at 1000 F

Deoxidized with 1.5 lb aluminum per ton (Structures A, B, and C) ————. Deoxidized with 0.5 lb aluminum per ton (Structures D, E, and F) — — —

nounced decrease in creep rate than the ferrite Widmanstätten or the fine ferrite-pearlite structures.

The creep behavior of these materials was also studied by plotting the creep rate during any given time period as a function of the applied stress. Such plots for the 500 to 1000 or 1000 to 2000-hr time periods were somewhat irregular and confusing, but those for the last time period, 2000 to 3000 hr, were quite regular. On this basis, two comparisons have been made, i.e., the creep behavior of various structures of a single material, and similar structures of the two different materials.

1 (*a*) *Creep at 1000 F of High-Aluminum Steel.* The 2000 to 3000-hr creep rate of the various structures of the high-aluminum steel has been plotted against the applied stress in Fig. 4(*a*). The coarse ferrite-pearlite structure *A* has the highest creep strength; the ferrite Widmanstätten structure *B*, formed from the same austenite grain size as *A*, is intermediate, and the fine ferrite-pearlite structure *C* is the weakest. On a comparative basis under 10,000 psi, *B* creeps twice as fast as *A*, *C* nine times as fast as *A*. While the coarse ferrite-pearlite structure *A* has higher creep strength than the ferrite Widmanstätten structure *B*, during the 2000 to 3000-hr time period, it should be noted (Table 2) that, under 12,000 and 15,000 psi, the intercept and total deformation at the end of 3000 hr was greater in *A* than in *B*.

(*b*) *Creep at 1000 F of Low-Aluminum Steel.* The creep behavior of this steel, Fig. 4(*b*) is similar to that of the high-aluminum steel just described, although the difference in strength between the various structures is not so great. The coarse ferrite-pearlite structure *D* is the strongest; the ferrite Widmanstätten structure *E*, somewhat weaker; and the fine ferrite pearlite structure *F*, is the weakest. Under 10,000 psi, *E* creeps 1.6 times faster than *D*, and *F* creeps 3.4 times faster than *D*. As in the high-aluminum steel, the coarse ferrite-pearlite structure *D* has slightly higher creep strength than the ferrite Widmanstätten structure *E*, but this advantage is offset by the fact that, under the same stress, the total deformation in 3000 hr is considerably greater in *D* than in *E*. This difference in amount of deformation is also noted in the magnitude of the intercepts.

2 *Comparison of the Two Steels on the Basis of Similar Microstructures.* The creep behavior of similar microstructures of the two steels is shown in Fig. 5. When both steels have a microstructure consisting of coarse ferrite-pearlite, Fig. 5(*a*), the low-aluminum steel appears to be more creep-resistant under stresses less than 12,000 psi than the high-aluminum steel. The same trend is indicated for the ferrite Widmanstätten structures of both steels, Fig. 5(*b*); the low-aluminum steel appears to be more creep-resistant than the high-aluminum steel, under stresses less than 15,000 psi. When both steels have a structure consisting of fine ferrite-pearlite, Fig. 5(*c*), the low-aluminum steel has higher creep strength than the high-aluminum steel under all stresses examined.

Creep Tests at 850 and 1000 F. In order to determine the effect of grain size on creep strength as a function of temperature, the coarse ferrite-pearlite structure *A* and the fine ferrite-pearlite structure *C* of the high-aluminum steel were tested at 850 F, as well as at 1000 F. The data are recorded in Table 2 and plotted in Fig. 6. It will be noted that the creep rates were taken for the 1000 to 2000-hr time interval, since two of the tests were run for only 2000 hr. The coarse ferrite-pearlite structure *A* (ferrite grain size 4) has considerably higher creep strength than the fine ferrite-pearlite structure *C* (ferrite grain size $7^1/_2$) at 1000 F, but somewhat lower creep strength at 850 F.

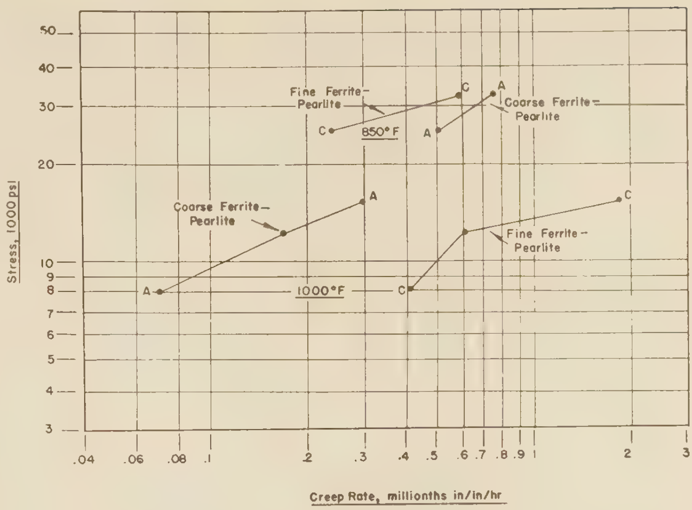

Fig. 6 Creep Tests at 850 and 1000 F on C-Mo Steel, Deoxidized With 1.5 Lb of Aluminum per Ton; 1000 to 2000-Hr Creep Rates

(*A*, Coarse ferrite and pearlite structure. *C*, Fine ferrite and pearlite structure.)

Hardness. The hardness of the creep specimens before and after test is shown in Table 2. In both steels, the ferrite Widmanstätten structure was found to be the hardest (142 to 151 VPN), the fine ferrite-pearlite structure was somewhat softer (118 to 131 VPN), and the coarse ferrite-pearlite structure the softest (102 to 109 VPN). Structure for structure, the low-aluminum steel appeared to be slightly harder (by about 10 points VPN) than the high-aluminum steel. Little or no change of hardness occurred in the specimens tested at 1000 F. There was a considerable increase in hardness of the specimens tested in creep at 850 F, presumably due to strain hardening.

Impact Strength. The results of modified Charpy impact tests on the various structures of the two steels before and after creep testing are listed in Table 2. In both steels the impact strength of the coarse ferrite-pearlite structure was the lowest ($11^1/_2$ to $12^1/_2$ ft-lb), the impact strength of the ferrite Widmanstätten structure was considerably better ($34^1/_2$ to 37 ft-lb), while that of the fine ferrite-pearlite structure was the best of the three (38 to 43 ft-lb). Structure for structure, the impact strength of the high-aluminum steel was a little higher (about $3^1/_2$ ft-lb) than that of the low-aluminum steel.

The impact strength of all of the structures decreased during the creep tests at 850 and 1000 F. Only a slight decrease was noted at 1000 F in the ferrite Widmanstätten and the fine ferrite-pearlite structures, but the decrease of impact strength of the coarse ferrite-pearlite structure was somewhat greater.

Change of Microstructure During Creep Test. Specimens of each structure, tested under the highest stress and for the longest time period at 1000 F, were examined for change of microstructure during creep test. A slight spheroidization, barely detectable under the microscope at 1000× and probably unnoticeable in a photomicrograph, was found in all of the specimens examined.

Discussion of Results

The results of the present investigation show that, even if the microstructure of the two steels is the same, the steel with the low aluminum addition (low austenite-grain-coarsening temperature) is more creep-resistant at 1000 F than the steel with the high aluminum addition (high austenite-grain-coarsening temperature), confirming the results of the tests by Cross and Lowther at Battelle Memorial Institute (9, 10).

Specification of the austenite grain size alone, without men-

tion of the cooling rate from the heating temperature, is insufficient to determine the creep behavior, as shown by the fact that the coarse ferrite-pearlite structure is more creep-resistant than the ferrite Widmanstätten structure formed from the same austenite grain size. While no spheroidized structure was included in this investigation, it has been clearly shown in a recent paper by Weaver (5) that specification of the ferrite grain size alone, without mention of the distribution of the carbide particles, is also inadequate to specify the creep behavior. In other words, the creep strength of a given material does not depend upon its grain size alone, but upon its entire microstructure and, thus, upon its entire heat-treatment. It should be noted that certain microstructures will spheroidize due to prolonged exposure to test or service temperatures (5, 11, 12) and hence the best structure for one temperature may not be the best at another temperature. No single grain size or microstructure produces optimum creep strength in a material over a wide range of temperature.

It should be noted that the terms "coarse-grain material" and "fine-grain material" have been avoided. These terms are ambiguous, since coarse-grain material could refer either to a steel with a low austenite-grain-coarsening temperature or to a steel with a coarse ferrite-grain size.

Summary

1 The steel deoxidized with 0.5 lb of aluminum per ton was found to have a low austenite grain-coarsening temperature, while that deoxidized with 1.5 lb of aluminum per ton had a high austenite-grain-coarsening temperature and a range of mixed coarse and fine "duplex" austenite grains. Coarse or fine ferrite grains could be produced in either steel by appropriate heat treatment.

2 When compared on the basis of similar microstructures, the low-aluminum steel had a slightly higher hardness and lower impact strength than the high-aluminum steel.

3 With a similar microstructure in both steels, the low-aluminum steel was found to be more creep-resistant at 1000 F under stresses less than 12,000 psi and to exhibit a more pronounced decrease in creep rate with time than the high-aluminum steel.

4 Comparison of the three types of microstructure in each steel showed that at 1000 F the coarse ferrite-pearlite structure was more creep-resistant and showed a decreasing creep rate more consistently than did the ferrite Widmanstätten structure, which in turn was more creep-resistant than the fine ferrite-pearlite structure. Under the same stress, the total deformation in 3000 hr was less in the ferrite Widmanstätten structure than in the other two, except in one case.

5 In either steel the coarse ferrite-pearlite structure had lower hardness and lower impact strength than the ferrite Widmanstätten structure which in turn was harder and had lower impact strength than the fine ferrite-pearlite structure.

6 The coarse ferrite-pearlite structure had higher creep strength than the fine ferrite-pearlite structure at 1000 F, but lower creep strength at 850 F.

7 While coarse ferrite-pearlite had the highest creep strength of the three structures at 1000 F, its initial impact strength was definitely inferior and deteriorated during the creep test. The fine ferrite-pearlite structure had the highest initial impact strength but the lowest creep strength. The ferrite Widmanstätten structure appeared to be most satisfactory at 1000 F; its creep strength was good and its total deformation the lowest of the three structures. Its initial impact strength was almost as high as that of the fine ferrite-pearlite structure, and there was little deterioration of the material, as evidenced by the small loss in impact strength during the creep tests.

8 Summarizing the difference between the two types of steel, it appears that when both steels are heat-treated from just above the A_3 temperature, the ferrite grain size of the low-aluminum steel would be coarser than that of the high-aluminum steel, and its creep strength would thus be superior at 1000 F. This is not the only reason for its superiority, however. The present investigation has shown that, even if each steel is heat-treated in such a manner that the grain size and microstructure of the two steels are similar, the creep behavior of the low-aluminum steel at 1000 F is better than that of the high-aluminum steel. The true difference between the two materials thus appears to be associated with the difference in the amount of aluminum added to the material.

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Discussion

Ernest L. Robinson.[6] The tests made under the direction of the author have been at specified stresses, with the result that his quantitative comparisons between one material and another are given on the basis of creep rates for a given loading rather than on the basis of creep strength for a given rate. While the author points out which materials are stronger or weaker, he does not make a quantitative comparison, based on a definite specified creep rate. However, Figs. 4, 5, and 6 of the paper enable such comparisons to be made by extrapolating the results on the stronger materials to somewhat higher stresses than those at which they were tested, and the weaker materials to stresses somewhat lower than those at which they were tested.

Results obtained by such extrapolation cannot be said to have the validity of actual tests, but they do enable us to make definite comparisons which may be said to represent the indications of the results of this program.

[6] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

The author's Figs. 4, 5, and 6 enable comparisons with a minimum of extrapolation at the middle creep rate of 5 per cent per 100,000 hr (0.5 millionth in. per in. per hr). In the case of the stronger materials, the tests have been actually carried down to rates less than 1 per cent per 100,000 hr (0.1 millionth in. per in. per hr). However, rather large extrapolation is required on the part of the weaker materials to determine what stress would cause a creep rate of 1 per cent per 100,000 hr in such materials.

Reading from the author's Figs. 4, 5, and 6, Tables 3 and 4 of this discussion have been prepared to show the creep strength in pounds per square inch at 1000 F and at 850 F.

TABLE 3 CREEP STRENGTHS IN POUNDS PER SQUARE INCH AT 1000 F[a]

	Rate, per cent per 100,000 hr	
	5 Per cent	1 Per cent
1.5 Lb aluminum, coarse, *A*	28000	10000
1.5 Lb aluminum, fine, *C*	7000	3000
0.5 Lb aluminum, coarse, *D*	18000	11200
0.5 Lb aluminum, fine, *F*	13000	7200

[a] Based on Fig. 4 or Fig. 5.

TABLE 4 CREEP STRENGTHS IN POUNDS PER SQUARE INCH AT 850 F[a]

	Rate, 5 per cent per 100,000 hr
1.5 Lb aluminum, coarse, *A*	25000
1.5 Lb aluminum, fine, *C*	30000

[a] Based on Fig. 6.

Coarse Versus Fine Structure. From these tables it appears that at 1000 F the steel with 1.5 lb of aluminum added is in the order of 4 times as strong in the coarse condition as in the fine for the faster creep rate of 5 per cent, and in the order of 3 times as strong for the 1 per cent creep rate. The steel with 0.5 lb of aluminum added is some 40 per cent stronger in the coarse condition at the higher creep rate, and more than 50 per cent stronger at the lower creep rate.

Variations With Aluminum Content. In the coarse condition, and at the higher creep rate, the material with 1.5 lb of aluminum is more than 50 per cent stronger than that with 0.5 lb of aluminum. However, at the lower creep rate, which is nearer to the design condition, the high-aluminum material is some 10 per cent weaker than the low-aluminum material.

In the fine condition at both creep rates, the material with the restricted amount of aluminum is two or three times as strong as that with the 1.5 lb addition.

The A.S.M.E. Boiler Code allows a working stress for this material of 5000 or 4400 psi, depending upon whether there is grain-size control. These figures are supposed to represent 0.8 of the creep strength, based on a rate of 1 per cent per 100,000 hr. Table 3 of this discussion shows a figure of 3000 for the fine material *C* with 1.5 lb of aluminum added. This figure is based on a long extrapolation giving the material the benefit of a good deal of doubt. A test might show a much lower value but probably not much higher. Certainly this material would be entirely inadequate to meet Boiler Code requirements.

At 850 F, the fine material is 20 per cent stronger than the coarse material for the higher rate of creep. No further comparisons are available at this temperature.

Thus, without being too precise as to the exact figures, the evidence of these tests indicates that the fine material may be in the order of 20 per cent better at the lower temperature, but at the higher temperature it may be only one quarter to one third as good. This evidence, therefore, shows that any comprehensive specification should provide for fine material at moderate temperatures and for coarse material at higher temperatures.

On the other hand, the evidence of these tests is equally clear that a blanket specification should call for a coarse structure, and especially so if the aluminum is not limited. A fine structure may be 20 per cent stronger at lower temperatures but at higher temperatures anywhere from one third weaker with low aluminum to only one quarter as strong with high aluminum. In any case, these tests indicate that careful limitation of the aluminum addition results in higher creep strength for the lower rates of creep corresponding to design loadings, regardless of whether the microstructure is coarse or fine.

A. E. WHITE.[7] The writer will discuss briefly three matters which the author has presented in his paper: i.e., (*a*) influence of grain size at 1000 and 850 F on high-temperature properties; (*b*) effect of aluminum on high-temperature properties (possibly through its effect on grain size); and (*c*) ferrite-pearlite versus ferrite Widmanstätten structures on high-temperature properties.

The findings of the author, with respect to the better high-temperature values obtained with large grain size at 1000 F, and with small grain size at 850 F, are quite in accord with the theory advanced by Dr. Clark and the writer.[8]

The author in his paper states: "The true difference between the two materials thus appears to be associated with the difference in the amount of aluminum added to the material." This is a perfectly guarded sentence, for it makes no commitments but only expresses trends. It is true that, from the results reported, the better high-temperature properties at 1000 F, with but one exception, were obtained with the steel to which the smaller amount of aluminum had been added. Yet, between the limits of aluminum additions of 1 to 2.25 lb per ton, White and Crocker, in a previous paper[9] failed to find a relationship between creep strength and aluminum additions. The work in question was done on a number of commercial heats of carbon-molybdenum steel purchased on the basis of the A-206 Specification. These steels, therefore, were essentially of the same composition. Also, all of these heats were given the same heat-treatment. Yet, in spite of the fact that these heats were essentially of the same chemical composition and had been given the same heat-treatment, the creep rate at 925 F, expressed in per cent per 100,000 hr, was found to be 2.1 per cent in a case in which 1 lb of aluminum per ton was added, whereas, when the amount of aluminum was increased to 1.2 lb per ton, a creep rate of 0.3 was found. In the case of two heats to each of which there was added 2.25 lb of aluminum per ton, the creep rate was 0.29 per cent in one case and 1.4 per cent in another case. Thus, within the range of aluminum additions of 1 to 2.25 lb per ton, there resulted no consistent changes in creep rates due to aluminum-addition differences.

We know that aluminum affects grain size and if we eliminated all variables but the amount of the aluminum additions, the influence of aluminum would be most potent. Yet the writer would point out, and in this respect he believes the author will agree, that there are factors other than aluminum which influence high-temperature properties.

The findings of the author that a coarse ferrite-pearlite structure is superior to a coarse ferrite Widmanstätten structure, from the standpoint of creep rate at 1000 F, are somewhat at variance with the findings at 925 F in White and Crocker's paper;[8] for in that paper the material with a coarse-grained Widmanstätten structure gave better creep values than the material with a coarse-grained ferrite-pearlite structure. The difference be-

[7] Director of Engineering Research, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

[8] Author's Bibliography (3).

[9] Author's Bibliography (4).

tween the findings of the author and those of White and Crocker may be due to the difference in the temperatures, 1000 F versus 925 F, or it may be due to other factors which have not as yet been explained.

It is interesting to note that the author is in agreement with the findings of White and Crocker in so far as the best possible all-round type of structure is concerned; that is, the author concludes: "The ferrite Widmanstätten structure appeared to be the most satisfactory at 1000 F."

In conclusion, the writer wishes again to congratulate the author on his contribution, for it is only through scientific work of the type reported in this paper that our knowledge with respect to the properties of metals, in this case carbon-molybdenum steel, can be increased.

Author's Closure

Mr. Robinson's analysis of the creep data is much appreciated, and the author is in general agreement with his conclusions. However, the purpose of the present work was to compare the behavior of the two materials under a variety of conditions, rather than to determine any specific creep rate. It has been our experience that stress-creep rate data do not always lie on a straight line on a log-log or a semi-log plot, and we, therefore, prefer to avoid extensive extrapolation of the data.

In regard to Prof. White's comments, the agreement between the present work and that carried out by White and Crocker might have been clearer had these investigators heat-treated their various carbon-molybdenum steels to produce substantially the same microstructure in all. Although their steels were all of the same general composition, the aluminum additions, and hence the austenite grain-coarsening characteristics were different; consequently, when all were given the same heat-treatment, different microstructures and correspondingly different creep characteristics resulted.

Static Friction

By WALTER CLAYPOOLE,[1] NEW YORK, N. Y.

A new machine is described by means of which static friction between contacting surfaces separated by lubricating films of thickness approaching molecular dimensions can be measured with considerable precision. In a test run, a number of spot determinations of the static coefficient are made at closely spaced points over a short rubbing track not exceeding a few thousandths of an inch in length. The individual "slips" at the contact may be as small as 0.0002 in. The unit pressure is of the order of 500 psi. A significant correlation between static friction and the molecular structure and dimensions of a pure hydrocarbon lubricant has been established. Exceedingly smooth contacting surfaces overlaid with monomolecular films of strongly polar hydrocarbons have exhibited the phenomenon of vanishingly low static friction.

Introduction

This paper presents the results of an experimental inquiry into some phenomena of static friction, particularly in a system where two relatively smooth surfaces, separated by a very thin lubricating film, are pressed together under load.

"Static friction" is measured by the tangential force required to initiate slip of one surface with respect to another. This generally accepted definition is faulty in that it presents a picture of absolute immobility until some critical value of tangential force is reached, after which one surface starts to slide visibly over the other.

It is of course well recognized that, in any specified friction system, the force necessary to cause slip may vary within rather wide limits and that such variation may be referred to the complex factors operating in the contact area. It is not so well recognized, however, that initial slip may be on a very minute scale observable only with the aid of a microscope. Granting this, it would naturally follow that on the basis of the accepted definition the value of static friction in a specified system may be much less than as determined under the usual conditions.

Another effect not commonly observed is a very slow creep of one surface over the other, the movement being initiated by a critically small increment of the applied tangential force at the contact.

Still another phenomenon observed occasionally under especially favorable conditions is that of vanishingly small static friction.

In a paper published elsewhere,[2] the postulate was advanced that two geometrically plane surfaces each overlaid with a monolayer of oriented molecules would slide over each other without hindrance, the only work absorbed being that referable to the distortion of the molecular field of force across the interface. Geometrically true planes are of course unattainable. However, in this paper experimental evidence will be presented in support of the theoretical argument.

Measurement of Static Friction

The determination of the "static coefficient" appears at first sight to be a very simple matter. One quantity must be known, i.e., the normal pressure or load between the test specimens. One quantity must be measured, i.e., the tangential force which is required to cause relative movement in the plane of contact. Visual or instrumental observation determines initial slip.

The classical tilting-track device is theoretically ideal. We do not even have to know the load. The plane of contact is tilted from the horizontal until slip occurs. The tangent of this angle is the "coefficient of static friction." It is not necessary here to discuss the practical limitations of this method as regards sensitivity and precision.

A very full literature exists on the subject of the measurement of friction under a wide variety of experimental conditions. In this paper, a new and sensitive machine is described which lends itself to the determination of static coefficients of the order of 0.0002, and of slips of the order of 1 μ (about 0.00004 in.).

The method involves the application of successive increments of tangential force at the contact between the test specimens. For each force increment the resultant slip is measured and the local value of the static coefficient is computed. The plot of coefficient against slip gives a picture of the variation of friction at closely spaced points along a short rubbing track.

It should be noted that in this investigation no attempt was made to determine static coefficients as such but rather to inquire into some of the basic phenomena of friction. To this end the experimental conditions were kept as constant as possible and were confined within narrow limits.

The Short-Track Friction Tester

A consideration of all aspects of the problem led to the formulation of certain specific requirements which must be met by a machine suitable for the purpose of the investigation. These are as follows:

1 The material used for the test specimens must be chemically inert toward the lubricant used.

2 The contacting surfaces must have as high a degree of smoothness as is practicable.

3 The geometrical form of the contacting surfaces must be such that the area of contact under a specified load may be calculated.

4 The working load, normal to the plane of contact, must be applied with minimum disturbance to the lubricating film.

5 It must be possible to apply successive small increments of force in the plane of contact and to determine the value of such increments to a high degree of precision.

6 Very small relative movement between the specimens at the contact area must be accurately measurable.

7 The suspension for the movable specimen must be such that only parallel travel is possible.

The first three of these requirements were met by the use of a glass lens of high optical quality opposed to a glass interferometer flat, the lens being the movable specimen. To meet Specification 7, the lens was mounted in a holder suspended by four light and

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[1] The Pupin Laboratory, Columbia University, on Fellowship established at Columbia University by The Texas Company, New York, N. Y.

[2] "The Nature of Static Friction," by Walter Claypoole and D. B. Cook, *Journal of The Franklin Institute*, vol. 233, no. 5, May, 1942, pp. 453–463.

Contributed by the Special Research Committee on Lubrication and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

flexible silver chains. Two of these chains were in the vertical plane containing the line of action of the applied force. This assured that the lens moved always in a horizontal plane.

To meet Specification 5 the use of a chemical balance first suggested itself. This would necessarily have to be of the chainomatic type since it is essential that the force increments be smoothly applied. This idea, however, was laid aside in favor of a torsion-wire device which proved both smooth and sensitive in operation. The limiting value of the applied force which can be measured using the dial vernier is of the order of 1 mg.

Specification 6 was met by the use of a microscope provided

Fig. 1 Short-Track Friction Tester

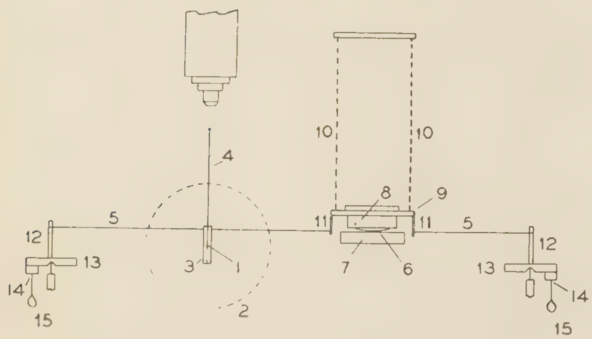

Fig. 2 Diagram of Short-Track Friction Tester

with an ocular micrometer. A light glass fiber mounted vertically on the linkage between the torsion wire and the suspended lens had its tip fused into a tiny sphere. The relation between the drift of the image of the needle tip in the field and slip at the contact was carefully determined. As before stated slip was easily measurable to 1 micron.

Fig. 1 shows the machine as assembled for test. For clarity the tube used for protecting the slip-indicator needle from drafts has been removed. The balance arm is in the form of a cross and widely spaced knife-edges are used to provide lateral stability. Underneath the main counterpoise, at the right, are two U-shaped weights made of wire and carefully adjusted to a value of 0.5 g each. Before a test these weights are on their hooks. A second small counterpoise on the balance bar provides a means of bringing the lens just into contact with the flat but without load. If one of the U-weights is now removed a load of 0.5 g is applied. If both are removed the load is 1 g.

Fig. 2 shows in diagrammatic form the essential features of the machine as it appears in front elevation. Item (1) is the torsion wire normal to the plane of the paper. Torque is imparted to the wire by turning a dial (2). This dial is graduated in 60 equal divisions and is provided with a vernier. Mounted at the middle point of the torsion wire is a light metal piece (3) comprising two rectangular plates screwed together, their ground inner faces serving as the jaws of a vise on the torsion wire. The glass needle (4) already mentioned is cemented in a small hole in the upper edge of the vise, and the complete assembly is made slightly bottom-heavy for mechanical stability. A silk thread (5) is cemented in a slot cut in the upper edge of the vise and passes horizontally to left and right. This thread has two functions:

(*a*) It transmits force from the torque system to the friction contact.

(*b*) It provides a means for maintaining the movable specimen in mechanical equilibrium.

The friction contact is between the lower surface of a convex lens (6) and a flat (7). The lens is mounted in a metal holder (8) supported in a hole in the suspension plate (9). The lens is held in place on a machined seat in the lower face of the holder by a touch of Duco cement and may easily be removed for cleaning.

The plate (9) is suspended from a similar plate mounted on the front end of the balance arm, as may be seen in the photograph, by four light silver chains such as used for costume jewelry. Two of these are shown at (10) and these lie in the plane of the thread. Two light metal lugs (11), the function of which will be later described, are attached to the edge of the plate (9) at the ends of a diameter coincident with the line of the thread.

The extreme opposite ends of the thread are cemented to pins (12) which rise from the upper faces of the metal disks (13), which ride on knife-edges. Sufficient tension to take up all slack in the thread is provided by two equal weights mounted at the points (14) near the outer edges of the disks. Also at the points (14) are silk loops (15) on which may be hung weights used for calibrating the machine.

Alignment

The last operation to be performed in assembling the machine is accurate alignment, particularly with respect to the thread. With the lens holder in place the balance arm is counterpoised to bring the lens just in contact with the flat. The arm is then locked in position and the lens holder removed. A free length of the thread to be used is now passed through the machine and held by external supports under normal tension, after which the following adjustments are made:

1 The thread is brought into the plane of the upper surface of the flat.

2 The thread is adjusted to lie in a plane normal to the torsion wire. This plane contains the point at which the lens will make contact with the flat.

3 Adjustment of the torsion assembly is made so that the thread lies truly in a narrow slot cut in the upper edge of the vise.

4 Adjustment of the lugs (11) is made so that the thread passes truly through narrow slots cut vertically in them.

5 The weighted disks (13), supported so that the pins (12) are vertical, are moved along their knife-edges until the thread just touches the side of the pins.

6 An adjustment is made to bring the glass pointer into the vertical.

7 A touch of cement is now applied to lock the thread at the lugs (11), at the pins (12), and in the vise slot.

8 When the cement has dried, the piece of thread between the lugs is cut away.

Calibration

Two calibrations are required, the first being the relation between torque and tension in the thread, and the second being the relation between torque and displacement at the contact point. By "torque" is meant the number of divisions through which the dial is turned from its working zero.

Calibration for Thread Pull in Terms of Dial Divisions. The balance-arm counterpoise is adjusted until contact between lens and flat is just broken. With the dial reading at its working zero, the micrometer fiducial line is brought to the edge of the image of the needle tip. A small known weight is then hung on the right-hand loop (15). The added known tension in this section of the thread produces a displacement of the needle tip observable in the field of the microscope. The dial is then turned by an amount sufficient to bring the image back to its original setting. This process is repeated by adding more small weights to the loop until a sufficient number of points have been obtained. Since the amplification factor of the mechanical system is known, the value of the thread pull in milligrams is also known. Fig. 3 is a graph of thread pull against dial divisions and shows a sufficiently accurate linear relationship over the working range. A factor of 2.75 mg per division was accepted.

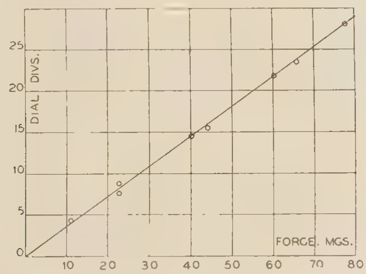

FIG. 3 DIRECT CALIBRATION GRAPH; APPLIED TANGENTIAL FORCE (MILLIGRAMS) AGAINST DIAL DIVISIONS (TORQUE)

Calibration for Slip in Terms of Dial Divisions. With the contact between lens and flat still broken, the dial is set at the working zero, and the fiducial line brought to the edge of the image. The ocular micrometer reading is noted, the dial turned through 1 division, and the micrometer reading again noted after bringing the edge of the image once more to the line. This process is repeated, turning the dial 1 division at a time until the working range is covered. This graph also was linear, the torque corresponding to 1 dial division giving a displacement at the contact of 3.24 μ. Since this amount of torque has been found equivalent to a thread pull of 2.75 mg, it is seen that, under conditions of zero friction (contact broken), a tangential force of 2.75 mg will produce a slip of 3.24 μ, or 1 μ per 0.85 mg. This factor was accepted.

Experimental Procedure

Cleaning the Glass Surfaces. After a test, the lens and flat are removed from their metal holders and immersed in acid cleaning solution to remove all traces of the old film. They are then washed in plenty of hot water to remove all acid and finally in distilled water. After this treatment, the glasses are dried in a jet of nitrogen. An air jet could be used if sufficiently clean.

Application of Lubricating Film. A very small amount of the test lubricant is spotted over the surfaces of the lens and flat. With a Kleenex pad this is then spread as uniformly as possible, the film at this stage being thick enough to be visible under suitable illumination. Three or more additional rubbings are now made with Kleenex pads, the object being to reduce the film thickness to molecular dimensions. After the rubbing-down process and under critical illumination no trace of a surface film can be seen. By breathing on the surface from time to time during the rubdown, the condition of the film with regard to uniformity of thickness can easily be determined. It is, however, impossible to estimate the actual thickness at any stage. If at the end of the rubdown water is poured on the surface it may be shaken off leaving the glass apparently dry. This definitely proves that a film of at least molecular thickness is attached to the surface. It appears to be experimentally proved that the removal of a material of polar nature can be carried down to molecular dimensions, but not further, by vigorous rubbing with a nonabrasive.[3]

In the tests reported in this paper, the qualifying term "monomolecular" means that the film on both lens and flat has been so vigorously rubbed down that its thickness must have been reduced to the order of molecular dimensions.

It would have been possible to apply true monolayers to the glass surfaces of certain of the materials used by using the Langmuir trough technique. This method was not adopted because many of the lubricants tested will not form condensed monolayers on the water surface.

Removing Accidental Surface Dust. It is very important that no dust or lint particles remain on the test surfaces. These may be blown off by a strong jet of nitrogen or clean air.

Removing Electric Charge. The rubbing process generates an electrostatic charge on the glass surfaces. If this is not completely removed accurate determinations of friction are not possible. The reason is that the apparent load is increased due to attraction between lens and flat at the contact spot. The force of attraction may be surprisingly large. Coefficients of friction in this investigation are computed on the basis of a load of 1 g.

On days when the humidity was low it was found that an upward pull of between 4 and 5 g was required to break contact after removal of the test load. This means that the computed coefficients were nearly 6 times too high, since the actual load was the sum of the test load and that due to the electrostatic attraction.

The charge may be removed by exposing the glasses to vapor arising from hot water through leakage over the condensed water film. During a test it is advisable to use a radioactive source close to the glasses and to cover the machine with a grounded metal shield.

Test for Zero Balance. The test surfaces are placed in the machine and the balance-arm counterpoise adjusted to bring the lens just in contact with the flat under no-load conditions. Observation of the motion of the image of the needle tip shows when this condition has been reached. With the dial set at its working zero, the test load is now applied. It is usually found that the image drifts a slight distance to one side or the other of the fiducial line. This is chargeable to a condition of unbalance of the various forces involved, and it is necessary to wait until equilibrium is established. A very slight tapping on the bench top will shorten this delay. The criterion for exact equilibrium is equal excursions of the image to right or left of the line when the dial is turned through the same small angle to left or right of the working zero.

The Friction Test

The ocular micrometer is set to bring the fiducial line in exact coincidence with the edge of the image and the reading noted. The dial is then turned slowly and smoothly through 2 divisions. If the image does not move, it means that initial static friction is

[3] "Built-Up Films of Barium Stearate and Their Optical Properties," by K. B. Blodgett and Irving Langmuir, *The Physical Review*, vol. 51, 1937, pp. 964–982.

greater than 5.5 mg, which is the value of the tangential force applied by virtue of the torque of 2 divisions. Conversely, if the image does move, it means that initial static friction is less than 5.5 mg, but it is not apparent how much less. With the standard load of 1 g the value of the coefficient at initial slip might be anything from just above zero to just below 0.0055. From the drift of the image the slip in microns is determined, and by reference to a correction table the coefficient at the end of the slip is computed. Data from an actual test are given in Table 1 and are used to illustrate the method of computing the local values of the static coefficient over a portion of the rubbing track.

TABLE 1 STEARIC ACID; MONOLAYER; GLASS LENS ON GLASS FLAT; LOAD, 1 G

Dial reading	Divisions, increase	Force increase, mg	Observed slip, μ
30	0	0	0
32	2	5.5	4
34	4	11.0	8
36	6	16.5	15
38	8	22.0	22
40	10	27.5	27
42	12	33.0	32
44	14	38.5	41

The first point to be plotted is 5.5 mg at zero slip. The question now is: Where must the second point be plotted? It is obvious that at the end of a slip the force on the thread is less than at the start of slip. It will be remembered that a factor of 0.85 mg per 1 μ slip was determined by direct calibration. Therefore, the correction to be applied to the first slip in this test is $4 \times 0.85 = 3.4$ mg, giving a net force of $5.5 - 3.4 = 2.1$ mg at the end of 4 μ slip.

The graph Fig. 4 shows, at A and B, the two points just determined. The system is now in equilibrium at the contact and will remain so until a tangential force in excess of the present value of static friction is applied. It has been found convenient, however, to turn the dial through exactly 2 divisions for every reading thus applying increments of force of exactly 5.5 mg. Referring to Table 1, it is seen that an increment of 5.5 mg at point B caused a further slip of 4 μ thus moving the contact a total distance of 8 μ along the track. The net value of the applied force at point C is $2.1 + 5.5 = 7.6$ mg.

A continuation of this procedure for the observed data gives a graph of a saw-tooth type. In a sense, the friction at the contact is of a "stick-slip" character and is reminiscent of that reported by Bowden[4] who regarded the "stick" as due to the formation of minute welds resulting from high pressure and high local temperatures developed during the rapid slip. In the present case, however, the slip speed is of the order of 50,000 times slower than in the Bowden experiments, which means that heat effects must be vanishingly small.

Coefficient of Static Friction

Since the load at the contact is 1 g, the same graph shows the local variation of the static coefficient along the rubbing track. The coefficients are obtained by dividing the values for friction (in milligrams) by 1000. The question arises: Which points on the saw-tooth graph, upper or lower, give the local values of the coefficient? As before stated, the upper points merely indicate the net values of the applied force at the contact for a succession of points along the track. At point C, for instance, the net force was 7.6 mg following the addition of an arbitrarily chosen increment of 5.5 mg. This increment may have been more than was necessary to initiate further slip, in which case the computed coefficient 0.0076 would be too high. Indeed, it is safe to assume that all the peaks on the coefficient graph are too high. What is now the situation with regard to the lower points? During slip, the force at the contact is being very slowly reduced and when it has fallen to a value approximating the friction at that point in the track, motion will cease. Very obviously, the local values of the static coefficient cannot fall below the lower points of the saw-tooth graph, but if, as is likely, kinetic "overshooting" is negligible, they will not lie much above these points.

A number of experiments have shown very conclusively that slip may be initiated by force increments very much smaller than were applied in the routine test reported in Table 1. Fig. 5 is a graph of the observed data in such an experiment. The material used had a high degree of so-called "oiliness" which accounts for the low frictional coefficients observed. The feature of direct interest, however, is the fact that only very small increments of force were required to initiate measurable successive slips. The graph is still of the saw-tooth type but if, as in graph B, the same ordinate scale is used as in Fig. 4 it is immediately seen that the lower points of any saw-tooth friction graph may, without serious error, be taken as the local values of the static coefficient. The dotted line in Fig. 4, connecting the lower points, is regarded as a sufficiently accurate picture of the variation of static friction

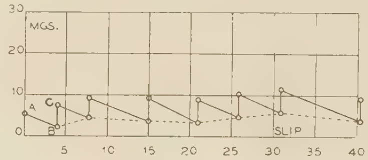

Fig. 4 Saw-tooth Friction Graph

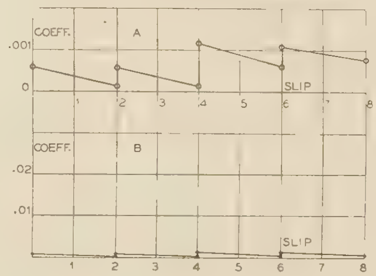

Fig. 5 Graph A: Open-Scale Saw-Tooth Friction Graph Obtained With Small Increments of Applied Force. Graph B: the Same Graph With Standard Scale for Ordinates

along the rubbing track. This method of plotting has been adopted as standard for tests made on this machine.

Data Relating to the Contact

The lens has a radius of curvature of 5 in. The diameter of the contact circle under the normal load of 1 g as computed from the Hertz formula is 0.0023 in. The average pressure is about 550 psi, and the maximum pressure in the center of the contact circle about 820 psi.

It will be noted that the diameter of the contact area is approximately $^1/_3$ of the length of the working track. (0.001 in. = 25.4 μ.) This scale is shown in Fig. 8.

Interpretation of Friction Graphs

A study of the friction graphs of three typical materials tested in the short-track machine will help to clarify the picture of what happens at the contact between lens and flat. Specifically, such a study will show that different types of lubricants afford different degrees of protection to the rubbing surfaces.

In Fig. 6, A is the graph obtained with monomolecular films of palmitic acid on the glass surfaces. The molecule of this lubri-

[4] "The Nature of Sliding and the Analysis of Friction," by F. P. Bowden and L. Leben, Proceedings of the Royal Society of London, series A, vol. 169, 1939, p. 386.

cant has the very active carboxyl group at one end and will bond itself strongly, even on a surface of glass. In this same illustration *B* is the graph of an exactly similar test made with ethyl palmitate, the ester of palmitic acid. The molecule of this material is much less active since the carboxyl group has been replaced by the ethyl radical C_2H_5. Finally, Fig. 7 shows the friction graph for Nujol, a highly refined hydrocarbon oil having negligible molecular bonding activity. In this test, the film applied to the lens and the flat was thick enough to be visible. This was deliberately done with the object of making the friction as low as was possible with such a poor lubricant. The actual thickness under the pressure conditions in the contact area is, of course, greatly reduced and may reach molecular dimensions toward the center.

Significant differences between the lubricants are revealed by a study of these graphs. The protecting value of palmitic acid is evidently very high as evidenced by the fact that the coefficient of static friction is fairly constant over the entire track at the low average of 0.005. The sharp variation in the local values of the coefficient in this and in all of the graphs is believed to be due mainly if not entirely to variations of smoothness of the surfaces. A general upward trend in the coefficient is referable to progressive thinning of the film under rubbing attack. The less strongly the molecules of the lubricant are bonded to the base surfaces, the more easily are they removed and the steeper will be the rise of the coefficient along the track. Graph *B*, Fig. 6, is illustrative of this. The starting value of the coefficient is the same for ethyl palmitate as for palmitic acid, but the end value is nearly 6 times higher. Evidently, the ester affords much less protection than does the acid. If the test had been carried further, it is to be expected that a point would be reached where the graph would rise very steeply, indicating that the film had been thinned to the danger point.

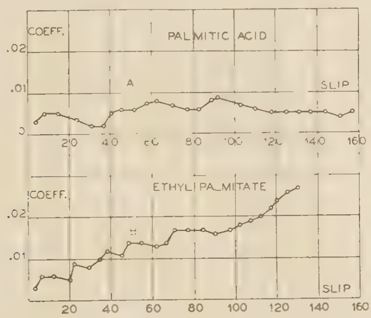

FIG. 6 GRAPH *A*: PALMITIC ACID; MONOMOLECULAR FILM. GRAPH *B*: ETHYL PALMITATE; MONOMOLECULAR FILM

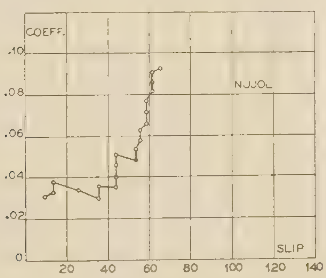

FIG. 7 NUJOL; MULTIMOLECULAR FILM

Nujol, the third typical material selected, provides an example of an oil having practically no protective value. In Fig. 7, the friction ordinate has been compressed to one half the scale of the two previous graphs. It is to be noted first that the starting value of the coefficient is much higher than for the other two lubricants studied. The basic reason for this lies in the fact that the molecules of Nujol do not orient themselves with respect to the surface, and in consequence, a monolayer of this oil is thinner than a monolayer of a lubricant such as palmitic acid. Such a material forms a carpet of closely packed molecules, the pile of the carpet being comparable to the length of the molecule. On the other hand, a monomolecular carpet of Nujol has no pile. The molecules are lying in random fashion on the base surface, which means that its thickness is much less than if they were oriented.

A further inspection of the Nujol graph shows that, after a travel of about 30 μ the coefficient begins to rise significantly, and after 50 μ, very sharply indeed, to an end value exceeding 0.09. If the test had been carried over the same track length as before, the friction would have reached such a high value that the surfaces would have been damaged.

PHENOMENON OF "VANISHING" FRICTION

Abnormally low values of friction have occasionally been observed when testing monomolecular films of highly active materials between the lens and flat, and under the working load. It is believed that an essential condition for the occurence of this phenomenon is an extreme degree of smoothness of both surfaces meeting at the contact spot. It is to be remembered that no surface can be made smooth in the absolute sense, but it is legitimate to assume that on the best surfaces very small areas may exist which approach the ideal. If now, two such abnormally smooth areas on the lens and flat could be brought together under load, there being present on each only a single layer of strongly bonded molecules, it may be expected that the static friction would also be abnormally low. Since it is a matter of chance for two areas of the necessary high degree of smoothness to come into exact opposition, the phenomenon of vanishingly low friction is only rarely observed.

Experimental. A massive concrete pier rising from bedrock was available for mounting the friction tester. The absence of vibration sufficient to cause visible ripple on the surface of clean mercury was proved. However, when the needle tip was observed under the microscope, after breaking contact between lens and flat, it was immediately seen that serious and variable vibration was present. The instrument was then mounted on a heavy cast-iron plate between which and the concrete pier was a thick pad of absorbent cotton. The free swing of the image was now very much less in amplitude and did not appear to be affected by ordinary activities in the building. There is some possibility that the irreducible vibration now existing may be of seismic origin.

The amplitude of free swing, i.e., under no-load conditions, does not exceed plus or minus 1.5 μ, referred to the contact spot at the lens. The movement of the image is perfectly smooth and is probably sinusoidal. When the load is applied, the image movement is arrested. As before mentioned, the system may not at this moment be in exact balance. A stable zero can, however, eventually be obtained. In the great majority of cases, the image will remain permanently at this final reading, but occasionally it will be seen to move through a few micrometer divisions, again coming to rest. After one or two seconds there will be another small excursion, perhaps in the opposite direction. This behavior has been observed to persist over a period of an hour or more but, after a sufficiently long time, the image comes permanently to rest. The maximum amplitude of the excursions is comparable to that of the free swing, which means that a comparable amount of vibrational energy is required for maintenance of the motion in both cases. The fact that the motion is not smooth is proof that friction, however small, exists at the con-

tact. If there were no friction whatever, the needle tip would vibrate as freely with load at the contact as with load removed.

Conclusions

On the evidence, it would appear that the following general conclusions may be drawn:

1 A small force, having its origin in the basic vibrational energy received by the system, operates at the contact. This force is alternating in direction. There is no basis for assuming that its root-mean-square value is constant.

2 Since the force is alternating the direction of slip is also alternating. Slip will only occur when the instantaneous value of the force at the contact rises above the instantaneous value of static friction.

3 Since the work done on the film is exceedingly small in amount, there can be only very slow deterioration. The general level of static friction remains for a long time below the maximum peak values of the force at the contact, and, while this condition obtains, the "stop-and-go" slip behavior will persist.

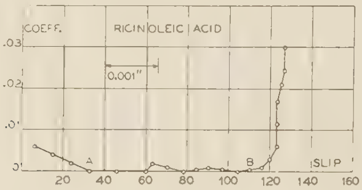

Fig. 8 Ricinoleic Acid; Monomolecular Film; Vanishing Friction

4 With any but perfect surfaces, however, there is a tendency toward the exposure of tiny denuded spots in the contact area and the simultaneous occurence of small peak values of friction. It is quite possible that the first time this happens the critical level of friction may be exceeded, in which case there will be no further slip.

Fig. 8 shows the graph of a friction test made with a film of ricinoleic acid of molecular dimensions. The corrected values of the static coefficient between the points A and B were: zero, zero, zero, 0.002, 0.001, 0.0002, 0.0006, 0.001, 0.0005, 0.0001, 0.0005.

In the early part of this test, the image of the needle tip was observed to move in the erratic manner previously described, but, after a few more readings had been made, the image became stabilized, indicating that the friction had risen above the critical level.

The steep, almost vertical, rise in the coefficient at the end of the test suggests a virtually complete removal of lubricant from the surfaces, and it is somewhat puzzling to account for this sudden destruction of the film.

If the surfaces of the lens and flat were perfect at the contact area the film might be expected to last indefinitely, since there would be complete separation at all times. Further, the friction might be expected to be below a measurable value. With ordinary surfaces, however smoothly finished, a film of molecular dimensions will inevitably break down sooner or later, the destructive agency being the rubbing together of tiny irregularities on the opposed surfaces. The qualifying term "tiny" means extremely small in the gross sense, but quite large on the molecular scale. When as a result of mechanical attack such irregularities are sheared off they carry with them relatively large numbers of bonded molecules of the lubricant. The effect of this loss may not be immediately apparent but at some later stage of the rubbing process, there will be a rise of the friction coefficient, gradual at first but becoming steeper at an increasing rate.

Suppose now, as appears to be the case in the experiment under discussion, the surfaces in contact happen to approach perfection in smoothness. This means that such surface irregularities as are present approach molecular dimensions, and a corollary of this is the rate of loss of lubricant due to attrition is now proportionately reduced. This state of affairs is reflected in a low and fairly constant coefficient of static friction over a considerable length of track. Occasionally, as will be noted in the graph, there may be a local increase in the friction which may be reasonably attributed to the chance opposition of irregularities somewhat larger than those specified.

The ultimate breakdown of any film of molecular dimensions may be expected to be initiated at the point of maximum pressure, i.e., at the center of the contact area. This is particularly true in the case under consideration since, in regions out from the center, irregularities of the order of molecular dimensions are subject to rapidly decreasing shearing forces. It thus becomes reasonable to assume that the status quo of the film will be maintained until the loss of lubricating molecules at the center of the contact area is such as to raise the friction sensibly above the critical value. From this moment, the damaging effect of attrition proceeds rapidly because of the very small area involved.

Discussion

M. E. Merchant.[5] The careful work of the author, reported in this interesting paper, has brought to light important facts on the frictional phenomena occurring at the very beginning of relative sliding motion between two boundary-lubricated contacting surfaces. Information on these phenomena is noticeably lacking in the existing literature on friction. The paper is therefore an important step toward the filling of an evident gap in our knowledge of the fundamentals of the friction of boundary-lubricated surfaces.

The author's discovery of almost vanishingly small values of the coefficient of friction, occasionally, is of particular interest to the writer and his associates, for in it we find further confirmation of a general equation for the coefficient of static friction, derived as the result of an extended experimental and theoretical study of friction.[6,7] According to this equation and accompanying theory the coefficient of static friction can be expected to become vanishingly small only if the mating surfaces are perfectly smooth and are covered with a complete film of adsorbed material having negligible shear strength. These ideal conditions are surely closely approached under the best conditions of the author's experiments.

The author apparently feels reasonably sure that practically all of the apparent contact area between the optically smooth lens and plate, given by the Hertz formula, is in actual "contact" or bearing. We will not attempt to argue this point, but would not be surprised if, even with these very smooth surfaces, the distribution of pressure is far from regular. The deformation of the film of lubricant by the surface pressures certainly cannot be entirely uniform. Nevertheless the conditions prevailing in these tests probably come as close to involving true elastic contact (as opposed to the usual plastic flow of contact areas) as is at present possible. Therefore, it would have been most interesting to have had data presented on the dependence of the coefficient of friction on load. For, as has been shown elsewhere,[6,7]

[5] Physicist, Research Department, Cincinnati Milling Machine Co., Cincinnati, Ohio.

[6] "Mechanism of Static Friction," by M. E. Merchant, *Journal of Applied Physics*, vol. 11, 1940, p. 230.

[7] "Surface Friction of Clean Metals," by H. Ernst and M. E. Merchant, Proc. Summer Conference on Friction and Surface Finish, Massachusetts Institute of Technology, Cambridge, Mass., June, 1940, pp. 76–101.

the coefficient of friction can no longer be expected to be independent of load when the area of contact is produced mainly by elastic deformation of the surfaces (unless, of course, the shear strength of the adsorbed film is actually equal to zero). Any data which the author may obtain on the effect of variations in load, during the course of his program of investigation with the present apparatus, should prove most interesting.

The author has assumed that the minimum points on the saw-tooth friction graphs are very nearly equal to the true static-friction coefficient. This assumption should be quite correct if, as is often the case with good boundary lubricants, the static-friction coefficient is less than the kinetic. If, however, the kinetic-friction coefficient is less than the static, as is usually the case with poor boundary lubricants, and if the dynamics of the system are such that true "stick-slip," or relaxation oscillation, can occur, then it can be seen that the minimum points on the saw-tooth will not be equal to the static coefficients but will in fact be less than them by an amount equal to nearly twice the difference between the values of the static and kinetic friction coefficients. We believe it should not be difficult to detect the existence of this latter condition with the author's apparatus. It should merely be necessary to rotate the torque dial slowly and smoothly an appreciable distance while simultaneously observing the needle tip through the microscope. If the needle tip does not move smoothly while the dial is being rotated but proceeds by a series of jerks or "sticks" and "slips," then it is evident that true stick-slip sliding is occurring for the particular lubricant under investigation.

In conclusion, we wish to point out an item in the technique used by the author in applying the lubricating film to his specimens which may possibly influence the chemical purity of the material so applied. We notice that the film was spread and rubbed down with Kleenex pads. Now, it was found in the course of our investigation of friction,[8] that if a metal surface previously rendered clean enough to be freely wet by water was merely wiped once with a fresh Kleenex pad, the surface could not then be wet by water, and the friction coefficient of the surface so treated would be found to have fallen to about $^1/_{10}$ its original value. Evidently the Kleenex tissue contains organic material which is readily and strongly adsorbed on a clean surface. Therefore, there is some question as to whether the Kleenex pads used by the author did not dilute or contaminate his applied lubricant during the rubbing-down process, resulting in a final film of mixed and partially unknown composition. It would probably be necessary to use a tissue thoroughly extracted with ether and other solvents and tested for freedom from adsorbable materials by the water wetting method just described, to insure against contamination occurring in the rubbing-down process.

S. J. Needs.[9] The writer was much impressed by the considerable amount of careful experimental work done by the author. However, this work appears to be open to criticism because no direct observations were made of the actual slip of the slider. The author measured the exaggerated movement of a point on the cord through which tangential force was applied to move the slider. Movement of the cord is not necessarily the same as movement of the slider and, in the absence of direct measurement of the latter, relationship between the two movements should be established.

The concept of vanishingly small or zero friction is not easily understood but the phenomenon does not seem impossible. In a paper[10] read before this Society several years ago, it was shown that when thin films of oil between two optically smooth plane steel disks are sheared by relative rotation of the disks, the torque required is directly proportional to the rate of shear. This relationship held true at all loads available, the greatest being 800 psi. As the rate of shear approached zero, the torque and friction coefficient also approached zero. This was true, however, only when the rate of shear was sufficient to prevent the building up of rigidity in the film, due apparently to the influence of the steel surfaces.

The author's Fig. 8 seems to show an analogous effect. Starting with a fairly high value, the friction coefficient drops to zero. This might be due to removal of film rigidity by movement of the slider, as in the tests with the plane surfaces. The zero or near-zero friction coefficient persists as long as the film is intact and the formation of rigidity is prevented by shear of the film. The sharp upturn of the curve, at the right of Fig. 8, is probably due to rupture of the film, as suggested by the author. No similar rupture was noted in the experiments with the plane disks even after continuous shear over periods of 50 hr or longer.

In Figs. 7 and 8, the author shows several values of friction coefficient for the same value of slip. This seems to imply a determination of static-friction coefficient with no slip. Perhaps the vertical scales represent force rather than friction coefficient. Also, it is not clear how slip was produced with no applied force, which seems to be the case in the observations of zero friction coefficient in Fig. 8. A more complete description of the experimental technique would probably explain these interesting points.

Author's Closure

Mr. Merchant raises the very interesting question of the possibility that the coefficient of friction may vary with the load. If we assume, as seems probable on the experimental evidence, that static friction may become vanishingly small in the entire absence of minute surface irregularities, we are compelled to regard the mechanism of friction as dominantly associated with the presence of such irregularities. The slightest movement of one surface with respect to the other tends to shear off the tops of the closely contacting irregularities on the mating surfaces, thus exposing clean material which cannot be recovered by a new film of lubricant since this is present only as a strongly adsorbed monolayer on contiguous areas. We thus have a condition where, possibly, a large number of weak molecular bonds are created between very minute clean spots on the opposed surfaces. To rupture these bonds requires the application of tangential force, the measure of which we call static friction.

Now, as the load increases so does the number and the size of these contacting spots, and so also does the force necessary to rupture the bonds formed. In other words, friction increases with the load, but whether the increase is linear is not established by the argument.

Experiments are now in progress to determine how friction varies with load under conditions of constant area of contact, and, conversely, how friction varies with contact area under conditions of constant load.

Mr. Merchant questions the justification for the assumption that the minimum points on the saw-tooth graph are only slightly below the true values of the static coefficient, intimating that the assumption would not hold with highly active lubricants exhibiting a static coefficient of lower value than the kinetic. The author agrees, with reservations. As was pointed out when the paper was presented, if the tangential force is applied with much smaller increments the upper and lower points are much

[8] "Surface Friction of Clean Metals," by H. Ernst and M. E. Merchant,[7] p. 87.

[9] Service Manager, Kingsbury Machine Works, Inc., Philadelphia, Pa. Mem. A.S.M.E.

[10] "Boundary Film Investigations," by S. J. Needs, Trans A.S.M.E., vol. 62, 1940, pp. 331–345.

closer together, supporting the assumption that no serious error is introduced by accepting the values of the lower points.

With regard to phenomena analogous to "stick-slip," it has occasionally been observed, particularly with lubricants of the less active type, that the image of the needle tip attains its maximum displacement by a series of small jerks, suggestive of a series of ruptures and formations of molecular bonds. The author agrees that the technique of the Kleenex rubdown of the film on the glass surfaces is not ideal but believes the possibility of contamination is largely nullified by the prior presence of a strongly adsorbed layer on the clean and active surface. This view is justified by the fact that the friction graphs correlate with known structural differences between the lubricants tested.

Mr. Needs mentions that no direct measurements of actual slip appear to have been made. As a matter of fact, this was done, although not reported in the paper. A second microscope was focused on a glass index rigidly attached to the suspended member of the assembly and lying in the plane of slip. It was found that the actual slip agreed with the slip as determined by the movement of the vertical needle.

The author agrees that the graphs, Figs. 7 and 8, appear to show several values of the coefficient for the same slip. A vertical element of a graph means that no further slip has been caused by several successive increments in the applied force. The points plotted on the vertical are not, as Mr. Needs points out, values of the static coefficient. They are merely values computed in the same manner. The real meaning of a point on a vertical element of a coefficient-of-friction graph is that static friction at the corresponding point of the rubbing track exceeds the force which has so far been applied.

In Fig. 8, it would appear from the graph that slip was caused without the application of tangential force. This is not, however, the case. It is to be remembered that the plotted points are the minimum values of a saw-tooth graph, the upper points of which are not shown.

Effects of Continued Heating on Mechanical Properties of Molded Phenolic Plastics

By T. S. Carswell,[1] D. Telfair,[2] and R. U. Haslanger[3]

The results of previous investigations have indicated the influence of temperature on the mechanical properties of molded phenolic compositions, which data are essential if these plastic materials are to be selected properly and designed for structural applications where elevated temperatures are involved either intermittently or continuously. To supplement these data the authors undertook research work to determine the change of impact and flexural strengths with prolonged heating up to 500 hr at temperatures of from 110 to 225 C for six molded phenol-formaldehyde plastics. The compositions studied, the method of testing, and the results in the form of tables and curves, constitute the subject matter of the present paper.

Data on the serviceability of molded phenolic compositions at elevated temperatures over long periods of time are important, if these plastic materials are to be properly selected and designed for structural applications. There are numerous uses for phenolics which require that they withstand operating temperatures up to 80 to 90 C (172 to 194 F) either intermittently or continuously.

Some investigations have been made to determine the influence of temperature on mechanical properties of phenolics by testing at a range of temperatures (1, 2, 3, 4, 5, 6, 7).[4] This previous work has indicated, for phenolic materials conditioned only long enough to bring them to the temperature of testing that:

1 Tensile and flexural strengths decrease gradually over the temperature range, asbestos-filled material showing less deterioration than organic-filled or pure resin.

2 Impact strengths of organic-filled materials show an increase or remain constant up to 140 to 160 C (285 to 320 F) and then fall off sharply, while the pure resin and asbestos-filled compositions remain practically unchanged even at 240 C (465 F) (1).

Azam (8) has reported on the effect of prolonged heating on the impact strength of phenolic laminated muslin and glass fabric at 150 C (300 F). Houwink (9) cites similar work by Nitsche and Salewski (10) on a number of phenolic materials. Delmonte (11) described the effect of continued heating at 200, 300, and 400 F on the impact strength of some laminated phenolics.

Strength of Molded Phenol-Formaldehyde Plastics

To supplement these data research work was undertaken to determine the change of impact and flexural strengths with prolonged heating, up to 500 hr, at temperatures of from 110 to 225 C (230 to 437 F) for a number of molded phenol-formaldehyde plastics. Creep properties are also being studied over a range of temperatures and will be reported at a later date.

Six different phenolic molding compositions were studied. Three materials containing organic fillers, wood flour, macerated fabric, and cotton cord, and representing three grades of impact strength, were evaluated. These were composed of approximately 50 per cent filler and 50 per cent resin. Two inorganic-filled compositions were evaluated. One contained 50 per cent asbestos fiber and 50 per cent resin, and the other contained 60 per cent mica and 40 per cent resin. An unfilled pure phenolic resin was also included in the study.

The two high-impact-grade materials, containing macerated fabric and cord fillers, were prepared by blending the resin and filler in a wet-mix process to assure a uniform mixture and to obtain optimum strengths. The remainder of the materials were processed on a set of differential rolls in the manner customary for the preparation of phenolic molding compositions. All six materials were from standard plant production and are typical of the classes which they represent.

Impact and flexural test specimens, $^1/_2$-in. × $^1/_2$-in. × 5-in. bars, of the impact-grade materials were molded in a single-bar mold according to A.S.T.M. "Tentative Specifications for Molds for Test Specimens of Moldings Materials Used for Electrical Insulation" (D647-41T).[5] The other four materials were molded in a 5-bar gang mold as specified in A.S.T.M. Specification D647-41T.

Impact data were obtained on a Baldwin-Southwark pendulum-type impact tester having a capacity of 4.0 ft-lb. Tests were run according to the A.S.T.M. "Tentative Methods of Test for Impact Resistance of Plastics and Electrical Insulating Materials" (D256-41T)[6] using the notched Izod method in which the stress is applied perpendicular to the molding pressure.

Flexural data were obtained on a Tinius Olsen universal tester, a screw-type machine with a constant rate of crosshead motion (0.05 in. per min) and 10,000 lb range. Flexural tests were run according to the procedure outlined in A.S.T.M. "Tentative Method of Test for Flexural Strength of Electrical Insulating Materials" (D650-41T).[7]

Specimens to be tested at 25 C (77 F) were conditioned, according to A.S.T.M. "Tentative Methods of Preconditioning Plastics and Electrical Insulating Materials for Testing" (D618-41T),[8] for 48 hr at 50 C ± 3 C (122 ± 5.4 F) and placed in a desiccator after removal from the oven. Other specimens upon which the effect of heating was to be studied were heated in a circulating oven held to within ± 5 C of the desired temperature. Specimens were baked for 2, 6, 18, 54, 162, and 500 hr at temperatures of 110 C (230 F), 140 C (284 F), 170 C (338 F), 200 C (392 F), and 225 C (437 F). After the desired baking time the specimens were cooled to 25 C (77 F) in a desiccator and tested.

The impact values reported represent the average of from 10 to 20 individual observations, while the flexural-strength values

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[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Rubber and Plastics Group of the Process Industries Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[5] 1941 Supplement to A.S.T.M. Standards, Part III, pp. 317–319.

[6] Ibid., pp. 339–344.

[7] Ibid., pp. 336–338.

[8] Ibid., p. 320.

TABLE 1 IMPACT-STRENGTH DATA AT 110 C

(Foot-pounds per inch of notch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	0.220 ± 0.01	0.280 ± 0.02	2.86 ± 0.27	5.90 ± 0.28	....	....
2	0.230 ± 0.016	0.257 ± 0.014	2.68 ± 0.18	6.40 ± 0.18	....	....
6	0.236 ± 0.018	0.263 ± 0.015	2.64 ± 0.16	5.77 ± 0.43	....	....
18	0.201 ± 0.007	0.270 ± 0.013	2.67 ± 0.21	5.52 ± 0.50	....	....
54	0.213 ± 0.010	0.271 ± 0.020	2.57 ± 0.12	5.20 ± 0.56	....	....
162	0.226 ± 0.010	0.271 ± 0.016	2.42 ± 0.10	5.30 ± 0.10	....	....
500	0.205 ± 0.007	0.260 ± 0.011	2.22 ± 0.11	5.36 ± 0.23	....	....

TABLE 2 IMPACT-STRENGTH DATA AT 140 C

(Foot-pounds per inch of notch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	0.220 ± 0.010	0.280 ± 0.020	2.86 ± 0.27	5.90 ± 0.28	0.316 ± 0.007	0.280 ± 0.010
2	0.254 ± 0.018	0.268 ± 0.021	2.69 ± 0.13	5.88 ± 0.20	0.315 ± 0.006	0.262 ± 0.013
6	0.222 ± 0.010	0.249 ± 0.015	2.60 ± 0.17	5.65 ± 0.34	0.328 ± 0.008	0.271 ± 0.007
18	0.202 ± 0.009	0.264 ± 0.018	2.57 ± 0.19	5.49 ± 0.26	0.303 ± 0.011	0.283 ± 0.009
54	0.215 ± 0.025	0.290 ± 0.019	2.31 ± 0.15	5.66 ± 0.53	0.310 ± 0.015	0.275 ± 0.022
162	0.205 ± 0.066	0.274 ± 0.013	2.16 ± 0.12	4.77 ± 0.11	0.286 ± 0.014	0.300 ± 0.010

TABLE 3 IMPACT-STRENGTH DATA AT 170 C

(Foot-pounds per inch of notch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	0.220 ± 0.010	0.280 ± 0.020	2.86 ± 0.27	5.90 ± 0.28	0.316 ± 0.007	0.280 ± 0.016
2	0.210 ± 0.008	0.258 ± 0.019	2.58 ± 0.15	6.26 ± 0.11	0.330 ± 0.013	0.294 ± 0.013
6	0.199 ± 0.003	0.274 ± 0.010	2.29 ± 0.10	5.50 ± 0.30	0.322 ± 0.016	0.293 ± 0.010
18	0.195 ± 0.009	0.259 ± 0.014	2.20 ± 0.10	4.30 ± 0.42	0.316 ± 0.016	0.312 ± 0.009
54	0.202 ± 0.004	0.238 ± 0.019	1.81 ± 0.06	3.24 ± 0.30	0.326 ± 0.016	0.338 ± 0.007
162	0.193 ± 0.005	0.231 ± 0.017	1.37 ± 0.210	0.65 ± 0.08	0.298 ± 0.013	0.308 ± 0.008

TABLE 4 IMPACT-STRENGTH DATA AT 200 C

(Foot-pounds per inch of notch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	0.220 ± 0.010	0.280 ± 0.020	2.86 ± 0.27	5.90 ± 0.28	0.316 ± 0.007	0.280 ± 0.010
2	0.200 ± 0.005	0.280 ± 0.060	....	5.10 ± 0.600	0.320 ± 0.020	0.320 ± 0.010
6	0.202 ± 0.013	0.260 ± 0.020	....	4.30 ± 0.500	0.320 ± 0.010	0.300 ± 0.010
18	0.200 ± 0.006	0.220 ± 0.020	....	0.91 ± 0.090	0.340 ± 0.020	0.330 ± 0.020
54	....	....	....	0.29 ± 0.020	0.310 ± 0.010	0.340 ± 0.010
162	....	....	....	....	0.310 ± 0.007	0.357 ± 0.080

TABLE 5 IMPACT-STRENGTH DATA AT 250 C

(Foot-pounds per inch of notch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	0.220 ± 0.010	....	....	....	0.316 ± 0.007	0.280 ± 0.010
2	0.184 ± 0.005	....	....	....	0.295 ± 0.014	0.279 ± 0.010
6	....	....	....	....	0.283 ± 0.007	0.299 ± 0.007
18	0.185 ± 0.005	....	....	....	0.282 ± 0.010	0.288 ± 0.007
54	....	....	....	....	0.297 ± 0.012	0.277 ± 0.017
162	....	....	....	....	0.281 ± 0.010	0.296 ± 0.008
500	....	....	....	....	0.253 ± 0.006	0.273 ± 0.005

are the average of from 5 to 10 individual results. The accompanying tables include the limits within which the observed value may be expected to lie 9 times in 10, limits of uncertainty, calculated according to the methods outlined in the A.S.T.M. "Manual on the Presentation of Data."[9]

Results of Impact- and Flexural-Strength Tests

Tables 1, 2, 3, 4, and 5 present the impact data, and Tables 6, 7, 8, 9, and 10 present the flexural data gathered at the various times and temperatures. The curves presented in Figs. 1 and 2 illustrate the change of impact and flexural strength with conditioning time at a given temperature. In some instances, evaluations at the higher temperatures were not made, because it was obvious that the material had already reached a limit of serviceability at a lower temperature. All of the materials stand up well at 110 C (230 F). Even after 500 hr there is no appreciable loss in either impact or flexural strength, except in the case of the pure resin, which exhibits a loss in flexural strength of about 20 per cent. The materials show only a slight tendency to blister or distort during conditioning.

At 140 C (284 F) a loss in impact strength is apparent in the fabric-filled material after 54 hr and in the cord-filled after 162 hr. The flexural strength of the cord-filled specimens falls off gradually after 6 hr at 140 C (284 F) but after 162 hr has decreased by only 16 per cent from the original value. The strengths of the pure resin, as well as of the materials containing wood-flour, mica, and asbestos fillers are unaffected even after 162 hr at 140 C (284 F).

The wood-flour-filled and pure resin specimens begin to show a gradual loss in impact strength after 6 to 18 hr at 170 C (338 F) while the flexural strength of the pure resin and of the fabric-filled specimens shows a drop after 6 hr.

The mica-filled product begins to show a slight loss in both impact and flexural strength at 225 C (437 F), but the asbestos-filled material stands up well even after 500 hr at this temperature. There is a tendency for these materials to blister at this temperature during heating, indicating a deterioration of the filler, as well as of the resin.

Figs. 3, 4, 5, and 6 are presented to show the effect on impact and flexural strengths of heating at several temperatures for 162 hr. From these curves it may be seen that the organic-filled materials remain practically unchanged up to 130 to 140 C (266 to 248 F), at which point the strengths fall off rapidly. In some instances the materials will withstand temperatures up to 170 C (338 F) for a relatively short period of time before breaking down. The asbestos and mica-filled compositions show no appreciable loss in strength at temperatures below 200 to 220 C (392 to 428 F).

Table 11 lists the six materials tested, together with the approximate limiting temperatures. The latter are arbitrarily taken as the temperatures at which a 10 per cent reduction in strength occurs after 162 hr of heating. It is quite obvious that the filler plays a major part in determining the limits of serviceability. The organic fillers show a definite deterioration, as exhibited both by a lowering of the strength characteristics and by a visual disintegration of the molded specimen, at much lower temperatures than do the inorganic- or mineral-filled materials.

The limits for the organic-filled compositions are only slightly below those previously reported by the authors (1) for short-time temperature effects. On the other hand, they are well above the limits established by the Army and Navy for molded parts for military applications, which require that the piece be serviceable at temperatures up to 70 C (160 F). The materials can in fact be used in structural applications where heat from

[9] Issued as a separate publication of the A.S.T.M.

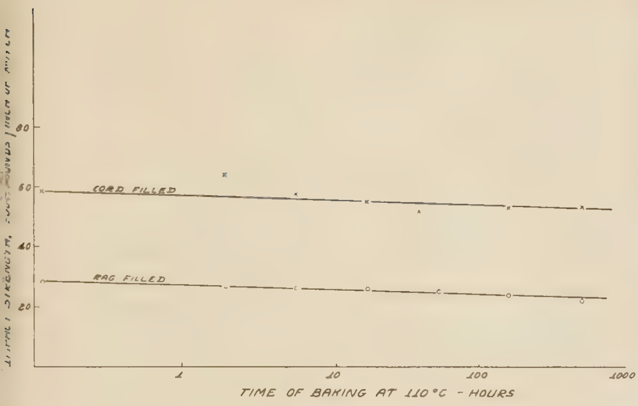

Fig. 1

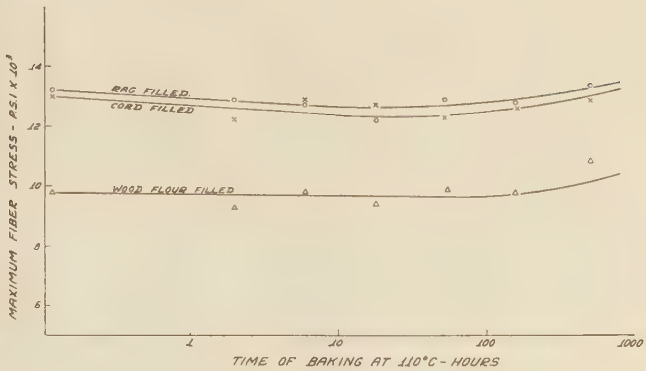

Fig. 3

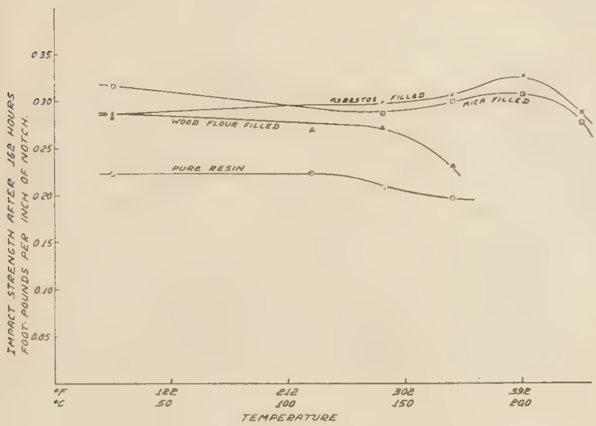

Fig. 2

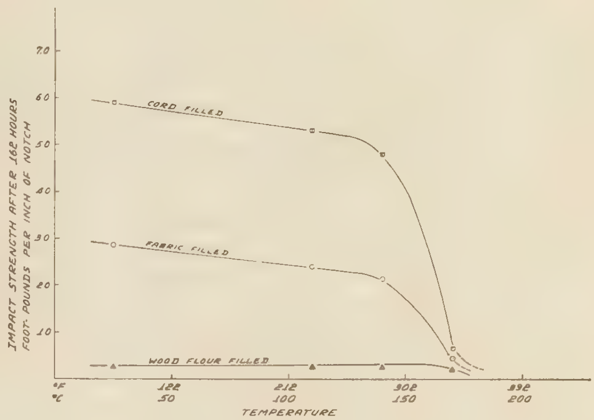

Fig. 4

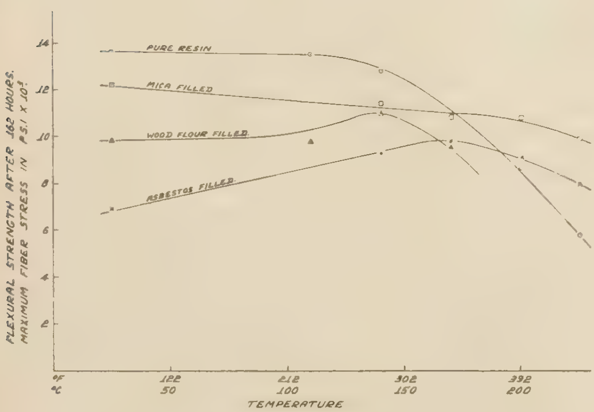

Fig. 5

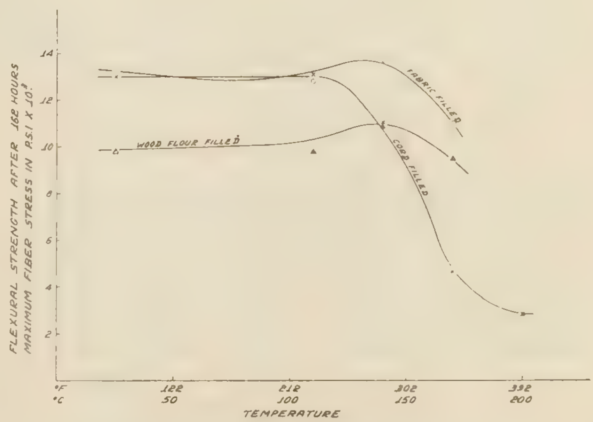

Fig. 6

engines and sources other than normal atmospheric conditions will elevate the temperature up to the limits reported.

The limits on the mica-filled, and especially the asbestos-filled, materials at 200 to 220 C (392 to 428 F) are sufficiently high to indicate their use for numerous applications where very high temperatures are likely to prevail for extended times. While the impact strengths of these products are such that they may be considered only as semistructural materials, they can be used where resistance to elevated temperatures is the prevailing factor.

In work of an empirical nature, such as is discussed here, it must be remembered that the results obtained are based on standard A.S.T.M. methods, and the actual numerical values serve principally as a comparative basis for evaluating specific mechanical properties.

Such other factors as molding conditions, form factors, combined stresses in application, and many others have a marked influence on the serviceability of the molded article. The final evaluation must therefore be based on actual service tests of the finished product.

TABLE 6 FLEXURAL-STRENGTH DATA AT 110 C

(Pounds per square inch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	13600 ± 500	9790 ± 370	13200 ± 830	13050 ± 710		
2	13000 ± 1550	9290 ± 390	12900 ± 270	12200 ± 900		
6	13900 ± 2040	9780 ± 390	12700 ± 1010	12800 ± 650		
18	13000 ± 1070	9390 ± 240	12200 ± 820	12700 ± 720		
54	13300 ± 1210	9880 ± 640	12900 ± 810	12300 ± 700		
162	13500 ± 1850	9780 ± 420	12800 ± 1060	12600 ± 820		
500	10800 ± 2800	10900 ± 470	13400 ± 540	12900 ± 750	....	.

TABLE 7 FLEXURAL-STRENGTH DATA AT 140 C

(Pounds per square inch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	13600 ± 500	9790 ± 370	13200 ± 830	13000 ± 710	12200 ± 720	6960 ± 330
2	13400 ± 850	10500 ± 410	12500 ± 870	12600 ± 760	10900 ± 560	7320 ± 600
6	12900 ± 940	10100 ± 460	12500 ± 750	11000 ± 680	11100 ± 500	7260 ± 600
18	13800 ± 1030	10600 ± 230	12400 ± 840	11500 ± 860	10900 ± 370	7450 ± 780
54	12830 ± 1230	10400 ± 400	12700 ± 770	10400 ± 1070	10800 ± 410	8870 ± 380
162	12810 ± 1120	11000 ± 320	13700 ± 700	10900 ± 360	11400 ± 550	9330 ± 710

TABLE 8 FLEXURAL-STRENGTH DATA AT 170 C

(Pounds per square inch)

Time, hr	Pure phenolic resin	Wood-flour filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	13600 ± 500	9790 ± 370	13200 ± 830	13000 ± 710	12200 ± 720	6960 ± 330
2	12900 ± 800	10000 ± 490	12600 ± 750	10800 ± 840	10600 ± 530	7690 ± 530
6	12000 ± 880	10400 ± 550	12200 ± 790	9790 ± 620	10700 ± 560	8200 ± 430
18	11900 ± 980	10800 ± 130	11100 ± 600	9260 ± 350	10600 ± 700	9190 ± 280
54	10800 ± 490	10200 ± 400	11200 ± 840	8110 ± 730	10800 ± 640	9350 ± 300
162	9470 ± 440	9610 ± 270	11050 ± 670	4580 ± 400	10900 ± 420	9780 ± 370

TABLE 9 FLEXURAL-STRENGTH DATA AT 200 C

(Pounds per square inch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	13600 ± 500	9790 ± 370	....	13000 ± 710	12200 ± 720	6960 ± 330
2	11300 ± 1000	10400 ± 350	....	10000 ± 670	10200 ± 450	9230 ± 390
6	11900 ± 940	10400 ± 490	....	9210 ± 730	10500 ± 310	8700 ± 600
18	9060 ± 540	9600 ± 430	....	8440 ± 700	10300 ± 730	9130 ± 450
54	....	8620 ± 340	....	5130 ± 160	10100 ± 770	9200 ± 430
162	8680 ± 980	....	....	3280 ± 230	10800 ± 700	9100 ± 210

TABLE 10 FLEXURAL-STRENGTH DATA AT 250 C

(Pounds per square inch)

Time, hr	Pure phenolic resin	Wood-flour-filled phenolic	Fabric-filled phenolic	Cord-filled phenolic	Mica-filled phenolic	Asbestos-filled phenolic
0	13600 ± 500	....	....	....	12200 ± 720	6960 ± 330
2	10300 ± 720	....	....	....	9740 ± 280	8310 ± 540
6	9630 ± 940	....	....	....	9560 ± 580	9590 ± 580
18	7620 ± 530	....	....	....	10500 ± 420	9740 ± 500
54	10400 ± 500	....	....	....	10800 ± 900	8590 ± 710
162	5830 ± 2470	....	....	....	9880 ± 840	7980 ± 440
500	5880 ± 1850	....	....	....	10100 ± 710	5440 ± 420

TABLE 11 APPROXIMATE LIMITING TEMPERATURES, ON BASIS OF 10 PER CENT REDUCTION IN STRENGTH, AFTER 162 HR OF HEATING

Material tested	Flexure		Impact	
	Deg C	Deg F	Deg C	Deg F
Pure resin	140	284	140	284
Wood-flour-filled	170	338	150	302
Fabric-filled	150	302	130	266
Cord-filled	130	266	130	266
Mica-filled	200	392	200	392
Asbestos-filled	220	428	220	428

Acknowledgment

The authors are indebted to R. A. Klucken who conducted the laboratory tests upon which the data and discussions presented are based.

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Drying of Textiles

By A. C. Walker,[1] New York, N. Y.

In 1937 a textile-drying research project was initiated by the United States Institute for Textile Research and supported by the textile industry, for the purpose of evaluating the effects of temperature and humidity on the physical and chemical properties of important textile fibers. The present paper gives a comprehensive report of the results obtained and points out the basis upon which the industry must proceed in problems relating to textile drying. In all cases, consideration should be given to the theories relating to the form in which the moisture is distributed within the fiber structure and to data of the type discussed by the author.

Drying-Research Objectives

The first objective of the textile-drying project by the United States Institute for Textile Research was the determination of the amounts of moisture retained by important textile fibers when brought to equilibrium at a series of temperatures in the range between 37.8 C (100 F) and 150 C (302 F), with a sufficient number of humidities at each temperature to characterize the shape of the moisture content–relative humidity relation. This information provides a basis for determining the limiting moisture contents for textile fibers which may be approached in any specified industrial drying process under definite atmospheric conditions.

Effects of Heat and Humidity on Physical Properties of Textiles. Another phase of this research was the investigation of the effects of humidity on these fibers when maintained for different periods of time under various humidity conditions at elevated temperatures. A somewhat unexpected result came from this work, of undoubted value to industry. It was found that when a textile is exposed to a temperature between 200 and 300 F for several hours and, at the same time, is in contact with a high relative humidity, the fiber is damaged more than if the humidity is lower. Also, the moisture regain after being dried under such conditions is always less than if the material is dried at such temperatures, but in contact with a lower humidity. The extent of the damage appears to be a function of the humidity of the heated air, temperature, and time of exposure.

Effects of Finishing and Scouring Agents on Moisture Relations of Textiles. A third objective of this research was the determination of the effects of some of the commonly used sizing and scouring agents on the moisture regain of the fibers when exposed to commercial-drying temperatures and at several humidities.

Package Textile Drying. The final objective was a study of some of the limiting conditions of temperature and air flow which may be important in the package drying of cotton, the investigation of the effectiveness of a closed air-circulating system during drying, as compared with an open system, and the demonstration that the moisture content of a textile may be brought to a predetermined uniform value from the wet state by conditioning the drying air in accordance with information available from the moisture-relations study.

The results of these studies have now been published with the exception of the data on package-drying and a thermodynamic treatment of the moisture-relations data. Based upon the evidence secured from this project, it is now possible to form some practical estimates of limiting conditions of temperature, humidity, and time of exposure in order to dry textiles with a minimum of damage. In some cases it has been found that textiles may be dried at higher temperatures than had been previously considered safe.

Discussion of Moisture-Relations Data

Figs. 1 to 4 show curves representing the data secured on four of the ten different fibers in the moisture-relations study at elevated temperatures.[2] It will be noted that these data are somewhat incomplete, particularly at the lower humidities and temperatures. With the aid of certain derived functions, conveniently linear over limited but practically important ranges, and with some application of thermodynamic theory, it is possible to extend these data in the missing ranges, to correct some obvious experimental errors and to provide commercially useful charts for expressing the relations between humidity, temperature, and moisture content.

This treatment of the moisture-content data involved a large amount of work correlating the several factors studied for the ten fibers. Some illustrations are given here of the methods employed so that interested industrial laboratories may be able to extend advantageously the use of the published material.

Fig. 5 illustrates the relation between vapor pressure and the reciprocal of the absolute temperature ($1/T$) in degrees C. The vapor pressures correspond to relative-humidity values taken at unit moisture contents from 1 per cent to 5 per cent for the purified-cotton curve of Fig. 1. This is considered to include all moisture contents likely to be left in textiles after being satisfactorily dried at elevated temperatures. The experimental points fit straight lines or two intersecting straight lines of very nearly the same slope up to temperatures as high as 150 C ($1/T = 0.00236$). This chart, therefore, is of value in interpolating desired intermediate temperature and vapor-pressure points at unit moisture contents and to a limited extent may be useful in extrapolation. On this chart is included International Critical Table data on pure water vapor. It will be seen that the slopes of the experimental curves for vapor pressure of water in equilibrium with cotton yarn tend to approach that for pure water at the higher moisture contents, consistent with thermodynamic considerations.

Fig. 6 shows the logarithmic relation between the percentage relative humidity and the percentage moisture content for soda-boiled cotton, between the limits of 5 per cent and 50 per cent relative humidity for each 10-deg C interval between 10 and 110 C. In this range the logarithmic function of the moisture relation, shown in Figs. 1 to 4, is conveniently expressed as two intersecting straight lines for each constant temperature, the point of intersection being taken at about 20 per cent relative humidity. This set of curves, considered as perhaps the best experimental evidence available on the temperature effect of

[1] Member of the Technical Staff, Bell Telephone Laboratories, Inc., and Chairman of Administrative Committee for Drying Research, United States Institute for Textile Research.

Contributed by the Textile Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

[2] "Moisture Relations of Textile Fibers and Elevated Temperatures," by J. G. Wiegerink, U. S. Bureau of Standards, *Journal of Research*, vol. 24, 1940, pp. 645–664.

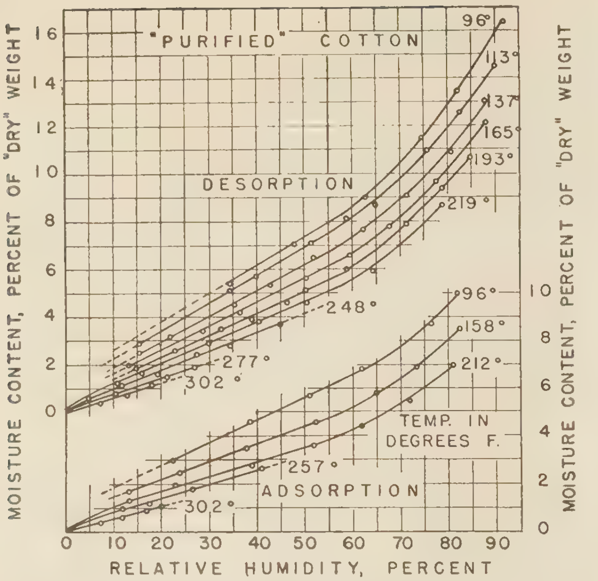

Fig. 1 Purified Cotton: Effect of Temperature on Moisture-Humidity Relation; Wiegerink

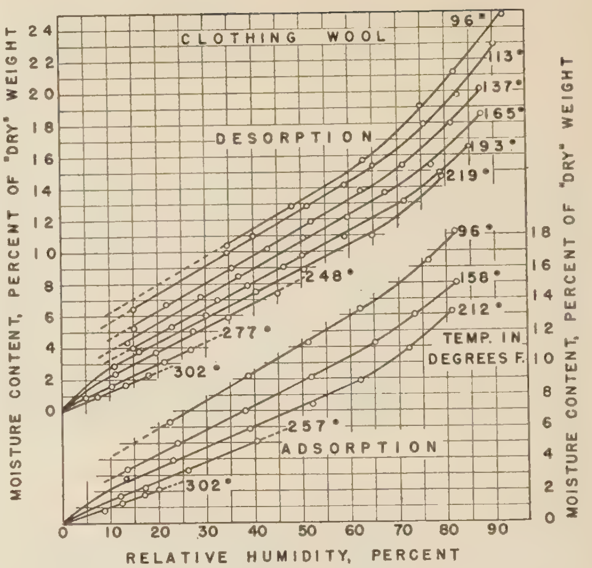

Fig. 3 Clothing Wool: Effect of Temperature on Moisture-Humidity Relation; Wiegerink

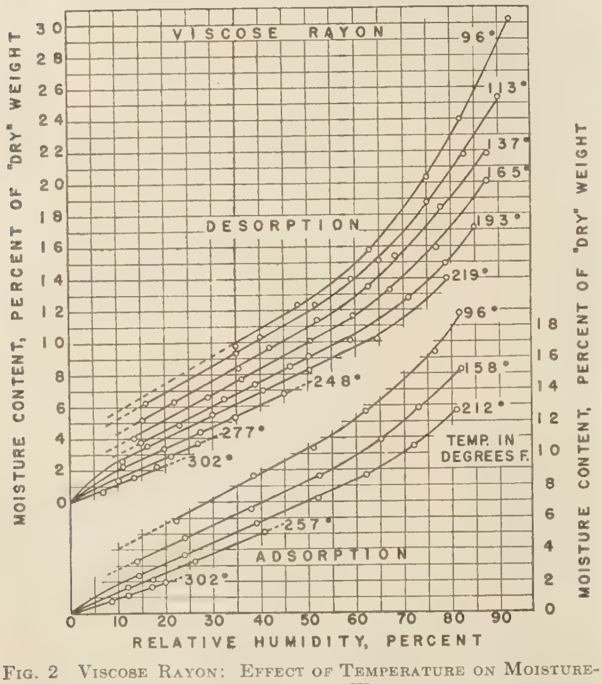

Fig. 2 Viscose Rayon: Effect of Temperature on Moisture-Humidity Relation; Wiegerink

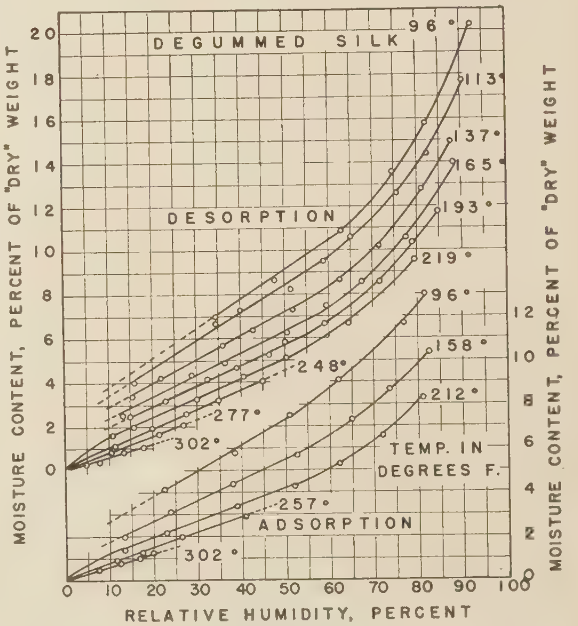

Fig. 4 Degummed Silk: Effect of Temperature on Moisture-Humidity Relation; Wiegerink

moisture adsorption in a textile, was prepared from the data of Urquhart and Williams.[3] Since these straight lines adequately describe the experimental data of these workers within reasonable practical limits, it is considered that similar plots for each of the ten fibers studied by Wiegerink may be used with confidence to extend the experimental data to lower moisture contents at lower temperatures, and to correct some obvious experimental errors.

[3] "Moisture Relations of Cotton," by A. R. Urquhart and A. M. Williams, *Journal of the Textile Institute*, vol. 15, 1924, pp. T559–572.

Two other interesting relations may be derived from these data. Fig. 7 is a plot of the log percentage relative humidity versus $1/T$ for the purified-cotton data by Wiegerink, at unit moisture contents. Similar curves for soda-boiled cotton[3] are included for comparison. It can be shown that the slopes of these curves bear a definite thermodynamic relation to the corresponding slopes of Fig. 5, for a given textile.

Fig. 8 illustrates the relation between the log percentage moisture content versus $1/T$ for selected percentage relative humidities between 5 per cent and 50 per cent. Similar charts for the soda-boiled data are included for comparison. It will be observed that up to about 100 C (212 F) the curves for cor-

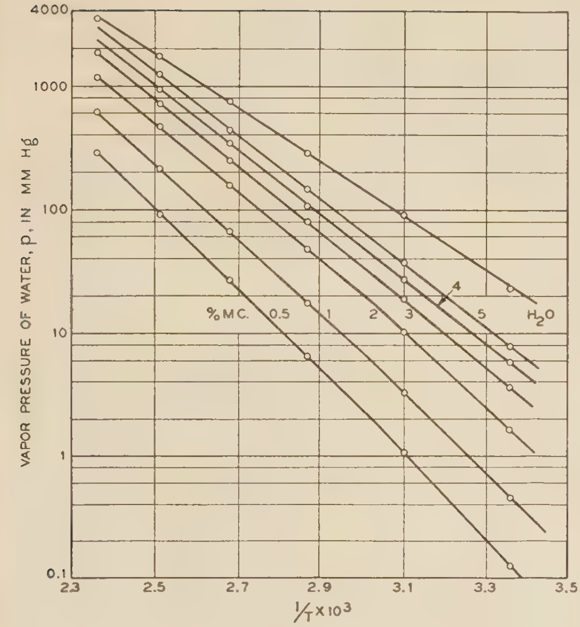

FIG. 5 PURIFIED COTTON: VAPOR PRESSURE–TEMPERATURE RELATIONS FOR MEAN MOISTURE SORPTION

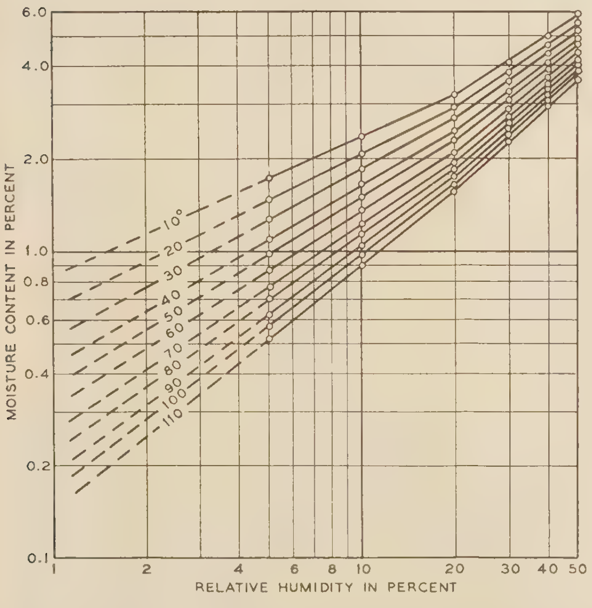

FIG. 6 SODA-BOILED COTTON: LINEAR MOISTURE-HUMIDITY RELATIONS, PLOTTED AS LOG FUNCTIONS; URQUHART AND WILLIAMS

responding relative humidities for the two kinds of cotton, and the data by different observers, have approximately the same slopes.

This set of four charts, Figs. 5 to 8, inclusive, thus represents several methods of expressing the moisture relations of textiles as a series of straight or intersecting lines for each condition. Sufficiently accurate linear functions may be derived from these charts for most industrial purposes. Their thermodynamic significance is beyond the scope of this paper, but it might be mentioned that only the mean values of adsorption and de-

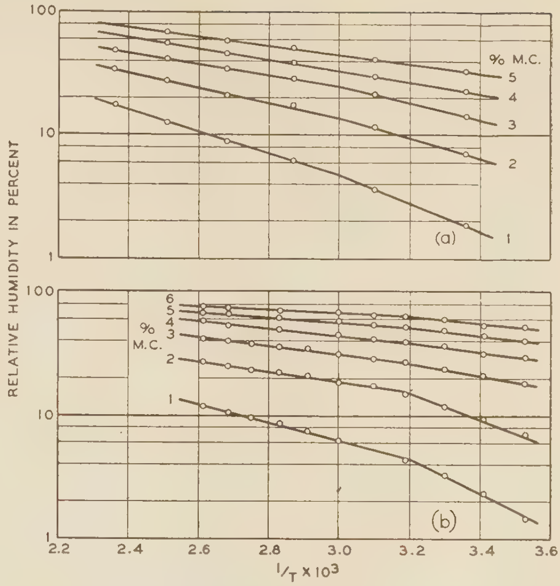

FIG. 7 TEMPERATURE–HUMIDITY RELATIONS FOR MEAN MOISTURE SORPTION

(*a*, Purified cotton; Wiegerink. *b*, Soda-boiled cotton; Urquhart and Williams.)

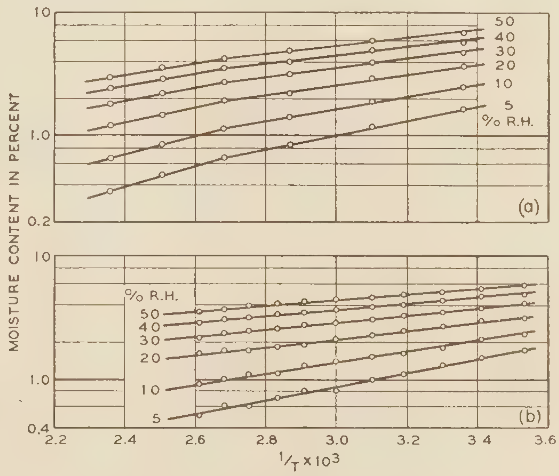

FIG. 8 TEMPERATURE–MOISTURE CONTENT RELATIONS—MEAN SORPTION

(*a*, Purified cotton; Wiegerink. *b*, Soda-boiled cotton; Urquhart and Williams.)

sorption were used in preparing these charts. This avoids certain difficulties which may be involved in such treatment, particularly that pertaining to the hysteresis effect in the sorption[4] of moisture by textiles.

HYSTERESIS EFFECTS

Certain aspects of the hysteresis effects may be discussed to advantage in connection with the drying of textiles. The well-known hysteresis effect in the moisture regain exhibited by fibrous materials is illustrated in Fig. 9, for cotton. This chart shows that cotton always retains more moisture when

[4] Adsorption is here defined as the taking up of water by a textile, desorption as the giving up of water, and sorption as the general process without special indication of direction.

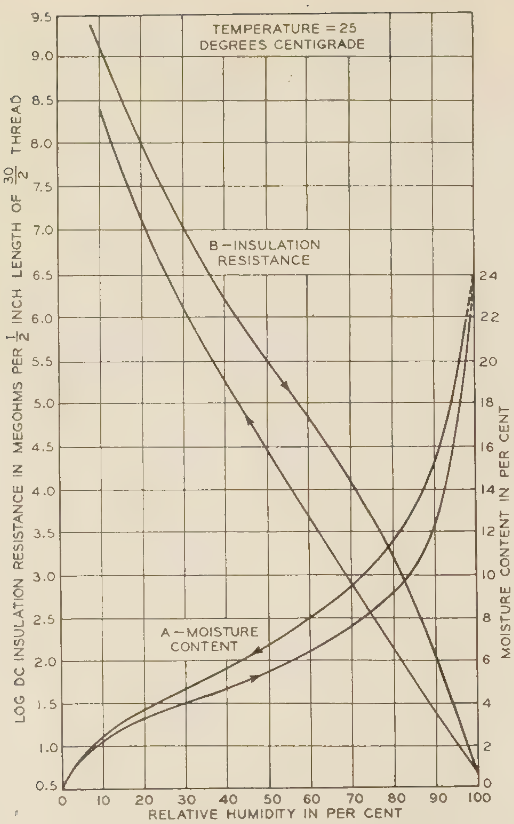

Fig. 9 Hysteresis Effects in Humidity–Moisture and Humidity–Electrical Resistance Relations for Cotton

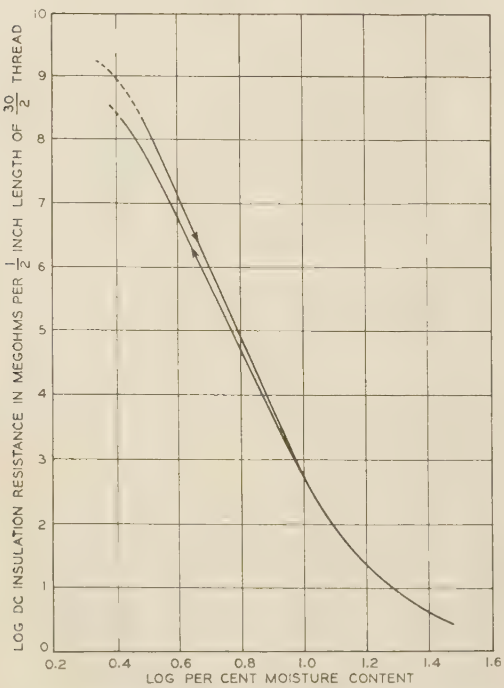

Fig. 10 Hysteresis Effect in Moisture–Electrical Resistance Relation for Cotton

brought to any equilibrium atmospheric humidity from the wet state than from the dry. There is a corresponding hysteresis in the relation between insulation resistance and relative humidity, also shown in Fig. 9. This relation demonstrates the critical dependency of electrical insulation resistance upon moisture in such fibers, and upon the direction of approach to equilibrium.

Another type of hysteresis is shown in Fig. 10, which illustrates the influence of "previous history" on the moisture relations. It is not so well known and is less well understood than the first type of hysteresis illustrated in Fig. 9, but it is important in relation to textile drying. Fig. 10 is a chart of the log percentage moisture content versus the log of electrical-insulation resistance. The upper curve on this chart was obtained by drying the cotton at a high temperature, above 100 C (212 F), directly from the wet state. The lower curve was obtained after subsequently exposing this cotton to a high humidity, near saturation. There are at least two different insulation-resistance values for the same moisture content. A still lower curve may be obtained by drying the material from the wet state at room temperature instead of at an elevated temperature.

This behavior suggests that even if cotton is brought to the same atmospheric-test condition from the same direction to avoid the well-known hysteresis effect, Fig. 9, different samples may still have different moisture contents, and therefore different properties such as, for example, the electrical-insulating quality. Such differences are dependent upon the previous history of the material, particularly the manner in which it is dried. These differences may amount to as much as 1 per cent in moisture content. Indeed in some cases where the previous history might be assumed to be the same, differences may occur. For example, consider the data in Table 1.

TABLE 1 MOISTURE CONTENT OF TEST SAMPLES

Sample no.	Moisture contents, per cent: 87.5 Per cent relative humidity	84.3 Per cent relative humidity
1	10.95	10.1
2	10.8	10.0
3	10.7	9.9
4	10.8	9.8
5	11.1	10.2
6	11.0	9.9
7	11.8	10.8
8	10.7	10.1
9	10.85	9.8
10	11.0	10.0
11	10.7	9.9

In this test, 11 samples of cotton were removed successively from the same package of yarn, which had initially been dried from the wet state at 110 C. Before testing, the samples were redried from atmospheric conditions in a current of dry air at room temperature and equilibrated under carefully controlled conditions at 87.5 per cent relative humidity at 100 F, giving the data in the second column of Table 1. Again they were dried at room temperature and several days later were equilibrated at 84.3 per cent relative humidity at 100 F. Small, but definite differences are to be seen in the moisture contents of these samples, persisting even between the two tests. Sample No. 7 preserved a marked difference in moisture content. That it adsorbed nearly 1 per cent more moisture than the others suggests that this portion of yarn may have been incompletely dried initially in the oven.

Since a difference of but 0.1 per cent moisture content may cause a difference of as much as 25 per cent in electrical-insulation resistance, these data are considered significant in the testing of textiles used for insulating purposes. It is clear that electrical measurements constitute a sensitive tool for investigating the moisture relations of textiles, but in this connection

it should be emphasized that one essential is very close control of atmospheric humidity during such tests.[5]

Because of these peculiarities in the behavior of cotton, dried under different conditions, it was considered worth while to study the combined effects of heat and moisture on this and other textiles in the drying-research project. By limiting the experiments to yarns made from the common fibers, freed from the natural nonfibrous materials which would be removed from them during manufacture, it was possible to obtain data which are considered basic to all types of textile drying.

Combined Effects of Heat and Humidity on Textiles During Drying

In this phase of the research, three temperatures were used, 221, 257, and 302 F, and three humidities were supplied at each temperature. At 221 F the humidities were 1 per cent, 48 per cent and 79 per cent, of saturation; at 257 F, they were 0.5 per cent, 24 per cent, and 42 per cent; and at 302 F, they were 0.2 per cent, 12 per cent, and 20 per cent. At this highest temperature, a humidity much above 20 per cent was not possible without using a pressure system. Times of exposure were up to 6 hr. Fig. 11 is a summary of some of the more important experimental results of this investigation. The quality-index function used in these plots is a combined product of the tensile strength and elongation percentages retained after each heat-treatment. Other properties, e.g., fluidity of solutions of the textiles in suitable solvents, and alterations in the dyeing properties, both sensitive indications of heat degradation, are reported in the original paper.[6]

[5] "Purified Textile Insulation for Telephone Central Office Wiring," H. H. Glenn and E. B. Wood, Trans. A.I.E.E., vol. 48, 1929, pp. 576–581.

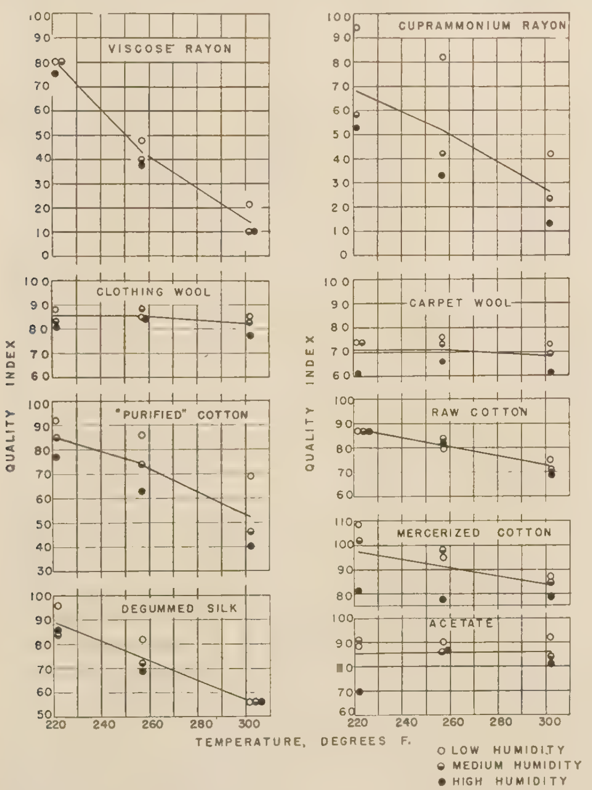

Fig. 11 Effects of Heat and Humidity on Percentage Tensile Strength-Elongation Product Function—Quality Index—of Various Textiles

Heat has a greater effect on the quality indexes of rayons than on those of the other fibers, except possibly purified cotton. High humidity at any temperature is more harmful than a low humidity, this being particularly true for purified cotton and cuprammonium rayon. The natural wax on raw cotton appears to be a definite protection against both heat and humidity. Carpet wool shows the degrading effect of humidity somewhat more than clothing wool, as does mercerized cotton and acetate rayon. The data on acetate rayon are particularly interesting in this connection. The breaking strength of this fiber decreases appreciably only at the lowest temperature, 221 F, in conjunction with the high humidity of 79 per cent. Its elongation at the different temperatures decreases progressively with humidity in a more or less regular manner. Therefore, it is not surprising that the quality index, a product of tensile strength and elongation factors, is lowest at the lowest temperature and highest humidity. This condition would be most favorable for hydrolysis of the fiber, since it is more difficult for moisture to evaporate from the yarn during drying. One of the wool samples showed this same effect. Degummed silk appears to be affected by moist heat in much the same way as purified cotton, except at the highest temperature, where the humidity effect is relatively negligible.

All of the textiles dried as noted in Fig. 11 failed to regain the amount of moisture held originally. Yarns heated at high humidity regained less moisture than those heated at low humidity, with the exception of the acetate rayon. The moisture content of this fiber, on conditioning after heating, showed little effect of humidity, but the heat effect was important. For example, acetate heated at 302 F for 6 hr had a moisture content of but 5.3 per cent on conditioning at 65 per cent relative humidity at 70 F, as compared with 7.3 per cent before heating.

This humidity effect during drying is of significance in connection with the electrical as well as the physical properties of textiles. Rapid drying from the wet state under low ambient humidity conditions should give a material of a relatively high moisture-regain capacity, whereas slow drying under poorly ventilated conditions favors hydrolysis and, as seen by these experimental data, results in a low moisture-regain capacity. Such a low moisture content would be favorable to high electrical-insulation resistance under any given atmospheric condition but at the same time would be unfavorable to the strength of the material. Thus, different drying conditions will give textiles of different properties, and this common denominator of all textile processing should be studied carefully with regard to the properties most desired in the finishing product.

Finishing and Scouring Agents

One or two important points may be mentioned in connection with the study of the effect of chemical and sizing agents on the moisture relations of textiles after drying. Wool treated with 0.5 per cent or with 4.5 per cent sulphuric-acid solutions, centrifuged and dried for 2 hr at 220 F, retained less water than the untreated material. With the 4.5 per cent acid solution, the reduction in moisture regain amounted to 2.5 per cent at 65 per cent relative humidity at 70 F. With the 0.5 per cent solution, the reduction was 2 per cent. It is known that wool adsorbs acids very strongly, even from dilute solutions, and the corresponding decreases in moisture regain appear due to some salt formation with amino groups which otherwise would attract water molecules. A similar behavior but to a lesser extent is

[6] "Effects of Drying Conditions on Properties of Textile Yarns," by J. G. Wiegerink, U. S. Bureau of Standards, *Journal of Research*, vol. 25, 1940, RP 1337, pp. 435–450.

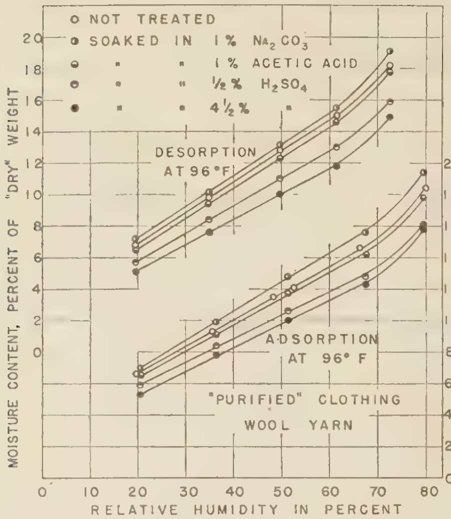

Fig. 12 Effects of Scouring Agents on Moisture Sorption of Wool

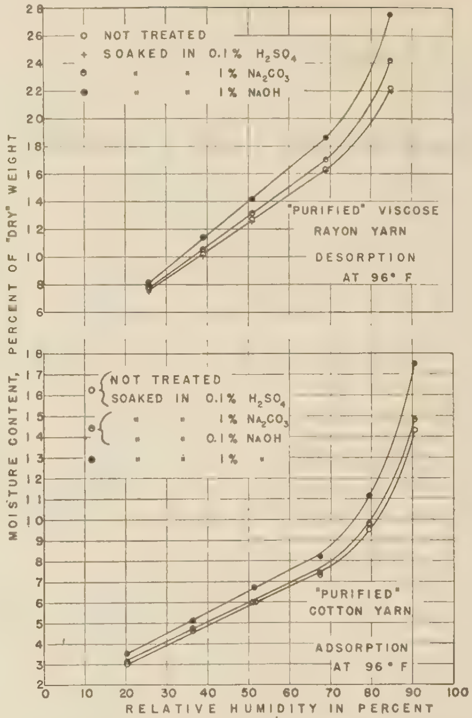

Fig. 13 Effects of Scouring Agents on Moisture Sorption of Rayon and Cotton

observed with acetic acid. Alkali, on the other hand, increases the moisture adsorption of wool under similar drying conditions. Purified cotton treated with dilute solutions of acids apparently is practically unaffected as regards moisture regain.

Washing of wool after treatment with acid solutions may not effect complete removal of the acid, and the change in moisture sorption of a wool might be one criterion of acid retention. This possibility is based on the fact that the effect of moist heat during drying is likely to reduce the moisture regain of wool by not more than 0.75 per cent, whereas the acid effect may easily be of the order of 2 per cent. See Figs. 12 and 13, with reference to this discussion.

Package-Drying of Cotton

Perhaps the most economical method of drying packaged cotton is in a hurricane oven or so-called cabinet drier. Usually the centrifugally extracted packages are dried in this way in from 10 to 18 hr. Attempts to reduce this drying time have been made, and a relatively successful commercial drier employs compressed heated air blown through the loose-wound packages mounted on the same perforated spindles used in the dye kier. A time range of $2^1/_2$ to 6 hr seems to be usual with such driers, this time depending upon the type of apparatus, size and density of package, type of yarn and temperature, and rate of flow of the air supply. Preliminary studies of this method of drying indicated that it should be possible to reduce this time still further, a highly desirable objective, under present conditions, since it would not only increase production but would help in preventing the hydrolysis previously discussed.

Fig. 14 illustrates two rather important results, not previously published, which have been found in the course of the package-drying research project. One of these is that such yarn may be dried in a fraction of 1 hr under suitable commercially practicable conditions, and the other result is that such drying may be carried out under conditions where the material may be brought to a predetermined moisture content without overdrying. Some other results have come out of this limited study of package drying of cotton, but they are beyond the scope of this discussion.

The numbers from 1 to 4 in Fig. 14 refer to thermocouples located at equally spaced intervals through the cotton package. Couple No. 1 was located on the inside of the package and No. 4

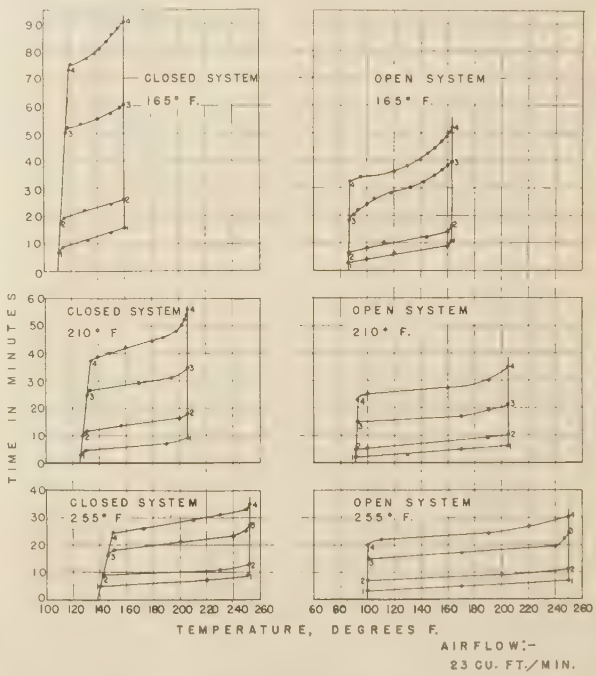

Fig. 14 Effect of Temperature on Drying Rate in Package Drying of Cotton at High Air Flow

on the outside. When heated air was passed through the package from inside to outside, No. 4 was the last couple to reach the air-supply temperature. Thus the temperature of this couple is considered an index of the time required to dry the package.

By "open system" is meant that the vapor-laden air blown out through the surface of the package was exhausted directly into the room. By "closed system" is meant that the package was enclosed in a tight container, and the air was blown through the package in either direction, recirculated throughthe blower, and mixed with sufficient dry air so that the humidity during drying could be controlled as desired.

This series of tests, representing but one of a number, was made with an air flow of 23 cfm, several times the maximum rate understood to be used in present commercial equipments of this type. While this increase in air rate no doubt represents a considerable increase in power cost, the results may be more than worth this cost increase both in production output and in textile quality.

Theory of Moisture Distribution in Textile Fibers

It is known that the removal of all but a few tenths of a per cent of water from a fibrous material is likely to result in more or less permanent changes in its molecular structure and therefore changes in its physical properties. One example of this is the difference in moisture regain of two samples of the same cotton, one of which was dried at room temperature and the other dried rapidly in a well-ventilated oven at 220 F. When reconditioned so as to avoid hysteresis, it was found that the air-dried material adsorbed about 1 per cent more moisture than the other. This difference in moisture-adsorbing capacity may be retained, under certain conditions, for a period of years.

To secure a better understanding of the reasons for such alterations in moisture-regain properties and in the general principles underlying drying of textiles, it is worth while to consider the structure of the textile fiber and the mechanism of moisture adsorption and distribution within this structure.

It is now generally accepted that textile fibers are built up of very long chainlike molecules. In certain regions, these molecules lie parallel with one another in such closely packed regular formation that they possess many of the properties of crystals. In other regions the orientation is poor and the molecules lie in such a disordered state that here the material may be termed amorphous. It is considered that the fiber is built up of alternate regions of well-oriented and poorly oriented molecular aggregates, with some of the single long chainlike molecules running continuously through both regions.

Mark[7] has provided an excellent series of pictures to help explain this arrangement, but it is only necessary here to point out that the fiber may be regarded as an irregular flexible net built up of crystallized and amorphous areas, exhibiting at certain places large holes, where the amorphous condition is most pronounced, with spaces and crevices at other places, and even smaller pores between contiguous surfaces, such as might well be found in the more highly crystalline regions. The well-oriented aggregates are relatively inert to chemical attack and to penetration by water, dyestuffs, or other reacting substances. On the other hand, these materials may enter into the amorphous areas to a greater or less degree depending upon the density of the structure at these points and to the accessibility of the cracks, crevices, and pores.

It is now generally accepted that the adsorption of moisture and gases on the internal surfaces of such fibrous materials is due to formation of monomolecular or multimolecular films on these surfaces at low pressures. Cotton, for example, will retain not more than about 25 per cent of moisture by weight, when in equilibrium with a saturated atmosphere. More than twice this amount may be held in the fiber structure when the cotton hair is wet and swollen with liquid water. Considerations of the heat of wetting of dry cellulose and the dielectric constant of cellulose of various moisture contents indicate that the first moisture adsorbed by dry cotton is more firmly bound than subsequent amounts, has much greater effect on the elastic properties of the fiber, and is more difficult to remove. At higher moisture contents much of the water is relatively free and simply fills the capillary spaces or holes in the fiber. These facts suggest that there is a greater affinity between water and the hydroxyl groups on the cellulose surface than between water molecules already adsorbed on this surface and additional water molecules which may be adsorbed subsequently.

Considering the picture of the fiber structure just described, and the affinity of water for hydroxyl groups on the cellulose surfaces, multimolecular layers of water may be held on all internal surfaces which are available for moisture sorption. These are not to be considered as continuous layers of liquid water completely filling the spaces between contiguous surfaces in pores, or more open surfaces in cracks or crevices, but are believed to exist as columns or chains of water molecules. One end of such a chain remains within the sphere of influence of a hydroxyl group in the cellulose surface, thus being more or less anchored. The remaining portion of the chain extends outward from this surface and the multimolecular layer of these chains is analogous to the vertical fibers in a pile fabric like velvet.

If moisture is removed during drying, down to a point where the initial monomolecular layer begins to be depleted, the energy residing in the hydroxyl groups freed of water may be neutralized by active groups on contiguous surfaces, resulting in a local cementing or "spot-welding" action. This would be likely to impair the lubricating effect of the continuous films of moisture between adjacent surfaces and affect the elastic properties of the fiber. If such an "overdried" material is permitted again to adsorb moisture, these cemented points not only are likely to interfere seriously with the uniform dis-

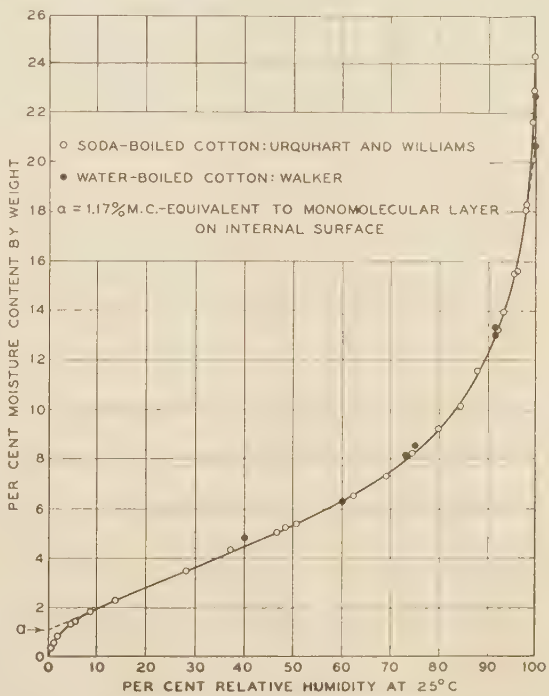

Fig. 15 Estimation of Internal Surface of Cotton

[7] "Structure of the Rayon Fiber," by H. Mark, *Nature*, vol. 144, 1939, pp. 313–314.

tribution of water on the former surfaces but may prevent as much moisture being adsorbed as before. Thus, such a conception of the internal structure and the behavior of adsorbed moisture admirably accounts for the increased harshness of overdried textiles and also the reduced moisture-regain capacity.

The amount of water required to form a monomolecular layer over all the internal surface of a cotton fiber is readily estimated from the moisture-adsorption isotherm for the material, Fig. 15.

The intercept of the substantially linear middle portion of this curve with the moisture-content axis, shown by the dotted line, gives the amount of water (a) required to cover the internal surface of the fiber. In this case it is seen to be slightly more than 1 per cent moisture. With other fibers, the amount of internal surface is different and the intercepts also differ.

There is much evidence to indicate that there are marked differences in the properties of a fiber at moisture contents below and above this monomolecular-layer figure. In a previous publication,[8] a series of simple linear relations was formulated to show the relation between electrical-insulation resistance, moisture content, and relative humidity for cotton. These equations were of the form

$$\text{log insulation resistance} = -A(X) + B$$

where A and B are constants determined by the textile and its previous history, and X is some function of moisture content or relative humidity. For moisture contents between 1 per cent and 6 per cent, the following linear equation holds

$$\text{log insulation resistance} = -A(\text{per cent moisture content}) + B.$$

Fig. 16 shows that this equation holds for a cellulosic material, in this case, rag paper. The important fact to be emphasized in connection with the present discussion of textile drying is that below 1 per cent moisture content, the electrical-insulation resistance increases much more rapidly than is consistent with this relation between 1 per cent and 6 per cent moisture content. It appears reasonable to explain the very rapid increase in the

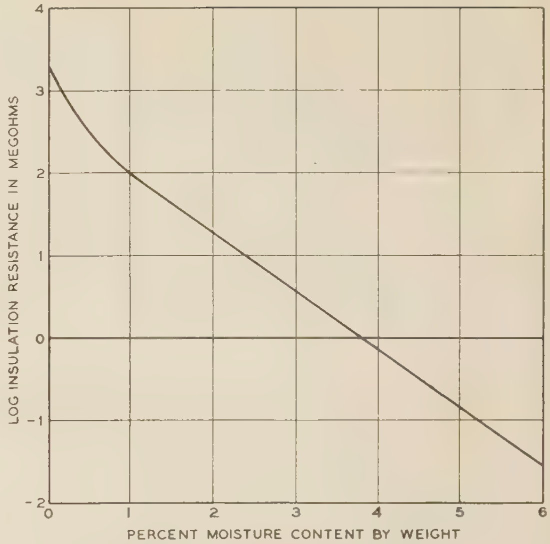

Fig. 16 Moisture–Electrical Resistance of Cellulose
(Significance of monomolecular moisture layer.)

resistivity of the cellulose with decreasing moisture contents below 1 per cent as being due to discontinuities in the monomolecular layer of moisture adsorbed on the internal surfaces.

Conclusion

It is seen from the foregoing discussion of recent experimental data on the moisture relations of textiles, and the discussion of the theory of moisture sorption and distribution in cotton and other cellulosic fibers, that it may be highly undesirable to remove all or part of the initial monomolecular layer of moisture from the internal surfaces of such materials. Too thorough or uneven drying may impair the valuable properties of flexibility, softness, and strength. Therefore, in any problem involving textile drying, consideration should be given to the theories relating to the form in which the moisture is distributed within the fiber structure and to data of the type discussed in this paper.

[8] "Moisture Content and Electrical Conductivity of Cotton," by A. C. Walker, *Journal of the Textile Institute*, vol. 24, 1933, pp. T145–160; also *Bell System Technical Journal*, vol. 12, 1933, pp. 431–451.

The Effect of Installation on the Coefficients of Venturi Meters

Final Report

By W. S. PARDOE,[1] PHILADELPHIA, PA.

THIS paper may be considered as the continuation and final report on the effect of installation on the coefficients of Venturi meters. Articles in Transactions of the A.S.M.E., November, 1936, Vol. 58, page 677, and Transactions of the A.S.M.E., November, 1937, Vol. 59, page 753, being progress reports giving data on the (11.956 × 8.2285-in. $\beta = 0.688$), the (7.810 × 5.00465-in. $\beta = 0.640$), and the (8.060 × 3.3759-in. $\beta = 0.394$) Simplex Valve and Meter Company Herschel type Venturi meters.

In the accompanying curves, Fig. 1 to Fig. 50, are given similar data for the (8.074 × 5.9988-in. $\beta = 0.743$) Simplex Valve and Meter Company Venturi meter and the (8.036 × 4.0005-in. $\beta = 0.498$) Builders-Providence, Inc., Venturi meter, both being of the Herschel type.

In Figs. 1, 12, 20, 28, and 43 are collected the coefficient installation curves from all three papers.

The method of correlating the data is shown in Fig. 51, which is the inked-in work sheet for installation shown in Fig. 12. In the table, column 1 is the value of x; columns 2, 4, 6, and 8 are the coefficients obtained from Fig. 12 for the four Venturi meters; columns 3, 5, 7, and 9 are the errors or difference between these coefficients and the normal coefficient C_n. These errors are plotted against the corresponding values of β and smooth curves run in on which values for $\beta = 0.8$. 0.75 — 0.4 are interpolated or extrapolated, giving the information for plotting Fig. 53. Figs. 52, 54, 55, and 56 were obtained in a similar manner.

When the coefficient curves became flat, or C constant, as they do in most cases, this value was used. If the value of the coefficient did not become constant, as in Figs. 18 and 19, the value at 17 ft per sec throat velocity was used.

Figs. 52 to 56 should be used with some caution, particularly for the high-ratio Venturi meters, in which case it would be well to refer to the experimental curves.

The entire investigation can be summed up as follows:

1 Low pipe factors, high coefficients

2 High pipe factors, low coefficients

This is evident from the formula

$$C = \sqrt{\frac{1 - \beta^4}{\alpha_2 - \alpha_1\beta^4 + k}}$$

3 Any vortex soon becomes a free vortex and lowers the coefficient.

R. B. Smith's formula

$$C_w = C\sqrt{\frac{1 + \beta^2}{1 + \beta^2 + \beta^2 \cot^2\alpha}}$$

is confirmed with respect to effect of β.

Hence the various fittings, in order of the distance over which their effects are noticeable, may be given as follows:

1 A single short-radius elbow is felt for about 6 diameters. See Fig. 54.

2 A 6 to 8-in. standard increaser is felt for close to 6 diameters. See Fig. 53.

3 A 24 to 8-in. series of decreasers produces an effect up to 12 diameters. See Fig. 52.

4 Two short-radius elbows in planes at right angles give first a positive error at low values of β due to a low pipe factor after the elbows, and then a negative error due to the formation of a vortex. The effect of this is not eliminated in thirty diameters. See Fig. 55.

5 Cross straightening vanes, 6 diameters long, eliminate the vortex and restore the normal coefficient. See Fig. 56.

6 For all setups the high-ratio tubes are most profoundly affected, but the effect of a vortex does not disappear in the low-ratio meter in 30 diameters.

7 Low-ratio tubes are but slightly affected by non-vortex-forming disturbances.

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[1] Professor of Hydraulic Engineering, Department of Civil Engineering, University of Pennsylvania.

Contributed by the Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 3, 1942, of The American Society of Mechanical Engineers.

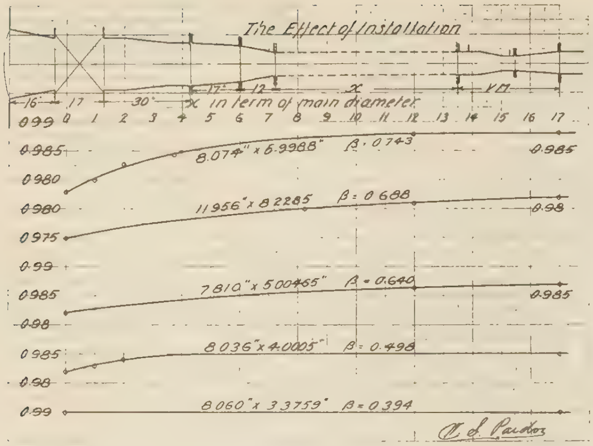

FIG. 1

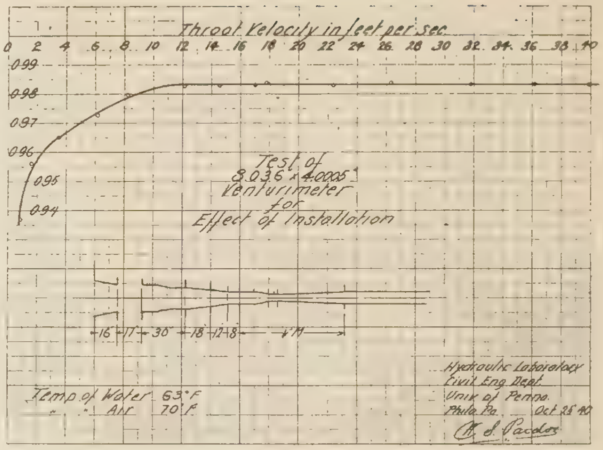

FIG 4

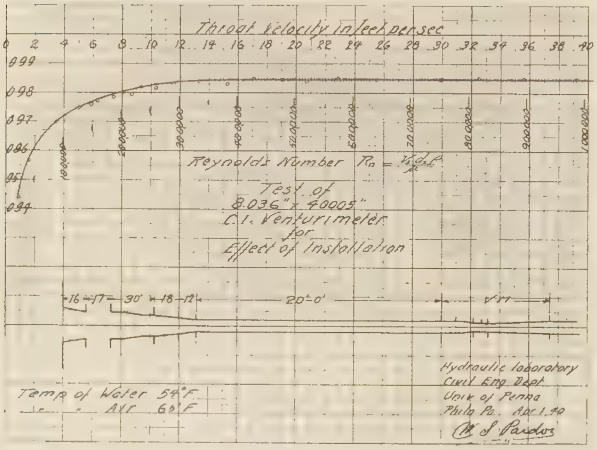

FIG. 2

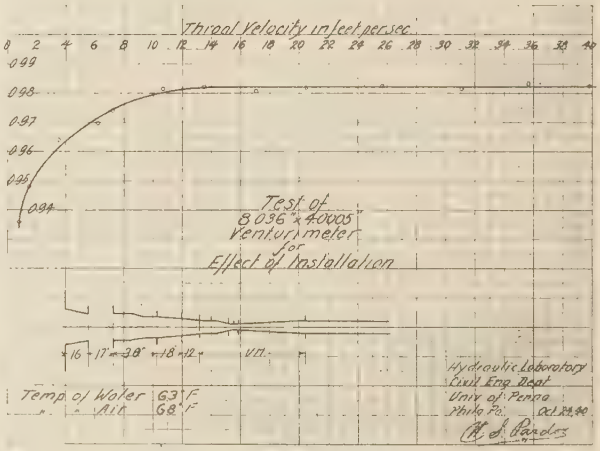

FIG. 5

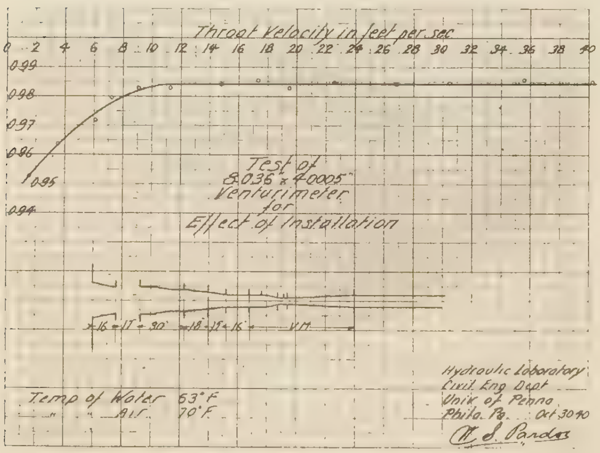

FIG. 3

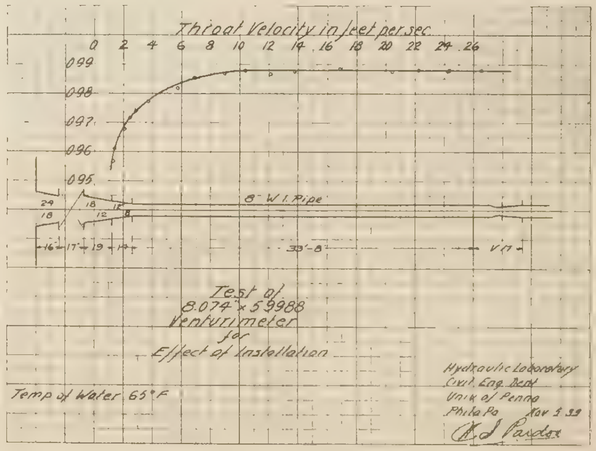

FIG. 6

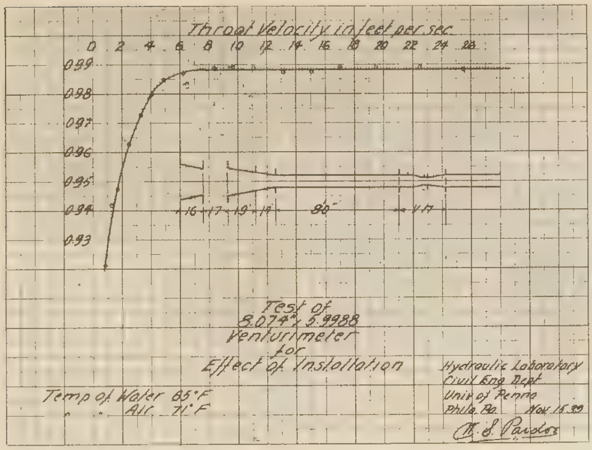

FIG. 7

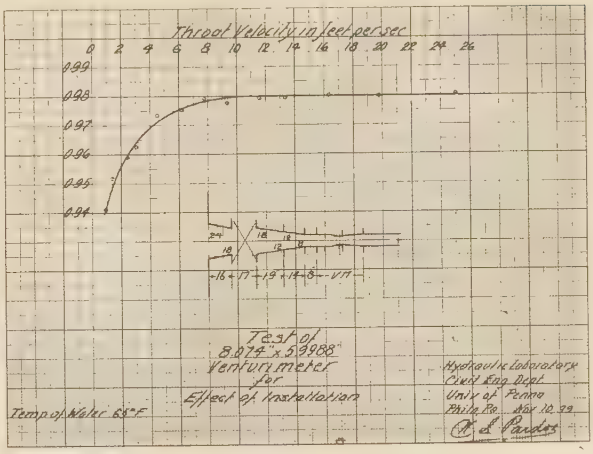

FIG. 10

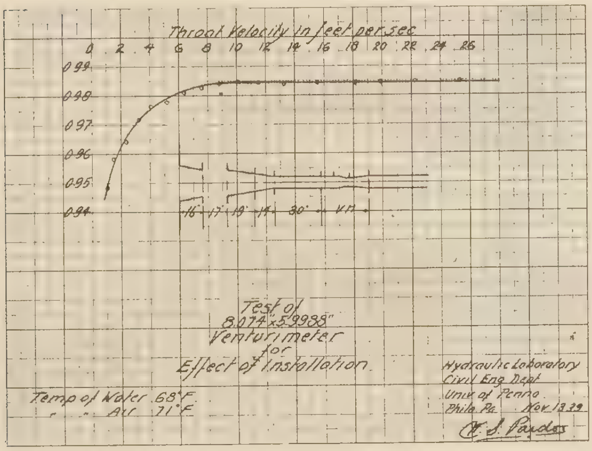

FIG. 8

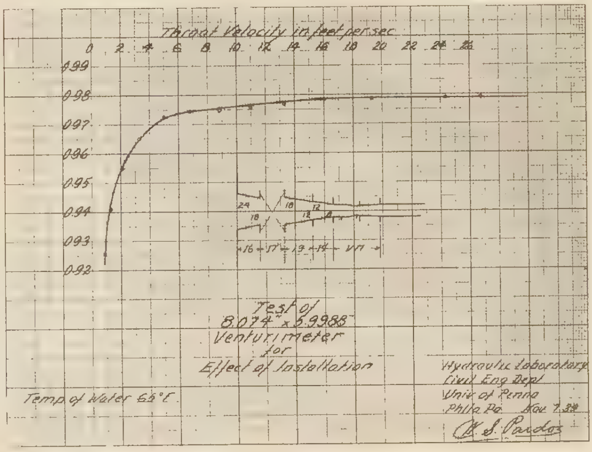

FIG. 11

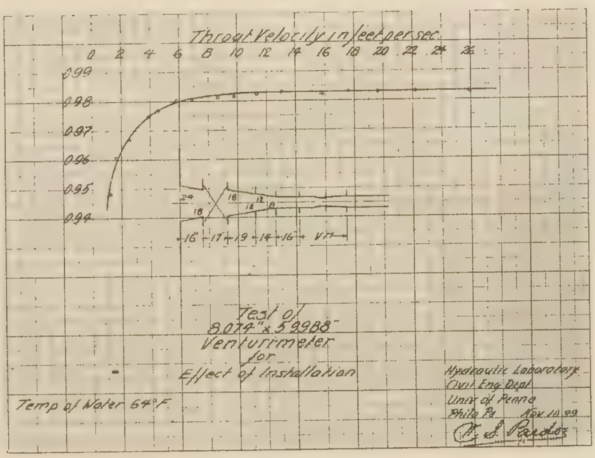

FIG. 9

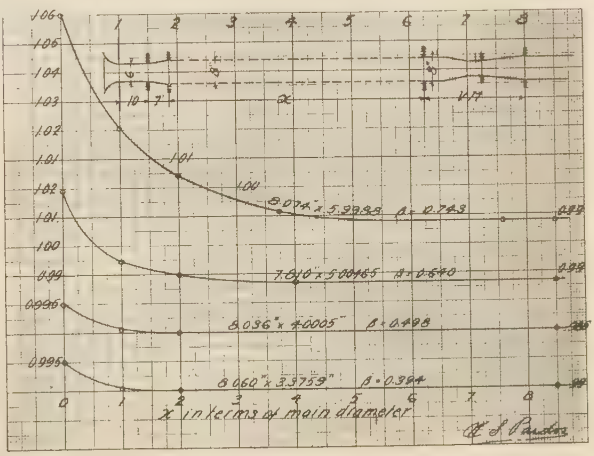

FIG. 12

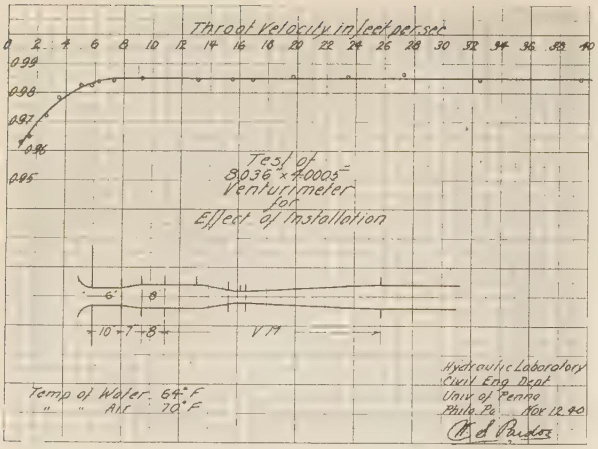

Fig. 13

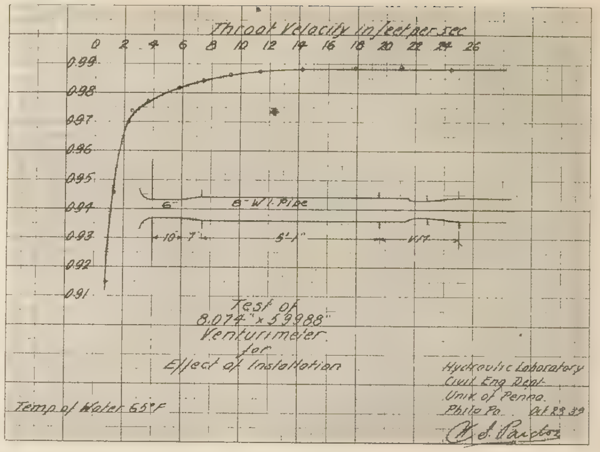

Fig. 16

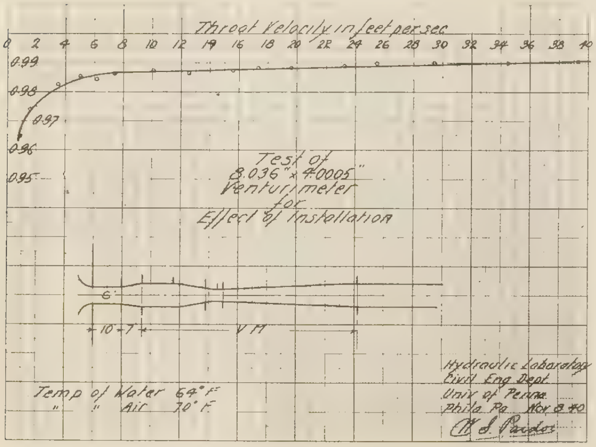

Fig. 14

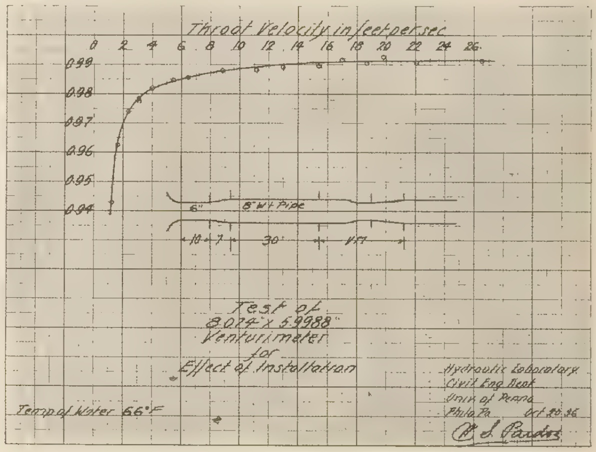

Fig. 17

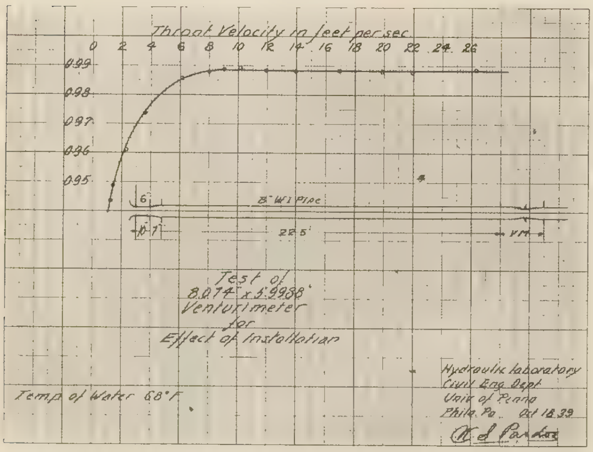

Fig. 15

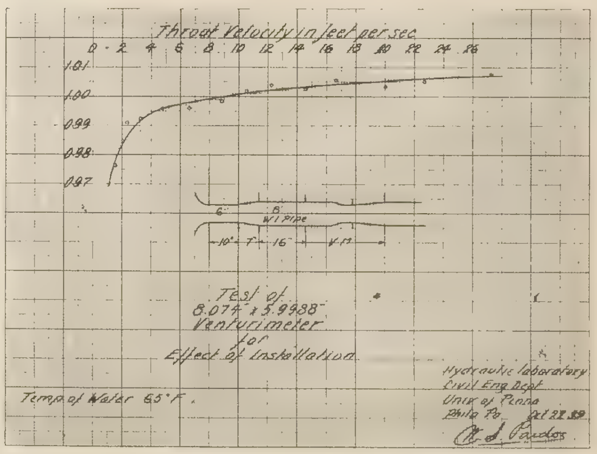

Fig. 18

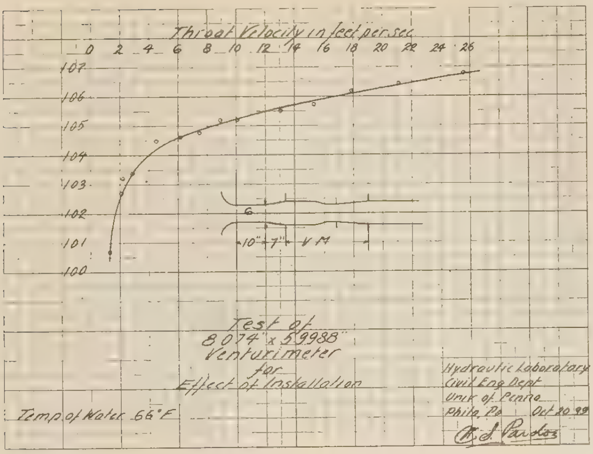

FIG. 19

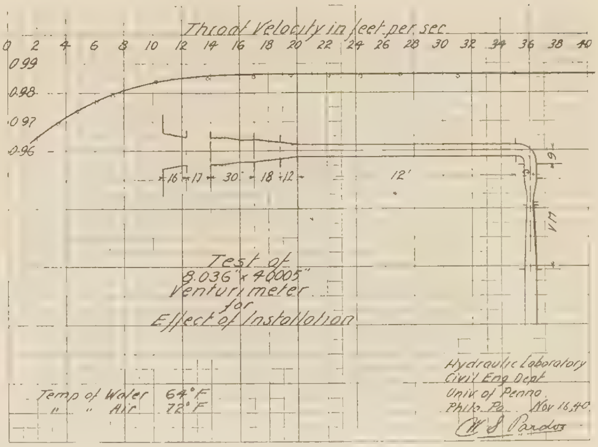

FIG. 22

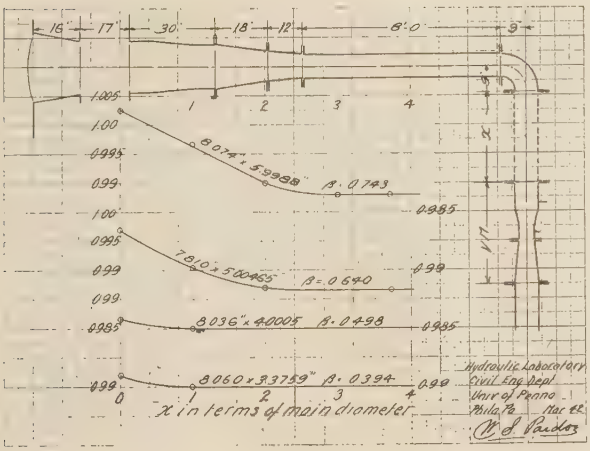

FIG. 20

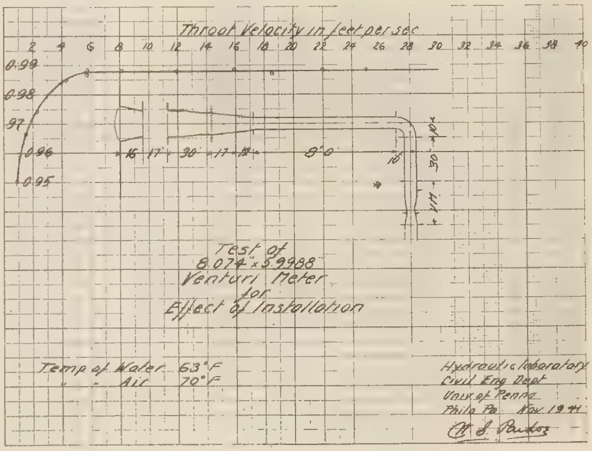

FIG. 23

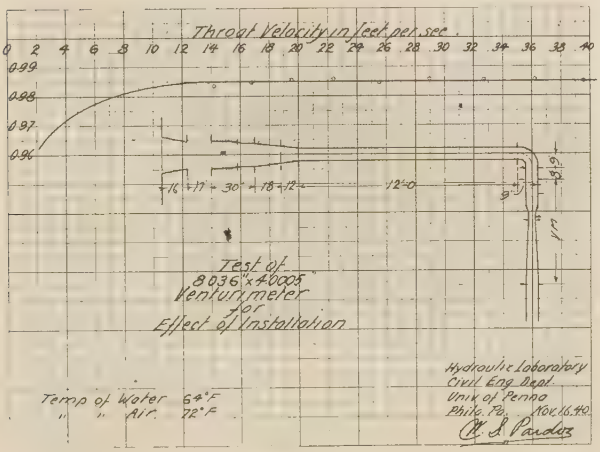

FIG. 21

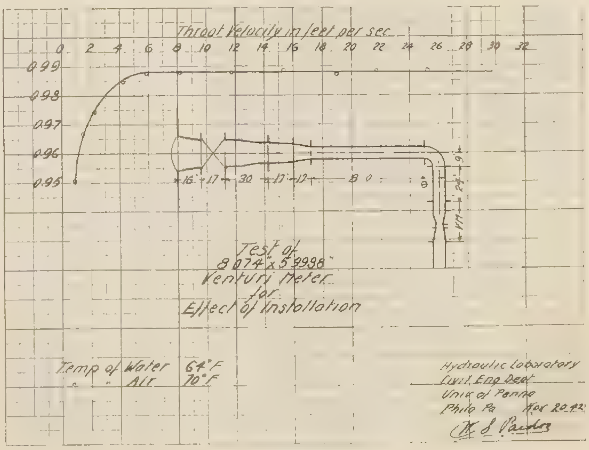

FIG. 24

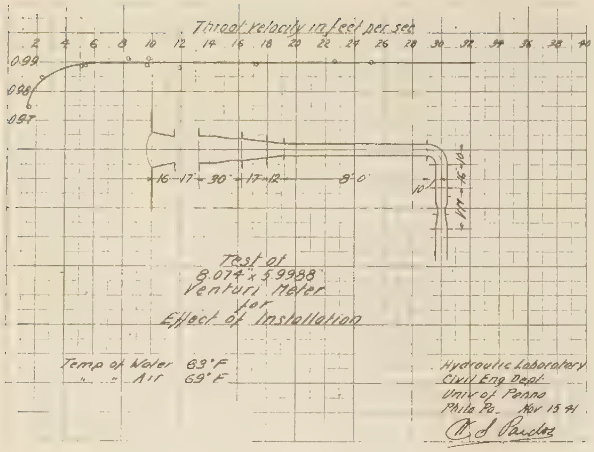

FIG. 25

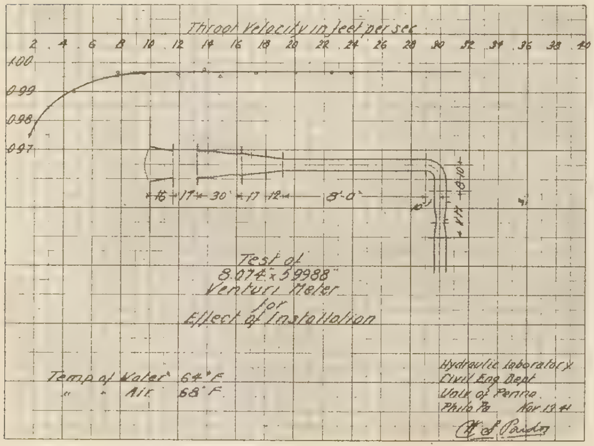

FIG. 26

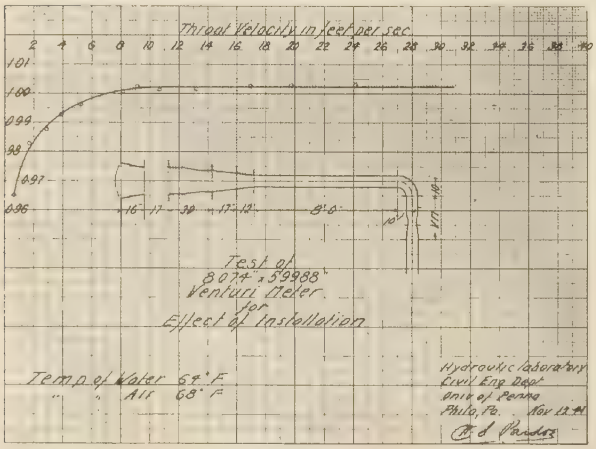

FIG. 27

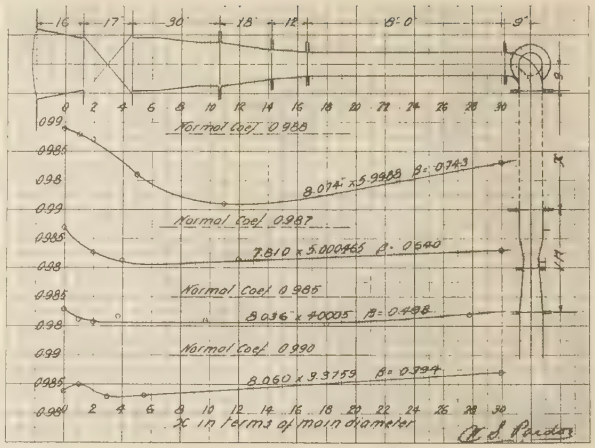

FIG. 28

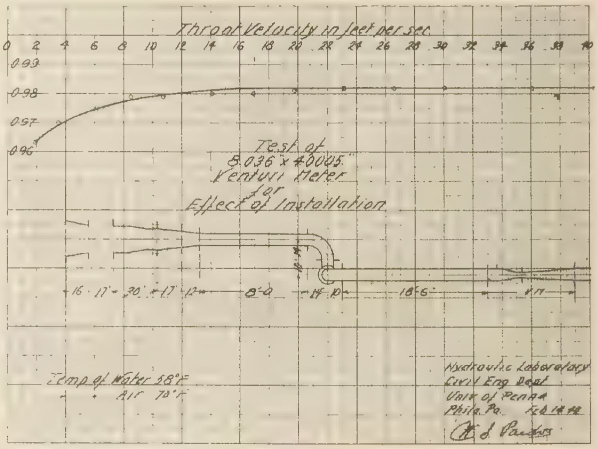

FIG. 29

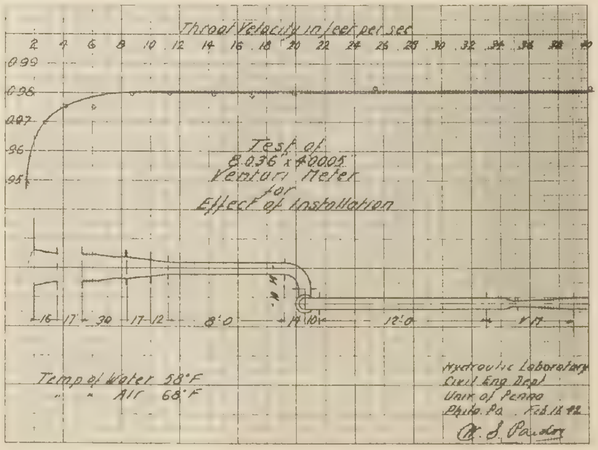

FIG. 30

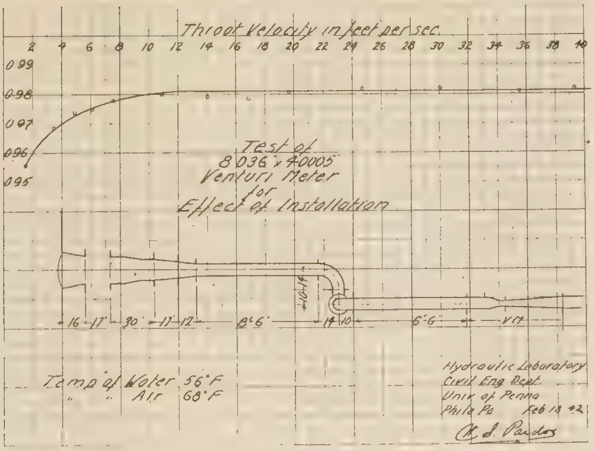

FIG. 31

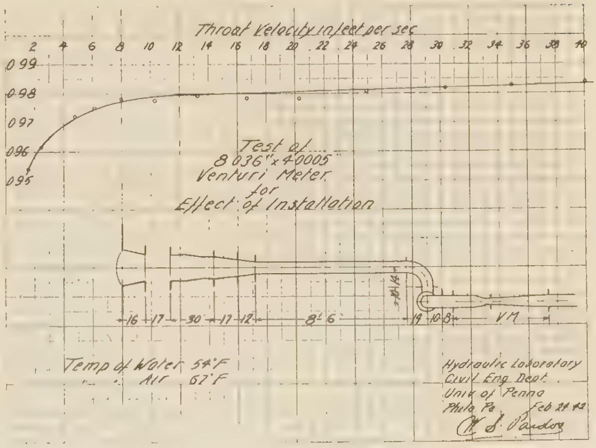

FIG. 34

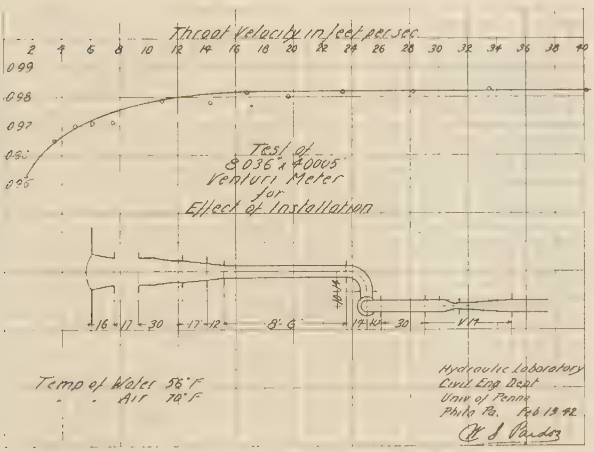

FIG. 32

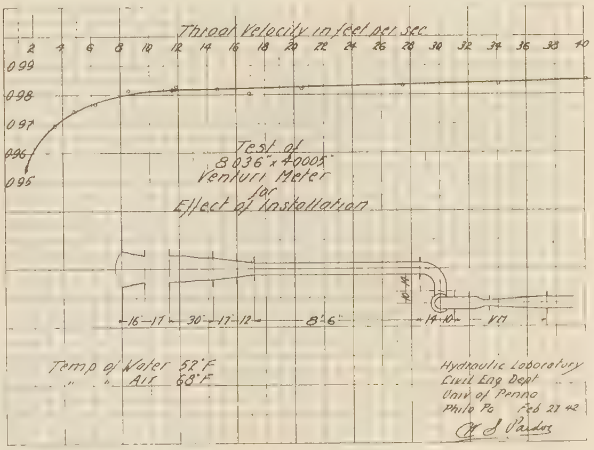

FIG. 35

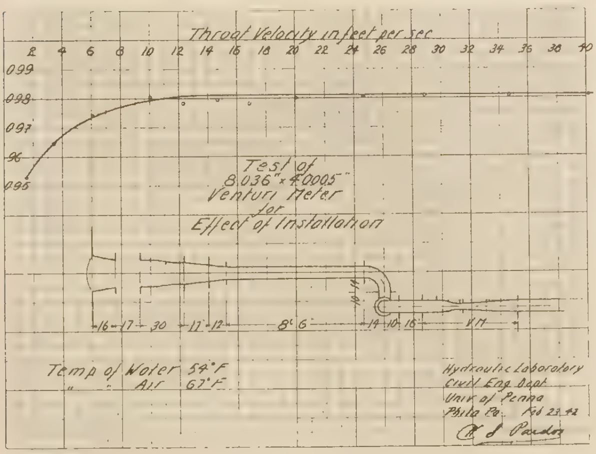

FIG. 33

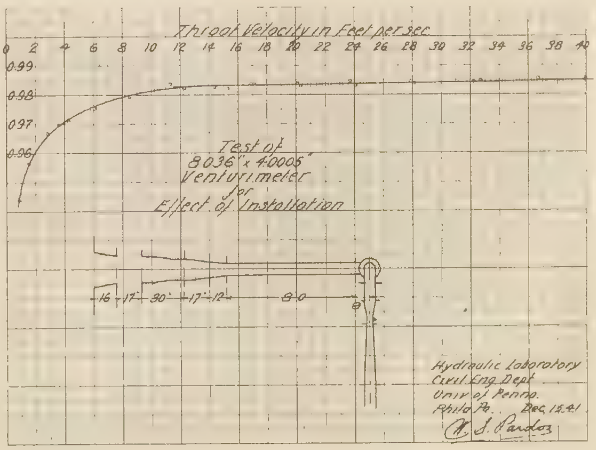

FIG. 36

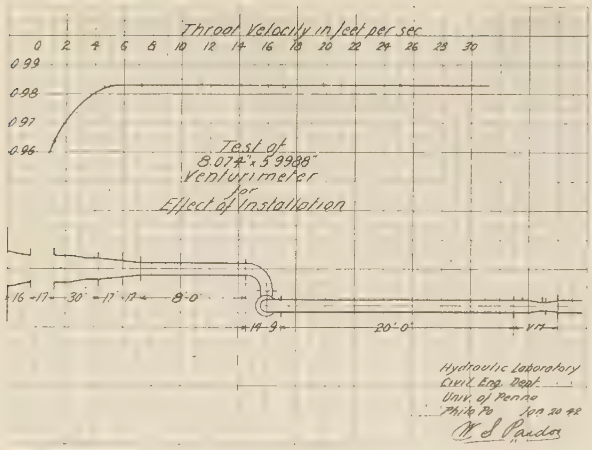

Fig. 37

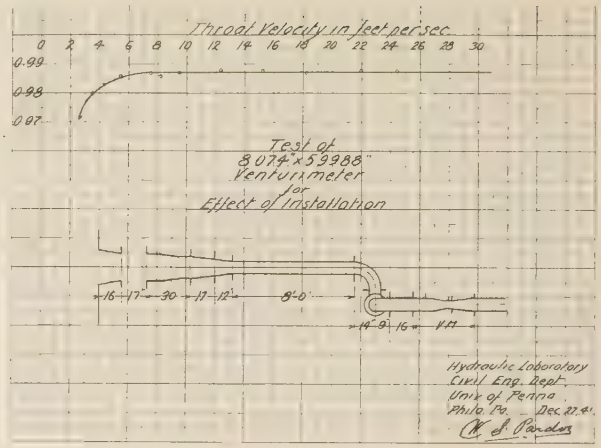

Fig. 40

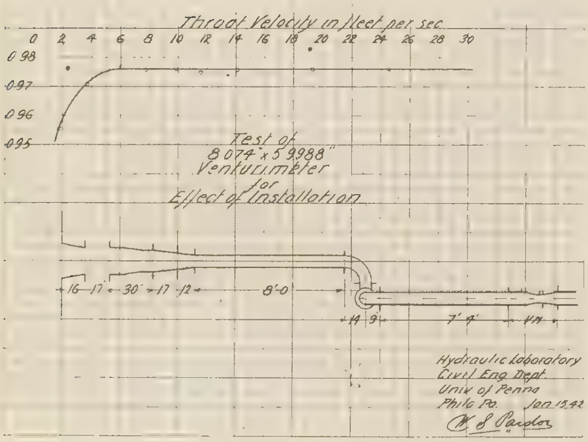

Fig. 38

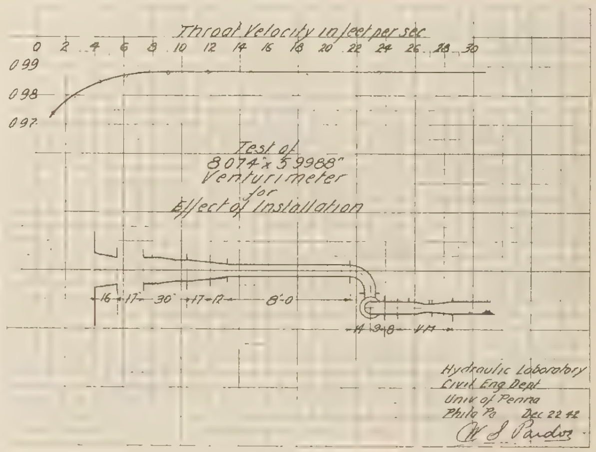

Fig. 41

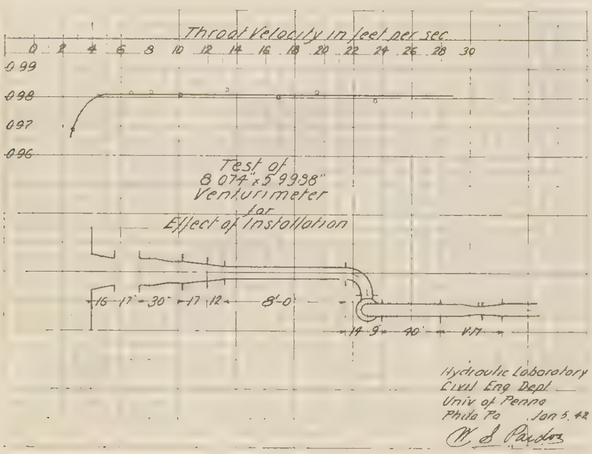

Fig. 39

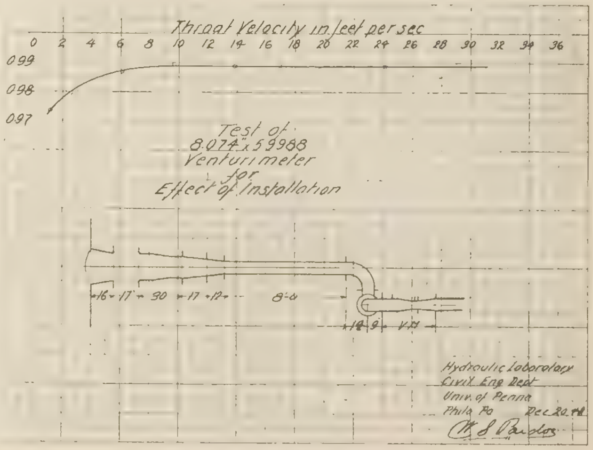

Fig. 42

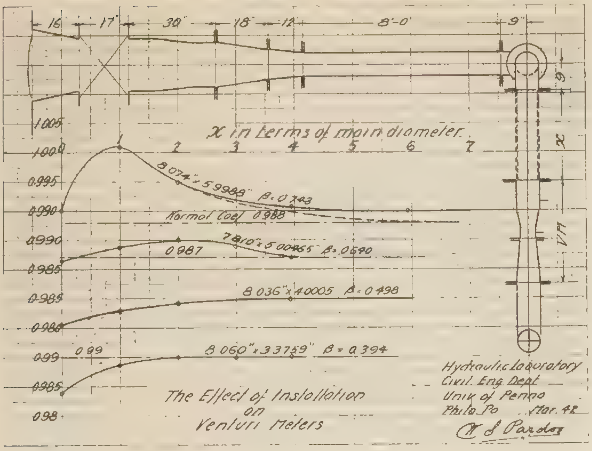

Fig. 43

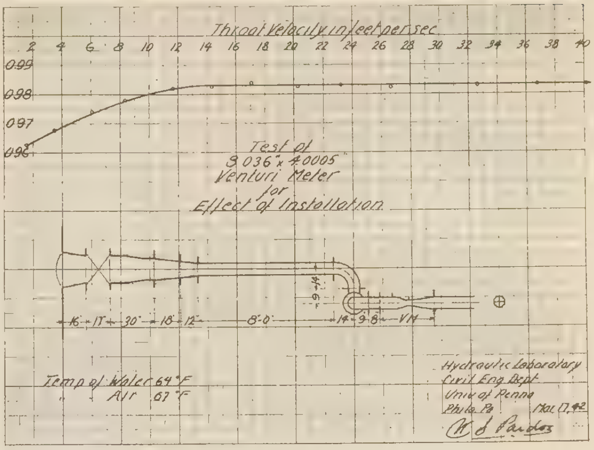

Fig. 46

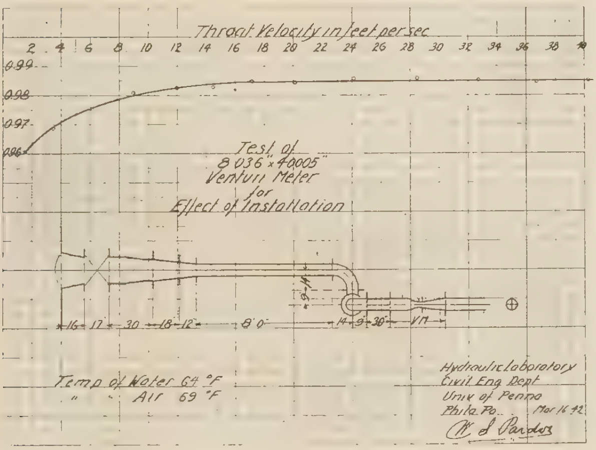

Fig. 44

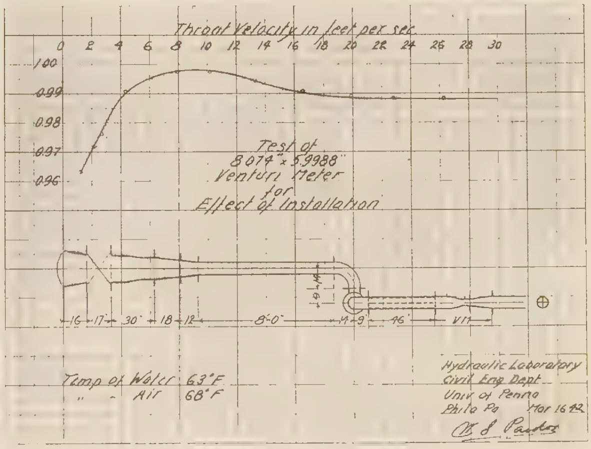

Fig. 47

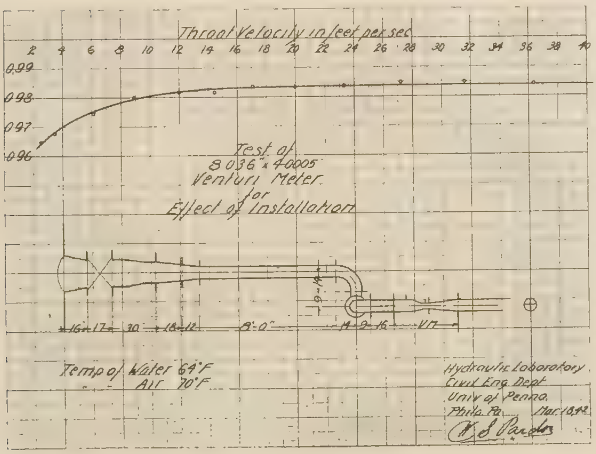

Fig. 45

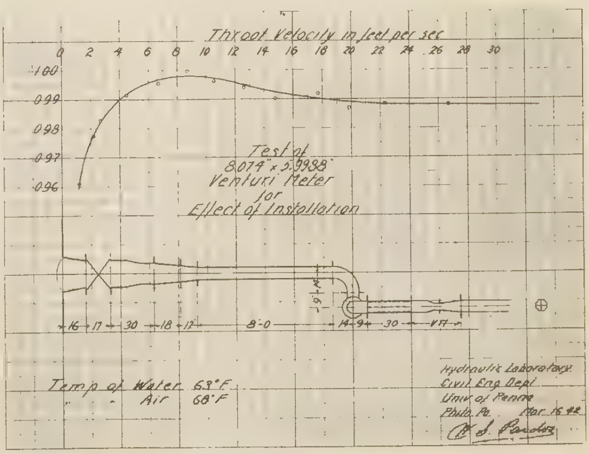

Fig. 48

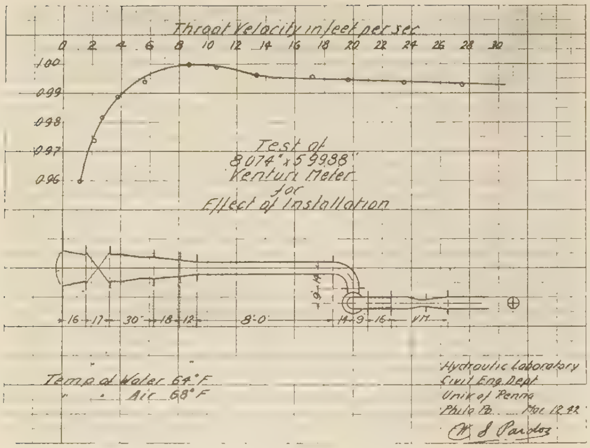

FIG. 49

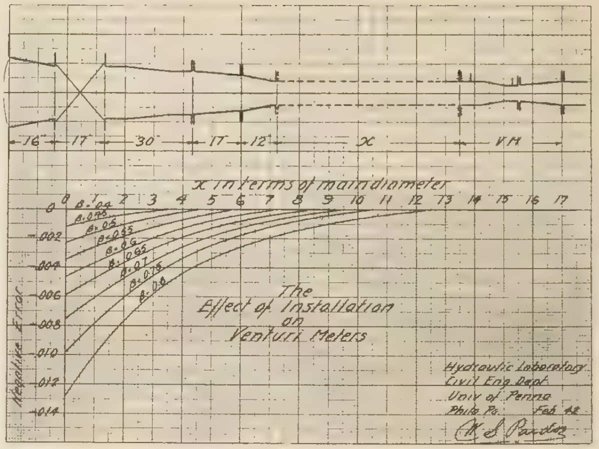

FIG. 52

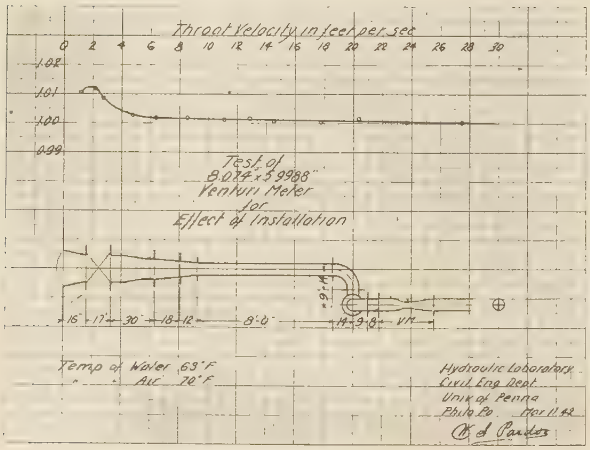

FIG. 50

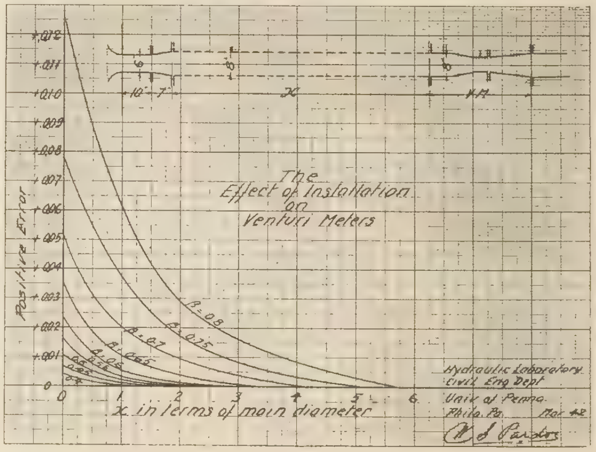

FIG. 53

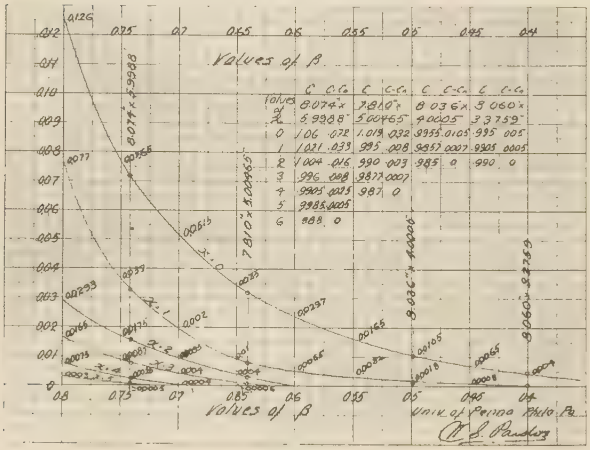

FIG. 51

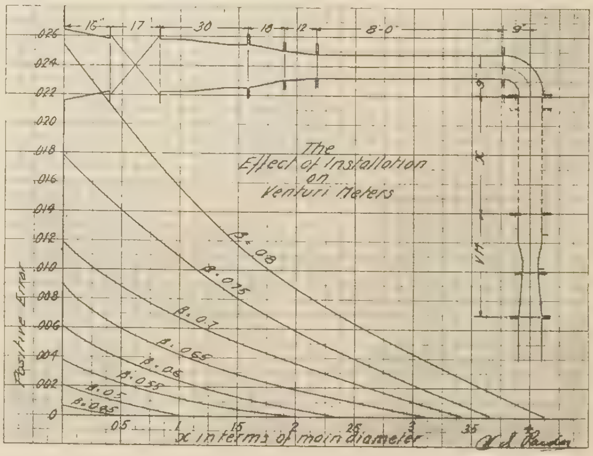

FIG. 54

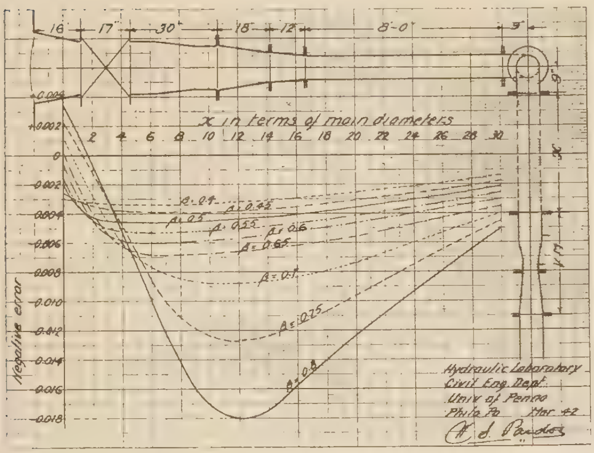

FIG. 55

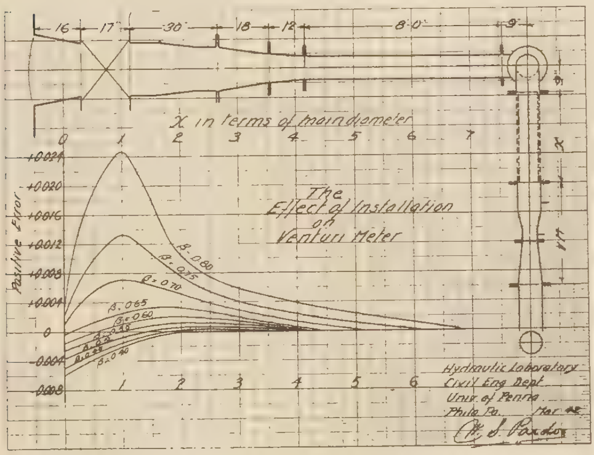

FIG. 56

Discussion

E. S. SMITH.[2] The author has done considerable testing work on the over-all coefficients for Venturi tubes and, in his penultimate paper, presented data showing temperature errors under certain conditions. This writer's discussion of that paper was intended to provide several tests for the Goffian hypothesis released by the author. However, his closure rejected the tests and hypothesis without providing any rational explanation for the errors. His present "final" paper fails to clear up this point. The writer accordingly urges the author to dispose of the cause for the error in his closure to this paper as necessary to its claimed finality.

L. K. SPINK.[3] The distinction made by the author between vortex flow and abnormal velocity distribution is of particular interest at this time. A review of the recommendations on the use of straightening vanes with this distinction in mind might be in order now when the saving of critical material is of such importance.

It is observed that the author used straightening vanes only to eliminate the vortex. There is no apparent reason why a straightening vane should correct an abnormal velocity traverse, unless constructed to offer the highest frictional resistance at the points where the velocity is abnormally high. Since special designs to fit specific flow patterns are not feasible for general commercial practice, there is a reasonable question whether the use of vanes should be recommended except to eliminate the vortex type of disturbance.

The American Gas Association–A.S.M.E. tests at Buffalo indicated that the error due to the swage, which causes an abnormal velocity traverse, was actually increased by the use of a straightening vane. An article[4] by H. S. Bean shows, in some instances, a greater approach run to be required with vanes than without, which adds further doubt of the advisability of using vanes on certain types of disturbances.

The analysis by Mr. Bean on the subject of installation effects emphasizes the inconsistency of the data by various experimenters. Some of this might be due to discrepancies in the arrangement, spacing, and physical design of the disturbance-producing fittings. In some instances, irregularities in the interior of the pipe and fittings or differences in alignment of the elbows might result in a change to vortex flow. The distinction observed by the author prompts the suggestion that better co-ordination of disturbance test results might be obtained by observing the type of flow by some means such as the Schlieren apparatus or glass-pipe sections rather than grouping all disturbances (for instance, those consisting of two elbows in the same plane) under the same classification. A partially closed gate valve is apparently capable of producing a vortex as well as an abnormal velocity distribution.

Item 4 of the author's summary should read "at high values of β." The value of β for the 8.06-in. $\times$ 3.3759-in. Venturi should be 0.419, as indicated in previous reports. The fact that the value of 0.394 was used in the compilation of the final curves has little effect, however, on the shape of the curves.

Loose use of the term "error" should be avoided in information presented to an organization composed largely of commercial users of head meters. The user purchases a primary device, the coefficient of which has been determined under conditions which permit accurate measurement, If the commercial installation results in a low actual coefficient, measurement by the predetermined coefficient is high; hence the error is positive.

We find no velocity-traverse data or other evidence to support the conclusion that the first error caused by elbows at right angles is due to a low "pipe factor," and that the error observed at greater distances is due to formation of a vortex. Although this conclusion appears logical, we believe that such statements should be accompanied by suitable proof.

The writer's experience has indicated that disturbed flows are usually accompanied by unsteady differentials. These require a damping means or shutoff device to obtain satisfactory readings on a hook-gage such as described in the 1936 report. We should be interested in further details of this portion of the test equipment.

We should also be interested to know the reason for basing the analysis on a throat velocity of 17 fps. The test data provide information for analysis of the effect of Reynolds number on installation errors, if the lower velocity readings are of equal reliability.

R. E. SPRENKLE.[5] The author has done industry a great service in exploding the commonly accepted idea that Venturi tubes can be used in any piping arrangement with no diminution of

[2] Eclipse Aviation, Bendix, N. J. Mem. A.S.M.E.

[3] The Foxboro Company, Foxboro, Mass.

[4] "Installation Requirements for Head Meters," by H. S. Bean, *Heating Piping & Air Conditioning*, vol. 13, 1941, pp. 744–746.

[5] Mechanical Engineer, Bailey Meter Company, Cleveland, Ohio. Mem. A.S.M.E.

metering accuracy. His composite curves, Figs. 52 to 56, inclusive, show that Venturis are as vulnerable as any other type of primary element to the arrangement of piping preceding them, and that due care must be experienced in placing them in such piping systems as will give accurate and dependable results.

It would appear that a reducer, Fig. 52, requires about twice the length of straight pipe as an expander, Fig. 53. Is not most of this decrease of length with the expander due to the use of a bellmouthed inlet which almost immediately precedes the expander? A series of air-flow tests conducted in the summer of 1942 under the writer's supervision, using a bellmouthed atmospheric intake, showed that best accuracy was obtained with from 5 to 6 diameters of straight pipe following the bellmouth and preceding the orifice. In these tests, no expander was used.

Some peculiar results are shown for the combination of three bends at right angles to each other, Figs. 29 to 35, inclusive, and Figs. 37 to 42, inclusive. These indicate that for the 8.036-in. × 4.0005-in. Venturi tube, as good accuracy is obtained with the length of 30 in. as with the length of 18 ft 6 in. immediately preceding the Venturi and, further, that the error at these lengths is about 0.3 per cent. One would have expected much larger errors with the shorter length of piping. Further, with the 8.074-in. × 5.9998-in. tube, there is no error with the Venturi immediately following the third bend, whereas a definite error of about 0.5 per cent is shown with the 20 ft of straight pipe preceding the Venturi. Is there any logical explanation for these seemingly inconsistent results?

It is never good metering practice to install a straightening vane immediately following a bend or immediately preceding any primary element, as indicated in Figs. 43 to 50, inclusive. There should be at least several diameters between the bend and the entrance to the vane, as well as between the exit of the vane and the primary element. It is not surprising, therefore, to note the peculiarly shaped calibration curves shown in Figs. 43 and 47 to 50, inclusive, for the high-ratio Venturi tube with this particular piping arrangement.

The reversal of errors by the use of a straightening vane following two bends at right angles to each other, as shown in Figs. 55 and 56, is indeed interesting. Are we correct in assuming that, in the case of the straightening-vane installation, Fig. 56, the over-all length of the straightening vane was in each instance equal to the total distance *X*?

Authors' Closure

Mr. E. S. Smith first suggested convection currents as a contributing cause. The author put the 8-in. × 3³/₈-in. meter in a vertical line, as shown in Fig. 57 of this closure. The curve still passed through 0.969 at 3 fps, indicating no effect. Now he seems peeved that the author, who closed the discussion of his "penultimate" paper, did not follow his suggestion of exploring the sublaminar layer with a Pitot tube and a thermocouple. As the laminar boundary layer is about 0.0006 ft thick, the job of getting temperature and velocity traverses of the sublaminar boundary layer is to say the least a little difficult, if not impossible. Nevertheless, if Mr. Smith will supply the Pitot tube and thermocouple, the author will be glad to assist him in making the traverses. It seems to the author the explanation as given in the original paper is rational and, in his closure, he computed the inside-wall temperature and found that the temperature difference varied inversely to the velocity, as one might expect.

Mr. L. K. Spink is quite right in his conclusion that straightening vanes should be used to eliminate vortex flow only. They should not be used to correct an abnormal velocity traverse; in fact they may perpetrate the abnormality. The author considers two cross-straightening vanes as good as any and these are most easily constructed. Mr. Spink called for Pitot-tube traverses at

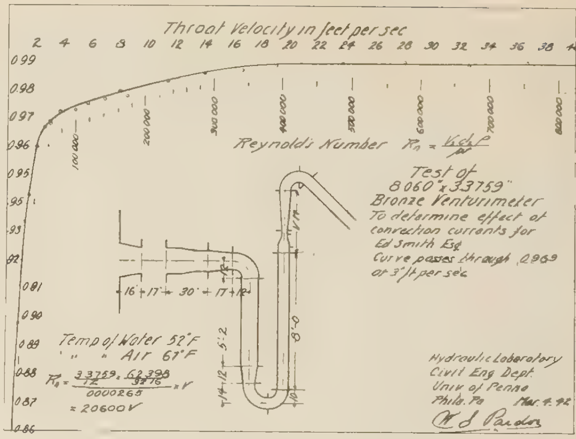

FIG. 57

the outlet of the two elbows in different planes to prove an explanation he considers "logical." Some time in the blue future, he may find some unused time to devote to this question.

The high values of the coefficient for low values of ∝ might be due to the low pipe factor offsetting the effect of the vortex but could also be explained if we assume that the two elbows in planes at right angles produced a temporary forced vortex, i.e., $v \alpha r$ instead of $v \alpha \frac{1}{r}$ as in the free vortex. Using the same method as Mr. R. B. Smith[6]

$$\frac{V_1^2}{2g} + \frac{U_1^2}{2g} + \frac{P_1}{w} = \frac{V_2^2}{2g} + \frac{U_2^2}{2g} + \frac{P_2}{w}$$

$$\left(\frac{P_1}{w} - \frac{P_2}{w}\right) 2g = V_2^2 - V_1^2 + U_2^2 - U_1^2$$

From continuity $r^2 V_2 = r_1^2 V_1$

The angular velocity $\omega = \frac{U_2}{r_2} = \frac{U_1}{r_1}$

$$2gh_v = V_2^2 \left[1 - \left(\frac{r_2}{r_1}\right)^4\right] + U_1^2 \left[\left(\frac{r_2}{r_1}\right)^2 - 1\right]$$

Let $U_1 = V_1 \cot \alpha = \left(\frac{r_2}{r_1}\right)^2 V_2 \cot \alpha$

from which

$$2gh_v = V_2^2 \left[1 - \left(\frac{r_2}{r_1}\right)^4\right] \left[\frac{1 - \left(\frac{r_2}{r_1}\right)^4 \cot^2 \alpha}{1 + \left(\frac{r_2}{r}\right)^2}\right]$$

$$V_2 = \frac{C}{\sqrt{\left[1 - \left(\frac{r_2}{r_1}\right)^4\right] \left[\frac{1 - \left(\frac{r_0}{r_1}\right) \cot^2 \alpha}{1 + \left(\frac{r_2}{r_1}\right)^2}\right]}}$$

$$\sqrt{2gh_v} = \frac{Cw}{\sqrt{\left[1 - \left(\frac{r_2}{r_1}\right)^4\right]}} \sqrt{2gh_v}$$

[6] "The Effect of Insulation on Coefficients of Venturi Meters," by W. J. Pardoe; discussion by R. B. Smith, Trans. A.S.M.E., vol. 59, 1937, pp. 750–751.

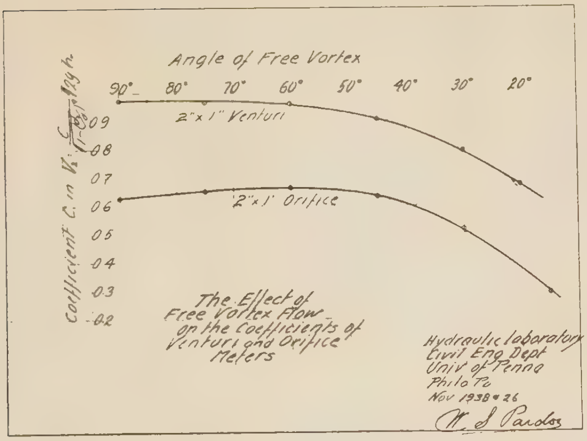

FIG. 58

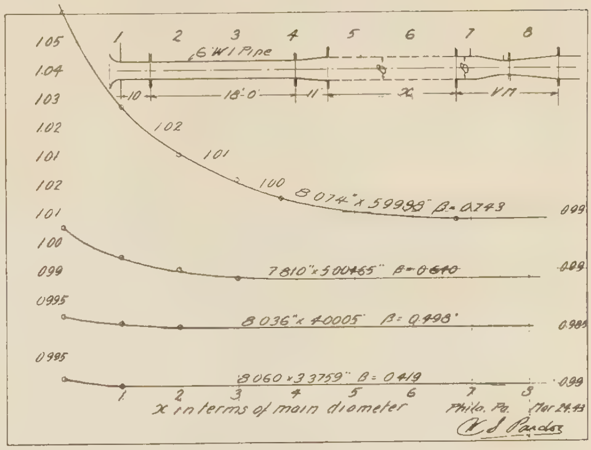

FIG. 59

or

$$\frac{Cw}{C} = \sqrt{\left[\frac{1 + \left(\frac{r_2}{r_1}\right)^2}{1 + \left(\frac{r_2}{r_1}\right)^2 - \left(\frac{r_2}{r_1}\right)^4 \cot^2 \alpha}\right]}$$

From which it is evident that Cw increases as α decreases from 90 deg. Hence, if the elbows produce a forced vortex, there will be a temporary high coefficient. The forced vortex soon changes to a free vortex, and the coefficient falls below normal, as indicated in Fig. 55 of the paper.

The Hook gage is used only up to 0.6 ft. As this paper deals only with the flat part of the curve, the Hook gage reading did not effect the result. Gage readings do fluctuate more than for normal flow but not abnormally, as is indicated by the deviations of the points from the curve.

The velocity 17 fps is a mean between 9 and 25 fps, a usual range for meters.

Mr. R. E. Sprenkle has it just the wrong way around. The author has never remotely suggested that the Venturi tube is not affected by installation. The late Mr. Spitzglass, on one occasion, stated that the pipe orifice was not affected by a free vortex. The author, believing otherwise, made the tests shown in Fig. 58 of this closure. It will be observed that the coefficient of the Venturi tube falls as the angle α decreases and that of the pipe orifice rises, becomes normal at $\alpha = 45$ deg, and then falls off as in the case of the Venturi tube. This suggests the inadvisability of using the information in this paper for pipe orifices.

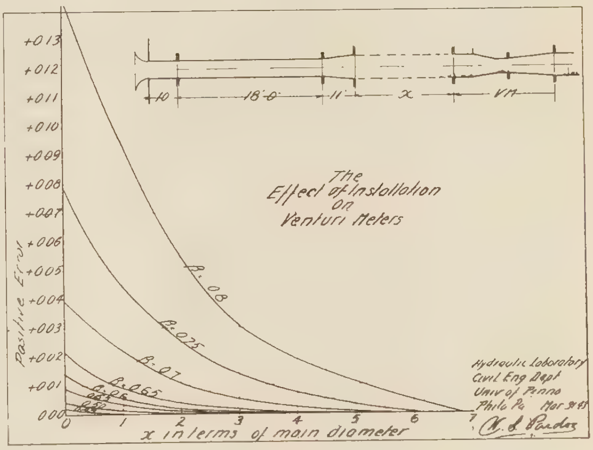

FIG. 60

The author thanks Mr. Sprenkle for drawing his attention to the effect of the bell mouth and hence has conducted the tests of installation shown in Figs. 59 and 60 of this closure. The necessary length is evidently increased from 5 to 7 diam for $\beta = 0.75$. When there is plenty of length, the author agrees with Mr. Sprenkle about the location of straightening vanes, but the tests were made to determine the minimum length of X and strengthening vane to obtain the normal coefficient. The cross-straightening vane in all cases was equal to the distance X.

Results of Tests on Volumeters for Liquid Hydrocarbons

By R. J. S. PIGOTT,[1] E. E. AMBROSIUS,[2] and E. W. JACOBSON[3]

This paper gives the results of the continuation of a test program begun in May, 1939, by the A.S.M.E. Special Research Committee on Fluid Meters, to determine the characteristics of displacement-type mechanical meters for measuring liquid hydrocarbons. Subsequently a Special Subcommittee to guide the volumeter research work was appointed and later expanded to include members from the Petroleum Institute and the meter-manufacturing companies. Meter-calibration test work, which is the first phase of the volumeter research program now under way, has been virtually completed. The essentials of the preliminary study of test results are contained in this paper. Later complete supporting data for the results will be given. The test work conducted shows that displacement meters are subject to viscosity and temperature effects, and if close measurement is to be obtained, suitable calibration equipment must be provided to test each meter at the viscosity, near the temperature, and throughout the flow range at which it is to be operated.

THE A.S.M.E. Special Research Committe on Fluid Meters started a test program in May, 1939, to determine the characteristics of displacement-type mechanical meters for measuring liquid hydrocarbons. The need for this work has grown in importance as the work has proceeded. Because of the increased interest of the American Petroleum Institute, a special Joint Committee of the two societies was formed to guide the volumeter research program. Recently, this committee has been expanded to include representatives of the various meter-manufacturing companies interested. The meter-calibration test work, which is the first phase of the volumeter research program, has been essentially completed and a preliminary study of the results has been made. There is urgent need that pertinent information from this preliminary study be disclosed and it is for this purpose that this paper is presented. Because of the great amount of information to be drawn from the preliminary study and the voluminous nature of the supporting data, for the sake of brevity no supporting data are included in this paper. Instead, the committee intends to publish the entire data in the form of a report later.

Manufacturers contributed 22 volumeters for the tests. Table 1 gives the number and range of the meters in each classification.

Each meter was tested for accuracy of measurement and pressure absorption through its entire flow range with three different oils at two temperatures for each liquid temperature. Table 2 gives data on the oils and gasoline used in the tests.

TABLE 1 NUMBER AND RANGE OF VOLUMETERS

Sealing type	Design type	Number tested	Tested range, gpm
Film-sealed	Cylindrical piston	1	0–100
		1	0–325
	Nutating piston	2	0–160
		1	0–450
	Square piston	1	0–200
		1	0–350
	Oscillating piston	1	0–125
		1	0–300
		1	0–325
		1	0–400
	Bucket-vane rotary	1	0–120
		1	0–450
	Straight-vane rotary	1	0–350
	Turbine-wheel rotary	1	0–250
		1	0–400
	Two-lobe rotary	1	0–425
	Three-lobe rotary	1	0–200
	Three-lobe rotary	1	0–425
Pack-sealed	Four-piston single-acting	1	0–120
	Single-piston double-acting	1	0–120
	Five-piston single-acting	1	0– 60

TABLE 2 DATA ON OILS AND GASOLINE USED IN TESTS

Order tested	Nominal viscosity	Viscosity at given temperature: Centipoises	Deg F
1	Heavy oil	199.4	84
		17.24	176.5
2	Light oil	3.002	84
		1.46	176.5
3	Medium oil	16.61	84
		3.84	176.5
4	Gasoline (specific gravity at 60/60 = 0.715)	0.440	68
		0.375	95

The oil tests were conducted on open weigh-tank-type apparatus at the University of Oklahoma, Norman, Okla. This equipment and its operation were described in a paper[4] presented before the American Petroleum Institute in 1939. The gasoline tests were conducted on closed volumetric water displacement-type apparatus at the U. S. Naval Experiment Station, Annapolis, Md. The type of equipment used for the gasoline tests was described in a paper[5] presented before this society in 1941 at Kansas City.

Conclusions derived from the preliminary study of the calibration and absorption curves are presented under general headings depending upon the major classification of subject matter.

General

All the volumeters tested show definite viscosity and temperature effects which must be considered if close measurement within 0.1 per cent error is desired. Most of the meters will measure within ± 1 per cent on oils above 1 centipoise viscosity, if viscosity and temperature effects are neglected, and the flow rate is held between 15 per cent and 85 per cent of their rated capacity. A few of the meters will measure within ± 0.5 per cent under the foregoing conditions. Nearly all meters show wide spread of error between the oils and gasoline, especially at flow rates below 20 per cent of rated capacity. All meters should be calibrated on the fluid at the viscosity and near the temperature at which it is to be used, if measurement within 0.1 or 0.2 per cent is to be possible.

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Contributed by the Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[4] "Fluid Meter Research at The University of Oklahoma," by W. H. Carson and E. E. Ambrosius, *Oil and Gas Journal*, vol. 39, November 14, 1940, pp. 143, 144, 146, 149, 150, 153, 154.

[5] "Calibration of Displacement Meters on Volatile Liquid Petroleum Fractions," by E. W. Jacobson, Trans. A.S.M.E., vol. 63, 1941, pp. 701–704.

Meter error is definitely a function of flow rate. Most meters show unstable performance at rates of flow below 15 per cent of their rated capacity, with large errors in excess. Even the best measuring meters have greater slip in the lower 10 per cent of their flow range. As the rate of flow increases above 10 per cent, some meters have rising error curves, while others have falling error curves. All meters should be calibrated at flow rates approaching the operating rates if measurement within 0.1 per cent is to be obtained. Most volumeters should be operated at rates of flow above 15 per cent of their rated-flow range if consistent predictable performance is to be expected.

Some of the meters showed very marked changes in measuring characteristics with different temperatures and viscosities. Some meters are better measurers on one temperature-and-viscosity combination than on other combinations. Some meters are definitely heavy-oil meters; others do better on the lighter oils. Only a few meters show a low error spread in the upper 85 per cent of the rated-flow range with gasoline.

If a meter has a combination of poor sealing and high absorption, it will have a rising error curve at higher flow rates. If the absorption is low, even with average sealing, the error curves are relatively good. The term absorption is used here as commonly in the meter industry and means pressure loss across the meter to attain flow. A free-running meter may not be a consistent measurer since the error curve of such a meter is sensitive to the register load and change in internal friction. A slight change in register load, internal friction, etc., will have a greater percentage effect than if the absorption of the meter were higher.

There was evidence during the testing that most meters have a wearing-in period during which their measuring characteristics are unstable. After a short period of operation, the performance became stable. This should be considered when new meters are being used.

Meters should be carefully designed to avoid air-trapping. Test experience has shown that air can be trapped in some meters with unpredictable results.

Viscosity Effects

As pointed out in a previous paper,[6] the basic design of displacement meters is divided into two classes in regard to type of sealing, namely, film- or capillary-sealed meters, and pack-sealed meters. The film-sealed metering elements depend upon very close working clearances between moving and stationary parts filled with a thin film of the fluid being metered. The pack-sealed metering elements make use of leather, composition, or metallic ring-packed pistons. In the film-sealed meter, the viscosity of the liquid being metered affects not only the slip but also the absorption (pressure drop across the meter). In general, the absorption increases with increase in viscosity because of the increased friction in the sealing film as well as greater liquid resistance to flow through the meter; the slip decreases with increase in viscosity of the metered fluid because of the better sealing effect of the higher-viscosity film. All but three of the meters tested were of the film-sealed type and exhibited general characteristics as described.

In many of the meters tested, the viscosity of the heavy oil controlled the shape and position of the error curves more than the temperature. As the viscosity of the fluid becomes less, the effect of viscosity change on the error curves becomes less, and the influence of temperature then becomes the controlling factor in these meters.

Effects of viscosity are very pronounced at rates below 15 per cent of rated capacity. At rates above 15 per cent and below 85 per cent, viscous effects are still evident, although to a lesser degree than below 15 per cent of rated capacity.

A great many meters have a relatively narrow range at some point between 15 per cent and 85 per cent of rated capacity where the viscosity has minimum effect. This is obviously the operating rate near which the meter should be used to give best results if the meter is to be used to measure fluids at fairly wide variance in viscosity such as in a products pipe line.

For a given sealing condition in a meter, there is an optimum value of viscosity below which slip will increase rapidly.

At constant flow rate and constant temperature, the absorption has a tendency to drop sharply as the viscosity is decreased. At constant absorption and temperature, the flow increases as the viscosity is decreased. At constant absorption and temperature, there is a critical value of viscosity below which the flow increases very rapidly with decrease in viscosity.

If close measurement is to be secured, meters should be calibrated on fluids of the same viscosity as those to be measured. This becomes increasingly important for fluids of viscosity below 1 centipoise. Meters for measuring gasoline, propane, etc. should be calibrated on these liquids if measurement closer than 0.5 per cent is expected.

Temperature Effects

Temperature has an effect on all meters which is difficult to evaluate, since it is dependent upon clearances, types of materials used, and inherent design features. The effect of temperature is about as varied as there are meters, some designs being much more sensitive to temperature changes than others. In general, the poorer sealed meters are less sensitive than the well-sealed meters. Meters should be calibrated on fluids near the temperature at which they are to operate if close measurement is desired.

The general effect of an increase in temperature is to make the meter seal better, as the meters are now constructed. This is particularly true in the meters using metals of different expansion coefficients. It may be entirely possible by careful design to largely lessen the effect of temperature as a factor in meter performance.

Absorption

Absorption is a function of flow rate. The higher the flow rate, the higher the absorption. For a given flow rate, the higher the viscosity, the higher the absorption. The effect becomes very pronounced with high-viscosity liquid. As the viscosity is decreased, a point is reached where the absorption becomes independent of the viscosity. If, however, the viscosity is decreased to a point where the lubrication is lost, the absorption in some meters actually increases, and the error curve rises.

Most of the meters show that the absorption does not reach zero as the flow rate approaches zero. This is apparently due to the fact that the coefficient of friction of all rubbing surfaces increases as speed decreases.

While holding the flow rate and viscosity constant, the absorption depends upon the temperature. For the more viscous oils, the higher the temperature, the higher the absorption becomes probably due to change in clearances.

Size Effects

It is an interesting fact that meters of the same design and make but of different size have nearly the same absorption at their maximum capacity. For instance, a meter of 100 gpm maximum capacity and of the same design and make as a meter of 300 gpm maximum capacity will have nearly the same absorption at 100 gpm as the larger meter at 300 gpm. This indicates rather good geometrical similarity in the sizes.

[6] "Some Fundamentals in the Design and Application of Displacement Meters," by E. W. Jacobson, *Oil and Gas Journal*, June 20, 1940, pp. 36, 39, 40.

In general for meters of different size, yet of the same design and make, the larger meter has better error-curve characteristics than the smaller. This is undoubtedly due in the larger meter to the load from friction, register, etc. being a smaller proportion of the power of the metering element than in the smaller meter. It follows that for the larger meter, less effect of change of register load through wear, corrosion, etc. will be evident. Size effects should be considered when selecting a meter for a particular application.

Index of Register

The meter index or register needs discussion because proper calibration and consistent performance during continued use depends so much upon a properly designed mechanism for recording the motion of the metering element. It has been pointed out in a previous paper[6] the great importance of having a register which does not overload the metering element or change load materially with use. In the light-running meters, the register load may be the greater portion of the load on the metering element and a slight change in the friction in the register caused by corrosion, dust, lubrication failure, and the like may cause the light-running meter to change calibration sharply. A meter having a relatively high absorption will ordinarily not be so sensitive to a similar small change in register load. When selecting a meter for a particular application, the extent of the register load should be considered. Both makers and users of displacement meters should give serious consideration to register design and application as affecting meter performance.

Since meter performance is influenced by register load, each meter should be calibrated and operated with the same register; a meter should not be calibrated with one register and put into service with another register. A meter can be calibrated only as close as its register can be read. If a meter is to be calibrated to 0.1 per cent, the register must be capable of being read at least to 0.1 per cent of the volume passed through the meter on each calibration run, and preferably less. For instance, if a 300-gal prover is being used, the meter register must be capable of being read to 0.1 per cent of 300 gal, or 0.3 gal, preferably to 0.2 or 0.1 gal. During the test work, it was difficult to get smooth or consistent results on some meters because the index could not be read closely enough.

Some meters are equipped with cams, slip clutches, etc., for making adjustments in the drive connection between the metering element and the register to change calibration. For calibration purposes, these should be eliminated or constructed in such a manner that their influence will not be noticed when a meter is calibrated on small quantities. Positive drive between metering element and register is essential if consistent performance and close calibration are to be obtained.

So-called temperature-compensating devices should be avoided as they only add to the complication of the registering mechanism and cannot possibly compensate adequately for temperature effects on both the metered fluid and the meter itself. The test results show that meters of the same design show different temperature effects on the meter characteristics, so that, to correct for these alone, each meter would need a separately designed and adjusted temperature compensator if close measurement is desired. The best temperature compensation is to calibrate the meter at temperatures which are near those at which it is to be operated.

Calibration

Much of the success in use of displacement-type meters depends upon the careful design and use of meter-calibrating equipment. Displacement meters for measuring liquid-petroleum fractions are by the nature of their use precision instruments and must be treated as such. They are mechanical in nature and dependent upon careful design, correct material combinations, and intelligent handling in use to operate with characteristics necessary to give the close measurement required. This is just as true for fluid measurement by displacement meters as it is for pressure measurement by precision gages, or electrical measurement by precision meters. In most cases, the precision of measurement demanded of displacement meters for gasoline and oil are much closer than that required in any other type of commercial measurement. As the precision of measurement increases, so does the cost of calibration, operation, and maintenance.

It is quite evident from the data that meters for gasoline must be calibrated on gasoline if close measurement is desired. One large user of displacement meters in pipe-line service has reported to the committee that slight differences are found in calibration from one gasoline to another. Calibration curves for a meter on oils cannot be used for gasoline measurement and vice versa.

Included in the test work was a short series of tests to determine relative accuracy of "start-and-stop" proving against "on-the-run" proving. These tests showed there was no measurable difference in the accuracy of the two methods when the same care was used in making the tests.

The same thing is true for use of volumetric provers as compared to weigh-tank provers. If there is careful design, construction, and use of either type, the precision of results is the same. If meters are to be calibrated to close precision, both the meter-register readings and the measured volumes or weights must be determined with better precision than the required measuring precision.

The test work has shown that displacement meters are subject to viscosity and temperature effects and, if close measurement is to be obtained, suitable calibration equipment must be provided to test each meter at the viscosity, near the temperature, and throughout the range of flow rates at which it is to be operated. If this equipment is available and each meter is carefully tested, the differences in measuring characteristics between one meter design and another are eliminated from the situation, and any design will be found satisfactory provided it has reasonably good wear characteristics and does not change very much from one calibration to the next.

Acknowledgment

This paper is presented as a part of the program of the A.S.M.E. Fluid Meters Committee. The committee wishes to acknowledge and thank the many contributors for the support to the extended test program which has made this paper possible. Of special mention are the staffs of the National Bureau of Standards, Washington, D. C., and the University of Oklahoma, Norman, Okla., for their technical guidance; the U. S. Works Progress Administration for its labor contribution; the Engineering Foundation, the oil companies and meter-manufacturing companies which contributed equipment and funds; the U. S. Naval Experiment Station and the University of Oklahoma for the use of facilities; and the American Petroleum Institute for its co-operation and joint sponsorship.

Developments in the Measuring of Pulsating Flows With Inferential-Head Meters

By S. R. BEITLER,[1] E. J. LINDAHL,[2] and H. B. McNICHOLS[3]

The measurement of pulsating flows with inferential-head meters has been rather undependable because of the fact that it was known that these measurements might be greatly in error due to pulsation, and no satisfactory instrument was available for measuring the intensity of the pulsation waves. This paper tells of two types of instruments which have been developed for measuring the frequency and amplitude, or the amplitude alone, of the differential pulsation wave which is the difference between the amplitude of the upstream and downstream pulsations and tells how these instruments are used. The results of considerable research are given to show how meter error varies with differential pressure and amplitude of pulsation waves.

In the introduction of a previous paper in 1938, by one of the authors,[4] it was suggested that the purpose was to inspire additional investigations on the effect of pulsations on orifice meters. Following that presentation, supervision of the investigation was assumed by a subcommittee of the A.S.M.E. Special Research Committee on Fluid Meters under the chairmanship of T. H. Kerr, who placed J. E. Overbeck, of the Columbia Engineering Corporation, and the senior author of the present paper in direct charge of the work. Arrangements were made to conduct tests during the summer of 1939, and these have been in progress continuously since that time. While there is considerably more to be accomplished, it is felt that a current report on the subject would be of great value to users of meters, and this is the purpose of the present paper.

Since the present method of approaching the problem of measuring pulsating flows is different from that previously employed, the reasoning used in arriving at this method will be outlined.

Any inferential head meter will measure correctly, if the secondary device will measure accurately the differential pressure across the primary element, and if the average of the square roots of the instantaneous readings can be determined. In the case of steady flow, these operations are comparatively simple but, when the flow is at all unsteady, this is not true.

If the flow is irregular as a result of rapid variations in velocity, the meter is inaccurate because of the fact that most secondary devices are too sluggish to follow the actual changes in differential pressure caused by rapidly changing flow. Furthermore, the differential pressure, indicated by the secondary device, is an approximate average of the maximum and minimum differential pressures. When the square root of this average is taken, the resulting value is higher than the average of the square roots of the instantaneous differentials, and the flow as determined from the meter will be too high.

The tests made by Prof. Horace Judd at Ohio State University in 1922, as well as some of those reported in 1938, where the setup was relatively simple and where it was possible to compute the effect of velocity variations, showed errors sometimes as much as 20 times the error which was due to the variation in velocity. Professor Judd concluded that these large errors were caused by the effect of pressure waves which were carried by the fluid in the pipe.

The preliminary studies made in this present investigation indicated that, in addition to the primary pressure waves in the fluid between the disturbance and the primary element, there are also pressure waves of different amplitude existing at the primary element on the side opposite from the disturbance, and suggested that these waves might be slightly out of time with the waves on the other side, so that the effect of pressure variation is multiplied at times. Then, in addition to the primary waves, there are harmonics which are affected by the size and shape of the conduit on both sides of the primary element. The differential across the primary element, where pulsation exists, is thus a complicated varying pressure made up of many elements.

If the true pressure differential could be measured instantaneously and if the square roots of these true pressures could be averaged, it is probable that a meter would indicate the rate of flow accurately. This would be true because, while the velocity through the primary element does not vary directly with changes in pressure waves, the energy present, due to these waves, will be used either in accelerating or decelerating the fluid passing through the primary element, so that the decelerations should balance the accelerations. The average flow will be indicated by the average of the square roots of the instantaneous differential pressures across the primary element.

In the practice of metering, the secondary element will not measure the true differential because it will not follow the rapidly fluctuating pressure waves. There are lines connecting the primary element which act as reservoirs and change the wave forms between the primary and secondary elements, and it is not practically possible to take the square roots instantaneously. For this reason, it is impossible to calculate the pulsation error produced on any given meter even though the exact differential wave form were known.

Because of the facts just mentioned, it was decided at the beginning of the investigations that all results must be in the direction of trying to find the limits of pulsation which would be permissible without affecting appreciably the accuracy of the meter. It was realized that in order to find such limits it would be necessary to have some method of measuring the pulsation waves, and the first operation was to develop instruments for measuring these waves. It was also decided to try to develop instruments for measuring the pulsations which could be employed by the actual users of the meter and not require the services of experts in their usage.

Pulsation-Measuring Instruments Used

Two types of instruments were developed; the first electrical, the second mechanical. The original electrical instrument used

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[4] "The Effect of Pulsation on Orifice Meters," by S. R. Beitler, Trans. A.S.M.E., vol. 61, 1939, pp. 309–312.

Contributed by Special Research Committee on Fluid Meters and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

was a recording instrument which gave both the shape and amplitude of the pulsation waves, while the later electrical and mechanical instruments developed gave the amplitude of the waves only.

The original electrical instrument consisted of two diaphragms of stainless steel of about 1 in. diam and $^1/_{32}$ in. thick which were connected to both sides of the primary metering element by very short connections and so constituted that a pressure variation would be indicated by a very slight movement of the diaphragms. Piezoelectric crystals were used to indicate the motion of the diaphragms, and the voltage produced was amplified and recorded on a recording voltmeter. The whole apparatus gave a record of the wave form at both the inlet and outlet pressure taps. The recorder was capable of responding to very rapid fluctuations of pressure, and the complete record of the form and amplitude of the pressure variations was made on a high-speed chart. The entire apparatus was constructed of standard types of instruments manufactured by the Brush Development Corporation of Cleveland.

With the equipment as originally used, it was possible to obtain a record of the pressure variations on either side of the primary element. However, a study of these records (along the line of reasoning mentioned) indicated that a knowledge of the differential pressure was desirable, and a special mixing switch was designed which made it possible to secure a record not only of the inlet or outlet pulsation waves but the differential pulsation waves as well. Fig. 1 shows some of the records taken with this instrument under various conditions of pulsation.

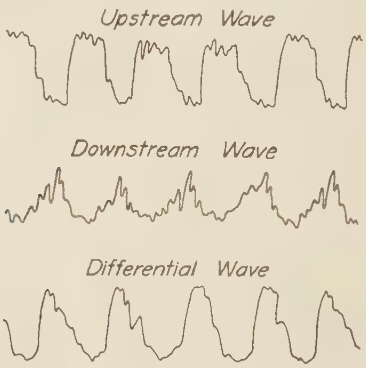

Fig. 1 Typical Pulsation Records Taken With Original Electrical Instrument

A study of these records will show that there is no regularity of pulsation waves, no apparent relation between the shape and amplitude of the waves on the two sides of the primary element, and that the differential wave bears no similarity to the other waves. These records were made at a gas-compressor station where the pulsation was produced by one compressor running at almost constant speed. The meter was placed directly on the outlet of the aftercooler so that the pulsations would be as nearly constant as possible. Because of the fact that the waves were so irregular it would be practically impossible to include the effect of wave form in the study of this problem, so the decision was made to develop a simpler piece of apparatus which would measure the amplitude of the wave only.

A second electrical instrument using piezoelectric crystals was then developed in which the amplitude of the pulsation wave was measured by an indicating voltmeter mounted on the top of a small box containing the diaphragm holders, the amplifier, and the mixing switch, so that the entire apparatus was self-contained. The power for amplification was supplied by dry cells so that the apparatus was portable. This instrument was also developed and manufactured by the Brush Development Corporation.

Operating experience indicated that there was some question about the calibration of the electrical apparatus, and it was decided to construct a mechanical device which would give the amplitude of the differential pulsation waves. This device, shown diagrammatically in Fig. 2, is a variation of the indicator principle which has proved very satisfactory in operation. Most of the later data have been correlated using the readings taken with this mechanical instrument.

In its present form the mechanical pulsameter consists of two cylindrical volumes, A and B, Fig. 2, separated by a lightweight piston C in a cylinder of smaller diameter than the volumes. The volumes and the cylinder have a common axis. Since the piston is sealed by means of two thin flexible diaphragms, D and E, there will be no leakage between the two volumes which are connected to the inlet and outlet pressure connections of the meter, and as closely to the primary element as possible. A coil spring F is mounted in volume A which volume is connected to the high-pressure tap. The tension on this spring can be changed by turning the handwheel G which is outside the instrument. The scale J indicates the elongation of the spring so the force on the piston tending to hold it against its stop K can be determined. Electrical connections are arranged in volume B which is connected to the low-pressure tap, so that any motion of the piston away from its stop K caused by differential pressure will be indicated by a light L located on the outside of the instrument. In operation, the tension of the spring is gradually increased until the light goes out, which indicates that the differential pressure has been equalized by the spring tension. When this occurs, the tension of the spring should represent the differential pressure due to the flow plus the maximum amplitude of the differential pulsation pressures.

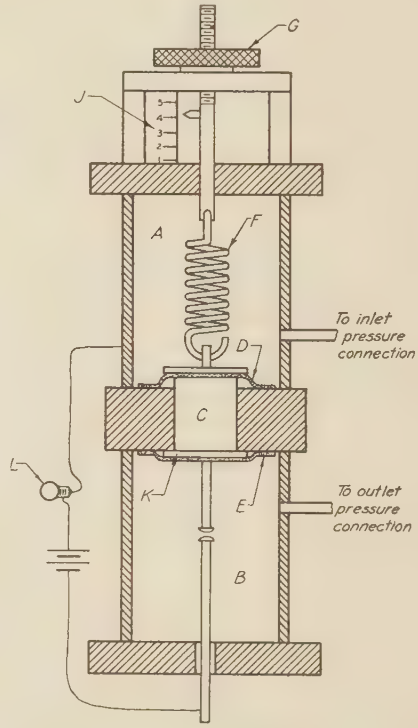

Fig. 2 Diagrammatic View of Mechanical Pulsameter

The instrument spring can be calibrated by comparing its scale readings with any differential gage where there are no pulsations present. When measuring pulsation amplitude the instrument is almost totally unaffected by inertia, since the piston is practically at rest when the determination is made. For practical use all pulsameters must have the same general dimensions as the one used to determine the effect of pulsation, since the shape and dimensions will undoubtedly have some effect on the instrument reading. Pulsameters, such as the one used in the test work, are now being built and sold by the Refinery Supply Company, Tulsa, Okla.

Description of Tests

A special test station was constructed at the Crawford No. 2 Station of the Ohio Fuel Gas Company. This test station was built with two orifice meters in series, one on the discharge and the other on the suction line of a large gas compressor. Gas was supplied to the first orifice directly from the outlet of the aftercooler of one compressor and, after passing through the second orifice meter, was discharged back into the suction of the same compressor. With this arrangement, pulsation could be maintained as nearly constant as possible and the other measurement conditions could be easily changed to give the pressure and rates of flow desired. Throttle valves (plug-type valves) were placed in the line between the two meters and also between the compressor and the inlet and outlet of the test setting. By means of these valves, it was possible to regulate the pressure on either one of the meters so that it would not be subjected either to suction or discharge pulsations from the compressor. This meter was then used as a standard or reference meter and the measurement error due to pulsation was determined by comparing the quantity indicated by the meter subjected to pulsation (or test meter) against that indicated by the standard meter. The test meter was equipped with both "flange" and "pipe" taps, and readings were taken on both simultaneously in order to determine what effect the location of pressure taps might have on the test results.

In order to eliminate the effect of small variations in orifice plates and runway construction, so-called unity tests were run for each arrangement of orifice plates. In making these tests, the flow into and out of the setting was throttled to the extent that there was no pulsation on either meter. By comparing the quantities as measured by the two meters under these conditions, an indication of how close the meters would check each other when no pulsation exists was obtained.

Since changing the speed of the compressor would change the type of pulsation, a by-pass was built into the setting between the inlet and outlet so that it was not necessary to change the speed of the compressor in order to change the rate of flow through the setting. The compressor used was a two-cylinder double-acting Ingersoll-Rand, having $18^1/_2$-in. × 48-in. cylinders and operating at approximately 70 rpm.

Test Results

The curve shown in Fig. 3 was determined from the tests made at this station. In preparing this curve, the results of about 500 tests on orifices with diameter ratios of from 20 per cent to 80 per cent in 2-in., 4-in., and 6-in. lines were studied. As previously explained, it was decided not to attempt to determine any correction factors but to concentrate on the determination of the conditions under which meters would measure accurately. Taking many facts into consideration, it was decided that a meter subject to pulsation would be normally within the limits of commercial accuracy if the error was less than 1 per cent. With spot tests, made with commercial setups and recording gages, it was felt that the probable accuracy of any individual test was within range of ±0.5 per cent. All of the test points were plotted on a large sheet, and the percentage of error was marked on the points. The line was then drawn through the points where the error was approximately 1.5 per cent. Because of the inherent inaccuracy of the tests there were three or four points with an error greater than 1.5 per cent below the curve and ten or twelve points with less than 1 per cent error above the curve. It will be noted that the same curve applies to both "flange" and "pipe" taps.

This curve can then be used to determine whether or not a meter measuring pulsation flow is accurate. It is necessary only to attach the pulsameter to the pressure taps and take a reading on it at the same time that the differential pressure is determined.

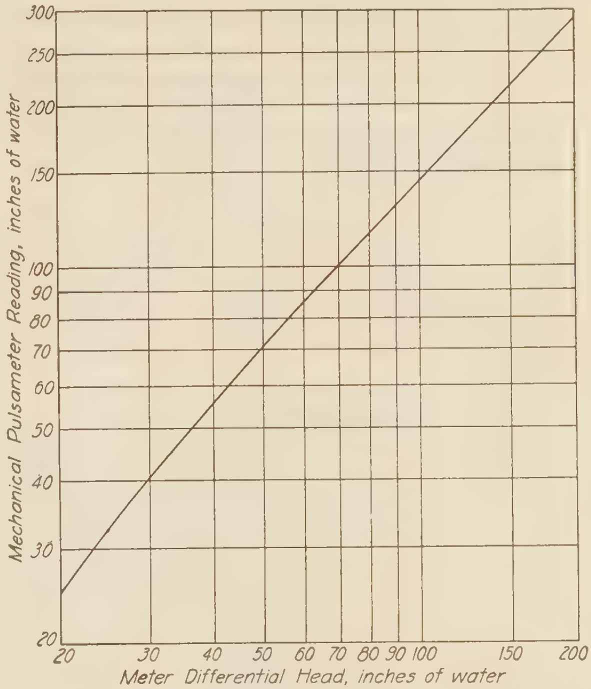

Fig. 3 Preliminary Pulsation-Error-Limit Chart for Flange and Pipe Taps

Method of Using Chart

[(1) Locate pulsameter reading on left margin. (2) Trace to right to perpendicular line representing meter differential on bottom scale. (3) If this point is below diagonal line, pulsation effect is within tolerance of 1 per cent. (4) If this point is above diagonal line, pulsation effect is greater than 1 per cent.]

If the point, when plotted on Fig. 3, falls below the curve, then the pulsation error is less than 1 per cent and the meter is probably within the ranges of ordinary commercial accuracy. If the point falls above the curve, then the pulsation error is greater than 1 per cent, and the measurement with the meter will not be accurate.

As previously mentioned, the location of the curve, drawn in Fig. 3, was determined by experiment, and it was thought that the results might be better understood if there was a rational explanation for drawing it in this manner. In order to see the relationship of the empirical curve to some more or less rational curves, Fig. 4 was plotted. In this figure, the dotted line represents the curve from Fig. 3, while the lower solid line represents the values of pulsameter and differential-head readings for 1 per cent error, computed with the assistance of several assumptions.

These assumptions are (1) that the error of the meter is due to the fact that the flow calculated from the meter readings is the

square root of the average differential, (2) that the reading of the pulsameter represented the true maximum value of the differential pressure, and (3) that the differential pressure when plotted against time would be a sine wave. Using these assumptions, the true rate of flow calculated from the average differential and the maximum pulsation wave would be

$$Q_a = K \frac{1}{2\pi} \int_0^{2\pi} (h + p \sin \theta)^{1/2} \, d\theta$$

where Q_a = quantity flowing in proper units
K = meter constant
h = average differential pressure as read by secondary device on meter
p = maximum height of pulsation differential pressure measured above average differential pressure and is equal to pulsameter reading minus reading of differential gage
θ = phase angle of differential pulsation wave

Since the flow indicated by the meter secondary device is

$$Q_m = K h^{1/2}$$

the error for any values of h and p can be computed by dividing $(Q_m - Q_a)$ by Q_a. There will be some definite relation between h and p for any percentage error which will not change with the absolute values of h and p.

Curve I, plotted in Fig. 4, represents 1 per cent error with

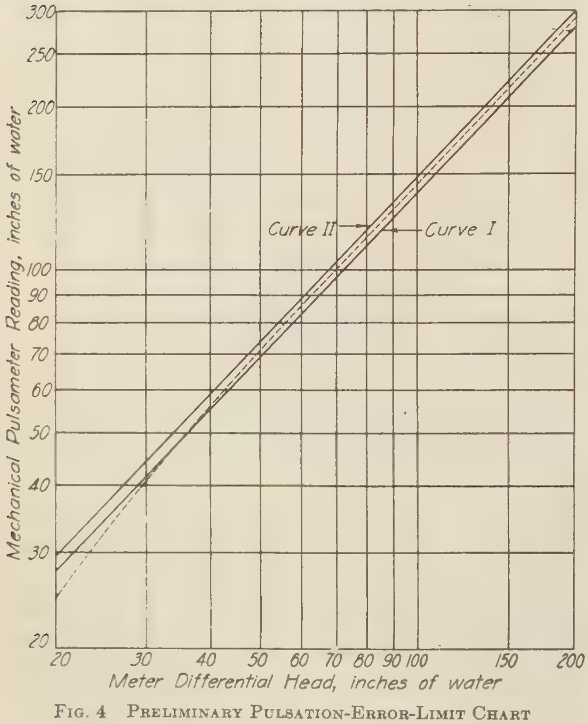

Fig. 4 Preliminary Pulsation-Error-Limit Chart

these assumptions and curve II represents 1.5 per cent error. It can be seen that the empirical curve lies between the two calculated curves for all differential heads greater than 32 in. of water. This indicates that the assumptions as to the effect of differential pulsation are reasonably correct. For differential pressures less than 32 in. of water, the experimental curve is intentionally drawn below the 1 per cent rational curve, since operating experience has shown that measurements made at low differentials, when the flow is pulsating, are very sensitive to changes in pulsation, and it was felt that the line should be lower than normal to allow for the effect of these accidental variations. Because of erratic results the empirical curve was not drawn for differentials below 20 in. of water and, if pulsations are present, it is believed better not to attempt to meter the flow where the differentials are as low as that. These curves were plotted on logarithmic paper so that points with the same percentage of error would be approximately the same distance from the curve for any value of the differential head.

A curve similar to Fig. 3 was drawn for the electrical instrument but its results could not be correlated as closely, because of the fact that calibration of the instrument was much more difficult. It is possible for many studies, especially where it is desired to obtain a record of the pulsations, that this instrument in one of its forms offers several features which make it better than the mechanical instrument but, at the present stage of development, the mechanical instrument is the one which gives the most consistent results.

Summary

From the results of the computations, it might seem possible to determine the percentage of error for any meter where the readings of the pulsameter were so high as to preclude the meter's reading accurately. The amount of work done to date indicates that this will not be practical because it has been found that, if any appreciable error is present, the results of the tests are very erratic, and it is difficult to predict the amount of error for any given condition. This was not true for the tests where the error was less than 1 or 2 per cent, which is the range of error upon which Fig. 3 is based.

This limit curve, Fig. 3, at present is tentative only, and it is possible that further tests will indicate that its shape should be altered slightly. It was determined using natural gas as the metered fluid and with pulsations of one general type only. The instrument has been used on compressed air, however, and at several different gas-compressor stations. The results of these tests seem to indicate that the results will apply to any other conditions.

No work has been done with steam or water meters, or with any other meter using a liquid in the pressure connections. It is therefore not possible to say whether this method of analyzing the accuracy of flow will apply to these types of meters. It was hoped to be able to continue these investigations during the past summer, but conditions made it impractical to attempt to carry out any comprehensive research program. The time was used in taking the instrument around to various gas-metering stations where pulsation was present and finding out if the instrument indicated any error. In one or two of the stations where there was error, it was possible to determine approximately the amount of the error. The tests made at these stations indicated that the curve gave accurate indications as to whether or not error was present. It is hoped that this work can be carried on to completion when conditions are such that research can be resumed.

The material presented in this discussion does not represent the complete results of the investigation to date. It does, however, represent the most important elements, because it gives a method indicating whether or not a meter is accurate.

Utilizing Pulverized Coal in the Metallurgical Industries

By C. F. HERINGTON,[1] PITTSBURGH, PA.

Following a brief outline of the early history of pulverized-coal firing, the author describes in some detail typical equipment which constitutes a modern installation, as best adapted for use in metallurgical furnaces. The relative effectiveness of burners is dealt with by examples. Coal-distribution systems including the pneumatic-transport system, are discussed, special note being taken of advantages of the latter type. Control systems are also considered, and the paper concludes with the presentation of operating data and cost figures.

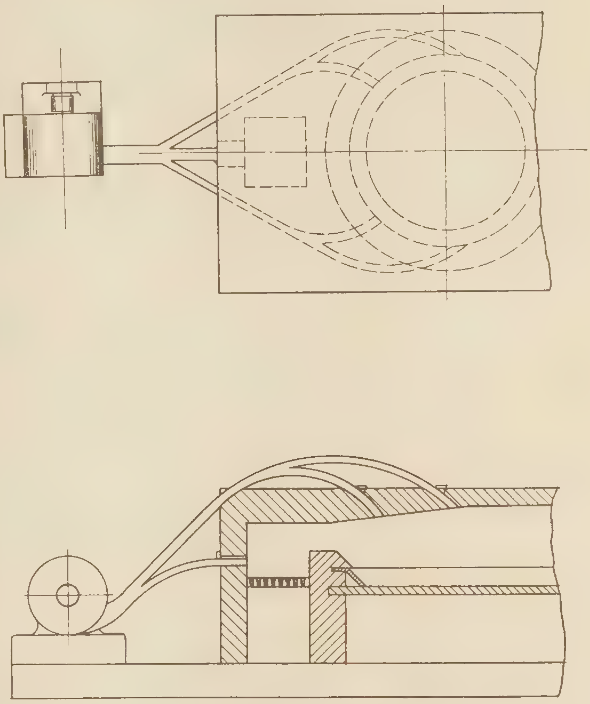

Fig. 1 Whelpley and Storer Apparatus

History

One of the earliest discussions relating to the use of pulverized coal in metallurgical furnaces took place at a meeting of the Engineers Society of Western Pennsylvania in 1913, when James Lord, president of the American Iron & Steel Manufacturing Company, Lebanon, Pa., presented a paper,[2] covering the installation and operation of pulverized-coal-burning equipment, which had been started in 1903.

Going back still further, one of the first patents granted in connection with pulverized coal was that of Messrs. Whelpley and Storer in 1866, Fig. 1. This patent was granted to cover a simple operation of feeding pulverized coal in a manner to cause it to come in contact with other burning gases and to assist solid fires already burning in the furnace. It was not intended to supplant the usual hand-fired furnace, but merely to augment it and to increase fuel economy.

In 1894, a series of experiments on the use of pulverized coal was begun by the Atlas Portland Cement Company. These were in charge of Messrs. Hurry and Seaman, chief engineer, and superintendent, respectively. The experiments led to many discoveries and to the invention of various devices and finally to the commercial development of the art.

The first commercial installation of any considerable size in a boiler plant was made at the Oneida Street plant of the Milwaukee Electric Railway and Light Company at Milwaukee, where five Edge Moor boilers, each of 468 nominal horsepower capacity, were equipped in 1918. This plant is still in operation, furnishing steam for most of downtown Milwaukee.

Pulverized-Coal Equipment

It will be assumed that most engineers are familiar with the essential equipment of a pulverized-coal plant. However, for purposes of discussion, a brief description will be given of a central-coal-plant installation.

Coal Handling and Drying. Formerly the coal was deposited in a bin, where it was delivered into a rotary coal drier in which the surface moisture was removed. Next it was elevated to the bins serving the pulverizers. Now the correct procedure is to supply the pulverizer with hot air up to 350 F and dry the coal in the mill as it is being ground. The drying is more complete, more uniform, and more economical.

These air heaters require but a small amount of space in the coal plant and are very efficient. They may be steam-, gas-, or oil-fired. The heaters are attached to the pulverizer air intake, and the hot air at temperatures ranging from 200 to 350 F flows into the base of the mill, then passes into the annular space and mixes with the air which is carrying the fine coal out of the mill into the cyclone collector or separator, Fig. 2. The mixture is discharged by the vent fan into twin concentrators, which separate the pulverized coal from the air, the air finally passing out into the atmosphere. Usually the vent fan under maximum conditions draws out the necessary volume at 120 F. This is based upon the coal entering the pulverizer having moisture of 5 per cent and being reduced to 1 per cent.

Pulverizing. The ideal size of pulverized coal for the best operation in an industrial furnace should be such that all the coal will pass through a 100-mesh screen. In selecting a mill to pulverize the coal, a unit is chosen which will deliver a minimum of 85 per cent of the total quantity desired of such size that it will pass through a 200-mesh screen, and 98 per cent through a 100-mesh screen. This is considered a reasonable and dependable practice. For boilers, a 65 to 70 per cent minimum has been

[1] Consulting Engineer, Pulverized Coal Division, The Amsler Morton Company.

[2] "The Use of Pulverized Coal in Metallurgical Furnaces," by James Lord, Proceedings of the Engineers Society of Western Pennsylvania, vol. 29, no. 7, Oct., 1913.

Contributed by the Fuels Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

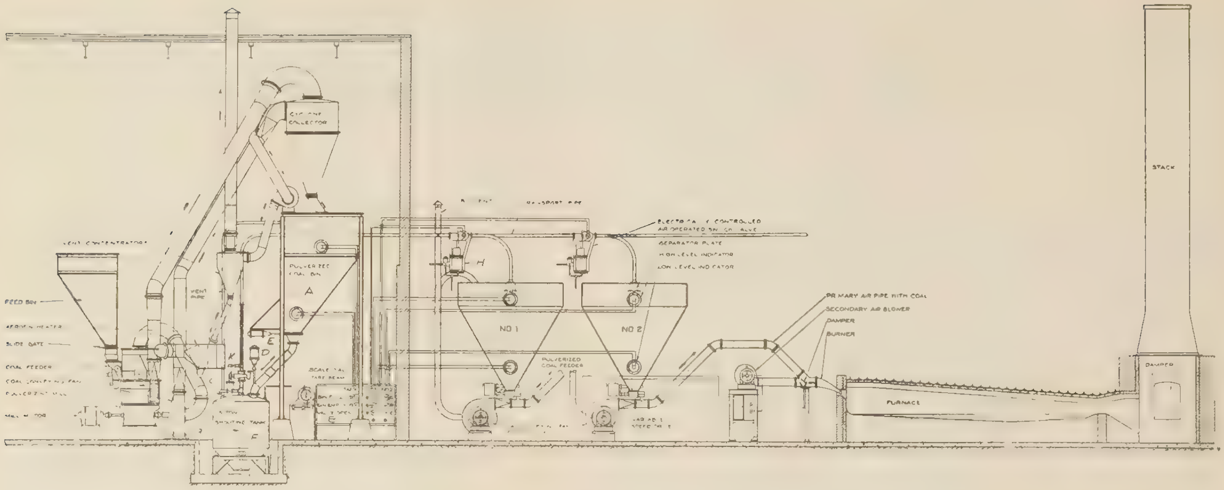

Fig. 2 Pulverized-Coal System
(Water-cooled)

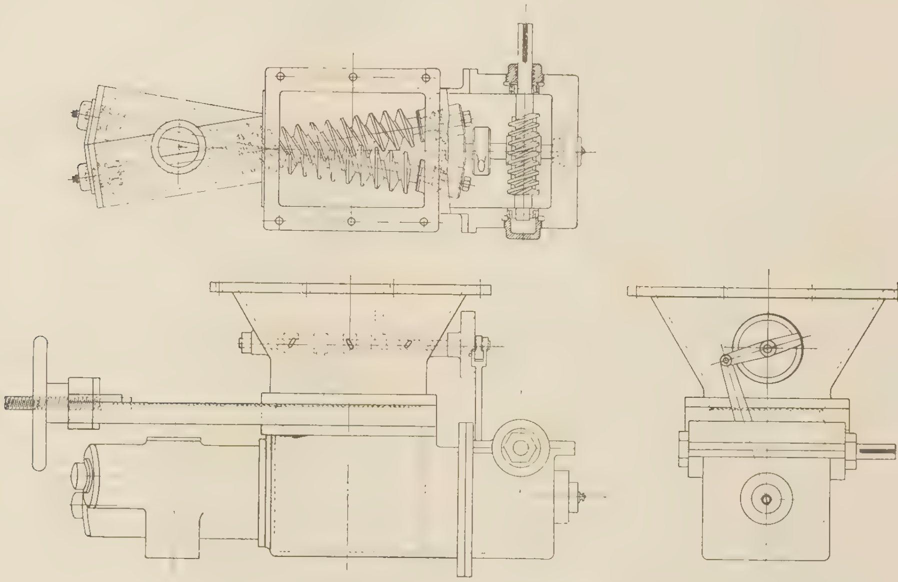

Fig. 3 Pulverized-Coal Feeder

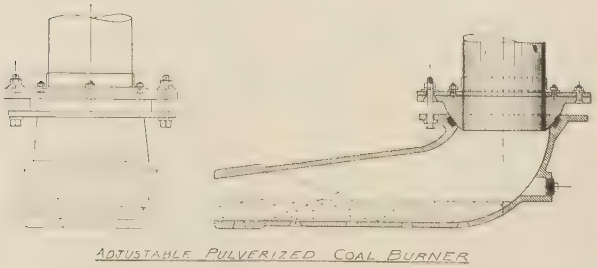

Fig. 4 Adjustable Pulverized-Coal Burner

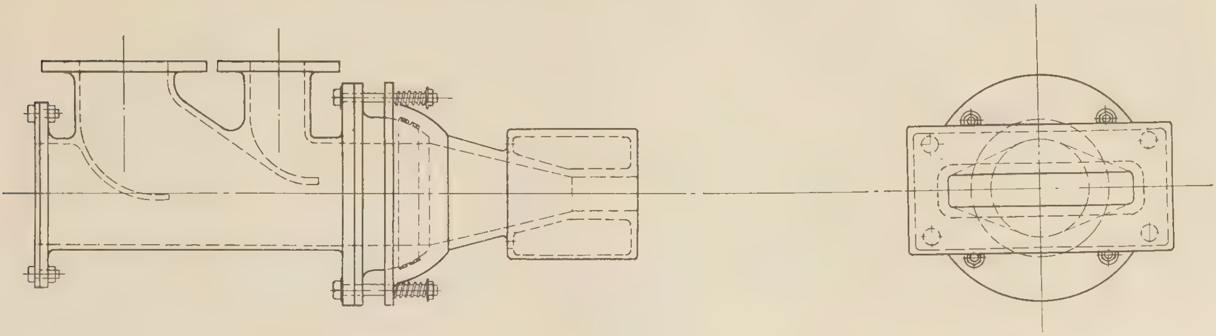

Fig. 5 Adjustable Pulverized-Coal Burner

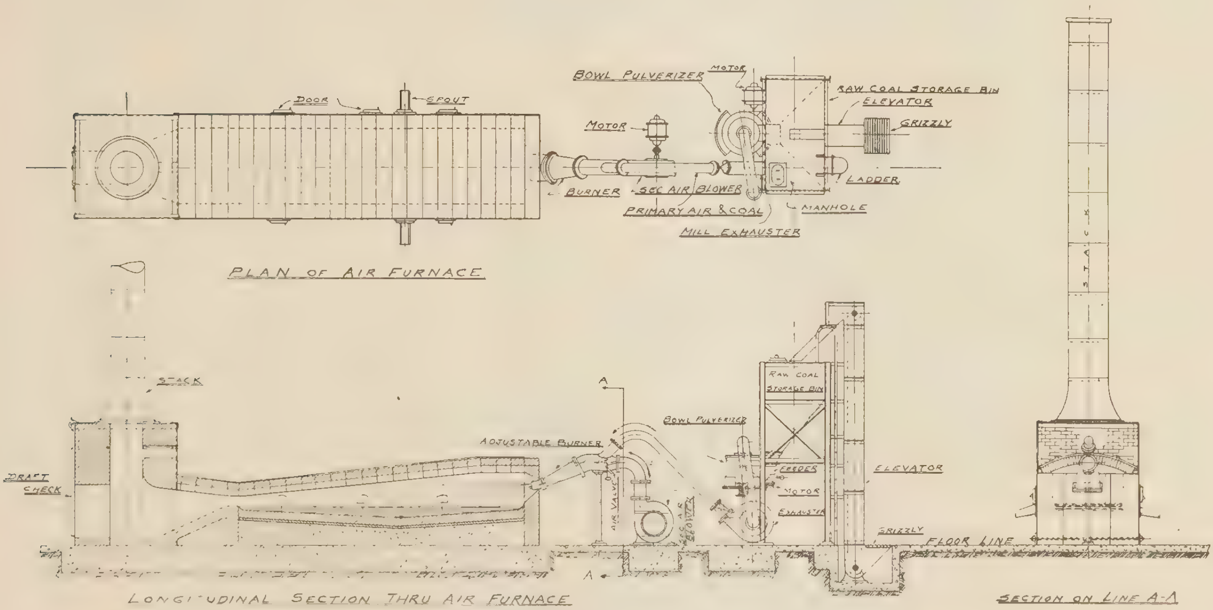

Fig. 6 Diagrammatic Application of Pulverized Coal to an Air Furnace for Melting Iron

used; most recent installations are raising the minimum to 75 per cent.

Lately it has been found that, by selecting a coal according to its grindability factor, a better determination of the correct size of the mill can be given to meet this characteristic of the coal. This insures that the basic unit in the system will be adequate for the delivery of the coal desired and in the degree of fineness predetermined for the work.

Coal Feeders. The twin-screw pulverized-coal feeder, Fig. 3, provides a nonflushing and positive-type unit. It consists of two forged-steel screws running in a cast-iron trough and is equipped with a mechanically operated agitator which prevents the coal from arching over the screws. A brass-plate cutoff gate is inserted between the agitator casting and the feeder. Each feeder is equipped with a variable-speed drive and is remotely controlled by the operator or heater at each furnace.

Burners. A burner of simplest possible design is the best for pulverized-coal use. Typical burners are shown in Figs. 4 and 5. Thousands of dollars have been expended by manufacturers to devise burners which would create and cause thorough mixing of the coal and air, and other burners in which the air surrounds the coal as it enters the furnace. Such types of burners have not proved practical.

A typical example of such an installation will serve to indicate the shortcomings of such units. A copper-melting furnace of over 200 tons capacity was fitted with three burners designed to admit the secondary air in a circle enclosing the core or center of the pulverized-coal mixture. As this mixture of coal and primary air entered the furnace chamber, the coal dust would fall into the lower half of the circle of secondary air, while the upper half of the circle of air as it entered the furnace would immediately rise away from the stream of coal. An excess of secondary air was then required in order to obtain complete combustion.

In another copper plant in which a furnace of the same capacity, heating the same kind of material under the same conditions, was installed, the author took steps to overcome the difficulty mentioned, by using a burner consisting of a circular pipe flattened at the end, having an opening of $1^3/_4$ in. $\times$ 14 in. wide. Directly below the burner mouth the secondary air was admitted through an opening 6 in. $\times$ 14 in. wide.

With one burner of this type placed where the center of three burners would be, a ratio of 160 to 180 lb of coal per ton of copper

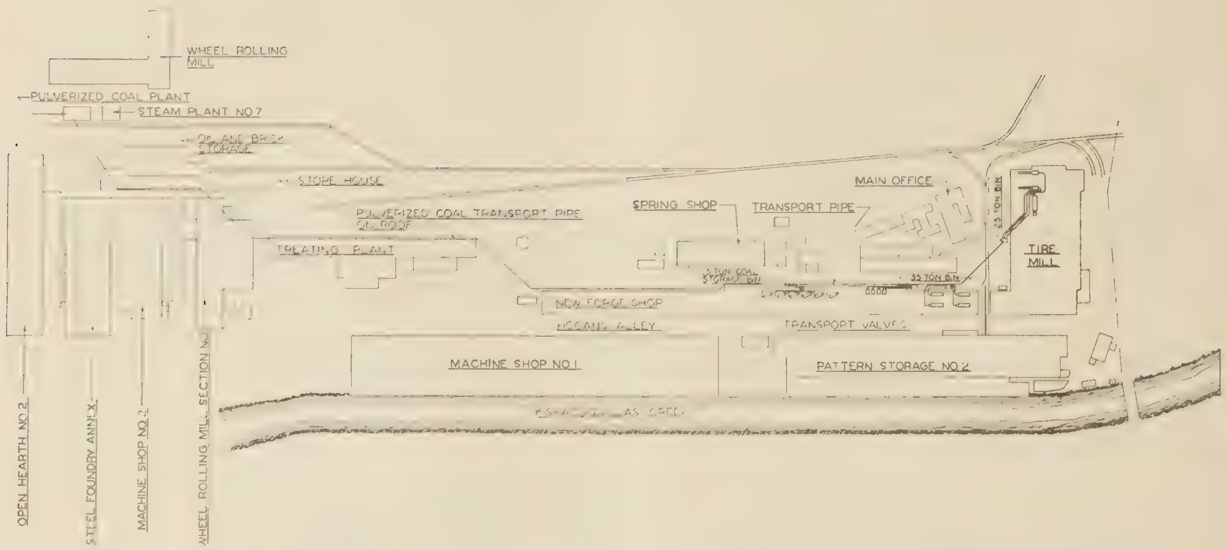

Fig. 7 Pneumatic Transport System to Convey Pulverized Coal to Forge Shop and Tire Mill; Distance, 4000 Feet

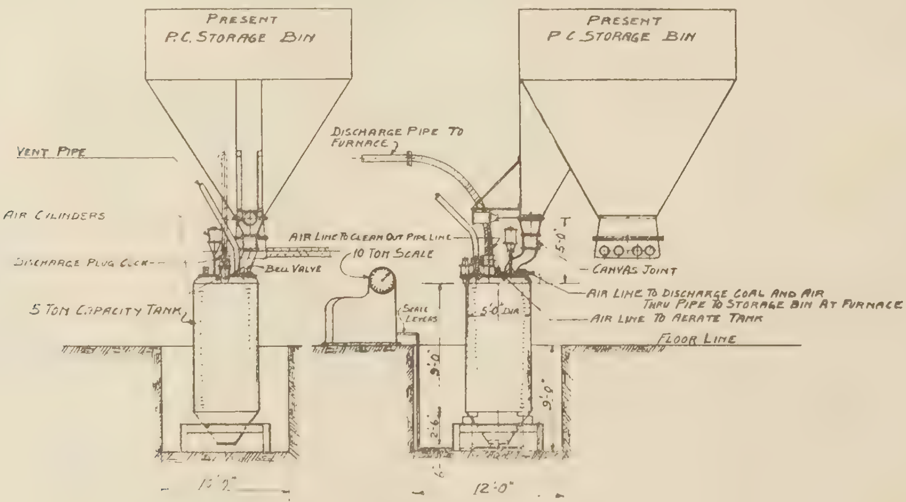

Fig. 8 Shooting or Transport Tank

melted was obtained, whereas with three circular burners in a row, the ratio secured was 250 lb of coal per ton of copper, besides the extra oxidation. Also, the velocity of the coal stream was increased with the flat burner, so that very little ash was deposited on the copper bath.

Central or Unit System. Original pulverized-coal-firing systems may be designated as falling into two groups, the central system and the unit system, Fig. 6.

The central system consists of four divisions, namely (1) receiving, crushing, and conveying raw coal; (2) drying, pulverizing, and storing pulverized coal; (3) delivering coal to the furnaces; and (4) the firing equipment. In this system all three divisions of the work are carried out in a building called the central coal plant.

The unit system eliminates the central coal plant and requires the delivery of the raw coal from the point of supply to each of the furnaces. Unit pulverizers which grind and deliver the coal directly into the furnace are placed at each of the furnaces.

Improved System. Improvements in the system are made by placing the coal plant at the point where the raw coal is received in the works, without regard to the location of the furnaces. The raw coal is received in crushed or slack form and is conveyed to a storage pile or large bin. From this bin, the coal is conveyed to a feed bin for the pulverizer or pulverizers.

The operating cycle begins at this point, as shown in Fig. 2. The coal is fed into the pulverizers, ground, and dried in the mill by the admission of hot air at a temperature of 200 to 300 F, and placed into a pulverized-coal storage bin. With this method, the pulverizer need only be operated the number of hours required to grind a 24-hr coal supply for the furnaces, so that in no case will the pulverized coal remain in the storage bin over 48 hr.

The pulverized coal is then available for gravity feed to a transport tank, where a scale is provided to apportion the desired quantity to each furnace. It is then delivered to substation storage bins through pipes of 3, 4, and 5 in. diam by means of compressed air. The prepared and dried coal is then measured and delivered to an air duct by a special feeder. The blower supplies sufficient air to convey the coal at the correct velocity to

the burner, where a secondary-air fan provides the additional air required for complete combustion.

In a central system, it has been found to be good practice to install equipment capable of supplying pulverized coal for 1 day's operation in 1- or 2-hr turns, leaving one turn at least for a shutdown of the plant.

Pneumatic-Transport System. Most transport systems involve apparatus having practically no moving parts which may be easily controlled. The control must be positive so that any predetermined quantity of fuel may be transported to any point in the entire system. This is an important consideration in installations where an accurate knowledge of operating conditions and detailed costs are required.

TABLE 1 TIME AND PRESSURE CHANGES DURING DISCHARGE OF TANK

Time (March 19, p.m.)	Pounds of coal	Tank pressure, psi	Reservoir pressure, psi
2:295	14000	..	90
2:30	Aerating the coal	..	..
2:315	Valve opened	80	88
2:32	10800	70	88
2:325	7600	65	88
2:33	5000	64	86
2:335	2500	60	88
2:3375	Air turned off	40	88
2:34	450	40	90
2:345	Coal all out of tank; time, 5 min		..
(March 19, p.m.)			
2:525	14000	..	90
2:5325	Aerating the coal	..	..
2:545	...	80	88
2:55	11300	67	88
2:555	8200	65	88
2:56	5800	60	88
2:565	3500	58	88
2:57	1800	55	88
2:575	500	30	90
2:58	Coal all out of tank; time, 5 1/2 min		..

In transporting pulverized coal through a pipe, it is essential that the coal is being conveyed with a minimum amount of energy, and that no conditions arise during or after the movement to cause delay or other difficulty.

piping are used for the tank-venting and discharge pipes. In this system, the coal is discharged from the top of the enclosed tanks, then through the bottom as in the other two systems described. Gate and bell valves in these pipes are operated from the control room which contains the scale dial registering the amounts of coal in the tanks. From this control room, by means of a panel board and remote-control valve, the coal-plant operator may also regulate the distribution of the coal.

Table 1 shows the time consumed in emptying a shooting tank, the pressure changes as the coal and air leave the tank, and also the pressure of the air being delivered to the tank. The line varies in distance from 1500 to 2100 ft with curves and varying elevations involving bends not calculated in the linear distances cited.

Each furnace storage bin in the installation under discussion is fitted with electric indicator switches arranged to operate when the bin is empty and when it is filled. These switches are connected to electric lights on the panel board.

Advantages of System

In an improved transport system without a screw pump, installed in 1930, the user, after 3 years of operation indicates that it is possible to convey 1 ton of coal for distances up to 1500 ft with 700 cu ft of air at 60 to 70 lb gage pressure. During the 3-year period, this operator has transported 73,715 tons of coal, at a total cost for repair and maintenance of $287.40, or $0.0038 per ton. It may be noted that the coal was transported in 5-in. pipes in temperatures as low as 22 deg below zero. This system is in regular daily operation at the present time.

The operator of a similar system (using 3-in. pipe) for the last 11 years reports that the total cost of transporting coal has been 1 cent per ton. During this period, no major repairs have been necessary.

After 25 years of experience with this fuel, the author feels no hesitancy in asserting that the correct application of pulverized coal to any heating operation requires a knowledge of the kind of heat best adapted to the work to be done; it is then necessary

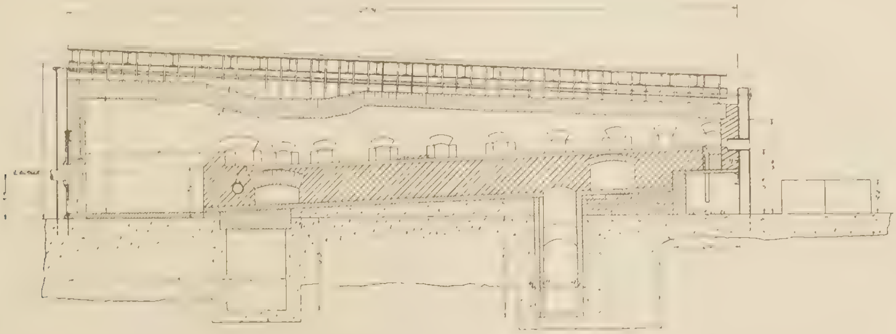

Fig. 9 Old Arrangement of Rolling-Mill Furnace

The transport system may have single or multiple units, varying in capacity to meet the conditions existing at the plant under consideration. Details of a typical transport system (Fig. 7) are somewhat as follows:

In the coal plant beneath the pulverized-coal storage bin, into which all of the pulverizers discharge their product, two enclosed transport or shooting tanks (Fig. 8) are installed. These tanks rest on scales for weighing the amounts of coal being transported. They may be apportioned to individual furnaces or to a group of furnaces to which the pulverized coal is to be delivered.

The upper portions of these shooting tanks and the flexible

Fig. 10 New Arrangement of Rolling-Mill Furnace

to design the furnace so that all the heat units released by a correct application of pulverized-coal-firing will be utilized in the furnace. The combination may be elevated to the level of a precision heating machine by applying automatic control of temperature, fuel-air ratio, furnace pressure, and of coal feed and fineness of pulverization. By such means may achievements in the use of pulverized coal be accomplished.

What can be accomplished in a modern automatically controlled plant is demonstrated by the following example:

Fig. 9 shows a longitudinal billet- or rail-heating furnace operating on two-beater-unit pulverizers. The operating company was persuaded to blast out the entire furnace including a solid concrete foundation and then to build a new furnace with new flues, as shown in Fig. 10. An unusual feature of the new installation was the short period required for construction. Starting on May 31, 1940, while the furnace walls were hot, all the old equipment, including the complete furnace, steelwork, pulverizers, coal-handling equipment, and coal bin were completely demolished and scrapped. The new installation was made so that within 10 days, on Sunday night June 16, at 6:20 p.m., after drying the furnace with wood fires for 42 hr, the pulverizer was started and in 20 min had a charge of rails heated. On Monday morning, June 17, regular rolling operations were started.

The former practice on the old furnace and pulverizers was as follows:

1 Start heating the furnace at 6:30 a.m.
2 Start rolling the rails at 7:30 a.m.

Now, with the new furnace and automatic controls, the following schedule is used:

At 6:50 a.m. city-gas pilot burners are lighted. At 7:00 a.m. the coal pulverizer is started and the coal is fed into the furnace where it ignites immediately.

Time, a.m.	Temperature recordings, deg F
7:00	1000
7:15	1800
7:25, start rolling	2200

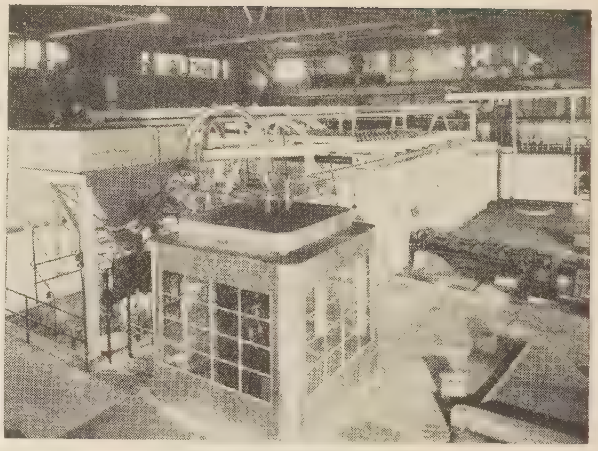

Fig. 11 Continuous Rail-Heating Furnace Showing Unit Coal-Pulverizer Plant, Automatic Temperature and Pressure Control, and Panel Enclosure

The foregoing figures indicate a time saving of 35 min in starting, attributable to better coal pulverization, more efficient burners, and better combustion.

The furnace, operating at the maximum speed of the mill, delivers about 120 tons of 15-ft (350-lb) rails per $7^1/_2$-hr day. The fuel consumption is approximately 210 lb of coal per ton of steel heated. This makes an average of 16 tons per hr, there being from 75 to 80 rails in the furnace at all times.

With the old furnace, using two-beater-type pulverizers, each being divided by a flap valve into two round burners inserted into the furnace 4 ft above the floor, and with pulverized coal having a fineness of 75 per cent through a 200-mesh screen, 340 lb of coal were required per ton of steel heated.

At the present time, the fuel consumption is 180 lb of coal per ton of steel. The fineness is 99.9 per cent through a 50-mesh screen and 90.1 per cent through a 200-mesh screen. The new

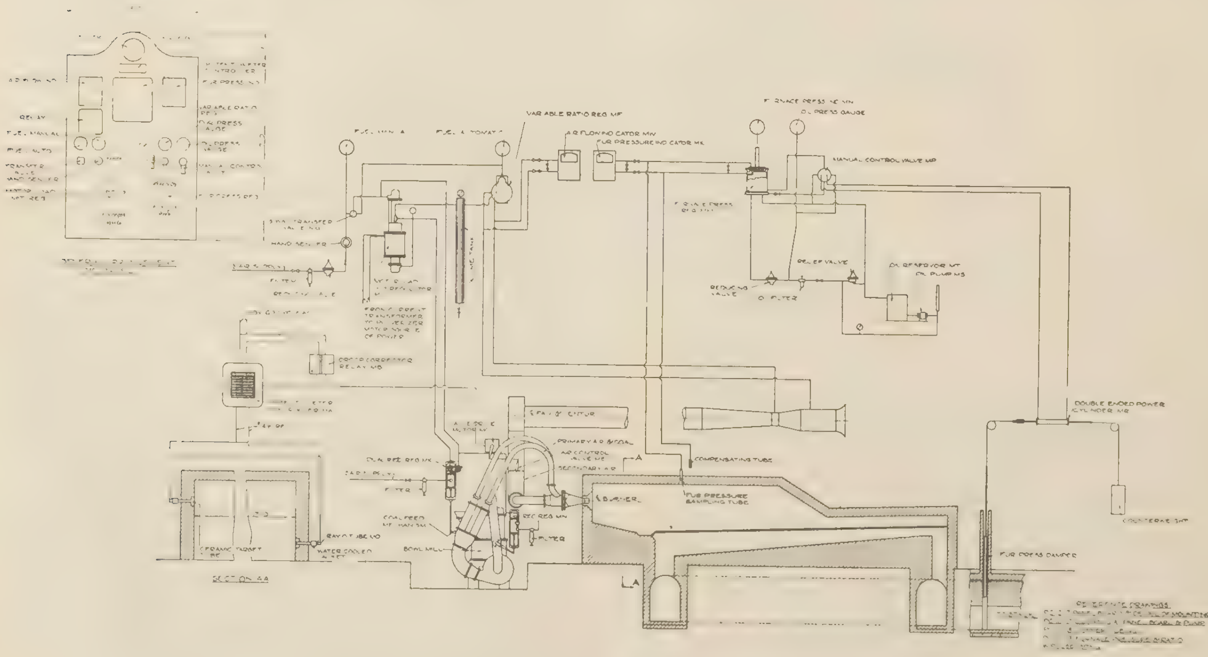

Fig. 12 Rolling-Mill Furnace and Automatic Control

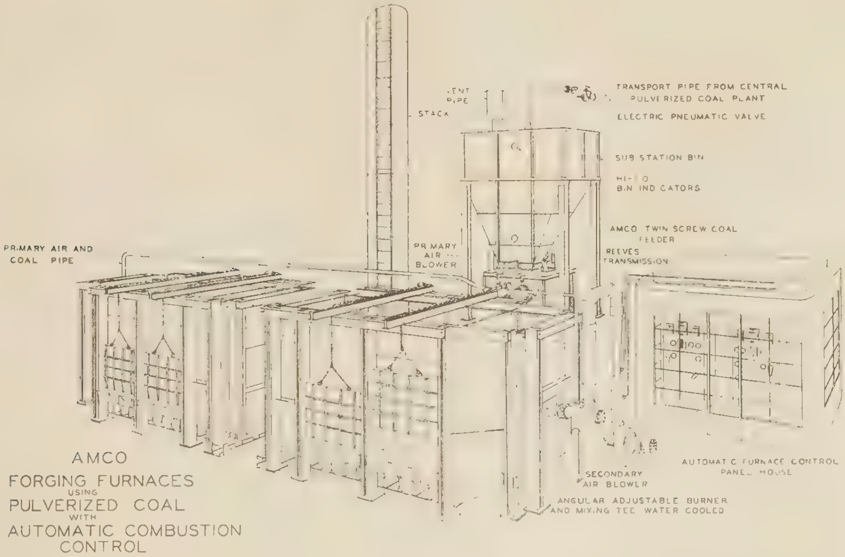

Fig. 13 Forging Furnaces Using Pulverized Coal With Automatic Combustion Control

high-velocity burners and the design of the furnace accomplish recirculation of the hot gases in a manner not previously achieved in continuous-heating-furnace design, and sweating and washing of the steel are eliminated.

The control system, shown in Fig. 12, operates by using an inlet-measuring element, such as a venturi tube, through which the secondary air necessary to complete combustion passes. A temperature-controlled power unit actuates a regulating valve to measure the air flow in accordance with the furnace requirements. An air-fuel-ratio regulator, connected across a venturi tube, measures the air flow and generates a loading pressure, varying from 0 to 50 psi, in accordance with the actual quantity of air supplied.

This loading impulse is transmitted to two receiving-type power cylinders, one connected to the mill-exhauster-fan inlet-damper control, and the other to the coal-feeder-drive control of the pulverizer. These two receiving-type power cylinders assume definite positions in accordance with the loading pressure, and the proper relations between primary air and fuel feed are maintained constant in any adjusted position.

The coal-feeder control operates a lever on the pulverizer, to deliver a variable stroke which actuates a revolving-slot feeder.

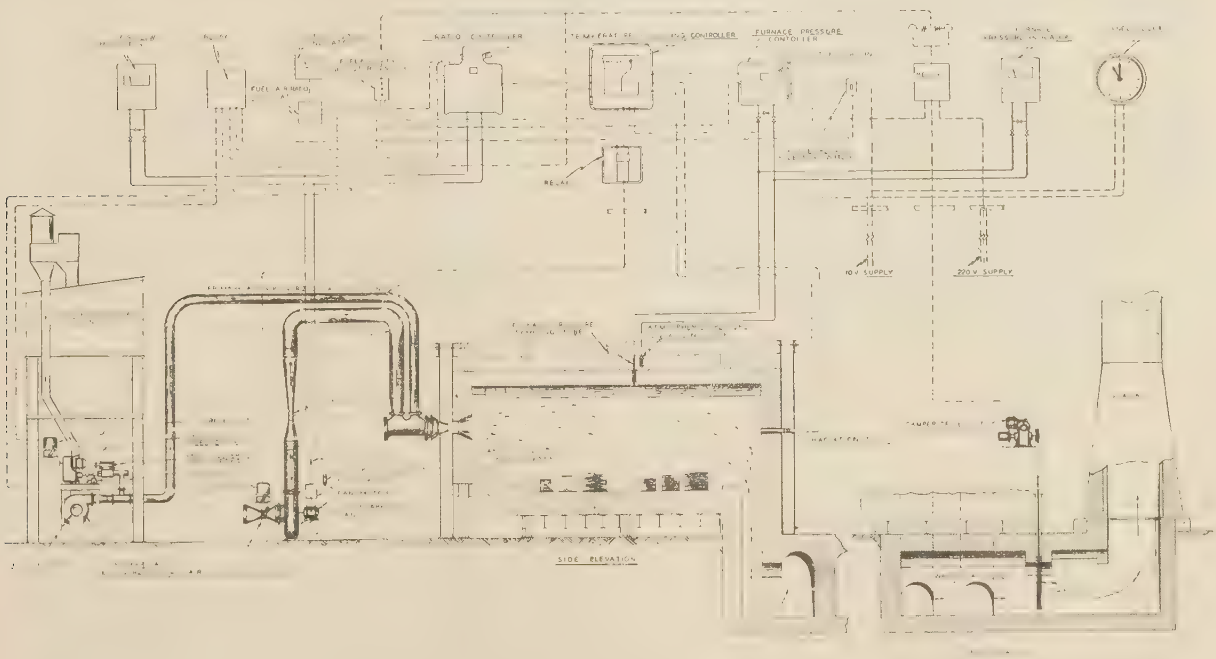

Fig. 14 Pulverized-Coal-Burning System With Diagrammatic Layout of Control Arrangement, Two-Forge Furnaces

This control is of the dual-acting type, having two receiving bellows which move the power cylinders in the desired direction; one is connected to the air-fuel-ratio regulator and the other to a mill-charge corrector. The mill-charge corrector measures the actual load on the pulverizer driving motor and generates a loading-impulse pressure which is transmitted to the second bellows unit of the dual-feeder driving control.

With the bowl-type pulverizer installed, there is a definite amount of coal within the mill for any firing rate and fineness. Hence there is a definite load on the mill driving motor. The mill-charge collector measures this load and readjusts the feeder-drive-control unit whenever necessary to maintain the desired connections. With this arrangement, inaccuracies in the feeder mechanism which might result from varying sizes of coal fed, or from other factors are automatically compensated.

The control of furnace pressure is accomplished in the usual manner by an oil-pressure power cylinder operating a water-cooled slide damper in the waste-gas flue, whereby heating conditions and furnace atmosphere remain constant.

Fundamentally the control system for any pulverized-coal-fired furnace is the same and operates the same as the system for a similar furnace fired with gas or liquid fuel. The input of fuel or air is controlled from the temperature within the furnace chamber, and the other element of the combustible mixture is held in a predetermined ratio.

In the installation shown in Fig. 14, we insert a closed-end radiation element through the furnace wall opposite the burner where the measured temperature is as closely related to that of the steel as possible. When this temperature reaches the control point, the flow of secondary air is reduced by the efforts of the temperature controller to reduce the firing rate. This change in air flow is immediately sensed by the ratio controller which measures the pressure drop across the venturi tube or orifice. The pressure drop is transformed into an electrical impulse in the ratio controller, which balances a similar impulse generated by the tachometer attached to the coal-feeder mechanism. Then, as the ratio controller senses the change in air flow, the electrical balance is disturbed and the controller restores the balance by reducing the speed of the coal feeder, which accomplishes the reduction in heat input called for by the temperature controller.

This automatic reduction of fuel input will continue until it becomes stabilized at the rate required to satisfy radiation and stack losses. At that time, it may be considered that the thermal requirements of the steel are satisfied and the billets or slabs are ready for forging.

Furnace pressure is controlled in the same manner as in the case of a gas- or liquid-fuel-fired furnace and with the same equipment. The pressure is simply measured at a convenient point in the furnace and that pressure is held to a constant value by means of a standard regulator and suitable damper-drive mechanism.

It has been found that pulverized coal is as easily controlled to the fine degree required in high-temperature furnace work as any other fuel—in fact more easily than some. The results obtained show that the equipment more than pays for itself in reduction in fuel rate and furnace maintenance, not to mention uniform quality of heating.

Operating Data

In an installation of pulverized coal on a copper-wire bar furnace, the following comparison was made between the use of fuel oil and pulverized coal:

The amount saved by the use of pulverized coal over that of fuel oil was 38.03 cents per ton of copper; \$4943.90 saved per month; and \$59,326 saved per year. These savings were based upon two furnaces pouring 13,000 tons per month.

On a malleable-iron air furnace used for melting pig iron, scrap and sprue, the substitution of pulverized coal for coal on grates has increased the tonnage of the furnace from 35 tons to 55 tons melted in the same time. The coal consumption was reduced from 900 lb per ton of melt to 600 lb per ton.

Using pulverized coal in malleable-casting annealing ovens, to replace hand-firing on grates, the fuel requirement has been reduced from 1000 lb of coal per ton of castings annealed to 500 lb per ton.

In addition, the annealing cycle was shortened because of the ability to bring the furnace up to soaking temperature more quickly. The coal-pulverizing costs are: Power, 12.314 cents; maintenance, 8.91 cents; labor, 33.68 cents, and supplies and overhead, 11.37 cents. The total cost per ton of coal used is thus 66.27 cents. These figures are based upon a central pulverized-coal plant, where a total of about 200 tons of coal is used each 24-hr day. When the output is 300 tons per day, the cost per ton is decreased to 43 cents. In a Canadian installation, pulverizing over 500 tons of coal per day, the total cost per ton of coal is 21 cents.

Pulverized coal can be used with success equal to that of other fuels from a thermal and control standpoint on any or all of the following heating operations:

Annealing	Drop forge
Air or malleable-iron melting	Forge
Bloom	Ingot
Busheling	Lead kettles
Billets	Rail heating
Car wheel	Rivet making
Copper anode	Roasting ore
Copper billet	Soaking pits
Copper cathode	Shell
Copper reverberatory	Steel
Continuous heating	Tool
Cement kilns	Nut
Cement driers	Open hearths

The Dynamic Viscosity of Nitrogen[1]

By W. L. SIBBITT,[2] G. A. HAWKINS,[3] and H. L. SOLBERG[4]

After discussing the existing data on the dynamic viscosity of nitrogen, as reported by various investigators, the authors present the results of a determination of the dynamic viscosity of nitrogen. A nickel capillary 118 ft long was used in the investigation. Data are reported which cover 45 calibration tests and 395 tests on nitrogen at pressures up to 1020 psia and 923 F. An equation is presented which expresses the test results over the entire range of pressures and temperatures covered by the investigation.

In many instances it is essential to know the value of the dynamic viscosity of a fluid in order to determine many of the important dimensionless ratios used in the solution of problems in the fields of heat transfer and fluid mechanics. In making a survey of the literature preparatory to an investigation in heat-exchanger design, the authors found that the experimental data on the dynamic viscosity of nitrogen gas were very meager or nonexistent at the temperatures and pressures which were to be considered. The authors decided, therefore, to utilize part of the apparatus which had previously been developed under the sponsorship of the A.S.M.E. Special Research Committee on Critical-Pressure Steam Boilers for the purpose of determining the viscosity of steam (1)[5] and to conduct experiments which would yield data on the viscosity of nitrogen gas.

In 1938, Rigden (2) and Majumdar and Vajifdar (3) determined the viscosity of nitrogen at room temperature. Trautz and Baumann (4) measured the viscosity of nitrogen gas up to 482 F by means of a short capillary tube. They found the value of "Sutherland's constant" to be somewhat lower than previous investigators had reported. Ribaud and Vasilesco (5) measured the viscosity of commercial nitrogen gas from 32 to 2912 F by means of a short capillary tube. They found that the viscosity could be represented by the Sutherland approximation if the Sutherland constant was made a function of the temperature.

The data reported by Boyd (6) and by Michels and Gibson (7) are apparently the only sources of high-pressure viscosity measurements on nitrogen gas as found by the author's survey of the literature. Boyd's results agree with those of Michels and Gibson within approximately 10 per cent, but they show a wide deviation between individual measurements. Michels and Gibson used a modification of the Rankine design to obtain data up to 1000 atm and 167 F. Boyd used a short capillary tube for measurements up to 191 atm and 158 F. In this type of apparatus, end corrections may be 10 or 20 per cent.

Requirements of a Capillary-Tube Viscometer for Measuring Dynamic Viscosity of Nitrogen Gas

In order to obtain accurate data, the authors decided to use a capillary tube designed and operated in accordance with the following principles:

1 The apparatus should be suitable for use over a wide range of pressures and temperatures.

2 In order to minimize the possible sources of error the resistance effect should be directly measurable.

3 The apparatus should be designed so that corrections are either negligible or of the minimum possible magnitude.

4 The capillary should be of such a length as to (*a*) give a pressure differential at the maximum operating pressure which could be measured accurately; (*b*) make the kinetic-energy-correction terms for the ends negligible.

5 The capillary should be of sufficient strength to retain its form at the highest operating temperature and pressure.

6 The capillary should be of a corrosion-resistant material such that its dimensions would not change during the period of operation.

7 The diameter of the capillary should be large enough to make the slip-correction term negligible and give a rate of flow which could be accurately measured and yet give a pressure differential of such magnitude that it could also be measured accurately.

8 A uniform temperature along the entire length of the capillary should be maintained.

9 All readings should be obtained after a steady-flow state has been attained as indicated by the manometer readings and the total pressure gages.

10 The capillary constant should be checked at periodic intervals during the investigation.

Description of Apparatus

Essentially, the apparatus consisted of a long nickel capillary, drier, heaters and coolers, collecting burette, and with the necessary valves to control the flow, and with gages, thermometers, precision potentiometer, controllers, and thermocouples required to record the data. A schematic diagram of the apparatus is shown in Fig. 1.

The capillary was made from 118 linear ft of seamless nickel tubing having inside and outside diameters of 0.09294 in. and 0.25 in., respectively. The inside diameters of random lengths of tubing were checked by means of a gage. The sections were connected into one continuous length by facing each end square,

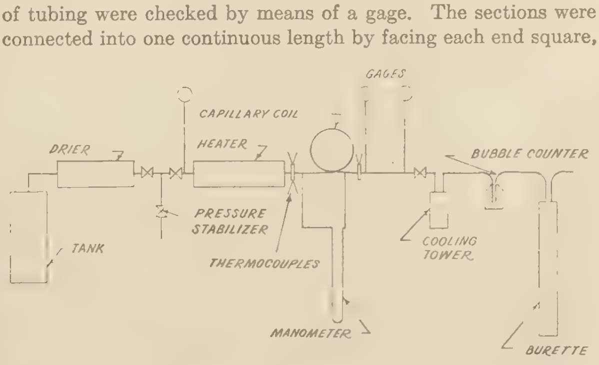

Fig. 1 Diagrammatic Arrangement of Apparatus

[1] Based on a doctorial thesis, Purdue University, 1942, by one of the authors.[2]

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[5] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Heat Transfer Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

butting it against the end of the next piece in a snugly fitting nickel sleeve, and welding the sleeve to the tube ends. Each joint was examined by radiograph in order to determine the actual position of the ends of each section and to verify that the channel was free from obstructions and abrupt changes in direction. All welds were tested for strength by subjecting the entire capillary to a hydrostatic pressure of over 2000 psia. Three piezometer-ring pressure connections were made on the capillary; one near each end and one in the middle. The entire length of the capillary (106.2 ft between outside pressure connection) was fastened rigidly to the rim of a steel wheel in order to give a mean diameter of the capillary coil of 2 ft. Since deformation of the cross-sectional area of the capillary could have occurred when the coil was bent to a 1-ft radius, sections were cut perpendicular to the tangent; these sections were polished, and enlarged photographs were made of the capillary cross sections at 100 diam. The photographs did not give any evidence of deformation of the circular capillary cross section.

Seventeen iron-constantan thermocouples were cemented to the capillary coil and special low-thermal-capacity thermocouples were placed in the flow channel at the coil entrance and exit. The coil was surrounded by an electric heater in such a manner that the capillary and thermocouples were shielded from the direct radiant energy of the heater elements. The complete coil unit was thoroughly insulated. The power input to the heating element was controlled automatically by a controller and magnetic switch.

Three low-pressure manometers were constructed from standard pressure-gage glasses, using carbon tetrachloride as a manometer fluid. The high-pressure two-fluid manometer was constructed from special gage glasses having inside and outside diameters of $^1/_8$ in. and 1 in., respectively. The manometer liquids were altered from time to time for various temperature-and-pressure combinations so as to give as large a difference of liquid level as possible. By doing this, the accuracy of reading the differential pressure was materially increased. The liquid combinations used consisted of a mixture of carbon tetrachloride and kerosene and an aqueous sodium-chloride solution to which a fluorescent indicator and a corrosion inhibitor were added. The high-pressure and low-pressure manometers were always checked by comparison against a standard manometer containing water. Before the high-pressure manometer was filled the solutions were shaken together, and then the densities of the two layers were determined with a Westphal balance at the temperature at which the manometer was used. Densities of the solutions in the manometers were checked periodically during the investigation.

The water used in the calibration tests was heated to the same temperature as the capillary temperature before entering by means of a simple heat exchanger. After discharging from the capillary, the water was cooled to room temperature in a second heat exchanger. By means of this arrangement it was possible to maintain the temperature of the entering calibrating fluid identical with the temperature of the coil.

The nitrogen gas taken from standard cylinders was passed through a calcium-chloride drier before discharging into the heater ahead of the capillary tube. After passing through the gas heater and capillary tube, the nitrogen was cooled by direct contact with a saturated sodium-chloride solution in a spray chamber. A submerged coil in the spray chamber maintained the circulating solution at a constant temperature. The cooled gas was then collected over a saturated sodium-chloride solution in a 4-l burette which had been calibrated by weighing the water delivered. Trial tests indicated that the gas-collecting-and-measuring unit gave a reproducibility of 1 sec in 300 when the pressure variation was less than 1 cm of water. During the actual tests the pressure in the measuring burette was easily held within a maximum variation of 0.5 cm of water. The burette and accessories were all enclosed in an insulated cabinet.

Viscosity Equation for the Capillary

Poiseuille's law which applies to the viscous flow of a fluid in a straight capillary, with correction factors for the kinetic energy and end losses, may be stated as follows

$$\mu = \frac{\pi r^4 \,\Delta P}{8Q(l + \lambda)} - \frac{M\rho Q}{8\pi(1 + \lambda)} \qquad [1]$$

where
μ = viscosity
r = radius of capillary
ΔP = pressure differential
Q = volume of flow per unit of time
l = length of capillary
M = constant
ρ = density of fluid

For a straight tube, in which the kinetic energy and end corrections are negligible, the equation reduces to the following form-

$$\mu = \frac{\pi D^4 \,\Delta P}{128 l W v} \qquad [2]$$

where
W = weight rate of flow
v = specific volume of fluid
D = diameter of capillary

For a given capillary, the diameter and length values may be combined with $(\pi/128)$ to form a constant. Equation [2] then may be written

$$\mu = \frac{C_1 \Delta P}{Wv} \qquad [3]$$

μ may be expressed in centipoises if consistent units are used throughout, and the necessary conversion factors are embodied in C_1.

For curved capillaries Equation [3] must be modified to account for the effect of curvature of the capillary. This is accomplished by introducing a factor φ which is a function of the Reynolds number and the curvature of the capillary and diameter of the bore. Equation [3] corrected for curvature is represented by

$$\mu = \frac{C_1 \varphi \Delta P}{Wv} \qquad [4]$$

According to White (8), the general hydrodynamic equation for evaluation of φ is presented in the following form

$$\varphi = 1 - \left[1 - \left(\frac{k_0}{k}\right)^x\right]^{1/x} \qquad [5]$$

where
x = a constant
$k = \mathrm{R}\left(\frac{D}{d}\right)^{1/2}$
D = inside diameter of capillary
d = diameter of coil

Water-Calibration Tests

Water was used in the tests to determine the factor φ, since the dynamic viscosity of water is known for moderate temperatures. Each test consisted of passing water at a predetermined rate through the capillary and recording the pressure differential, temperature, and weight rate of flow.

The value of the Reynolds number was computed from test data by means of the following relation

$$\mathbf{R} = \frac{DV\rho}{\mu} = \frac{C_2W}{\mu} \qquad \text{[6]}$$

where V = velocity of flow

The results of the calibration tests gave the following values for the various items in Equation [5]:

$$x = 0.442$$
$$k_0 = 12.6$$
$$\mathbf{R}_0 = 202.5$$

Fig. 2 represents a plot of the curvature function φ, against

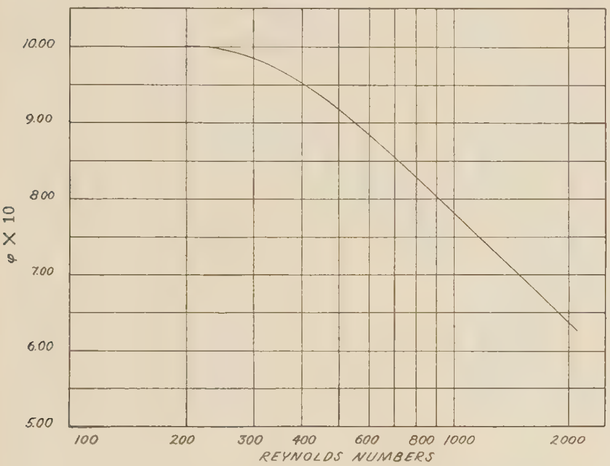

Fig. 2 Calibration Curve for the Capillary

the Reynolds number $\mathbf{R}$, as determined from 43 test points. The average deviation of the 5 worst points was 5 per cent.

Nitrogen-Viscosity Tests

By means of the calibration data the final viscosity values were determined independently of the actual value of the capillary diameter. However, the accuracy of the results is dependent upon the water-viscosity values used. The values used in this program were taken from the work of Dorsey (9).

The test procedure was similar to that followed during the calibration; however, special care was necessary to insure steady-state flow conditions.

The value of the viscosity at a specific temperature and pressure was determined by successive approximation from Equations [4] and [6]. It was necessary to employ the compressibility isotherms of nitrogen gas (10) and to extrapolate the compressibility data for the higher temperatures.

Test Results

The viscosity of nitrogen gas was determined at temperatures and pressures ranging from 68 F to 923 F and 14.7 psia to 1020 psia.

The Reynolds numbers varied from 10 to 1000 during the nitrogen-gas tests. However, due to the more difficult experimental technique at the high Reynolds numbers most of the tests were carried out at Reynolds numbers below 200. By conducting the tests at very low Reynolds numbers the testing technique was simplified, and the effect of curvature of the capillary was eliminated.

In order to correlate all of the data from 395 nitrogen tests, it was necessary to compute temperature and pressure coefficients in order to transform the data to a uniform basis. For example, for a series of tests at 500 psia, the pressure, although constant for one test, would vary slightly from one test to another.

All of the viscosity values were corrected for the increase in diameter of the capillary due to thermal expansion of the metal at the high temperatures.

The curves, shown in Figs. 3 and 4, represent the smoothed

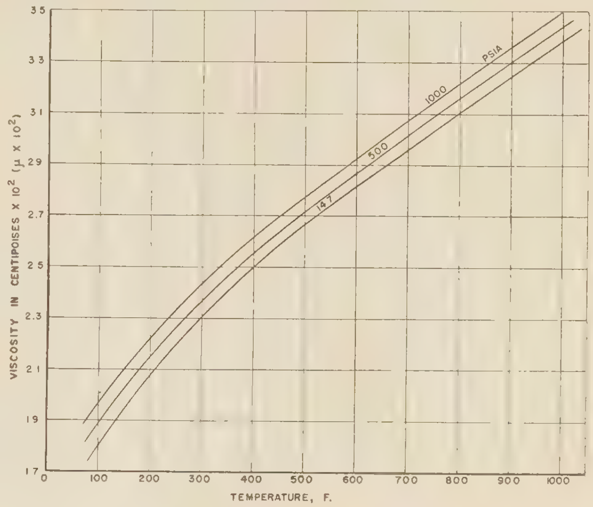

Fig. 3 Dynamic Viscosity of Nitrogen Gas

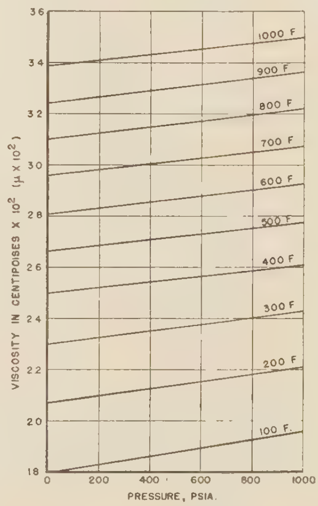

Fig. 4 Dynamic Viscosity of Nitrogen Gas

plots of data for the 395 tests. The mean deviation of test points was 2.7 per cent with one point deviating 10 per cent from the average. The next greatest deviation occurred for one test point which was 6.6 per cent.

The large amount of experimental data collected during this investigation has made it impossible, because of space limitations, to present all of the tables of test data. However, these data are available at Purdue University for examination by anyone who is interested. Thirty different manometer fluid combinations were employed in order to obtain large pressure differentials over the entire range of tests conducted. This variable alone increased the number of tables required to formulate the data.

Examination of Figs. 3 and 4 reveals that the temperature influences the viscosity of nitrogen gas far more than does the pressure. An increase in temperature from 100 to 1000 F at atmospheric pressure changes the viscosity from 1.8 (10^{-2}) to 3.38 (10^{-2}), while a change in pressure from atmospheric to 1000 psia at a constant temperature of 100 F causes an increase in viscosity from 1.8 (10^{-2}) to 1.96 (10^{-2}).

The value obtained by Rigden (2) at atmospheric pressure and a temperature of 62.6 F agrees within 2 per cent of the value obtained by the authors. Rigden did not experiment at temperatures in excess of 62.6 F. The reported values at 73.4 F and atmospheric pressure agree within 2 per cent of the value reported by Majumdar and Vajifdar (3). The Trautz and Baumann (4) results over the temperature range from 61 to 482 F at atmospheric pressure agree within less than 2 per cent of the authors' values.

The results of Boyd (6) covered ranges of temperatures and pressures from 86 to 158 F and 1050 to 2810 psia. His results at a pressure of 1050 psia and at temperatures of 86 F and 158 F are approximately 7 per cent higher than the corresponding values reported by the authors. The values presented by Michels and Gibson (7) at pressures of 226 and 850 psia through a temperature range from 77 to 167 F agree within 1 per cent of those reported in this paper.

An attempt was made to express the results herein presented by means of the equation used for determining the viscosity of gases, as based on the kinetic theory. The usual form of the equation is

$$\mu = \frac{C}{1 + \frac{1}{C}\sqrt{T}} \qquad [7]$$

The value of C is known as the "Sutherland constant;" however, it is not a true constant as it is usually expressed as a function of the temperature T.

Basing the Sutherland constant on the data obtained in the temperature range of 100 to 200 F and at atmospheric pressure, it was found that the equation gave results which were approximately 5 per cent too high at 1000 F. While an equation of this type could be used for approximating the results, an empirical equation was finally developed which fitted the results with a maximum deviation of 1.6 per cent over the entire range of pressures and temperatures investigated. This equation may be expressed as follows:

$$\mu = (A + E) + [BT + CT^2 + DT^3] \qquad [8]$$

$$E = (p - 14.7)\,[aT + bT^2 + cT^3] \qquad [9]$$

$A = -8.80 \times 10^{-3}$
$B = 7.00 \times 10^{-5}$
$C = -4.67 \times 10^{-8}$
$D = 1.283 \times 10^{-11}$
$a = 8.08 \times 10^{-9}$
$b = -1.20 \times 10^{-11}$
$c = 4.76 \times 10^{-15}$
T = absolute temperature, R
p = pressure, psia

It is hoped that the results herein presented will aid the engineer in making more accurate predictions relative to the viscosity of air and other gases, for which the viscosity data are lacking or not entirely satisfactory. The results should give a better understanding of the effect of pressure on the viscosity of gases similar to nitrogen.

ACKNOWLEDGMENT

The authors are particularly indebted to the Purdue Research Foundation for making funds available for carrying out this work.

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Discussion

K. A. Gardner.[6] In connection with the viscosity of gases at high pressure, attention should be called to a paper by Comings and Egly,[7] which presents an empirical correlation from data on many gases of the ratio μ/μ_a in terms of reduced temperatures and pressures; μ is the viscosity at high pressure and μ_a is the viscosity at atmospheric pressure and the same temperature.

In the equation previously given by Dr. Leib, which is due to Enskog,[8] the quantity b is considered constant. This being the case, if the ratio μ/μ_a for any given substance be plotted against ρ, a single curve should be obtained for all temperatures. Actually, this is not the case for Michels and Gibson's data on nitrogen for, although they report excellent agreement with Enskog's equation at any one temperature, the writer finds that separate curves are obtained for each of the three temperatures at which their data were taken. In an attempt to reduce these results to a single curve, the writer assumed that the sole cause of the variation of the viscosity ratio with temperature is the reduction of the effective diameter of the molecules with increasing temperature, thus causing a variation in b; this is consistent with Sutherland's explanation of the fact that the viscosity of gases does not vary directly with the square root of the absolute temperature, although kinetic theory indicates that it should.

Enskog and others give the following equations

$$b = \frac{2\pi\sigma^3}{3m} \qquad [10]$$

$$\mu_a = \frac{0.499m\bar{c}}{\pi\sqrt{2}\sigma^2} \qquad [11]$$

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[6] Engineer, The Griscom-Russell Company, New York, N. Y.

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$$\bar{c} = 0.921\sqrt{\frac{3kT}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [12]$$

where
σ = diameter of molecule
m = mass of molecule
$\bar{c}$ = average molecular velocity
k = Boltzmann constant

From these it may be deduced that

$$b = \frac{1.783}{M^{1/4}}\left(\frac{\sqrt{T}}{\mu_a}\right)^{3/2} \cdot 10^{-7} \quad \ldots\ldots\ldots\ldots [13]$$

where
M = molecular weight
T = absolute temperature, K
μ_a = viscosity at 1 atm and T K, poises
b = viscosity covolume, cc per g

Good correlation is obtained by plotting μ/μ_a versus $b\rho$, where b is determined from Equation [13], Fig. 5 of this discussion. The foregoing is independent of the exact form of Enskog's equation for μ/μ_a; however, a comparison may be made by reducing this equation to a series, which results in

$$= 1 + 0.175b\rho + 0.8651\,(b\rho)^2 + \ldots\ldots\ldots\ldots [14]$$

The lack of terms beyond the third should cause little error at moderate densities, so the available terms of Equation [14] are shown as the solid line in Fig. 5; it can be seen that good agreement with the nitrogen data is obtained. However, the data for carbon dioxide do not fall on the same curve and those for n-butane do not fit either one.

Inasmuch as the temperature range of Michels and Gibson's data was only from 20 C to 75 C, the test of the correlation suggested is not very rigorous, and it would be interesting to learn from the authors how their high-temperature data compare with Fig. 5 of this discussion. Should the agreement prove satisfactory, Fig. 5 could be used with confidence for nitrogen at any temperature and pressure and probably also for other diatomic gases such as oxygen, carbon monoxide, etc. As mentioned, however, the same curve does not seem to apply to more complex molecules.

M. Jakob.[9] Viscosity is one of the essential physical properties used in heat-transfer work. It occurs in the Reynolds, Prandtl, and Grashof numbers and in some other dimensionless groups. It is gratifying that the influence of high pressure, about which little only was known before, has now been determined in a reliable manner for steam and nitrogen by the group of men working together at Purdue University, and it is hoped that data for other gases will follow.

The authors corrected their values for the influence of the curvature of their coil using the results of C. M. White. It is not so generally known that according to White the critical Reynolds number increases with the ratio D_t/D_c where D_t is the inner diameter of the tube and D_c the mean coil diameter. Interpolating the values given in White's paper, the writer estimates that for the authors' coil the critical number would be about 4500, i.e., more than double the ordinary value. It might be worth while to check this by experiments at different pressures, and the writer suggests that the authors undertake this additional work.

E. F. Lype.[10] The authors present an empirical correlation of their experimental results in their Equation [8]. Nitrogen at high pressure is in the state of a dense gas. A formula for the dynamic viscosity of dense gases is available and was derived from Enskog's theory of the molecular collisions in a dense gas. The authors' data are in close agreement with this formula which, therefore, may be quoted.[11]

The ratio of the viscosity μ' at any pressure to the viscosity μ at atmospheric pressure and the same temperature is

$$\frac{\mu'}{\mu} = b\rho\left(\frac{1}{b\rho\chi} + \frac{4}{5} + 0.7614\,b\rho\chi\right)\ldots \text{(loc. cit. p. 286)}$$

where
ρ = density
$b = \frac{{}^2/_3\pi\sigma^3}{m}$ van der Waals' covolume... (loc. cit. p. 274)
m = mass of one molecule, from molecular weight
σ = closest approach of center of molecules during collision, nearly equal to diameter of molecule, but not exactly a constant
χ = a factor greater than unity, accounting for reduction of space available for motion of a molecule (loc. cit. p. 275)

When the p, v, T relations of the gas are known, the combined quantity $b\rho\chi$ is obtained from a modified van der Waals' equation of state

$$p + a\rho^2 = RT\rho(1 + b\rho\chi)\ldots\ldots\ldots \text{(loc. cit. p. 288)}$$

by differentiation
$$\left(\frac{\partial p}{\partial T}\right)_\rho = R\rho(1 + b\rho\chi)$$

When the change of pressure with temperature is known from observation, the quantity $b\rho\chi$ can be calculated and substituted in the viscosity equation. Usually, the variation of σ or, respectively, b with pressure is not known, and the molecular diameter is substituted for σ if the viscosity relation is used to calculate μ'/μ. If the viscosity ratio is known from observation the variation of b or σ with pressure can be calculated from the viscosity equation. This determines a in the equation of state.

To calculate the viscosity μ' directly, a relation for viscosity at atmospheric pressure and various temperatures is needed. Various modifications of Sutherland's formula have been suggested. The following formula gives satisfactory results:

$$\mu = \frac{5}{16\sigma^2}\left(\frac{km}{\pi}\right)^{1/2} \cdot \frac{T^{3/2}}{S + T}\ldots\ldots\ldots\ldots \text{(loc. cit. p. 223)}$$

where k = Boltzmann's constant.

Attention may be called to the relatively small effect of pressure on the viscosity of a substance in the state of a dense gas, as compared with the considerable effect of pressure on the viscosity of a substance in the vapor state of moderate density, e.g., superheated steam.[12]

Authors' Closure

The authors wish to express their sincere appreciation for the instructive written, as well as oral, discussions which have been presented relative to this work.

A reliable formula for the dynamic viscosity of gases over a wide range of pressures and temperatures is highly desirable, but additional data at various temperatures and pressures should be obtained for a number of other gases before a general equation can be developed upon which reliance may be placed.

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[9] Research Professor of Mechanical Engineering, Illinois Institute of Technology and Armour Research Foundation, Chicago, Ill. Mem. A.S.M.E.

[10] Analytical Research Engineer, DeLaval Steam Turbine Company, Trenton, N. J. Dr. Lype has previously written papers for the Transactions under his former name, E. F. Leib.

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[12] "The Viscosity of Superheated Steam," by G. A. Hawkins, H. L. Solberg, A. A. Potter, and discussion by E. F. Leib, Trans. A.S.M.E., vol. 62, 1940, pp. 677–684.

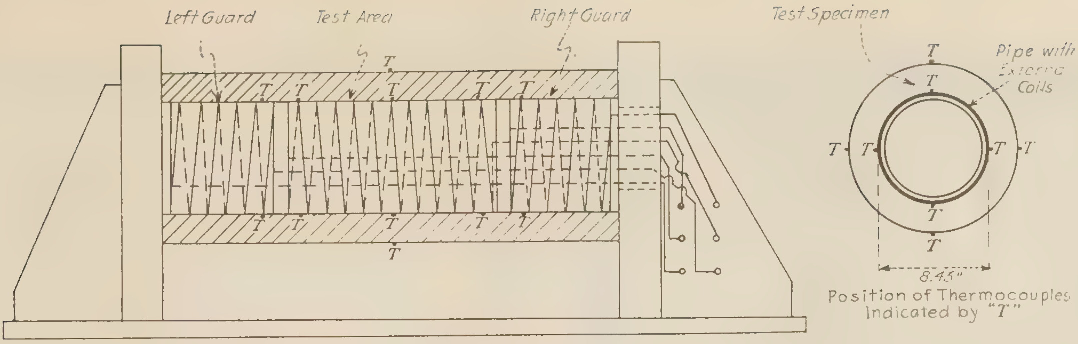

FIG. 1 DIAGRAM OF 8-IN. TESTING APPARATUS

Tests of Steam-Pipe Insulation

By E. A. ALLCUT,[1] TORONTO, CANADA

In connection with various war projects some questions arose which made it necessary to design an apparatus suitable for testing the heat transmission through cylindrical samples of various kinds of thermal insulating material used for protecting hot pipes. With this apparatus the properties of spun rock wool (which is used as an alternative to magnesia) were investigated with regard to the effects of density, binder, and thickness. Other experiments were made on glass wool and corrugated asbestos. The influence of wind and the protection of the outer surface of the insulation by metallic coverings were investigated, as also was the protection of fabric coverings by paints of various kinds and colors. It was found that even bright metals such as copper and aluminum made but slight difference in the heat losses, and that most paints increased the losses as compared with those from the unpainted fabric. The details are given in the seven tables and seventeen curves accompanying this paper.

Introduction

PROBLEMS which arose in connection with the construction and operation of plants for war industries made it desirable to obtain, by actual test, the heat-transmission characteristics of some of the materials used for the insulation of pipes conveying steam and other hot fluids. Many of these pipes were exposed to wind and weather and therefore it was desirable that the length of the test specimen be restricted to 3 ft or less, so that air could be drawn across its outside surface by means of a fan which was available for that purpose.

The obvious difficulties of controlling steam pressures and temperatures, and the objection to the use of a long pipe, prompted the decision to employ electric heating which was easier than steam to install and control. Moreover, an apparatus that was small, light, and self-contained could be made portable and could be placed in any convenient location. The apparatus as finally designed and constructed, Fig. 2*a*, fulfilled these conditions, was provided with electrical energy from an ordinary lighting fixture, and could be operated with a single measuring instrument, i.e., a portable potentiometer. It was realized that with so short a pipe length the greatest experimental difficulties were likely to be experienced with specimens of large diameter, and therefore a pipe diameter (external) of 8.43 in. was selected for the first series of tests. The weights of the insulation and wiring were supported by a porcelain tube 8 in. diam, supported at its ends on two transite blocks, each 3 in. thick, Fig. 1. On the outsides of these were mounted cork slabs, each 2 in. thick, to reduce "end losses." The heating coils were wound on the tube in three sections, the central one being 15 in. long and each of the two end sections $7^1/_2$ in. long, leaving a space of 1 in. between each pair of coils.

All measurements of heat losses were made on the central coil, the end, or guard, windings being separately controlled by rheostats, so that there was no drop in temperature at the two ends of the 15-in. section. Thus no heat traveled along the test specimen, and the electrical energy supplied to the center section indicated the quantity of heat flowing radially through the material. This was controlled by means of five sets of four thermocouples each.

Two sets of these were situated $^3/_8$ in. inside and $^3/_8$ in. outside the central test section at each end, respectively, with one set in the center, Fig. 1. The temperatures were adjusted by wire rheostats which varied the current through each of the three windings. The temperature of the outer surface of the pipe covering was measured in the center by four shielded thermocouples placed at equal distances around the circumference. Preliminary tests showed that this number was sufficient, as there was practically no change in temperature along the 15 in. length, provided that the internal temperatures were properly adjusted.

The tests were made at different internal temperatures both with natural air circulation and with wind velocities up to 20 mph, Table 1. It was found that with magnesia pipe covering $1^1/_2$ in. thick, there was little difference in the heat losses with wind speeds varying from 11 to 20 mph, and therefore most of the remaining "wind" tests were made at air speeds of about 15 mph. The position of the apparatus was also varied to determine if the change of natural air circulation with the pipe held horizontally in various positions affected the results materially, but no appreciable difference was observed.

In view of previous experiences with tests on flat samples of in-

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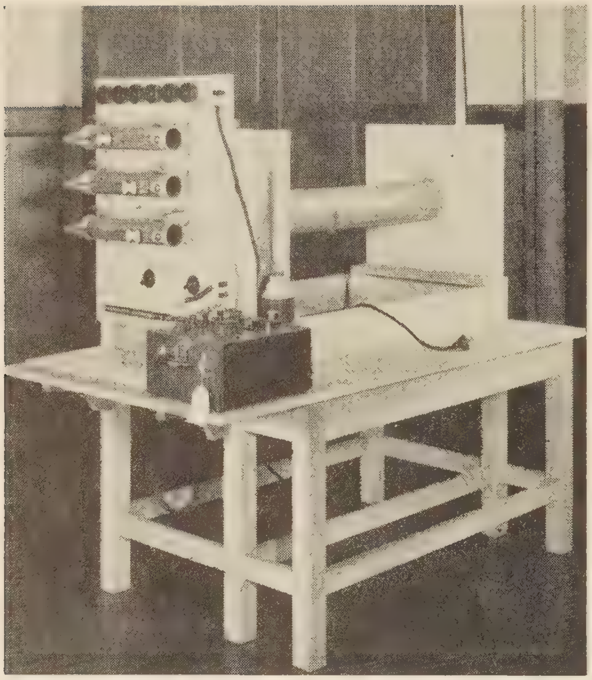


Fig. 2(a) View of 2-In. Testing Apparatus

thermocouples to be used. This expectation was realized in practice, and the 2-in. apparatus was used exclusively in the later tests, Tables 4 to 7.

Comparison of Results on 8-in- and 2-in-Diam Apparatus

The first tests were made on the 8-in. apparatus with 85 per cent magnesia pipe covering $1^1/_2$ in. thick, covered with white canvas, and the results obtained are shown in Fig. 3. The mean temperature in all curves is the arithmetic mean between the surface temperatures of the inside and outside of the pipe covering.[3] This is a convenient basis of comparison, because the temperature gradient through the material is very nearly a straight line (2). The "conductivity" k is expressed in British thermal units per hour per degree temperature difference between the inside and outside surfaces, per inch of thickness per square foot of logarithmic-mean area. This is a method frequently employed for evaluating these materials but is of doubtful utility from a comparative standpoint. Conductivity should be solely a characteristic of the material tested, but the results of this calculation on a given material vary also with external conditions.

The sample of magnesia used in these tests had a density of 10.5 lb per cu ft, and the results obtained were so close to those of Gard (3) for magnesia of 12 lb per cu ft, that it was impossible to draw two curves between them, Fig. 3 (curve C). Similar results obtained by Jakeman (4) curve D, and others published by the Atlas Asbestos Company, (5) curve E, are also included for comparative purposes. Other curves given by McMillan (2), McAdams (6), and others, curve A, and from the latest edition of Kent's Handbook (7), curve B, show higher conductivities than

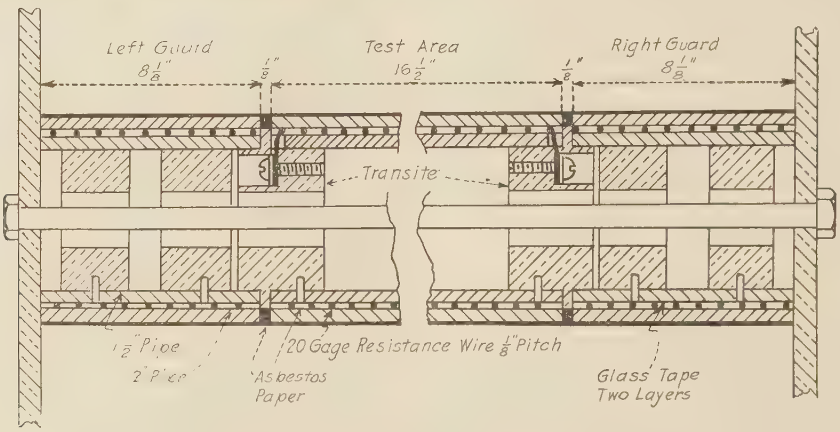


Fig. 2(b) Construction of Heating Elements in 2-In. Testing Apparatus

sulating material (1),[2] it was questionable as to whether the results obtained on large pipe coverings would also be applicable to small ones. For this reason, a similar apparatus was constructed to test coverings suitable for pipes of 2 in. ID. As trouble had been experienced due to cracking of the porcelain tube in the larger apparatus, this was replaced by concentric sections of 2-in. and $1^1/_2$-in. steel tubing supported on transite blocks and held together by a central bolt, Fig. 2(b). The resistance windings were placed between these two steel tubes, and the central section, which was $16^1/_2$ in. long, was separated from the end sections by $^1/_8$-in. gaps filled with asbestos paper. It was hoped that this arrangement would be more durable than the porcelain tube and that the high conductivity of the metal would assist in the uniform distribution of heat and would enable a smaller number of

these, but the latter probably apply to pipe coverings averaging about 17 lb per cu ft, as this was the density generally used in American practice.

Further tests were made in the 2-in. apparatus on 85 per cent magnesia pipe covering having a density of 12 lb per cu ft and about 1.2 in. thick; the results of these, Table 4, are included in Fig. 3, curve F. The two curves F and C are so close together that the difference between them is negligible. For convenience, these two curves are reproduced to a larger vertical scale in Fig. 4. The consistency of these and other test results appeared to indicate that the method employed was correct within reasonable commercial limits.

Other tests were made on spun rock wool, the sample being machine-made, 1.1 in. thick, and having a density of 10.3 lb per

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

[3] For convenience, the approximate pipe temperature for an air temperature of 70 F is also included.

TABLE 1 RESULTS OF TESTS ON 8-IN-DIAM PIPE INSULATION

Air Vel. Miles per Hour	t_a Air Temp. °F.	t_1 Pipe Temp. °F.	t_2 Surface Temp. °F.	$t_1 - t_2$ Pipe to Surface Temp. Diff. °F.	$t_1 - t_a$ Pipe to Air Temp. Diff. °F.	Mean Temp. °F.	BTU per sq.ft. per °F. per in. per hr. K	BTU per sq.ft. per hr. Pipe Surface	Rel. Hum. %	$t_2 - t_a$ Surface to Air Temp. Diff. %	BTU per sq.ft. per hr. External Surface
(A)	85% Magnesia, Canvas Covered; Approx.thickness, 1.5 in.; External diameter, 11.40 in. Density, 10.5 lb.per cu.ft.										
0	69	340	107	233	271	223	.437	80	59	38	59
0	81	447	129	318	366	288	.458	114	43	48	84
0	73	446	125	321	373	285	.472	119	38	52	88
0	86	538	143	395	452	340	.486	151	52	57	111.5
0	80	657	162	495	577	410	.525	204	..	82	151
0	98	987	226	761	889	607	.633	407	..	128	300
18	76	325	84	241	249	204	.427	81	39	8	60
14.6	71	328	78	250	257	203	.412	81	53	7	60
11.3	72	335	85	250	263	210	.432	85	53	13	63
15	78	399	89	310	321	245	.430	105	45	11	77
20	80	431	92	339	351	262	.446	119	35	12	88
15	87	519	103	416	432	311	.469	153	40	16	113
15	70	587	89	498	517	339	.487	190	64	19	140
15	86	640	110	530	554	375	.504	208	50	24	153.5
(B)	85% Magnesia, Aluminium Covered; Approx.thickness, 1.5 in.; External diameter, 11.40 in. Density, 10.5 lb.per cu.ft.										
0	83	340	132	208	257	238	.415	67.8	49	49	50
0	88	500	168	332	412	334	.457	119	61	80	88
0	84	616	188	428	532	400	.492	165	42	104	122
15	85	490	100	390	405	295	.451	138	38	15	102
15	86	705	116	589	619	411	.512	236	45	30	174
(C)	85% Magnesia, Copper Covered; Approx.thickness, 1.5 in., External diameter, 11.40 in. Density, 10.5 lb.per cu.ft.										
0	89	515	173	342	426	344	.460	123	60	84	91
0	93	623	201	422	530	412	.495	164	55	108	121
0	98	1019	306	713	921	660	.699	390	53	208	288
x	82	485	157	328	403	321	.455	117	55	75	86
x	84	629	188	441	545	409	.488	167	61	104	123
15	92	501	105	396	409	303	.454	141	50	13	104
15	90	617	109	508	527	363	.471	188	53	19	139
15	85	946	121	825	861	532	.600	389	42	36	287
(D)	Spun Rock Wool Blanket; Iron Covered, Aluminium Painted; Approx.thickness, 1 in. External diameter, 10.40 in., Density 12.0 lb.per cu.ft										
0	75	170	85	85	95	128	.291	28	..	10	23
0	85	400	141	259	315	270	.410	120	..	56	97
0	83	710	216	494	627	463	.548	306	..	133	252
0	83	978	298	680	895	638	.745	571	..	215	463
15	75	165	72	93	90	119	.304	32	51		26
15	79	397	82	315	318	239	.382	136	36	3	110
15	76	714	95	619	639	405	.506	354	20	19	287
15	71	985	111	874	914	548	.656	648	39	40	525
(M)	Spun Rock Wool Blanket; Iron Covered, Aluminium Painted; Approx.thickness, 2 in. External diameter, 12.49 in., Density, 12.0 lb.per cu.ft										
0	73	190	83	107	117	136.5	.327	21.1	..	10	14
0	70	266	84	182	196	175	.354	38.9	..	14	26
0	71	378	96	282	307	235	.389	65.2	..	25	44
0	75	380	101	279	302	239	.401	67.5		26	45
0	83	829	183	646	746	506	.663	258	52	100	174
0	82	1021	223	798	939	622	.825	397	65	141	268
15	76	631	85	546	555	358	.495	163	39	9	110
15	70	829	87	742	759	459	.613	274	63	17	185
15	77	981	98	883	904	539	.721	384	60	21	258
(G)	Spun Rock Wool Blanket; Iron covered, Aluminium painted; Approx.thickness 2.9 in. External diameter, 14.20 in., Density 7.62 lb.per cu.ft										
0	75	220	82	135	145	151	.352	21.6	56	7	13
0	83	680	135	545	597	407	.630	156	48	52	92.5
0	86	998	190	808	912	594	.963	354	46	104	210
(R)	Spun Rock Wool, Machine Made, Canvas covered; Approx.thickness, 1.2 in. External diameter 10.75 in., Density 9.4 lb.per cu.ft.										
0	87	185	94	91	98	139	.317	29.2	..	7	23
0	93	431	133	298	338	282	.415	126	42	40	99
0	81	717	179	538	636	448	.555	304	42	98	238
0	91	891	222	669	800	556	.710	481	..	131	377
(P)	Spun Rock Wool, Machine made, canvas covered; Approx.thickness, 1.? in. External diameter 10.82 in., Density 10.5 lb.per cu.ft										
0	76	187	84	103	111	135	.318	31.2	30	8	24
0	81	417	123	294	336	270	.436	122	25	42	95
0	79	712	180	532	633	446	.600	303	45	101	236
0	81	900	227	673	819	564	.746	478	55	145	372
(N)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness, 1.5 in. External diameter 11.40 in., Density 13.0 lb.per cu.ft										
0	65.5	165.5	70	95.5	100	118	.33	24.7	..		
0	81	423	111	312	342	267	.414	101	..	30	75
0	79	703	156	547	624	429	.575	247	..	77	183
0	89	718	166	552	629	442	.595	258	37	77	191
0	100	974	221	753	874	599	.804	475	39	121	351
(L)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness, 2.0 in. External diameter 12.49 in., Density 7.5 lb.per cu.ft										
0	76	188	76	112	112	132	.343	23.2	57		
0	83	480	111	369	397	295	.452	101	46	28	68
0	90	776	155	621	636	465	.630	276	33	65	159
0	97	980	200	780	883	590	.798	376	45	103	254
(K)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness, 2.2 in. External diameter, 12.83 in, Density 14.0 lb.per cu.ft.										
0	74	203	76	127	129	139	.386	27.7	43		
0	82	191	81	110	109	136	.383	23.8	35		
0	84	417	105	312	333	261	.469	82.7	51	21	54
0	89	748	148	600	659	448	.632	214	55	59	140
0	88	976	185	791	888	580	.775	346	25	97	226
0	87	1014	191	823	927	604	.799	371	35	104	243
(H)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness, 2.3 in. External diameter 13.07 in., Density 9.0 lb.per cu.ft.										
0	82	199	87	112	117	143	.369	22.4	..	5	14
0	85	454	112	342	369	283	.462	85.5	..	27	55
0	85	698	141	557	613	419	.585	177	41	56	
0	98	944	188	756	846	566	.791	324	43	90	209
(E)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness, 312 in. External diameter 14.80 in., Density 13.0 lb.per cu.ft										
0	72	220	74	146	148	147	.379	23.4	31		
0	82	461	98	363	379	279	.479	73.5	41	16	42
0	92	975	162	813	883	569	.762	261	..		148
(F)	Spun Rock Wool, Machine made, Canvas covered; Approx.thickness 3.25 in. External diameter 14.92 in., Density 9.7 lb.per cu.ft.										
0	82	245	85	160	163	165	.363	24.2	39		
0	87	381	97	284	294	238	.401	47	62	10	26.5
0	95	766	133	633	678	450	.585	154	52	38	87
0	91	1027	159	868	936	593	.760	272	63	68	153

x Indicates tests in which the specimen was removed from the wooden frame so that convection was entirely unimpeded. This did not produce any great difference in the results obtained.

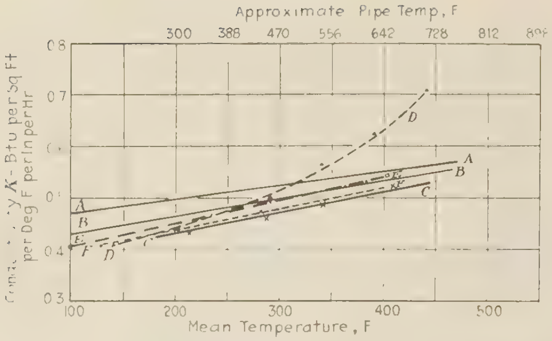


Fig. 3 Comparative Tests on 85 Per Cent Magnesia Pipe Covering

A McMillan; 17 lb per cu ft density (approx)
B Results from Kent's Handbook; 17 lb per cu ft
C Gard, 6 in. diam and Allcut, 8 in. diam; approximately 10–12 lb per cu ft
D Jakeman
E Atlas Asbestos Company
F Allcut, 2 in. diam; approximately 10–12 lb per cu ft

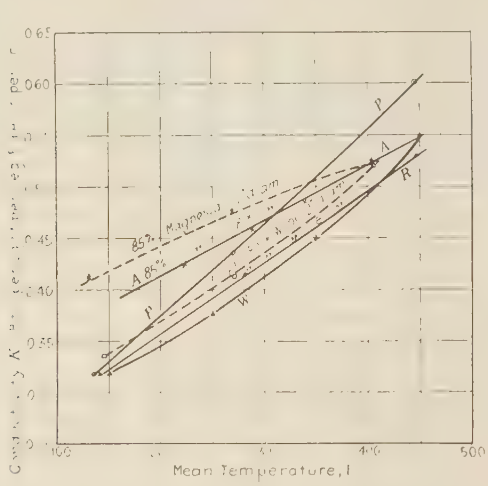


Fig. 4 Comparative Tests on Magnesia and Spun Rock Wool

P Spun rock wool, 8 in. diam; 1.2 in. thick, 10.5 lb per cu ft
R Spun rock wool, 8 in. diam; 1.2 in. thick, 9.4 lb per cu ft
W Spun rock wool, 2 in. diam; 1.1 in. thick, 16.5 lb per cu ft

cu ft. This is compared, in Fig. 4, with the results obtained on similar samples of spun rock wool[4] (series *P* and *R*, Table 1), using the 8-in. apparatus. Here again, there appears to be no material difference in the values of k, as calculated from observations made on the two sizes of specimen.

It is noteworthy, Fig. 3, that apart from the Jakeman (4) results, curve *D*, the curves for magnesia are approximately parallel, indicating that the temperature coefficients are about the same, irrespective of differences of size and density.

For convenience, curves *A* and *C* have been reproduced on several of the other diagrams, as they are considered to be representative of the results obtainable on 85 per cent magnesia of 17 and 12 lb per cu ft density, respectively.

Spun Rock Wool; Effects of Density and Thickness

The curves in Figs. 5 and 6 show the results obtained on samples of spun rock wool when tested in the 8-in. apparatus, and the letters on the curves refer to the series letters in Table 1. In Fig. 5, the heat lost per square foot of pipe surface (internal area of pipe covering) per hour has been plotted against the mean temperature of the material. The quantity of rock wool present in each "thickness" group is about the same, differences in density being due mostly to the varying amounts of "binder" present. Curves *P* and *R* refer to thicknesses of about 1.2 in. and, as their densities are similar, there is no great difference in the results obtained. The apparent difference in their respective conductivities, indicated in Fig. 4, is partly due to the large vertical scale, and possibly also to differences in the physical structure of the two samples (8). Curves *L*, *K*, and *H* form another series for specimens varying from 2 to 2.3 in. thick with densities varying from 7.5 to 14 lb per cu ft. It is evident that the difference in density does not affect the results seriously in this material.

A similar observation applies to curves *E* and *F* for samples 3.2 in. thick and different densities. A high density increases the weight and stiffness of the heat insulation but, within the limits of these tests, does not appear to influence considerably its heat-transmission properties. It was noted, however, in the case of samples heated internally to temperatures of about 1000 F that those materials containing large amounts of binder became more brittle inside than did those with smaller amounts of binder.

The curves marked *N* and "magnesia," respectively, give comparative results obtained on spun rock wool and magnesia of about the same thickness and density. There is little difference between them up to a mean temperature of about 320 F (pipe temperature approximately 500 F), spun rock wool having a slight advantage. Magnesia is slightly better above that temperature, the difference between them at 375 F (pipe temperature approximately 600 F) being about $5^1/_2$ per cent. The rock wool, however, can apparently be used over a wider range of temperatures, as prolonged heating (without vibration) for about 30 hr at 1000 F did not appear to produce appreciable disintegration, unless there was an excessive amount of binder present.

The influence of thickness on the "conductivity" of rock wool is indicated in Fig. 6. Here the specimens were of the same internal diameter, but had thicknesses varying from 1.16 to 3.25 in. The curves are all of similar form, indicating an increasing temperature coefficient as the mean temperature rises. Curves *A* and *C* are reproduced from Fig. 3 to give a comparison between the various samples of rock wool and 85 per cent magnesia of different densities and 1.5 in. thickness. The results obtained indicate that the method of packing or arranging the fibers has a more important bearing upon the conductivity than does the thickness. Curves *H* and *L* refer approximately to the same thickness, and the lighter material *L* gives slightly lower conductivities than does the heavier *H*, but the difference between them is very small. Curves *S*, *T*, and *U* were obtained on the 2-in. apparatus, and in this series the conductivities were lower in spite of the higher density of the material.

The curve for the light magnesia crosses the rock-wool curves for the 8-in. apparatus in the mean temperature range 290 to 410 F (pipe temperature 450 to 650 F), and those for the heavy magnesia at 380 to 460 F (pipe temperature 600 to 750 F). Below these temperatures rock wool has a lower conductivity than magnesia.

TABLE 2 APPROXIMATE PERCENTAGE INCREASE OF HEAT LOSS FROM 8-IN. INSULATED PIPE WHEN SURFACE IS EXPOSED TO A WIND OF 15 TO 20 MPH

Series	Description	Temperature difference, pipe to air, deg F				
		250	350	450	550	750
A	$1^1/_2$-in-thick magnesia, canvas covered	11	9.5	8	7	..
B-C	$1^1/_2$-in-thick magnesia, aluminum or copper covered	..	..	15	15	..
D	1-in-thick spun rock wool, iron covered, aluminum paint on outside	16	13.5	12.5	12	12
M	2-in-thick spun rock wool, iron covered, aluminum paint on outside	..	..	..	4.5	2.5

[4] See Appendix.

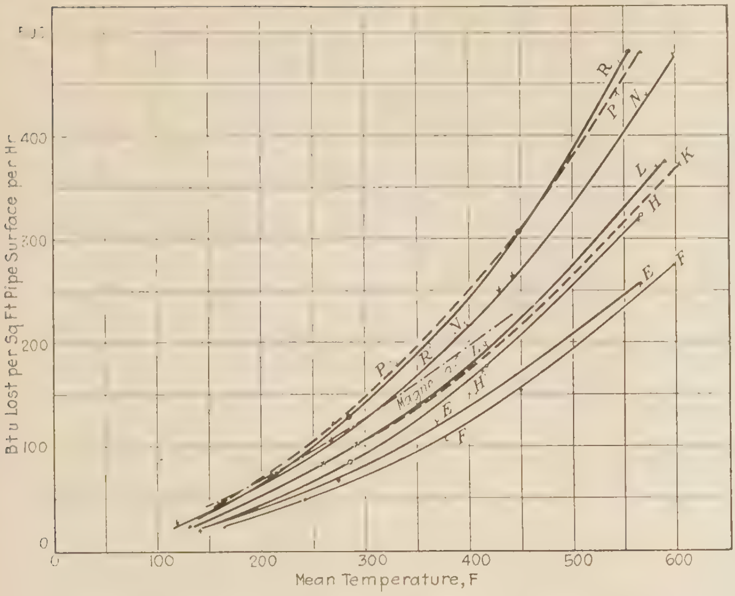


Fig. 5 Effect of Thickness and Density on Heat Transmission of Spun Rock Wool; 8 In. Diam

R Thickness 1.2 in.; density 9.4 lb per cu ft
P Thickness 1.2 in.; density 10.5 lb per cu ft
L Thickness 2.0 in.; density 7.5 lb per cu ft
K Thickness 2.2 in.; density 14.0 lb per cu ft
H Thickness 2.32 in.; density 9.0 lb per cu ft
E Thickness 3.12 in.; density 13.0 lb per cu ft
F Thickness 3.25 in.; density 9.7 lb per cu ft
N Thickness 1.5 in.; density 13.0 lb per cu ft
Magnesia 1.5 in.; density 10.5 lb per cu ft

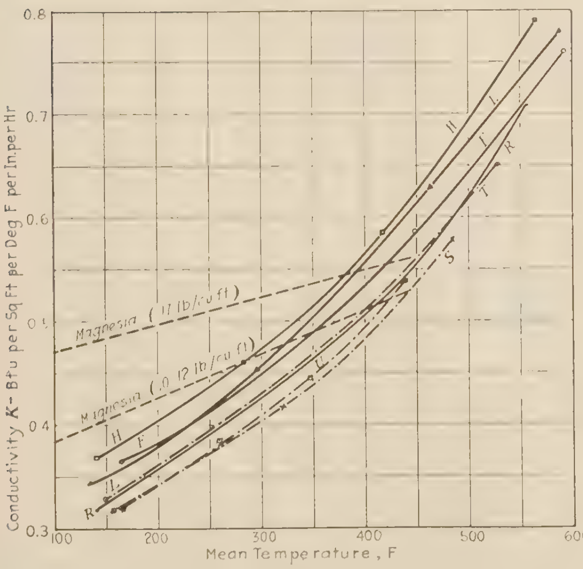


Fig. 6 Effect of Thickness and Size on Conductivities of Machine-Made Spun Rock Wool; 8 In. and 2 In. Diam

H Thickness 2.3 in.; density 9.0 lb per cu ft; 8 in. diam
L Thickness 2.0 in.; density 7.5 lb per cu ft; 8 in. diam
F Thickness 3.25 in.; density 9.7 lb per cu ft; 8 in. diam
R Thickness 1.2 in.; density 9.4 lb per cu ft; 8 in. diam
S Thickness 1.07 in.; density 16.6 lb per cu ft; 2 in. diam
T Thickness 1.10 in.; density 16.1 lb per cu ft; 2 in. diam
U Thickness 1.10 in.; density 16.9 lb per cu ft; 2 in. diam

Effect of Wind

The results of experiments, made with air approaching the pipe at a velocity of about 16 mph, and with various kinds of metallic protection on the outside of the insulation, are given in Fig. 7, in which the heat loss per square foot of pipe surface (inner surface of covering) is plotted against the difference in temperature between the atmospheric air (t_a) and the mean temperature of the inner surface (t_1) of the insulation. The comparative losses with still air and with air approaching the surface at a mean velocity of 15 to 20 mph are given in Table 2. Curves *A* refer to 85 per cent magnesia (series *A*, Table 1) covered with canvas. The tests were repeated with the same samples covered with thin aluminum foil (series *B*) and electrolytic copper (series *C*), respectively. As might be expected, it makes little if any, difference which of these metals is employed, as only one curve can be drawn through the points. Curves *D* and *M* refer, respectively, to spun-rock-wool blankets approximately 1 and 2 in. thick, surrounded by a sheet of 28-gage black iron, painted inside and out with one coat of rust-resisting paint and one coat of aluminum paint. The flanged joint of this cover was placed downstream to avoid interference with the air flow. The distribution of air velocities was taken with a pitot tube and is shown in Fig. 8. The readings on the table at the right of Fig. 8 were taken with a pitot tube about 16 in. in front of the pipe covering and indicate a mean air velocity of 16.5 mph. The readings and velocities on the left side of Fig.

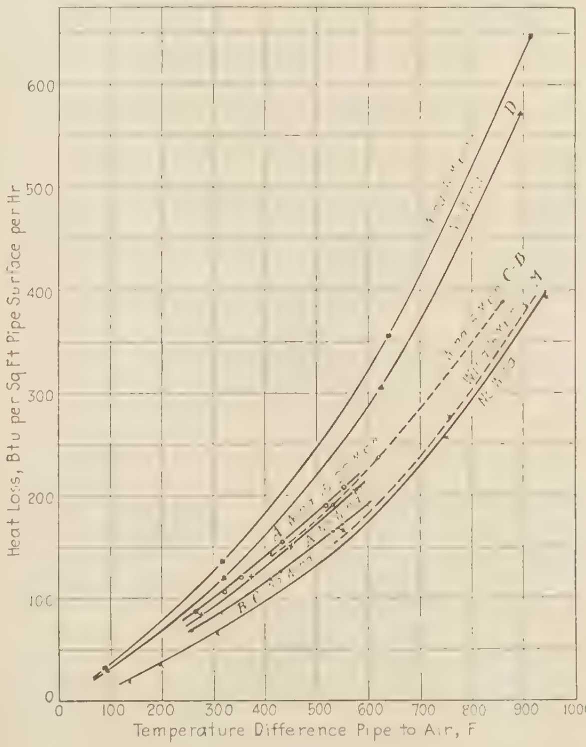
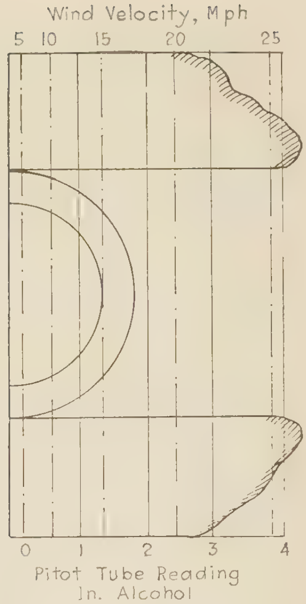


Fig. 7 Effect of Wind on Heat Losses From Insulating Covering on 8-In. Pipe

D Spun rock wool 1 in thick, aluminum paint on sheet iron
A Magnesia 1½ in. thick, canvas covered
B Magnesia 1½ in. thick, aluminum covered
C Magnesia 1½ in. thick, copper covered
M Spun rock wool 2 in. thick, aluminum paint on sheet iron



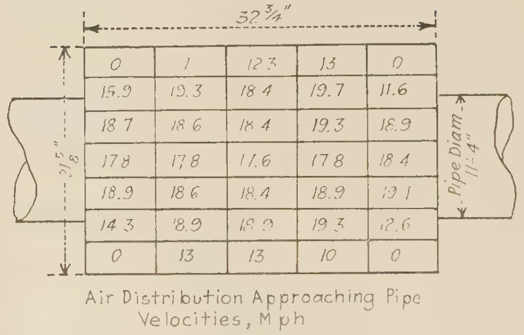


Fig. 8 Distribution of Air Velocities for Wind Tests

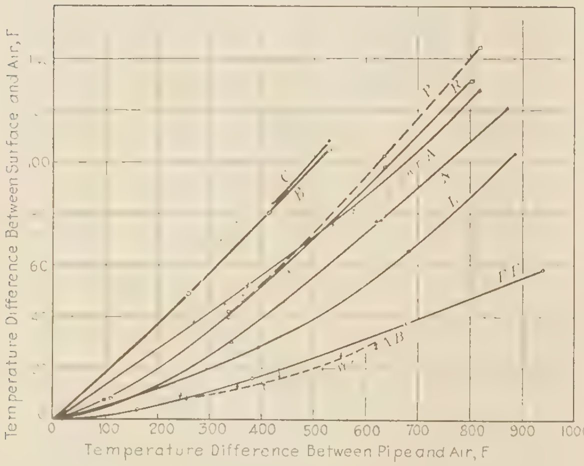


Fig. 9 Inside and Outside Surface Temperatures (Test Designations as in Table 1)
(8-in-diam pipe. Symbols as in Table 1.)

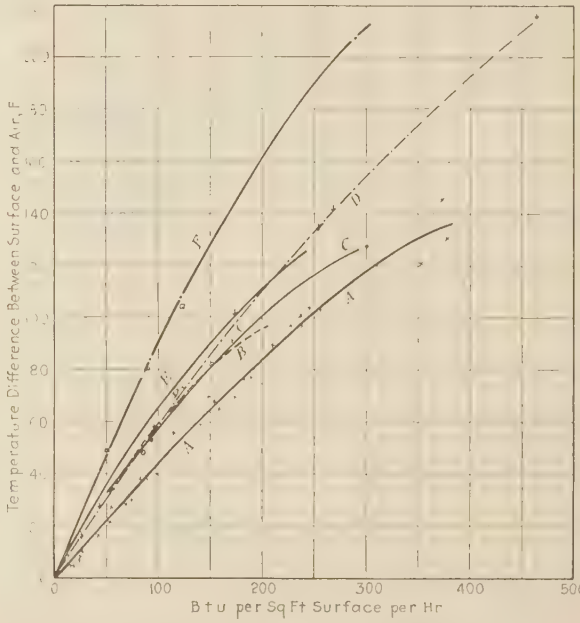


Fig. 10 Heat Losses From Outside Surfaces of 8-In-Diam Pipe Covering

A Spun rock wool, cotton cover
B Magnesia (Heilman), canvas cover
C Magnesia (Allcut), canvas cover
D Spun rock wool, aluminum paint on sheet iron
E Magnesia (McMillan), canvas cover
F Magnesia (Allcut), copper and aluminum covers

8, were taken at the minimum areas available for air flow above and below the pipe covering.

The effect of wind in reducing the mean temperature of the outside surface is shown in Fig. 9, where the difference of temperature ($t_2 - t_a$) is plotted against ($t_1 - t_a$) for the different series of experiments (Table 1). Separate curves are drawn for canvas covered (*A*) and aluminum covered (*B*) magnesia with still air, and one average curve (*AB*) is drawn for both of them with wind velocities of about 16 mph. When exposed to wind the temperature difference between outer surface and air was reduced to 20 to 25 per cent of its "still-air" value. This reduction was greater in the case of metal coverings than with canvas covers.

Surface Effects in Still Air

The other curves in Fig. 9 give the appropriate temperature differences for various thicknesses of spun rock wool (machine-made samples), as detailed in Table 1. The temperature differences between surface and air become less with increasing thick-

TABLE 3 REDUCTION OF HEAT LOSS FROM SURFACE WHEN METALLIC COVERING IS SUBSTITUTED FOR CANVAS ON OUTSIDE SURFACE

Material	Temperature difference between surface and air, deg F	Btu per sq ft per hr transmitted from outer surface: Canvas	Metal	Reduction of superficial heat loss due to metallic covering, per cent
85 per cent magnesia.....	50	87	53	39
(Curves *C* and *F*, Fig. 10).	75	136	81	40
(Covering, aluminum or	100	197	112	43
Copper)............	125	285	146	49
Spun rock wool..........	50	114	86	25
(Curves *A* and *D*, Fig. 10)	75	175	130	26
(Covering, iron sheet	100	242	179	26
painted with aluminum)	125	325	230	29

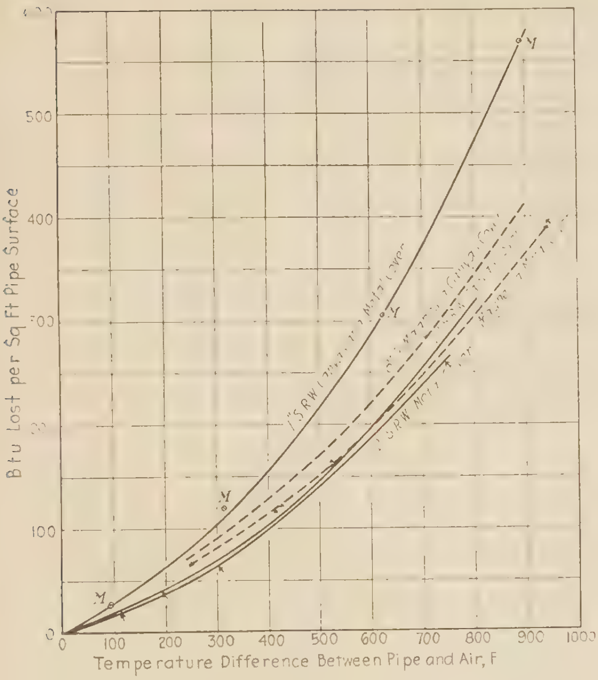


FIG. 11 HEAT LOSSES FROM OUTSIDE SURFACES OF 8-IN-DIAM PIPE COVERING

(M Spun rock wool 1 in. thick; aluminum paint on sheet iron)

ness, and the figures given enable the surface losses to be calculated from canvas coverings to still air (with natural convection).

The heat losses per square foot of external surface with various values of $(t_2 - t_a)$ are given in Fig. 10. Curves B and E, respectively, give comparative results obtained by Heilman (9) (9.5 in. OD) and McMillan (2) (7 to 11 in. OD) on samples of 85 per cent magnesia. The results obtained in the 8-in. apparatus (11.4 in. OD) are shown in curve C, and these agree very closely with Heilman's results. The effect of substituting a copper or aluminum covering for the canvas cover on 85 per cent magnesia is indicated by the differences between curves C and F (Fig. 10 and Table 3).

Average results obtained, respectively, on spun rock wool with canvas and iron covers painted externally with aluminum, are shown in curves A and D, and the approximate reduction of superficial heat loss resulting from the substitution of the iron-aluminum surface for canvas is also given in Table 3.

While this is interesting scientifically, engineers are more concerned with the quantity of steam that will be condensed per hour under operating conditions. This is indicated by replotting the same results on the basis of the temperature difference $(t_1 - t_a)$, as shown in Fig. 11, which indicates that the saving in heat loss obtained by using bright copper or aluminum sheet over the canvas cover surrounding $1^1/_2$ in. of magnesia is only about 11 per cent. The corresponding saving in the case of 2 in. of rock wool covered with aluminum-painted iron is less than 8 per cent, and for a thickness of 1 in., there is no saving at all, as the two curves are identical. The reason for this is, apparently, that for any given value of t_1 the lower emissivity of the metallic outer covering is compensated for, wholly or partially, by the greater temperature difference $(t_2 - t_a)$. Thus an external metallic covering does not affect the heat loss greatly, its value, if any, being mechanical rather than thermal.

It was then decided to try the effects of painting the external surface of the fabric covering. A series of tests was made with the 2-in-diam apparatus on three samples of spun rock wool (machine made) 1.1 in. thick, having densities of approximately 16.5 lb per cu ft, Table 5. When the values of conductivity against mean temperature were plotted, they agreed so closely with each other and with the results on the 8-in. specimens that it was decided to draw a mean curve to indicate the thermal characteristics of these samples. They were then painted externally as follows:

Sample 1; one coat of size and one of flat brown paint
Sample 2; one coat of size and one of flat white paint
Sample 3; one coat of size and one of aluminum paint

The results of these tests are given in Table 5. It was found that the calculated conductivities for the painted specimens in all cases were higher than those for the unpainted specimens. This increase in samples 1 and 2 varied from 13 per cent at a mean temperature of 200 F to 30 per cent at 400 F. Sample 3, painted

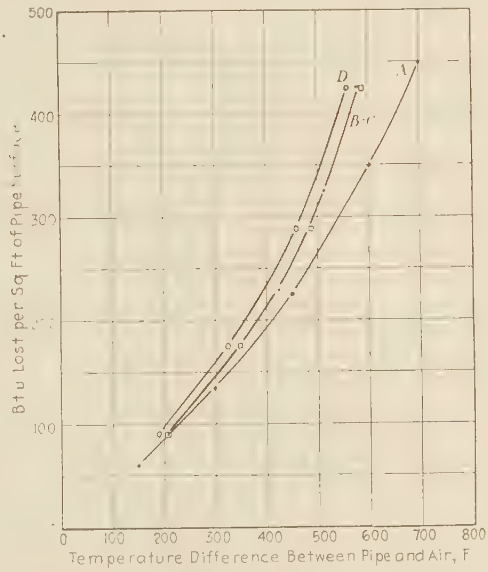


FIG. 12 HEAT LOSSES FROM PAINTED AND UNPAINTED SURFACES ON 2-IN-DIAM PIPE COVERING

A Average of unpainted specimens
B Flat brown paint, turpentine base
C Flat white paint, turpentine base
D Aluminum paint, turpentine base

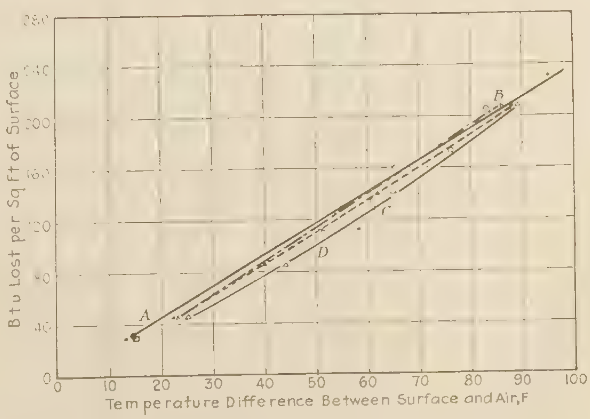


FIG. 13 HEAT LOSSES FROM PAINTED AND UNPAINTED SURFACES ON 2-IN-DIAM PIPE COVERING
(Symbols as in Fig. 12.)

TABLE 4 RESULTS OBTAINED ON SPECIMENS OF INSULATION FOR 2-IN-DIAM PIPE,[a] "STILL-AIR" CONDITION

t_a Air temp, F	t_1 Pipe temp, F	t_2 Surface temp, F	$(t_1 - t_2)$ Pipe to surface temp. diff, F	$(t_1 - t_a)$ Pipe to air temp diff, F	Mean temp, F	Conductivity, k	Btu per sq ft per hr, pipe surface	Relative humidity, per cent	$(t_2 - t_a)$ Surface to air temp diff, F	Btu per sq ft per hr external surface
85 Per Cent Magnesia[b]										
73	180	82	98	107	131	0.410	49	45	9	24
76	416	116	300	340	266	0.474	173	..	40	85
78	657	147	510	579	402	0.520	323	70	69	159
Spun Rock Wool[c]										
..	203	89	114	...	146	0.336	49	..	..	25
76	426	114	312	350	270	0.407	164	52	38	84
79	659	149	510	580	404	0.520	341	72	70	174

[a] Outside diameter of pipe, 2.31 in.
[b] 12 lb per cu ft; 4.7 in. OD; canvas cover.
[c] Machine made, 10.3 lb per cu ft; 4.52 in. OD; cotton and asbestos cover.

TABLE 5 RESULTS OBTAINED ON SPECIMENS OF INSULATION FOR 2-IN-DIAM PIPE, "STILL-AIR" CONDITION[a]

Specimen no.	t_a Air temp, F	t_1 Pipe temp, F	t_2 Surface temp, F	$(t_1 - t_2)$ Pipe to surface temp diff, F	$(t_1 - t_a)$ Pipe to air temp diff, F	Mean temp, F	Conductivity, k	Btu per sq ft per hr, pipe surface	$(t_2 - t_a)$ Surface to air temp diff, F	Btu per sq ft per hr, external, surface
Spun Rock Wool: Machine-made; cotton-covered; average density = 16.5 lb per cu ft. Three samples Nos. 1, 2, 3; OD = 4.50 in.										
2	72	219	87	132	147	153	0.331	56	15	29
1	73	222	86	136	149	154	0.316	56	13	29
3	67	272	62[b]	209	205	167	0.321	87	..	45
2	65	419	87	332	354	253	0.395	170	22	87
3	71	428	89	339	357	259	0.384	170	18	87
1	65	522	123	399	457	322	0.416	218	58	113
3	70	585	110[b]	475	515	347	0.446	274	40[b]	141
2	77	661	153	508	584	406	0.513	336	76	173
3	70	726	153	573	656	439	0.540	400	83	205
1	70	801	172	629	731	486	0.580	476	102	247
2	64	858	192	666	794	525	0.649	561	128	288
Spun Rock Wool: Machine-made, cotton-covered; 16.6 lb per cu ft. OD = 4.45 in., ID = 2.20 in. Treated with one application of size and one coat flat brown paint										
1	72	279	95	184	207	187	0.380	91	23	44
1	75	498	126	372	423	312	0.473	228	51	112
1	72	587	137	450	515	362	0.561	326	65	160
1	71	652	157	495	581	404	0.661	424	86	208
Spun Rock Wool: Machine-made, cotton-covered; 16.1 lb per cu ft. OD = 4.50 in., ID = 2.20 in. Treated with one application of size and one coat of flat white paint										
2	72	281	95	186	209	188	0.381	91	23	44
2	75	426	115	311	351	270	0.445	176	40	86
2	76	554	138	416	478	346	0.543	288	62	141
2	73	664	161	503	591	413	0.660	424	88	208
Spun Rock Wool: Machine-made, cotton-covered; 16.9 lb per cu ft. OD = 4.51 in., ID = 2.20 in Treated with one application of size and one coat of aluminum paint										
3	71	263	96	167	192	180	0.426	91	25	44
3	68	394	112	282	326	253	0.493	176	44	86
3	74	532	139	393	458	335	0.575	288	65	141
3	75	637	164	473	562	401	0.702	424	89	208

[a] Outside diameter of pipe, 2.31 in.
[b] Probably in error (low reading).

TABLE 6

t_a Air temp, F	t_1 Pipe temp, F	t_2 Surface temp, F	$(t_1 - t_2)$ Pipe to surface temp diff, F	$(t_1 - t_a)$ Pipe to air temp diff, F	Mean temp, F	Conductivity k	Btu per sq ft per hr, pipe surface	$(t_2 - t_a)$ Surface to air temp diff, F	Btu per sq ft per hr external surface
(*F*) *Glass Wool:* Cotton-covered; machine-made; air still; density 8.9 lb per cu ft. OD = 4.56 in. ID = 2.25 in.									
75	287	97	190	212	192	0.370	91	22	45
78	432	117	315	354	274	0.435	173	39	87
78	571	135	436	493	353	0.514	282	57	142
78	699	154	545	621	426	0.604	414	76	204
(*J*) Same specimen as (*F*), painted with three coats of commercial aluminum paint									
76	285	102	183	209	194	0.385	89	26	45
77	424	122	302	347	273	0.455	173	45	87
79	569	148	421	490	358	0.532	282	69	142
80	700	174	526	620	437	0.624	414	94	204
(*G*) *Glass Wool:* Cotton-covered; machine-made; air still; density 8.36 lb per cu ft. OD = 4.61 in. ID = 2.25 in.									
73	251	93	158	178	172	0.372	73	20	36
72	388	108	280	316	248	0.430	150	36	74
73	536	126	410	463	331	0.492	250	53	122
75	533	127	406	458	330	0.495	250	52	122
73	665	148	517	592	406	0.590	380	75	186
76	694	149	545	618	422	0.608	414	73	208
(*H*) Same specimen as (*G*), painted with five coats of special aluminum paint									
77	284	102	182	207	193	0.390	89	25	45
76	421	114	307	345	267	0.450	173	38	86
76	563	141	422	487	352	0.536	282	65	141
74	689	164	525	615	427	0.634	414	90	208

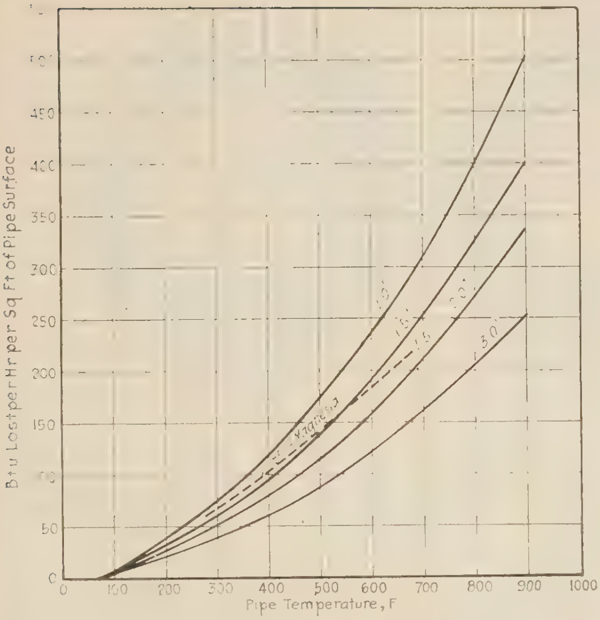


Fig. 14 Characteristic Curves of Heat Losses Through Spun-Rock-Wool Pipe Coverings of Different Thicknesses

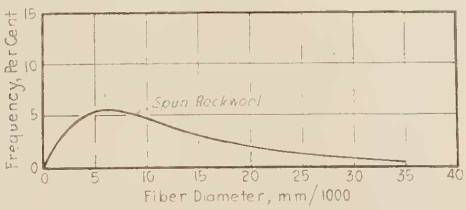


Fig. 15 Fiber Diameters of Machine-Made Spun-Rock-Wool Pipe Covering

with aluminum, gave an increase of 28 per cent at 200 F (mean temperature) to 35 per cent at 400 F. The smaller rate of increase in this case may possibly be due to the lower emissivity, which becomes increasingly important as t_2 rises. Similar results were obtained when the rate of heat loss per square foot of pipe surface was plotted against $(t_1 - t_a)$, in Fig. 12. Samples 1 and 2 gave increased losses from the painted surface varying from 3 per cent at 200 F to 20 per cent at 550 F. Again, the aluminum paint

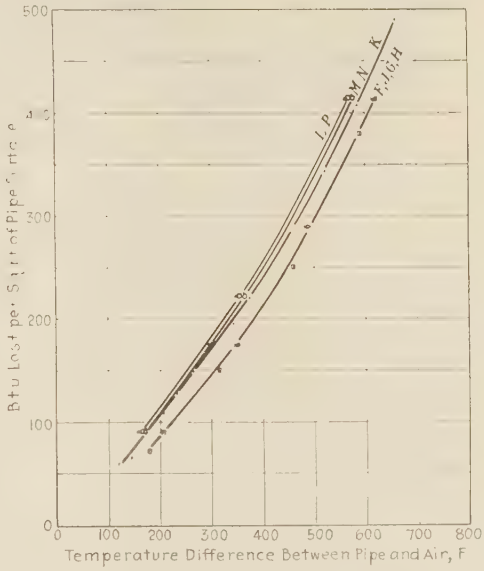


Fig. 16 Glass Wool and Corrugated Asbestos With Painted and Unpainted Surfaces

FG Glass wool with unpainted canvas cover
JH Glass wool, painted aluminum surface
K Corrugated asbestos, unpainted canvas cover
L Same as *K*, one coat aluminum paint
M Same as *L*, five coats aluminum paint
N Same as *M*, covered with fine dust
P Same as *M*, with extra coat of gray paint

TABLE 7

t_a Air temp, F	t_1 Pipe temp, F	t_2 Surface temp, F	$(t_1 - t_2)$ Pipe to surface temp diff, F	$(t_1 - t_a)$ Pipe to air temp diff, F	Mean temp, F	Conductivity k	Btu per sq ft per hr, pipe surface	$(t_2 - t_a)$ Surface to air temp diff, F	Btu per sq ft per hr, external surface
(K^1) *Corrugated Asbestos:* Cotton-covered; density 17.3 lb per cu ft OD = 4.51 in., ID 2.25 in.									
76	197	92	105	121	145	0.432	58	16	29
81	226	101	125	145	163	0.455	73	20	36
78	253	100	153	175	176	0.452	89	22	44
78	287	106	181	209	196	0.465	108	28	53
76	308	107	201	232	208	0.495	128	31	62
81	346	114	232	265	230	0.500	150	33	73
77	372	115	257	295	243	0.522	173	38	86
79	375	118	257	296	246	0.522	173	39	86
73	418	117	301	345	268	0.508	196	44	96
77	451	127	324	374	289	0.533	222	50	109
75	524	136	388	449	330	0.563	282	61	141
76	599	147	452	523	373	0.590	344	71	168
74	662	157	505	588	410	0.635	414	83	203
77	739	171	568	662	455	0.673	490	94	240
77	815	182	633	738	498	0.706	575	105	282
(*L*) Same specimen as (*K*), painted with one coat of "University" aluminum paint									
74	240	95	145	166	168	0.475	89	21	44
78	431	123	308	353	277	0.562	222	45	109
80	648	164	484	568	406	0.665	414	84	203
(*M*) Same specimen as (*L*), painted with five coats of "University" aluminum paint									
78	246	98	148	168	172	0.464	89	20	44
75	439	120	319	364	279	0.542	222	45	109
70	651	147	504	574	399	0.636	414	70	203
77	654	153	501	577	403	0.640	414	76	203
(*N*) Same specimen as (*M*), dust sprinkled on top of aluminum paint									
76	241	95	146	165	168	0.475	89	19	44
79	436	127	311	357	283	0.558	222	48	109
79	652	154	498	573	403	0.643	414	75	203
(*P*) Same specimen as (*N*), three coats of gray oil paint on top of aluminum paint									
74	236	93	143	162	164	0.485	89	19	44
75	429	118	311	354	274	0.560	222	43	109
76	645	148	497	569	397	0.645	414	72	203

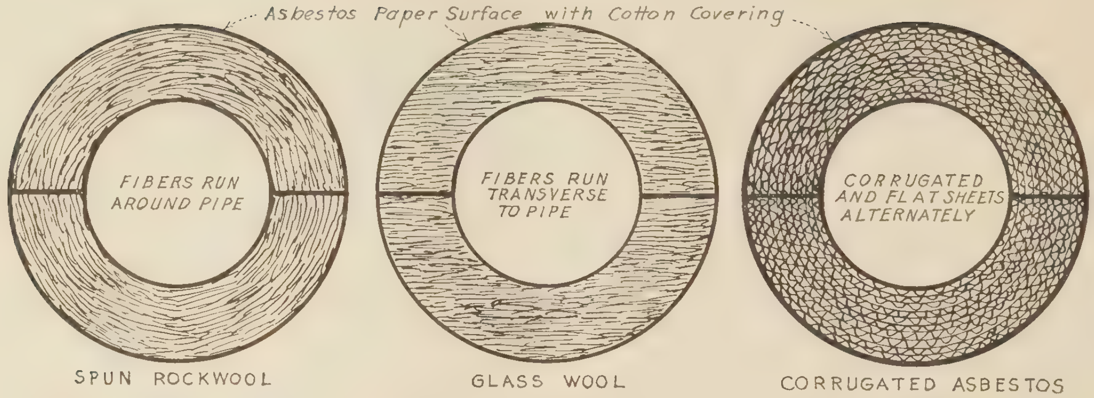


Fig. 17 Structural Arrangement of Glass-Wool, Spun-Rock-Wool, and Corrugated-Asbestos Specimens

(turpentine base) gave 15 per cent greater heat loss than the unpainted surface at 200 F, increasing to about 31 per cent at 550 F. The painted samples in all cases had higher surface temperatures than the unpainted ones for the same pipe temperatures. The difference increases with t_1 and varies from 3 to 5 per cent at 300 F to 10 to 16 per cent when $(t_1 - t_a)$ is 600 F. If $(t_1 - t_a)$ be plotted against $(t_2 - t_a)$ the resulting curves are all practically straight lines. Fig. 13 indicates the relationship between the rate of heat loss from the outside surfaces of the painted and unpainted specimens, respectively, when plotted against $(t_2 - t_a)$.

The lowest results, in this case, were obtained with aluminum paint, but the curves are close together. The large increases in the heat losses from the painted surfaces, as compared with the unpainted ones in this series, were entirely unexpected and are difficult to explain save, possibly, on the grounds that the outer painted surfaces were rough. The results were too numerous and uniform, however, to allow them to be ignored with safety.

Further tests were made with the 2-in. apparatus on two samples of glass wool which were cut out of the solid mass, with the result that the fibers were arranged as shown in Fig. 17. This structure is frequently used for small diameters. The results of these tests are given in Table 6 and Fig. 16, *F* and *G*. The curves are so close together that they can only be indicated by a single line which is evidently representative of this material when covered with unpainted canvas. One of the specimens was then painted with three coats of commercial aluminum paint (turpentine base) and another with five coats of aluminum paint specially prepared with an acetone base, the number of coats in both cases being chosen to give a smooth, bright surface. The results obtained are given in Table 6 and Fig. 16, *J* and *H*, and they are identical with those for the unpainted surfaces.

Another series of tests was made on a 2-in. sample of corrugated asbestos, Fig. 17, and the results of these tests are given in Table 7. They also are plotted in Fig. 16, where curve *K* refers to the specimen with an unpainted canvas surface. This specimen was then covered successively with one coat of commercial aluminum paint, obtained from the University stores (curve *L*) and with five coats of the same paint (curve *M*). The heat losses in both cases were somewhat higher than those from the unpainted sample, as follows:

Temperature difference between pipe and air.......	200 F	550 F
Increase of heat loss over unpainted sample:		
(*A*) One coat of aluminum paint, per cent......	6.5	6.7
(*B*) Five coats of aluminum paint, per cent.....	2.8	4.6

The surface was then covered with dust to simulate the actual operating conditions, and the rate of loss was practically unaffected (Table 7 and Fig. 16 *N*). The same specimen was next cleaned, three coats of gray paint were applied on top of the aluminum, and the tests repeated (Table 7 and Fig. 16 *P*). The losses in this case were slightly higher, but the curve was practically identical with that obtained with one coat of aluminum paint (curve *L*). Thus in the tests made on corrugated asbestos, the application of paint to the outer surface increased the heat losses from 3 to 7 per cent, and the use of aluminum paint, in this case, did not change the heat losses materially, as compared with an ordinary oil paint.

Conclusions

1 Short lengths of pipe insulation may be tested by the electrical method described and will give results which are comparable with those obtained by other methods.

2 The results obtained on similar insulating materials when applied to pipes 8 in. and 2 in. diam, agree within practical limits.

3 At pipe temperatures below 500 F, the conductivity of spun rock wool is slightly below that of 85 per cent magnesia pipe insulation. Between 500 and 600 F, there is little difference between the two materials.

4 Variations in density, produced by different proportions of silicate binder have little influence on the conductivity of spun rock wool. The conductivity also appears to be practically independent of variations in thickness of material. These remarks refer only to the range of the present series of tests.

5 Exposure to winds of 15 to 20 mph gives increased heat losses up to 16 per cent, as compared with natural convection. This increase is small for pipe insulation more than $1^1/_2$ in. thick, but appears to be greater with metallic coverings than it is with fabric.

6 In still air the surface temperature of metallic coverings is greater than that of fabric coverings. This counteracts the lower emissivity of the metal so that there is little, if any, saving of heat if metallic outer coverings are used.

7 Painting the outer surface of canvas coverings with a single coat may increase the heat loss substantially as compared with unpainted surfaces. This includes aluminum paint with a turpentine base. If a sufficient number of coats be applied to give complete surface protection, the use of paint does not appear to affect the heat losses materially.

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Appendix

The insulating value of a fibrous material such as rock wool depends upon the number and diameter of the fibers arranged within a given volume, and its resilience depends to some extent also upon the length of the fibers. Most varieties of rock wool are produced by blowing air or steam into a stream of molten rock, but spun rock wool is made by directing the stream onto the surface of a rapidly rotating disk. The coarse particles are thrown by centrifugal force to the wall of the surrounding chamber, the "wool" is removed by suction and is conveyed in the same way to the storage bin. The fibers produced are relatively coarse, Fig. 15, and long, so that either they may be made into a "mat" or "batt" for covering flat surfaces or they may be arranged in concentric layers round a pipe form by means of a machine, as shown in Fig. 17. The latter form is described in the paper as "machine made" and in it the fibers are held in position by a silicate binder. Most of the specimens tested on the 8-in. apparatus, and all of those on the 2-in. apparatus, were of this type.

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8 "Properties of Heat Insulating Materials," by E. A. Allcut, *Engineering Journal*, vol. 24, 1941, pp. 514–524; and Bulletin No. 169, School of Engineering Research, University of Toronto, 1941.

9 "Heat Losses From Bare and Covered Wrought-Iron Pipe at Temperatures up to 800 F," by R. H. Heilman, Trans. A.S.M.E., vol. 44, 1922, pp. 299–310; also "Surface Heat Transmission," by R. H. Heilman, Trans. A.S.M.E., vol. 51, 1929, FSP-51-41, pp. 287–298.

Discussion

C. B. Bradley.[5] The author's choice of an electrically heated pipe-insulation test apparatus is believed to have been wise. In so far as we know, all investigators long ago gave up the steam-condensation method. McMillan[6] in 1915, Heilman[7] in 1926, and Griffiths[8] in 1927 described electrically heated pipe-insulation test apparatus. The McMillan Thermal Conductivity Laboratory of the Johns-Manville Research Laboratories has for many years used electrically heated apparatus for testing cylindrical specimens of thermal insulations. These are equipped with calibrated end caps instead of guarded ends, and long experience has proved that they give highly satisfactory results, closely checking results obtained on flat samples by means of the guarded hot plate. In addition, we believe that they are much easier to operate, especially when a number of tests must be carried on simultaneously by one or two operators.

We believe, however, that the author's 8-in. apparatus, as illustrated in Fig. 1, is subject to rather severe criticism. Apparently there is no separation between the central test section and the guard sections of the porcelain tube, which serves as the core for the heater windings as well as for the test pipe. The thermal conductivity of the porcelain is so high (approximately 20 times that of the insulation being tested) that a comparatively small temperature difference between the guard and test sections would result in a flow of heat which might well be an appreciable part of the total measured heat dissipated by the center heater windings. Even though the temperature balances between the sections were held sufficiently close to prevent heat flow along the test specimen, heat might still flow along the porcelain tube between guard and center windings.

Contact between the test samples and the heater wires, as is the case if we interpret Fig. 1 correctly, would inevitably result in a nonuniform distribution of temperature along the inner surface of the insulation. As a result, a large number of thermocouples would be required to give even a good average inner-surface temperature.

The foregoing criticisms do not apply to the 2-in. apparatus, which has low-conductivity connections between the center and guard sections of both heater core and test pipe. The checks between the author's results and those of other investigators on somewhat similar materials, and between his own results obtained on the 8-in. and the 2-in. apparatus, again on similar but not identical materials, do not seem convincing enough to establish the validity of all results obtained on the 8-in. apparatus.

The author states that he expresses his "conductivity" values in terms of "British thermal units per hour, per degree temperature difference between inner and outer surface, per inch of thickness per square foot of logarithmic-mean area," but that this "is of doubtful utility from a comparative standpoint." Using the actual thickness and logarithmic-mean area results in exactly the same expression for conductivity as using the actual pipe area and a logarithmic "equivalent" thickness of $r_1 \log_e \frac{r_2}{r_1}$ where r_1 and r_2 are the radii of inner and outer surfaces of the insulations, respectively. The meaning of this is that a flat slab of the same material must have an actual thickness equal to this "equivalent" thickness if the heat loss per unit area is to be the same as that through the cylindrical sample with the same temperature difference between the two faces. The conductivity coefficient thus obtained is the same as would be obtained on a flat sample of the same material if tested in a guarded hot-plate conductivity apparatus. Thus if the conductivity determined in the guarded hot-plate apparatus has utility, and we believe it does have, that obtained on a cylindrical apparatus will have the same utility.

The author states: "Conductivity should be solely a characteristic of the material tested, but the results of this calculation on a given material vary also with external conditions." Conductivity should be, and is, a characteristic of the material, and differences in the calculated results must be ascribed to some characteristic of the test method. The results of many hundreds of conductivity determinations both on the guarded hot-plate and on pipe-insulation apparatus have convinced us that the term "conductivity" can properly be applied to all reasonably homogeneous materials.

In this connection, we are completely at a loss to understand the reported increase in the measured conductivity resulting from painting the canvas covers (see Table 5 of the paper). It appears that the values for the conductivities of these painted samples are incorrectly calculated from the data shown in the table. When recalculated, the conductivity of sample 1 shows increases upon painting of from 11.7 per cent at 150 F mean

[5] Physicist, Johns-Manville Research Laboratories, Manville, N. J.

[6] "The Heat-Insulating Properties of Commercial Steam Pipe Coverings," by L. B. McMillan, Trans. A.S.M.E., vol. 37, 1915, pp. 921–969.

[7] "Determination of the Thermal Conductivities of Insulation for Temperatures up to 1000 Deg Fahrenheit on Other Than Flat Surfaces," by R. H. Heilman, *Mechanical Engineering*, vol. 48, 1926, pp. 1297–1306.

[8] "Loss of Heat From the External Surface of a Hot Pipe in Air," by E. Griffiths, *Engineering*, vol. 123, 1927, pp. 1–4.

temperature to 8.6 per cent at 450 F. (The calculated value at 404 F is so out of line with the rest of the values that we have disregarded it. There appears to be an error in the data as given.) Sample 2 increases in conductivity from 5.4 per cent at 150 F to 17.4 per cent at 350 F (using recalculated values), while on the same basis the conductivity of sample 3 increases from 25 per cent at 150 F to 28 per cent at 350 F. It is quite unreasonable to suppose that painting the surfaces affects the conductivities in this manner. We are compelled to question the accuracy of the conductivity determinations.

We would suggest a careful check upon the correctness of all calculated values in this paper. We have recalculated, from the data given in Table 1, the conductivity values for P and R. We find the values for P correct, but the values of R as given in the table are low by from 2.8 to 4.1 per cent.

Curve A, Fig. 3 of the paper, taken from McMillan is for 85 per cent magnesia of 16.5 lb per cu ft density.[9] This curve represents the same values as Johns-Manville's standard figures for 85 per cent magnesia of 15 lb per cu ft density. However, the tendency in the United States has for some years been toward the manufacture of lower-density magnesia. Periodic tests on regular production of Johns-Manville 85 per cent magnesia over the past 3 years reveal an average density for this period of about 13 lb per cu ft. The average of 24 thermal conductivity tests on the same material is as follows:

Mean temperature	100	200	300	400
k, J-M average............	0.407	0.445	0.481	0.517
k, Allcut's curve, "2 in. diam," Fig. 4....................	...	0.443	0.487	0 521

The Johns-Manville average figures include results of tests on pipe insulation of various sizes and on blocks. The comparison given in the table certainly does not indicate any marked difference between present-day American and Canadian magnesia. The 85 per cent magnesia, having a density of 17 lb per cu ft, is still manufactured for special purposes but this certainly is not the "density generally used in American practice."

Curves A and C, Fig. 10 of the paper, should be the same if the canvas covers are similar. The heat loss from the surface is a function only of the nature of the surface, as indicated by the surface coefficient of heat transfer, and of the temperature difference between the surface and the surrounding air. The surface coefficient of heat transfer is determined solely by the nature of the surface, the air velocity, and the temperature of surrounding bodies to which the surface may radiate. Two similar surfaces, tested under the same surrounding conditions, should always give the same results, regardless of the nature of the material behind the surface. The difference between curves A and C must necessarily be due to differences between the canvas covers or in the techniques or surrounding conditions as between Heilman's and Allcut's tests.

M. Jakob.[10] Owing to the great amount of material presented in this paper the writer had difficulty in following some of the author's conclusions at the meeting. In particular, the statements on the insulating effect of bright metallic surfaces invite some clarification.

This effect manifests itself in a different manner according to whether the pipe-surface temperature or the heat loss is considered as invariable. The first case is approached in steam pipe lines. Covering the insulation with a bright metal sheet reduces the heat loss owing to the small emissivity of metals. However, the reduction of emissivity is partly compensated by an increase of the temperature of the sheet. Eventually a steady state will exist in which the heat flow by conduction through the insulation equals the heat loss by convection and radiation to the environment.

For the second case which is realized in constant electrical heating from inside, metallic covering will produce a much higher surface temperature, since the total heat developed inside must be given up to the environment by convection and radiation.

H. B. Nottage.[11] While it is true that this paper contains a great deal of potentially interesting data, a direct and assured application of the results to the practical problems of accurate performance prediction for systems differing in any significant details whatever from the particular test arrangement employed is hampered by two outstanding unanswered questions; these are:

1 How well did the test system maintain what might be termed its "calibration" over the entire range of operation in yielding total rates of heat loss and corresponding temperature data for the center section which were truly independent of any installation or end effects? To answer this properly would require the determination and establishment of a complete heat balance on the system, a consideration of the accuracy and reproducibility of the instrumentation employed, and a check on the comparative similarity of the different insulation specimens themselves.

2 What would the over-all effects reported reduce to when analyzed in the accepted terms of individual heat-transfer resistances or conductances for the several components of the over-all heat-flow path? Such a procedure would be essential before any general and extensive interpretation or application of the data in this paper could be successfully undertaken. As the data exist, they indicate some rather interesting possibilities which must, however, remain in a speculative shroud until treated analytically by more exhaustive and comprehensive studies.

V. Paschkis.[12] The author mentioned that, in some instances, he has pushed the tests to a temperature of 1000 F. He states that the samples showed considerable increase in brittleness but that the thermal conductivity did not change.

The writer questions whether the length of the test which was mentioned to be 30 hr was enough entirely to justify this statement.

The increase of brittleness indicates a change in structure. Obviously, it takes time to obtain such an effect. Let us assume that the change in brittleness takes place after 25 hr. If the change would also cause a change in thermal conductivity, then the heat losses would tend to increase, starting with the twenty-fifth hour, and 5 hr later (i.e., 30 hr after the experiments started) the increase might not yet be noticeable on the cold side.

Author's Closure

The author thanks Mr. Bradley for his long and comprehensive contribution to the discussion. The objections to the use of a porcelain tube were realized in the first place, but at that time it was the only usable material that was available in the size required. This tube also suffered from the disadvantage that heat-

[9] "Recent Improvements in the Manufacture of 85 Per Cent Magnesia Insulation," by L. B. McMillan, Letter to the editor, *Journal*, A.S.M.E., vol. 40, 1918, p. 970.

[10] Research Professor of Mechanical Engineering, Illinois Institute of Technology and Armour Research Foundation, Chicago, Ill. Mem. A.S.M.E.

[11] Assistant Project Engineer, Pratt & Whitney Aircraft, East Hartford, Conn. Jun. A.S.M.E.

[12] Research Laboratory, Mechanical Engineering Department, Columbia University, New York, N. Y.

ing and cooling had to be performed slowly, to avoid breakage. This fact unduly lengthened the periods between successive tests but, in spite of these precautions, cracking occurred. For these reasons, the design was changed when the 2-in. apparatus was made, and a reconstruction of the 8-in. apparatus along similar lines will be undertaken as soon as possible. Nevertheless, it is claimed that, with the large number of thermocouples used, the agreement of these results with those obtained by other observers using different apparatus indicates that, with care, reasonably accurate results can be obtained. It should be explained that this apparatus was designed originally for making commercial comparisons only, and it was an agreeable surprise to find that wider applications of the results were possible.

There was no contact between the test samples and the heater wires, as the latter were covered with cement and asbestos paper.

The author has objected for many years to the use of the term "conductivity" to designate results calculated from hot-plate and similar tests, but made use of the term as a matter of convenience in the various figures and tables. The objections and proofs have been given in detail in previous publications.[13]

The calculated values included in the tables were all checked for correctness before the paper was presented, but in some of the tables the approximate thickness only is given, whereas the conductivities were actually calculated from the mean values of the thicknesses as obtained from the circumferential measurements. It was considered unnecessarily meticulous to state in the tables the mean thickness to several places of decimals when the actual measured values varied at different points in the length. Hence, these figures were intended as guides only and not as bases for the calculation of conductivities.

The author agrees (and stated in the paper) that the increased conductivities of the painted surfaces (Table 5) are incomprehensible, but the figures were included because it was considered inadvisable to ignore them, and it was hoped that the discussion would suggest a reason for them. The consistency of these results seems to rule out any variable inaccuracies and experimental errors appear to be excluded by the fact that later tests on different materials made on the same apparatus in the same way gave results that were quite reasonable.

The close agreement between our results and those obtained by the Johns-Manville Research Laboratories on 85 per cent magnesia is gratifying. With reference to the question of density, the author notes Mr. Bradley's correction to the effect that the average density now used in American practice is about 13 lb per cu ft, but the figures published in American papers and reference books generally give representative conductivities for the 17 lb density only.

With reference to Professor Jakob's remarks, the explanation of the comparatively small effect of a bright metallic surface when the pipe temperature is constant is that, owing to the small emissivity of the surface, the superficial temperature is raised. This, in turn, increases the rate of heat loss both by convection and radiation, so that comparisons made on the basis of a constant temperature difference between surface and air, Fig. 10, are not valid. As a result, the saving produced by using a bright metallic surface on the outside of pipe insulation is considerably less than would normally be expected.

A complete analysis or heat balance for the apparatus at different temperatures, as suggested by Mr. Nottage, is evidently desirable and will be undertaken as soon as circumstances permit.

Dr. Paschkis is under a misapprehension regarding the duration of the tests. The statement made in the paper regarding brittleness after heating for 30 hours referred to a special test for determining physical properties only. The thermal tests for which results are tabulated frequently lasted for several days and were continued until the temperatures became steady.

[13] Bulletins Nos. 149 and 169, School of Engineering Research, University of Toronto, and *Canadian Journal of Research*, vol. 17, November, 1939.

Measurement of High Temperatures in High-Velocity Gas Streams

By W. J. KING,[1] WEST LYNN, MASS.

Conventional methods of measuring temperatures are subject to considerable errors, when applied to gas streams at velocities around 500 fps and temperatures of the order of 1600 F, which conditions occur, for example, in the exhaust-gas pipe of an airplane engine. An investigation of this problem the results of which are reported in this paper was suggested by the Special Subcommittee on Exhaust Gas Turbines and Intercoolers of the N.A.C.A. and was carried out with the assistance of the Subcommittee on Exhaust Gas Temperature Measurement of the Committee on Industrial Instruments and Regulators. Tests were conducted on the thermocouple, which at present is the most satisfactory temperature-measuring device available, to develop a shield and mounting that would achieve maximum accuracy and minimum size and weight for airplane application. The results indicate that something better than the ordinary thermocouple mountings is required for even a fair degree of accuracy with either the high velocities or temperatures considered. A few tentative devices are suggested which will serve until further studies have been made and more adequate solutions provided.

RECENT experience, particularly in the field of aeronautics, has established the fact that conventional methods of measuring temperatures are subject to very considerable errors when applied to gas streams at velocities around 500 fps and temperatures of the order of 1600 F. Such conditions may obtain in the exhaust-gas pipe or "tail stack" of an airplane engine. With standard equipment, discrepancies of 50 F or more have been observed in this region and, with somewhat higher temperatures and lower velocities, there is evidence that the error may exceed 200 F.

At the present time, the thermocouple seems to be the most satisfactory temperature-measuring device available for use in this field. Electric-resistance thermometers have been used very successfully for measuring air and oil temperatures on aircraft, but as yet no resistance element suitable for high-temperature service hasappeared on the market. Most of the following remarks on the thermal and mechanical features of the thermocouple will be applicable also to resistance elements.

A typical thermocouple-pyrometer system is divided into the following elements: (*a*) Hot junction, (*b*) leads, (*c*) selector switch, (*d*) cold junction, (*e*) instrument (potentiometer or millivoltmeter). Each of these elements may contribute to the error if not properly used. References (1, 2, 3, 4)[2] contain extensive discussions of these factors under ordinary conditions. The larger errors, encountered under the special conditions considered herein, are associated primarily with the hot-junction mounting and the present paper will therefore be concerned specifically with this element.

Theoretical Considerations

The chief sources of error associated with the hot junction are:

1 Radiation to cooler surroundings.
2 Conduction to cooler surroundings:
  (*a*) Along the thermocouple wires.
  (*b*) Along the protecting tube or mounting.
3 Contamination of thermocouples due to chemical effects of hot gases.
4 Velocity effects, due to incomplete conversion of kinetic energy into thermal effects (significant only at extremely high velocities).

From the thermal standpoint (neglecting velocity effects), the fundamental fact to be kept in mind is that the temperature indicated by a thermocouple is merely an "equilibrium temperature," representing the point at which the heat transferred from the gas to the couple is balanced by the heat transferred from the couple to the surroundings. This is represented by the equation

$$\text{Heat flow, Btu per hr} = A_1C_1G^n(T_g - T) = \frac{kA_2}{L}(T - T_w) + C_3A_3\epsilon\,(T^4 - T_w^4) \qquad [1]$$

$$\begin{Bmatrix}\text{Heat transfer}\\\text{from gas to}\\\text{couple by}\\\text{convection}\end{Bmatrix} = \begin{Bmatrix}\text{Heat transfer}\\\text{from couple}\\\text{to surround-}\\\text{ings by con-}\\\text{duction}\end{Bmatrix} + \begin{Bmatrix}\text{Heat transfer}\\\text{from couple to}\\\text{surroundings by}\\\text{radiation}\end{Bmatrix}$$

where

A_1 = total exposed surface area of hot junction (or adjacent portions of protecting sheath, at the tip), sq ft
A_2 = cross-sectional area of wires, insulation, and sheath, sq ft
A_3 = effective radiating area of hot junction (or sheath tip), sq ft
for a simple surface, this is the same as A_1, but if there are fins, etc. A_3 is the "envelope," which is less than A_1
C_1, C_3 = constants, for the present purpose
G = mass velocity of gas, lb per sq ft per sec (product of linear velocity, V, and density ρ)
n = exponent varying from roughly 0.5 to 0.8, depending upon size and shape and direction of flow of gas relative to surface
T_g, T, T_w = temperatures of gas, thermocouple, and surrounding walls, respectively. Degrees F absolute are used for the sake of the radiation term
k = mean effective thermal conductivity of wires, insulation, and sheath, Btu per hr, per sq ft, per deg F, ft
L = length of heat path from hot junction to wall, ft
ϵ = emissivity factor

[1] General Electric Company.
[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by Committee on Industrial Instruments and Regulators of the Process Industries Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

From Equation [1]

$$(T_g - T) = \frac{kA_2(T - T_w)}{L\ A_1C_1G^n} + \frac{C_3A_3\epsilon\ (T^4 - T_w{}^4)}{A_1C_1G^n} \ldots\ldots [2]$$

where $(T_g - T)$ is the error in the temperature measurement, and Equation [2] shows how it varies with the principal factors involved.

Equation [2] indicates that, in general, the error can be reduced by reducing all of the factors in the numerators, or increasing all of the factors in the denominators, of the two terms on the right. But there is obviously no advantage in increasing A_1 if A_2 and A_3 also increase correspondingly; it is the "area ratio" that is significant in each term.

The conduction term

$$\frac{kA_2(T - T_w)}{L}$$

brings out the importance of the immersion L, although in some respects the local temperature gradient (dt/dL) away from the junction is more significant. Anything that reduces this gradient, as by increasing T_w or promoting the heating of the lead wires near the junction, will reduce the error.

Thick-walled sheaths or heavy lead wires have an adverse effect by increasing A_2. Fortunately, the alloy steels used in the sheaths for this high-temperature service have relatively low heat conductivities (about $1/3$ the value of k for ordinary mild steel). Ceramic tubes are better in this respect, but by the same token they tend to insulate the couple junction from direct contact with the gas stream. Even with metallic sheaths it is important to obtain good thermal contact between the actual junction and the exposed surface of the sheath tip, to avoid a temperature drop through a poor conductor. However, with reasonable attention to these details it is fairly easy to avoid appreciable errors due to conduction effects.

The real crux of the present problem of minimizing the value of $(T_g - T)$ lies in the second term of Equation [2]

$$\frac{C_3A_3\epsilon\ (T^4 - T_w{}^4)}{A_1C_1G^n}$$

In the case of a gas the rate of heat transfer from the stream to the couple (represented by the denominator) is relatively low, whereas the radiation through it becomes very intense with high values of T, due to the fourth-power law. The principal objective is therefore to reduce the radiation from the couple and promote convection to it as much as practicable. For many applications, particularly in aircraft, it is essential to keep the size and weight of the device to an absolute minimum consistent with reasonably accurate results.

Referring further to the second term of Equation [2], C_3 is a constant[3] and A_3 cannot be reduced without correspondingly reducing A_1, so that the only effective means for reducing the radiation is to reduce the emissivity ϵ of the surfaces or to increase the temperature T_w of the surroundings.

The emissivity (or "black-body coefficient") conceivably could be reduced by using suitable polished metal surfaces but, in most cases, the efficacy of such surfaces would soon be lost due to tarnish and oxidation. The most effective recourse is therefore to increase T_w. In fact it may be seen that the error is completely eliminated (apart from velocity effects) if $T_w = T$.

In the typical case with which this discussion is concerned T_w is the temperature of the walls of a pipe or vessel containing the gas; and T_w is less than T_g by virtue of heat loss from the outer surface of these walls. It is theoretically possible to insulate the walls sufficiently to maintain $T_w = T = T_g$, but in practice this would usually be very difficult or cumbersome. Unlike the case of saturated steam or liquids, there is apt to be a sharp temperature drop between the gas and the walls $(T_g - T_w)$, due to the combined effects of the gas-film resistance on the inner surface and the relatively high heat loss from the outer surface, due to the high temperatures. In some cases, as in the tail stack of an airplane, the use of insulation is objectionable because of the high temperatures and thermal stresses which are set up in the pipe itself. Covering a bare pipe carrying hot gas or air may raise the wall temperature by 500 or 600 F. But, even when the walls are nearly as hot as the gas, the radiation effect may still be very considerable. For example, if $T = 1800$ and $T_w = 1750$ F, the radiation from the couple to the wall would be the same as from a hot surface at 763 F to surroundings at zero. This is because

$$(1800 + 460)^4 - (1750 + 460)^4 = (763 + 460)^4 - (460)^4$$

Radiation Shields

Since in general it is neither permissible nor sufficient to reduce the radiation by insulating the pipe, some other means must be adopted to limit the error. A very effective recourse, which has been widely employed for this purpose, is to interpose a number of radiation screens or shields, in the form of concentric cylinders, between the thermocouple tip and the pipe walls. The experimental studies to be described were concerned very largely with the determination of the optimum size and arrangement of these shields.

The efficacy of such radiation shields is due to two circumstances: First, leaving out conduction and convection effects and considering radiant-heat transfer alone (as in a vacuum), interposing one screen between a radiating body and its surroundings will reduce the radiation by one half, or to $1/(n + 1)$ if there are n screens.[3] This results from the fact that the screen surrounds the object with a new surface having a higher value of T_w. In a hot gas stream there is a second effect in the additional heating of the screen, or screens, by convection, which has a marked influence upon the error by causing a further rise in the effective value of T_w. If the shields are made of a poorly conducting material, such as porcelain, there is a further benefit because of the insulating effect, depending upon the thickness.

Ordinarily the effect of convection upon the shields is very similar to its effect upon the thermocouple tip, as indicated in Equations [1] and [2]; the higher the rate of convection the lower the error. The controlling factor here is the mass velocity G, which is proportional to the linear velocity V (fps) if the density is constant. Secondary factors are included in the constant C_1 and the exponent n, which represent such effects as the properties of the gas and the size and orientation of the heat-transfer surface. Convection rates per unit of surface are always higher for small than for large objects and higher for transverse than for parallel flow of the gas stream.

Application of Principles to High-Velocity Thermocouple

On the basis of the foregoing principles, it has been fairly common practice in the past to "swamp out" the radiation error in several similar applications simply by increasing the gas velocity G to a sufficiently high value. The ordinary sling psychrometer is an example (the radiation here is from the surroundings to the thermometer). A more pertinent case is the high-velocity thermocouple which has been used for measuring gas temperatures for many years and has recently been described in two

[3] This is a practical simplification which is justified for the present purpose. For more extensive and rigorous treatment of heat-transfer factors see Bibliography (5).

excellent papers by H. F. Mullikin.[4] This device has been applied chiefly to slow-moving streams of hot gases, such as boiler flue gases. It employs one or more cylindrical radiation shields through which a sample of the gas is drawn at high velocities by means of an external source of suction, such as a pump or aspirator. The conditions of the present problem are quite similar, with the important distinction that the velocity of the gas stream is already very high and in the typical case no other means for increasing it is necessary or permissible. For this reason considerable attention has been given to the question of how far the size of the radiation shields could be reduced without choking off too much of the gas flow through them.

Most of this study has been concerned with velocities below 500 fps, where the kinetic-energy effects mentioned are not very appreciable. In this region the error should be reduced continuously as the velocity is increased and, according to Equation [2], the temperature T indicated by the thermocouple should continuously approach the true gas temperature T_g. As a matter of fact in using the high-velocity thermocouple, it has been assumed quite generally that $T = T_g$ when the indicated temperature ceases to rise with further increase of the gas velocity. There is, however, a considerable amount of recent evidence to indicate that even with moderately high gas temperatures and a fair amount of shielding the true temperature cannot be approached in this manner no matter how much the gas velocity is increased. This is a consequence of the kinetic-energy effect, which has been studied extensively abroad and more recently in this country (6, 7, 8, 9, 10, 11).

From elementary physics, if a body of gas of mass m is accelerated from rest to a velocity V (as in the flow through a nozzle) the resulting kinetic energy is acquired at the expense of the temperature of the gas, as indicated by the energy equation

$$^1/_2\, mV^2 = mgJc_p(T_t - T_s) \qquad [3]$$

where

m = mass in poundals
V = velocity, fps
g = acceleration of gravity, fpsps
J = mechanical equivalent of heat, 778 ft-lb per Btu
c_p = specific heat at constant pressure
T_t = "total" temperature of gas at rest, deg F
T_s = "static" temperature of gas, hypothetically measured by a thermometer moving at same speed as gas stream, deg F

From this the temperature drop is

$$(T_t - T_s) = \frac{V^2}{2gJc_p} = b\left(\frac{V}{1000}\right)^2 \qquad [4]$$

where

$$b = \frac{10^6}{2gJc_p}$$

Substituting the values of the constants and using Heck's specific-heat data (12), values of b for dry air may be computed for various temperatures, as given in Table 1.

TABLE 1 SPECIFIC HEAT AND VALUES OF b FOR DRY AIR

Mean temperature, deg F	c_p	$b = (T_t - T_s)/(V/1000)^2$
0	0.240	83.2
200	0.2415	82.6
400	0.2450	81.5
600	0.2503	79.7
800	0.2565	77.8
1000	0.2628	76.0
1200	0.2690	74.1
1400	0.2741	72.7
1600	0.2788	71.5

[4] Refer to Bibliography (2), pp. 775 and 805.

It will be observed that the values of b in Table 1 represent the temperature drop for an air-stream velocity of 1000 fps and that the drop for any other speed will be proportional to the square of the velocity. It is also apparent that Equations [3] and [4] and Table 1 apply as well to the temperature rise which must result when a gas stream, moving with an initial velocity V, is stopped adiabatically. Conceivably, the kinetic energy could be completely recovered or reconverted into thermal energy manifested by the temperature rise $(T_t - T_s)$. Actually any thermometer or thermocouple immersed in such a gas stream would substantially stop the flow in the layer immediately at the surface, due to impact at stagnation points and friction elsewhere. The indicated temperature T would rise above T_s, approaching T_t. However, the process would not be adiabatic since there is a simultaneous tendency toward heat transfer back into the gas stream, or a convective cooling action, because T_s is less than T. In other words, entirely apart from radiation effects, any ordinary thermometric device will indicate a temperature part way between T_s and T_t due to the incomplete recovery of the kinetic energy. The "recovery coefficient," α, of the device is, therefore, defined as the ratio $(T - T_s)/(T_t - T_s)$, which is usually less than unity. Neglecting radiation and conduction, the temperature attained by any surface at any particular point is then

$$T = T_s + \alpha\,(T_t - T_s) \qquad [5]$$

or

$$T = T_s + \alpha b\left(\frac{V}{1000}\right)^2 \qquad [6]$$

Obviously when $\alpha = 1$, the indicated temperature is equal to the total temperature, or $T = T_t$. Otherwise, the error is

$$T_t - T = (1 - \alpha) b\left(\frac{V}{1000}\right)^2 \qquad [7]$$

Total and static temperatures are closely analogous to total and static pressures. But while an ordinary impact tube, connected to a manometer, will read total pressures directly, an ordinary thermometer will not read total temperatures since $\alpha < 1$, due to the loss of heat back into the air stream, as just mentioned. Such a thermometer is analogous to a manometer which reads something less than the true total pressure because of a leak in the line from the impact tube.

Theoretical and experimental studies described in the references cited indicate that the recovery coefficient for air in parallel or lengthwise flow along flat plates, cylinders, or wires is about 0.85. For transverse flow across wires and small cylinders, the value of α appears more variable, ranging from about 0.55 to 0.8, according to various observers, depending considerably upon the velocity and the diameter of the cylinder. For the present purpose a value of $\alpha = 0.6$ has been assumed for hot air or exhaust gases flowing across a thermocouple sheath. At a temperature of 1000 F and a velocity of 1000 fps, this would result in an error of $0.4 \times 76 = 30.4$ F, according to Equation [7] and Table 1, giving values for b.

Diffuser-Type Temperature Probe

In order to avoid such errors, Franz (9) developed the diffuser-type "temperature probe," shown in Fig. 1, in which the high-velocity gas is brought substantially to rest in a region protected from the cooling action of the main stream. The small downstream bleeder holes allow a little gas to flow over the couple junction to prevent cooling by conduction to the walls, etc. Franz's data show a value of $\alpha = 1$ for air velocities from 150 to 600 fps, at ordinary room temperatures, but there is considerable

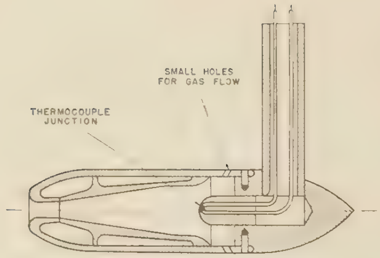

Fig. 1 Franz Type Probe for Direct Measurement of Total Temperature of Gas Stream

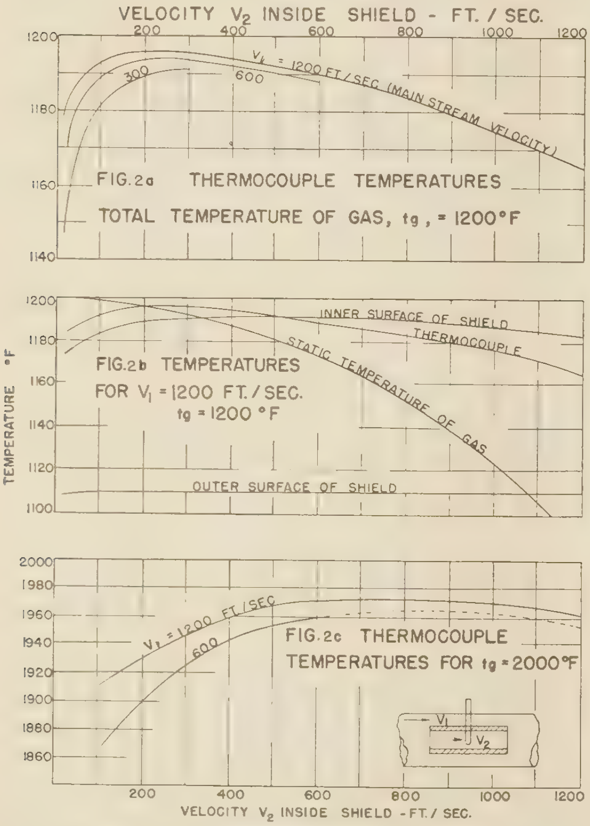

Fig. 2 Thermocouple Temperatures as Affected by Gas Velocities Inside and Outside of Shield; Calculated Values

evidence that these results were probably in error. Franz's own tests on a similar device with a thermometer in place of the couple show values of α increasing from about 0.8 to 0.92 over this range, and Eckert's tests (6) on an equivalent thermocouple probe indicated an average value of $\alpha = 0.99$. A number of tests on Franz-type probes have been made in various laboratories in this country, consistently yielding values of α appreciably less than unity, for room-temperature air. Unfortunately, further reference to these studies cannot be given at the present time, because of their restricted classification. Probes of this type are unsuitable for high temperatures due to their inadequate shielding.

It is now possible to take into account the combined result of the thermal effects represented by Equation [2] and the velocity effects represented by Equation [7]. Fig. 2 shows the results of computations for a shielded thermocouple in an airplane exhaust pipe, exposed to an external air stream at —48 F and a speed of 400 fps. In all cases, the emissivity ϵ of the couple, shield, and pipe was assumed to be equal to 0.9; the value of α was taken as 0.85 for the shield and 0.60 for the couple, while the conduction along the couple was neglected. In Figs. 2 (a and b), the total temperature of the gas was taken as 1200 F and the shield insulation was assumed to be a $^1/_2$-in. thickness of relatively low conductivity ($k = 0.08$). The curves of Fig. 2(a) show the variation of the indicated temperature with variations of the velocity V_2, inside the shield for several different values of V_1, the main stream velocity of the hot gas in the pipe. (V_2 could be varied by inserting various restrictions in the exit end of the shield.) The significant fact brought out here is that the error due to incomplete recovery of kinetic energy becomes appreciable, as the velocity over the couple increases, before the radiation error is swamped out by the convection. It is also evident that there is an optimum velocity over the couple, for the higher values of V_1, which will reduce the error materially, relative to the unrestricted flow when $V_2 = V_1$.

Fig. 2(b) represents a further study of the conditions obtaining in the case of the upper curve of Fig. 2(a), for $V_1 = 1200$ fps. It is interesting to note that at the higher values of V_2 the temperature of the inner surface of the shield is above that of the thermocouple, because of the higher recovery coefficient for the shield. As a result the thermocouple error is less than that due to velocity effects alone, since the couple is warmed by radiation from the shield.

In Fig. 2(c) the conditions have been changed as follows: The total temperature of the gas is taken as 2000 F; the thickness of the shield is reduced to $^1/_4$ in., and the conductivity k is increased to 0.24. This is to bring out the magnitude of the error with more intense radiation and poorer shielding. The dotted extension of the lower curve for $V_1 = 600$ represents the effect of increasing the velocity over the couple beyond the main stream velocity, which might be done by applying external suction to the shield, as is done in the typical high-velocity thermocouple. In the past the experimental observation that the temperature ceases to rise with further increases in velocity beyond a certain point has commonly been taken to mean that the indicated temperature has substantially reached the true gas temperature. Fig. 2(c) indicates that, under these conditions, no amount of raising or lowering the gas velocity will suffice to reduce the error below about 40 F.

Experimental Results

An extensive series of tests has been carried out by the General Electric Company to study the magnitude and nature of these effects and to develop a thermocouple shield and mounting

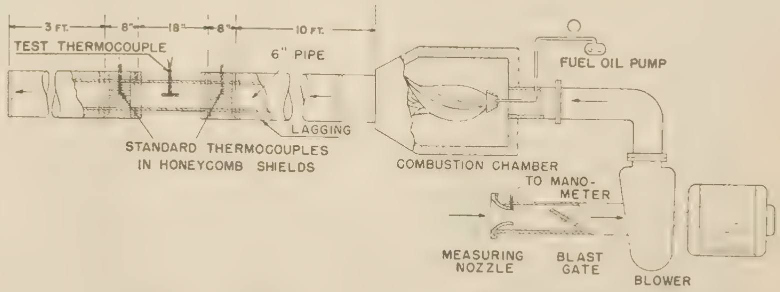

Fig. 3 Thermocouple Calibration-Test Apparatus

representing a practical compromise between maximum accuracy and minimum size and weight. In the work done at Lynn during the past 2 years most of the tests were run in the apparatus shown in Fig. 3, which is largely self-explanatory, although several modifications were used at various times. The gases were ordinary oil-furnace flue gases, diluted with considerable amounts of excess air. Velocities were usually from 200 to 500 fps, or roughly 250 to 625 psf per min. The "true" gas temperature was taken as the average of the readings of the couples in the two "honeycomb" type shields (Fig. 4) or in some cases two of the large quadruple shields shown in Fig. 5. There is ample evidence to indicate that either of these devices will reduce the actual error to a negligible amount (5 or 10 deg up to 1800 F) under these conditions, particularly when backed up by a heavy layer of insulation around the pipe.

Since it would be impracticable to describe the numerous tests and data in complete detail, the following is a summary of the more significant results and conclusions.

Fig. 6 shows the results of a series of tests to determine the magnitude of the error with unshielded thermocouples and the effect of depth of immersion in the gas stream. The latter variable brings out the effect of conduction of heat to the pipe wall. Obviously this is of secondary importance, so long as the wires

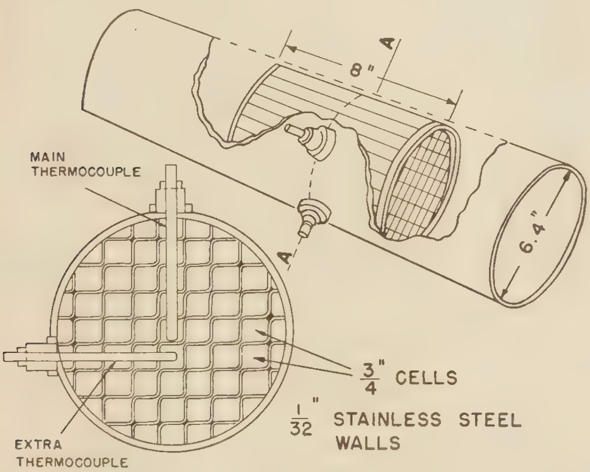

Fig. 4 "Honeycomb" Type Thermocouple-Shield Assembly

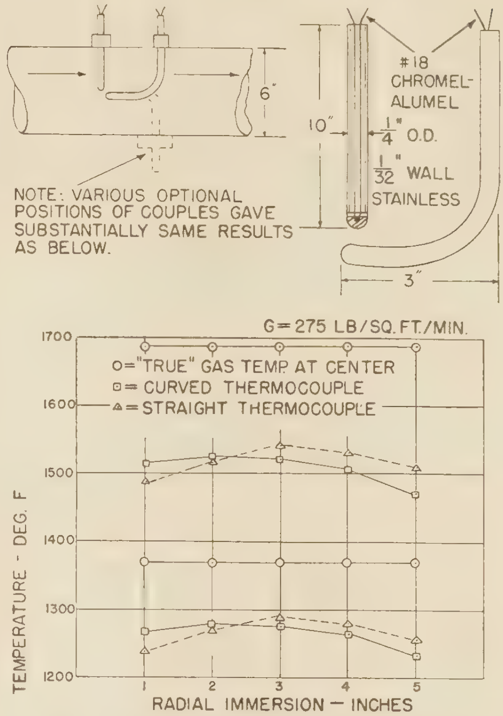

Fig. 6 Effect of Depth of Immersion Upon Temperatures by Unshielded Thermocouples

and sheath have the general proportions shown in the illustration. Other tests showed materially greater errors when the wires and sheaths were heavier and shorter. The most surprising result, disclosed in Fig. 6, is that the straight thermocouple consistently read higher than the curved one at the same radial depth beyond the first 2 in. It was confidently anticipated that the curved couple would always read higher because of its greater length of immersion for a given depth. However, these results were confirmed by three separate series of tests with totally different couples and other equipment and with the couples interchanged in various positions. The explanation is presumed to lie in the fact that the boundary layer around the axial portion of the curved couple must have built up to a greater thickness than in the case of the transverse flow across the straight radial couple. This would retard the convection of heat to the former and establish a temperature gradient away from the junction.

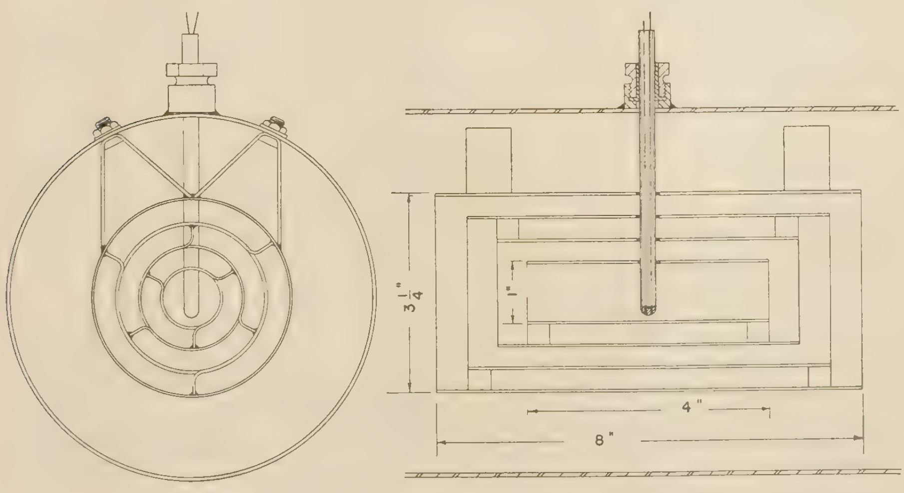

Fig. 5 Large Quadruple-Shield Thermocouple

Fig. 7 represents a preliminary set of tests to measure the effect of various numbers of shields upon the error. The difference between the data for the $^{13}/_{32}$-in. single shield and an earlier test on the 1-in. shield is due to the fact that the temperatures of the couple junction and the smaller shield were practically equalized by

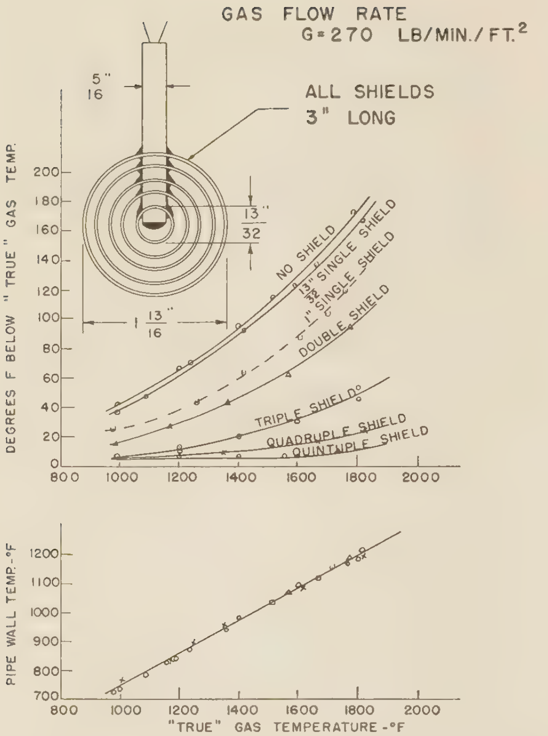

Fig. 7 Effect of Various Numbers of Shields Upon Thermocouple Error in Hot Gas Stream

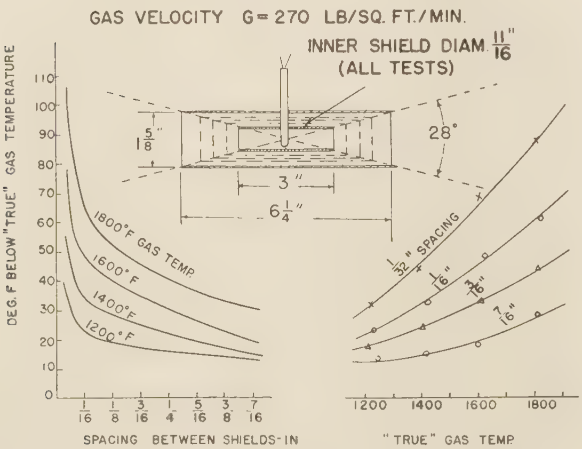

Fig. 8 Effect of Spacing Between Shields Upon Error With Double Shielded Thermocouple

conduction through the very short path separating them. Welds between the couple sheath and shields were eliminated in later tests.

Preliminary tests with double shields, to determine the effect of the spacing between shields, indicated a marked effect due to the varying unshielded angles at the ends, if the length was held constant. A simple calculation in geometry showed that there is little advantage in the latter respect if the length-to-diameter ratio is increased beyond 4. This ratio was therefore held constant in the tests represented in Fig. 8. Since both the viscosity and thermal conductivity of air increase markedly at these high temperatures, it is possible that the large increase in the error with the smaller spacings is due to heat conduction between shields, as well as to the retarded convective heating in the annular passage. With the $^1/_{32}$-in. spacing the two shields behave almost as one.

The tests of Fig. 9 were run to determine the extent to which the thermocouple sheath would obstruct the flow through a relatively small inner shield. Small steel spheres of various sizes were used to represent the sheath. The velocity through the tube was measured by means of a small impact tube near the exit end and the velocity past the sphere was calculated from the area ratio. In general it seems that the size of the inner shield should be dictated more by the advantage of having a fair length of sheath fully shielded near the tip than by any regard for maintaining V_b to insure good convection.

Recommend Shield Designs

As a result of these and other tests the small quadruple-shielded hermocouple of Fig. 10 was evolved as a practical compromise

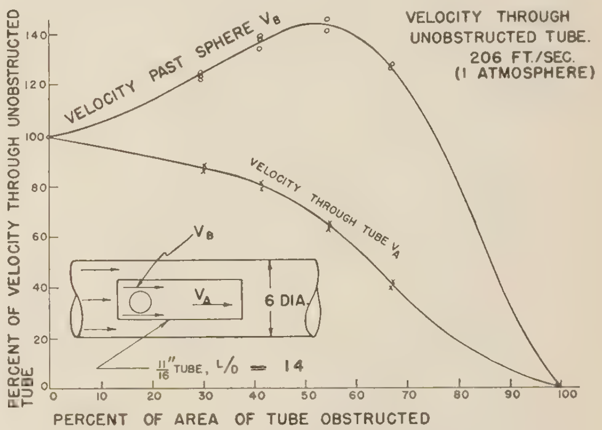

Fig. 9 Effect of Spherical Obstruction Upon Flow of Air Through Tube

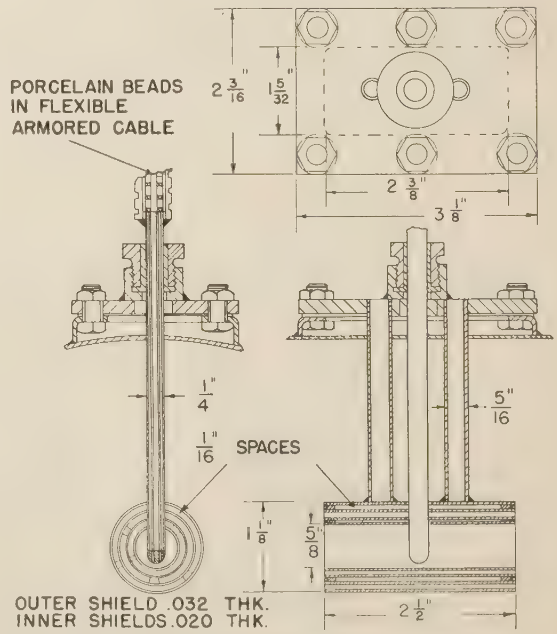

Fig. 10 Small Quadruple-Shielded Thermocouple

design for measuring temperatures up to about 1850 F in pipes from roughly 2 to 10 in. diam (varying the length of the couple and supporting tubes to suit). The thermocouple wires are No. 18 chromel-alumel, insulated by porcelain tubing, protected by a metal sheath. A high-temperature alloy known as KA2SMO (18-8 stainless stabilized with moly) has been found very satisfactory for the metal parts. A considerable number of tests has indicated that the error will not exceed 20 deg at 1800 deg, or 10 deg at 1200 deg with air or exhaust gases at velocities of the order of 100 to 400 fps (G = 200 to 500) in a bare pipe.

This type of mounting has been used in a number of applications for both flight and ground testing during the past year and has given a good account of itself, from the standpoint of practical serviceability, as well as consistent accuracy. Somewhere in the vicinity of 1900 F, they begin to disintegrate fairly rapidly, in an oxidizing atmosphere, and a very short exposure at 2000 F will burn out the inner shields and sheath.

The design of Fig. 11 was the result of several attempts to develop a still smaller mounting, for which there is a persistent demand. This construction, with the $^1/_4$-in. layer of fuller's earth in lieu of the metal shields, gave substantially the same degree of accuracy as the Fig. 10 version, but it was found very difficult to insulate the thermocouple wires in the small sheath with any material that would stand up more than 1 hr or so in service. However, this general design has considerable merit and it may be entirely possible to solve this problem. Decreasing the thickness of the fuller's earth in the shield was found to increase the error very rapidly. It is impracticable to apply the thermos-bottle construction here, because of the great difficulty in holding even a rough vacuum between metal walls above a red heat.

Fig. 12 is included to illustrate a type of design which may be used to advantage in special cases where direct exposure of the thermocouple wires to the gas stream can be tolerated. Ordinarily this will result in contamination of the metals at the junction by chemical action which will cause erratic readings after a

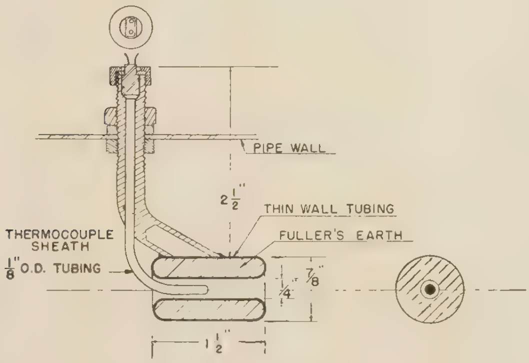

Fig 11 Small Insulated Shield Thermocouple Mounting

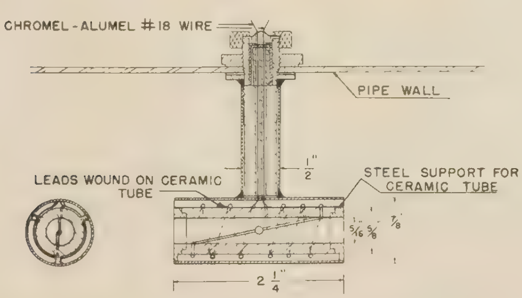

Fig. 12 Exposed-Junction Thermocouple for Inert Gases

few hours of use at high temperatures. This is especially true of small wires in reducing atmospheres. There may be occasions, however, where the need for minimum size and rapid response to changes in temperature will justify the necessity for frequent calibration or replacement which such a design will entail. Note the care that has been taken to minimize the temperature gradient in the wires away from the junction. A short radial sheath of the type used in Fig. 10 would cause large conduction errors in a design of these proportions.

In order to obtain experimental confirmation of the predicted effect of high velocities, as shown in Fig. 2, one of the small quadruple-shielded mountings (Fig. 10) was fitted with a simple butterfly valve arranged to vary the opening at the downstream end of the inner shield. This was mounted in a modification of the Fig. 3 setup, in which a 2-in. orifice was installed in the 6-in. pipe at a distance of 1 in. upstream from the inlet end of the shields. With a pressure ratio of about $2^1/_4$ to 1 across the orifice the thermocouple was thus exposed to a blast of hot gas moving at the speed of sound. At 1600 F, this amounted to roughly 2000 fps.

The results of the first tests with this arrangement are shown

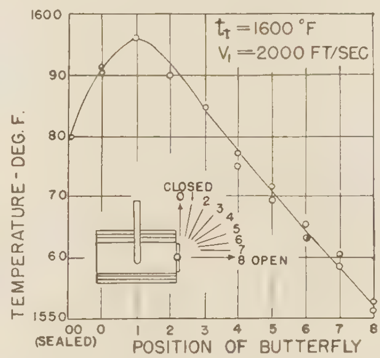

Fig. 13 Effect of Varying Gas Velocity V_2 Through Thermocouple Shield; Test Results

in Fig. 13. The tests were completed just prior to going to press and as a matter of convenience no attempt was made to duplicate the conditions of Fig. 2, quantitatively. The test section of the 6-in. pipe was completely covered with 1 in. of lagging to reduce radiation effects. Since an appreciable amount of leakage was observed with the butterfly in the "closed" position, an extra run was made at the "00" point after sealing the valve with cement. As may be seen, these data confirm the fact that there is an optimum velocity of the gas over the thermocouple if reasonable accuracy is to be secured under these conditions.

General Comments

It should be evident from the foregoing that something better than the ordinary commercial thermocouple mountings are required for even a fair degree of accuracy with either the high temperatures or high gas velocities considered herein. In general it will not suffice simply to calibrate the conventional devices, except for very restricted usage, since the errors may vary so widely with such factors as gas density, velocity, and temperatures, as well as with variable radiant emissivities, and other heat-transfer effects.

A further investigation of this subject might provide an appropriate research project for a properly equipped laboratory, since there is, at the present time, a rapid extension of significant activities requiring the use of such techniques for industrial test-

ing operations. Specifically, more adequate data are needed on actual recovery coefficients of thermometric devices under various conditions; there is still a need for smaller shielded mountings, preferably with some automatic or inherent feature to provide optimum values of V_2; and there is a need for suitable techniques for measuring still higher gas temperatures. Possibly it would be worth while to establish the absolute magnitude of these errors more accurately, since the "true" gas temperatures cited refer to an arbitrary standard assumed to be "reasonably" accurate on the basis of indirect evidence.

Acknowledgments

The author is indebted for suggestions and advice to a number of individuals in the General Electric organization in addition to the members of the two subcommittees mentioned. He particularly wishes to acknowledge the contributions of Messrs. E. G. Allen, W. S. Thompson, and S. W. Mikulka in carrying out the tests and calculations.

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Discussion

E. D. Haigler.[5] This study is an excellent example of the development of theory, the establishment of magnitudes experimentally, and then the evolution of an adequate device. While the immediate object of the study was specific and military, both the theory and the results have much more general application.

Industrial temperature measurement presents many problems which call for careful design and placement of temperature-sensitive elements if large conduction errors, and at high temperatures radiation errors also are to be minimized. In these days of large-scale emergency-plant construction and conversion, it is particularly important that engineers understand these principles so that specification and installation errors may be largely avoided or quickly found. The experimental data presented, being in the medium-temperature field, are useful not only directly in solving specific measurement problems in the combustion and heat-treating fields, but also as examples to give an idea of the order of magnitude of conduction and radiation errors at these and other temperatures.

W. W. Johnson.[6] It is easy to believe that the errors in measuring high temperatures in high-velocity gases can be very large if devices are used which are not suited to the conditions.

The question naturally arises, however, as to whether the standard thermocouples in "honeycomb" sheaths actually did register the true total temperature of this high-temperature high-velocity gas stream. The author apparently recognizes the possibility of some doubt in this respect.

Referring to Fig. 3, apparently no provision was made for mixing the gas from the combustion chamber in order to eliminate stratification, which might exist to a degree large enough to cause some errors, especially in the tests with varying immersion, Fig. 6. The question of stratification might have been settled by comparing the two standard thermocouples, one fixed at the pipe center and the other moved in steps across the pipe. More data would be desirable as to the agreement between the upstream and downstream standards. Some difference between them might be expected because of radiation.

It is possible that the standard thermocouples themselves read too low a temperature at high velocities, since the downstream side of a cylindrical sheath or well under this condition is at a distinctly lower temperature (and pressure) than the upstream side. This effect could have been avoided or reduced by increasing the pipe diameter at the standard thermocouple stations and by "streamlining" the cross section of their sheaths.

If this condition existed, the conclusion would be that the errors of thermocouples without screens are even larger than those disclosed by these tests. There seems to be no reason to doubt the relative differences between the test thermocouples and the standards.

J. H. Marchant.[7] Mr. King's paper represents a very generous contribution of the General Electric Company to disseminate information where it is vitally needed. However there are points in this paper which the writer believes require further illumination. They are as follows:

1 In item 4 under "Theoretical Considerations," the author points out parenthetically that velocity effects are significant only at extremely high velocities. On the contrary, velocity effects may be significant at any velocity depending upon the accuracy required.

2 In Equation [1], the effect of gaseous radiation has been neglected. Since the author is primarily concerned with the determination of significant engine-exhaust-gas temperatures, it would seem in order for him to justify this assumption in cases where the more highly radiating gaseous components of the exhaust have been increased as, for example, if detonation suppression were effected in the engine by water injection.

3 Referring to Equation [1], it might have been more enlightening if the author had pointed out that C_3 is not constant but varies, among other things, with any change in the shape of

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[6] Turbine Engineering Department, General Electric Company, Lynn, Mass. Mem. A.S.M.E.

[7] Engineer, Pratt & Whitney Aircraft, East Hartford, Conn. Mem. A.S.M.E.

the probe. Such a statement would have made the experimental work more logical.

4 In this paper, the author is proposing the use of instruments, which are calibrated under steady-flow conditions, to measure a significant temperature of a violently unsteady gas stream.

It is questionable whether or not the response of these steady-flow indicators under unsteady conditions has any true thermodynamic significance, and if so, what it is.

It would also seem desirable to investigate the response of all circuits involved, thermal, electrical, etc., under the unsteady-flow conditions of usage.

5 The author states: "It is theoretically possible to insulate the walls sufficiently to maintain $T_w = T = T_g$, etc." It seems if $T_w = T_g$, that $T > T_g$ unless the gas velocity is zero.

6 The example of the sling psychrometer may lead to some confusion on the part of the reader, since it is equally as complicated as the subject of the paper; and, in addition, it involves relatively large mass-transfer effects which are now not well understood.

7 Immediately preceding Equation [3], the author says that (as in flow through a nozzle) the resulting kinetic energy is acquired at the expense of the temperature of the gas.

This statement is misleading in that this kinetic energy is acquired as a result of a decrease in the enthalpy of the gas which involves both internal (temperature) energy and pressure (elastic potential) energy.

8 Referring to Equation [3], M = mass of gas in pounds, not poundals as the author says; and g is not the acceleration of gravity, but a pure number which is dimensionless and numerically equal to the standard acceleration of gravity.

9 The author points out that a recovery coefficient of 0.6 has been assumed for the thermocouple.

While the writer agrees that this value of α is reasonable, he would like to know whether or not α has ever been measured for the probes described in the paper.

10 At another point, the author states: "Probes of this type are unsuitable for high temperatures due to their inadequate shielding."

On the other hand, it should be pointed out that diffuser-type probes, similar to those described by Franz and Eckert, have been developed by Pratt and Whitney Aircraft. By means of these probes, gas temperatures up to approximately 450 F and velocities corresponding to a Mach number of 0.95 have been determined to within less than 0.25 per cent.

It should be pointed out in this connection that the type of probe described by the author is primarily useful at high temperatures and comparatively low velocities.

Work done by Pratt and Whitney Aircraft indicates that α has appreciable Reynolds number effects at the higher velocities.

In conclusion the writer would like to compliment the authors on performing a very difficult piece of research which is complicated by many pitfalls which may not be apparent to the reader.

H. F. Mullikin.[8] The depression of the temperature reading of a thermocouple due to kinetic-energy effects, dealt with in this paper, must certainly be considered when high accuracy at elevated temperatures is required.

It has been pointed out that by definition the static temperature of a flowing gas is below the total temperature by an amount equivalent to a velocity temperature (which is due to the kinetic energy of motion). This is illustrated by Fig. 2 (*b*) of the paper, which also indicates that for high velocities a thermocouple will read between the total and the static temperatures.

[8] Instructor, College of the City of New York, New York, N. Y. Mem. A.S.M.E.

It is ordinarily implied that the true gas temperature is the total temperature. It seems to the writer that the true gas temperature can be either the true total temperature or the true static temperature, depending upon which is meant.

The static temperature of a gas is defined as the temperature hypothetically measured (i.e., true static temperature which should be indicated) by a thermometer moving at the same speed as the gas stream. This is the actual temperature of the gas which is radiating heat and is the temperature to be used in formulas governing radiant-heat exchange. The fact that the gas would have another temperature (true total temperature) if brought to rest does not alter this fact.

It does not follow that a stationary temperature-measuring device should necessarily read true total gas temperature just because it is not moving with the gas stream. It seems to the writer that the ideal temperature-measuring device might also be defined as one which would read the true static temperature, since this is the actual temperature of the gas that is radiating heat. From this point of view it could be said that the temperature-measuring thermocouple is in error because it reads above the actual (static) temperature (as well as below the total temperature). While the actual (static) temperature is of interest as regards radiation phenomena, the total temperature is of importance in considering the heat capacity or heat content (enthalpy possessed by the gas.)

This merely emphasizes the author's statement as to the desirability of determining the relation between the true total temperature and the reading of the temperature-measuring device but points out that the actual (static) temperature is also of importance.

In measuring hot gas temperatures in steam boilers, the partial velocity-temperature error mentioned is, as a rule, subordinate to other sources of graver error. That is to say, it is a small error among some large errors, these being due mostly to slag in gases resulting from pulverized-fuel combustion. Due to the cleaner gases in the exhaust from internal-combustion engines, it should be possible to work to closer accuracy so that the refinements discussed in the paper are justified.

In the high-velocity thermocouples used by the writer which were all of a definite type of construction, we were not able to note a decrease of the indicated temperature with increased velocity as shown in Fig. 13 of the paper. This was undoubtedly due to the fact that the construction used resulted in sufficient gas-flow friction to prevent the attainment of extremely high velocities.

In the definition of m in Equation [3], mass is given as poundals. Since poundals is a force unit we are inclined to believe that pounds mass are meant.

N. F. Murphy.[9] In connection with Fig. 6 of the paper, would the author care to comment on the possibility that, in the case of the straight thermocouple, the error is primarily due to conduction losses, while in the case of the curved thermocouple, the error is primarily due to radiation losses when the immersion length is approximately 2 in. or less? In the case of the curved thermocouple, there is approximately 3 in. more length of surface area subject to the effect of the pipe wall. The writer will be interested to learn if tests were made to prove or disprove this assumption.

I. M. Stein.[10] Mr. King has condensed into a relatively brief paper a great deal of valuable information on a very complex subject. The data submitted in the paper comprise practical

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[10] Director of Research, Leeds & Northrup Company, Philadelphia, Pa. Mem. A.S.M.E.

and much needed refinements in several generally used arrangements for measuring high temperatures, and also some data pertaining to the general problem, in connection with which the author made his temperature measurements. The writer's only criticism of the paper is that, as a result of the considerable condensation, the author has not distinguished clearly enough between the consideration of temperature measurement as such and the other considerations involved in his general problem.

Specifically, there is some confusion between error in temperature measurement and the error which results from using a temperature measurement to indicate something else.

The author defines T_t as the "total" temperature of the gas at rest, and defines T_s as the "static" temperature of the gas, hypothetically measured by a thermometer moving at the same speed as the gas stream. He finds that the best thermocouple arrangement so far devised indicates a temperature intermediate T_t and T_s. Later he implies that T_t should be taken as the true temperature of the gas, and that any departure from T_t represents an error in the temperature measurement.

Actually, all that any thermocouple, free from error in itself, could be expected to measure is the temperature T_s, not T_t. Such a perfect thermocouple, mounted in a fixed position, in a high-velocity stream of gas would indicate too high a temperature because of the partial conversion of the kinetic energy of the gas stream. Hence the error in temperature measurement would be positive and not negative.

What the author probably wished to point out is that a moving thermocouple to measure T_s is impractical, and since a stationary one will not measure T_s accurately, it would be very helpful in his particular problem if the thermocouple could be made to indicate T_t; and he has shown how nearly he has succeeded in making it do that. Temperature T_t, however, would never be the true temperature of the gas at the point of measurement.

The true temperature must always be T_s. Even if all of the kinetic energy in the gas stream at the "point" of measurement could be converted into heat energy, and there were no error in the thermocouple itself, the measured temperature could not be translated accurately into the "total" temperature, unless the only reduction in temperature between the point of zero velocity and the point of measurement were truly represented by the velocity of the gas. In other words, any leakage of heat from the gas stream between these two points would vitiate the assumed relation.

In this same connection, in the section "Recommended Shield Designs," it would be desirable to indicate clearly whether the errors stated are departures from true temperatures or from assumed total temperatures.

Guenther von Elbe.[11] It appears that the thermocouple designs shown in the paper approach the optimum compromise between high accuracy and small size. Concerning the concluding paragraphs of the paper, one may inquire with what means further research of this subject can be carried out. Certainly the electrical-resistance method of temperature measurement offers no attractive possibilities here, since the problems of shield design and recovery coefficients would be substantially the same as in the thermocouple method.

There are, however, a number of methods by which the static temperature T_s could be measured directly; such measurements would supplement the thermocouple studies since, if the gas velocity were known, they would allow determination of recovery coefficients by Equation [6] and also yield values of the total temperature T_t which could be compared with the values obtained by methods described in the paper.

Methods of measuring static temperatures are based on observation of properties, such as thermal radiation or gas density, by various optical means without introducing a fixed test body into the gas stream. They cannot compete with the thermocouple method in simplicity and ruggedness of design, but this does not detract from their value in calibration work.

The best known methods are those based on thermal radiation. It must be admitted that they have only been applied to temperatures higher than 1600 F, but the limit of applicability, which is a question of sensitivity of brightness observations, can probably be extended to lower temperatures.

Among methods based on density changes, the observation of the refractive index is the most promising. It has been little used thus far, but it seems to be adaptable to many problems; and one would like to see further developments of this technique since it offers a means of optical temperature readings below the range of brightness observations.

In calculating total temperature T_t from static temperature T_s, it should, in principle, be considered that a part of the kinetic energy is stored in the turbulence of the stream. It is believed that, in a channeled stream, this contribution is insignificant and Equation [6] of the paper is substantially valid, with V denoting the axial velocity.

[11] (a) Published by permission of the Director, Bureau of Mines, United States Department of the Interior.
(b) Physical Chemist, Central Experiment Station, Bureau of Mines, Pittsburgh, Pa.

Author's Closure

The author is sincerely grateful to Messrs. Haigler, Johnson, Marchant, Mullikin, Murphy, Stein, and von Elbe for their constructive and helpful comments on this paper.

In reference to Mr. Johnson's remarks, the question of stratification was investigated, very much as he suggests, and the evidence indicated that satisfactory mixing was effected by the 20 diameters of pipe ahead of the test section. As he surmises, the downstream standard thermocouple always reads considerably lower than the upstream standard. The "true" temperature at the plane of the test thermocouple was taken as the average of these two readings, assuming a straight-line temperature gradient. The latter was justified by the fact that the axial temperature gradient was small relative to the difference between pipe and ambient temperatures.

Dr. Marchant's comments are welcomed as contributing a higher degree of preciseness to several passages which were written originally in engineering rather than in rigorous scientific terms. But in most of these instances, the practical significance of the distinction is negligible under ordinary circumstances. For example, in his item 5, Dr. Marchant urges that when the pipe wall is insulated so that its temperature T_w is equal to that of the gas T_g, the indicated temperature T is greater than T_g, unless the gas velocity is zero. This is literally true if T_g is assumed to be the static temperature. The original condition wherein $T_w = T = T_g$ was mentioned in the first section of the paper in a discussion "from the thermal standpoint (neglecting velocity effects)." Even when the velocity is as high as 100 fps, the difference between T and T_g is less than 1 deg.

With reference to his item 4, the significance of the unsteady nature of the gas flow appear somewhat overemphasized. The gas flow in the main tail-stack of a modern multicylinder aircraft engine is usually so nearly steady that very special instruments would be required to measure the pulsations. It is granted, however, that the effect becomes very pronounced in the case of the discharge from a single cylinder. There appears to be no adequate method available for measuring the gas temperature in this case.

In answer to Dr. Marchant's question in his item 9, the actual recovery coefficients were not measured for the probes described

in the paper. In an earlier series of tests, values of α in the vicinity of 0.6 were obtained for ordinary mercury thermometers with transverse air flow.

As Messrs. Mullikin and Stein very properly point out, the "true" temperature of the gas is not necessarily the total temperature T_t. Under certain circumstances, possibly in most cases, the static temperature T_s may be regarded as representing the condition of the gas more appropriately. This is certainly true, for instance, in the measurement of the ambient-air temperature by means of an instrument mounted on an airplane flying at high speed. But it is somewhat like asking whether the static or the total pressure more properly represents the "true" pressure of a gas stream; it depends upon which property of the stream is most significant at the moment. In the various applications with which the author has been concerned, the total temperature has been of prime importance, since this is the maximum temperature which may be attained by any object or apparatus exposed to the gas stream.

The use of the term "poundals" instead of "pound mass" in defining m of Equation [3] of the paper was due to an oversight.

In reply to Mr. Murphy's question, the tests, represented by Fig. 6, were run primarily to demonstrate the fact, indicated by several other tests, that the error due to conduction is small relative to that due to radiation, under these general conditions. It may be seen that, even when the radial immersion was less than 2 in., the error was about the same with the straight or curved thermocouple, in spite of the fact that the total effective immersion was nearly 3 in. greater in the case of the latter.

Incidentally, the significance of Fig. 6 is not as clear as it might be. Two separate sets of data are represented: In one the "true" gas temperature was held at 1690 F, and the temperatures measured at various depths were in the vicinity of 1500 F; whereas in the other the "true" temperature was 1370 F, and the measiured temperatures varied between 1235 and 1290 F.

Dr. von Elbe's remarks concerning various methods of measuring T_s are very pertinent, for there is a definite need for better techniques of this sort. There is considerable occasion, however, for further work on recovery coefficients of thermocouple sheaths and similar shapes, which are significant in the precision testing of steam and gas turbines.

Since the paper was written, the author has found what is believed to be the first published reference to the distinction between total and static temperatures, in a paper by Sanford A. Moss.[12] The paper includes an interesting discussion of temperature and pressure phenomena in high-speed gas streams.

[12] "The Impact Tube," by S. A. Moss, Trans. A.S.M.E., vol. 38, 1916, pp. 761–797.

Process Lags in Automatic-Control Circuits

By J. G. ZIEGLER[1] and N. B. NICHOLS,[2] ROCHESTER, N. Y.

Methods are given for quantitative determination of time lags in automatically controlled processes. The area under recovery curves is taken as a direct measure of process difficulty, and this area is shown to vary as the second power of the time lag. A "recovery-factor" term, part of a complete expression for controllability, is introduced which makes possible a classification of processes in dimensions of the process itself, regardless of controller or valve mechanism used. Values of this recovery factor from various industrial applications are given in tabular form. Several processes are examined for the time lag, and means of reducing this unfavorable characteristic are demonstrated. It is felt that this paper will be useful to engineers who are interested in improving the controllability of the processes which they design.

The importance of automatic controllers in the operation of modern plants is increasing yearly if the number of controllers used is any indication. Knowledge of instrument characteristics is also increasing; the theoretical action of each control effect has been expressed as a mathematical equation, and the newer instruments follow the equations very closely. Adjustment dials are even calibrated in terms of the constants appearing in the equations which describe the responses. Industry's demands for closer and closer control have forced the development of the refined control effects which the instrument manufacturer has supplied. Now it appears that the picture has become top-heavy. The science of instrument design has exceeded the study of process design for controllability.

In the application of automatic controllers, it is important to realize that controller and process form a unit; credit or discredit for results obtained are attributable to one as much as the other. A poor controller is often able to perform acceptably on a process which is easily controlled. The finest controller made, when applied to a miserably designed process, may not deliver the desired performance. True, on badly designed processes, advanced controllers are able to eke out better results than older models, but on these processes, there is a definite end point which can be approached by instrumentation and it falls short of perfection.

The chronology in process design is evidently wrong. Nowadays an engineer first designs his equipment so that it will be capable of performing its intended function at the normal throughput rate plus a factor of safety. The control engineer or instrumentman is then told to put on a controller capable of maintaining the static equilibrium for which the apparatus was designed. The control engineer faced with this do-or-die ultimatum recommends a type of controller basing his judgment on experience with similar jobs. If his analogy is good, the correct controller is selected. When the plant is started, however, it may be belatedly discovered that, in spite of the correct equipment design for steady-state conditions and the correct instrument selection, control results are not within the desired tolerance. A long expensive process of "cut and try" is then begun in order to make the equipment work. Both engineers realize that some factor in equipment design was neglected but generally they can neither identify the missing ingredient nor correct it in future design.

The missing characteristic can be called "controllability," the ability of the process to achieve and maintain the desired equilibrium value. Design for steady-state conditions is not enough if exact maintenance of variables is necessary. Control action consists of continuous correction of process changes, tending to destroy equilibrium at the desired value and, as such, its study involves not steady-state but transient characteristics of the process and controller.

A tubular heater for raising milk to the pasteurizing temperature may be designed with ample heating surface, and the steam supply may be adequate, but the maintenance of a constant milk outlet temperature by steam-valve manipulation is very difficult if milk flow or incoming temperature vary suddenly. A good controller will be able to bring the temperature back to the correct value following one of these disturbances but only at the expense of some deviation for a certain length of time. During the recovery period a loss results, since any increase in temperature spoils the "cream line" of the product and any drop requires reprocessing. These deviations are so important in milk pasteurization that most of the equipment now used has been designed to make excellent control results possible.

The problem of equipment design for controllability involves transient conditions and transients usually involve exponential curves and an order of mathematics not at the finger-tips of the average engineer. Even if he were able to deal with transient phenomena, he would not know where to start, since to the authors' knowledge no complete formula for controllability has ever been published. A great many of the factors affecting controllability have been identified and investigated in the numerous papers sponsored by the A.S.M.E. Committee on Industrial Instruments and Regulators. All of these factors affect controllability; no one is a complete solution to the problem, and each factor uncovered increases the certainty that the problem is a complex one, not to be solved in a day. As it now stands the plant designer is almost justified in disregarding the entire matter, hoping that the magic quantity, controllability, is included in his apparatus but turning the burden over to his instrument engineer.

Sooner or later, however, these factors affecting process controllability will have to be smoked out and reduced to definite "good-practice" rules which will be as much a part of equipment design as safety factors. Furthermore, establishment of rules is not enough; simple methods of applying the rules must be developed at the same time so that the complex mathematics involved will not be the stumbling block. It was possible to calculate the equilibrium conditions existing in a fractionating column before the McCabe-Thiele method of graphical analysis was developed but that method reduced the time required to a reasonable figure.

Unfortunately, the authors are not able to give a formula for controllability. It appears that when such a formula is devised it will consist of several factors. One might be called the "recovery factor," the ability of the process to recover from the maximum change in demand or load. Another, a "load factor," must take into account the point in the process at which the disturbance occurs. That expression will cover the thing called

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"supply and demand side capacity" (1).[3] The third might be called the "mobility factor," the ability of the process to follow demands for different values of the variable. This factor would be important when controller set points had to be changed suddenly or changed gradually as on a "time-schedule" control problem. The failure of some time-schedule temperature-control applications has been due to lack of mobility, not lack of the recovery factor. Added to these three may be other secondary factors as yet unexplored.

In this paper, a tentative formula is set up for the first or "recovery factor" involving three process characteristics. One of these, time lag, is shown to be of primary importance and simple methods are given for approximating the time lag on actual examples of industrial control installations. This paper then attempts to deal with *only one* term in *only one* of the factors affecting controllability—the time-lag term in the recovery factor.

A Control Circuit

Illustrated in Fig. 1 is a control circuit consisting of a controller and a process (2). Note that no control valve is shown, it being considered a portion of the process. Between pen and output lies the controller which transforms pen behavior into appropriate output behavior. The output effects the process, changing some variable which is translated into a pen movement through the measuring portion of the circuit at the right. If for every output there were a definite pen position no controller would be necessary and manual control would suffice. The purpose of the controller is to keep the pen at the desired point in spite of the load changes which are shown entering the process at several points. It is these load changes which require altered output in order that the same pen position be maintained.

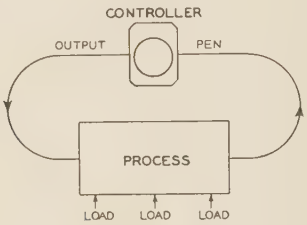


Fig. 1

Between output and pen lies the relatively uncharted portion of the control circuit, the process. It is with the latter that this paper is chiefly concerned, though a brief résumé of the control effects present in modern controllers and their characteristics must be included if only to establish a terminology. Air-operated instruments will be considered simply because they are the type most familiar to the authors, so output will be given as an air pressure in pounds per square inch. Pen movement will be given in inches in most cases.

In the process examples to follow it will be assumed that one set point is to be maintained regardless of process load conditions, so a controller with proportional and automatic reset (proportional-speed-floating) responses will be used. The first of these two control effects, proportional response, gives an output change proportional to pen movement; the magnitude of this response will be called "sensitivity," and the unit of sensitivity will be the output-pressure change per inch of pen movement. Automatic-reset response detects the deviation of the pen from the desired set point and gives a rate of output-pressure change proportional to the discrepancy. The magnitude of automatic-reset response will be called "reset rate," the number of times per minute which it duplicates the proportional-response output change caused by the discrepancy between pen and set point. A third control effect called "pre-act" or "derivative" response is often used on processes with long time lags. This response in its pure form gives an output-pressure change proportional to the rate of pen movement and its unit has been called the "pre-act time" in minutes. This response will be considered in this paper only to the extent of pointing out the processes on which it could be used effectively.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

Process Reaction Curve

The magnitude of controller responses can be determined by impressing various pen movements and noting the resulting output-pressure behavior. It would appear reasonable then that some process characteristics could be identified by impressing an output-pressure change and noting the resulting pen behavior. The authors (5) have found that the "reaction curve" drawn by the pen in response to a sustained change in output pressure can be analyzed to give a fair picture of the process from the standpoint of optimum controller settings.

In order to visualize a process-reaction curve, consider the control circuit of Fig. 2 in which an actual process replaces the blank box of Fig. 1. Cold water flowing to a tank is heated by steam injection and flows at constant rate through a pipe line to the bulb of a temperature controller located some distance away. The pen moves in response to temperature changes at the bulb, and the output pressure alters valve opening and the corresponding steam flow to the tank. The principal load change on the process comes from variations in the temperature of incoming cold water. This process is chosen because it exhibits the least complex type of time lag, notably a "distance velocity" (3), or dead-period lag. A definite length of time is required for water from the tank to flow to the bulb; consequently, the temperature of water at the bulb will lag the tank temperature by this interval.

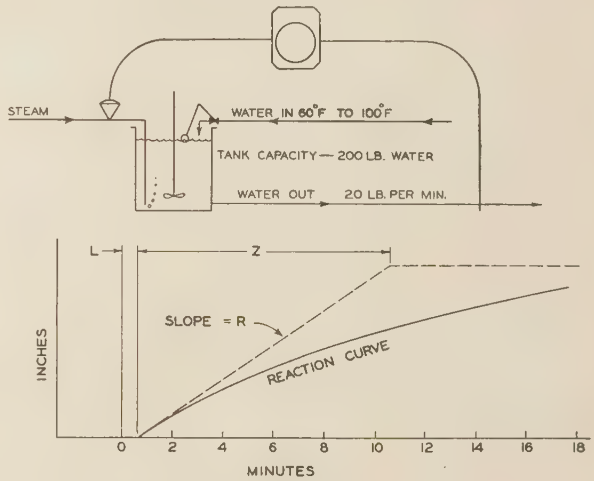


Fig. 2

Actually there are other lags, in the control circuit of Fig. 2, such as the lag of the bulb to changes in water temperature, the lag of moving the valve from one position to another, and small lags in the controller itself. In addition, the heat content of tank walls and of the pipe leading to the bulb would have an effect but it can be assumed that these factors are negligible in the example.

If the system Fig. 2 were in balance at a constant temperature and a small sustained change in output pressure of F pounds per square inch were made which opened the steam valve slightly, the tank temperature would immediately start to increase to-

ward a new balance point. After the interval necessary for water to flow from tank to bulb, the pen would move accordingly. This process-reaction curve is shown in Fig. 2. Two characteristics of the reaction curve are used to determine the optimum controller settings, the "lag" L (minutes) and the maximum rate of pen movement caused by the impressed output change which is called the reaction rate R (inches per minute). Experimental work has shown that optimum settings for a controller with proportional-and-automatic-reset responses are approximately

$$\text{Sensitivity} = \frac{0.9\,F}{RL}\,\text{psi per in.}$$

$$\text{Reset rate} = \frac{0.3}{L}\text{ per min}$$

These settings appear to be very nearly correct on the processes tested for wide variations in the values of R and L. On those infrequent processes in which L becomes greater than Z_0, Fig. 2, the settings given are too conservative.

There are two drawbacks to the use of experimental reaction curves for process analysis. In the first place the disturbance caused by running a reaction curve, can seldom be tolerated on a continuous process. In the second place, it is necessary to have a process on which to run the test, and this the designer does not have since pilot-plant and full-scale units will usually have widely different control characteristics. Nevertheless, reaction curves are very practical because they give a simple pictorial representation of a process and an explanation of process difficulties which is almost impossible to reach by chasing air, steam, and temperatures around the control circuit. In addition, process-reaction curves can often be calculated quite easily as will be shown. In fact, it is often easier to calculate a reaction curve than to believe that so simple a picture actually gives an indication of controller settings.

Calculated Reaction Curve

In order to calculate the controller settings required for the process of Fig. 2, it is only necessary to find values of F, R, and L. Assume the following data:

Water in tank, lb.	200
Water in line between tank and bulb, lb.	12
Water flow lb per min.	20
Steam flow (maximum) lb per min.	6
Incoming-water temperature (minimum), F.	60
Incoming-water temperature (maximum), F.	100
Outlet-water temperature, F.	160

Diaphragm-operated valves normally require a pressure change of about 12 psi to give full stroke. Each pound per square inch change in output will make $^1/_{12}$ of the total steam flow of 6 lb per min or $^1/_2$ lb per min. This assumes that the valve has the linear characteristics which are correct for this process (4, 5). If the tank temperature were constant at 160 F and a 2-psi change in output were made, increasing the heat flow by 1000 Btu per min the tank temperature would start to rise $^{1000}/_{200}$ or 5 F per min. Assuming 1 in. on the instrument chart or scale equivalent to 25 F, the reaction rate R, resulting from a 2-psi change in output, would be 0.2 in. per min, and the unit reaction rate R_1 for a 1-psi change would be

$$R_1 = \frac{R}{F} = \frac{0.2}{2} = 0.1\text{ in. per min per psi}$$

The time lag of the process will be the time necessary for the water flow of 20 lb per min to displace the 12 lb of water between tank and bulb

$$L = \frac{12}{20} = 0.6\text{ min}$$

The controller settings for proportional-and-automatic-reset responses will then be

$$\text{Sensitivity} = \frac{0.9\,F}{RL} = \frac{0.9}{R_1L} = 15\text{ psi per in.}$$

$$\text{Reset rate} = \frac{0.3}{L} = 0.5\text{ per min}$$

In terms of output pressure, the maximum load change, ΔF, which can occur in the process would be the difference in heat input required to raise 20 lb per min of 60 F water to 160 F and that required for 100 F inlet water divided by the valve constant of 500 Btu per min per psi

$$(20)(160 - 60) = 2000\text{ Btu per min}$$
$$(20)(160 - 100) = \underline{1200}$$
$$\text{Maximum load change} = 800/500 = 1.6\text{ psi} = \Delta F$$

Now let us see how this calculated reaction curve can be translated into one measure of process controllability.

Recovery Factor

On this process, a controller adjusted to the foregoing values of sensitivity and reset rate would correct the maximum load change of 1.6 psi (incoming-water change from 60 to 100 F) at the expense of a recovery curve similar to that shown in Fig. 3. The

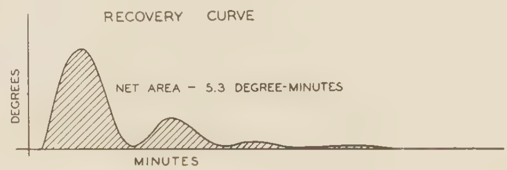


Fig. 3

shaded area under the curve can be taken as a measure of maximum process difficulty inasmuch as recovery curves for load changes smaller than 1.6 psi would enclose less area. In order to determine the area, it is only necessary to remember that automatic-reset response changes controller output at a rate proportional to the distance between pen and set point, the latter converted by proportional response into an output change. It can be shown that the "net area" under the recovery curve of Fig. 3 will then be equal to

$$\text{Net area} = \frac{\Delta F}{(S)\,(RR)}\text{ in-min} \qquad [1]$$

where ΔF = load change, psi
S = sensitivity, psi per in.
RR = reset rate, per min

The worst load change of 1.6 psi which can afflict the process considered will then produce an area of 0.21 in-min or 5.3 deg-min.

It is obvious from Equation [1] that lowering either controller setting will increase the net area. Likewise, increasing the sensitivity will increase the amplitude ratio of oscillations in the recovery curve and also increase the area. Raising the sensitivity from $0.9/R_1L$ to $2/R_1L$ would give an oscillation which would never die out and the area would become infinite. An increase

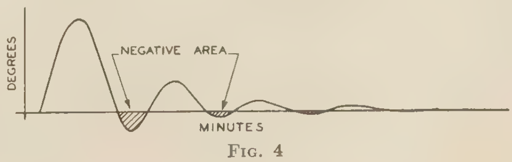


Fig. 4

in reset rate would allow the recovery curve to swing on both sides of the set point adding a negative area as shown in Fig. 4. This reversed swing would undo an equal portion of the work done by automatic reset while the pen was above the set point and the total area would become

$$\text{Total area} = (\text{net area}) + (2)\ (\text{negative area}) \qquad [2]$$

In the authors' experience the settings given reduce the total area to about the minimum possible without introducing an additional control effect such as pre-act response.

If the optimum controller settings are then

$$\text{Sensitivity} = \frac{0.9}{R_1 L} \qquad [3]$$

$$\text{Reset rate} = \frac{0.3}{L} \qquad [4]$$

These values may be substituted in Equation [1] to give

$$\text{Net area} = 3.7\,\Delta F R_1 L^2 \text{ in-min} \qquad [5]$$

In the recovery curve of Fig. 3 where the net area is equal to the total area, the latter is also equal to $3.7\,\Delta F R_1 L^2$. This holds true for processes similar to that shown in Fig. 2 where the load change occurs at essentially the same point in the circuit as that at which the output-pressure effect takes place. This is the same as saying that processes in which the reaction curve, caused by a sustained change in load, and the reaction curve caused by an equivalent valve change are identical, the net area is equal to the total area, and the total area is equal to $3.7\,\Delta F R_1 L^2$. Quite often, however, load changes occur at other points in the circuit, and the recovery curve swings on both sides of the set point, even at the optimum controller settings. This would be the case in the process of Fig. 2, if the principal load change were not the temperature of incoming water but a heat gain or loss in the pipe line between tank and bulb. In that event the total area would be greater and a "load factor" would replace 3.7 in Equation [5] when solved for the total area under a recovery curve from maximum load change. The total area can be expressed as

$$\text{Total Area} = (\text{Load Factor})\ (\text{Recovery Factor}) \qquad [6]$$

where both the load factor and recovery factor are characteristics of the process. The effect of load changes at various points in the process has not been completely investigated by the authors so they cannot quantitatively fix the load factor except for the limited case noted in which it is equal to 3.7. This factor has been qualitatively investigated by others as the relation between supply and demand side capacities (1). The recovery factor, $\Delta F R_1 L^2$, equal to 0.0575 in-min or 1.44 deg-min in the given example, has been used by the authors as a means of process classification and appears to be a good yardstick for evaluating this phase of process controllability.

In a previous paper (5), it has been pointed out that values of R_1 and L can be determined during adjustment of a controller on an application. The proportional-response sensitivity, which just gives sustained oscillation, is called the "ultimate sensitivity" S_u, and the period of oscillation at this sensitivity is called the "ultimate period" P_u. If S_u is taken in pounds per square inch per inch, and P_u in minutes, R_1 and L are determined by the formulas

$$R_1 = \frac{8}{(P_u)\ (S_u)} \text{ in. per min per psi} \qquad [7]$$

$$L = \frac{P_u}{4} \text{ min} \qquad [8]$$

The difference between the controller-output-pressure readings at minimum and maximum loads is equal to ΔF pounds per square inch. The recovery factor is then

$$\text{Recovery factor} = \Delta F R_1 L^2 = \frac{(\Delta F)(P_u)}{2 S_u} \text{ in-min} \qquad [9]$$

Miscellaneous values of ΔF, R_1, L, and the recovery factor taken from various applications are given in Table 1 only to show the range of recovery factors. Note that on the temperature-control applications the recovery factor is also converted to

TABLE 1 FACTORS FOR VARIOUS APPLICATIONS

	ΔF	R_1	L	$\Delta F R_1 L^2$, in-min	$\Delta F R_1 L^2$	
Ammonia absorber	10	0.07	8.7	53	1100	Deg F-min
Fractionating column	5	0.06	7.5	17	410	Deg F-min
Superheater	3	0.38	2.1	5	240	Deg F-min
Oil-tube still	3	0.13	3.1	3.7	180	Deg F-min
Wet bulb	6	0.02	4.5	2.4	45	Deg F-min
Dry bulb	2	0.22	0.77	0.26	5	Deg F-min
Milk heater	0.5	0.60	0.67	0.13	3	Deg F-min
Canners' retort	2	0.17	0.03	0.0003	0.008	Deg F-min
Column vent	0.5	0.2	0.08	0.0006	0.036	Psi-min
Air pressure	0.2	37.5	0.002	0.00003	0.0006	Psi-min
Water flow	4	0.56	0.12	0.032	0.55	Gal

"degree-Fahrenheit-minutes," and the pressure-control applications given as "psi-min." Temperature and pressure applications can only be compared on the "inch-minute" basis. Most of these values are calculated from ultimate sensitivity, period, and ΔF readings, taken during instrument adjustment. Some are taken also from experimental reaction curves.

Determination of ultimate sensitivities and periods during controller adjustment and subsequent notation of maximum and minimum output-gage readings provide a ready means of arriving at process characteristics in terms of the recovery factor. It is hoped that industrial plants will tabulate these data for all control applications so that a rational classification of processes will someday result.

Process Design to Reduce Lag

The recovery factor has been identified as one of the important characteristics determining the controllability of industrial processes. Let us now turn to the question of process redesign to reduce this factor. In the process of Fig. 2 a reduction in size of the maximum load change would reduce the recovery factor although generally load changes are a "death and taxes" sort of quantity and cannot be avoided. Assuming this is the case in the process of Fig. 2, our efforts will have to be directed at R_1 and L. An increase in tank size will reduce R_1 since the heat storage will be increased. The effect of doubling the tank size would be to halve the unit reaction rate and consequently double the proportional-response sensitivity. Fig. 5 shows that each wave in the new recovery curve would have just one half the amplitude as before so the area would be halved. Obviously a reduction in valve size

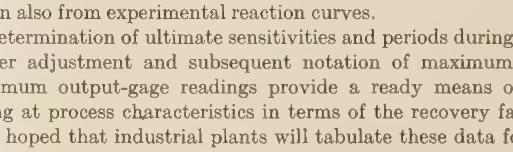


Fig. 5

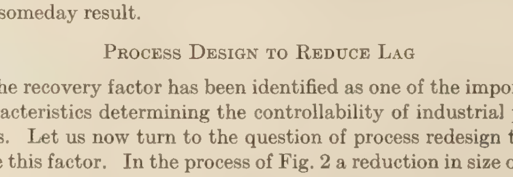


Fig. 6

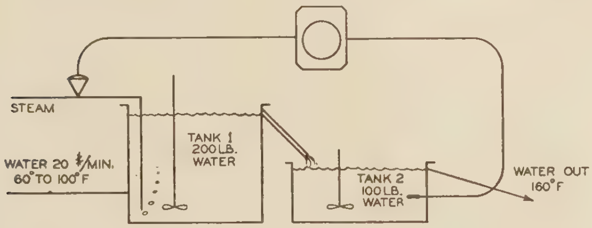
would reduce R_1 but increase ΔF proportionately. In this case, and generally, reduction of $R_1\Delta F$ in a process entails increase in the physical size and cost of the apparatus.

Moving the bulb to a position one half as far from the tank would halve the lag of our process and allow both sensitivity and reset rate to be doubled. The new recovery curve. Fig. 6 would have one half the amplitude and one half the period of the former process and consequently only one fourth the net area. This is shown by the recovery factor which varies as the second power of the lag. Reduction of process lag usually means only a process rearrangement and has a greater proportionate effect than comparable change in R_1. The remainder of this paper will therefore consider only means of reducing process lags, leaving a study of reaction rate for a future paper.

Lag of Multiple-Capacity Circuits

It will be relatively easy for the process designer to identify the simple distance-velocity lags and take steps to reduce them to a minimum. Faced with the process of Fig. 2, he would move the temperature bulb as close as possible to the tank. Unfortunately, however, most processes are made up of a series of resistances and capacities and the effective lag is a complex function of the number and size of these RC (resistance-capacity) units (3). Exact determination of lags is not usually necessary and it is believed that the following method of approximation is sufficiently accurate for practical purposes:

In Fig. 7 another process is shown in which a constant flow of



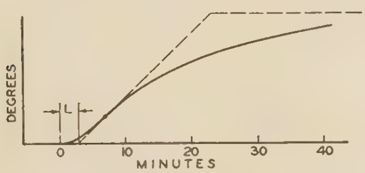


Fig. 7

water is heated in one tank and overflows to a second tank. The reaction curve for this process would be S-shaped. It has been found by experiment that this reaction curve can be approximated by the two dotted lines in Fig. 7. The slanting line is drawn tangent to the point of inflection and intersects the original temperature a time L after the output change was made; this time of L min being considered the effective time lag of the circuit.

In order to determine the lag in this circuit, it is first necessary to evaluate the time constant of the two principal capacities

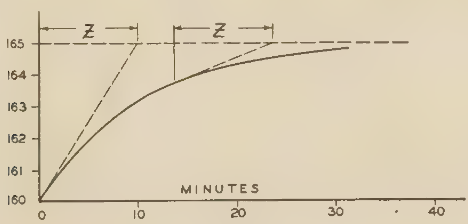


Fig. 8

separately. A sudden change in steam flow of say 100 Btu per min to the first tank would cause its temperature to rise as shown in Fig. 8, rapidly at first and then slower and slower as it approached the new equilibrium point. This curve is an exponential curve and has one characteristic in which we are interested. At any instant, the temperature is rising at a rate proportional to the remaining temperature difference, so it would always reach equilibrium in a definite length of time if it continued at that rate. Note that the two tangents to the curve of Fig. 6 reach the final temperature in the same interval. This time has been called the "time constant," "characteristic time," "lag," etc. (2, 6) of the exponential curve. Here it will be referred to as the "impedence," Z, of the RC unit. The sudden introduction of 100 Btu per min to tank 1 over and above the amount of heat necessary to maintain 160 F will first cause the tank to rise at a rate of 100/200 or 1/2 F per min. Eventually the 20 lb per min of incoming water will be heated 100/20, or 5 deg to 165 F and the system will again be at balance. The impedence, Z, of the curve is then 5 deg divided by $^1/_2$ deg per min or 10 min. Any other change in heat flow could have been used still giving the same value. In like manner the second tank alone would respond to a sudden change in its incoming-water temperature by giving a similar curve with an impedence z of 5 min. The response of both tanks together to a sudden change in heat input of 100 Btu to tank 1 is shown in Fig. 9. Tank 1 rises on an exponential curve with a 10-min time; the temperature of tank 2 rises on the S-shaped curve. At any time A it is rising at such a rate that it would reach the corresponding incoming temperature B, 5 min later at C. This curve is complex mathematically but the lag L can be readily determined from Fig. 10. The ratio of z/Z is 0.5, showing a factor of

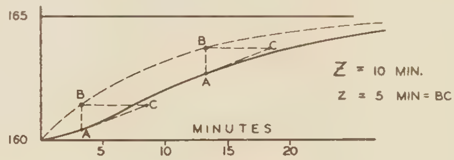


Fig. 9

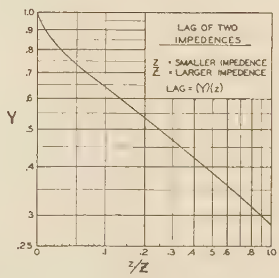


Fig. 10

0.39. The smaller z multiplied by this factor gives L for the two impedences of 5 and 10 min as (0.39) (5) or 1.95 min. Actually L for the entire circuit would include an impedence due to the valve motor and one due to the bulb but in this case they are small by comparison and can be neglected.

Reduction of L in the circuit can be accomplished by reducing the impedence of either unit, preferably the smaller. Making tank 2 one half as large would cut its time to $2^1/_2$ min, and L would become 1.25 min, while halving the size of tank 1 so that both tanks had 5-min impedences would only reduce L to 1.41 min. Thus tank 2 should be considered first and the greatest possible reduction made in its capacity.

If tank 2 could be completely removed and the bulb placed in tank 1, the system would apparently have only one capacity and

consequently no lag. A great deal of work has been done on these single-capacity systems (7), although they are only approached in industrial circuits since other impedences become appreciable. In this case the impedence of the valve would enter the picture as small z.

The response of diaphragm valves to a sudden output change at the controller is not a true exponential, although an approximate impedence can be used which varies with motor size, controller-relay-valve capacity, and the friction in the line between controller and valve. Figures between 0.05 min and 0.5 min are found on different sizes of valves with a multiplying factor of 2 or 3 for long connecting lines. A normal figure might be 0.15 min. Eliminating tank 2 would then leave two major impedence units again, tank 1 with 10-min impedence, and the diaphragm valve with 0.15-min impedence. The lag from Fig. 10 would be 0.13 min. This lag represents about the lowest limit for considering the use of the derivative (pre-act) responses now available. Some reduction in lag could be accomplished by reducing the size of valve motor or length of connecting line though this is not always possible.

Further improvement then would consist of complete elimination of tank 1, making the process simply a steam-water mixer, at which time another impedence would become appreciable, that of the bulb. A definite amount of heat is required to raise the temperature of a bulb and there is only so much area through which the heat can flows, so the controller bulb in the flowing water would have a definite impedence. This time for bulbs has been investigated quite thoroughly and data are available (6, 8). With water flowing at good velocity past a bulb its time is roughly 0.05 min. The remaining combination of $Z = 0.15$ for the valve and $z = 0.05$ for the bulb would give, from Fig. 10, a lag of 0.022 min or 1.3 sec. If the bulb were located a short distance downstream of the mixing point, a small distance-velocity lag might exist which could be added to the 0.022-min figure. Successive reduction in the number and size of impedences in the circuit of Fig. 7 has made possible a 100 to 1 reduction in L. Note that only the time-lag term in the recovery factor is being considered. In certain cases the increased unit reaction rate which may attend reduction in lag can overbalance the good results although in general any reduction in lag will improve controllability.

The complete reaction curve for the circuit of Fig. 7 is not quite the same as that of Fig. 9 since the former has the actual valve and bulb impedences included. With small error the lag shown in Fig. 7 can be calculated by adding the two small times of 0.15 and 0.05 to the 1.95-min lag of the two principal impedences giving a total circuit lag of 2.15 min. This problem of approximating the circuit lag when several impedences are present divides itself into three groups:

1 The lag of a circuit consisting of one very large and several very small impedences will be very nearly equal to the sum of the small ones. Example: One 10-min and three 0.05-min impedences would give a lag of approximately 0.15 min.

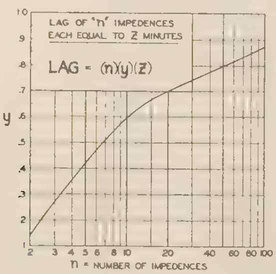


Fig. 11

2 The lag of a circuit consisting of two large and several very small impedences is approximately equal to the lag of the two major times determined from Fig. 10 plus the smaller times. Example: The lag resulting from one 8-, one 3-, and two 0.1-min impedences would be the lag of the two larger ones, 1.3 min, plus 0.2 min, or 1.5 min.

3 The lag of a circuit consisting of several equal impedences is found from Fig. 11. Example: Five impedences each with a time Z of 0.8 min would give a lag equal to $5 \times 0.42 \times 0.8$ or 1.68 min. This case is found in fractionating columns, absorbers, etc., where each plate from the point of measurement up to a regulated liquid reflux flow constitutes an impedence equal to the volume of liquid held on each plate divided by the reflux volume. Several impedences much smaller than the value of the large equal ones can be added directly to the lag found from Fig. 11, without much error, e.g., two 0.1-min impedences would increase the foregoing lag to 1.88 min.

Pressure Control

The simple pressure-control circuit shown in Fig. 12 can have three appreciable impedences, namely, valve, tank, and measurement.

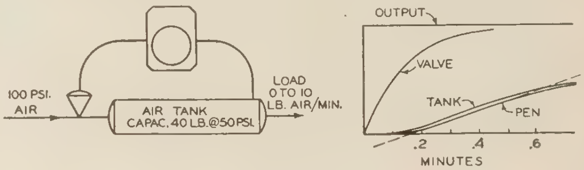


Fig. 12

The valve impedence can be taken as 0.15 min. The measurement impedence varies widely and depends upon the length and size of connecting tubing, upon the material in the line, and upon the volume displacement of the measuring element in the controller. For air pressures measured by Bourdon springs with $^3/_{16}$-in-ID connecting tubing the time will be about 1 sec per 100 ft of tubing. In this problem we will assume a 0.01-min measuring impedence. The tank impedence will not be constant at all loads but will be a maximum at the no-load condition. With no outflow, the slightest valve opening will cause the tank pressure to rise toward the supply pressure of 100 psi, so under these conditions the impedence will be essentially infinite. At any rate it will be a great deal longer than either of the other two impedences in the circuit, so rule 1 applies and the circuit lag will be equal to $0.15 + 0.01$ or 0.16 min.

A pressure-control application can often be improved by reducing the length of tubing between tank and instrument or filling the tubing with a less viscous medium. Using a smaller valve motor and shortening the air connection between instrument and valve can reduce this impedence. The "booster relays" offered by some manufacturers are designed to give faster valve action. As a general rule, damping introduced in either the measurement or output connections will increase the lag since these applications follow rule 1.

Flow Control

In a liquid-flow-control circuit such as that of Fig. 13 there are

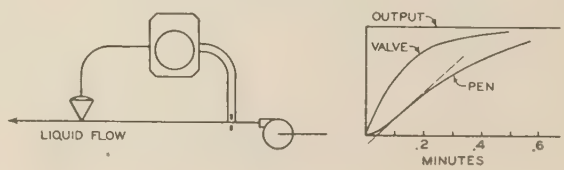


Fig. 13

only two appreciable impedences, those of valve and measurement. Except for small inertia effects in the flowing liquid its flow rate follows valve movement. Measurement time varies with manometer displacement, length of connections, and material moved in the connecting lines. The response of the mercury manometers used in industry while not a true exponential can be given an effective impedence which is generally not less than 0.15 min, even with short connecting lines filled with material of low viscosity. Combining this with a normal valve impedence of 0.15 min the lag shown in Fig. 10 is 0.04 min. This checks well with actual practice since the 0.04-min lag would give an ultimate period (5) of 4 times this value or 10 sec. Industrial flow controls rarely show periods less than this figure. Reduction of either valve or measurement impedence improves a flow-control application; the former has been covered under the pressure-control example. Manometer impedence can be lessened by opening the mercury-damping valve with which most are equipped and locating the manometer as close to the orifice as possible using remote transmission systems if it is necessary to carry the indication of flow to a central panel. Recent development of so-called "aneroid" manometers is a step in the right direction, as they have less inherent resistance to change and less displacement than the corresponding mercury type.

Air Heater

Control of conventional air-heating apparatus represents a rather difficult control application when air temperatures must be held within close limits and must recover quickly from changes in load. A good example of current interest is the problem of controlling carburetor air temperature in aircraft-engine testing where the multitude of readings cannot be taken until carburetor air temperature is correct. The wide change in heating load occurring when engines are "gunned" must be corrected quickly as time is an important factor.

Conventional design of air-heating equipment usually neglects all consideration of the priceless ingredient, controllability, and as a result adequate control is often not attained. Fig. 14 shows the system normally used and its reaction curve. A tubular steam-heated surface is placed in the air duct, a bulb installed downstream, and a control valve located on the steam line to the heat exchanger. Three appreciable impedences are present, the valve, the heating surface, and the bulb. The data are given in Fig. 14; an air flow of 10 to 40 lb per hr at some temperature between 0 and 70 F is to be heated to 80 F with steam at 50 psi.

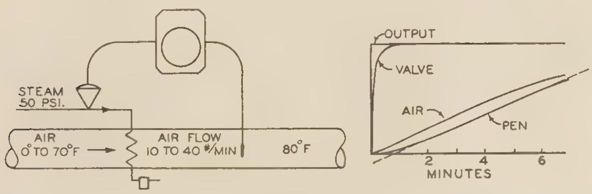


FIG. 14

The heating surface necessary for maximum load of 40 lb per min of zero-deg air would be approximately 75 sq ft, after including a factor of safety. The weight of metal in the heating surface would be in the neighborhood of 150 lb which would have a heat capacity of about 15 Btu per deg.

The impedence of the heating surface will vary with load but will be greatest under minimum load. In this case it can be most easily estimated by calculating the equilibrium existing at minimum load and finding the rate of fall toward incoming-air temperature with no heat inflow. To heat 10 lb per min of 70-deg air to 80 requires 23.7 Btu per min. The temperature of the 75-sq ft heating surface will only be a few degrees above the air temperature, say 83 F. If, from this equilibrium, the steam were suddenly shut off, the heating-tube temperature would fall toward 70 F, the incoming air temperature. The initial heat flow would be 23.7 Btu per min which if continued would dissipate the 15 $\times$ (83—70) or 195-Btu content of the metal in 195 /23.7 or 8.2 min. The heat-content figure of 195 Btu assumes that all the metal in the coil is at the 83-deg skin temperature, which is essentially correct inasmuch as the air film constitutes the largest resistance to heat flow. It also disregards the heat content of the steam in the tubes but the 8.2-min figure for surface impedence is sufficiently exact to show the weakness of the system.

Impedences of bulbs in air are quite large and depend upon bulb diameter and air velocity. The time in min for a $^1/_2$-in-diam bulb is about $100/U^{0.5}$ where U is the air velocity in feet per minute. The constant of 100 for $^1/_2$-in. bulbs becomes 72 for $^3/_8$-in. bulbs, 23 for $^3/_{16}$-in. bulbs, and 12 for $^1/_8$-in. capillary bulbs. If 1000 fpm is taken as maximum duct velocity in this problem, minimum velocity would be 250 fpm and the impedence of a $^3/_{16}$-in-diam bulb would be 1.4 min. The small steam valve would have an impedence of about 0.1 min.

The circuit lag can then be evaluated by combining the three impedences of 8.2, 1.4, and 0.1 min according to rule 2 which gives a lag of 0.88 min. Sudden changes of air flow through the duct of Fig. 14 will cause disturbances from which the system can recover only after a considerable time has elapsed. The period of oscillation (5) will be about 5.7 L, or 5 min, and if two appreciable waves are required in the recovery curve before the correction is essentially complete, at least 10 min will have elapsed. This delay is generally intolerable in aircraft-engine testing.

The process of Fig. 14 could be improved somewhat by installing a bulb of smaller diameter although even the $^1/_8$-in. bulb would still leave a process lag of 0.64 min. Pre-act response included in the controller would reduce the effective lag by perhaps 40 per cent to 0.4 min. Even so the period of oscillation would be 2.3 min. So instead of patching up this poor process let us consider a complete redesign keeping our eyes on controllability.

In Fig. 15 a rearranged process is sketched which has only two appreciable impedences instead of three. Air flows through two ducts in parallel; one contains the heating coil, the other is a by-pass. Temperature is controlled by operating a damper to mix cold and hot air. Since this system is designed for controllability, the smallest-diameter bulb available will be used, even though it is no longer the controlling (smaller) impedence. The motor for damper operation will be somewhat larger than before so its time can be estimated at 0.15 min. The bulb will have an impedence of 0.76 min, which combined with the 0.15-min valve impedence gives a circuit lag of only 0.08 min. This compares favorably with the 0.88-min lag of the first system. Further reduction in lag could be accomplished by using a "booster relay" to lower the valve impedence. Decreased bulb impedence would have only a small effect on the lag.

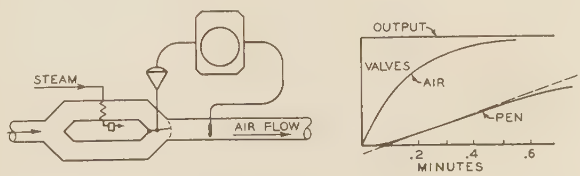


FIG. 15

The increased controllability of this process over that shown in Fig. 14, if measured in terms of the recovery factor, would be as the ratio of the lags squared or 120 to 1. This is not an insignificant improvement. It should also be noted that ΔF in the re-

covery factor would also be reduced by the redesign because very little repositioning of the damper would be required to compensate for a large increase in the air flow. The ratio of hot and cold air would remain essentially constant as long as incoming temperature remained the same.

Conclusion

In this paper the authors have attempted to show the quantitative effect of process time lag on control. One yardstick for the measurement of controllability, the recovery factor, has been introduced which seems to show that controllability varies as the square of the lag. Equations have been given for determination of the recovery factor on control circuits during instrument adjustment, so that this characteristic of processes may be easily found and tabulated. Rapid solution for the effective lag of multiple-capacity circuits is made possible by Figs. 10 and 11, and illustrative problems have been solved.

The few examples, illustrating lag reduction by process redesign, should point the way toward a better understanding of this important step in control improvement. Methods must also be made available for evaluating the other factors affecting controllability, as well as the terms other than lag in the recovery factor. Analysis of tubular liquid heaters for lag is possible, but an example was omitted from this paper, as the authors feel that their present method can be simplified considerably. It is hoped that the picture of controllability, and the methods developed in this paper for lag determination, will be of some immediate assistance to the conscientious process engineer who is anxious to include greater controllability in the equipment he designs.

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Discussion

W. F. Hickes.[4] Ever since the instrument industry learned to make controllers with two and three types of response, the on-off controller has been regarded as a "poor relation" not to be mentioned in the same breath with its betters. Yet every practical instrument man knows that on-off control is the best control for the many processes which are essentially single capacity and substantially free from distance-velocity lag. It is interesting to see the result obtained by applying the authors' methods of calculating controller settings to such a process. It was stated that for optimum performance

$$\text{Sensitivity} = \frac{0.9}{RL}$$

However, for a single-capacity process without distance-velocity lag, the reaction curve has its maximum slope at the origin and $L = 0$. Substituting this value in the previous equation, we find that the optimum sensitivity is infinite. In like manner, we may calculate reset rate with a similar result; the optimum reset rate is infinite. Since a controller having infinite sensitivity or infinite reset rate is an on-off controller, we have confirmed the conclusion reached through practical experience that, for the case under consideration, the on-off controller is the best controller. Thus, the lowly on-off controller is not a cheap substitute that will "get by" but is actually supreme in its proper field.

It should be added in passing that this applies only to a true on-off controller, one without appreciable dead space or throttling action. Also, even the most perfect on-off mechanism will be worthless if a sluggish thermal system or inadequate control-relay capacity makes a single-capacity process effectively multicapacity.

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P. W. Keppler.[5] The fact is brought out in this paper that the recorder (or indicator) type of flow controller in many cases suffers excessively from the inertia accompanying the metering element. The authors have made no mention of the nonrecording (and nonindicating) type of flow controller in which the motion of the measuring elements is made negligible, thereby eliminating fluid displacement (as well as other bad effects due to the motion of the sensitive measuring elements). Nonrecording and nonindicating flow controllers are of course used very extensively.

The authors recommend placing the metering element of the indicator close to the orifice and using a device for remote indicating. This, however, only eliminates fluid displacement in connecting tubing, not that in the meter, nor the inertia of the meter.

A much better solution would probably be to make the instrument servo-operated and thereby derive an inertia-free flow measure for controlling. For example, the flow differential could be converted into air pressure by a suitable pilot-valve arrangement, essentially without fluid displacement. This air pressure could in turn be used for controlling as well as for recording or indicating. This would probably cost no more than a device for remote indicating, and would appear to offer a much better solution.

Besides eliminating inertia, an essentially motionless sensitive element should also be much easier to make accurate and durable.

These thoughts would seem to apply to a greater or lesser extent to all controllers and instruments where improvements can be economically justified.

A. A. Markson.[6] This paper must be rated as a substantial addition to the Society's literature on automatic control. The method of attack is empirical and experimental and will probably find a certain disfavor among "control mathematicians." What is not generally realized is that the differential equations of highly irreversible processes are well handled by systematic empiricism. This is a platitude to workers in the fields of heat transfer, hydraulics, and aerodynamics, to name several. From this point of view, the writer considers the authors' work as a valuable step in this direction. However, empiricism, which is not carefully founded on the fundamental equations of a science, should be handled cautiously. While the authors' formulas for "reset rate" and "sensitivity" are undoubtedly very useful, the writer believes it worth their trouble to set these formulations up in dimensionally consistent form. For example, reset rate is given the dimension "per minute," which is at the least somewhat confusing.

The examples given to show how controllability may be improved are worth study. The interesting case of where control

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[6] Hagan Corporation, Pittsburgh, Pa. Mem. A.S.M.E.

may be improved by a change in the controlled variable or by the use of auxiliary variables has not been considered, doubtless as being outside of the scope of the paper. One illustration of the former will serve to show how, in certain very practical applications, the choice of the proper controlled variable radically alters the controllability of the process. Let us suppose it is desired to control the temperature in a deaerating water heater by bleeding live steam to the heater. This control may be effected either from the heater water temperature or from the heater pressure, since the two are uniquely related. Both practice and the principles of the paper show that the pressure control is usually preferable.

The matter of terminology is a troublesome one in the control field. Use of coined words such as "pre-act" and "impedence" will naturally encounter the purist's scorn. Yet they have much to recommend them because, like a good trade-mark, they identify some very definite phenomena with their proper field of technology. The use of the term "sensitivity" to denote proportional response is unfortunate. To bring this point out clearly, consider the use of the term as applied to galvanometers, the sensitivity of which is often referred to as deflection per microvolt. If we take a relatively crude galvanometer and add a relay or magnifier to it, we can give it the same deflection per microvolt as a better instrument. Thus, the two instruments have now the same "sensitivity," as defined by the paper. Yet changes can occur which will cause absolutely no response whatever in the poorer instrument.

It does not seem right to appropriate a term long used in metrology as a figure of instrument merit merely to denote relay action, especially since there is still a definite need for reserving it as a figure of merit even in the control field. The use of this term for proportional response goes back, without doubt, to the days when controllers "hunted" because they were too "sensitive." The term we employ to denote proportional response is "gradient." When used in a resetting controller, it is denoted as "temporary gradient." This but adds one more expression to the terminology of control which is badly in need of standardization.

J. B. McMahon.[7] This paper discusses how process lags affect process "controllability," in an attempt to make it possible for process designers to incorporate "controllability" in their designs, as well as the other necessary factors. However, the basic assumption is made in the paper that "controllability" is a matter of degree, or difficulty, and that all processes are controllable to some extent.

This is frequently not the case. Before the difficulty of controlling a process can be checked or calculated, it is necessary to determine whether it is controllable at all. This question has never been widely discussed in the literature on automatic control but has been mentioned by the writer.[8]

Briefly, the factors to be considered are as follows:

(*a*) Susceptibility to measurement.
(*b*) Significance of measurement.
(*c*) Susceptibility to automatic control.
(*d*) Magnitude of process lags.

(*a*) Many factors, such as chemical composition, which may be measured in the laboratory, are not susceptible to continuous measurement by an industrial instrument.

(*b*) Many factors capable of being measured are not satisfactory criteria of changes in process conditions, e.g., the temperature of vapors, leaving a column fractionating a binary mixture, is not a satisfactory criterion for heat supply to the column.

(*c*) The obvious way of satisfying (*b*) may be very uneconomic. The response to corrective action must be consistent. The significance of the measurement must be consistent, e.g., both above and below ebullition temperature is a significant measure of the heat content of water or steam, but not at the point of change of state.

(*d*) The final result, such as vapor pressure of the end product of a fractionating column, may be capable of being measured so as to satisfy requirements (*a*), (*b*), and (*c*), but the process lags introduced by the method of measurement may be so great as to preclude all hope of compensating successfully for any variations in operating conditions.

It may be thought that the foregoing considerations are only rarely of importance, and, considering the great bulk of all automatic-control applications, that is true. However, this is because most applications are repetitions of jobs which have already been handled successfully. However, when new processes are developed, or old ones are redesigned, such considerations become vital. Attempting to redesign a process so that it becomes more controllable may readily result in its becoming entirely unmanageable, due to neglect of these factors.

Another factor which is important is that of the self-regulation of the process itself. All of the process examples cited in this paper show definite positive self-regulation. However, numerous applications exist in which there is no tendency for the process to balance out, after an upset, within reasonable limits; and in many cases, an upset tends to accentuate itself. Exothermic chemical reactions may be very violent in their unbalancing tendencies.

The writer feels that the art of automatic control has not yet reached the point of progress where it is possible to lay down very definite rules with respect to preadjustment of automatic controllers, or with respect to process design, except on the basis of experience. Experience may lead to rearrangement of apparatus so that better control results are possible, but in practically all process designs, efficiencies, heat exchange, recoveries, etc. will continue to be the dominating factors.

It may be thought that this is an argument for doing nothing with respect to process controllability. It is actually a plea to go slowly. Twenty years ago, before automatic controllers reached their present state of development, many elaborate automatic-control installations were made, which in a short time proved to be more decorative than useful, and many of them were far from beautiful. The results proved very harmful to the industry, and the effects have not yet died out in many places. If process designers become too sold on the possibility of design for controllability, and the results are disappointing, the art can very readily receive another such setback.

J. C. Peters.[9] From the standpoint of exact quantitative solutions, most practical process-control problems fall into one of two classes; one class in which the solution is fairly easy but scarcely worth while, and another in which the solution would be of considerable interest but which involves great practical difficulty.

The authors have directed their attention to what might be considered as short-cut methods for practical use. The value of such methods is determined by the extent to which they fit actual cases. While the authors have presented but little evidence that their methods have wide application, they are probably awaiting the reports of others before making too definite statements on this point. The writer is glad to report that he has applied the equation relating "reset rate" and "lag" with encouraging results.

[7] Application Engineer, Republic Flow Meters Company, Chicago, Ill. Mem. A.S.M.E.

[8] "Mechanical Engineers' Handbook," by Lionel S. Marks, fourth edition, McGraw-Hill Book Company, Inc., New York, N. Y., section on "Automatic Control," by J. B. McMahon, pp. 2116–2123.

[9] Research Engineer, Leeds & Northrup Company, Philadelphia, Pa. Mem. A.S.M.E.

In the present paper, a rather unusual case of temperature control is taken as the basis for calculating the area under the control curve. The authors indicate that they fully realize that the "load factor" may be considerably different for different types of processes. The writer has determined this factor for a particular case of temperature control and obtained a value of 15, as compared with the value of 3.7 applying to the process of Fig. 2 of the paper.

It is to be pointed out that, if a "measure of process difficulty" is to be considered as a measure of control difficulty, the equation for it should include a factor dependent upon the nature of the disturbances to which the process is subjected. When disturbances always take place very slowly, excellent control may be obtainable, whereas the same process may be practically uncontrollable if subjected to sudden changes.

Process designers may well pay particular attention to the emphasis placed upon the importance of the elimination of what the authors refer to as "lag." As is pointed out, lag may result from the fact that a material must be transported over a certain distance before its effect is felt, or it may result from the combined effect of capacity and resistance to flow. In the case of a thermal system, this lag may be reduced to a minimum by seeing to it that the adjusted heat supply is given as favorable a thermal relation to the point of measurement as circumstances permit.

While, in general, control terms are not sufficiently standardized to justify quibbling about them, it seems proper to call the authors to task for the use of the term "impedence" while speaking of the time-constant of an RC circuit. If electrical analogies are to be used, certainly the well-established term, "time-constant" is a natural one to employ. As an electrical engineer, the writer usually thinks of impedences measured in ohms, and to him, to express "impedance" in minutes, seems intolerable.

In conclusion, the writer wishes to express his appreciation to the authors for the great amount of thought which they have put into this paper. Its fresh and practical approach may well lead the way to a more rational analysis of control-application problems.

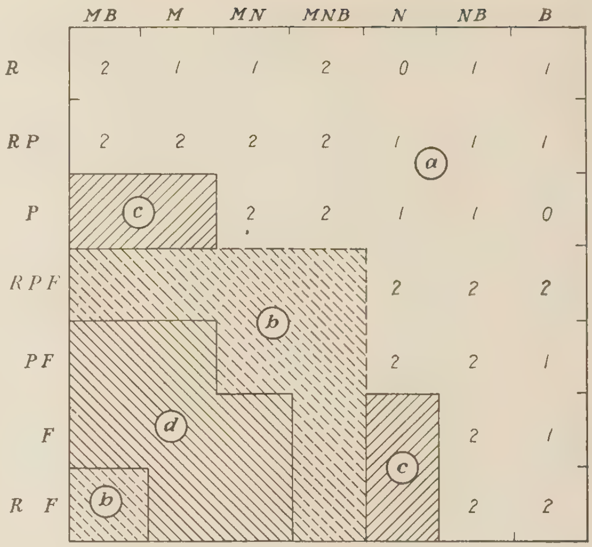
Ed S. Smith.[10] Instead of being the single entity urged by the authors, "controllability" seems to this writer to consist of different elements in different cases and not to be properly a blanket concept at all.

A stable regulated system, comprising a plant and its regulator, may be aptly considered as forming a chain whose length increases with the number of lags (or "capacities") in series, the chain having to extend the whole distance between its two ends. As long as a regulator either supplies missing links or strengthens too-weak ones, it controls equally regardless of the location of the links. The shorter chain with a missing link is no more controllable than a longer one also having a missing link. The correct link must be supplied in each individual case, and its location is lost by the use of RL or any other blanket controllability factor.

The diagram, Fig. 16 of this discussion, shows the effect on stability of missing elements, or links, in a plant or process having inertia (mass) M, damping N, and/or self-regulation B, when controlled by a regulator having rate R ("pre-act" in the paper), proportional P, and/or floating F ("reset" in the paper) components, the rate component alone being incomplete as a regulator.

From Fig. 16, it appears that there can be no correlation between any controllability factor such as RL of the paper and the performance of regulated systems generally. In other words, there is a fatal lack of correspondence between the mathematics of the paper and the physical system, which keeps its method from being generally applicable.

[10] Eclipse Aviation, Bendix, N. J. Mem. A.S.M.E.



Symbol	No. Cap System	Performance
a	0–2	Always Stable
b	3	Stability Depends on **RPF** Values
c	2	Unstable, Steady Hunting
d	3	Unstable, Increasing Hunting

Fig. 16 Stability Diagram Showing Complex Nature of Controllability

The authors' method is not suitable, without extensive modification, to floating regulators and plants. Its use in such fields is questioned as being outside of the field of proportional control in which its use has been tested.

The authors should test the limits of usefulness of their method on plants, respectively having the following relations

$$\ddot{X} + \dot{X} + X = 0 \text{ and } \ddot{X} + X = 0$$

The first plant would not need any regulator at all, while the second would hunt steadily without one.

A rational use of either reaction curves or known plant coefficients with reasonably representative differential equations is believed by the writer to furnish a sounder basis for control engineering than the rules proposed by the authors. The differential equations are handier than their solutions because of the fact they are "always true," while the solutions of course depend upon initial conditions and take many forms. However, families of typical curves are available which enable the differential equations to be evaluated and used conveniently for control purposes.

Two incidental notes are as follows:

1 The method of Fig. 9 of the paper appears to be limited to the case of real time constants and thus not to apply to the most interesting case of damp oscillations with their complex time constants.

2 Offhand, there appears to be something wrong with Equation [2] of the paper for Fig. 4, since areas below the set point should, as a matter of physical common sense, have equal weight with those above it, an improper use of signs possibly being involved.

The authors are requested to include, preferably as an appendix, a copy of their mathematical developments so that each reader will not have to supply them himself and to increase the use of the method suggested within its limits.

This paper has performed a service in directing attention to-

ward the usefulness of reaction curves as diagnostic symptoms in prescribing the remedial regulator.

Authors' Closure

As Mr. Hickes points out, an "on-off" controller, one having very high proportional-response sensitivity, is theoretically and actually ideal for processes which are essentially single-capacity. Since no thermal systems are infinitely fast in their response, and no relays have infinite capacity, some time lag will exist in all temperature-control circuits, nevertheless, they can approach the desirable characteristics of $R_1L = 0$, when R_1 is very small. Even though a finite R_1L intercept is present on a process, an "on-off" controller often gives stable control, simply because the controller sensitivity is not infinitely high as the name implies but lower than $1/(R_1L)$.

The authors accept and appreciate Mr. Keppler's discussion regarding the use of pressure elements with small displacement. It is hoped that the paper will help point the way toward such useful improvements in instrument design. Servo-operation of measuring devices does not necessarily eliminate measurement lag inasmuch as some lags are always introduced in each stage of amplification of the servo-mechanism.

Mr. Markson's confusion over the "per minute" units of reset rate might be eliminated if he considered it as being something dimensionless per minute. Actually, it is the number of times per minute at which automatic reset response duplicates the proportional-response output change caused by the pen deviation. Instead of expressing the magnitude of reset response by saying that it produces so many pounds per square inch per minute per inch deviation, it is given as pounds per square inch per minute per pounds per square inch change in proportional-response output. This leaves the unit as 1/min or "repeats per minute." All equations are dimensionally consistent if F is expressed in pounds per square inch, R in inches per minute, and L in minutes. Term R_1 is then inches per minute per pound per square inch.

Terminology is troublesome. The authors feel that the magnitude of proportional response is correctly expressed by "sensitivity;" also, that the deflection of a galvanometer per microvolt is correctly called its sensitivity. It is thought that the threshold of potential necessary to overcome friction in a galvanometer is something else which might be called "sensibility," defined as "that quality of an instrument which makes it indicate very slight changes of condition." "Gradient" is a word worthy of consideration as an alternate for sensitivity.

In the application of automatic-control instruments to a process, consideration must certainly be given to the points brought out by Mr. McMahon. In this paper, the authors chose to bypass the first two, which only involve measurement and not control. Generally, the object of automatic control is to hold a pen at one set point which represents an optimum condition in the process, whether it is indicated as temperature, pressure, or some other variable. Generally also, under point (c), response to output change is reasonably consistent, at least to the extent that a positive output-pressure change from equilibrium causes a pen to move in a direction opposite to that caused by a negative-output change. There are cases, for example, in azeotropic distillation, where increased reflux flow may cause a temperature deviation in either direction, depending upon the equilibrium conditions in the column. If composition is the desired quantity in this case, temperature is really a nonsignificant measurement of composition.

Mr. McMahon questions whether controllability is a matter of degree, and rightly, since it entails a definition of terms. The authors, in this paper, arbitrarily took the area under a recovery curve as one measure of controllability. This is only one of many possible bases for comparison of control results. For example, the maximum amplitude of an oscillation resulting from a load change might be taken as the sole criterion. This is possibly the basis of Mr. McMahon's contention that some processes are not controllable or synonymously "unmanageable." Probably even the processes to which he refers are controllable but not within the required tolerance.

It might be well to explain the reason for choosing this basis and the significance. On most control applications, the set point simply represents optimum conditions in a process. Deviations in either direction increase processing cost either by increasing steam cost or producing an inferior product, which must be reprocessed or sold at a lower price. If process thru-put is uniform, a plot of processing cost per minute against deviation from the set point might be a probability curve for the average process. This curve of the form $y = (1 - ae^{-bx^2} + c)$ with minimum cost at the set point is shown in Fig. 17 of this closure. It can be seen

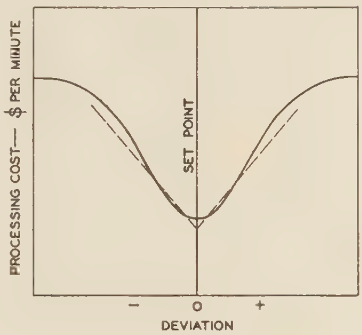


Fig. 17

that, around the set point, this curve is fairly well approximated by the two straight lines which represent the process evaluation used in this paper, i. e., that processing cost increases at a uniform rate with deviation.

Obviously, a probability curve does not represent all processes. Many would be discontinuous at some point. For example, an increase in temperature beyond a certain point might increase processing cost very suddenly, if at that point a reaction started which blew a wing off the plant. Again, considerable level variations about a set point might cause no trouble at all until a tank overflowed or a pump lost suction.

Self-regulation of a process appears to be one of the very minor considerations on processes, such as those discussed in the paper, those in which L is less than Z_0 (Fig. 2). If the 8.2-min "impedence" of Fig. 14 were made infinitely large, the process would not be self-controlling but the lag would only be increased from 0.88 to the sum of the remaining impedences or 1.5 min.

Mr. Peters brings out a very important point which cannot be overlooked in future work on the "load factor," and that is the type of load change, or rather, the rate at which the load change occurs. The authors carefully side-stepped this in their analysis of the process shown in Fig. 2. In this process, the load change was located so that it affected the process in exactly the same manner as a change in controller output. It should be apparent that, under these conditions, the rate of load change makes absolutely no difference in the area under a recovery curve. The recovery curve for a gradual load change would deviate a small amount and remain away until the load ceased to change; the pen distance from the set point being just sufficient to make the rate of output change from automatic reset response correspond to the rate of load change. When the load change was complete, the pen would return exponentially to the set point. In this case as well as that of an equal sudden load change, the area under the recovery curve would be equal to $\Delta F/(S)(RR)$.

On the other hand, a sudden load change, such as a flow of water entering just upstream of the bulb, Fig. 2, would cause a very rapid initial deviation and give a recovery curve with a large negative area. The load factor would be in the neighborhood of 65. But a gradual load change, even at this unfavorable location, would shrink the load factor back toward 3.7. Thus the load factor, as Mr. Peters indicates, depends upon both location and rate of load change.

The authors do not choose to defend their use of the term "impedence" in place of the more bulky "time constant of a resistance capacity unit." The "ence" ending was used to distinguish it from electrical "impedance." It is felt that a happier word might be chosen to describe this concept which is so necessary in process evaluation.

The exact mathematical solution of automatic-control problems, as championed by Mr. Smith, is certainly a desirable goal. However, the first paragraph of Mr. Peters' discussion expresses the authors' opinion in the matter.

Negative areas do have equal weight with positive areas so the signs of Equation [2] are correct.

The authors are glad to append equations for Figs. 10 and 11 of the paper, even though it is thought that the approximation which they express can be determined graphically more easily and with sufficient accuracy.

Equation for Fig. 10

$$Y = \frac{L}{z} = 1 + \frac{1}{X} - \frac{1}{X^{\frac{1}{1-X}}} - \frac{\log_e X}{1 - X}$$

where $X = \dfrac{z}{Z}$

Equation for Fig. 11:

$$y = \frac{L}{nZ} = 1 + \frac{1}{n} + \frac{n-2}{n(n-1)} + \frac{(n-2)(n-3)}{n(n-1)^2}$$
$$+ \frac{(n-2)(n-3)(n-4)}{n(n-1)^3} + \ldots + \frac{(n-2)!}{n(n-1)^{n-2}}$$
$$- \frac{(n-2)!}{n(n-1)^{n-1}} e^{n-1}$$

An Analysis of Gas-Pipe-Line Economics

By H. C. LEHN,[1] BUFFALO, N. Y.

In designing gas pipe lines the aim is to develop characteristics which will result in the lowest operating cost. The customary approach to this problem is to utilize formulas for line capacity and compressor horsepower as a basis for line design and from it the expected cost of operation. By trial the effect of varying the line and compressor characteristics can be observed. This paper shows that sufficiently accurate equations, involving all essential factors, can be developed and that these equations can be readily solved to determine the specifications of a line of theoretical minimum cost. Adjustments to commercial requirements, principally the factors of pipe diameter and station spacing, are possible with only slight divergence from theoretical results.

Nomenclature

The following nomenclature is used in the paper:

A_1 = coefficient-of-flow formula
A = coefficient-of-flow formula, including line efficiency factor
a = factor in initial-line-cost equation, dollars per mile
B = cost of operating compressor stations, dollars per I.C.H.P. per year
b = ratio $\dfrac{\text{Operating cost}}{\text{Initial cost}}$ of line only
C = coefficient of line-annual-cost equation
C_1 = coefficient of line-initial-cost equation
C_0 = constant of line-initial-cost equation, dollars per mile, independent of other factors
c = constant of equation for initial cost of pipe per ft
D = inside diameter of pipe, in.
E = efficiency factor in flow formula
e = exponent in supercompressibility equation for gas flow
f = exponent in supercompressibility equation for horsepower
F = constant of final equation for diameter
g = fuel consumption, cu ft per I.C.H.P. hr
H = I.C.H.P. per M^2 cu ft per 24 hr
H_1 = theoretical adiabatic horsepower per M^2 cu ft per 24 hr
I.C.H.P. = indicated compressor horsepower
j = factor in initial-line-cost equation, dollars per in. of pipe diam per mile
K = ratio $\left(\dfrac{\text{Gas required, including fuel gas}}{\text{Gas delivered}}\right)$
L = station spacing, miles
L_1 = total length of line, miles
M = number of compressor stations, fuel gas included
M^2 = 1,000,000
m = exponent of diameter in flow formula
N = number of compressor stations, fuel gas not included
n = exponent of pressure/length factor in flow formula
P = compressor discharge pressure, psia
p = compressor suction pressure, psia
p_0 = base pressure in flow and horsepower formulas, psia
Q = gas delivery M^2 cu ft per 24 hr
q = gas delivery $\dfrac{M^2}{100}$ cu ft per 24 hr
r = compression ratio
S = pipe stress, psi
s = pipe stress, $\dfrac{\text{psi}}{10{,}000}$
T = cost of pipe per ton, dollars
t = pipe thickness, in.
x = exponent in diameter and station-spacing equations $= m - n(2 + e)$
Y = total annual cost of line and stations, dollars per mile per year
Y_1 = total annual cost of line only, dollars per mile per year
Y_2 = total operating cost of all stations, dollars per year
Y_{11} = initial cost of line only, dollars per mile
Z = supercompressibility factor

Introduction

The design of gas pipe lines assumes the determination of line and compressor characteristics which will result in the lowest possible annual cost for the stipulated conditions. Since the line diameter and the compressor horsepower vary inversely for a given delivery, there will be certain values of the line and station annual costs for which the total annual cost will be a minimum. The formulas for line capacity and compressor horsepower are sufficient as a basis for determining the line design and from it the expected operating cost, and by trial, the effect on the cost of varying the line and compressor characteristics can be observed. By this method, however, the effect of a change in any of the factors is not apparent, nor is the lowest cost necessarily obtained.

It is the object of this paper to show that sufficiently accurate equations involving all of the essential factors can be developed, and that these equations can be readily solved for the minimum. In these equations the relative weight of the various factors is observable. For those infrequent cases in which minimum initial investment instead of minimum annual cost is required as, for example, where the field is of short life, the solution can be obtained by a change of certain of the constants. It is to be understood, however, that unless otherwise stated, the analysis is directed to the solution for total yearly capital and operating costs.

On a proposed new line, the following factors are stipulated:

Q = delivery, M^2 per 24 hr at a stipulated base pressure and temperature
L_1 = total length, miles

The values of the following factors will be selected on the basis of engineering judgment:

S = pipe stress, psi
t = pipe thickness, in. (tentative, and subject to possible revision after other factors are determined)

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[1] Consulting Engineer, Worthington Pump & Machinery Corporation. Mem. A.S.M.E.

Contributed by the Petroleum Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

There is then required to be determined from the foregoing factors the following factors resulting in minimum cost:

D = inside diameter of pipe
N = number of compressor stations, from which
$L = \frac{L_1}{N}$ = number of miles between stations is obtained
P = station discharge pressure, psia
p = station suction pressure, psia

Putting Y = total yearly operating cost per mile, the value of these factors is to be determined for which YL_1 total yearly capital and operating cost of the line is a minimum. This cost is expressed in its simplest form by

$$YL_1 = Y_1L_1 + Y_2 \qquad [1]$$

where Y_1 = yearly capital and operating cost of line only per mile
Y_2 = total yearly capital and operating cost of all stations

Derivation of Y_1, Yearly Operating Cost of Line Only

The total annual cost of the line consists of the fixed charges on the initial investment plus the direct operating expenses. The latter are comparatively small, and it is common practice to estimate them as a percentage of the initial cost. Accordingly, the entire line operating cost can be calculated on the basis of a percentage of the initial line cost. The first step then is to derive a formula for that cost in a form suitable for the solution for a minimum.

Various formulas for the line cost have been devised, some segregating the pipe cost and the laying cost, others involving this segregation but expressing it in terms of the pipe diameter and of the unit pipe weight. Using the first method and considering first the cost of the pipe, an analysis of quotations on pipe from 8 to 24 in. diam resulted in $ct^{0.8}D^{1.06}$ as a formula for the cost of the pipe per foot, where c is a constant depending on the price level. For the period during which the quotations were obtained, the value of c was 0.275. The addition of a constant which would not enter into the calculations for a minimum, or a slight change in the exponents or coefficient, or both, would result in a formula of this type which would fit any schedule of pipe prices. The cost of laying the line is sometimes estimated as a percentage of the pipe cost, and when this method is used, the foregoing expression can be applied to the total line cost by proper adjustment of the coefficient. However, it is the more general practice to estimate the pipe cost on a straight per ton basis with possibly some implicit allowance for the slight variation in cost with variation in diameter. Accordingly the per ton basis will be used, and it will be assumed to include mill inspection and an allowance for freight.

The various items included in the laying of the line, and which with the pipe cost make up the total cost of the line, may be separated into the following groups:

(1) a = dollars per ton-mile of pipe for those items varying principally with the weight of the pipe such as unloading, hauling, and stringing.

(2) j = dollars per inch diameter per mile for those items varying principally with the diameter such as ditching, laying, backfilling, welding, painting, etc.

(3) C_0 = dollars per mile for those items which are practically constant per mile and independent of the size of the line, comprising the right of way, surveying, telephone, etc.

No rigorous segregation of the items between groups (1) and (2) is possible, as the assumption of proportionality to weight or to diameter is not exactly true and the absolute values will vary with terrain, labor, and other conditions. Group (3) is more definite but, while a factor in the total cost, it is not involved in the calculation for the minimum as it does not vary with the line dimensions.

The assignment of dollar values to a, j, and C_0 thus involves engineering judgment for each particular case. However, analyses of actual estimates of line costs indicate a normal range of their values, and it will be seen later that precise actual values are not necessary for the purpose of the analysis.

A normal range of the variables comprising the total line cost is as follows, where T is pipe cost per ton:

T = \$60 to \$80
a = \$8 to \$12 per ton-mile
j = \$200 to \$275 per inch diameter per mile
C_0 = \$1500 to \$2500 per mile

It will now be found that the total initial cost per mile, estimated on the foregoing basis, is given very closely by the formula

$$Y_{11} = C_1t^{0.7}D + C_0 \qquad [2]$$

where

$$C_1 = 19.5T + 20.5a + 2.6j \qquad [3]$$

The minimum value of C_1 for the minimum values of T, a, and j as listed is about 1800, and for the maximum about 2500.

The total yearly capital and operating cost per mile is then

$$Y_1 = bY_{11}$$

where

$$b = \left(\frac{\text{Fixed-charge percentage plus operating-charge percentage}}{100}\right) \qquad [4]$$

or putting

$$bC_1 = C$$

$$Y_1 = Ct^{0.7}D + bC_0 \qquad [5]$$

In the past, a value of b of 0.14 was in common use, in which was included 0.015 to 0.02 for the direct line operating expenses. Considering the recent limitation by regulatory commissions of rate of return or interest, it might well be not over 0.10 or 0.11.

Derivation of Y_2, Station Operating Cost

Both the initial and annual costs of the compressor station are generally considered as varying directly with the installed horsepower. It is necessary first then to develop an expression for the total horsepower required on the line.

The compression of natural gas is adiabatic or nearly so, and the theoretical adiabatic horsepower is

$$H_1 = 14.2p_0(r^{0.213} - 1) \qquad [6]$$

In Equation [6], H_1 is the theoretical compressor horsepower per million cubic feet of gas at the base pressure p_0, psia, and at the temperature of the suction gas, r is the compression ratio. The constant and exponent are for methane. Both of the latter will vary slightly with the gas analysis, but for the low compression ratios used on pipe lines, this variation is negligible.

The horsepower required to overcome the valve and other losses determining the compressor efficiency is independent of the pressures or compression ratios as used on pipe lines and may be considered constant for a particular compressor. The complete equation is then

$$H = 14.2p_0(r^{0.213} - 1) + k_1 \qquad [7]$$

Equation [7] may be written

$$H = 14.2p_0(r^{0.213} - k) \qquad [8]$$

Term k will vary from about 0.98 for a highly efficient compressor to 0.96 for one of low efficiency.

The effect of supercompressibility is to reduce the horsepower in direct proportion to the supercompressibility factor, as determined by the suction pressure. The exponent of compression is not appreciably affected as for natural gas, and for the range of compression ratios involved, the effect of the increasing pressure on the supercompressibility factor is counteracted by the effect of the increasing temperature.

The supercompressibility factor for the horsepower equation is then

$$Z = \frac{1.455}{p^f} \text{ (see Appendix 1), or since } p = \frac{P}{r}$$

$$Z = \frac{1.455 r^f}{P^f} \dots\dots\dots\dots [9]$$

Introducing Equation [9] in [8], there is obtained for the horsepower with supercompressibility included

$$H = (14.2 \times 1.455) p_0 \frac{r^f (r^{0.213} - k)}{P^f} \dots\dots\dots [10]$$

Multiplying Equation [10] by Q for the horsepower per station by B as will be defined and by $\frac{L_1}{L} = N$, the number of stations, an equation for the total yearly operating cost of the stations is obtained, which is

$$Y_2 = (14.2 \times 1.455) \frac{BQp_0 r^f (r^{0.213} - k) L_1}{P^f L} \dots\dots [11]$$

in which B = station capital and operating cost in dollars per indicated compressor horsepower per year.

The value of B will vary approximately from \$20 to \$25, depending upon the initial station cost per horsepower, the magnitude of the fixed charges, fuel, attendance, and maintenance items. Corresponding initial costs per horsepower are approximately from \$90 to \$125. There is no object in segregating these total costs for the present problem. In general, the lower costs will apply to the smaller stations with compressors of the angle type and the higher costs to larger stations with horizontal units, with some reduction in cost with increasing station size for each type. This variation could be approximated by writing an exponent of Q in Equation [11] of slightly less than 1.

It is to be noted that, throughout, the term horsepower refers to indicated compressor horsepower. To convert to builders' ratings in brake horsepower, the following can be used

$$\text{Bhp angle-type units} = \frac{\text{I.C.H.P.}}{0.90}$$

$$\text{Bhp horizontal-type units} = \frac{\text{I.C.H.P.}}{0.95}$$

These adjustments should be borne in mind in estimating station costs.

Equation of Total Cost

Equation [1] total yearly capital and operating cost of the line can now be expressed in terms of Equations [5] and [11] or

$$Y = Ct^{0.7}D + bC_0 + (14.2 \times 1.455) \left[\frac{BQp_0 r^f (r^{0.213} - k)}{P^f L} \right] \dots [12]$$

Since L_1, total length of the line, appears only as a fixed factor it is omitted in Equation [12], and the problem simplifies to one of determining a minimum unit cost per mile.

The variables in Equation [12] are four in number, D, P, r, and L. It will be found possible to eliminate the first three by successive steps, leaving one variable L, in the final form which Equation [12] will take, and in which form it will be differentiated to obtain the minimum value of Y.

It will first be shown that there is a certain value of r, depending upon the compressor efficiency and on the supercompressibility effect, but independent of all other factors, for which the total horsepower on a line of any arbitrary design is a minimum. Since there is no restriction in the diameter, discharge pressure, or delivery, this value of r must hold for the line designed for minimum cost.

Assume a line L_1 miles long of diameter D and operating at discharge pressure P, to determine the compression ratio r for which the total horsepower will be a minimum. Since the station annual cost is assumed to vary directly with the horsepower, the cost will also be a minimum.

Referring to Equation [11], which is the equation for total station annual cost and which is equal to total horsepower on the line times B, two variables appear, L and r, but by means of the flow equation, L can be expressed in terms of the fixed factors of design and of the remaining variable r.

Solving the general flow Equation [52] for L and introducing the resulting expression in Equation [11] and dropping B for the moment

$$H_t = (14.2 \times 1.455 \times 1.40) \left[\frac{(p_0 Q)^{\frac{n+1}{n}} L_1 r^{2+f}}{A^{\frac{1}{n}} D^{\frac{m}{n}} P^{2+e+f}} \right] \left[\frac{r^{2+f}(r^{0.213} - k)}{r^2 - 1} \right] \dots [13]$$

Since all of the factors except r are now constants for any given line, Equation [13] may be written

$$H_t = H_2 \frac{r^{2+f}(r^{0.213} - k)}{r^2 - 1} \dots\dots\dots [14]$$

and it is required to find the value of r for which H_t will be a minimum.

Substituting the value of $f = 0.074$, differentiating, and equating to zero, there is obtained

$$\frac{dH_t}{dr} = \frac{2.287 r^{0.213} - 0.287 r^{2.213}}{2.074 - 0.074 r^2} - k = 0 \dots\dots [15]$$

Neglecting supercompressibility, $f = 0$, Equation [15] becomes

$$\frac{dH_t}{dr} = 1.1065 r^{0.213} - 0.1065 r^{2.213} - k = 0 \dots\dots [16]$$

Application of the rule for discrimination between maximum and minimum shows Equation [15] and Equation [16] to be minimums. These equations determine the compression ratio for which the total cost is a minimum. Equations [15] and [16] are plotted in Fig. 1, showing the variation of r with compressor efficiency and the effect of supercompressibility.

The variable r is now eliminated from Equation [12]. The symbol will be retained, however, and a numerical value selected after the final equation is obtained. To eliminate D, solve the general flow Equation [52] for D, obtaining

$$D = \left(\frac{p_0 Q}{A} \right)^{\frac{1}{m}} \left[\frac{1.40 r^2 L}{P^{2+e}(r^2 - 1)} \right]^{\frac{n}{m}} \dots\dots\dots [17]$$

The variable P is eliminated by use of the stress formula, $S = \frac{PD}{2t}$ (see Appendix 2) from which $P = \frac{2tS}{D}$, then

$$P^{(2+e)} \text{(in Equation [17])} = \frac{(2tS)^{(2+e)}}{D^{(2+e)}} \dots\dots [18]$$

Substituting Equation [18] in [17] and reducing, there is obtained

$$D = \frac{\left[1.40 \times \frac{r^2}{(r^2 - 1)}\right]^{\frac{n}{x}} p_0^{\frac{1}{x}} Q^{\frac{1}{x}} L^{\frac{n}{x}}}{A^{\frac{1}{x}} (2tS)^{\frac{n(2+e)}{x}}} \quad \ldots\ldots\ldots [19]$$

where the exponent $x = m - n(2 + e)$.

Term D is now in terms of the stipulated factors and of r, the variable of which the value for minimum has been determined, and of the variable L; and P has been eliminated. In the same manner, P is eliminated from the third term of Equation [12] by $P^f = \frac{(2tS)^f}{D^f}$, obtaining the expression for D^f from Equation [19].

The complete annual-cost Equation [12] can now be written in terms of known factors and the remaining variable L, as follows

$$Y = Ct^{0.7}\left(\frac{1.40r^2L}{(r^2 - 1)(2ts)^{2+e}}\right)^{\frac{n}{x}}\left(\frac{p_0Q}{A}\right)^{\frac{1}{x}} + bC_0$$

$$+ \frac{(14.2 \times 1.455 \times 1.40^{\frac{nf}{x}})\, Bp_0Q^{\frac{x+f}{x}}\, r^{\frac{f(2n+x)}{x}}\, (r^{0.213} - k)}{A^{\frac{f}{x}}\, (2tS)^{\frac{f(x+n(2+e))}{x}}\, (r^2 - 1)^{\frac{nf}{x}}\, L^{\frac{x-nf}{x}}} \quad \ldots\ldots\ldots [20]$$

Putting J and K for the known factors in the first and third terms of Equation [20], respectively

$$Y = JL^{\frac{n}{x}} + KL^{\frac{nf-x}{x}} + bC_0 \ldots\ldots\ldots\ldots [21]$$

Differentiating Equation [21] and equating to zero

$$\frac{nJL^{\left(\frac{n}{x}-1\right)}}{x} + \frac{(nf - x)}{x} KL^{\left(\frac{nf-x}{x}-1\right)} = 0 \ \ldots\ldots [22]$$

Changing signs and reducing

$$\frac{nJL^{\frac{2x-nf}{x}}}{xL^{\frac{x-n}{x}}} = \frac{(x - nf)}{x}K \ldots\ldots\ldots\ldots [23]$$

from which

$$L = \left(\frac{x - nf}{n}\right)\left(\frac{K}{J}\right)^{\frac{x}{x+n(1-f)}} \ldots\ldots\ldots\ldots [24]$$

Equation [24] is the value of L for the minimum cost.

Raising Equation [24] to the $\frac{n}{x}$ power and substituting in Equation [19], restoring the values of J and K, and of the exponent x, there is obtained

$$D = F\left[\frac{Q^{1+n}}{S^{n(2+e+f)}t^{n(2.7+e+f)}}\left(\frac{B}{C}\right)^n\right]^{\frac{1}{m-n(1+e+f)}} \ \ldots [25]$$

where

$$F = \left\{\left[\frac{(14.2 \times 1.455 \times 1.40)(m - n(2 + e + f))(r^{0.213} - k)r^{2+f}}{n(2^{2+e+f})(r^2 - 1)}\right]^n \left(\frac{p_0^{\,1+n}}{A}\right)\right\}^{\frac{1}{m-n(1+e+f)}} \ldots\ldots\ldots [26]$$

Term D as given by Equation [25] is the diameter for minimum cost.

A similar expression can be obtained for P, but more simply

$$P = \frac{2tS}{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [27]$$

An equation for the station spacing can also be obtained in a similar manner from Equation [24], but again more simply by rearrangement of the flow equation, by which

$$L = \left(\frac{A}{p_0Q}\right)^{\frac{1}{n}} \frac{D^{\frac{m}{n}} P^{2+e} (r^2 - 1)}{1.40r^2} \ \ldots\ldots\ldots\ldots [28]$$

From these three Equations [25], [27], and [28], the specifications of the line for minimum cost are obtained.

In Equations [25] and [26], all of the literal factors in F and in the exponents are retained so that formulas can be set up, using any flow formula, with or without consideration of supercompressibility, and for any value of r as given in Fig. 1. As an example, assume the Miller flow formula,[2] in the form of Equation [52], which, while not given in that form, is closely approximated by it with $m = 2.624$ and $n = 0.541$, and a constant of 800, corresponding to a line efficiency of approximately 0.93 and a base pressure of 14.4 psia. Term A in Equations [25] and [26] then becomes $\frac{(800 \times 14.4)}{1,000,000}$. Assume a compression ratio of 1.34, cor-

[2] "Determining Gas Line Capacity," by B. Miller, *Gas*, vol. 13, Nov., 1937, pp. 22–26, 72–74.

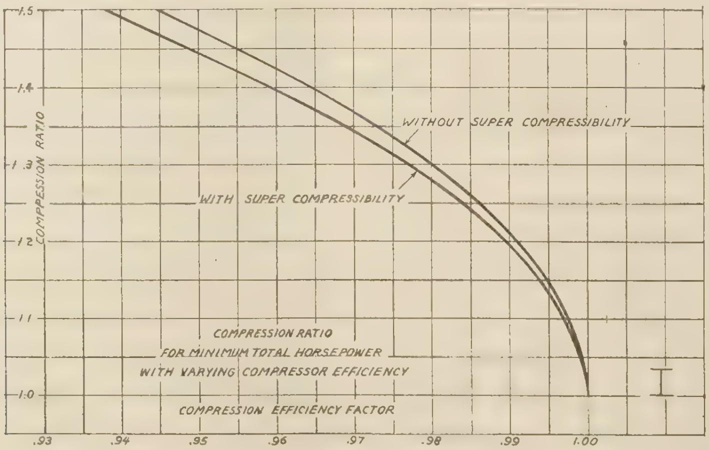

Fig. 1 Compression Ratio for Minimum Total Horsepower With Varying Compressor Efficiency

TABLE 1 COMPARATIVE ANNUAL COSTS PER MILE[a]

Compression ratio	Compressor efficiency factor	Outside diam, in.	Inside diam, in.	Discharge pressure, psia	Suction pressure, psia	Station spacing, miles	Annual costs per mile Line	Stations	Total
1.34	0.97	22.33	21.70	719	536	82.7	$2547	$ 852	$3399
1.34	0.97	24	23.38	669	498	99.7	2726	711	3437
1.34	0.97	22	21.38	731	546	77.72	2513	908	3421
1.25	0.97	24	23.38	669	535	81.1	2726	715	3441
1.275	0.98	19.92	19.29	814	638	50.12	2281	1075	3356
1.60	0.97	22	21.38	731	457	106.5	2513	960	3473
1.60	0.97	24	23.38	669	418	137.3	2726	752	3478

[a] Conditions: Delivery = 200 M² cu ft per 24 hr; Stress = 25,000 psi; Pipe thickness = $^5/_{16}$ in.; C = 240; B = \$20; C_0 = \$2000 per mile; b = 0.12 (see Equation [5]).

responding to a compression efficiency factor k of 0.97 which is a fair average for commercial compressors. Also put $q = \dfrac{Q}{100} =$ delivery in hundreds of millions and $s = \dfrac{S}{10{,}000}$.

Terms q and s can then be read in the more accurate range of a log-log slide rule. The equations then become

$$D = 17.44\,\frac{q^{0.768}}{s^{0.577}\,t^{0.765}}\left(\frac{B}{C}\right)^{0.270} \qquad [29]$$

The equation for the station spacing obtained from Equation [24] is

$$L = 264.0\,\frac{q^{0.289}\,s^{0.46}}{t^{0.065}}\left(\frac{B}{C}\right)^{0.75} \qquad [30]$$

Substituting in Equation [21], and restoring the constant

$$Y = 23.85\,\frac{q^{0.768}\,C^{0.73}\,B^{0.27}}{s^{0.577}\,t^{0.065}} + bC_0 \qquad [31]$$

If $B = aC$, then Equation [31] becomes

$$Y = 23.85\,\frac{q^{0.768}\,Ca^{0.27}}{s^{0.577}\,t^{0.065}} + bC_0 \qquad [32]$$

or from Equations [29] and [32]

$$Y = 1.352t^{0.7}CD + bC_0 \qquad [33]$$

Inspection of these equations shows that, of the factors, the values of which are optional, the stress predominates. The pipe thickness is a considerable factor in determining the diameter but is practically negligible in its effect on the cost as well as on the station spacing. While the exponent of t is shown the same in the station-spacing and cost equations, it will be slightly different if extended to greater accuracy. In the cost equation, it could be made to vanish by changing the exponent in the initial-line-cost equation, $Y_1 = Ct^{0.7}D$, to 0.8. Hence it may be considered that the pipe thickness has a negligible effect on the cost.

The values of the ratio $\dfrac{C}{B}$ for the range of the factors in the initial-line-cost Equation [3] and for a value of b, ratio of line annual cost to initial line cost of 0.12, are given in the following tabulation:

Laying cost of line	High		Low	
Station annual cost B	\$25	\$20	\$25	\$20
$\dfrac{C}{B}$ For pipe cost \$60 per ton	10.2	12.8	8.9	11.1
For pipe cost 70 per ton	11.1	14.0	9.8	12.3
For pipe cost 80 per ton	12.0	15.2	10.8	13.5

The high laying cost of line corresponds to the maximum values and the low laying cost to the minimum values of the factors a and j in Equation [3].

A fair average value of C constant in the formula for operating cost per mile is 240 and for B, 20, then $\dfrac{C}{B} = 12$. The diameters and pressures for minimum operating cost are shown in Figs. 2, 3, and 4, for the foregoing values of C and B, for stress of 20,000, 25,000, and 30,000 psi, respectively, and for pipe thicknesses of $^3/_{16}$, $^1/_4$, $^5/_{16}$, and $^3/_8$ in.

Fig. 5 shows the station spacing and total yearly cost per mile, including the constant C_0 at \$2000 per mile. Because of the small effect of the pipe thickness Fig. 5 is plotted only for $^5/_{16}$-in. pipe. The curves are extended slightly beyond the range of commercial pressures and pipe thicknesses.

In Table 1, the operating cost per year for optimum design is compared with corresponding costs for larger and smaller diameters and compression ratios. The calculations are for a delivery of 200,000,000, and other data as listed. The data for a line with the compressor efficiency factor increased from 0.97 to 0.98 are also given. It will be seen that the increased efficiency decreases the minimum total operating cost about 1.25 per cent.

Comparison of Initial and Annual Costs

As stated, these equations can be used for either minimum initial investment or minimum annual cost. While it is seldom proposed or desirable to design a line for minimum initial investment instead of minimum annual cost, the relation between the two will be of interest. Using the previously established values for B and C, assuming that the initial cost of the station is five times the annual cost, which is in reasonable proportion, and that the line annual cost is 12 per cent of the initial cost; then as C and B will be the only factors that will change, for the purpose of comparison Equation [19] can be written

$$D = K_1\left(\frac{B}{C}\right)^{0.27} \qquad [34]$$

for minimum annual cost and

$$D_1 = K_1\left(\frac{0.12 \times 5B}{C}\right)^{0.27} \qquad [35]$$

for minimum first cost. Then

$$\frac{D_1}{D} = 0.6^{0.27} = 0.86 \qquad [36]$$

Thus the line diameter for minimum first cost is 14 per cent smaller than for minimum annual cost, and from $P = \dfrac{2ts}{D_1}$, the pressure is 16 per cent higher.

It is obvious that a line designed for minimum annual cost will have a higher initial cost and conversely. The ratio of these costs also can be obtained. In obtaining the cost Equation [31] from Equation [20], the following intermediate form occurs

$$Y = 0.73 \times 23.85\,\frac{Q^{0.768}C^{0.73}B^{0.27}}{S^{0.577}t^{0.065}} + 0.27 \times 23.85\,\frac{Q^{0.768}C^{0.73}B^{0.27}}{S^{0.577}t^{0.065}} \qquad [37]$$

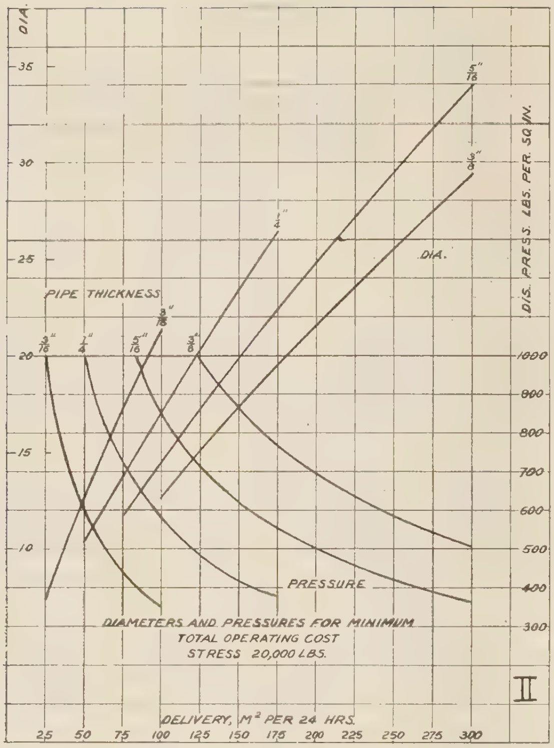

FIG. 2 DIAMETERS AND PRESSURES FOR MINIMUM TOTAL ANNUAL COST
(Stress = 20,000 lb.)

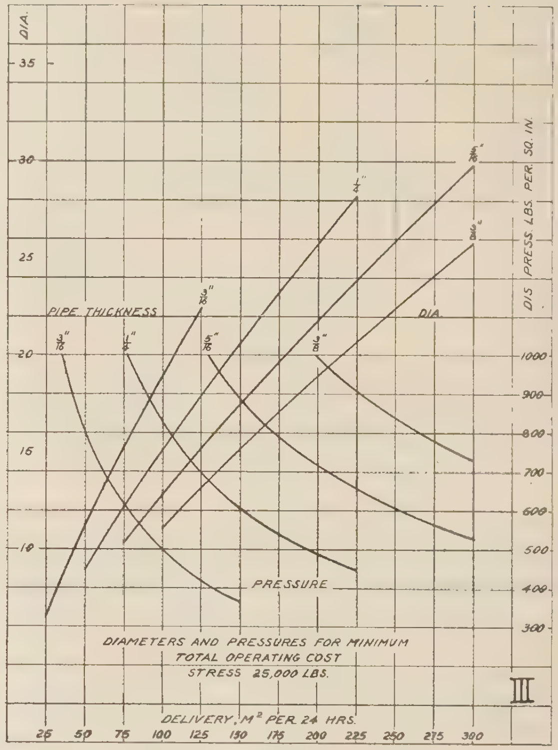

FIG. 3 DIAMETERS AND PRESSURES FOR MINIMUM TOTAL ANNUAL COST
(Stress = 25,000 lb.)

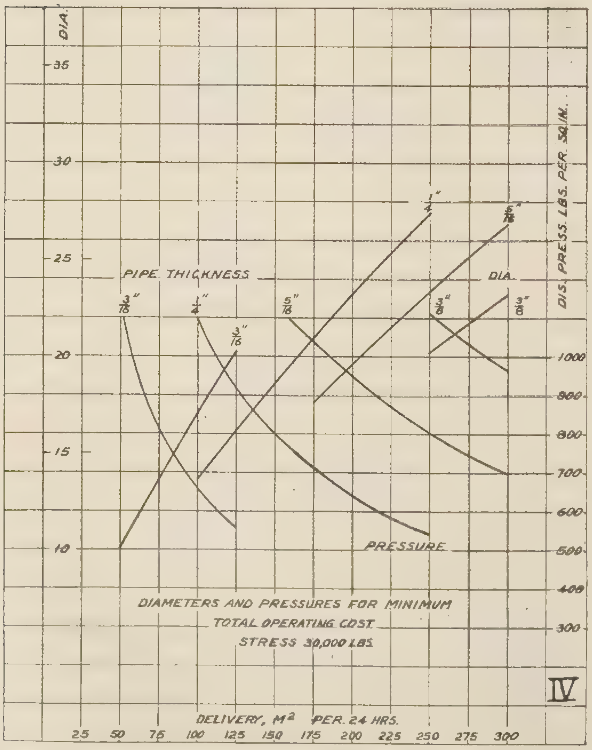

FIG. 4 DIAMETERS AND PRESSURES FOR MINIMUM TOTAL ANNUAL COST
(Stress = 30,000 lb.)

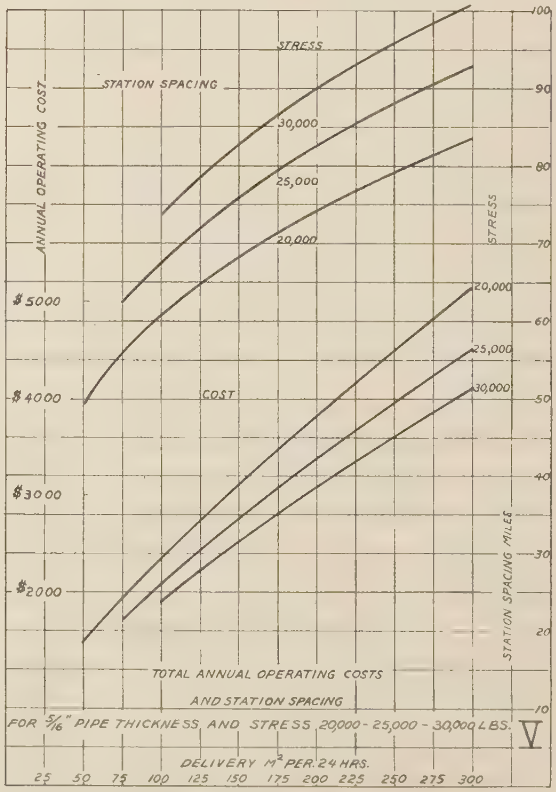

FIG. 5 TOTAL ANNUAL COSTS AND STATION SPACING
(For 5/16-in. pipe thickness and stress of 20,000–25,000–30,000 lb.)

In Equation [37] the first term is the line cost and the second term, the station cost. It is interesting to observe that these costs are in the ratio of the exponents of C and B. Equation [37], in which the line and station cost for minimum total annual cost are separated, can be written

$$0.73K_2 + 0.27K_2 \qquad [38]$$

The corresponding initial cost is

$$\left(\frac{0.73}{0.12} + 5 \times 0.27\right) K_2 = (6.08 + 1.35)K_2 = 7.43K_2 \qquad [39]$$

If designed for minimum initial cost Y_a, then

$$Y_a = 0.73K_2\left(\frac{5^{0.27}}{0.12^{0.73}}\right) + 0.27K_2\left(\frac{5^{0.27}}{0.12^{0.73}}\right)$$

$$= (5.30 + 1.96)K_2 = 7.26K_2 \qquad [40]$$

Then

$$\frac{Y_a}{Y} = \frac{7.26}{7.43} = 0.975 \qquad [41]$$

The line designed for minimum initial cost is only $2^1/_2$ per cent lower in first cost than the line designed for minimum annual cost.

The annual cost of the line designed for minimum initial cost is

$$\left[(5.30 \times 0.12) + \frac{1.96}{5}\right] K_2 = 1.028K_2 \qquad [42]$$

or about 3 per cent higher than that of the line designed for minimum annual cost.

It is to be noted that the ratios of diameters and initial and annual costs are independent of the ratio B/C, varying only with the percentage of the line first cost assumed for the line annual cost, and with the ratio of station-annual to initial cost. These latter ratios are subject to slight variations only. It follows that considerable variation in line diameter is permissible without raising appreciably either initial or annual costs above the possible minimum.

Effect of Change in the Variables

Figs. 2 to 5 show the diameters and pressures for minimum annual cost through a range of stress and pipe thickness and for an average ratio of station to line cost. The extremes of this latter ratio will affect the diameter as follows: The range of C for the range of the factors entering the pipe-line cost as previously given is 1800 to 2500. Assuming a minimum of 10 per cent and a maximum of 12 per cent for the line annual cost, and a minimum and maximum station total annual cost per horsepower-year of \$20 and \$25, there is obtained

$$\frac{C}{B}\text{ maximum} = \frac{2500 \times 0.12}{20} = 16.7 \qquad [43]$$

$$\frac{C}{B}\text{ minimum} = \frac{1800 \times 0.10}{25} = 7.2 \qquad [44]$$

Then for the diameter range

$$\frac{\text{Maximum diam}}{\text{Minimum diam}} = \left(\frac{16.7}{7.2}\right)^{0.27} = 1.25 \qquad [45]$$

and for the range of station spacing

$$\frac{\text{Maximum spacing}}{\text{Minimum spacing}} = \left(\frac{16.7}{7.2}\right)^{0.75} = 1.87 \qquad [46]$$

The effect on the diameter and, from it, the effect on the station spacing and discharge pressure caused by changes in the other factors, can be calculated by changes in the corresponding constants in Equations [25] and [26]. For example, if consideration of supercompressibility is omitted, the numerical factors 1.455 and 1.40 become 1 and the factors and exponents e and f vanish. A change in the compressor efficiency factor k will also change r in accordance with Fig. 1, but will affect only the constant F.

None of these factors has a sufficiently large range to produce any but a minor and practically negligible change in the diameter.

Of more marked effect is the substitution of a flow formula with different values of the exponents m and n. In order to show the effect of a change in the flow formula, as well as of the range of the station-line cost ratio $\frac{B}{C}$, Figs. 6 and 7 have been prepared. In Fig. 6, the diameters and pressures, and in Fig. 7, the station spacing and total operating costs for the flow formula used in Figs. 2 to 5, are compared with those for the Weymouth formula.[3] Both are through the range of the $\frac{B}{C}$ ratio and for 25,000-psi stress and $^5/_{16}$-in. pipe. Curves for the Weymouth formula omit supercompressibility, as it is not ordinarily considered in using that formula. The Weymouth formula, as used in the charts is

$$Q = \frac{870}{1{,}000{,}000} D^{2.667}\left(\frac{P^2 - p^2}{L}\right)^{0.5} \qquad [47]$$

The increase in total horsepower caused by deviation from the optimum compression ratio is shown in Fig. 8, which also represents the percentage increase in initial or annual cost of the stations. Since the station cost on a line of optimum design is of the magnitude of 25 per cent of the total annual cost, the effect on the latter of a change in the compression ratio is approximately one fourth of the percentage shown on the curve.

Additional Factors of an Actual Line

These formulas show diameter, pressure, and station spacing of a theoretical line for minimum operating costs. On an actual line the following additional factors occur, some of which modify the results given by the formulas:

1 Available commercial-pipe diameters.
2 Available sizes of compressor units.
3 Ratio of the actual length of the line to the theoretical station spacing which, in general, will not be an integral number, and which also involves the increased spacing possible at the delivery end of the line.
4 The field suction pressure which, if low, might require reduction of the discharge pressure for minimum total horsepower.
5 Fuel gas, which must be added to the net delivery capacity of the line (see Appendix 3).
6 Load factor, herein assumed to be 100 per cent.
7 Deliveries from the line between field and terminal.

Of these, the effects of the station spacing, including that at the terminal, and field suction and fuel gas are dependent in varying degrees upon the length of the line, hence cannot be included in general expressions for costs per mile. Of the remaining items, the diameter may be changed to a commercial size without greatly affecting the costs, as was shown in the comparison of initial and annual costs. The question of compressor-unit size will in some cases result in increased horsepower installation,

[3] "Problems in Natural Gas Engineering," by T. R. Weymouth, Trans. A.S.M.E., vol. 34, 1912, pp. 185–234.

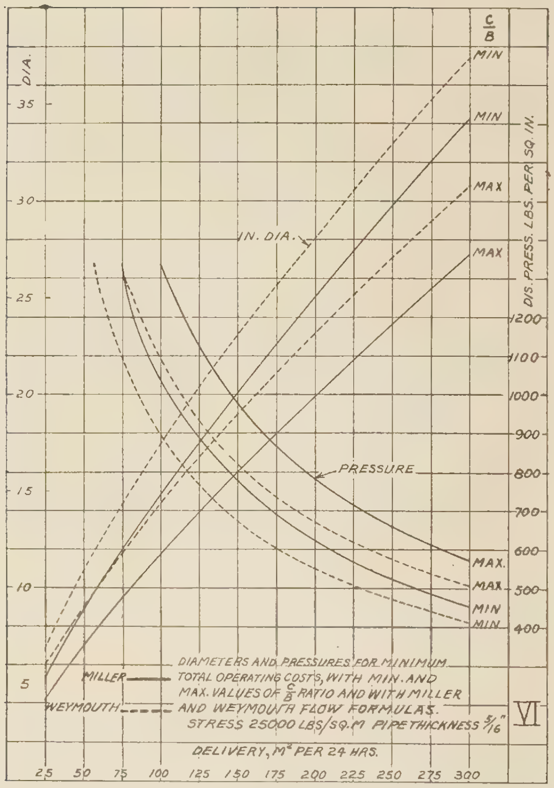

FIG. 6 DIAMETERS AND PRESSURES FOR MINIMUM TOTAL ANNUAL COSTS, WITH MINIMUM AND MAXIMUM VALUES OF C/B RATIO AND WITH MILLER AND WEYMOUTH FLOW FORMULAS

(Stress = 25,000 lb per sq in.; pipe thickness = $^5/_{16}$ in.)

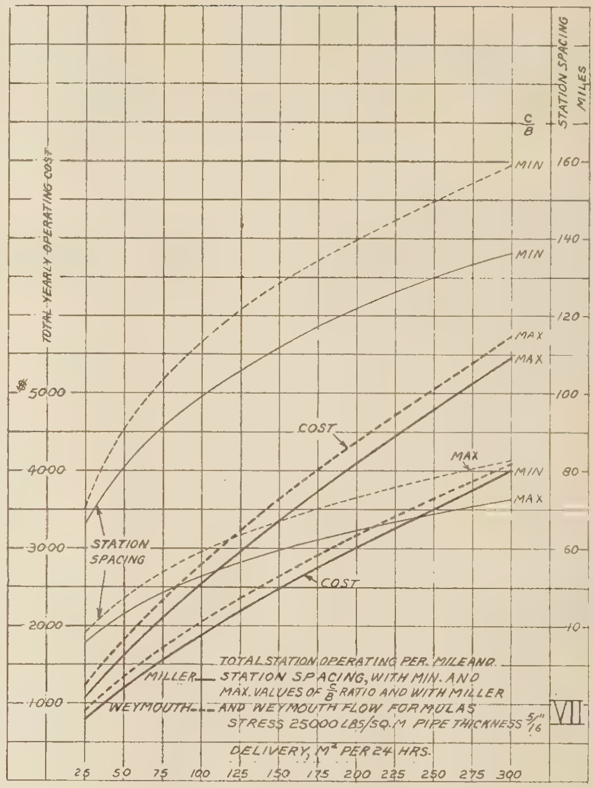

FIG. 7 TOTAL STATION ANNUAL COST PER MILE, AND STATION SPACING, WITH MINIMUM AND MAXIMUM VALUES OF C/B RATIO AND WITH MILLER AND WEYMOUTH FLOW FORMULAS

(Stress = 25,000 lb per sq in.; pipe thickness = $^5/_{16}$-in.)

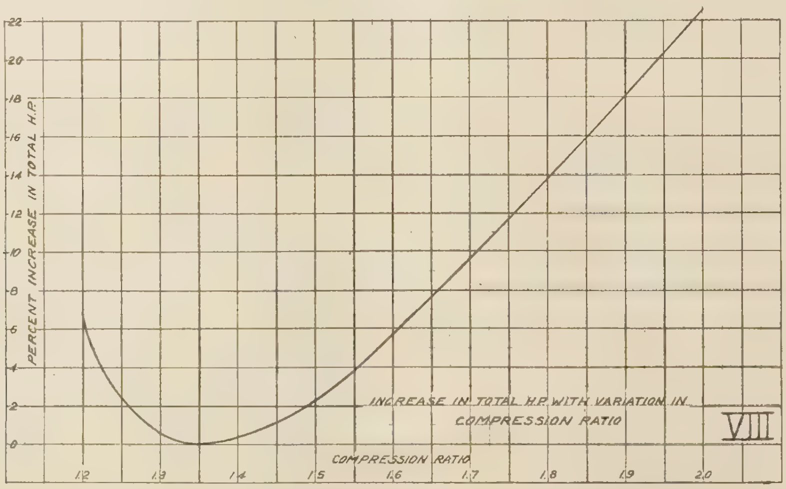

FIG. 8 INCREASE IN TOTAL HORSEPOWER WITH VARIATION IN COMPRESSION RATIO

which may be considered as spare power. It is evidently equivalent to increasing the value of B. The fuel gas can be included in the manner shown in Appendix 3. The additional horsepower required at the field station to some extent offsets the increased terminal spacing, and both are of decreasing importance with increasing line length.

The load factor in modern lines is generally at least 80 per cent and in many cases higher. With these values, its effect is small, as it reduces only the fuel-consumption item in the station operating cost.

Take-offs along the line of a considerable percentage of the final delivery can be arranged in groups, and the line considered as a series of shorter lines.

While all of these modifications can be made, it is questionable whether they are justified, as the nature of the problem hardly warrants the resulting implied degree of accuracy.

Summary

The equations derived in the foregoing determine the specifications of a line of theoretical minimum cost. The method consists in the stipulation of unit line and station costs, of the pipe stress, and of a tentative pipe thickness. From these factors, annual-cost equations are developed, involving the variables of pipe diameter, compression ratio, compressor discharge pressure, and station spacing. It is then shown that the compression ratio for minimum cost is independent of all factors. By substitution in terms of known factors, the diameter and pressure are then eliminated leaving one variable, the station spacing in the final equation.

Differentiating this final equation and equating to zero, the station spacing for minimum cost is obtained and from it the corresponding pipe diameter and discharge pressure.

Since all of the stipulated factors appear in the final formulas, the effect of a change in any one of them can be at once determined. Thus, if the selected pipe thickness is found to be unsuitable for the calculated diameter, the proper correction can be made by changing only the factor involving the pipe thickness in the final formula.

In the equation for diameter, the stipulated unit costs appear as a ratio of line-to-station cost. The exponent of this ratio depends upon the flow formula used and will never exceed 0.3, so that its effect on the diameter is small. Further, it is possible to estimate an approximate value of this ratio without recourse to detail line or station unit costs (see tabulation of $\frac{C}{B}$ values). It is also to be noted that a proportional change in line and station unit costs, such as might be due to changing material and labor costs, will not change the ratio.

There is also shown the effect of variation in the stipulated factors through their normal range, from which it is evident that the curve of costs is fairly flat, and considerable variation of all of the factors is possible without greatly increasing the cost above the calculated minimum. Accordingly, adjustments of the results to commercial requirements, principally of pipe diameter, and to a station spacing for an integral number of stations are possible with only slight divergence of practical from theoretical results.

Appendix 1

The Flow Equation

For a long period the Weymouth formula was used exclusively in pipe-line calculation and its results were accepted as the best approximation available. Recently other formulas have been developed, based, in general, on more complete consideration of all factors involved. These new formulas show higher capacities under normal conditions than the Weymouth formula, but they also differ among themselves.

The result is that the increased knowledge of the laws of gas flow has produced a degree of uncertainty, cleared to some extent by various pipe-line companies modifying the new formulas to conform with their experience. In some cases, this has resulted in retaining the Weymouth formula.

All of these formulas are of the general form

$$Q = \frac{A_1 D^m}{p_0}\left(\frac{P^2 - p^2}{L}\right)^n E \qquad [48]$$

where E is a line efficiency factor.

By writing $p = \frac{P}{r}$, Equation [48] becomes

$$Q = \frac{AD^m P^{2n}(r^2 - 1)^n}{p_0\, r^{2n}\, L^n} \qquad [49]$$

In Equation [49] $A = A_1E$.

The effect of supercompressibility can be included by multiplying general Equation [48] by the factor $\frac{1}{Z^n}$.

The data of Kvalnes and Gaddy[4] for the supercompressibility of methane at 60 F is very nearly equivalent to

$$Z = \frac{1.455}{p^{0.074}} \qquad [50]$$

Equation [50] is used in the formula for total line horsepower. In the flow formula the average factor is required and the corresponding equation based on the optimum compression ratio of 1.34 is

$$Z = \frac{1.40}{P^{0.066}} \qquad [51]$$

These formulas are sufficiently accurate for pressures between 250 and 1200 psi, the present range of pipe-line practice. Within this range, the maximum divergence from the Kvalnes and Gaddy data is 0.7 per cent. The formulas can be adjusted for less divergence for a shorter range or for a higher or lower range by a slight change in the constant and exponent.

In the analysis, the exponents of P in Equations [50] and [51] are replaced by f and e, respectively.

Introducing Equation [51] in [48], the general flow equation becomes

$$Q = \frac{AD^m P^{n(2+e)}(r^2 - 1)^n}{p_0\,(1.40r^2L)^n} \qquad [52]$$

Equation [52] is used in this analysis.

Appendix 2

Stress Formula

The Barlow stress formula is based on the gage pressure and the outside pipe diameter. The inside diameter and the absolute pressure are used herein, thus eliminating plus and minus signs which would result in unsolvable equations. For the pressures used on pipe lines, the difference is negligible, particularly so, as the allowed stress is more or less an arbitrary figure. For large thin pipe the stress calculated by the foregoing method is slightly higher and for small thick pipe slightly lower than by the Barlow formula. For each pipe diameter, there is a thickness and pressure at which the calculated stresses are the same; in particular when

$$P = \frac{7.5D}{t}$$

For example, with $10^3/_4$-in-OD pipe 0.25 in. thick, and $p = 1000$ psi, the stresses are 20,500 and 21,000 psi, and for 20-in-OD pipe 0.3 in. thick at 500 psi, they are equal.

Appendix 3

Horsepower Required for Fuel Gas

With five or more stations, the amount of fuel gas is sufficient to require consideration in the calculation of the line.

Having determined the number of stations of equal horse-

[4] H. M. Kvalnes and V. L. Gaddy, *Journal of the American Chemical Society*, 1931.

power, equally spaced, for the terminal delivery Q, provision for the fuel gas may be made by decreasing the station spacing uniformly toward the field without changing the discharge pressure or compression ratio. The calculations will be consistent then with those for a line of minimum cost.

Consider now in a series of stations any two adjacent stations S_1 and S_2, the direction of gas flow being from S_2 to S_1, and let S_1 be delivering Q million cu ft per 24 hr, with H horsepower.

Let g = fuel consumption in cubic feet per I.C.H.P. per hr. Then the total fuel consumption for S_1 in M^2 per 24 hr is $0.000024gH$. Put

$$K = 1 + 0.000024g\frac{H}{Q} \qquad [53]$$

Then K is the ratio of gas delivered by S_2 to that delivered by S_1, and the gas delivered by S_2 is KQ.

Since the pressures are constant, the flow equation may be written

$$Q = \frac{A_1}{L^n}$$

and for the length of line between S_1 and S_2

$$KQ = \frac{A_1}{L^n}$$

from which

$$L = \left(\frac{A_1}{QK}\right)^{1/n} \qquad [54]$$

and the ratio of station spacing is $1/(K)^{1/n}$.

For a line of total length L_1 with N stations and uniform station spacing, L, without fuel gas, L remains as the distance of the last station S_1 to the terminus.

Let M be the number of stations with fuel gas, then

$$L\left[\frac{\dfrac{1}{K^{M/n}} - 1}{\dfrac{1}{K^{1/n}} - 1}\right] = L_1 \qquad [55]$$

$$\left[\frac{\dfrac{1}{K^{M/n}} - 1}{\dfrac{1}{K^{1/n}} - 1}\right] = \frac{L_1}{L} = N \qquad [56]$$

Solving Equation [56] for M there is obtained

$$M = 1 + \frac{\log[N - K^{1/n}(N-1)]}{\log K^{1/n}} \qquad [57]$$

The horsepower at each station increases in proportion to the total gas compressed, including the fuel gas, so the total horsepower on the line is

$$H_t = H\left(\frac{K^M - 1}{K - 1}\right) \qquad [58]$$

The ratio of the additional horsepower to the horsepower without fuel gas is

$$\frac{H_t}{HN} = \frac{K}{N}\left(\frac{K^{M-1} - 1}{K - 1}\right) \qquad [59]$$

The horsepower at the field station assuming the suction to be the same as on the line station is

$$H_f = HK^{M-1} \qquad [60]$$

The total gas delivered into the line which is the terminal delivery Q plus the fuel gas for all stations except the field station is

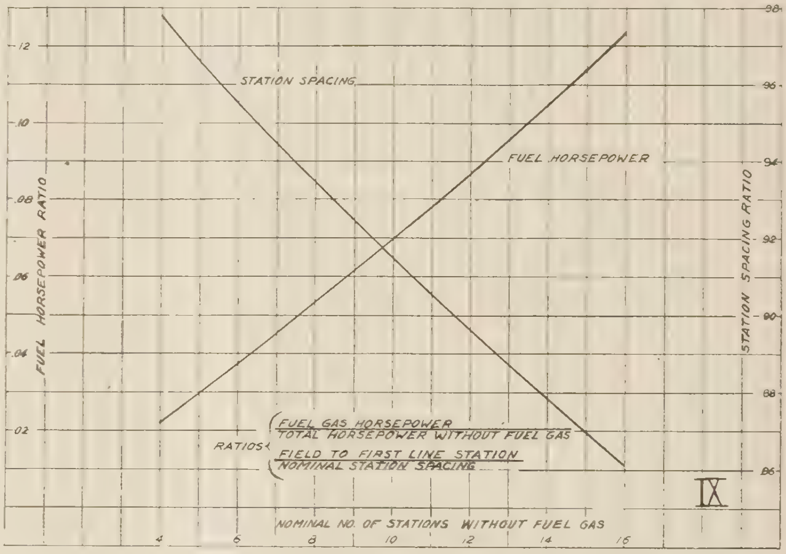

Fig. 9 Fuel Horsepower; and Station-Spacing Ratios

$$Q_t = QK^{M-1} \qquad [61]$$

From Equation [59]

$$\frac{L_f}{L} = \frac{1}{(K^{M-1})^{1/n}} \qquad [62]$$

where L_f is the distance between the field station and the first line station.

Referring to Equation [53], a conservative value of g is 11 cu ft per hp-hr. Term $\frac{H}{Q}$ is the horsepower per million and is 19.3 for the compression ratio of 1.34 for minimum cost, from which K = 1.0051.

The ratio of the additional horsepower required for the fuel gas to the horsepower required for the terminal delivery as given by Equation [58] is shown in Fig. 9, and also the ratio of the first station spacing, given by Equation [62], and for the foregoing value of K.

For any other compression ratio than 1.34 the horsepower for fuel gas will be increased in the ratio shown in Fig. 8.

In practice, the station-spacing and the compression ratio would be modified slightly to provide or compensate for the additional horsepower, these modifications being involved in the adjustment to an integral number of stations.

Appendix 4

Minimum Cost of Looped Lines

In adding a loop to an existing line, the same problem of division of total cost between line and compressing equipment occurs as in the calculation of a new line, but in a different form. The existing line may or may not be designed for or operating under

the minimum conditions, and this has an effect on the relative amount of line and horsepower for minimum additional cost. The same general method may be applied, with several modifications. The discharge pressure and the station spacing are now fixed, and the variables to be determined are the length of the loop and the suction pressure, which will change from that with which the line is operating without a loop.

The practical application of the solution is much more limited than in the case of a new line and is not given here. The development leads to an equation for p, the new suction pressure, from which the amount of loop can be calculated. An analysis of the results leads to the following conclusions.

The range of increased capacity through which the conditions for minimum cost can be applied is limited.

A loop of smaller diameter than the original is less expensive than one of larger diameter, provided both the loop and the original line costs are on an equal basis and are stressed equally.

The additional horsepower decreases as the total capacity increases, and fairly rapidly. This makes it impossible to design for increased capacity at minimum cost in steps as then there would be excess horsepower on the line when looping for the ultimate capacity is done. The logical procedure would seem to be to install the additional horsepower required for the ultimate capacity as the first step, followed by looping as required.

It is impossible to double the capacity of the line without departing from the requirements for minimum cost, and for this extreme condition a loop larger in diameter than the original line will probably tend toward lower costs.

Increase in cost is less than in the case of the original line for corresponding variation from the requirements for minimum, so that there is considerable latitude in proportioning the increase between looping and horsepower.

Discussion

C. R. Hetherington.[5] The writer is in general agreement with the paper, but while certain aspects of the problem are treated in a helpful simplified manner, based on reasonable assumptions, other means to simplify the mathematical manipulations and expressions apparently complicate the use of the final equations. Assuming the operating charges to be a constant percentage of the initial cost is helpfully simplifying, but the approach in which the pipe thickness t is chosen constant when it actually depends upon the pressure and diameter makes application more involved than using economic equations in which the thickness has been eliminated.

The low optimum compression ratio independent of the line-design variables is in agreement with recent publications.

Benjamin Miller.[6] The author develops a method of designing a gas pipe line to give a minimum total cost of transportation. The writer has been engaged for many years in gas-pipe-line design and is pleased to see a paper in which gas-transportation costs are analyzed. However, the method of analysis used by the writer is fundamentally different from that developed in this paper. The basic assumptions used in the paper are so different from those which the writer has found valid that it is not possible to compare the conclusions given in the paper with the conclusions which the writer has reached.

Because a comprehensive discussion of this subject would run to considerable length, the writer's comments will be confined to certain features of the paper having primarily to do with gas-compressing stations. The author's association with one of the great manufacturers of gas compressors makes what he says about them of interest to everyone working in the field of gas transportation. It is hoped that in his closure he will shed further light on some of the points treated only briefly in the paper.

The paper correctly points out that, with compressors having losses which are independent of the compression ratio, there will be a certain value of the compression ratio for which the total horsepower of any gas line is a minimum. It does not follow from this, however, that the line designed for minimum cost (including in "line" the compressing stations) will necessarily employ this compression ratio. The reason for this is that the cost of operating compressing stations decreases as their size increases, as the author points out. It is therefore quite possible that the total cost of operating a few stations, each of large size, will be less than the total cost of operating many stations, each of small size, even though the total horsepower in the many small stations is less than the total horsepower in the few large stations. The author shows that, even with the assumption of operating cost per horsepower independent of station size, the increased cost of station operation, in going from the compression ratio for minimum power to a compression ratio 20 per cent larger, is only about 5 per cent. This increase could easily be overcome by the lower cost of station operation per horsepower which would be expected from the increased size which goes with a 20 per cent increase in compression ratio, if the delivery is relatively small.

Consider the second and seventh lines of Table 1 of the paper. The second line has an annual station cost of $711 per mile. This means 35.55 indicated compressor horsepower per mile, because at the bottom of Table 1, B is stated to be $20, and B is defined as station capital and operating cost in dollars per indicated compressor horsepower per year. The definition follows Equation [11]. The station spacing on the second line is 99.7 miles, so that the total indicated compressor horsepower per station is the product of 35.55 and 99.7, which is 3545. Checking this product against Equations [8] and [10] shows that Equation [10] has been used, that is, this part of Table 1 was calculated with allowance for deviation from Boyle's law.

The seventh line of Table 1 has an annual station cost of $752 per mile. This means 37.60 indicated compressor horsepower per mile because B is $20. The station spacing on the seventh line is 137.3 miles, which makes the total indicated horsepower per station the product of 37.60 and 137.3, which is 5163. A check of this product against Equations [8] and [10] shows that Equation [10] was used here also. Therefore, allowance was made for deviation from Boyle's law in this case also.

If the station spacing were 139.6 miles, the compression ratio would be 1.623 and the suction pressure would be 412 psia. According to Equation [10], the indicated compressor horsepower would be 26.43 per million, or the indicated compressor horsepower per station would be 5286 to handle 200,000,000 per day. If there were a stretch of line 698 miles long, five such stations would be required, for a total of 26,430 indicated compressor horsepower.

The job could also be done with seven stations, each of 3545 indicated compressor horsepower, as just shown for a compression ratio of 1.34. The total indicated compressor horsepower would then be 24,815.

According to the unnumbered equation following Equation [11], the indicated compressor horsepower for angle-type units is 90 per cent of the brake horsepower. Therefore, if the line had five stations, it would require six units per station, each of 1000 bhp, while if the line had seven stations, it would require four units per station, each of 1000 bhp.

Now considering the things which have to go into a station in addition to the units, such as buildings, water-supply system, station and yard piping, etc., and considering the operating ex-

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[6] Gas Advisers, Inc., New York, N. Y.

penses which are not directly proportional to the number of units, such as supervision, it is obvious that a station with six units will cost less than 150 per cent of the cost of a station with four units, and that the total cost of owning and operating a station with six units will be less than 150 per cent of the total cost of owning and operating a station with four units. The author is aware of this, for, in the paragraph following Equation [11], there occurs this sentence: "In general, the lower costs will apply to the smaller stations with compressors of the angle type and the higher costs to larger stations with horizontal units, *with some reduction in cost with increasing station size for each type.*" (Emphasis supplied by the writer.) But the author does not seem to appreciate that this reduction in cost per horsepower-year with increasing station size vitiates his primary assumption. It is true that there is a certain compression ratio which requires the minimum total line horsepower. But it does not follow that the choice of this compression ratio will give the minimum cost. The reduction in cost per horsepower-year with increase in station size may overcome the effect of increasing total horsepower, so that the total cost may be greater with the minimum total horsepower than it would be with somewhat greater total horsepower, but concentrated in fewer stations.

Reverting to the example: If B is \$20, the total cost per brake horsepower year is \$18. Suppose this applies to the station with four units. Then the total cost per station is \$72,000 per year and the total cost for the seven stations is \$504,000 per year. Suppose further that the reduction in cost per horsepower-year for a 50 per cent increase in station size is 7 per cent, then the total cost per brake-horsepower-year for the six-unit station would be only \$16.74. Hence, the total cost per station would be \$100,440 per year, and the total cost for the five stations would be \$502,200 per year.

Since we are concerned with minimum cost, and not minimum power *per se*, the author's method would fail. The author suggests that the variation in cost per indicated-compressor-horsepower-year with increase of station size could be approximated by applying to Q in Equation [11] an exponent slightly less than 1. But this would not take care of the situation, because Q is fixed, and only the number of stations is to be chosen. Nor would it be proper to make an allowance by applying an exponent to r, as the unit cost for stations of the same size but different compression ratios should not be changed. The only solution is to make B a function of station size. This would complicate the analysis, but simplicity without accuracy is not desirable.

The writer does not suggest that the decrease in cost per horsepower-year, due to a 50 per cent increase in station size, would be 7 per cent, but does suggest that, in order to make the proper choice, it is necessary to consider the several factors which make up station initial cost and station annual cost. Therefore, the writer cannot agree that the problem studied by the author can be solved without considering the individual items of cost.

In the example, no consideration was given to spare units. The writer doubts that many would be willing to build a long line intended for 100 per cent load-factor operation at 200,000,000 cu ft per day and equip that line with angle units without having at least one spare unit per station. And with one spare unit per station, the total installed brake horsepower would be 35,000 in either case; five stations with seven units per station or seven stations with five units per station. Thus the five-station plan would certainly deserve consideration, even though it did require greater operating horsepower and greater fuel consumption.

A particularly interesting part of the paper is the section on cost of compressing stations, both initial cost and total operating cost. The total station cost is said to range from \$90 to \$125 per indicated horsepower. The lower cost is said to apply to smaller stations using angle units, and the higher cost is said to apply to the larger stations using horizontal units. Just why the less costly angle units are not used in the larger stations is not explained. To the station costing \$90 per indicated horsepower, an operating cost of \$20 per horsepower-year is assigned. This figure is used in the example of Table 1 and suggests that the author is recommending the use of the lower-cost stations of the angle type. The \$20 per horsepower-year includes fixed charges, fuel, attendance, and maintenance. Since the indicated horsepower of the angle units is said to be 90 per cent of the brake horsepower, this is \$18 per brake-horsepower-year without allowance for spare capacity. It is customary to have at least one spare unit in each station for a line which is intended to operate at high load factor.

If the figure of \$20 per indicated-horsepower-year is intended to be applied to the *installed* indicated horsepower, the station costs, shown in Table 1, are too low, as these station costs, as already indicated, are calculated on the *required* indicated horsepower.

Also, as shown previously, the four-unit stations, applying to the second line of Table 1, would have annual costs of \$72,000 per year. Taking 8760 hr per year, the total brake-horsepower-hours would be 35,040,000. This makes the total cost a little over 2 mills per bhp-hr (2.055 mills is the calculated figure). Stations whose records are available to the writer show operating costs, exclusive of fuel and fixed charges, ranging upward from 1 mill per bhp-hr. The writer is of the opinion that 1 mill per bhp-hr is a low figure for operating costs, exclusive of fuel and fixed charges, for angle-type units.

The author uses 11 cu ft per ihp-hr as the fuel consumption. This is a little over 12 cu ft per bhp-hr, which is, as the author indicates, on the high side of reasonable. At the very low figure of 7 cents per M cu ft, a fuel cost of 0.8 mill per bhp-hr for angle-type units is minimum. Adding this to the 1 mill gives a total of 1.8 mills, exclusive of fixed charges. This leaves 0.255 mill per hp-hr for fixed charges, or \$2.23 per hp-yr for fixed charges. At \$81 per installed bhp, as given by the author, the fixed charges are only 2.76 per cent of the initial cost, which seems very low; so low that the writer hopes the author will reconsider and detail the costs of station owning and operating.

The author points out that load factor lowering affects only station fuel and this in general is true, although it is sometimes possible to shut down every other station for operation at 70 per cent load factor. However, if the purpose is to design for minimum total cost of operation, it may be better to design for operation at the load factor which seems most likely. This may result in a higher total cost of operation when operating at 100 per cent load factor. In other words, design for minimum cost of operation at 100 per cent load factor may not give minimum actual cost of operation over the life of the enterprise.

The foregoing comments have been based, as the paper has, on the implicit assumption that gas-engine-driven reciprocating compressors are employed. This is almost universally true, but the analysis shows that it may not continue to be true. In the early days, it was common to use a compression ratio in the neighborhood of 3 for the reason that there is a compression ratio at which a compressor requires maximum power, and this maximum-power compression ratio is approximately 3. The point was to guard against stalling the engine by changing suction or discharge pressures. Analysis such as that given in the paper indicates that lower compression ratios could be used with resulting lower total horsepower requirements. That there is a compression ratio which requires minimum total horsepower is due to the characteristic of the reciprocating compressor of having losses which are independent of compression ratio.

If it were possible to make a compressor the efficiency of which did not decrease rapidly as the compression ratio decreased,

there would be no compression ratio for minimum total power. This can easily be seen by reference to Fig. 1 of the paper. But even if there were no compression ratio for minimum total horsepower, there would still be a compression ratio for minimum total cost.

The author has developed the compression ratio for minimum total horsepower by differentiation. A numerical example may show more clearly that it is the inherent feature of the reciprocating compressor which leads to this result and not anything in the theory of gas transportation.

Neglecting in the following any deviation from Boyle's law, consider the effect of changing the compression ratio from 1.3 to 1.1. The figure 1.3 is the compression ratio for minimum total horsepower, according to the equations given in the paper, if the compression efficiency factor k has its maximum value of 0.98. In other words, if the objective is to keep the total horsepower to a minimum, the compression ratio must be at least as high as 1.3, and higher if the compressor is not of maximum efficiency.

From Equation [6] of the paper, it can be shown that, with the compressor of maximum efficiency, the adiabatic efficiency at a compression ratio of 1.3 is 70.5 per cent. This may not appear to be a high efficiency to those who are using piston compressors at high compression ratios, but the writer has compared this figure with estimates of other manufacturers and has found that it is on the high side, as the author says.

It may be shown that, if the adiabatic efficiency remained constant as the compression ratio decreased, a line having stations operating with a compression ratio of 1.1 would require 17 per cent less total horsepower than a line with stations operating at a compression ratio of 1.3, but the adiabatic efficiency of a piston compressor at a compression ratio of 1.1 will not exceed 48.1 per cent according to the equations given in the paper. It follows that the compression ratio of 1.1 would make it necessary to use 25 per cent more total power than a compression ratio of 1.3 would require, even though theoretically 17 per cent less power would be sufficient.

It is hoped that the author will include in his closure some consideration of the possibility of improving the efficiency of the piston compressor at low compression ratios, or consider the possibility of using some other type of compressor which has higher efficiency at low compression ratios than the piston compressor has. Perhaps the centrifugal compressor will be more suitable. According to Avery:[7] "For estimating purposes, a round figure of 70 per cent may be taken as the adiabatic efficiency of good uncooled centrifugal compressors at designed load conditions."

While the principal applications of centrifugal compressors have been to compression of atmospheric air, Avery[8] further says: "In some cases, the compressors are required to boost gas from a high pressure to a still higher pressure. Typical conditions which have been met include pressure rises of 120 to 180 (1422 to 1458) [3300 to 3650] lbs per sq in. abs for inlet volumes of 2500 (360) [435] cfm."

The writer appreciates that the adiabatic efficiency is only one factor and that many others must be considered in evaluating the possibilities for improving gas-pipe-line economy by substituting for the piston compressor some other type of compressor perhaps better adapted for operation at low compression ratio. Certain it is that any such change would require a complete re-examination of the relationship for minimum transportation cost developed in the paper.

[7] "Centrifugal Compressors," by John Avery, a section of "Mechanical Engineers' Handbook," edited by L. S. Marks, McGraw-Hill Book Company, Inc., New York, N. Y., 1941, p. 1936.

[8] Ibid., p. 1944.

G. I. Rhodes.[9] During the past 10 or 12 years the author has more than once explained to the writer his mathematical studies into the economics of natural-gas pipe lines. His interest was to compare his results with our own practice in pipe-line design. There are so many variables that the task must have seemed all but hopeless. Yet he persisted in the face of criticisms, not always constructive, and has now developed some formulas and convenient charts which have fair mathematical support, and which constitute a useful guide in the economic design of natural-gas pipe lines.

These charts should not be used to find the cost of transporting gas. The author would be the last to use them for such a purpose.

The charts should be used merely as a guide in the preliminary choice of pipe dimensions and of number and power of compressor stations. The cost would then be determined to fit the conditions of the particular project in hand. Doubtless, variations in design would be indicated with redeterminations of cost until the best design had been found. Resort to the author's charts thus saves the voluminous preliminary work required in the layout of any natural-gas project.

The mathematical analysis which led to these formulas and charts was based on the premises (a) that the size and spacing between like compressor stations on a gas pipe line are the primary or independent variables, (b) that the pipe diameter and line pressure are secondary or dependent variables, and (c) that the thickness of and permissible stress in the pipe wall and the resultant line pressure are dependent only upon considerations such as physical properties of the steel pipe, desired factors of safety, and resistance to corrosion. The relative importance of the several factors within the choice of the designer is well shown by the formulas.

The mathematics are not rigorous. For instance, an empirical equation is used to represent the cost per mile of a pipe line as a convenient function of diameter and wall thickness, with appropriate constants to reflect the cost of steel and construction conditions. A different form of equation may well be needed to reflect varying conditions in the pipe market or the construction conditions affecting any given project. Also a uniform cost per horsepower is assumed independent of station size. Yet for any given type and size of compressor unit, the small station usually costs more per horsepower than the large station.

In spite of these convenient simplifications and others of less importance, it seems to be a fact that the most economical compression ratio is wholly independent of the size or the cost of the pipe line itself. All compressor stations on the line are assumed to be alike; they all operate on the same discharge and suction pressures, with the same pressure loss in the sections of line between stations. When the cost per ton of steel in the form of a pipe line is high, as compared with the cost per station horsepower, then the stations are relatively close together, and with cheap steel and expensive stations they are further apart. But the best compression ratio is fixed solely by factors relating to the stations themselves.

It is shown in Fig. 1 of the paper that the best compression ratio is fixed by a so-called compression-efficiency factor. This is not the true compressor efficiency ratio; it is merely a correction in the formula to reflect the effect of valve losses, etc., which are fixed irrespective of the compression ratio. When this factor k is 0.97 representing average practice, then these losses amount to about 6 hp per million cu ft of gas pumped per day, expressed in volume occupied at atmospheric pressure. The total power required for a compression ratio of 1.34 is about 19 hp per million cu ft per day.

Fig. 1 of the paper shows that, by increasing the valve and

[9] Vice-President, Ford, Bacon & Davis, Incorporated, New York, N. Y. Mem. A.S.M.E.

other losses from those corresponding to an efficiency factor of 0.97 to those corresponding to a factor of 0.94, the best compression ratio is increased from 1.34 to about 1.5. In other words, the cost of an additional fixed valve loss of 6 hp per station per million cu ft of gas per day makes it most economical to operate at a pressure ratio of 1.5, which results from fewer stations on the line spaced at greater distances.

The fixed cost of station-site development has a similar effect on the best compression ratio. This fixed cost may well exceed that of 6 hp per station per million cu ft per day, with the resulting increase in best compression ratio to 1.5. This well accords with established practice on pipe lines of high load factor. During occasional extreme peaks, the necessary spare equipment is operated, and the ratio sometimes exceeds 1.8. It would not be economical to install compressor power solely to develop such a compression ratio but it certainly is economical to use otherwise necessary spare equipment for the purpose. Here we have an explanation of the reason why modern pipe lines during extreme loads are designed to operate at compression ratios considerably higher than the seemingly best ratio of 1.34.

Equation [32]of the paper is of particular interest. It gives the minimum cost per mile for transporting a given quantity of gas q, when pipe-wall thickness t and the stress in the steel s are fixed. Thickness appears to the inverse 0.065 power. This means that increasing the pipe wall from $^1/_4$ in. to $^1/_2$ in., for instance, reduces the cost of transporting the gas, but only to the negligible extent of about 4 per cent. On the other hand, stress in the steel appears to the inverse 0.577 power. This means that increasing the steel stress from 12,500 to 25,000 psi decreases the cost of transporting gas about 33 per cent.

Here is the explanation of the most important advance in natural-gas pipe-line design that has taken place in the past 20 years. With former lap-weld pipe 12,500 psi was the maximum safe stress due to the low yield point of 25,000 to 30,000 psi, and to the inherent weakness of the weld. Now with seamless or electric-weld pipe, yield points of 45,000 to 50,000 psi can easily be obtained. Due to the absence of the weak lap weld, it is quite as safe to design for a stress of 25,000 to 28,000 psi as for the 12,500 psi of 20 years ago.

The present method of pipe-line design may be briefly summarized into four steps:

1 Choose pipe-wall thickness to meet the requirements of corrosion resistance or other physical requirements.

2 Choose a pipe with the highest feasible yield point consistent with good welding qualities since most lines now built are with welded joints.

3 By repeat computations find the size of pipe, the pressure, the size, and the spacing of compressor stations required to transport the total market requirements of gas in the most economical manner.

4 Provide for the installation of that part of the total compressor power required to serve the immediate market needs.

Resort to the author's formulas and charts reduces the work required in making step 3. They indicate a fair approximation to the correct answers, and comparatively little added computation should be required to secure any desired degree of refinement.

L. F. Terry.[10] I have long been a follower of the theory and practice of gas-pipe-line design and construction, and in years past have presented papers on the subject.[11]

[10] Second Vice-President, Chase National Bank, New York, N. Y.

[11] (a) "Design of High-Pressure Gas Pipe Line," by R. E. Davis and L. F. Terry, Trans. American Institute of Mining & Metallurgical Engineers, vol. 82, 1928–1929, pp. 598–613.

(b) "Economics of Pipe-Lines for Natural Gas," by L. F. Terry, Proceedings, Engineers' Society of Western Pennsylvania, 1933, p. 1.

For the last 5 or 6 years, I have been interested in the natural-gas industry in my present connection with the Chase National Bank. With this background, I would like to call attention to the value and interest which may attach to this paper. I believe it is the first attempt publicly to present to the industry under the auspices of any national engineering body a process for the design of gas pipe lines in one operation, i.e., other than by trial and error.

Author's Closure

Replying to Mr. Hetherington; the thickness factor t is retained in the formulas principally for the reason that the selection of the pipe thickness is a matter of engineering judgment, based on the factors mentioned by Mr. Rhodes in his discussion following. The selection of a tentative pipe thickness and an approximate stress in advance of the calculations is the normal procedure in pipe-line design and, in the subject analysis results, in directly solvable equations, not possible when additional variables are retained. As shown by Equation [31] and pointed out in the paragraph following Equation [33] of the paper, the pipe thickness has no appreciable effect on the cost, and hence adjustment to commercial practice can be made with negligible change in the results.

Mr. Miller has contributed a constructive and most interesting discussion. The writer agrees with his remarks regarding the decrease in cost with increase in size of compressor stations and already had investigated to some extent the possibility and desirability of including factors representing this variation. In particular, and as referred to by the discusser, a reduction of the exponent of Q in the horsepower equation to some value less than 1 is suggested; not to provide for the increase in horsepower due to increased compression ratio, but for the variation in horsepower with variation in quantity delivered.

Decrease in the cost due to increased compression ratio can be included by substituting B_1/r^x for B in the station-cost equations, and this will lead to a new value for the optimum compression ratio. Since the formulas hold only for this optimum ratio, the objection that unit station costs of the same size but different compression ratios would show different costs will not be valid. An objection is that the value of the exponent will depend upon the rate at which the station cost decreases, and this rate will vary considerably, requiring a different exponent for each case. An average value for the exponent perhaps could be used. The suggested value of 7 per cent decrease in annual costs for 50 per cent increase in size should apply for some particular condition. If this be taken, the value of the exponent x is approximately 0.3, leading to an optimum compression ratio of approximately 1.45 for $k = 0.97$. However, in the example given in the discussion, the reduction in cost by using the higher compression ratio is only 0.36 per cent. This implies a degree of accuracy hardly to be expected in an analysis of this type where base costs are subject to variation in absolute and relative values and, of necessity, enter in the form of empirical equations.

Further, and affecting actual station costs, is the fact that commercial total station horsepower is not a continuous function but varies in steps of the unit horsepower of the particular compressors selected. Thus, in a station with a small number of units, conditions can be such that the lowest cost would be shown with a lower compression ratio. Another item which should be considered in increasing the total horsepower over the minimum, particularly on a long line, is the increased fuel gas required.

It is agreed that a higher degree of theoretical accuracy would be obtained by including a factor for decreasing station cost with increasing size. This factor, however, must be of a form which does not increase the complexity of the equations to a degree

where they would be no longer directly solvable. Recourse to cut-and-try methods would then become necessary, which to the author's mind would defeat the purpose of the analysis.

With or without a factor for relation of station cost to size, it is the intention that the value of B be selected largely on the basis of engineering judgment, giving due consideration to the size of the station, the type of compressors, and the number of spare units. That variation in the value of B is of slight effect is observable from Equation [31].

The values of B of \$20 and \$25 used in Table 1 of the paper and in the table of C/B ratios following Equation [33] may be segregated as follows:

Fixed charges, 10 per cent	\$10.00		\$12.00
Fuel, 10 cu ft per hp-hr, 80 per cent yearly load factor, @ 6c/M	4.1	@ 9c/M	6.2
Attendance	5.2		6.0
Lubricating oil, etc	0.7		0.8
	20.0		25.0

In the foregoing tabulation, the fuel consumption is taken at 10 cu ft, which should be attained in a well-operated station. In the analysis of the fuel-gas effect (Appendix 3), a unit of 11 cu ft is taken in order to insure conservative results.

The figures, in Table 1 of the paper, are submitted as examples only. As stated, there is no attempt to recommend or suggest either a particular flow formula or particular basis of costs, the final formulas being developed in such form that any flow formula or any basis of cost may be used. On the other hand, the horsepower formula is the result of field tests conducted under the author's direction over a considerable period of time and is based upon the measurement of nearly 2000 indicator cards. The actual horsepower, which will be required on a particular proposed line, is subject to less accurate predetermination than in the case of other commercial-compressor installations and will frequently vary between different stations on the same line. This is due to the surge generally encountered in varying degree in both suction and discharge lines adjacent to the compressor. The effect of the surge was segregated in the analysis of the indicator cards, and an average value included in the empirical constant k.

The reason for considering the higher costs of stations as applying to the horizontal units, with the inference that the angle units are not used in the large stations, is that so far, in the author's experience, the preponderance of sentiment is for the horizontal units in main-line stations. This situation may or may not change with future developments.

While not particularly pertinent to the subject, an error in the discussion regarding the compression ratio for maximum horsepower should be corrected. This is not approximately 3, as stated, but may be anywhere between 2.3 and 1.6, depending upon the clearance volume and to a less degree upon the compressor efficiency. The theoretical ratio for zero clearance and 100 per cent efficiency is $n^{\frac{n}{n-1}}$ where n is the exponent of the gas. For methane, with $n = 1.27$, the theoretical compression ratio is 3.07. On one of the principal pipe lines, the compressors have a total clearance volume including unloaders of 50 per cent, and the maximum horsepower occurs at a compression ratio of about 1.85.

The author concurs with the discusser in his statement of the importance of high efficiency for compressors operating on the low compression ratios required on pipe lines, and much has been done toward this end. The principal and practically the only method, but a very effective one, is to decrease the velocity through the valves and this is not difficult. However, its importance has not been fully appreciated until recently.

The value of k is based largely upon the valve velocity, and $k = 0.97$, used in the analysis, is a fair average for compressors built in the past and now in service. In more modern compressors, the k value is approximately 0.975, and 0.98 or higher can be attained. Space limitations prevent a detailed analysis showing the actual economic gain, but it is appreciable. The discusser calls attention to the fact that if it were not for this constant loss causing the actual efficiency to vary with the compression ratio, there would be no optimum ratio. With the constant loss, increased efficiency lowers the optimum compressor ratio. Analytically, by Equations [15] and [16], as k approaches the limit 1, or 100 per cent efficiency, the compression ratio approaches its limit of 1, or suction and discharge pressure equal, station spacing zero, and number of stations infinite. Obviously, the same result will be obtained with a type of compressor having a constant efficiency instead of a constant loss. Such a type is approached very closely by the centrifugal compressor, as stated in the discussion.

Centrifugal compressors have been given consideration for pipe-line service from time to time, and one motor-driven unit was installed on a major pipe line. Aside from a lack of flexibility, as compared with reciprocating compressors, probably not a serious disadvantage considering the manner in which pipe lines are now operated, there does not seem to be any reason why the centrifugal compressor in itself would not meet the requirements of pipe-line service. The difficulty seems to be the lack of a suitable driver. Electric power is not looked upon with favor, assuming that it could be obtained at comparable cost. A steam-turbine plant of comparable efficiency shows no economic advantage in first cost, nor in annual cost, except possibly in localities where cheap coal is available; and such localities are not very frequently those where stations are required. Diesel engines driving centrifugal compressors through step-up gears would attain a high fuel economy, offset by higher initial cost. The gas turbine would be an ideal driver from the mechanical standpoint, but until more experience has accumulated and much lower fuel consumption has been obtained than at present, this type of prime mover cannot be considered for pipe-line service.

The possibility of the centrifugal compressor showing to economic advantage thus depends to a considerable extent upon the availability and relative cost of some other fuel than the line gas. An extension, in a very generalized form, of the analysis of horsepower required for fuel gas (Appendix 3) shows that on a long line the maximum economy would be obtained by abandoning the use of the line gas for fuel somewhere along the line. In particular, with the total annual cost of the alternate type of station not using line gas equal to that of the gas-engine station, the use of the latter should be discontinued at the tenth station. Such a situation would be to the advantage of the centrifugal compressor, but obviously would occur only on an extremely long line. The probability of many such lines being built is remote, so it would seem that, except for special conditions, the present type of compressor equipment will prevail for some time.

Mr. Rhodes emphasizes concisely the process and the objectives of the analysis and, of equal importance, points out what it is not supposed to do.

Attention is called to the fact that, under different market conditions, a different formula for line cost would be required. This would result in a variation of the constants and exponents of Equation [2] of the paper, but the form would be unchanged.

As previously described, variation in station cost per horsepower with the size of the station could be included by changes in the exponents of the symbols for quantity delivered and for the compressor ratio. This variation actually is due to the fact that certain of the items making up the total station cost do not vary directly with the horsepower, either varying at a lower rate, or remaining constant regardless of the size of the station. For the purpose of the analysis, all of these items can be included in a single group as a fixed charge. The discusser constructively

points out that this fixed charge is equivalent to a certain number of horsepower per 1,000,000 cu ft per day and hence can be included in the factor k. This is a simple and fairly accurate method of including the variation of station cost with size, and as shown, results in solutions for optimum compressor ratio in agreement with those at which many of the large pipe lines are operated.

The discusser's comment, concerning Equation [32], the final equation for total annual cost, illustrates a second objective of the analysis, which is to show the relative weight of the various factors determining the specifications of the line. The importance of the stress factor is brought out in the discussion.

To Mr. Terry, the writer expresses appreciation of his favorable comment.

Acknowledgment

The author expresses his appreciation to Mr. George I. Rhodes, vice-president, Ford, Bacon & Davis, New York, N. Y.; Mr. L. F. Terry, vice-president, Chase National Bank, New York, N. Y.; and Mr. Paul Diserens, Worthington Pump and Machinery Corporation, Harrison, N. J., for their valuable suggestions in the preparation of his paper.

1825-Lb-Pressure Topping Unit With Special Reference to Forced-Circulation Boiler

By F. S. CLARK,[1] F. H. ROSENCRANTS,[2] and W. H. ARMACOST[3]

Because of current War Department regulations, the name and location of the power company whose additional facilities are discussed in this paper cannot be divulged at the present time. Originally the power station, established in 1923, incorporated turbine-generator capacity in two units of 75,000 kw, supplied by 375-psi, 750 F steam at the throttles by five boilers having a continuous steam-generating capacity of about 1,000,000 lb per hr. Early in 1940, economic and engineering studies were undertaken by Stone & Webster Engineering Corporation, leading to improvements in the economy of the station and additional generating capacity. The result is the installation of a 25,000-kw superposed turbine generator with one boiler having a continuous steam-generating capacity of 650,000 lb per hr at 1825 psi and 960 F at the superheater outlet. This forced-circulation boiler is the first of its type ever installed in this country and is 85 per cent greater in capacity than any of similar type installed in European plants. The authors discuss comprehensively the details of this boiler and its operation.

THE power company, whose steam-generating facilities are to be discussed, was organized in 1923, to supply power to three owning companies and to buy and sell secondary power under an agreement with one additional company not included in the group. The station was established because of the desire of the three owning companies "to secure an economical and assured supply of electric power at minimum cost, in addition to the capacity of their respective steam stations," which were old and incapable of most economical extension.

The power station is located on tide water and receives fuel in vessels up to 10,000-ton capacity. Up to the time of the present installation, it contained two turbine-generators of a combined capacity of about 75,000 kw and five boilers having a continuous steam-generating capacity of about 1,000,000 lb per hr. Steam conditions at the turbine throttles are 375 psi and 750 F. The boilers are equipped to burn both oil and pulverized coal, the latter being prepared in a separate building.

Early in 1940 the company requested Stone & Webster Engineering Corporation to undertake the necessary engineering and economic studies to determine the best type of capacity addition to make to its power station. It was desired to improve the economy of the station and to provide additional generating capacity. These studies covered 20,000- and 40,000-kw condensing units, also a superposed 25,000-kw unit. The steam conditions for the condensing units were 850 psi and 900 F; for the superposed unit, 1800 psi and 950 F at the throttle, and 400 psi and 603 F at the exhaust. As a result of these studies, it was decided to install the superposed turbine-generator with one boiler having a continuous steam-generating capacity of 650,000 lb per hr at 1825 psi and 960 F at the superheater outlet. In combination with the two 375-psi turbine-generators, the resultant gross output is 72,800 kw.

Layout studies and estimates indicated that the new boiler should be installed in the vacant space in the boiler room and the superposed unit in a separate room adjacent to the new boiler. Fig. 1 shows the arrangement in plan and in relation to existing equipment.

The column footings are on hard rock, the basement floor being at grade el 118.0, the forced- and induced-draft-fan floor at el 186.5. Economy in structural costs and the desire to maintain existing operating levels dictated the advisability of installing a steam-generating unit that could be erected within the limits set by the existing structure. This meant a so-called "low-head" boiler, with furnace dimensions such that the wet-bottom type had to be used, whereas the existing furnaces have dry bottoms. A study of the coals normally available showed that no operating difficulties or restrictions as to choice would result from using the wet-bottom furnace.

The fact that at 1850 psi water at the boiling point weighs about 40 lb and saturated steam 4.75 lb per cu ft presented a problem in connection with natural circulation in a low-head boiler, having a distance from the bottom of the main drum to the furnace floor of about 60 ft. Studies of natural-circulation designs submitted left doubts, possibly unjustified, as to satisfactory operation under the conditions imposed. The forced-circulation boiler was then considered, with the result that after most thorough study its use was decided upon.

The design submitted was supported by experience with several hundred units operating in Europe, on land and sea, running in capacity up to 350,000 lb per hr. The fact that the new unit is the first of its type to be installed in this country and that, when compared with European practice, is 85 per cent greater in capacity, is for higher pressure and higher steam temperature, and that it involves steam reheat, does not place it in the sphere of radical pioneering. A number of European installations were investigated. The performance and design data, associated with developments up to date, were made available to the engineers of the American licensee. The greater capacity, high steam pressure and temperature, and the reheat feature are new only in their association with a unit of the forced-circulation design.

Generating and Electrical Equipment

The new generator, Fig. 2, is rated 20,000 kw, 14 kv, but is capable of generating 25,000 kw continuously. No new step-up transformers were added at this time. In order to handle the increased output, the capacity of the two existing main transformer banks, having a combined capacity of 82,500 kva, has been increased by the addition of blower equipment on the radiators to approximately 107,250 kva.

The additional generating capacity has necessitated increasing the rupturing capacity of the 14-kv oil circuit-breakers. These

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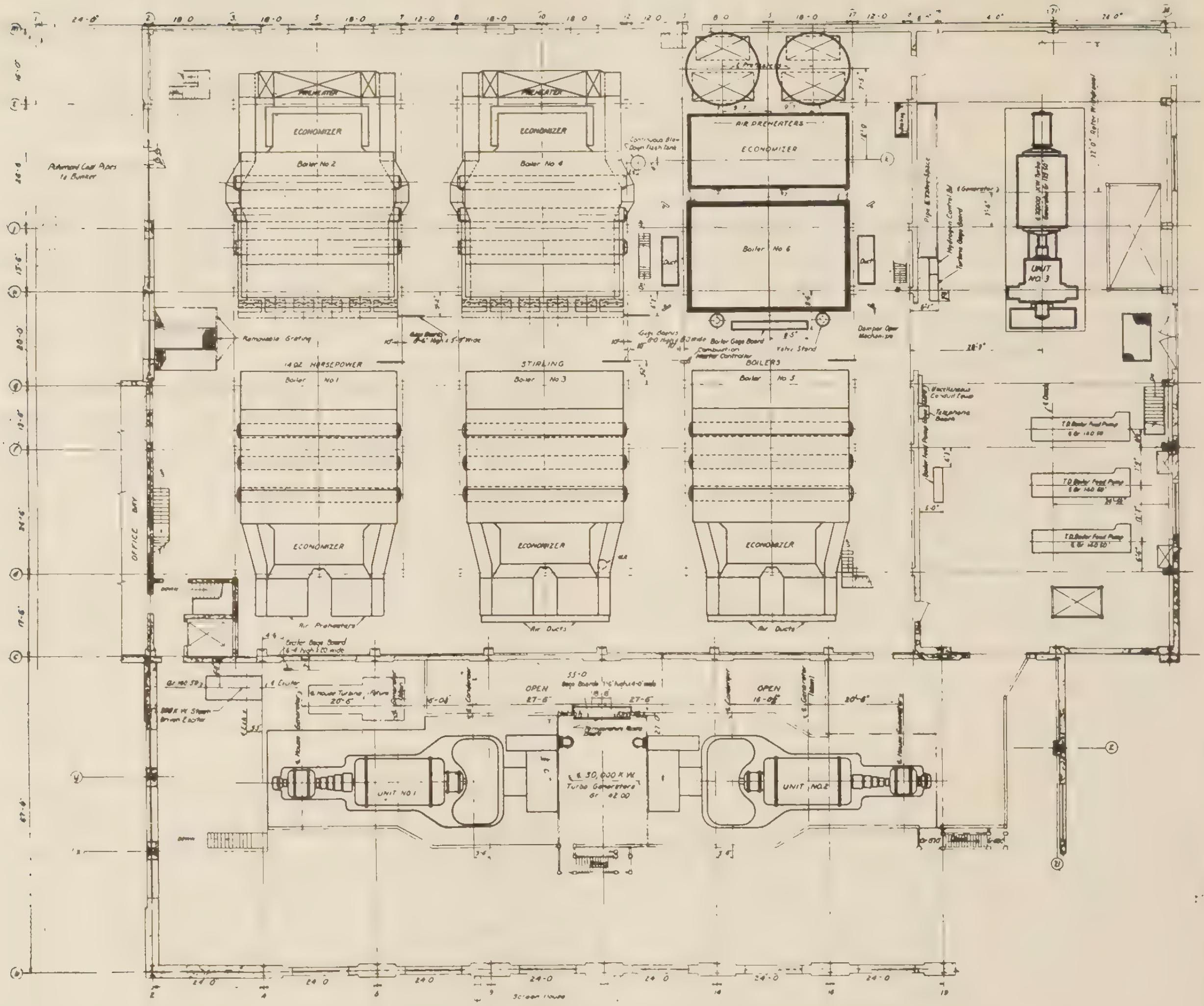

Fig. 1 Plan Showing Location of New Boiler and Turbine Unit

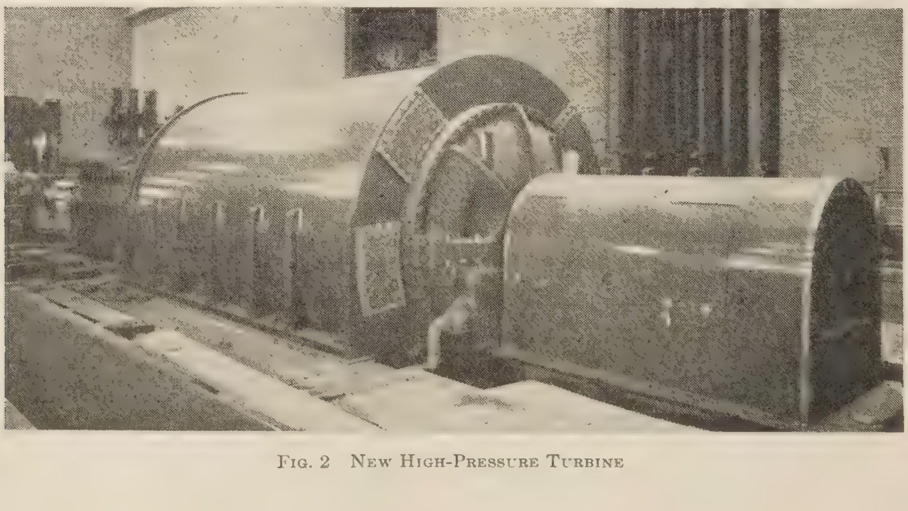

Fig. 2 New High-Pressure Turbine

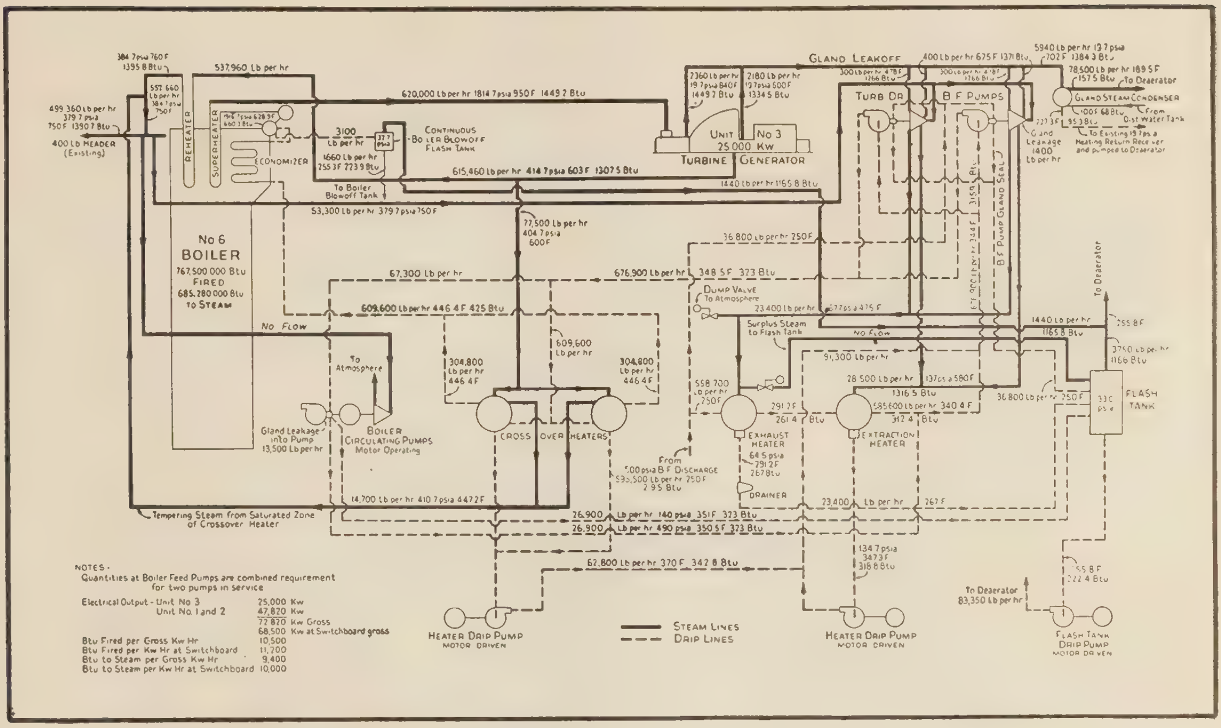

Fig. 3 Heat-Balance Diagram

were rated at approximately 500,000 kva. To meet the new requirement of 1,000,000 kva, eight breakers have been rebuilt with modernization parts, increased current-carrying-capacity contacts, and subcell disconnecting switches. In all cases, modern high-speed operating mechanisms have been substituted for the existing ones. Additional copper has been added to transformer circuits and silver plating of contacts has been used to increase current carrying capacity of connections and disconnecting switches where required.

Heat Balance

Fig. 3 shows the heat-balance diagram for the high-pressure installation with the "topping" unit generating 25,000 kw. Under this condition, the boiler delivers 620,000 lb per hr of which 615,460 is exhausted from the turbine at 400 psi and 603 F. Of the exhaust, 77,500 lb is used in two crossover heaters as a final or third stage in feedwater heating. The remaining 537,960 lb goes to the reheater in the boiler setting where its temperature is raised to 765 F. Of the reheated steam 53,300 lb is used to drive the high-pressure boiler feed pumps. Approximately 500,000 lb, including tempering steam used for temperature control, passes into the header supplying the two existing generating units. This is sufficient for the generation of about 47,800 kw.

Exhaust steam from the high-pressure boiler-feed-pump turbines is used for the first stage of feedwater heating. The second stage of heating is with steam extracted from these same turbines at about 122 psi. Drips from the extraction and crossover heaters are pumped to the high-pressure boiler-feed-pump suction header.

High-Pressure and Reheat Steam Piping

The high-pressure steam pipe between the boiler and turbine is $12^3/_4$ in. OD, turned and bored, with walls 1.71 in. thick of forged carbon-molybdenum steel, and conforming with A.S.T.M. Specification A-206-39-T. There are two gate valves in the line, one located at the superheater outlet and the other near the turbine throttle. There is no nonreturn valve. The valves have carbon-molybdenum bodies and all seating surfaces are stellited. Each is equipped with a motor-operating unit, having two control stations, one located near the valve, the other at boiler-room floor level. All joints in the line are welded, stress-relieved, and Gamma-rayed.

A pressure-reducing-and-desuperheating station is installed between the high-pressure steam line and the exhaust line from the turbine to the reheater. This consists of a 10-in. gate valve, a quick-opening valve, pressure-reducing valve (1825-400 psi), and a spray-type desuperheater. A second similar valve, 4 in. in size, is installed in parallel for by-passing up to 100,000 lb per hr, thus avoiding wear on the main valve under low flow conditions. The piping below the reducing valves is protected against accidental rise in pressure by safety valves.

The 400-psi exhaust line from the turbine to the reheater and from it to the existing 375-psi steam line is 16-in-OD carbon steel with all joints welded. Safety valves protect the unit from rise in pressure should a gate valve in the line be inadvertently closed.

Boiler

Comparison With Natural Circulation. The boiler differs in construction from a conventional natural-circulation unit in two fundamental respects as follows:

1 Pumps, Figs. 4 and 5, inserted in the downcomer connections leading from the main boiler drum to the bottom furnace-wall headers serve to insure the volume of circulation established by the design.

2 An orifice, inserted at the entrance to each tube element, at the point where it emerges from the bottom furnace-wall header, serves to fix the flow of water at a predetermined proportion of the total volume in circulation.

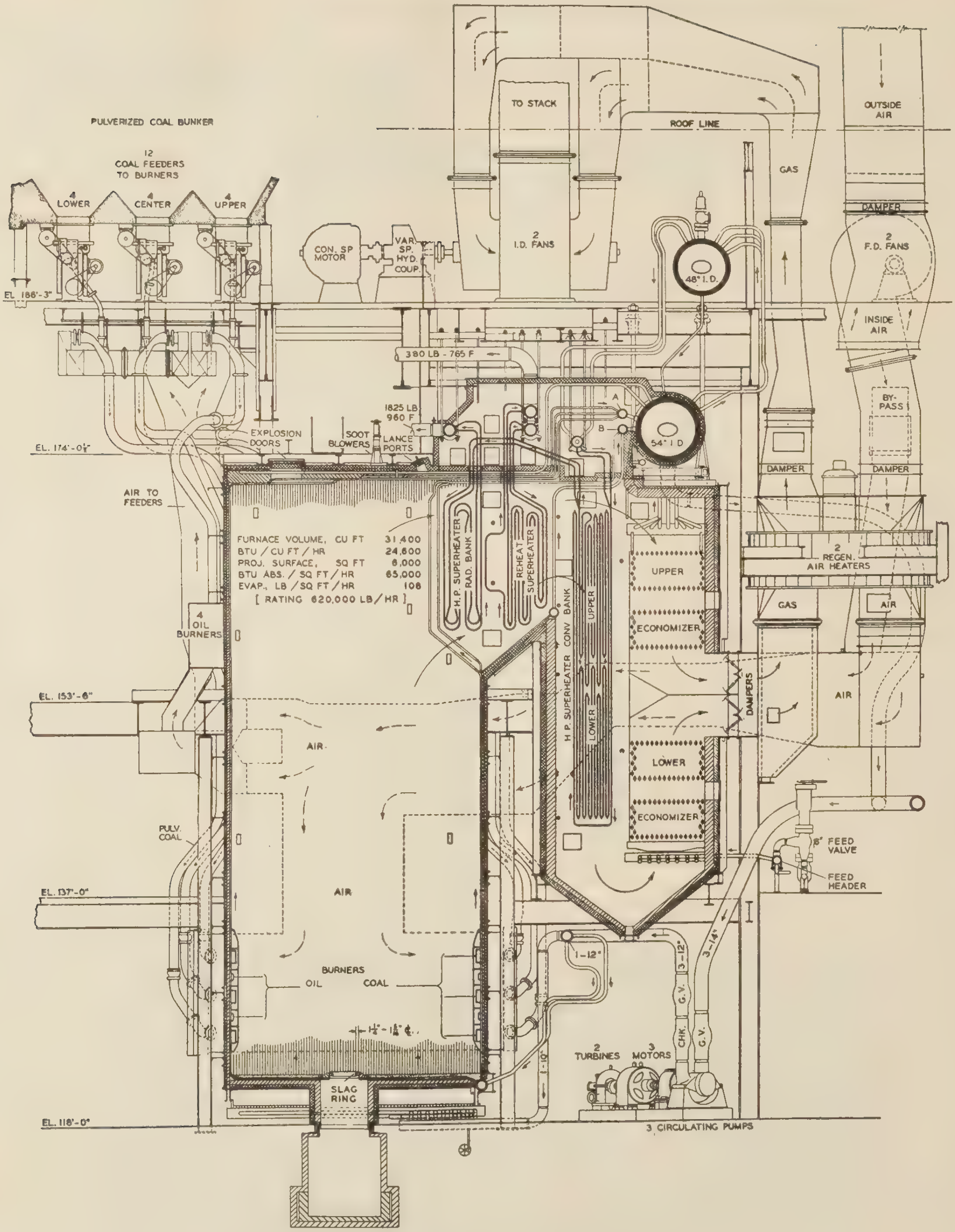

Fig. 4 Sectional Elevation of New Boiler Unit

The operation of the boiler is uninfluenced by the forced- and controlled-flow features. Burners, draft system, feedwater regulation, blowdown, etc. are operated and controlled in identically the same way as would apply to a natural-circulation unit of comparable design. The circulating pumps introduce minor variations in the routine of placing the unit in service, but having been placed on the line, the attention given is no more than that given to the forced- and induced-draft-fan units; in fact, it is less as no attempt is made to control the rate of delivery, either by throttling or speed control.

Protection Against Failure of Circulation. Two of the three circulating pumps are dual-driven and so arranged that, if the speed drops to a predetermined value due to motor failure, the steam turbine picks up the load automatically. Two pumps

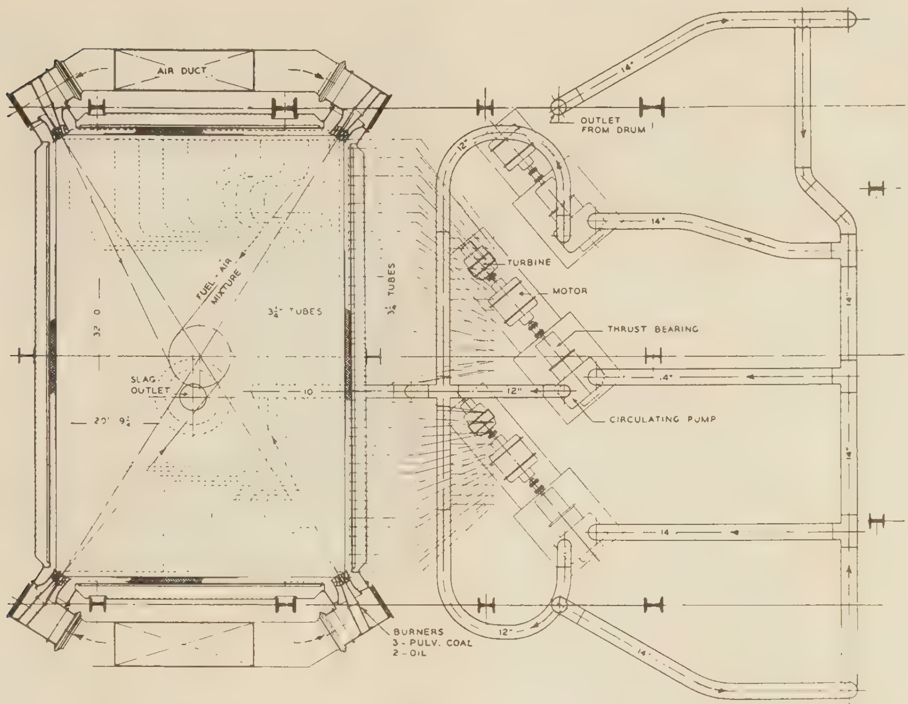

Fig. 5 Plan Arrangement, Showing Relation of Pumps and Circulating Piping to Furnace

will be in service at all times so at least one of the dual-driven units will always be in operation. With two motor drives in service, the source of electric power for each will be independent of the other. If for any reason the differential pressure between the suction and discharge headers falls to an established minimum, an automatic device acts to shut off the fuel supply. The worst that can happen, as a result of pump failure, is a shutdown, the same as would be the case in the event of failure of fuel or feedwater supply, or failure of either the forced- or induced-draft-fan equipment.

Main Operating Characteristics: Full load of 650,000 lb of steam per hr when firing coal:

Design pressure, psi	2000
Steam at superheater outlet, psi and deg F	1825 and 960
Steam at reheater inlet, psi and deg F	400 and 603
Steam at reheater outlet, psi and deg F	380 and 765
Feedwater temperature to economizer, deg F	446
Feedwater temperature to boiler, deg F	520
Final gas temperature, deg F	290
Total draft at air-heater outlet, in.	10.85
Total air pressure at air-heater inlet, in.	11.80
Efficiency of unit, per cent	89.3

Additional data shown graphically in Fig. 6

Fuel. The unit is fired with 12 (3 sets of 4) pulverized-fuel burners and alternately by 8 (2 sets of 4) oil burners arranged for tangential firing. Each set of 4 burners includes one burner in each corner of the furnace, all on the same horizontal plane. These burners are supplemented by four auxiliary oil burners in the upper wall opposite the superheater. Pulverized fuel is supplied by 12 feeders from a pulverized-fuel bunker, supplied in turn by an existing detached pulverizing plant. These feeders are divided into 3 sets of 4 to correspond with the sets of burners. Each set is driven by a constant-speed motor through a variable-speed unit.

The primary fuel is coal. Superheating surfaces are predicated on this fuel. When burning oil, the superheat and reheat will be held up to the design figures by burning the required amount of oil in the auxiliary burners. It is conceivable that these burners could also be used in combination with coal as a means of regulating superheat. Such operation was not contemplated, and operating experience proves it to be unnecessary.

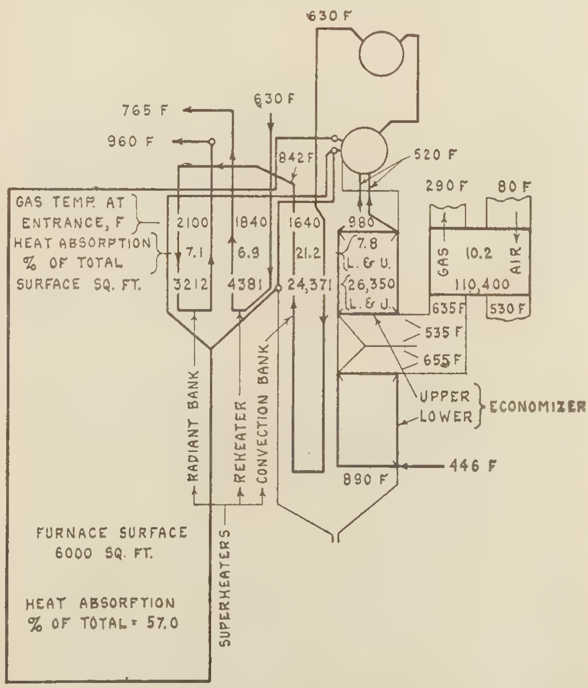

Fig. 6 Surfaces, Temperatures, and Distribution of Heat Absorption
(620,000 lb per hr rating.)

Furthermore, except in such instances as when the price of coal and oil happens to be on a parity, such operation would probably be uneconomical.

Forced Circulation. Any two of three 3500-gpm 50-psi-head pumps provide circulation up to full-load operation. One pump alone it is anticipated will give adequate protection to heating surfaces for an indefinite period but may possibly not give the desired margin of safety for continuous operation at full capacity. Each pump is provided with motor-operated gate valves at suction and discharge and a discharge swing check valve. Two 14-in. downcomer pipes, one from each end of the main steam drum, join through a cross header from which three 14-in. suction connections supply the three pumps. The three 12-in. discharge pipes supply a common header system of 12- and 10-in. pipe from which water is distributed through seventy-two $3^1/_4$-in. tubes to the bottom furnace headers. The suction and discharge pipes for each pump are short-circuited through a $^3/_4$-in. pipe connected just above the shutoff valves for the purpose of providing adequate circulation to keep them up to full temperature even though the pump may not be in service. Each of the 14-in. pipes is protected with an entrance screen where it leaves the drum.

The pump impeller is of the simple overhung type. Running

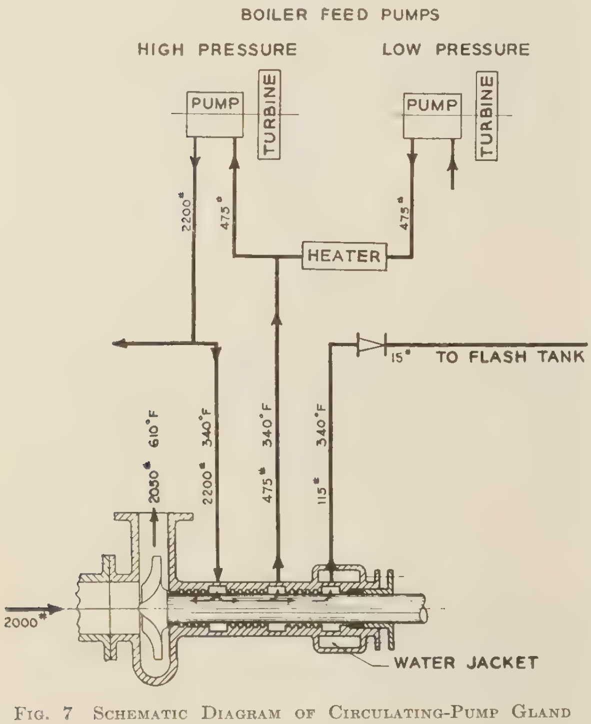

Fig. 7 Schematic Diagram of Circulating-Pump Gland

clearances provide for the large increment of temperature change which takes place when putting an idle pump into service. The end thrust, about 28,000 lb, is taken on a Kingsbury thrust bearing. The shaft gland is packed against a pressure of 115 psi; the pressure being maintained by a reducing valve discharging to a 15-psi flash tank. Fig. 7 clearly indicates the manner in which the 2000 psi is broken down and the means of taking care of the leak-off water, a system identical with practice long established in high-pressure boiler feed pumps.

The performance characteristic, satisfactory operation of the lubrication system, thrust bearing, gland seal, etc. were fully established on each pump by shop test.

The extreme simplicity of design, coupled with the ideal conditions of constant temperature, speed, and load and with the further fact that a relatively clean noncorrosive water is being handled should place the operation of the circulating pumps on a plane approaching absolute reliability.

With two pumps in service, the total water in the system is circulated in a period of 1 to $1^1/_2$ min, the exact time depending upon the level of water in the drum and the ratio of steam to water in the furnace-wall tubes. At 3500 gpm per pump two units circulate about 2,100,000 lb per hr or three times the full-load evaporation rate. Tests made at 2000 psi by the research department of Consolidated Edison Company,[4] on setups made with tubing of the same diameter and wall thickness as used in

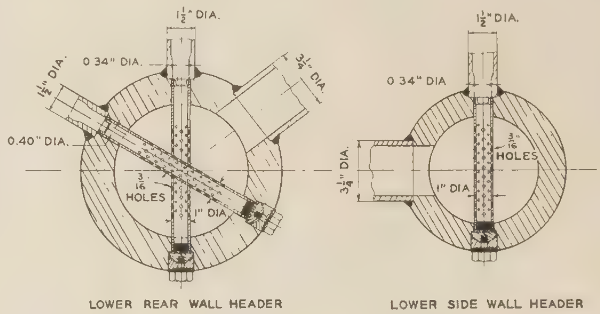

Fig. 8 Strainer-and-Orifice Assembly

the present furnace, confirmed European experience that this ratio of water to steam is well on the safe side.

The steam equivalent of the energy consumed by the circulating-pump drives is approximately 0.30 per cent of the steam generated.

Controlled Circulation. Equal in importance with positive minimum volume of circulation is the assurance that each element of the circulating system shall receive its proportionate share of the total. The nozzle and strainer, Fig. 8, inserted at the entrance to each of the furnace-wall elements is designed to give a pressure drop of about 15 psi. Actually, in establishing the pressure drop, the resistance offered to flow by the entire circuit between the lower header and the drum must be taken into consideration. Theoretically each variation of tube diameter, tube length, heat absorption with different locations in the furnace, etc. dictates a different size of orifice and a different orifice pressure drop. Actually, however, minor variations are unimportant, and the margin of safety is such that precision in design is not necessary. In the present design two sizes of orifice, 0.34 and 0.40 in. diam, take care of all the variations.

The holes in the strainers are $^3/_{16}$ in. diam with an aggregate area of 20 or more times the area of the orifice.

As part of the research program undertaken by Consolidated Edison Company,[4] its engineers co-operated with engineers of the boiler manufacturer in a thoroughgoing calibration of the orifices. A setup was made of an exact replica of a section of the header with screen-and-orifice assembly, and tests run on a series of five orifices covering the range from $^1/_4$ to $^1/_2$ in. diam, with pressures covering the range from 250 to 2500 psi. The water in all tests was only slightly below saturation temperature. Aside from their value as applying to the new boiler, these tests

[4] "Studies of Heat Transmission Through Boiler Tubing at Pressures From 500 to 3300 Psi," by W. F. Davidson, P. H. Hardie, C. G. R. Humphreys, A. A. Markson, A. R. Mumford, and T. Ravese, Advance Paper No. 16, presented at the 62nd Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., December 4, 1941. To be re-presented for discussion at the next meeting of the Society.

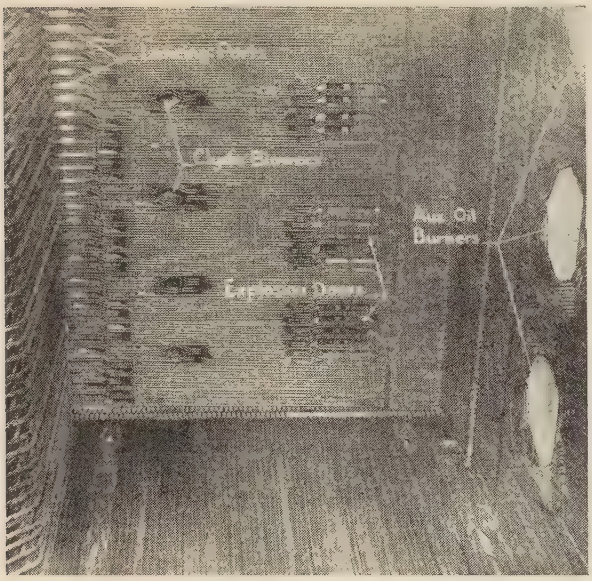

Fig. 9 Roof and Upper Zone of Furnace
(Note screen tubes and superheater tubes immediately behind them.)

prove that the coefficient of discharge of an orifice is uninfluenced by pressure and temperature; and hence orifices for future boilers may be calibrated before installation with water at low pressure and temperature.

Evaporating Surface. All furnace-wall tubing is $1^1/_4$ in. OD. At the point of leaving the bottom headers, a single $1^1/_2$-in. outlet is bifurcated into two $1^1/_4$-in. tubes, and all spacing arranged to give a furnace lining of $1^1/_4$-in. tubes on $1^5/_{16}$-in. center lines. Tubing of $1^1/_4$ in. was selected with due regard to (*a*) resistance to flow; (*b*) economic limitations on weight; and (*c*) the certainty of having a turbulent-flow condition at all points in all tubes without requiring a prohibitive volume of circulation.

Referring to Fig. 4, the heat-absorbing surface of the furnace may be divided into four groups of elements, in which all elements in each group are of similar design and which, because of similar location and exposure, absorb approximately the same amount of heat per element, as follows:

1 A group running from the lower rear header across the furnace bottom, up the front wall, across the furnace top into header (*A*) which is connected by twenty-five $3^1/_4$-in. nipples into the drum.

2 Two groups of identical design, one for each side wall, run from headers at the furnace bottom to similar headers at the top which in turn are connected to the steam drum through a total of thirty-four $2^1/_2$-in. tubes.

3 A group, making up 50 per cent of the rear wall tubes, running from the lower rear header is increased from $1^1/_4$ to $1^3/_4$ in. at a point just below the superheater and arranged to pass in front of the superheater in two-in-line rows on $6^9/_{16}$-in. centers. These tubes pass over the top of the reheat superheater and up to header (*B*) which in turn is connected through twenty $3^1/_4$-in. nipples into the drum.

4 The other 50 per cent of the rear wall tubes pass beneath the reheat superheater to a header from which the steam-and-water mixture passes through fourteen $3^1/_4$-in. tubes to the same header mentioned for group (3).

The $1^1/_4$- and $1^3/_4$-in. tubing described adds up to about 63,000 linear feet and on a projected basis the tubing exposed to heat is equivalent to approximately 6000 sq ft. It constitutes the entire evaporating surface. At 620,000-lb per hr capacity, this means roughly 108 lb of evaporation per sq ft of projected surface per hour. The water enters the circulating system from the economizer at about 530 F. Heat required per pound of evaporation, including 150.7 Btu to raise to the boiling point plus the latent heat of 480, is 630.7 Btu. The heat absorption per square foot of projected heating surface is therefore about 65,000 Btu per sq ft per hr.

Superheater; High-Pressure. The large convection superheater (24,371 sq ft) is made up of $2^1/_8$-in. tubes with the elements spaced on $3^9/_{32}$-in. centers across the boiler. The radiant bank (3212 sq ft) made up of 2-in. tubing is placed next to the furnace, so arranged that the loops carrying the coolest steam are placed closest to the furnace in the hottest gases. The elements are spaced on $6^9/_{16}$-in. centers in line with the $1^3/_4$-in. furnace screen tubes.

Superheater; Reheat. Located between the radiant and convection banks of the high-pressure superheater, the reheater elements made up of $2^1/_8$-in. tubes are spaced on $6^9/_{16}$-in. centers and in line with the radiant high-pressure and the furnace screen tubes ahead of it. (Reheat surface 4381 sq ft.)

Economizer. The economizer (25,854 sq ft) made up of 2-in. multiple-loop finned elements takes water into two bottom headers through a total of fourteen $3^1/_4$-in. tubes and discharges from two top headers through a total of eighteen $3^1/_4$-in. tubes into the bottom of the drum. Elements run from both sides to the middle; i.e., there are actually two sets of economizers. This arrangement was necessary to facilitate element removal. A horizontal plate, see Fig. 4, half way up divides the unit into upper and lower economizers through which gas flow is controlled by dampers at the outlet. The manner in which these dampers may be operated to control high-pressure-steam temperature is obvious.

Steam Drums and Connecting Tubing. The main steam drum,

Fig. 10 Bottom of Furnace
(Note burner, slag outlet, and lower rear header assembly. Bottom tubes and slag opening were subsequently protected with refractory coverage.)

TABLE 1 TEMPERATURES AND DISTRIBUTION OF HEAT ABSORPTION[a]

	Entrance temperature of gases, deg F	Per cent of total heat absorption
Furnace	—	57.0
Superheater, radiant bank	2100	7.1
Reheater	1840	6.9
Superheater, convection bank	1610	21.2
Economizer	890	7.8
Total		100.0
Air heater	640	10.2

[a] Note that the absorption by the air heater is returned to the furnace via the heated air and is a circulating element.

TABLE 2 PRESSURE-PART DESIGN DETAILS OF BOILER

Item	Diam, in.	Wall thickness, in.	Material specification	Material description
Drum Assemblies				
Dry drum	48 ID	4 3/16	A.S.M.E. S-44-Grade B A.S.T.M. A-204-39	70,000 T.S. Molybdenum steel
Main steam drum	54 ID	4 23/32	A.S.M.E. S-44-Grade B A.S.T.M. A-204-39	70,000 T.S. Molybdenum steel
Tubes from main to dry drum	3 1/4 OD	0.320	A.S.M.E. S-49 A.S.T.M. A210-40	Silicon-killed medium-carbon steel
Tubes from dry drum to sat. header	3 1/4 OD	0.320	A.S.M.E. S-49 A.S.T.M. A-210-40	Silicon-killed medium-carbon steel
Header A	8 5/8 OD	31/32	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Nipples from header A to drum	3 1/4 OD	0.375	A.S.M.E. S-8 Class II A.S.T.M. A-105-40	Forged steel
Header B	8 5/8 OD	31/32	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Nipples from header B to drum	3 1/4 OD	0.375	A.S.M.E. S8 Class II A.S.T.M. A-105-40	Forged steel
Circulating System				
Downcomers to pumps	14 OD	1.406	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Pump discharge pipes	12 3/4 OD	1.312	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Distribution header, across rear	12 3/4 OD	1 1/2	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Distribution header, under furnace	10 3/4 OD	1.735	A.S.M.E. S-4 Grade 1	Carbon steel
Pipe connecting headers (C) and (D)	10 3/4	1.000	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Tubes to lower furnace headers	3 1/4 OD	0.300	A.S.M.E. S-49 A.S.T.M. A210-40	Silicon-killed medium-carbon steel
Furnace				
Bottom rear header	10 3/4 OD	1 13/16	A.S.M.E. S-4 Grade 1	Carbon steel
Bottom side wall headers	8 5/8 OD	1 7/16	A.S.T.M. A106-40 Grade B	Silicon-killed medium-carbon steel
Upper side wall headers	8 5/8 OD	1 3/16	A.S.T.M. A106-40 Grade B	Silicon-killed medium carbon steel
Upper rear wall header	6 5/8 OD	27/32	A.S.T.M. A-106-40 Grade B	Silicon-killed medium-carbon steel
Connections at headers	1 1/2 OD	0.200	A.S.M.E. S-40 A.S.T.M. A-132-40	Silicon-killed low-carbon steel
Furnace wall tubes	1 1/4 OD	0.165	A.S.M.E. S-40 A.S.T.M. A-192-40	Silicon-killed low-carbon steel
Screen tubes below superheater	1 3/4 OD	0.220	A.S.M.E. S-40 A.S.T.M. A-192-40	Silicon-killed low-carbon steel
Tubes upper side wall header to drum	2 1/2 OD	0.260	A.S.M.E. S-49 A.S.T.M. A-210-40	Silicon-killed medium-carbon steel
Tubes upper rear wall header to header B	3 1/4 OD	0.340	A.S.M.E. S-49 A.S.T.M. A-210-40	Silicon-killed medium-carbon steel
Superheater Tubes				
High pressure convection bank				
Bottom half	2 1/8 OD	0.300	A.S.M.E. S-40	Silicon-killed low-carbon steel
	2 1/8 OD	0.320	A.S.T.M. A-192-40	
Top half	2 1/8 OD	0.300	A.S.M.E. S-48 A.S.T.M. A-209-40T Grade T-1	Silicon-killed carbon-moly steel
High pressure radiant bank				
Loops nearest furnace	2 OD	0.300	A.S.M.E. S-52 A.S.T.M. A-213-40T-16	Chrome-molybdenum titanium steel
Finishing loop	2 OD	0.380	A.S.M.E. S-52 A.S.T.M. A-213-40T-16	Chrome-molybdenum titanium steel
Reheater tubes	2 1/8 OD	0.135	A.S.M.E. S-17 A.S.T.M. A-83 38T	Low-carbon steel
Superheater Headers				
High pressure inlet	10 3/4 OD	1 1/2	A.S.M.E. S-18 Grade B A.S.T.M. A-106-40 Grade B	Silicon-killed medium-carbon steel
High pressure outlet	12 3/4 OD	1 5/8	A.S.M.E. S-45 Grade P-1 A.S.T.M. A-206-39T Grade P-1	Carbon-molybdenum alloy steel
Reheater inlet and outlet	12 3/4 OD	7/8	A.S.M.E. S-18 Grade B A.S.T.M. A-106-40 Grade B	Silicon-killed medium-carbon steel
Economizer Tubes and Headers				
Finned tubes	2 OD	0.280	A.S.M.E. S-40 A.S.T.M. A-192-40	Silicon-killed low-carbon steel
Feed header to inlet header	3 1/4 OD	0.340	A.S.M.E. S-49 A.S.T.M. A-201-40	Silicon-killed medium-carbon steel
Outlet header to drum	3 1/4 OD	0.340	A.S.M.E. S-49 A.S.T.M. A-201-40	Silicon-killed medium-carbon steel
Economizer headers	8 5/8 OD	1 1/2	A.S.M.E. S-18 A.S.T.M. A-106-40	Silicon-killed medium-carbon steel
Orifice and Strainer Assembly				
Orifice	0.34	1/32		Stainless
Orifice	0.40	1/32		Stainless
Strainers	1 OD	1/8		Low-carbon steel
Closure plugs	1 13/32	1 1/4	18% Chrome 8% Nickel	Stainless
Gasket	1 9/32	1/16	Soft iron	Brinell (80–100)

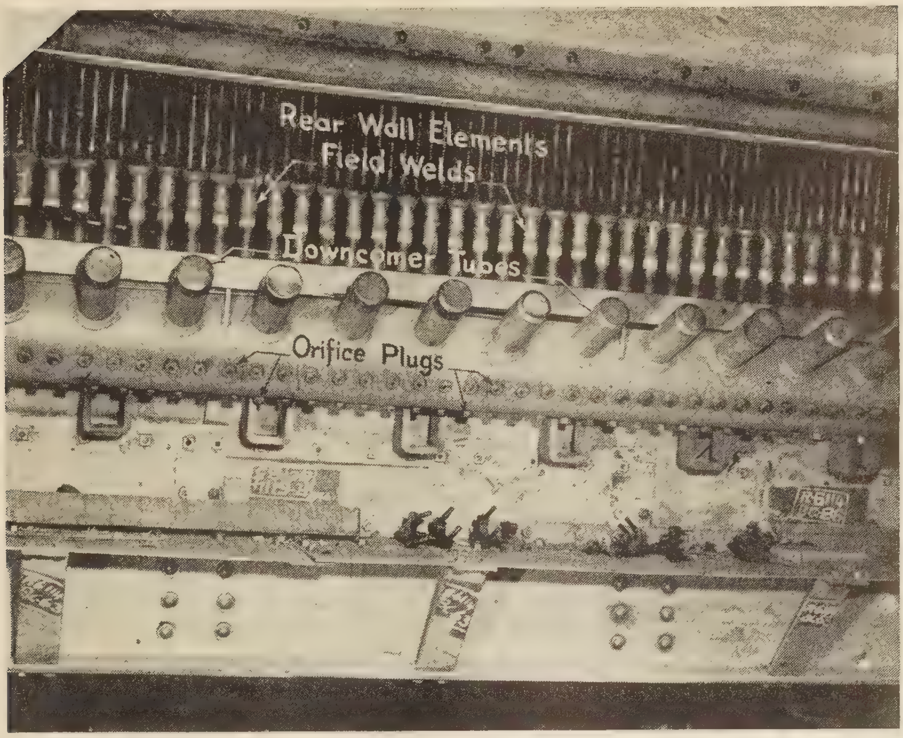

FIG. 11 LOWER REAR-WALL-HEADER ASSEMBLY

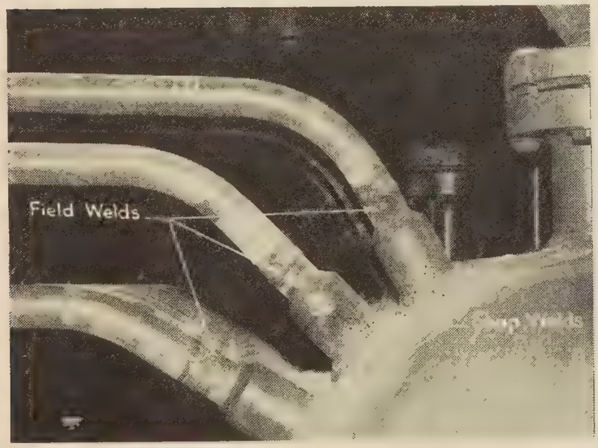

FIG. 12 SHOWING MEANS OF REINFORCING DRUMS AT TUBE ENTRANCE
(Large tubes go through drum and are welded inside and out.)

41 ft $3^{1}/_{2}$ in. long $\times$ 54 in. ID, is $4^{23}/_{32}$ in. thick. The dry drum, 24 ft 2 in. long $\times$ 48 in. ID, is $4^{3}/_{16}$ in. thick.

The manner of entering the tubes into the drums, Fig. 12, is such that the ligament efficiency is 90 per cent. Since this is the same as the design limit of strength (fixed by the A.S.M.E. Boiler Code) for the longitudinally welded seams, as compared with virgin metal, no increase in drum thickness is imposed.

A total of 66 tubes $3^{1}/_{4}$ in. diam take the steam from the main steam drum up to the dry drum; 40 tubes of the same diameter convey the steam from the dry drum to the saturated header of the high-pressure superheater.

The drum internals are fully illustrated in Fig. 13. Attention

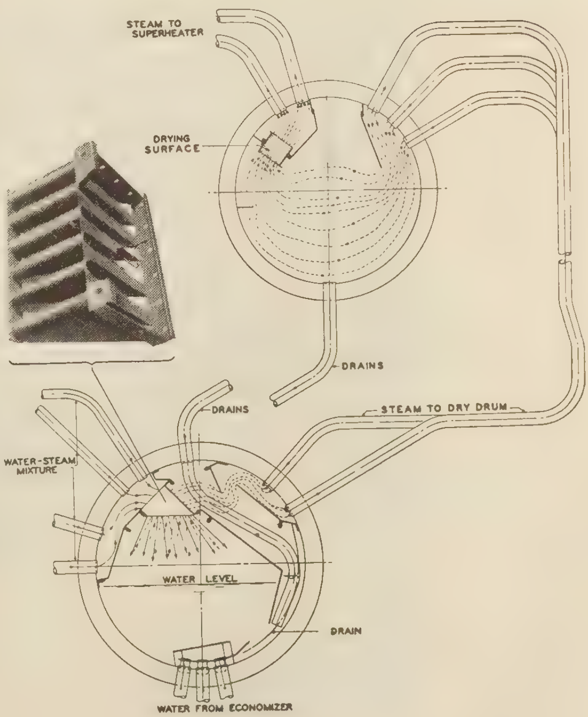

FIG. 13 DRUM INTERNALS

is directed to the low point at which steam is taken from the main drum up to the dry drum, and the internal arrangement provided for overcoming the apparent defect. Reference to Fig. 4 reveals the physical obstructions external to the drum which prevented taking the steam off at a higher level.

Air Heater. Two air heaters (55,200 sq ft each) complete the heat-absorbing equipment. They are equipped with by-passes for light-load operation. The active heating element, 58 in. in height over-all, is made up of three layers—a lower or hot layer 42 in. high, and a 10-in. intermediate layer of 24 gage open-hearth steel and a top cold layer 6 in. high of 22 gage Toncan iron. At full capacity, the air heaters lower the temperature of the gases from 640 to 290 F with a corresponding air-temperature range of 80 to 530 F.

Soot Blowers and Lancing Provisions. Soot blowers located in the furnace roof and walls are distributed as follows:

8 Revolving units in roof to clean screen and superheater section
3 Revolving units in front wall
3 Revolving units in rear wall
4 Revolving units in side walls (2 on each side)

In the superheater and reheat section, units on each side of the boiler are distributed as follows:

2 Retractable units between the radiant bank of the high-pressure superheater and the reheater
4 Revolving units ahead of the convection superheater
3 Revolving units behind the convection superheater

In the economizer section, there are 16 revolving units and 16 stationary units. The stationary units clean the ends of the loops beyond the barrier formed by the supporting structures. The economizers are divided into two vertical halves, thereby doubling the number of supports added to the number of soot blowers to reach all parts.

All of the furnace units take saturated steam at 750 psi through a reducing valve. The remainder of the blowers take steam from the reheater outlet at 375 psi and 750 F.

Access for hand-lancing of the screen and superheater section is provided through a row of lancing ports in the roof, permitting entry of a lance to each open lane. These lanes are on $6^9/_{16}$-in. centers.

Material Specifications and Thickness. Table 2, showing the diameter, wall thickness, material description, and specifications, as applying to the pressure parts, is of interest and is included as a reference and record.

Welded Construction; Pressure Parts. All tube connections to headers and to the boiler drums are welded. There are no rolled joints of any character. All joints of the 14, 12, and 10-in. circulating system are welded, stress-relieved, and Gamma-rayed. Welded construction is in no way a feature of the controlled-forced-circulation system but was decided on as desirable in view of the high pressure. The only joints which are not welded are the plugs opposite the orifice and strainer assemblies, the manholes in the drums, the bolted joints on the safety-valve flanges, the bonnets on the circulating-water pump valves, and the heads on the circulating pumps. In addition to welded joints made in the shop, there is a total of approximately 2720 field welds.

Supports. The dry drum is mounted on structural steel. The main steam drum is hung on two $4^3/_4$-in-diam carbon-steel U-bolts from steel at the same level.

The economizer is supported by rods running from cross members under the bottom section up over the main steam drum. These rods engage intermediate cross-members carrying the several sections. They are protected from excessive gas temperature by outer sleeves forming an annular space through which air from the forced-draft fan is blown.

The superheaters and reheater are supported by hangers from overhead steel independent of all other members of the boiler.

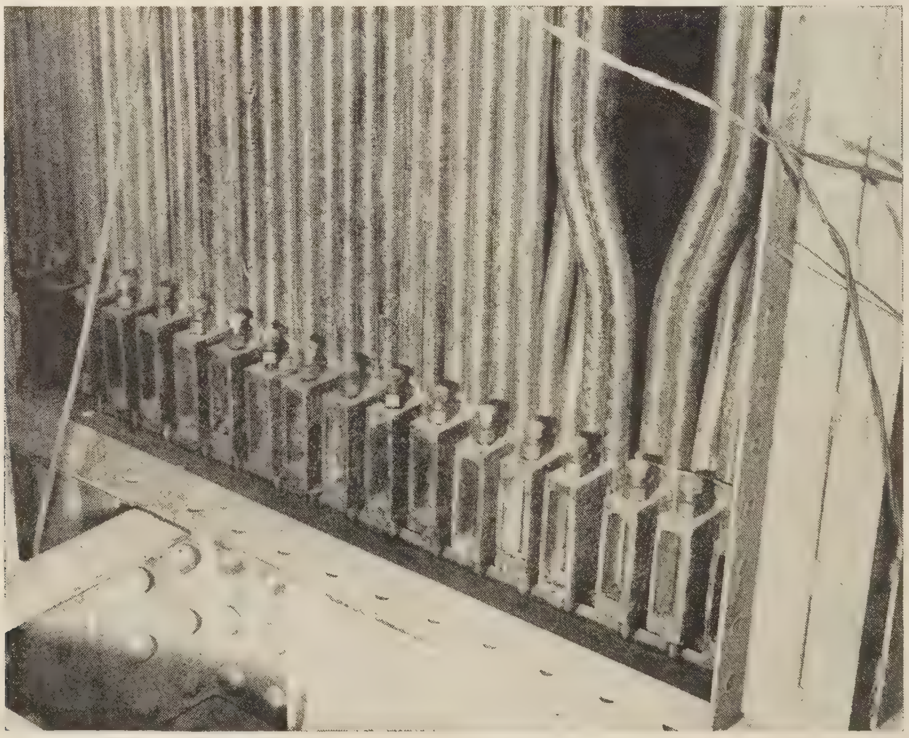

Fig. 14 Furnace-Tube Supporting Fixtures

The furnace tube elements are supported at the 153-ft 6-in. elevation by fixtures, Fig. 14, made adjustable for convenience in erection. All expansion of pressure parts is taken up by the flexibility of interconnecting tubing. No springs are used in the supporting system.

The magnitude of the expansion values to be provided for may be gaged, when it is stated that the vertical expansion of the 49-ft-high furnace, due to a temperature range of 80—630 F, is approximately 2 in.

Control

Boiler Operation. All boiler-operating controls are brought to a central panel board, which carries also all operating meters, draft gages, and valve and damper position indicators.

A pneumatically operated control system co-ordinates the operation of the various auxiliaries. Primary control of firing rate is from steam-header pressure.

The control system applies to all burners in the lower zone of the furnace. It is arranged to permit firing any one or all three sets of pulverized-coal burners, or either or both sets of oil burners. If desired, both oil and coal in combination can be burned, with provision for automatic control of all burners or base loading one or more sets, while retaining automatic control of the others from steam pressure. Provision is made also to permit any distribution of fuel desired among the various sets of burners while under automatic control from steam pressure.

The air for combustion is measured by metering separately and totalizing the flue-gas flow through the two economizer sections. Control of air flow is from a totalized measurement of all fuel fired, including both coal and oil. The rate of air supply is regulated by varying the speed of the hydraulic-coupling-driven forced- and induced-draft fans in parallel with readjustment of induced-draft-fan speed if necessary to maintain constant furnace draft.

Damper controls are provided in the air-supply ducts to the main and auxiliary burners. When the auxiliary burners are in use at low boiler ratings, the speed of the forced-draft fans is controlled from the air-pressure requirements of the auxiliary burners and the requirements for the main burners, regulated by means of dampers.

The controls at both forced-draft and induced-draft fan couplings include special features to insure that each fan carries its share of the load.

Safety devices include electrical interlocks with the fan-motor circuits arranged to close the outlet damper on any one fan when the corresponding fan is shut down or fails. If all fans fail, all dampers automatically open.

Safety-check devices sensitive to pressure and draft shut off all fuel on failure of either forced or induced draft due to motor failure or otherwise.

Load-limit devices restrict the maximum quantity of fuel that can be fired to the capacities of the fan equipment actually in service for delivering air and withdrawing products of combustion.

Steam-Temperature Control. Control of high-pressure steam temperature is obtained by controlling dampers regulating the quantity of flue gas passing over the high-pressure convection superheater. These dampers are automatically controlled in parallel with rate of air flow through the boiler unit with automatic readjustment from a thermostat in the high-pressure-steam outlet header. By design, full superheat, with coal fuel, is available down to a rating of 480,000 lb of steam per hr.

Reheat temperature is automatically limited by injection of saturated steam from the saturated zone of the crossover feed heater into the steam line leading from the reheater. In the event the high-pressure turbine is by-passed, the steam temperature to the reheater is reduced by spraying water under automatic control into the steam line leading to it.

The temperature of either the high-pressure or the reheated steam may be boosted by burning the required amount of oil in the upper burners under control of manually operated devices located on the operating panel.

Feedwater. The feedwater control regulates the water level and supply in and to the drum. This consists of two control valves, one in each feed line actuated by a thermostat and by the differential pressure between the drum and the superheater outlet. A master excess-pressure controller varies the speed of the feed-pump turbines to maintain a difference in pressure across the boiler-feed control valves sufficient to supply the required amount of water. The control also actuates a by-pass valve in the discharge from each feed pump for recirculating a portion of the feedwater at very light loads.

The boiler feed pumps and drives are designed for two pumps in service handling 325,000 lb of water each with the drives exhausting to a 50-psi closed heater. This requires approximately 1000 hp per pump. The drives are designed to be capable of carrying approximately 2000 hp if exhausting to the atmosphere. The control for this emergency condition functions from a pressure connection in the boiler-feed-pump turbine casing to start to open a dump valve to the atmosphere when the horsepower is something over 1000. This control will allow the dump valve to be wide open when the turbine horsepower reaches approximately 2000, allowing one boiler feed pump exhausting to the atmosphere to carry practically the entire boiler load.

In the event of excessive pressure, a Bourdon tube connected to the pump discharge operates a leak-off valve in the turbine-governor oil line and reduces the speed. This functions at about 100 psi in excess of normal maximum discharge pressure. A second Bourdon tube closes an electric circuit and trips the overspeed governor in case of complete failure of the control equipment.

Advantages of Forced-Circulation Control

The provisions incorporated in the boiler for insuring the volume and distribution of circulation eliminate the last important uncertainty present in the operation of all natural-circulation boilers. They reduce the design problems associated with circulation to a point approaching an exact science.

The merits of any boiler from an operating point of view may be classified under the following headings:

1. Efficiency of fuel consumption.
2. Operating labor.
3. Steam quality.
4. Availability.
5. Maintenance.

Efficiency, operating labor, and steam quality will be so nearly identical for the natural- and forced-circulation boilers that no advantage for either can be logically claimed. The fact, however, that the amount of water passing through the drum is known and that a substantial pressure loss may be tolerated in separating devices without jeopardizing circulation opens up possibilities for developments giving drier steam for the forced-circulation unit.

Availability and maintenance are so intimately related that they cannot be discussed separately to advantage. Furthermore availability is so dominating in importance that any departure from present-day design must justify its existence on this point above all others. Even a small amount of outage, either in terms of elapsed time or of frequency of occurrence, has a most important influence on spare equipment which must be available and the amount which must be maintained in service.

The most important causes of outage are leakage or failure of pressure parts, failure of auxiliaries, and excessive slag deposits. Any advantage from less slag deposit in favor of forced-circula-

tion must arise from the small closely spaced tubes on the furnace wall and the relatively flat surface which results. Experience does indicate some such advantage. The addition of circulating pumps in the case of forced circulation adds to the auxiliary-failure hazard. Their reliability must be established to a point approaching the "absolute." A large part of the justification for forced circulation must therefore be predicated on less outage and maintenance due to leaks and failures of the pressure parts.

Leakages occur at joints such as at the rolled-tube seats, handholes, flanged connections, etc. Pressure-part failures are largely a matter of water conditioning and circulation, associated with high concentration of heat application to localized areas.

The welded construction at the station eliminates tube-seat leakage, but this type of construction is not an advantage limited to forced circulation. It adds from 3 to 5 per cent to the cost of the unit and is perhaps not justified, particularly as applying to forced circulation. Tube-joint leaks are for the most part the result of stresses imposed because of temperature differentials which occur between different parts of the pressure structure during starting up and shutting down. These differentials are almost entirely eliminated with forced circulation, owing to the fact that circulation is fully established before lighting off the furnace, and the whole of the water removed and pumped back into the boiler about once a minute during the entire period of the pressure raising and lowering processes.

There are no handholes in the new unit. The small plugs, opposite the strainer and orifice assemblies, the manhole covers, and safety-valve flanges constitute the remaining points of possible leakage. They are not likely to leak as they are not subjected to sudden changes of temperature.

The combination of small-tube diameter and high velocity of flow results in a Reynolds number many times the critical range at which turbulent flow begins. Thus, a scouring action between the water or steam-water mixture and the inner tube wall exists in every square inch of the circulating system exposed to heat. No possibility of partially dry tubes or hot spots on otherwise relatively cool tubes, conditions which lead to deposits of sludge, aggravation of concentrated scale deposits, tube pitting, and kindred troubles, can exist.

Other advantages result from the small furnace tubes. The rise above saturation temperature of the outer surface due to the thin wall is less by 30 to 50 deg F, depending upon the rate of heat absorption, than for the corresponding rise for the thicker wall of the larger tubes essential to natural circulation. At high pressure, this is important as the outer-wall temperature at high rates of heat absorption begins to crowd the safe limit for carbon steel. The thermal capacity of the tubes plus their contained water being substantially less, the rate at which pressure may be raised with the same rate of heat application is correspondingly more rapid. It should be noted that the rate of heat application is limited by the superheater to about the same value for both types.

The small tubes, adaptable to forced-circulation boiler construction, very greatly simplify the construction problems imposed by expansion. They are flexible. The walls are thin. Also the internal temperature stresses in the metal imposed at high rates of heat absorption are very much less than for large-diameter heavy-walled tubes. The extent to which this factor may affect the life of the metal may possibly prove to be an important consideration.

The orifice-control feature of forced circulation eliminates any possibility of the phenomena of "hide-out" and of uneven distribution of solids present in most, if not all, natural-circulation boilers. To be complete in this respect, the orifice-control feature must be supplemented by forced circulation which gives the necessary head requirements and withdraws all water and returns it to the boiler about once a minute. Thus, thorough mixing and, hence, equal distribution of solids result. A further advantage comes from the fact that a sample of water taken from any part of the system consistently represents the condition of the water in the boiler—a great advantage and aid to the control of water conditioning. Advantage has been taken of this characteristic in the instrument equipment. An electrolytic cell is set up using boiler water as an electrolyte. The boiler water is supplied in a continuous stream through a needle valve. On its way to the cell, water passes through a small heat exchanger to reduce its temperature to approximately that of the surrounding atmosphere. The potential across the cell is recorded on a continuous strip chart and gives a continuous indication of the boiler water concentration.

Addendum

Operating Record

The turbine-generator became available for actual operation under load on October 16, 1942. From that time until this addendum was written, on November 20, the unit had been in practically continuous operation with pressure and load gradually increased, although neither the boiler nor the turbine had reached full design conditions. On November 20, the unit was carrying 17,500 kw and operating at a throttle pressure of 1725 psi. The boiler output is 480,000 lb per hr. It is considered unsafe to carry a higher load because the by-pass reducing valve has not proved reliable under full-automatic control. If the turbine should trip out, failure of the reducing valve to operate immediately would cause a loss of load by the low-pressure units in the station, with the possibility of a complete station shutdown. The valve manufacturer is working on this installation, and the defect should be eliminated at an early date. Only one of the two feed-water-level regulators is in reliable operation and this is also a handicap to full-load, full-pressure operation.

The boiler was available for preliminary operation for some time before the turbine, and this afforded an opportunity to discover and correct defects in the boiler and its auxiliaries and to make necessary adjustments by operating over a wide range of output, supplying steam to the low-pressure turbine-generators through the reducing valve. The major troubles encountered were leaks in auxiliary equipment, such as valve bonnets, and carry-over from the main drum to the dry drum under conditions of high water and high concentration. Modifications were made to the drum internals and there has been no subsequent difficulty from this source.

The design of the replacement-drum internals had its origin in a research program carried on for some years and still continuing in response to the more and more rigid demand for cleaner and still cleaner steam for service with high-pressure turbines. During the period intervening between the original design of the internals and the date of placing the unit in service, very definite improvements were developed and these were applied to the boiler as soon as entirely new internals could be constructed and installed.

There has been one tube failure. This was a pressure failure caused by defective tube metal; there was no evidence of overheating. A section about 5 ft in length was cut out and a replacement section was welded in. The tube failure was a wide-open rupture, and the fact that water level was maintained in the drum until the furnace was partially cooled is a point of interest.

Note: To secure the advantage of additional operating experience, the section on operation will be submitted as an addendum (copies to be available to those present) at the time of presenting the paper.

The principal valves in the circulating system have ring-gasket bonnet joints. It has been necessary to replace all of the original oval rings with octagonal rings. Since this replacement, no leaks have been experienced. No major leaks have occurred in the boiler or circulating system, except a dozen or so of the plugs opposite the screen and orifice assemblies at the lower furnace-wall headers and at the flanges on the heads of the circulating pumps. These, however, were made tight without reducing pressure or capacity. No leaks have occurred on any welded joints and none on the manhole covers or the safety-valve flanges.

Some difficulty was experienced with the labyrinth packing on one of the three circulating pumps, necessitating replacement. During the preliminary operation, it was found necessary to lap in the high-pressure safety valves and the water-column fittings in order to make them tight at full pressure.

Performance

No official tests have been run. Curves Figs. 15 to 17, inclusive, of this Addendum, show anticipated values of the functions plotted, and the points recorded thereon are values observed in normal operation. All readings were taken after several hours of operation at substantially constant load, with readings taken of all values as nearly simultaneously as possible. All temperature readings are by thermocouples, the accuracy of which was carefully checked. Fig. 18 shows the position of the three sets of dampers used in the control of superheat and outlet-gas temperature as established by observation during actual operation.

The values shown in Fig. 18 should be carefully considered when studying the values of superheater-outlet temperature, gas- and air-outlet temperatures from the air heater, draft at the air-heater outlet, and air pressure at the air-heater inlet. A wide range of control is possible through manipulation of these dampers. Referring to values taken at the 425,000-lb rating, it will be observed that the upper economizer-outlet damper, referred to in Fig. 18 as "superheater by-pass" damper, is 100 per cent open, and the lower economizer damper is just over 40 per cent open. To obtain maximum superheat, the openings would be 0.0 per cent and 100 per cent, respectively. Such a setting

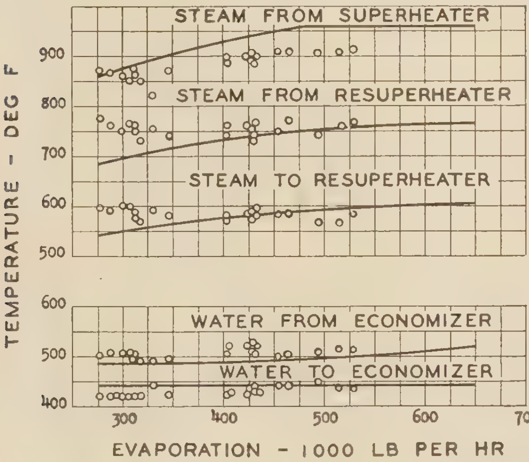

Fig. 15

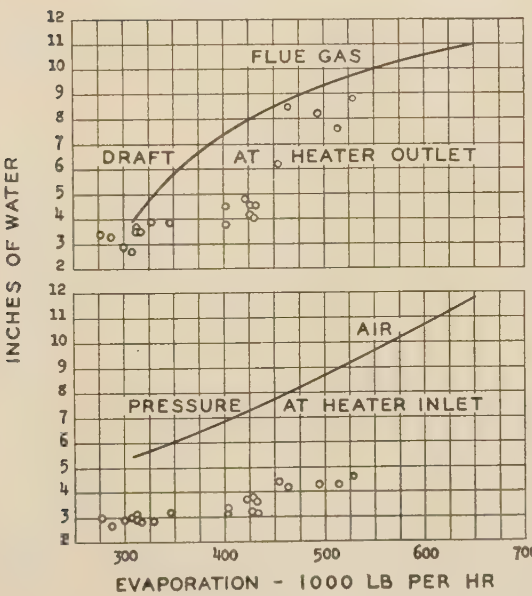

Fig. 17

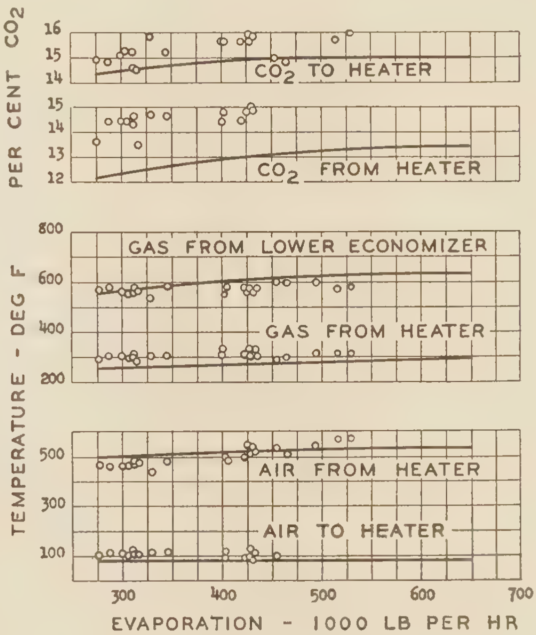

Fig. 16

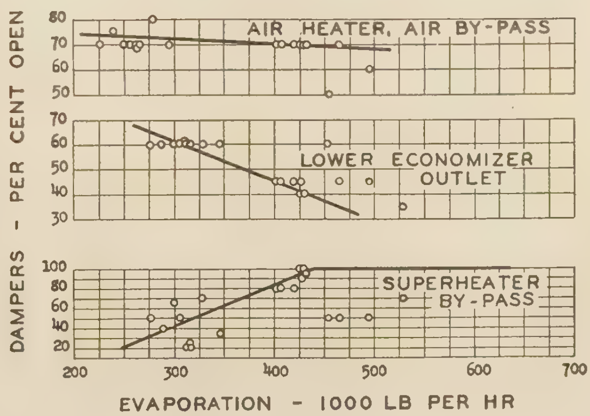

Fig. 18

would result in a temperature substantially above the anticipated curve value and incidentally reveals that full superheat is attainable at ratings substantially below the rating of 480,000 lb per hr, the lowest rating at which full superheat of 960 F was guaranteed. This extreme position of damper setting would also result in a substantial increase in draft at the air-heater outlet. It will also be noted that, at the 425,000-lb rating, the air-heater air-by-pass damper is 70 per cent open. The purpose of this is to maintain the outlet-gas temperature up to approximately 300 F and avoid all possibility of corrosion or deposits at the cold end of the air-heater elements. This results in a reduction of air pressure at the air-heater inlet. A comparison of the gas- and air-temperature ranges between air-heater inlet and outlet indicates that about 20 per cent of the total air is being by-passed.

The air pressure observed at the air-heater inlet is substantially less than indicated on the curves of anticipated performance. It should be stated, however, that, in addition to the influence on this value of by-passing air around the air heater, the air pressure at the burners was predicated on the requirements of the oil-burning equipment. Actual operating experience does not dictate the desirability of utilizing the full available pressure when burning coal. The fact that somewhat more gas is by-passed around the large convection superheater than was allowed for in proportioning the induced-draft fan accounts in some measure for the low value of induced draft as compared with anticipated values shown by the curves.

Many determinations of conductivity of steam samples taken simultaneously at the entrance to the dry drum and from the saturated superheater header have been made. Under conditions of normal water level and with boiler-water concentrations corresponding to conductivity up to 2600 micromhos (about 1300 ppm), the conductivity of a sample at the entrance to the dry drum has, in no instance measured, exceeded 2.5 micromhos with normal values about 2.1, both uncorrected for dissolved gases. The corresponding values in the saturated header have been only slightly less, showing that practically all separation took place in the main steam drum. By deliberately increasing the water level to a point 6 in. above normal, conductivity values at the entrance to the dry drum up to 27 micromhos have been measured with no increase, however, in values measured in the saturated header. Conductivity of samples from the saturated superheater header which had been incompletely degasified showed values under 1 micromho, corrresponding to something under 0.5 ppm.

It seems reasonable to presume that, had external obstructions permitted a more favorable steam take-off from the main drum and the installation of drying screens in this drum, equal values of steam quality would have been obtained without the dry-drum installation.

The heat balance constructed for the 425,000-lb rating, using observed temperature data, CO_2, etc., and based on a West Virginia coal of 18 per cent volatile content, shows the following losses and efficiency:

	Per cent
Dry gas loss	5.20
Hydrogen and moisture loss	3.20
Moisture in air loss	0.14
Carbon loss	0.50
Radiation loss	0.46
Total loss	9.50
Indicated efficiency by difference	90.50

In general, the observed performance reveals that the proportions of all elements of the heat-absorption equipment, and also the capacity of forced- and induced-draft fans, are on the generous side; also that the range of control provided permits regulation of important temperature values to the anticipated performance with some margin to spare. The period of operation has not been long enough to give assurance that all operating difficulties have been overcome but, thus far, troubles which have arisen have not been such as to cause concern for the future. Time remains the essential factor for proving the ultimate merit of the installation, and time will not be hurried.

Discussion

E. G. Bailey.[5] As the operating data presented in this paper are somewhat limited both with respect to time and high ratings, it was my intention to say nothing by way of discussion, until Mr. Moultrop mentioned the 2400-lb-pressure boiler at Twin Branch, operating since April, 1941, with natural circulation, and expressed a wish to hear some comments as to the relative merits of natural- and forced-circulation boilers.

Frankly, I see no advantage in arguing this point based upon data now at hand, because as time goes on the operation of units involving each method of circulation will finalize the answer in facts regardless of opinions or arguments.

Some of us will remember the many discussions and arguments on the pros and cons of burning coal on stokers and in suspension from pulverizers. Time has very largely settled that question especially as it relates to larger units, and I might say even on the smaller units also, the latter staying with stokers and the former going almost entirely to pulverized coal.

I am very glad that the authors have been able to install a large high-pressure forced-circulation boiler in this country, so that we will not have to depend further on relatively small high-pressure units or medium-sized low-pressure units of the forced-circulation type in Europe.

We have made several forced-circulation boilers mostly for restricted headroom and moderate sizes. The shape of the boilers would make it very difficult to design them for effective natural circulation. In the case of a large central-station unit, such as the one at Twin Branch, which has been quite completely described,[6] there has been no difficulty whatever due to circulation, and in fact, we have recently restricted the circulation about 14 per cent so that we would have less water to deal with in separating steam in the upper drum.

J. S. Bennett.[7] Several years ago the London Power Co., Ltd., installed a Lamont forced-circulation boiler in its Deptford West Power Station. This stoker-fired steam generator was designed for a maximum continuous evaporation of 350,000 lb of steam per hr at 375 psig pressure, 850 F total steam temperature, Fig. 19 of this discussion.

Although this unit has not been operated to date at its maximum continuous rated capacity, there have been no circulation problems which have dictated operation at lower steaming rates. The main difficulty is that the gas-temperature drop through the unit is not "steep," and the exhaust temperatures to the chimney are too high. Indications are that the straight-through gas flow results in by-passing an appreciable amount of the heating surface. This fact was substantiated by creating experimentally a slightly positive pressure in the furnace which results in a drop of 25 F in the exhaust gases.

The performance as a whole has been satisfactory, considering the inclusion of certain experimental features.

Some years ago, during the development of the water-cooled stoker, it was found convenient to extend the Lamont forced-

[5] Vice-President, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

[6] "Twin-Branch Extends High-Pressure Economies," by P. Sporn, *Electrical World*, vol. 16, Oct. 18, 1941, pp. 80–82.

[7] Manager of Sales, American Engineering Company, Philadelphia, Pa. Mem. A.S.M.E.

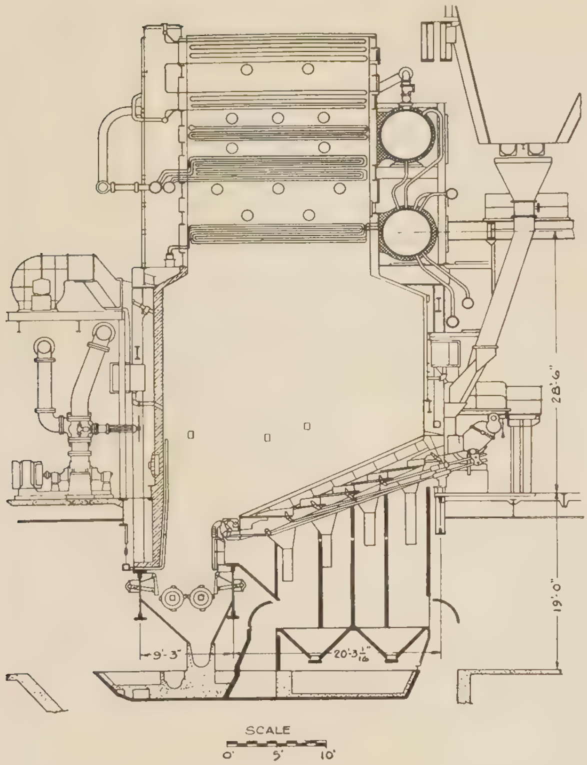

Fig. 19 Lamont Forced-Circulation Boiler at Deptford West Power Station, London

circulation waterwalls to provide water cooling for eight stokers. The first experimental design, Fig. 20, provided for headers at A and B with the water being pumped in at header A and leaving at header B. Several tube failures at point C occurred, presumably because the steam bubbles rose to the high point of the tube and were not swept away by the velocity of the water. This was corrected by reversing the flow of water. In this particular plant, the feedwater conditions were poor and periodically tube failures occurred because solid material would plug up the orifices between the header and the individual tubes. It was found, however, quite practical to make very quick and easy repairs by welding, because of the small size of the tubes.

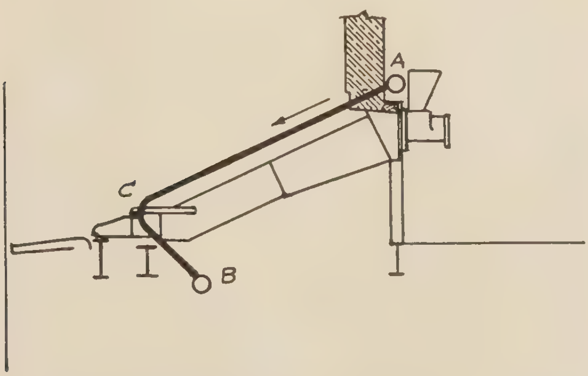

Fig. 20 Experimental Design of Waterwall Extension to Provide Stoker Cooling

W. E. Caldwell.[8] Prudent conservatism characterizes the boiler unit, aside from the application of forced circulation. The heat-release and absorption rates are consistent with conventional practice, although forced circulation appears to offer opportunities for increased heat-release rates and increased heat absorption within the furnace. The advantage of tangential firing in a slagging furnace has not been fully explored and it seems likely that forced circulation may remove one major obstacle from the path of this development.

In large interconnected power systems, economy considerations dictate sustained full-capacity operation for modern high-pressure boilers, which further aggravates common problems. The necessity of cleaning gas passages continuously under such conditions is due, largely, to adoption of 950 F steam, with consequent high gas temperatures and to fuel characteristics, and requires a substantial increase in operating labor. Maintenance of superheater elements, hangers, dampers, baffles, etc., exposed to the high gas temperature, is also a difficult problem. Since it is impossible to design a unit which is self-cleaning for all coals, accessibility for cleaning is of utmost importance Furnace-wall cleaning is usually necessary to keep superheat within range. Experience indicates that bifurcated-tube furnace walls clean more easily than fin tubes, and it seems likely that the boiler under discussion is a further improvement in this direction.

The firing circle appears small, and it would seem advantageous to increase the diameter, thereby utilizing the energy introduced into the furnace to force more heat into the walls, reducing gas temperature and draft loss. The permissible heat input into conventional bare-tube waterwalls is limited by considerations of circulation, as well as by size and thickness of tubes. Experiments have shown that very high rates of heat input are permissible in tubes so long as unidirectional turbulent flow is maintained in the water and steam mixture. With some coals, a large firing circle results in a self-cleaning furnace. Wall wastage, due to chemical attack, may temporarily discourage such considerations, but experience indicates that with conservative burner design and moderate coal velocity this problem is absent. Several low-pressure boilers have been operating many years with the flames scouring the walls most of the time without distress, other than burning away the fins. Experience indicates that trouble starts with burning rates somewhere above 3000 lb of coal per nozzle per hr, and injection velocity as well as nozzle shape are contributory influences.

Fig. 21 of this discussion illustrates three types of corner burners in three different installations, from which certain general deductions may be drawn, based on long experience with direct-firing a wide variety of coals. Burner (*a*) with two vertical coal nozzles, having a firing rate up to 6000 lb per nozzle per hr, causes acute wall-wastage trouble when adjusted to a large firing circle. Burner (*b*), with two coal nozzles almost square, having a firing rate up to 5000 lb per nozzle per hr, causes much less wall attack than the (*a*) type. Burner (*c*), with three horizontal rectangular coal nozzles, having a firing rate up to 3000 lb per nozzle per hr, causes no wall distress, regardless of the diameter of the firing circle.

With burner and combustion conditions arranged to avoid a highly reducing atmosphere associated with wall distress, it becomes possible to apply more heat to the walls, thus reducing cleaning labor requirements by lowering the gas temperature entering the tube banks or superheater.

Information on the following questions by the authors would be of interest to operators:

1 Duration of circulating-pump operation required after extinguishing fire.

[8] Mechanical Plant Engineer, Consolidated Edison Company of New York, Inc., New York, N. Y. Mem. A.S.M.E.

2 Minimum practical submergence required on circulating-pump suction.

3 Effect of smaller water storage or reduced accumulator effect on pressure changes with rapid load swings.

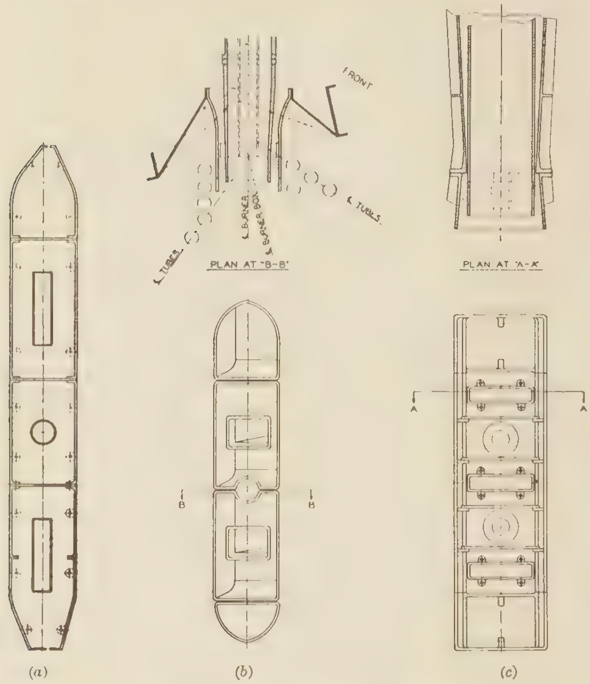

Fig. 21 Types of Burner Outlets

M. K. Drewry.[9] Reliability, as the authors stress, is the principal determining factor of boiler-unit desirability. Whether this new type has inherently high reliability can only be proved by service experience.

Heretofore, circulation passages in boiler units have always been kept large to prevent clogging by scale or other boiler-water solids. Whether all the $^3/_{16}$-in. holes of 20 times the total area of the 0.34-in. orifice will remain open more reliably than the unguarded orifice seems in question.

Thinner tubes are not entirely without demerit. External corrosion, not entirely unknown recently in furnace tubes, weakens small tubes faster than it does larger ones. For instance, 0.15-in. corrosion of a 3-in. tube may simply double its local stress without immediate distress, but equal corrosion of a $1^1/_4$-in. tube can cause prompt rupture.

Whether the industry is ready for acid cleaning of boiler tubes seems in question. That alternate mechanical and acid cleaning is necessary to remove some silica scales is understood. Further, the prevention of those scales is understood to be uncertain.

Francis Hodgkinson.[10] Forced circulation should, it seems, be a means of increasing heat-transfer rates in the evaporating surfaces and hence bring about a reduction of surface.

From the data in the paper, it appears that with two of the circulation pumps operating, and assuming the pumps receive water approximating that of saturation, about 3.6 lb of water are circulated per lb of steam delivered. Forced circulation has been practiced in Europe for some time, where the ratio of water circulation to steam has been greater; it is believed to be 6 to 8 times in the case of Lamont and 10 to 20 times in the case of "Velox" practice.

The writer's discussion concerns the circulating-pump gland principally. From Fig. 7 of the paper, it is assumed the gland comprises three sets of labyrinths, having two lantern chambers connected to feed-pump elements.

In the Loeffler system, as is well known, a circulating pump is also employed, but circulating saturated steam. In this case, the gland was a problem, having to be packed against 1900-psi saturated steam. Following some experiences, among which was that of sealing a labyrinth with oil of high viscosity, the method finally adopted was to employ labyrinths sealed with the superheated steam (about 930 F) delivered by the boiler. By this means, injury by erosion would be less, as compared with saturated steam. From observation of these pumps in operation, the glands are satisfactory.

The energy of a pressure drop with steam is much greater than the same pressure drop with water, but the greater volume and limitation, because of critical velocity (sonic velocity) incidental to a vapor, restricts the weight of flow.

It is to be expected that the weight of flow, in this case, would be about 10 times greater with water in the high-pressure labyrinth than with superheated steam expanding to atmospheric pressure. On the other hand, the thermal loss, whatever its actual value may be, compared with the work the superheated steam could do in the turbine expanding to condenser pressure, would be about 9 times greater than with the water seal, but there would be little fear of erosion.

It seems by no means certain that a bearing lubricated with water from the pump might be substituted for these labyrinths with reduction of overhang of the rotor, of cost, and of space. With ordinary bearing clearances, the flow and the loss could be made negligible. Of course bearing and journal must be made noncorrosive; the water would seem to be clean enough.

Were the writer responsible for the operation of this unit, it would be his purpose to ask funds to set up an experimental bearing alongside the circulating pumps, operating with the same water, with the idea of perfecting such a bearing.

J. C. Hobbs.[11] I hesitate to discuss this paper because my attention is primarily absorbed by the manufacture of chemicals, particularly following an eminent scientist like Mr. Bailey who has given such a clean-cut analysis. Mr. Bailey, as you know, first determines the fundamental facts by means of scientific instruments and with an exact knowledge of the conditions appears to be in the best possible position to determine whether mechanical or natural circulation is best suited for a particular installation. This discussion is concerned with the question whether the pumps shall be mechanically or thermally driven. The latter have much the greater reliability.

I have wondered why Mr. Bailey did not emphasize one of the fundamentals, namely, that all successful natural-circulation boilers depend upon thermal pumps which have none of the three weaknesses of the mechanical-circulation boiler. These were enumerated by the authors in the paper under discussion, namely, shaft leakage, failure of auxiliary power, and pump failure. Mr. Bailey did point out that the pounds of water per pound of steam in many of the natural-circulation boilers is already several times as much as the 3.6 lb of water per lb of steam stated for this forced-circulation installation. The 2200-psi unit with which I am familiar has a water-steam ratio of about 17 lb of water per lb of steam.

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[10] Consulting Engineer, New York, N. Y. Past Vice-President A.S.M.E.

[11] Vice-President in Charge of Manufacturing, Diamond Alkali Company, Painesville, Ohio, and Engineering Consultant on ships of advanced technical design, U. S. Navy. Fellow A.S.M.E.

There are many factors, of course, in the decision between mechanical and natural thermal circulation. In the great majority of cases, it is better to eliminate every possible source or cause of trouble. A large installation which depends for its successful continuous performance on factors of uncertain dependability will of course in time show a poorer performance than one in which all unreliable elements have been eliminated.

The Society is of course very much indebted to the authors of this paper because of the contribution they have made to this new subject.

G. E. Morgan.[12] The writer is impressed with the comparative simplicity and compactness of this interesting forced-circulation-boiler application, which may now be accepted as a standard means of salvaging the investment in existing "low-head" structures in connection with superposed installations. Not only are the steam temperature, pressure, and reheat feature within the established range of topping-unit practice but, perhaps more significant still, the final feedwater temperature at full load, 446 degrees F, appears to fall almost directly on the median curve of generally accepted values.

The obvious advantages inherent in the forced-circulation principle may well extend the present economic range of superposed installations, since former restrictions on higher pressures arising from circulation difficulties may now be relaxed. The circulating pumps are not only free from complications in design, but in addition the power consumption (equivalent to only 0.3 per cent of the steam generated) is strikingly low. However, this is natural, since the circulating pumps are required to effect an increase of but 50 psi over the suction pressure of 2000 psi.

[12] Head Steam Engineer, Tennessee Valley Authority, Knoxville, Tenn. Mem. A.S.M.E.

The authors state that experience indicates some advantage from less slag accumulation on the furnace walls with the relatively small and closely spaced tubes of the forced-circulation design, and the resultant flatness of the interior surfaces of the furnace. With respect to this feature, it would be of interest to the writer to learn from operating experience any indication of a relation between the lesser slag accumulation on the furnace walls and the deposits beyond the furnace (requiring more soot or slag blowing) or emitted as fly ash from the stack. Operating records at Watts Bar point to the existence of some such relationship.

The heat-balance diagram, Fig. 3 of the paper, indicates considerable improvement for the whole station over the heat rate for the older section of the plant and, of course, was taken into consideration in the studies for the new addition. While we appreciate that the paper was written with special reference to the forced-circulation boiler, it would also be of interest to have the authors say something further regarding the economics which led to the selection of 1825 psi pressure for the superposed unit.

The supplement to the paper relative to the operating record indicates no more than the usual "ironing out of wrinkles" for a natural-circulation unit. The section of the supplement devoted to "Performance," gives a heat-balance tabulation for the 425,000-lb rating constructed from the various calculated losses which total 9.5 per cent, with the difference from 100 per cent representing the efficiency of 90.50 per cent. The writer thinks that this construction may have the elements of some error, and an efficiency higher than justified in that no unaccounted-for losses are included. A figure commonly used for the unaccounted-for losses for a comparable unit is $1^1/_2$ per cent and with the modern combined slag-tap and dry-ash removal using sprays, the cooling or evaporation losses from both of which are difficult, if not impossible, to calculate, we think that even this $1^1/_2$ per cent for unaccounted-for losses may be a trifle optimistic.

Mechanical Tests of Cellulose Acetate—III

By W. N. FINDLEY,[1] URBANA, ILL.

Data are presented from additional mechanical tests of specimens taken from the same sheet of cellulose acetate for which tests have been reported in two previous papers by the author.[2,3] The tests were conducted in a laboratory maintained at a constant temperature of 77 F and a relative humidity of 50 per cent: (1) Repeated bending (fatigue) tests were made at different speeds of testing and different ranges of stress, which demonstrate that both of these variables are important. The effect of different shapes of fatigue specimens (circular, square, and rectangular cross sections) on the endurance limit is also discussed. (2) Static compression tests of preconditioned specimens tested at intervals of time up to 10 months show the effect of initial moisture content and time on the yield point and the weight. (3) Static torsion, tension, and compression tests, conducted at the same rate of strain, indicate the effect of rate of strain and type of loading on the strength properties of the material. (4) Static tension tests of a specimen containing a transverse hole show the effect of a stress concentration on the tensile strength.

THIS paper reports additional tests of a cellulose acetate for which tests have been reported in two previous papers.[2,3] The first paper reported static tension tests at a wide range of speeds of testing. It was shown that increasing the speed of testing increased the "static" strength of the acetate up to a certain point. Data showing the effect of stress on the time for fracture under a constant tension load were given, and fatigue tests with a repeated bending-type fatigue machine demonstrated the effect of notches upon the endurance limit, and also the effect of shape of specimen upon the endurance limit. Likewise the effect of stress on the temperature developed in the fatigue specimens was discussed. The second paper reported creep tests at constant temperature for several different stresses and indicated the relationship between stress and rate of creep.

The present paper gives the results of additional tests on the same sheet of cellulose acetate. Static tests are presented to show the effect of initial moisture content on results of compression tests at various periods of time. Tension, compression, and torsion static tests at three different rates of strain provide data on the effect of rate of strain, and on the comparative properties of the material under these three loading conditions. The effect of a transverse hole on the results of the static tension test is also shown. Additional data are presented to show the effect of shape of specimen (circular, square, and rectangular in cross section) on the endurance limit and the effect of the temperature of the specimen on the endurance limit. In addition, the effect of speed of testing on the endurance limit is shown for a range of speeds, from 42 to 2900 rpm as well as the effect of range of stress on the endurance limit. The last two variables mentioned are of particular importance, because material may be subjected to vibration at both very low and very high frequencies, so that a knowledge of its ability to resist vibration at these frequencies is important. Likewise, the range of stress is frequently not a complete reversal during the vibration of a member, so, in order to avoid failure, one should know whether the endurance limit for ranges of stress, other than complete reversal, is the same as for completely reversed cycles of stress.

The purpose of this investigation was (1) to obtain as complete information as possible on the mechanical properties of cellulose acetate and (2) to evaluate the effect of certain variables, such as speed of testing and others just mentioned, upon the results of mechanical tests. This latter type of information is needed in order properly to take account of such variables when setting up specifications for testing, and when designing members to resist loads under different conditions of speed, range of stress, etc.

A study of the properties of cellulose acetate reported in this paper required the control during the tests of a number of variables which are unimportant in the testing of metals under similar conditions. Among these were small changes in temperature and relative humidity, so that a special laboratory maintained at constant temperature, 77 F, and relative humidity, 50 per cent, were required for the tests.

Many of these tests also required the development of special machines and instruments since available equipment was unsuited to tests of this material.

Materials and Specimens

The cellulose acetate for these tests was supplied by the Plastics Division of the Monsanto Chemical Company. The material was a clear, transparent thermoplastic composed of medium-viscosity cellulose acetate of the acetone-soluble type, plasticized with about 26 per cent of phthalate and aromatic phosphate-ester plasticizers. The Monsanto formulation number was 2050 TV.

All specimens used in these tests were cut from the same sheet of cellulose acetate which was used for the tests reported in the two previous papers. The sheet was 0.3 in. thick and was made by the sheeter process, at a molding temperature between 200 and 250 F. The finished sheet contained less than 1 per cent of residual solvent and water and had a Rockwell hardness of about L 40 (at 77 F, 50 per cent relative humidity).

Details of the specimens used are shown in Fig. 1. All specimens were machined with the longitudinal axis parallel to the short side of the original sheet. The specimens were cut from the sheet on a jig saw and then milled or turned, as the case might be, to the shape shown in Fig. 1. The machined edges were then smoothed by hand with No. 0000 emery paper in order to remove all burrs and scratches, leaving the final polishing marks parallel to the axis of the specimen. The compression specimens were not smoothed after machining, because of the fact that slight surface scratches would not affect the results, inasmuch as the material did not fracture under a compressive load. All specimens were conditioned by allowing them to remain in the atmosphere of the testing laboratory for a period of at least two weeks prior to the start of their tests, and they remained in the laboratory continuously thereafter. The laboratory was maintained at a constant temperature of 77 ± 1 F,

[1] Associate in Theoretical and Applied Mechanics, College of Engineering, University of Illinois. Jun. A.S.M.E.

[2] "Mechanical Tests of Cellulose Acetate," by W. N. Findley, Proceedings of the A.S.T.M., vol. 41, 1941, p. 1231.

[3] "Mechanical Tests of Cellulose Acetate—Part II, on Creep," by W. N. Findley, Proceedings of the A.S.T.M., vol. 42, 1942, p. 914.

Contributed by the Rubber and Plastics Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

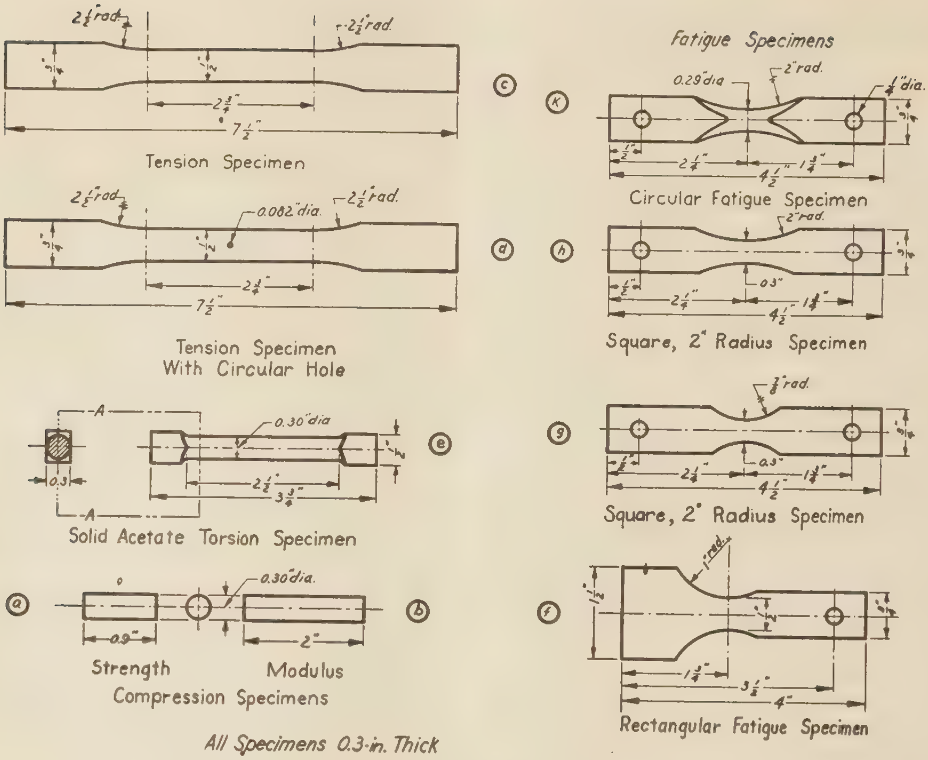

Fig. 1 Specimens

and 50±2 per cent relative humidity continuously throughout the duration of the tests.

Static Tests

Effect of Initial Moisture Content. Fifty specimens were machined, as shown in Fig. 1 (*a*), from the ends of specimens used previously for fatigue tests. This portion of the fatigue specimen had not been damaged in any way by the fatigue tests. These specimens were given the following treatment: One half of them were immersed in water for a period of 48 hr, and then removed to a rack where they remained until tested. The other half were placed in a desiccator over anhydrous calcium chloride for a period of 48 hr and then removed and placed on a rack. Both groups of specimens were then stored at constant temperature and relative humidity in the testing room for the duration of the tests. Two specimens from each group were set aside as control specimens, and the weight and length recorded both before conditioning and at intervals of time after the foregoing treatment. The remaining specimens were tested in compression at intervals for over a one-year period. One specimen was tested immediately after removal from the water bath, and another immediately after removal from the desiccator. The other specimens were tested at intervals in order to give a uniform distribution of data when the test results were plotted against the logarithm of the time elapsed from the time of removal from the conditioning medium. All tests were run at a no-load head speed of 0.06 in. per min. This resulted in a rate of strain of about 0.002 $[\text{min}]^{-1}$.

It was observed that a definite yield point in compression exists for cellulose acetate. This was determined by the fact that the load remained constant for a period of time during the test. Since fracture did not occur during the compression test, the yield point was used to measure the effect of moisture on the strength of the material. In Fig. 2 the compressive yield point for these tests is plotted against the elapsed time. It was observed that a time of about $1^1/_2$ months (1000 hr) was required for the compressive yield point to approach a stable value, and it is significant that the end point for both the moistened specimens and the dried specimens was nearly the same. Some scatter in this data was observed due in part to the fact that the air conditioning of the laboratory was interrupted for 4 or 5 hr on two or three occasions.

In the lower part of Fig. 2 is shown the average specific weight of the two control specimens, plotted against the lapse of time. These control specimens were weighed and measured at the same time that tests of the other specimens were made. The data, shown in Fig. 2, indicate that a time of about one month was required for the specific weight to return to the value which was obtained before the conditioning was started. No explanation is offered for the fact that the specific weight of the initially

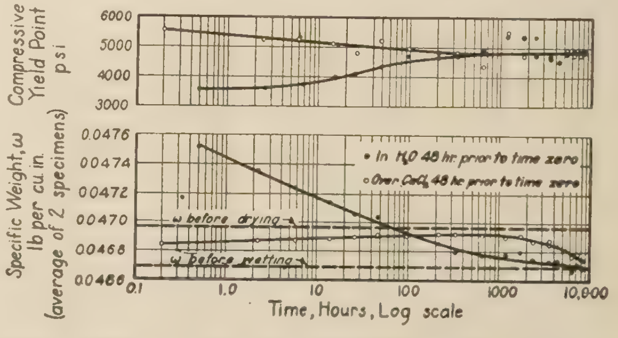

Fig. 2 Effect of Time and Conditioning on Yield Point in Compression; and Specific Weight

dry specimens decreased after 7000 hr, unless perhaps the specimen was losing plasticizer.

No appreciable change in length of the specimens was observed as a result of the treatment described. The wet specimens decreased about 0.4 per cent in length during the testing period, while no measurable change in length of the dried specimens was observed.

Effect of Rate of Strain on Tension, Compression, and Torsion Tests. Static tension, compression, and torsion tests were performed at about the same time on specimens, as shown in Fig. 1(*c*), 1(*b*), 1(*e*). These tests were performed in a universal testing machine, illustrated in Figs. 3 and 4. This machine was used for tension, compression, and torsion tests. It was a single-screw machine of 1500 lb capacity, arranged with pendulum weighing and equipped with a device for semiautographic recording of load-deformation curves. The machine was equipped for variable speed by means of a series of V-belt drives. For the tension tests, Templin wedge grips were used to hold the specimens. The compression tests were performed with a compression tool, Fig. 3. This tool was used in order to avoid the possibility of eccentric loading of the compression specimens. In this instrument, the specimen *A*, Fig. 3, was compressed between the upper platen *B* and the cylinder *C*. The cylinder was guided in the yoke *D*, so that the face of the cylinder was always parallel to the upper platen. Thus, if precautions are taken to machine the specimen ends parallel, and center the specimens on the cylinder, the amount of eccentric loading should be neglible.

These compression tests were made on specimens 2 in. long and about 0.3 in. diam, so the l/r ratio[4] was about 27. This length of specimen was necessary in order to accommodate the compressometer. The compressometer had a gage length of 1 in. and consisted of a 1 to 1 lever to which was attached a 1-thousandth dial. The instrument was attached to the specimen by means of pointed screws on opposite sides of the specimens, so that the strain measured was the average strain in the specimen. Compression tests performed on the 2-in. specimen were used to determine the modulus of elasticity and shape of the stress-strain diagram in compression. A still longer specimen would be desirable to increase the accuracy of strain measurement. The effect of buckling would, however, become more serious with longer specimens. In order to obtain the values of compressive yield strength, a shorter specimen, about 0.9 in. in length, was used as discussed in the previous article. The l/r ratio[4] for the short specimen was about 12. The yield point observed with the short specimens was 4900 psi at a rate of strain of about 0.002 $[\text{min}]^{-1}$.

[4] The ratio l/r refers to the ratio of the length of the specimen to the radius of gyration of the cross section of the specimen.

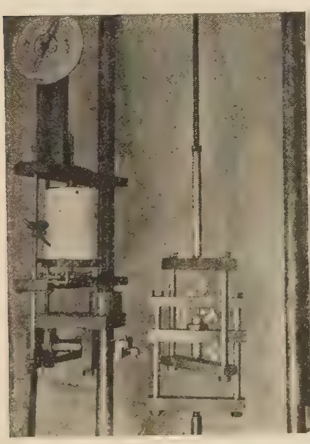

Fig. 3 Testing Machine With Compression Tool

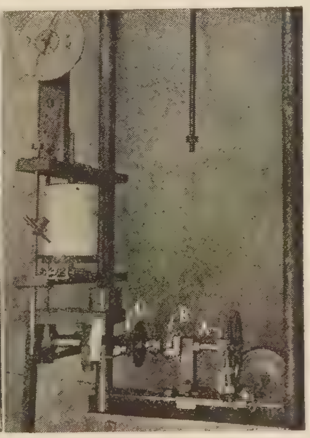

Fig. 4 Torsion Testing Machine

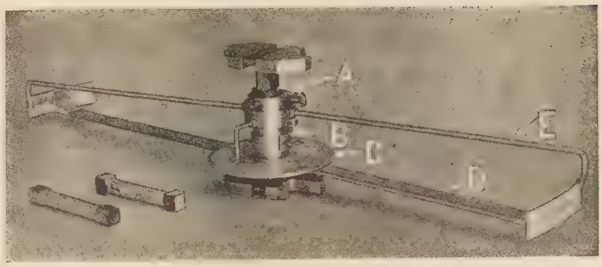

Fig. 5 Detrusion Gage

It was necessary to design and build special apparatus for torsion testing of plastics because machines of low capacity were not available. The apparatus used is shown in Fig. 4. The pendulum-weighing system of the tension-testing machine was used as the torque-measuring device for the torsion machine. This was accomplished by attaching to the tension machine a twisting head *A*, Fig. 4, driven by a double worm drive. A special chuck *B* was attached to the shaft of this twisting head, and another chuck *C* to the axis of the pendulum *D*. These chucks were designed to apply a torque to the specimen with little danger of bending the specimen at the same time. This was accomplished by mounting the specimen on centers and applying the torque as a couple by means of adjustable screws.

The gage used for measuring the shearing strain is shown in Fig. 5. It was designed to accommodate materials whose ultimate shearing strain was relatively small, and also materials which might twist 2 or 3 revolutions in a length of 2 in. The instrument consisted of two rings *A*, Fig. 5, which were slipped over the specimen and fastened to it by three adjusting screws in each ring. A gage length of 2 in. was obtained by the use of a removable spacer *B*. To one of the rings was fastened a circular scale *C*, for measuring large angles of twist. Two 10-in. arms *D* fastened to the same ring carried scales on the end which were used in measuring small shearing strains. Adjustable pointers *E* were attached to the other ring in such a way as to indicate the readings on their respective scales. Unfortunately, material available for these tests allowed machining of torsion specimens only 0.3 in. in diam, so that some bending of the specimen resulted from the weight of the detrusion gage. However, the effect of this bending probably did not seriously affect the results of the test.

Torsion tests differ from tension and compression tests in two important respects; in respect to the state of stress developed, and in respect to the stress gradient. The state of stress may be defined in terms of the ratio $\frac{\sigma_{max}}{\tau_{max}}$, i.e., the ratio of the maximum tensile stress to the maximum shearing stress at a point in the stressed member. In a tension member this ratio is 2, while in a torsion member it is 1. Thus it may be that some materials will behave much differently in the two tests. The stress gradient is a measure of the distribution of stress over the cross section of a member. It has a value of 0 in a tension member but can never be zero in a torsion member. The magnitude of the stress gradient in a torsion member depends upon the stress and the size of the member. The measured strength of a material may be influenced by the stress gradient so that different results may be obtained from the tension test than from the torsion test.

Results of Tension Tests. The tension tests were performed at three different head speeds ranging from 0.021 to 0.32 in. per

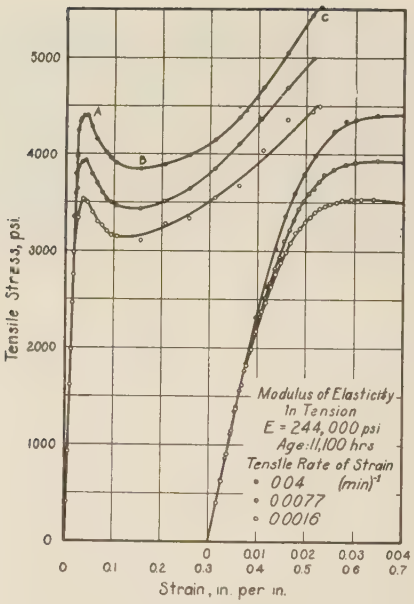

Fig. 6 Static Stress Versus Strain in Tension

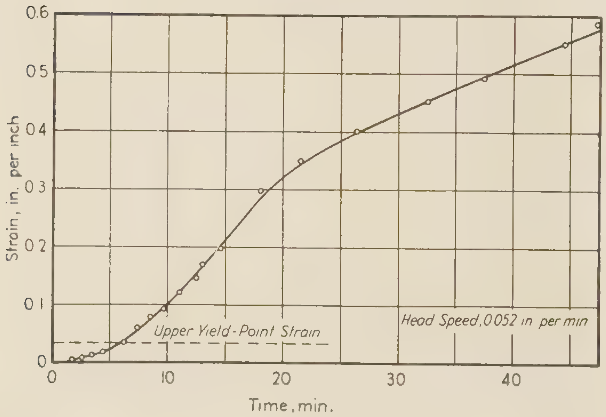

Fig. 7 Strain Versus Time in Tension

min. During the tests, readings of load, deformation, and time were taken. From these readings, the values of stress and strain were computed, using the original cross-sectional area for computation of stress. Fig. 6 shows the tensile stress plotted against tensile strain for these three tests. It is evident that the higher rates of strain produced higher values of stress in the region beyond the proportional limit. This was in accordance with observations previously reported.[2]

It was apparent from the diagrams plotted to an enlarged strain scale that the modulus of elasticity was the same for all three rates of strain. In this paper, the modulus of elasticity is designated as the ratio of stress to strain at the initial portion of the curve and is measured as the slope of the initial straight-line portion of the curve. The value obtained in the tension test was 244,000 psi. The upper yield point was designated as the stress corresponding to position A, Fig. 6. The upper yield point varied from about 3500 psi at a rate of strain of 0.0016 $[\text{min}]^{-1}$ to 4400 psi for a rate of strain of 0.04 $[\text{min}]^{-1}$. The lower yield point B, Fig. 6, ranged from 3200 to 3800 psi and the fracture stress ranged from 4500 to 5500 psi for the same rates of strain as given previously.

The rate of strain for these tests was determined from strain-versus-time curves, such as shown in Fig. 7, and was computed from the slope of the strain-time curve in the portion of the curve corresponding to the linear portion of the stress-strain curve. It was noticed that the rate of strain was not constant but consisted of three fairly distinct but separate straight lines. This change in the rate of strain during a test was due in part to the elasticity of the testing machine.[2]

Results of Compression Tests. Compression tests were run at head speeds from 0.008 to 0.13 in. per min. During each test, readings of load, deformation, and time were taken. From these data, the stress and strain were computed using, as before, the original cross-sectional area in computing the stress. The stress was plotted against the strain in Fig. 8, for three compression tests at rates of strain approximately the same as those used in the tension test. It was observed in these tests that the modulus of elasticity, as previously defined, was not the same for all rates of strain but increased from 204,000 psi at the lowest rate of strain to a value of 278,000 psi at the higher rate of strain, which was about 10 times as fast as the first rate of strain. The yield point observed in these tests increased with the rate of strain, from a value of 3400 to 4300 psi. The rate of strain for the compression tests was determined from strain-time diagrams, similar to that shown in Fig. 7, and the rate of strain was determined in the same way. The strain-time diagrams for compression tests were similar to those for tension, except that the test data were not obtained much beyond the yield point.

Results of Torsion Tests. Torsion tests were performed at three different head speeds, ranging from 0.017 to 0.33 rpm. During these tests, simultaneous readings of torque, angle of twist, and time were taken. The shearing stress at the surface of the cylindrical specimen was computed from the equation $\tau = \frac{Tc}{J}$ and the shearing strain was computed from the relationship $\gamma = \frac{c\theta}{l}$. It is recognized that the equation used for τ does not give the actual stress, except for stresses below the propor-

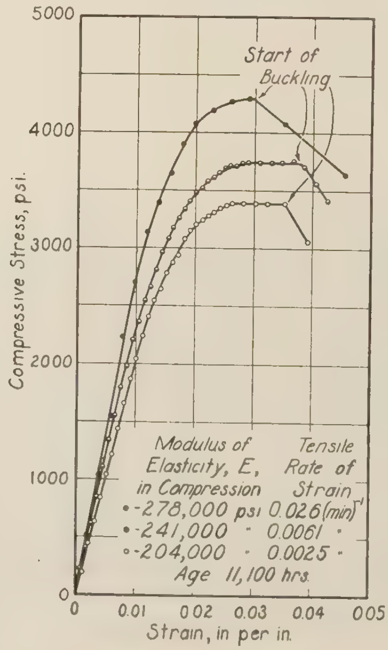

Fig. 8 Static Stress Versus Strain in Compression

tional limit. However, for comparison of the relative strength of the material under different conditions, the equation is sufficiently exact.

The shearing stress, as just discussed, plotted against the shearing strain is shown in Fig. 9, for three different rates of strain. The shearing modulus of elasticity was found to be the same for all three rates of strain and had a value of 78,300 psi. The values of shearing stress increased with the rate of strain, just as observed in the tension and compression tests. The upper yield point A, Fig. 9, increased from about 2600 to 3300 psi, with increase in rate of strain; the lower yield point increased from about 2500 to 3000 psi; and the torsional modulus of rupture increased from 4300 to 4900 psi.

The rates of strain were determined from strain-time diagrams, as shown in Fig. 10, and were computed from the slope of the curve which obtained during the linear portion of the stress-strain curve. In order to compare values of shearing strength with values of tensile and compressive strength, equivalent rates of strain should be used for all tests. To accomplish this, the rate of tensile strain was kept the same for all three types of tests. The maximum tensile stress occurring in a torsion member is equal to the maximum shearing stress, so that the tensile rate of strain can be computed from the shearing rate of strain by the relation $\frac{\dot{\epsilon}}{t} = \frac{\dot{\gamma}}{t} \cdot \frac{G}{E}$, where $\frac{\dot{\epsilon}}{t}$ is the tensile rate of strain; $\frac{\dot{\gamma}}{t}$ is the shearing rate of strain; and $\frac{G}{E}$ is the ratio of shearing modulus to tensile modulus. It was decided to use equal tensile rates of strain rather than equal shear rates of strain, because the material fractured along planes of maximum tensile stress under the conditions of loading studied.

Comparison of Results of Tension, Compression, and Torsion Tests. A comparison of the results of the three different tests shows that the upper yield point in tension and the yield point in compression (long specimen) were approximately the same. However, the yield point obtained with the short compression specimen was about 40 per cent higher than the upper yield point in tension. The upper yield point in torsion was about 75 per cent of that for tension. In the compression test, no difference was observed between upper and lower yield points. The difference between upper and lower yield points in torsion was much less than that in tension. The total strain required to rupture a specimen in torsion was 50 per cent greater than that required to rupture a tensile member, whereas for compression, rupture did not occur at all for very large strains.

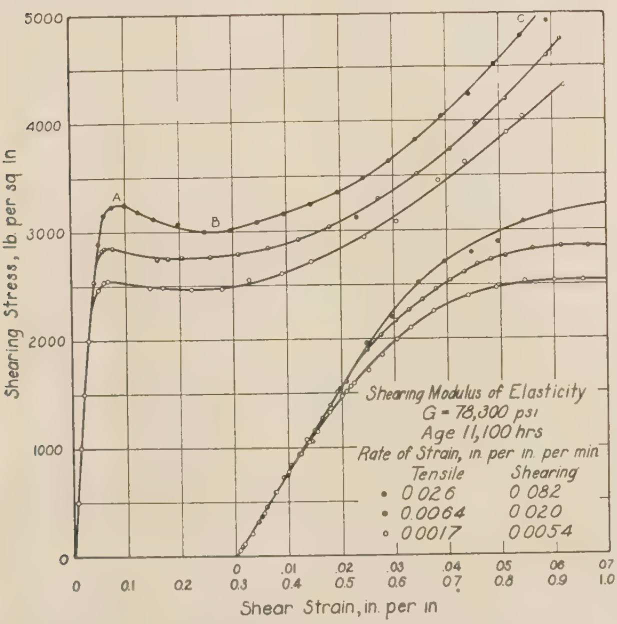

FIG. 9 STATIC SHEARING STRESS VERSUS SHEARING STRAIN IN TORSION

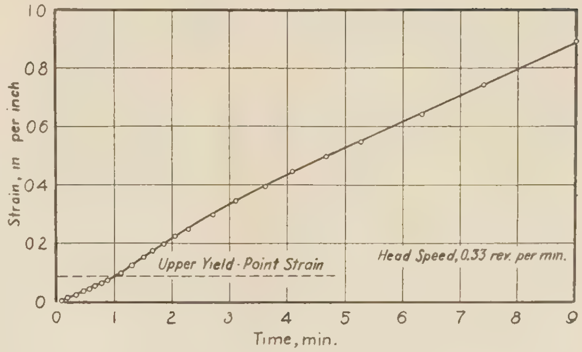

FIG. 10 SHEARING STRAIN VERSUS TIME IN TORSION

A considerable ductility of material was shown in all tests, as evidenced by maximum strains of the order of 40 to 50 per cent in tension. However, the fractures were those characteristic of brittle material, as usually considered from the standpoint of metals; that is, fracture occurred as a result of separation along the plane of maximum tensile stress, a plane perpendicular to the axis of the tensile specimen, and a 45-deg helix in the torsion specimen; and there was no evidence of local "necking-down" in the tension specimen. The modulus of elasticity in tension was about equal to the average modulus of elasticity observed in the compression tests, and the modulus of elasticity in shear was about $^1/_3$ of that of the tensile modulus.

Poisson's Ratio. No tests were made to determine Poisson's ratio μ by direct measurement, and the values of modulus of elasticity in tension and in shear were not accurate enough to calculate Poisson's ratio from elastic theory with good precision. However, the following results were obtained: The elastic theory gives the relation

$$\mu = \left(\frac{E}{2G} - 1\right)$$

where E is the modulus of elasticity in tension (or compression) and G is the modulus of elasticity in shear. The value of μ was about 0.53 to 0.56, depending upon whether the modulus E was obtained from the compression or the tension test. This value is slightly larger than the theoretical maximum value $^1/_2$.

Aging. A comparison of the results of the tension tests reported herein with the tests reported in the first paper,[2] for the same material, shows that the strength of the material has increased with age. The age of specimens at the time of tests reported in the first paper was about 2000 hr (measured from the start of the investigation), and the age at the time of the tension tests reported in this paper was about 11,000 hr. During this time interval of 9000 hr, the upper and lower yield points increased about 4 per cent at a rate of strain of 0.0016 $[\text{min}]^{-1}$ and about 8 per cent at a rate of strain of 0.04 $[\text{min}]^{-1}$. The fracture stress increased about 12 per cent for both speeds, and the strain at fracture increased about 4 per cent. An increase in the modulus of elasticity of about 15 per cent was also observed.

Effect of a Transverse Hole on Tension Test. Tensile specimens containing a circular transverse hole, 0.082 in. in diam, as shown in Fig. 1(d), were tested in tension to determine whether the

tensile properties of the material would be affected by the stress concentration or the stress gradient resulting from the hole.

Specimens were tested at three different head speeds which resulted in rates of strain (measured over a 2-in. gage length) equal to the rates of strain used in the tension tests described. Readings of load, deformation, and time were taken throughout the tests and, from these data, the rate of strain in a 2-in. gage length and the average stress at fracture were determined. The stress-strain diagram for the specimens with the hole differed from that for the solid specimens in that no yield point was observed. The load increased gradually to a maximum value at which fracture took place. The total extension of specimens with a hole (as measured over a 2-in. gage length) was only about 8 per cent of the elongation of the solid specimens. The ultimate strength was computed by dividing the maximum load by the net cross-sectional area at the hole. At a rate of strain of 0.04 $[\min]^{-1}$, the ultimate strength was 5090 psi. At a rate of strain of 0.0077 $[\min]^{-1}$, the ultimate strength was 4350 psi. At a rate of strain of 0.0016 $[\min]^{-1}$, the ultimate strength was 3920 psi.

As in the case of tests reported in the previous paper,[2] the effect of increasing the rate of strain was to increase the ultimate strength. The corresponding values of ultimate strength for unnotched specimens were 5520 psi, 5000 psi, and 4490 psi, respectively. Comparing these values with the values for the specimens containing holes, it is evident that the hole resulted in a decrease in ultimate strength of 8 to 13 per cent. The cause of this decrease may possibly be explained as follows:

It was shown in the previous paper[2] that the fracture in a tension test occurred on a plane at right angles to the axis of the specimen. Thus, failure resulted from separation of the material under a tensile stress, starting usually by a small fissure at the surface of the material. In tests of the specimen with the transverse hole, it was observed that all of the plastic deformation occurred in two wedge-shaped areas starting at the hole and spreading fanwise to the sides of the specimen. Under this condition, the deformation was approximately uniformly distributed over the cross section of the solid portion of the specimen, so that the total axial strain adjacent to the hole was much greater than that at the sides of the specimen, where the amount of plastically deformed material was the greatest. Thus the stress distribution was not uniform during plastic extension at the cross section containing the hole, i.e., the stress gradient was not zero. Stresses were much higher near the hole than at the sides. Since failure occurred by a crack starting at the hole and spreading rapidly when once formed, it might be concluded that the average stress at the time a crack started in the specimen with the hole was less than the average stress which caused a crack to start in the unnotched specimen. Thus, the ultimate strength of the notched specimen would be less than the ultimate strength of an unnotched specimen.

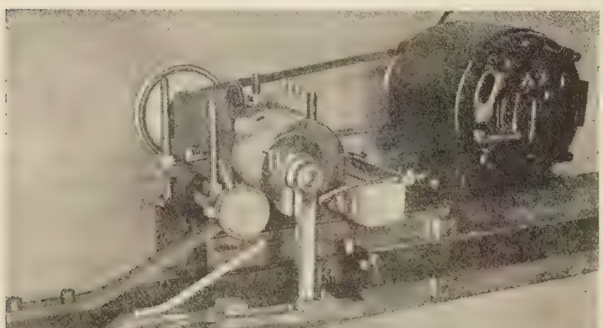

Fig. 11 Fatigue Machine

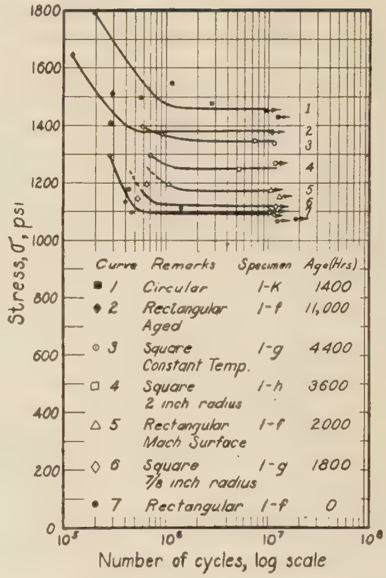

Fig. 12 Stress Versus Cycles ($\sigma - N$) Diagrams Showing Effect of Shape of Specimen

Fatigue Tests

Fatigue tests were conducted on repeated-constant-deflection-type fatigue machines. These machines were equipped with a V-belt drive to provide variable speed, as shown in Fig. 11. In this type of machine, the specimen *A* was repeatedly bent back and forth as a cantilever beam by the variable eccentric *B*. Both horizontal and vertical adjustment of the relative position of the spindle of the machine and the specimen vise was provided to allow a variety of different tests.

The method of conducting tests was as discussed in A.S.T.M. Designation D 671-42T.[5] In all cases, the stress in the specimen was computed from the equation $\sigma = \frac{Mc}{I}$. The bending moment M was obtained for most of the tests by calibration of the specimen as a beam by means of dead weights. The deflection of the specimen under load was measured by means of a dial gage. In a few of the tests, a dynamometer *C*, Fig. 11, was used for measuring the bending moment. The number of cycles to which the specimen was subjected was recorded on a counter *D*, Fig. 11, and a toggle switch was arranged to stop the machine when the specimen fractured. Thus, for each specimen placed in the machine, the stress corresponding to the deflection of the specimen during the test was calculated from the bending moment measured while the machine was at rest, and the number of cycles for fracture was obtained. These data were then plotted with stress as ordinates, and number of cycles as abscissa using semilogarithmic plotting. For the cellulose acetate a well-defined endurance limit was found.

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[5] "A.S.T.M. Tentative Method of Repeated Flexural Stress (Fatigue) Test of Plastics," A.S.T.M. Standards, Part III, 1943, p. 1251. This method was prepared by Section C on fatigue and repeated impact tests of plastics, of which the author was chairman and was based, in part, on experience gained by tests reported herein.

Effect of Shape and Age of Specimen. In Fig. 12, σ—N (stress versus cycles) diagrams are plotted for tests of specimens having several shapes: curve 1, a circular cross-section specimen shown in Fig. 1(k); curve 7, a rectangular specimen, Fig. 1(f); curve 6, a square specimen, Fig. 1(g) of $^7/_8$ in. radius; and curve 4, a square cross section of 2 in. radius. The endurance limit for these tests is shown in the third column of Table 1.

It was found by retests of the same material after a lapse of time of 1 year and 3 months that the endurance limit increased with the age of the material. The σ—N diagram for specimens aged 11,000 hr is shown in curve 2, Fig. 12. Comparing this endurance limit, 1380 psi, with the endurance limit, 1100 psi, for the same shape specimen tested 11,000 hr earlier, it was found that the endurance limit of the material had increased about 25 per cent in 11,000 hr (about 15 months).

In order to compare data obtained at different periods of time, the endurance limit for the several shapes tested was adjusted approximately to the value which was obtained at an age of 11,000 hr (measured from the time of the original test). The adjustment was made on the assumption that the change of endurance limit with time was nearly a linear function during the time interval considered. The age as just defined for each set of tests is shown in column 4 of Table 1, and the endurance limit adjusted to an age of 11,000 hr is shown in column 5. Comparing different shapes of specimens by means of column 5, it was found that the square specimen with $^7/_8$-in. radius had nearly the same endurance limit as the rectangular. The square specimen with 2 in. radius had a slightly higher endurance limit, an increase of about 4 per cent, whereas the circular cross-section specimen had an endurance limit markedly higher than either of the other two shapes, showing an increase of about 27 per cent over the endurance limit for the rectangular specimen. The reason for this is not known. A similar effect has been observed in some metals and not in others. It may be associated, in part, with the quantity of material subjected to high stress, compared to the quantity of material immediately adjacent to high-stress material which is available to receive additional stress, resulting from the redistribution of stress after a minute crack has formed.

TABLE 1 ENDURANCE LIMITS FOR DIFFERENT SHAPED SPECIMENS

Curve no.	Specimen	End limit (as obtained), psi	Age, hr	End limit at age,[a] 11000 hr, psi
1	Circular, Fig. 1(k)	1450	1400	1760
2	Rect. aged, Fig. 1(f)	1380	11000	1380
3	Const. temp, Fig. 1(g)	1350	4400	1540
4	Square 2-in. rad, Fig. 1(h)	1250	3600	1440
5	Machined, Fig. 1(f)	1175	2000	1410
6	Square $^7/_8$-in. rad, Fig. 1(g)	1120	1800	1350
7	Rectangular, Fig. 1(f)	1100	0	1380

[a] These values were obtained by correcting all endurance limits to the value which would be found if tested at an age of 11,000 hr. The correction was based on an assumption that the endurance limit increased linearly with time. The rate of increase was found from curves 2 and 7.

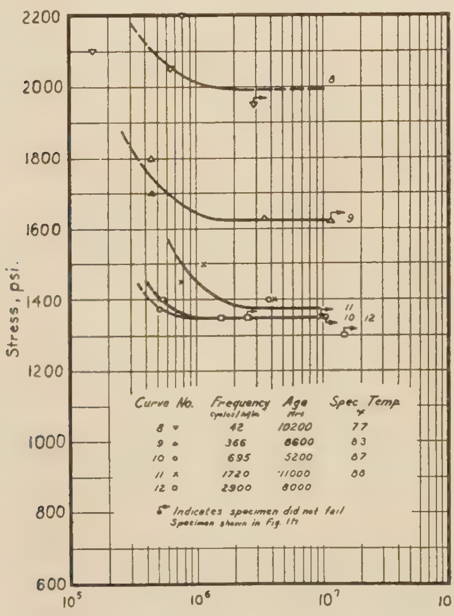

FIG. 13 STRESS VERSUS CYCLES (σ — N) DIAGRAMS FOR SEVERAL TESTING FREQUENCIES

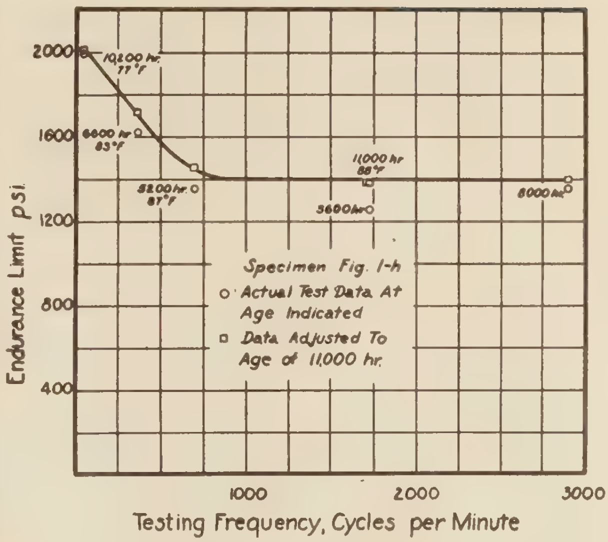

FIG. 14 EFFECT OF "SPEED" OF TESTING ON ENDURANCE LIMIT

Effect of Molded Surface. In order to determine the effect of the molded surface on the endurance limit of cellulose acetate, about 0.025 in. was removed from both surfaces of rectangular specimens, Fig. 1(f). The σ—N diagram obtained for these specimens is shown as curve 5, Fig. 12. It was found that the endurance limit, adjusted to an age of 11,000 hr, was about 2 per cent higher for the machined surface than for the molded surface (see Table 1, column 5). Apparently the molded surface has very little effect on the fatigue strength.

Effect of Specimen Temperature. In the first paper on cellulose acetate,[2] it was shown that the temperature of the specimen during a fatigue test was a few degrees above room temperature. It was also shown that a blast of air from a low-pressure high-velocity fan would reduce the specimen temperature nearly to room temperature. An endurance limit was obtained with specimens tested under an air blast of about 2500 fpm, under which conditions the temperature of the specimen as measured by a thermocouple taped to the top of the specimen was less than 1 deg above room temperature. The σ—N diagram for these tests is shown as curve 3, Fig. 12, for a square specimen, Fig. 1(g). The endurance limit for this test adjusted to age at 11,000 hr was about 14 per cent higher than the corresponding endurance limit obtained in still air (see Table 1, column 5).

Effect of Speed of Testing. Five different endurance limits

were obtained at various speeds of testing, ranging from a frequency of 42 cycles per min to a frequency of 2900 cycles per min. The σ—N diagrams for these tests are shown in Fig. 13. It will be appreciated that tests at 42 cycles per min require an extremely long time to complete. The endurance limit of curve 8 is not quite as well defined as for the other curves in this series.

Fig. 14 shows the effect of frequency on the endurance limit. In this curve, the endurance limit is plotted against the test frequency. The circles represent the actual value of the endurance limit as obtained. The square-plotted points represent the data corrected for the effect of aging on the basis of linear variation of endurance limit with time. For this series of tests, the age effect was determined from a different set of tests than in the former. Curve 11, Fig. 13, having an age of 8000 hr corresponds with curve 4, Fig. 12, at an age of 3600 hr.

The effect of age, as obtained from this set of tests, does not agree exactly with the effect of age in the previous tests. This may be due in part to the fact that humidity control of the laboratory was interrupted for a short time during the time interval covered by the first tests, or the difference may indicate that the effect of aging is not a linear function of time. The square-plotted points, shown in Fig. 14, represent the data adjusted to an age of 11,000 hr. It was found that the endurance limit markedly decreased with increasing testing frequency up to a frequency of about 750 cycles per min. From there on to a frequency of 2900 cycles per min (the extent of tests reported) the endurance limit remained substantially constant. Thus for low frequency the magnitude of the frequency has an appreciable effect upon the endurance limit, but above 750 cycles per min, the endurance limit is unaffected by change in frequency. The effect may in part be due to relaxation of stress during each cycle as a result of creep at the low frequencies of testing.

Effect of Range of Stress. All of the endurance limits mentioned were obtained for completely reversed stress cycles. It was known that the resistance to repeated loading of some materials was influenced by the range of stress, so that it seemed desirable to study the effect of this variable. In this paper, range of stress will be defined in terms of two quantities. A cycle of stress may be resolved into two components, a constant or "mean" value of bending stress σ_m, and an "alternating" stress σ_a, which is superimposed on the mean stress.

For the previous set of tests, the mean stress was zero, and the endurance limit was the magnitude of the maximum alternating

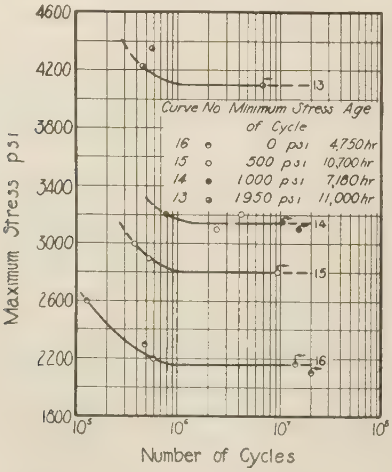

Fig. 15 Stress Versus Cycles (σ—N) Diagrams for Several Different Ranges of Stress

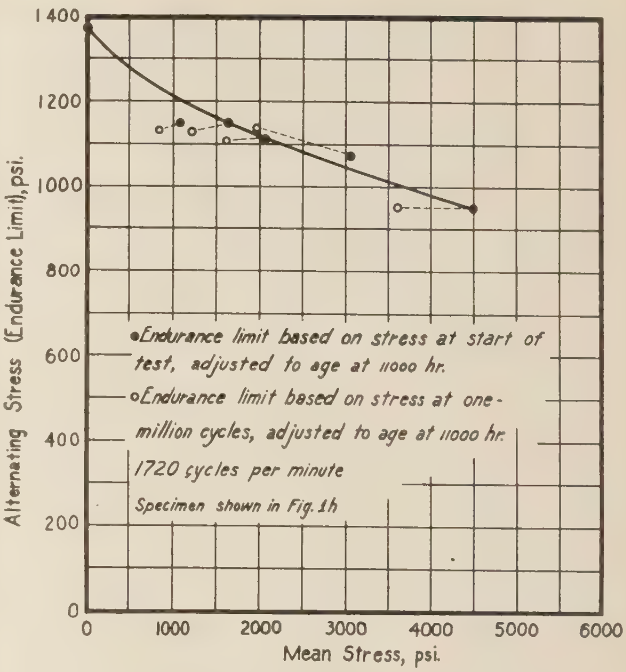

Fig. 16 Effect of Mean Stress on Endurance Limit for Different Ranges of Stress

stress which would not cause fracture after a large number of cycles. When the mean stress is not zero, the corresponding value of maximum alternating stress σ_a, which will not cause fracture, is here defined as the endurance limit of the material for that value of the mean stress. In Fig. 15, the maximum stress of the stress cycle for various values of the minimum stress of the cycle is plotted against the number of cycles required to cause fracture. The maximum stress was found to increase with increase in mean stress. However, when the endurance limit was plotted against the mean stress for different ranges of stress, the diagram, Fig. 16, was obtained. Fig. 16 shows that the endurance limit decreases with an increase in the mean stress of the cycle, when the mean stress is a tension stress. The ranges of stress shown are for tension mean stress, since it was found that the fatigue fracture originated on the tension side of the specimen.

It was found that, during the test of a specimen, the mean stress of the cycle decreased very rapidly during the first million cycles of the test (at a speed of 1750 cycles per min). Thereafter, the mean stress decreased very slightly with increase in number of cycles. This decrease in mean stress was due to relaxation as a result of creep under the action of the mean stress. In order to evaluate the effect of this relaxation on the diagram, Fig. 16, the value of alternating stress and mean stress, which obtained at 1,000,000 cycles, was plotted as open circles in Fig. 16. It was found that the relaxation of stress did not materially affect the endurance-limit-versus-mean-stress diagram. It should be mentioned that data, plotted in Fig. 16, were adjusted to an age of 11,000 hr.

Summary and Conclusions

The following conclusions may be drawn from tests of cellulose acetate, conducted in a room maintained at a constant temperature of 77 F and constant relative humidity of 50 per cent.

1 A time of about $1^1/_2$ months was required for the compressive yield point and the weight to approach equilibrium in an

atmosphere of constant temperature and relative humidity.

2 The yield point and fracture stress were found to increase with increasing rate of strain for tests in tension, compression, and torsion.

3 The modulus of elasticity in tension and torsion was found to be independent of the rate of strain for the values tested, but the modulus in compression was found to increase with increasing rate of strain for loading in compression. The following values of modulus were found: tension, E = 244,000 psi; torsion, G = 78,000 psi; compression, E varied from 204,000 to 278,000 psi.

4 At a tensile rate of strain of 0.04 in. per in. per min, the values of upper yield point were: tension 4400 psi; torsion 3300 psi; compression (long specimen) 4300 psi. The fracture stress was: tension 5500 psi; torsion (modulus of rupture) 4900 psi.

5 Aging of the material at constant temperature and relative humidity was found to increase the tensile strength about 4 to 12 per cent, and the modulus of elasticity in tension about 15 per cent during a time of about 1 year.

6 A transverse hole in a static-tension specimen was found to reduce the ultimate elongation in 2 in. to about 8 per cent of the elongation in a solid specimen. The ultimate strength (based on the net area) was found to be about 10 per cent less for a specimen with a hole than for the solid specimen.

7 Fatigue tests indicated that the endurance limit, obtained by computing stress from the flexure formula, varied with the shape of the specimen.

8 Aging of the material was found to increase the endurance limit about 25 per cent in a period of 15 months.

9 The endurance limit of the material was found to be decreased by the rise in temperature of the specimen resulting from internal friction.

10 The speed of testing was found to affect the endurance limit of the material. The endurance limit decreased with increasing testing frequency up to 750 cycles per min. From there on to 2900 cycles per min, the endurance limit was constant.

11 A change in the range of stress was found to cause a decrease in the endurance limit (defined in terms of the alternating component of the stress cycle), as the mean stress of the cycle becomes larger for values of mean stress in tension.

Acknowledgments

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The Holding Power and Hydraulic Tightness of Expanded Tube Joints: Analysis of the Stress and Deformation

By J. N. GOODIER[1] and G. J. SCHOESSOW[2]

The expanding process used for making the tubes of boilers tight and in other applications is idealized to form a problem in the theory of plasticity. This problem is solved in order to find out how far the factors taken into account in this theory are adequate to explain the results obtained in tests, mainly those reported in the companion paper of Grimison and Lee (19).[3] The pressure left between the tube and plate or seat, which gives it tightness and contributes to its strength, is the principal object of calculation. Its variations with the yield stresses of tube and plate material, with degree of expanding, and with the thickness of the plate, are obtained in graphical form. Numerical comparison with tests is made for six joints, with fair agreement in five, the theoretical values being within 12 per cent. All are on the low side, and this is to be expected from the fact that the theory disregards strain-hardening, stress in the direction of the tube axis, and possible differences between expanding by a three-roller tool and expanding by uniform pressure within the tube.

Introduction

If the end of a tube is inserted loosely into a hole in a plate (Fig. 1) and is then plastically expanded by internal pressure, removal of the pressure leaves the tube bearing tightly against the plate, and the result is called an expanded tube joint. This process is used in the construction of water-tube boilers. The joint must not only be tight against leakage but must have structural strength, to contribute to the support of dead load and to resist the stresses and strains of temperature changes. The tightness against leakage will evidently depend upon the surface conditions of the tube end and the plate hole, the uniformity of the expanding process, and the residual radial pressure left between tube and plate. The structural resistance to tension, torsion, and bending depends upon the friction in the joint. This in turn depends upon the surfaces and upon the residual radial pressure. The discussion of this pressure is the principal object of the present paper. One form of expanding tool, in which three rolls are pressed against the inner tube surface and travel around it, is illustrated in Fig. 2. Part 5 is a flaring roll to turn over the end of the tube.

It will be seen that the expanded joint resembles the shrink fit, or rather the expansion fit, where the slightly oversize tube would be chilled, inserted in the hole, and allowed to expand against the plate by warming up. In the expanded joint, the "oversize" of the tube is produced by the internal forces after the tube has been inserted. Whereas the shrink or expansion fit is normally such

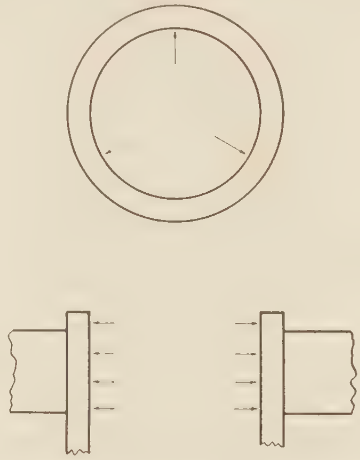

Fig. 1 Expanding Tube Plastically by Internal Pressure

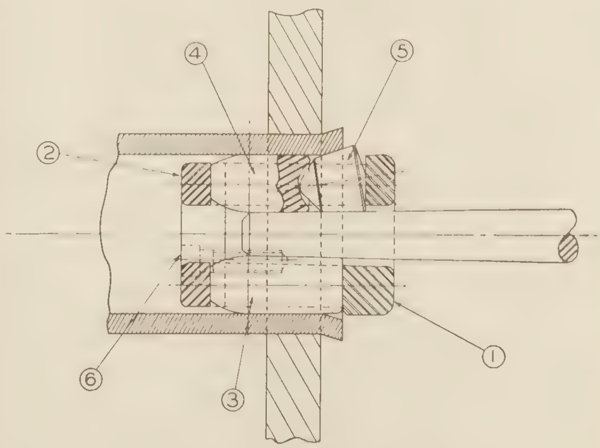

Fig. 2 Three-Roller Expanding Tool

that only elastic stresses are produced, and the oversize of the tube is under immediate control, the expanded joint on the other hand necessitates plastic stresses, at least in the tube, and the effective "oversize" produced is not under immediate control.

In seeking to understand how an effective oversize of the tube is created by the expanding process, it may be helpful to consider a strip of tube material confined in a perfectly rigid frame, Fig. 3(a), and subjected to the action of a single roll. The material under the roll, which is yielding plastically, tends to extend

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[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

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longitudinally and laterally. It is able to extend somewhat longitudinally by putting the remainder of the strip under elastic compression. If the roller is now removed, some of this compressive stress remains, the element under the roll retaining some permanent set. The rolling process may be imagined as the repetition of this process on the different elements.

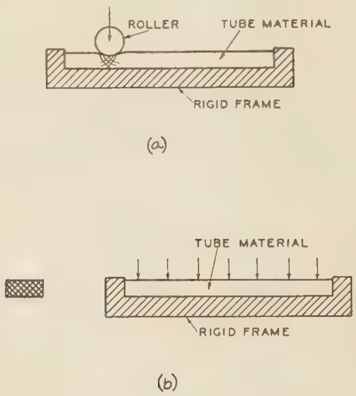

Fig. 3 Diagram Showing How Effective Oversize of Tube Is Created by Expanding

(*a*, Model to illustrate roll expanding; *b*, model to illustrate uniform-pressure expanding.)

When the strip, instead of being straight, is curved up into a circle, retaining its rigid backing, we have the case of a tube in a rigid plate. The residual longitudinal stress becomes a residual circumferential stress requiring a residual radial pressure for equilibrium.

If, instead of the roll, we have a uniform pressure, Fig. 3(*b*). only lateral, not longitudinal, extension is possible, and this condition is unfavorable for the development of a longitudinal residual stress. However, when the pressure is applied within a circular hole in a plate, it is possible to have permanent set created in a ring zone surrounding the hole. If the whole plate is elastic under the pressure, the radial and circumferential strains would be equal in magnitude but opposite in sign at any point, and both decrease outward. If the pressure is increased, these strains near the hole begin to exceed those elastically possible, and in the zone where this occurs there will be permanent set.

In the actual rolling, the pressure on the rolls gradually increases and the rolls make several circuits. From the first therefore an all-around pressure between tube and plate is being gradually built up, in virtue of the all-round permanent expansion of the tube already established. At any stage there is combined with this the local stresses created by the rolls wherever they happen to be. These localized stresses are absent in uniform-pressure expanding. The differences are well illustrated by Figs. 4(*a*), (*b*) from a paper by Thum and Jantscha (10), showing the patterns of slip lines in the two cases. In Fig. 4(*b*), owing to uniform pressure, only the well-known spiral system of slip lines appears. In Fig. 4(*a*), due to a roll expander, similar slip lines appear but accompanied by a narrow inner blackened zone, which is evidently a ring succession of zones of localized contact stress under a roll—the plastic zone of Fig. 3(*a*) repeated all around the ring.

Analysis of Stress During and After Expanding

Some knowledge of the state of stress both during and after expanding is essential not only for an understanding of the expanding process but also for the consideration of the plate under service stresses. The favorable circumstance that the strains created in expanding are not or rather do not need to be large, makes the problem an appropriate one for analysis by the mathematical theory of plasticity, ignoring rise of stress beyond the yield point. Typical physical characteristics of tube and plate materials and measurements of the strains are given in the paper of Grimison and Lee (19).

Only the results are given in this section. Derivations will be found in the Appendix.

It is necessary for reasons of mathematical difficulty to restrict the calculations to joints made by uniform internal pressure, as in some of the tests of Jantscha (11). Jantscha found, however, that the joints thus made were as good as his rolled joints. The differences between the stresses produced by the two methods have already been discussed qualitatively.

The law of plastic flow adopted is that of constant strain energy of shear (von Mises-Hencky hypothesis). The stress parallel to the axis of the tube is taken as zero (plane stress). The other extreme case, that of zero axial strain (plane strain), was also investigated, but the stress curves were not much different.[4]

Fig. 4 Patterns of Slip Lines in Expanded Tubes

[*a* (*upper*) Caused by roller expander; *b* (*lower*) caused by uniform pressure.]

Conditions are taken as uniform through the plate thickness.

We consider first the simplest case, where there is no clearance and tube and plate are of the same material. They can then be treated as one piece of metal, i.e., an infinite plate with a hole of diameter equal to the inner tube diameter. When pressure is applied to the inside of this hole, the stress distribution is at first elastic, but plastic flow begins at the hole, when the pressure reaches $s_0/\sqrt{3}$, where s_0 is the yield stress in simple tension. As the pressure is increased beyond this, the plastic zone spreads out from the hole, but under the assumption of plane stress, this does not continue indefinitely. When the radius of the plastic zone has reached 1.75 times the radius of the hole, and the pressure the

[4] Plane strain, however, does not set a limit to the pressure attainable in the hole, whereas in plane stress, this limit governs the maximum residual pressure.

corresponding values $2s_0/\sqrt{3}$ or 1.15 s_0, both cease to increase. Any attempt to increase the pressure further is defeated, in the idealized material, by axial extrusion. It seems entirely probable that a similar situation exists for the actual material.

Records of tests by The Babcock & Wilcox Company on $3^1/_4$-in-OD tubes of thicknesses 0.17, 0.29, 0.43, 0.50 in. show that the mill scale on the plates popped to distances of 0.6, 0.6, 0.5, 0.6 in. The diameters of the plastic zone thus estimated to the inner diameter of the tube are 1.53, 1.67, 1.78, and 1.98 as compared with the theoretical limit 1.75, ignoring clearance. In another case, thickness 0.17 in., hardness measurements in the plate as well as scale-popping observations gave a value 1.65.

Returning to the theoretical problem, the curves of the radial stress s_r and the circumferential stress s_t, due to the limiting pressure can be calculated. They are shown in Figs. 5 and 6. In Fig. 5, the stress beyond the limit of the plastic zone is of elastic form, although of course it is not the elastic stress corresponding directly to the pressure in the hole. The broken line shows how it would continue into the plastic zone. The actual plastic stress s_r is the full curve below it and is practically a straight line.

In Fig. 6, the curve BJ shows the stress s_t beyond the plastic zone, continuing as BA within the plastic zone. It will be seen that, even during the application of the pressure, the circumferential stress is compressive in a ring of thickness 0.20 a, where a is the radius of the hole.

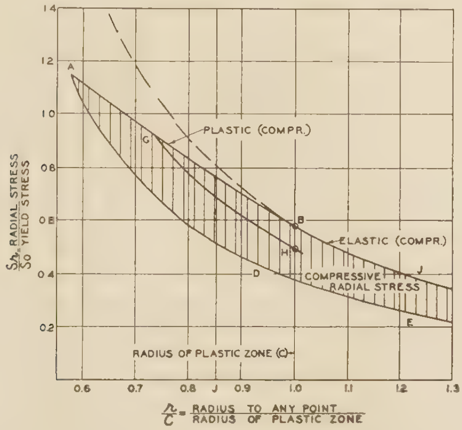

Fig. 5 Radial Stress Curves

In order to determine the residual stress remaining after the pressure in the hole has been removed, it is assumed that the changes of stress and strain during decrease of pressure are purely elastic. This is known to be characteristic of unloading in simple tension, compression, and torsion (17), and the assumption that it is also valid for compound stress is invariably made in the calculation of residual stress (18). We may then regard unloading as the superposition of an equal radial tensile stress in the hole, producing the elastic curves ADE in Fig. 5, FDE in Fig. 6. These are tensile and compressive stresses, respectively. They are, however, inverted in order to exhibit what stresses remain when they are superposed on the preceding stresses. The remainders, shown by the intercepts in the shaded areas, are the residual stresses. The maximum residual radial stress is 0.245 s_0 and occurs at a radius approximately 1.45 a. The circumferential stress s_t is compressive up to 1.58 a with a maximum value 1.73 s_0 at the hole. This value, however, exceeds the plastic limit of 1.15 s_0 which governs both principal stresses.[5] The values of the residual s_t for radii up to about 1.1 a also exceed this limit.

[5] See Appendix

It would appear therefore that the process of unloading cannot be a purely linear one as we have supposed. As it proceeds, it continually adds, near the hole, compressive circumferential stress to a stress of the same nature which was induced on loading. Elastic recovery may occur until the circumferential stress reaches 1.15 s_0, but after that, the recovery becomes inelastic. No attempt is made in this paper to take account of such an inelastic unloading process, and the residual circumferential stress of 1.73 s_0 remains as a defect in the theory. In spite of this, reasonably good agreement with test results is found.

To find the residual pressure between tube and plate, we have merely to take the intercept in Fig. 5 which occurs at an abscissa corresponding to the outside of the tube. If the outer radius is b the required abscissa is given by the value of b/c (c = 1.75 a). We are assuming here that the expanding pressure is always the

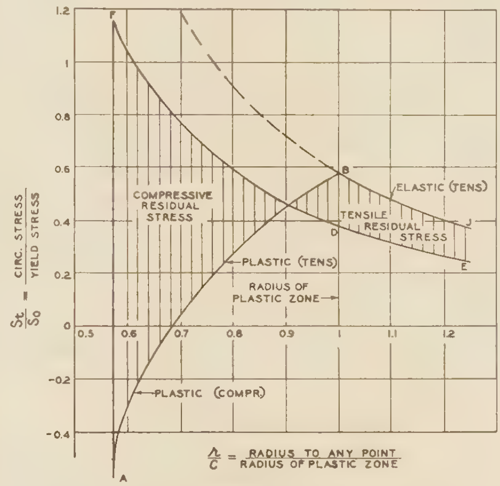

Fig. 6 Circumferential Stress Curves

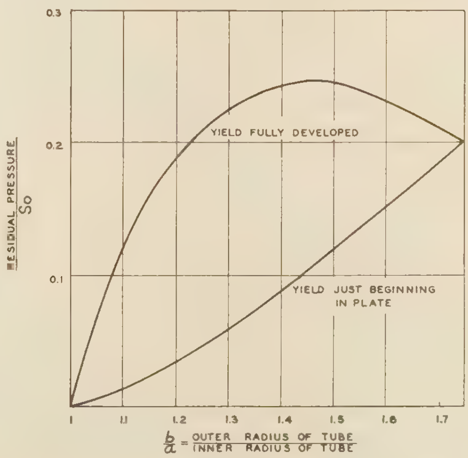

Fig. 7 Variation of Residual Pressure With Thickness of Tube

limiting pressure 1.15 s_0. It is thus possible to determine the residual pressures for a range of values of b/a. Such values are plotted in Fig. 7 (upper curve).

To examine the commonly held view that the best joint is obtained when the plate is stressed only up to its elastic limit, but not beyond, we may start from the point B in Fig. 5, which represents the limiting elastic radial stress. Whatever the tube thickness, the plastic curve will proceed from B as before. We may,

however, stop at any point G and take that as corresponding to the inner tube radius. Then the ordinate to G represents the highest pressure to be used in expanding if the elastic limit of the plate is not to be exceeded. The elastic unloading curve GH must then be drawn from G, and the intercept HB now gives the residual pressure. Again, this can be determined for various ratios of tube outer and inner radii. These values also are plotted in Fig. 7 (lower curve). They give much lower values than those obtained previously by allowing the pressure in the hole to reach the plastic limiting pressure, and the plastic zone to extend into the plate. Thus the view that the plastic zone, with its own inner zone of residual compressive circumferential stress, acts to shield the tube from the elastic reaction of the plate, does not justify the conclusion that the plastic zone should not extend into the plate. The shielding effect is present, but by not stopping when it begins we are able to go to a higher expanding pressure which more than offsets the shielding effect by producing a general increase in stress.

The implications of the theory as to overexpanding, i.e., the decrease of residual pressure and holding power beyond an optimum degree of expanding, may also be examined with the help of Fig. 5. The point B in all cases represents the boundary of the plastic zone. The corresponding radius c is now taken to be within the plate. We can choose any point G on the plastic part of the stress curve and take it as corresponding to the inner radius of the hole. We can thus obtain any ratio c/a, within limits, corresponding to some degree of expanding (radius of the plastic zone) and expanding pressure, less than the possible maxima. We may then take an abscissa J between G and B, to represent the outer radius of the tube, and read off the corresponding intercept at J between the curves GB and GH. The values thus obtained are plotted in Fig. 8. Each curve shows the variation of residual pressure with the extent c of yield in the plate, for a definite ratio b/a of the two tube radii. It will be seen that in all cases the residual pressure increases with c.

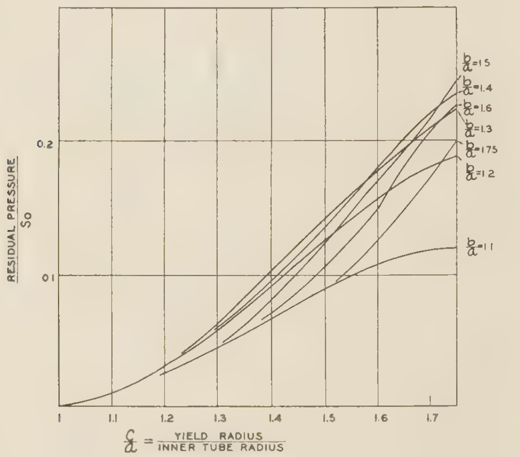

Fig. 8 Variation of Residual Pressure With Radius of Plastic Zone

Thus the theory, as far as it has been carried, yields no clue to the explanation of overexpanding. This negative result, however, serves to direct our attention to the factors which the theory has not taken into account. Of these the most suggestive is the thinning of the tube wall during expanding. If we treat this as a simple reduction of b/a, we conclude from the upper curve in Fig. 7 that this will explain some reduction of residual pressure, but only if b/a is less than about 1.4. For instance, in the case of a $3^1/_4$-in-OD tube, of 0.32-in. wall thickness, $b/a = 1.24$, and if the extrusion and, consequently, wall thinning is 15 per cent, the value of b/a after expanding is about 1.21. The reduction of residual pressure, from Fig. 7, is from 0.21 s_0 to 0.19 s_0, or 10 per cent.

The initial clearance between tube and plate has so far been ignored. It makes no difference, however, to the foregoing results, as far as the theory is concerned. When the expanding begins, the tube expands with a plastic stress distribution (the lowest curve in Fig. 9) different from that of Fig. 5. When contact is established, the pressure on the outer tube surface grows, and the stress distribution progressively changes. But the curves cannot rise beyond the curve of Fig. 5, and this is the final curve just as without clearance. The tests of Oppenheimer, however, showed a marked effect of clearance (15).

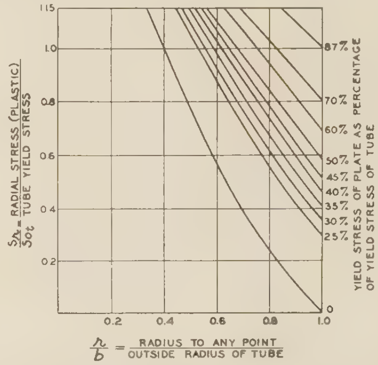

Fig. 9 Variation of Radial Stress Over Thickness of Tube for Limiting Internal Pressure, Various External Pressures

In actual joints the yield stresses of the tube and plate materials are usually different. The effect of this will therefore be considered before numerical comparisons are made.

Effect of Different Yield Points of Tube and Plate

Plate Yield Point Higher Than That of Tube. In the case already considered, of identical yield points of plate and tube, the best joint was obtained when the limiting pressure 1.15 s_0 was reached inside the tube. Since the radial stress due to the applied pressure invariably decreases along the outgoing radius, the radial stress in the plate never reaches the limiting value 1.15 s_0, except for a tube of infinitesimal thickness. For a tube of any given thickness, there is a radial pressure, defined by the appropriate point on the plastic curve AGB of Fig. 5, which is required on the outer tube surface. A plate of higher yield point is able to supply this pressure equally well. The stress distribution within the plate will be different. It becomes wholly elastic if the yield point of the plate is twice that of tube. But the plastic curve within the tube material is still that of Fig. 5, and the residual pressure is still found by the same construction—the intercept between the plastic curve AGB and the elastic curve ADE at the value of r/c which corresponds to the outside of the tube, that is, b/c. The s_0, however, which figures in the dimensionless ordinates, is of course that of the tube, not that of the plate. The two will be distinguished by the subscripts t and p, as s_{0t} and s_{0p}. The residual pressures for various values of b/a are therefore given by the upper curve of Fig. 7, the ordinates being values of residual pressure divided by s_{0t}.

We have the conclusion then that no advantage is gained by using a harder plate, as compared with a plate as hard as the tube, and this result is incorporated in Fig. 10. There, however, the ordinates are residual pressures divided by s_{0p}. Since we are considering increases of s_{0p} the ordinates of the curve of Fig. 7

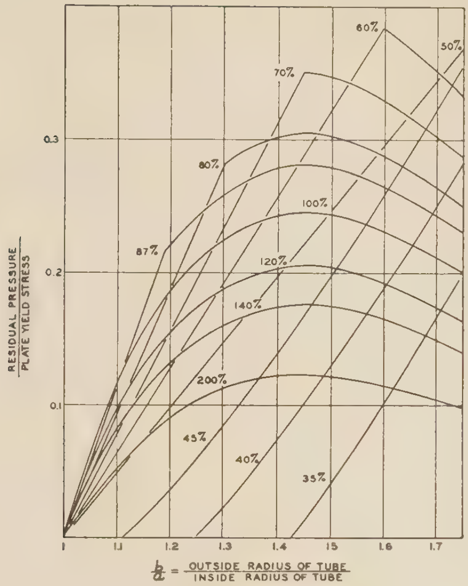

Fig. 10 Residual Pressures for Various Tube Plate Hardnesses and Various Thicknesses

must be reduced in proportion. Thus we obtain the curves marked 120, 140, 200 per cent. The curve marked 100 per cent is the same as that of Fig. 7. The percentages denote the ratio of plate yield point to tube yield point.

Plate Yield Point Lower Than That of Tube. It is necessary as a preliminary to consider the plastic stress in a tube when the internal pressure has the limiting value 1.15 s_{0t}, but the pressure on the outside of the tube has any value. This problem offers no difficulty and is solved in Nádai's "Plasticity"(18).[6] The solution is represented by the curves of Fig. 9. Starting at the right-hand end of a curve, we begin with the prescribed outside pressure. The plasticity law and the conditions of equilibrium then determine the curve completely, and it terminates at a pressure of 1.15 s_{0t}, and at a radius depending upon the outside pressure chosen. There is thus a lower limit to the inside radius. Any thicker tube will be in plastic equilibrium only for a lower external pressure. The lowest curve, for zero external pressure, has its inner terminus at $r/b = 0.335$. Thus if b/a exceeds 1/0.335, or 2.98, fully plastic equilibrium throughout the tube is not possible. The tube will be elastic in an outside zone.

Consider now, as an example, a plate whose yield point is 60 per cent of the tube yield point or 0.6 s_{0t}. Its limiting radial pressure is then 60 per cent of 1.15 s_{0t}, and this ordinate is marked 60 per cent on the right of Fig. 9. We may then follow the plastic tube curve which passes through this point and proceed inward to a value a/b of r/b corresponding to the inside of the tube. The stress reached on the curve is the highest internal pressure which can be applied without exceeding the limit set by the plate. From this point on the curve the elastic curve can be drawn, and the residual pressure read off as the intercept between it and the plastic curve at $r/b = 1$. This construction can be carried out for different values of a/b until the inner terminus of the curve is reached. Results are plotted as the straight-line part of curve marked 60 per cent in Fig. 10, which stops at $b/a = 1.6$, or $a/b = 0.625$, the abscissa of the inner end of the 60 per cent curve in Fig. 9.

For thicker tubes, the limiting stress of the 60 per cent plate is not reached, and we have to go to a lower curve, accepting a lower stress in the plate. We can retain the limiting stress 1.15 s_{0t} at the inside of the tube. The situation is thus no longer governed by the limiting stress of the plate, but by that of the tube. There is consequently a change in the nature of the curve for large b/a, as appears in Fig. 10. The curve in fact continues in the same way as Fig. 7.

The curves marked 50, 70, 80, and 87 per cent in Fig. 10 are determined in the same way. The same construction is used for the curves marked 45, 40, and 35 per cent. These, however, show here residual pressure below certain values of b/a. This feature begins at 50 per cent. An elastic curve (for the infinite plate) drawn through the right-hand point of the curve marked 50 per cent in Fig. 9 has, at that point, its tangent in common with the plastic curve, and, proceeding to the left, the elastic curve lies above the plastic curve. For the lower plastic curves, however, the elastic curves drawn through their right-hand ends lie, for some distance to the left, below the plastic curves. If we take a point on one of these plastic curves, within this distance, to correspond to the inner radius of the tube, the elastic curve drawn from this point will lie above the plastic curve at $r/b = 1$. The intercept will then give a negative residual pressure, a tension, which would exist if the tube adhered to the plate. It would thus indicate zero pressure when there is no adhesion. Thus according to the present theory, no joint can be made if the tube is much harder than the plate, unless the tube is sufficiently thick.

Fig. 10 gives a complete account of the residual pressures obtainable for the range 35 per cent to 200 per cent of s_{0p}/s_{0t} and the range 1 to 1.75 of b/a.

In order to exhibit more clearly the effect of varying the yield point of the plate, values from these curves are plotted in Fig. 11

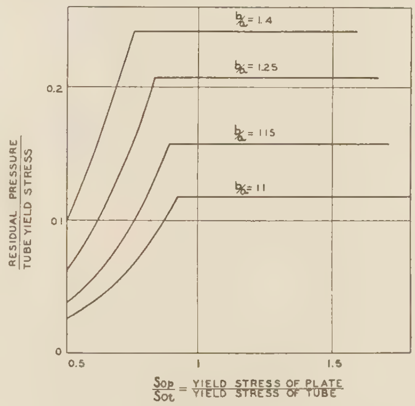

Fig. 11 Effect of Hardening Plate

in a different manner. The ordinates, residual pressure divided by s_{0t}, represent directly the residual pressure if s_{0t} is kept constant. The abscissas, s_{0p}/s_{0t}, correspond to changing s_{0p} under the same condition. We have then one curve for each thickness of tube. It will be seen that increasing the yield point of the plate has a very favorable effect so long as s_{0p}/s_{0t} is below a certain limit depending upon b/a. It is, for instance, 0.83 for $b/a = 1.25$. But any further increase in the yield point of the plate is without effect on the residual pressure, although it will mean a somewhat different stress distribution in the plate. The reason for this is of course that above this limit, the situation is governed by the limiting stress which the tube material can bear, and the plate is no longer called on for its limiting stress. The conclusion may be expressed by saying that if the yield point of the plate is substantially less than that of the tube, better results may be expected by using a harder plate. This state of affairs is the usual one. Quantitative comparisons are made in the text following.

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[6] Equation [30] on p. 191 contains an error. The corrected form is given in the Appendix of this paper.

The results recorded in Fig. 10 are again replotted in Fig. 12, to show the effect of changing the yield point of the tube when that of the plate is kept fixed. The ordinates then correspond to the residual pressure, and the abscissas to the yield point of the tube, and there is a curve for each value of b/a. The linear left-hand parts, corresponding to the softer tubes, are governed by the limiting stress of the tube material. The curved parts beyond

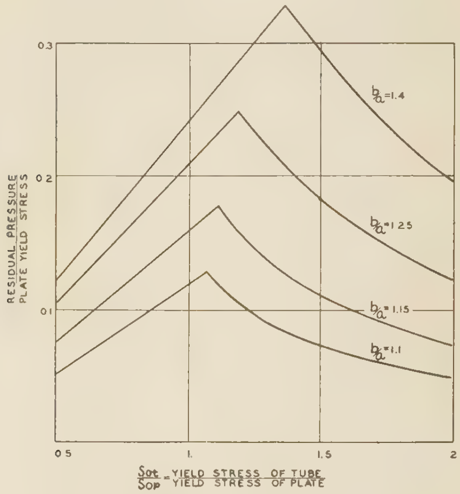

Fig. 12 Effect of Hardening Tube

the breaks are governed by the limiting stress of the plate material, which there does not permit that of the tube material to be reached. It will be seen that if the tube is substantially softer than the plate, the residual pressure can be raised by using a harder tube. But the use of a tube substantially harder than the plate may result in a smaller pressure.

Tests have been made by The Babcock & Wilcox Company on the holding power of soft tubes rolled in hard plates, and hard tubes in soft plates. The details and results are as follows:

Tube hardness	100 Bhn	320 Bhn
Plate hardness	302 Bhn	107 Bhn
Initial slip	18500 lb	11500 lb
Ultimate load	79000 lb	36000 lb

Tube, $3^1/_4$ in. OD; wall thickness 0.22 in.

No values of the yield points are available, but the hard plate would without doubt correspond to a point on the straight part of the appropriate curve in Fig. 11 ($b/a = 1.15$). The residual pressure would then be $0.158\ s_{0t}$. When the hard material is used for the tube, we would have to conclude from Fig. 10 that no joint is possible if s_{0p}/s_{0t} is less than about 0.45. The theoretical curves thus suggest that the hard plate - soft tube combination is very much better than the reverse, and this is in qualitative agreement with the results for the holding power.

If we consider two materials, one having twice the yield point of the other, with $b/a = 1.15$, the hard plate - soft tube combination gives a residual pressure of $0.158\ s_{0t}$ or $0.158\ s_0$ if s_0 is the yield point of the softer material. The hard tube - soft plate combination gives $0.074\ s_0$. The advantage is about 2 to 1 in favor of the hard plate - soft tube combination.

Comparison With Test Results

Table 1 shows the residual pressures obtained by the preceding theory for joints made and tested by Grimison and Lee (omitting one of alloy tubing showing no yield point). To illustrate the determination of the theoretical value, consider Case 2. The yield stress of the plate material, from test, is 31,500 psi, that of

TABLE 1 COMPARISON OF THEORETICAL RESULTS WITH MEASURED PRESSURES ON $3^1/_4$-IN-OD TUBES IN PLATES $1^1/_2$ IN. THICK

Case no.	Tube-wall thickness, in.	b/a	Plate yield stress s_{0p}, psi	Tube yield stress s_{0t}, psi	s_{0p}/s_{0t}	Greatest measured residual pressure, psi	Calculated residual pressure, psi
1	0.17	1.12	31500	55000	0.59	5600	2400
2	0.32	1.25	31500	40000	0.79	8100	7200
3	0.32	1.25	26500	35000	0.76	6400	5720
4	0.50	1.44	31500	37800	0.83	10100	9200
5	0.32	1.25	very high	40000	exceeds 0.83	8400	8160

the tube material 40,000 psi, so that $s_{0p}/s_{0t} = 0.79$. The value of b/a is 1.25. Using the corresponding curve of Fig. 11, we find that the ordinate for the abscissa 0.79 is 0.18, so that the residual pressure is $0.18 \times 40{,}000$ or 7200 psi.

With the exception of Case 1, the agreement with the test values is satisfactory. The theoretical values are all on the low side. This may be attributed to three causes, i.e., the neglect of the rise of the stress-strain curves beyond the yield point; differences between roll- and uniform-pressure expanding; and the assumption of plane stress. As to the latter, Grimison and Lee (19) find evidence that the axial frictional stress between tube and plate increases the residual pressure. Such stress is taken as zero in plane stress. There appears to be no evident reason for the large discrepancy of Case 1.

Prerolled Plates. The fact that in normal practice the tube has a considerably higher yield point than the plate (e.g., 35,000 psi as compared with 26,500), a difference which is increased by cold-working, leaves room for improvement by the use of a harder plate. The original plate can, however, be conveniently hardened, in the zone around the hole where hardness is required, by expanding the undersized hole in the plate by means of an ordinary expanding tool. This idea has been tested by The Babcock & Wilcox Company and, in the results available to date,

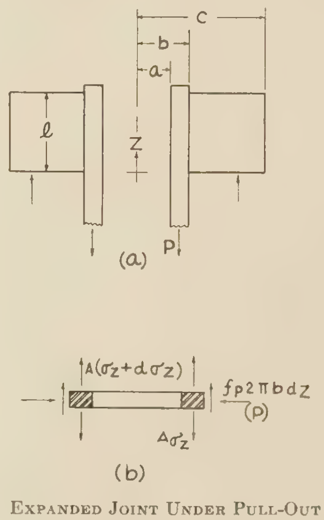

Fig. 13 Expanded Joint Under Pull-Out Force

a maximum residual pressure of 8290 psi was found for $3^1/_4$-in-OD tubes 0.32 in. thick. The maximum found for ordinary plates was 6400 psi, this being Case 3 of Table 1. The appropriate curve in Fig. 11, that for $b/a = 1.25$, shows that the use of plates such that $s_{0p}/s_{0t} > 0.83$, which correspond to the flat part of the curve, can raise the pressure to $0.208\ s_{0t}$, or, since $s_{0t} = 35{,}000$, to 7300. This falls short of the test value 8290 by 12 per cent, the same percentage defect as in Case 3 itself. It thus seems very probable that all the improvement available by using harder plates can be secured by prerolling.

The initial linear form of the curves of Fig. 10 for the lower

values of b/a is in agreement with conclusions from tests by Jantscha (11) and Ries (13).

Calculation of Frictional Holding Power

To calculate the frictional holding power the tube is treated as fitted into a plate, with an external radius c and hole radius b, with a uniform pressure p_0 at the common surface. The outer diameter is regarded as finite to correspond with test specimens. Tension being applied to one end of this cylinder will tend to reduce its diameter and so partially relax, in an axially non-uniform manner, the original fit pressure p_0. Let the actual pressure between tube and plate be p, a function of z (Fig. 13), when a pull P is applied to the tube. The corresponding axial stress in the tube σ_z, also a function of z, is taken as uniform over the thickness of the tube. Vertical equilibrium of the element of the tube shown in Fig. 13(b) gives

$$\frac{A d\sigma_z}{dz} + 2\pi b f p = 0 \qquad [1]$$

where $A = \pi(b^2 - a^2)$, the cross-sectional area of the tube, and f is the coefficient of friction.

On application of the pull-out force P, the pressure p_0 is reduced at the height z by $p_0 - p$. This permits a decrease of the radius of the hole in the plate by the amount

$$\Delta b_p = (p_0 - p)\frac{b}{E}\left(\frac{c^2 + b^2}{c^2 - b^2} + \nu\right) \qquad [2]$$

where E is Young's modulus and ν Poisson's ratio. This formula is taken from the thick-cylinder theory, for conditions uniform along the axis. It can also be established for linear variation of stress along the axis, provided certain shearing stresses on the tube ends are accepted. The variations ultimately found in the present case are nearly linear.

The reduction of the pressure by $(p_0 - p)$ permits an increase of external radius of the tube of

$$(p_0 - p)\frac{b}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu\right) \qquad [3]$$

but the tensile stress σ_z involves a decrease of $\nu\sigma_z b/E$. Equating the net decrease to the decrease of radius of the plate hole Equation [2], we obtained an equation which reduces to

$$\sigma_z = (p_0 - p)\frac{2b^2(c^2 - a^2)}{\nu(c^2 - b^2)(b^2 - a^2)} \qquad [4]$$

Equations [1] and [4] serve to determine σ_z and p as functions of z. Eliminating σ_z we find

$$\frac{dp}{dz} - \alpha f p = 0 \text{ where } \alpha = \nu\frac{c^2 - b^2}{b(c^2 - a^2)}$$

so that $p = Ce^{\alpha f z}$. Then Equation [4] gives

$$\sigma_z = \frac{2\pi b}{A\alpha}(p_0 - Ce^{\alpha f z})$$

At $z = l$, $\sigma_z = 0$ and at $z = 0$, $A\sigma_z = P$.

The first of these conditions gives $C = p_0 e^{-\alpha f l}$ and the second then

$$P = p_0\frac{2\pi b}{\alpha}(1 - e^{-\alpha f l}) \qquad [5]$$

and the distribution of pressure is now given by

$$p = p_0 e^{-\alpha f(l - z)} \qquad [6]$$

To gain some idea of the variations involved, consider the case $l = b . c = \infty$. Then

$$\alpha f l = \nu\frac{c^2 - b^2}{b(c^2 - a^2)} f l = \nu f$$

The value of ν will be about 0.3. The value of f will probably be between 0.3 and 1. Taking the latter value so as to get the most extreme variations of p with z, and P with l, we have $\alpha f l = 0.3$. The corresponding variation of p with z is sensibly linear. It would be practically linear for considerably longer joints.

Considering variations of l, keeping $f = 1$ and $\nu = 0.3$, the variation of P with l is found to be linear over the range of practical values of l/b. This agrees with the test results of Ries and Siebel (12, 13, 14).

The case of a push-out test may be obtained from Equations [5] and [6] by reversing the signs of f and P. The results may be written

$$P = p_0\frac{2\pi b}{\beta}(e^{\beta f l} - 1) \qquad [7]$$

$$p = p_0 e^{\beta f(l - z)} \qquad [8]$$

where P is the push-out force and $\beta = \nu(c^2 - b^2)/b(c^2 - a^2)$.

Appendix

The Plasticity Problem

The theoretical results obtained have been based on the plastic-elastic stress curves of Figs. 5 and 6, for pressure applied within a hole in an infinite plate, and the plastic stress curves of Fig. 9.

The equations for the plastic parts of all these curves have a common derivation (A. Nadai, "Plasticity," Chap. 28).[7] The law of plastic flow adopted, that of von Mises and Hencky, is in the general case of three principal stresses s_1, s_2, s_3.

$$(s_0 - s_1)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2 = 2s_0^2$$

where s_0 is the yield stress in simple tension. In the present problem we have the principal stresses s_r, s_t, radially and tangentially, in the plane of the plate, and the third principal stress is taken as zero. The plasticity condition then reduces to

$$s_r^2 - s_r s_t + s_t^2 = s_0^2 \qquad [9]$$

The curve showing this relation between s_r and s_t is an ellipse. The use of an eccentric angle θ of this ellipse as a parameter, by which the stresses are expressed as

$$s_r = \frac{2s_0}{\sqrt{3}}\sin\left(\theta - \frac{\pi}{6}\right),\quad s_t = \frac{2s_0}{\sqrt{3}}\sin\left(\theta + \frac{\pi}{6}\right) \qquad [10]$$

leaves the plasticity condition automatically satisfied. Both s_r and s_t are limited to lie between $\pm\frac{2s_0}{\sqrt{3}}$ or $\pm 1.15\, s_0$, and this is the limiting radial stress frequently referred to in the paper.

The equation of equilibrium is

$$r\frac{ds_r}{dr} + s_r - s_t = 0$$

Using the variables

$$s = \frac{s_r + s_t}{2} = s_0 \sin\theta,\qquad s' = \frac{s_t - s_r}{2} = \frac{s_0}{\sqrt{3}}\cos\theta \qquad [11]$$

this equation is transformed into

$$r\frac{d}{dr}\left[\sin\left(\theta - \frac{\pi}{6}\right)\right] = \cos\theta$$

[7] Ref. (18), Chap. 28, p. 185.

and the integral of this is

$$r^2 = A^2 e^{\sqrt{3}\theta} \sec\theta \qquad [12]$$

where A is the arbitrary constant. It is determined as soon as the stress s_r is given at some value of r. For the corresponding value of θ can then be found from Equation [10], and substitution in Equation [12] then gives A.

Curves which yield the solution of all the special problems may be obtained by plotting $\frac{r}{A}$ against θ from Equation [12], and s_r and s_t against θ from Equation [10]. Fig. 14 shows these curves over the range required in the problems of the paper. For a special case, as for instance $s_r = -0.6 \frac{2s_0}{\sqrt{3}}$ at $r = b$, we have $-\sin\left(\theta - \frac{\pi}{6}\right) = 0.6$ at $r = b$, or at $r/A = b/A$. Picking out the point on the curve of $s_r \sqrt{3}/2s_0$ which has the ordinate 0.6, we find that

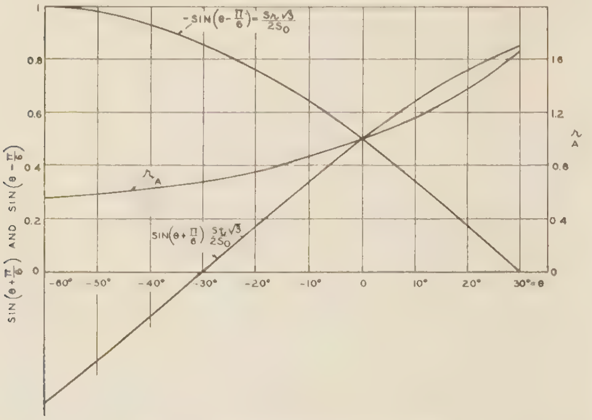

Fig. 14 Special-Problem Curves

it corresponds (for the same θ) to $r/A = 0.9$. Thus $b/A = 0.9$ and $A = 1.11b$. The curve of $s_r \sqrt{3}/2s_0$ versus r/A can now be converted into the curve of s_r/s_0 versus r/b, which appears in Fig. 9.

The problem of the infinite plate, with its elastic exterior zone, is solved by observing that since in the elastic zone the stresses s_r and s_t are equal in magnitude, though opposite in sign, the plasticity condition is satisfied at a boundary $r = c$ between the elastic and plastic zones if, from Equation [9] with $s_t = -s_r$, $3s_r^2 = s_0^2$ or $s_r = \pm s_0/\sqrt{3}$. If we then solve the plasticity problem as outlined with the boundary condition $s_r = -s_0/\sqrt{3}$ at $r = c$, the curve of s_r/s_0 can be plotted against r/c. This yields the s_r curve of Fig. 5, AGB, with abscissas r/c. The curve terminates at A, with the limiting stress $1.15\ s_0$ at $r/c = 0.567$. Thus the plate can only be made to yield as far as $c/a = 1/0.567 = 1.75$. Any value of the ratio a/c larger than 0.567 corresponds to the choice of a point between A and B for the hole and the pressure required to cause yield to the corresponding extent, $c - a$, is given by the ordinate.

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19 "Experimental Investigation of Tube Expanding," by E. D. Grimison and G. H. Lee. See pages 497–505 of this issue.

Experimental Investigation of Tube Expanding

By E. D. Grimison,[1] New York, N. Y., and G. H. Lee,[2] Ithaca, N. Y.

This paper gives results of an experimental investigation to determine (*a*) the fundamentals involved in tube expanding; (*b*) the various practical methods of measuring the degree of expansion; (*c*) the optimum degree of expanding; and (*d*) the ultimate strengths of expanded joints under various conditions of service. Characteristic relationships of seat pressure on tube to degree of expanding are given, which clearly show the optimum degrees of expanding. An explanation for the existence of the optimum is developed. Determinations of structural strengths of joints under instantaneous loading to failure are reported, as well as the results of repeated smaller loadings through 1000 cycles. The joint was not injured by these repeated applications of loads. The investigation has derived information which has led to a more positive procedure for rolling-in tubes.

Lack of a general understanding of the fundamentals of tube expanding and the confusing nature of the data available caused The Babcock & Wilcox Company to undertake a complete investigation of the problem. It was decided to examine both theoretically and experimentally the fundamental principles involved and to combine the knowledge thus gained with practical experience in the field. From the beginning of the investigation, it was planned to publish the results as the company's contribution to the development of knowledge of a problem vital to its own business and of great importance to its customers. The general objectives were:

1 An investigation of the fundamentals involved as an aid to analysis of the problem.

2 Investigation of the various practical methods of measuring the degree of expansion.

3 Determination of "optimum" degrees of expanding. By this is meant that degree of expansion which will create a joint of maximum strength and tightness.

4 To furnish data in the shape of ultimate strengths of joints under various conditions of service and suitable factors of safety applying to them.

These objectives are only partially fulfilled in the investigation reported in this paper. The investigation was well in progress when the war started, and it was decided to continue it to a point at least partially conclusive before stopping it for the duration, in the belief that the small expenditure in materials and manpower might be offset many times over by the savings which would result from a more positive procedure of rolling-in boiler tubes. At some time in the future, this work will probably be continued by the company and it is hoped by others as well.

Basis of Experimental Study

In a companion paper,[3] Goodier and Schoessow develop the theoretical background of the investigation reported here and include findings which extend the conclusions of this experimental study substantially. The equations which they develop for the strength of expanded joints are based on the assumption of purely frictional resistance to slip of a tube expanded in a plain seat. This involves the assumptions: (1) The surfaces in contact were originally cylindrical (before application of load) and (2) the pressure exerted by the seat on the tube was uniform before application of any load. The latter assumption is not important if the coefficient of friction is constant with pressure.

Referring to the equations given by Goodier and Schoessow the tube-seat radial pressure on the tube is a prime variable and the only variable likely to be influenced markedly by degree of expanding except, perhaps, the coefficient of friction. On the assumption that coefficient of friction is independent of degree of expanding, the carrying capacity of the joint in either axial loading or torsion is directly proportional to the tube-seat pressure.

According to this assumption, the plans for the investigation provided that a major portion of the program should be devoted to tube-seat pressure determinations. The object was primarily to define the degree of rolling for which a maximum of seat pressure is obtained. This maximum of seat pressure was believed to coincide with the maximum structural strength of the joint. This belief was to be checked by experiment and has been so checked with no reason forthcoming to modify it within limits. By thus attacking the problem, a useful design tool is furnished with a minimum of experimental labor.

It was believed that many expanded joints fail through repeated applications of loads lower than the ultimate strengths of the joints, particularly in torsion and bending. It is often difficult to introduce flexibility into long riser and downcomer connections between waterwall headers and drums, without subjecting the expanded joints to heavy torsion or bending loads each time the unit is put on or taken off the line. Determination of endurance limits was, therefore, included in the program. Due to the low frequency with which these loadings occur, the tests were carried through only 1000 cycles, approximately equivalent to 20 years of normal service. The results of these tests under repeated loadings did not support the original belief in an endurance limit less than the ultimate strength of the joints for the number of cycles of repeated loading used in these experiments.

Experimental Procedure

All tubing used in this investigation was $3\,^1/_4$ in. OD $\times$ 0.17, 0.32, and 0.50-in. wall thicknesses. The tubes were expanded into seat disks $1^1/_2$ in. thick, using three-roll propulsive expanders in all cases, with belling rolls removed. All the seats were bored on a lathe with arbitrary but constant boring speeds. All seats were plain, with no irregularities except the bore marks

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Contributed by the Power Division and the A.S.M.E. Research Committee and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[3] "The Holding Power and Hydraulic Tightness of Expanded Tube Joints: Analysis of Stress and Deformation," by J. N. Goodier and G. J. Schoessow, presented at the Annual Meeting, New York, N. Y., Nov. 30-Dec. 4, 1942, of The American Society of Mechanical Engineers. Published in this issue p. 489.

which were not disturbed in any way before use. No grooved seats were used in these experiments. All seats were bored to $3\,^9/_{32}$ in., giving a nominal clearance of $^1/_{32}$ in. on the diameter. Before making the joints, the tubes were buffed free of all rust and scale, but no attempt was made to eliminate occasional deep pits in the tubes. Tube and seat surfaces were cleaned with carbon tetrachloride to remove all oil and grease.

The seamless tubing used in this investigation was made to A.S.M.E. Code Specification S-17 Grade A, and was normalized to reduce variations in physical properties to a minimum. The seat-plate specifications were intended to be A.S.M.E. Code Specification S-1 in all cases, but some of the plate was more nearly of 70,000 psi tensile strength than 55,000 psi, as is indicated specifically in reporting the results of the experiments. A few tests were made using seat plates of heat-treated tool steel of approximately 500 Bhn and a few tests using tubes made by boring out tool-steel rod had a hardness of approximately 200 Bhn.

Three methods were considered for the determination of tube-seat pressures. All involved the use of the thick-cylinder theory, i.e., the problem was assumed to be one in which the deformations were symmetrical about an axis. After the expansion of the tube into the seat, both the tube and the seat are elastically restrained by the tube-seat pressure built up during the expanding operation. If either the seat is removed from the tube or the tube is removed from the seat, an elastic recovery will occur in the remaining member. The tube-seat pressures are calculated from the strains measured during the elastic recovery of either the tube or the seat.

The first method used for the determination of the tube-seat pressure involved the measurement of the strains occurring in the tube during its elastic recovery as the seat was removed. Many tests were conducted using this method before a comparison of results with those of the second method, involving the strains occurring in the plate upon removal of the tube, indicated an inconsistency that could arise only from an error in the application of one or both of the methods. Careful consideration of the factors involved led to the discard of the first method and the adoption of the second. As the first method is attractive in its apparent simplicity, it will be briefly described and the reasons for its discard stated.

After completion of the joint, six SR-4 Metalectric strain gages, described by de Forest and Anderson,[4] were bonded in alternately tangential and axial directions on the inside surface of the tube with their center lines coinciding with the middle of the joint. The unit strains measured on cutting away the seat were substituted into Equation [1] which is derived from the thick-cylinder theory (the Lamé formula),[5] to give the tube-seat pressure.

$$p_0 = \frac{E(e_t + \nu e_l)}{2(1 - \nu^2)} \left[1 - \frac{D_i^2}{D_0^2} \right] \quad \text{[1]}$$

The nomenclature used in Equations [1] and [2] is as follows:

a and D_i = inside radius and diameter, respectively, of tube, in.
b and D_0 = outside radius and diameter, respectively, of tube, in.
c = outside radius of seat plate, in.
e_t and e_l = tangential and longitudinal (axial) unit deformations, in. per in.
p_0 = radial pressure exerted at tube-seat interface, psi
r = radius, in.
u = radial displacement, in.
E = Young's modulus of elasticity, psi
ν = Poisson's ratio

After noting the discrepancy between the results of the two methods for the determination of the tube-seat pressure, a more careful consideration of the problem led to the conclusion that the thick-cylinder theory could not be applied to the elastic recovery of the tube. One of the assumptions upon which the thick-cylinder theory is based is that the radial displacements are constant along the length of the cylinder. Calculations showed that when the seat was removed from the tube, the walls of the tube did not have a radial-displacement constant along the length of the seat, but rather the tube walls "bulged" out. The longitudinal strain arising from the "bulging" was found to be of sufficient magnitude to introduce an error of the order of 50 per cent into the calculations of the tube-seat pressure. An accurate determination of the tube-seat pressure using the strains occurring on the inner surface of the tube would necessitate a knowledge of the distribution of the pressure along the seat.

Assuming two extreme pressure distributions the tube-seat pressure was determined by calculation. The pressure distributions assumed were (1) uniform over the length of the seat and (2) a triangular distribution with a maximum in the middle of the seat. The data used were taken from a test in which the tube was rolled to approximately 8 per cent total extrusion. The results were 7200 psi for the uniform pressure distribution, an average of 3800 psi for the triangular distribution, 2280 psi as determined using the thick-cylinder theory applied to the tube, and 4300 psi for elastic measurements made on the plate. This latter pressure was determined using the second method previously mentioned. The value of 4300 psi for the average pressure is seen to lie between the pressures as determined assuming the two extremes of pressure distribution. This is as expected, since it is reasonable to assume that the true pressure distribution lies between the two extremes. The results indicated, while not too definite, are sufficient to indicate the error encountered in applying the thick-cylinder theory to the problem of the elastic recovery of the tube upon removal of the plate.

The second method considered involved elastic measurements made in the plate during its elastic recovery upon removal of the tube. A special mechanical strain gage was used to measure radial displacements of gage points placed on the seat plate in sets of two. The points of each set were at diametrically opposite positions on a 6-in-diam circle concentric with the tube seat. The points were then approximately $1^1/_2$ in. from the seat, a distance sufficient to render negligible the effects of any self-equilibrating force system occurring in the seat. Four sets of gage points, equally spaced, were placed on each plate. Changes in the radial distances between points, upon removal of the tube, were the measurements made. Errors arising from the instruments were quite small, of the order of 1 per cent in measuring displacements of the order of 20×10^{-5} in.

The tube-seat pressure p_0 could be determined by substituting the measured radial displacements in Equation [2], which is derived from the thick-cylinder theory

$$p_0 = \frac{uE\,(c^2 - b^2)}{a^2 \left[(1 - \nu)r + (1 + \nu)\dfrac{b^2}{r} \right]} \quad \text{[2]}$$

This method for the determination of the seat pressure is theoretically sound and gave consistent results as shown later.

[4] "A New Lateral Extensometer," by A. V. de Forest and A. R. Anderson, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 63, Dec., 1941, p. A-152.

[5] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Company, Inc., New York, N. Y., vol. 2, 1930, p. 528.

The small displacements involved could be measured with sufficient accuracy, and the method was adopted.

The third method used measurements of tangential strains occurring on the periphery of the seat during the removal of the tube. The strains were too small to promise any acceptable degree of accuracy and the method was discarded.

The indicators of degree of rolling, considered in this investigation were the popping of mill scale around the seat, partial and total extrusion of the tube from the seat during the expanding operation, increase in inside diameter of the tube after it makes first contact with the seat during expansion, and the radial displacements of points on the seat plate adjacent to the tube seat. All of these indicators have certain advantages and disadvantages which are discussed later.

The popping of mill scale was measured by means of a linear scale, an average being taken of several readings around the descaled ring surrounding the seat. The partial extrusion of the tube is the extrusion as measured from either the tube end or the open end of the joint. This measurement on the tube end

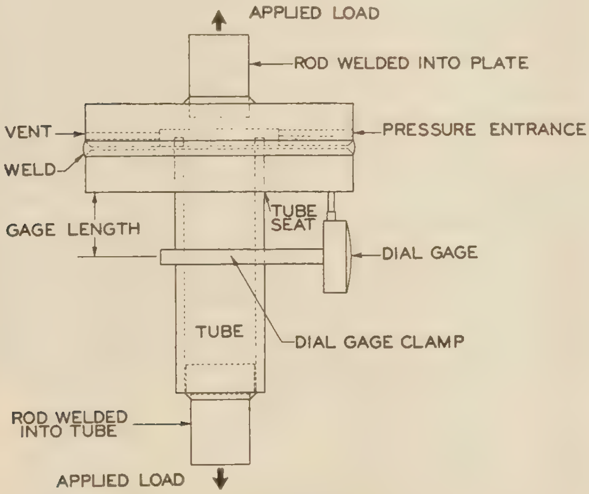

FIG. 1 SKETCH OF TENSION-TEST SPECIMEN SHOWING POSITION OF DIAL GAGE

of the seat was taken by means of a dial gage fastened to the tube with plunger resting on the plate. The dial clamp was sufficiently far from the seat to insure only the action of the extrusion being indicated. The extrusion of the open end of the tube from the seat was measured by a removable instrument consisting of a notched bar and dial gage, which was placed on the end of the tube in such a manner that the motion of the middle of the tube wall away from the plate was indicated on the dial. The total extrusion of the tube is equal to the sum of the partial extrusions. The increase in the inside diameter of the tube over that at measured tack rolling and calculated contact rolling was determined by averaging micrometer measurements taken around the tube. The radial displacements of points on the plate adjacent to the seat were measured by means of a mechanical strain gage capable of measuring accurately displacements of the order of 10^{-4} in. with an error of about 1 per cent.

All determinations of structural strengths were made on $3^1/_4$-in-OD tubing $\times$ 0.32-in-wall thickness. An internal hydrostatic pressure of 1800 psi was maintained, during all tests, which corresponds to the pressure permitted by the A.S.M.E. Code. Tension samples used were of all-welded construction except for the expanded joints, as shown in Fig. 1. The expanded joint was kept cool during welding by circulating water internally through the pressure vents. The samples were tested in a universal testing machine directly to failure, readings of the dial gage being made at set increments of load. The loads at first leak and at ultimate strength of the joint were recorded. Ultimate strength is defined as the maximum load, in either tension or torsion, the joint can withstand without slipping. The point of failure was unmistakable.

The type of test sample used in the torsion tests is illustrated in Fig. 2. The ultimate strengths of the joints were such that in some instances the elastic limit of the tube was reached before

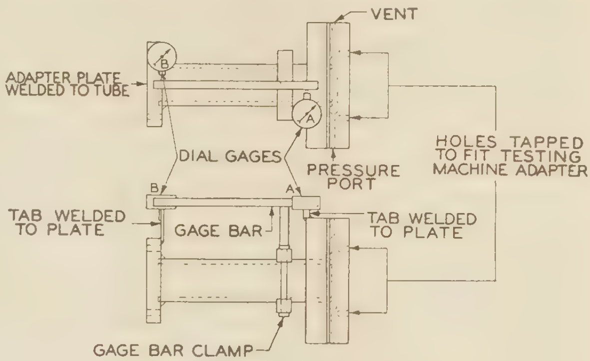

FIG. 2 TORSION-TEST SAMPLE
Sketch shows positions of dial gages and adapters to testing machine.

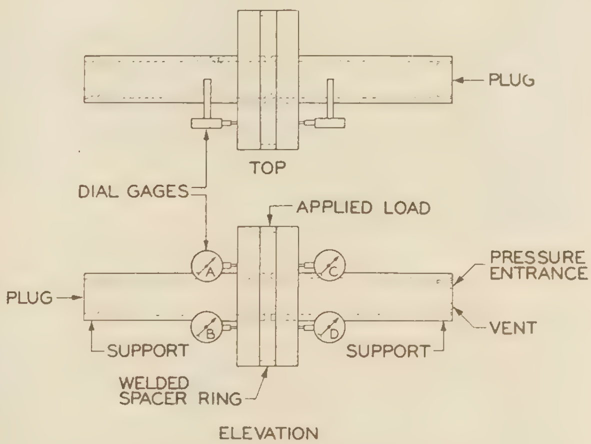

FIG. 3 TRANSVERSE-BENDING-TEST SAMPLE
(Sketch shows positions of dial gages and points of application of load.)

the joint failed. A comparison of the dial readings would give an indication of any slippage occurring in the joint. Several torsion tests were run in which the loads at first leak and at the ultimate strength of the joint were noted. Dial readings were taken at definite increments of loading. Additional tests were run under repeated loading to loads somewhat below the ultimate strength. In loading the sample repeatedly, the elastic deformation of the tube was used as an indication of the load applied in preference to the load indicator of the machine, aiding greatly in the facility with which the load could be repeated. Occasional checks were made between dial readings and actual torsional moment indicated by the machine. The load was not reduced to zero in these repeated-loading tests, but to 3000 in-lb minimum. Considerable time was thus saved.

The transverse-bending tests were made on samples of the type shown in Fig. 3. Two joints were tested simultaneously, the action of the one joint in no way affecting the other joint. The sample was of all-welded construction, the same precautions

against overheating being taken as in the construction of the other samples. The double sample was supported as a simple beam with the load applied in the middle of the span as indicated in Fig. 3. Dials were fastened to tabs welded to the tube at a distance of 2 in. from the joint. Due to the push-out action of the internal pressure, the motion of the top of the tube relative to the seat was different from that of the bottom of the tube, hence the two sets of dials per sample. A fifth dial was placed on the testing machine to indicate the total deflection of the sample. Several direct bending tests were run to first leak. The binding of the tube in the seat raised the structural strength of the joint far beyond the leak strength. Several additional tests were run in which the loading was repeated 1000 times. Again the object of these tests was to obtain, if possible, a failure of the joint somewhat below the ultimate strength of the joint.

Results of Tube-Seat-Pressure Determinations

The assumption was made initially that the tube-seat pressure was a direct indication of the structural strength of the joint. Experience and previous tests have indicated that, as the degree of rolling progresses, the tensile strength of the joint increases to a maximum and then decreases. According to the foregoing assumption, the tube-seat pressure plotted against the degree of rolling should give a similar maximum. Three factors were thought to influence the shape of the curve, i.e., the decrease in the wall thickness due to rolling; a change in the friction between the tube and the seat; and finally the ring of material surrounding the seat, plastically deformed during the expanding operation, acted as a cushion for the tube against the elastic recovery of the plate. The first was found not to enter into consideration here since its action is felt only after excessive rolling of the tube. The second factor, i.e., the change in the friction between the tube and the seat, is the dominant influence, as will be shown later. The third factor has been shown by Goodier and Schoessow, in a companion paper,[3] not to affect the tube-seat pressure materially.

The first series of tests was run on $3^1/_4$-in-OD × 0.32-in-wall-thickness tubing of an average hardness of 130 Bhn rolled into $1^1/_2$-in-thick plate of an average hardness of 105 Bhn. The seat pressures determined are plotted as curve (1) in Figs. 4 and 5, as functions of the percentage increase in the inside diameter of the tube over that calculated at first uniform contact of tube and seat, and the total extrusion of the tube, respectively. The tube inside diameter at first contact of tube with the seat is calculated on the basis of an unchanged cross-sectional area of tube. The correlation between the tube-seat pressure and the percentage increase in the tube inside diameter over that measured at tack rolling is very poor, due primarily to the sensitivity of the latter to slight errors in measurement, and so was not represented by plot. The serial numbers of joints tested refer to tabulated data.

The unexpected sharp break-off in the tube-seat pressure with increased degree of rolling at about 4 per cent total extrusion caused considerable speculation as to influencing factors. A maximum in the curves was expected, but not the rapid drop in tube-seat pressure. An examination of the seat after removal of the tube indicated a definite answer to the questions. Top views (magnified 6 times) of sections of the tube seats indicated are shown in Fig. 6. The views extend from the center of the seat to the edge. Reference to Figs. 4 and 5 will place the seats illustrated with reference to degree of rolling and tube-seat pressure as indicated by joint numbers.

The bore marks of seat No. 67, Fig. 6(*a*), are apparently undamaged; the illustration indicates that the original roughness is not materially changed. Reference to Figs. 4 and 5 will indicate that the tube was slightly underrolled. The bore marks of

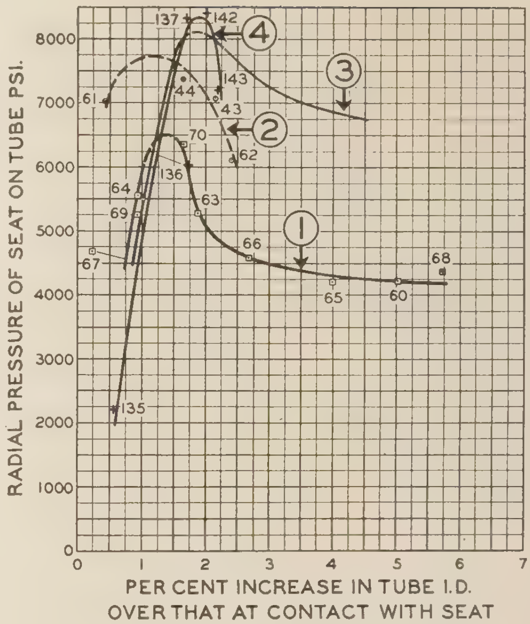

Fig. 4 Influence of Increase in Tube Inside Diameter, Over That at Contact, on Seat Pressure on Tube

($3^1/_4$-in-OD tubes; 0.32-in-wall thickness. ① Tubes 130 Bhn, seats 105 Bhn. ② Tubes 118 Bhn, seats prerolled to 130 Bhn from 105 Bhn as received. ③ Tubes 118 Bhn, seats 128 Bhn. ④ Tubes 118 Bhn, seats 413–578 Bhn.)

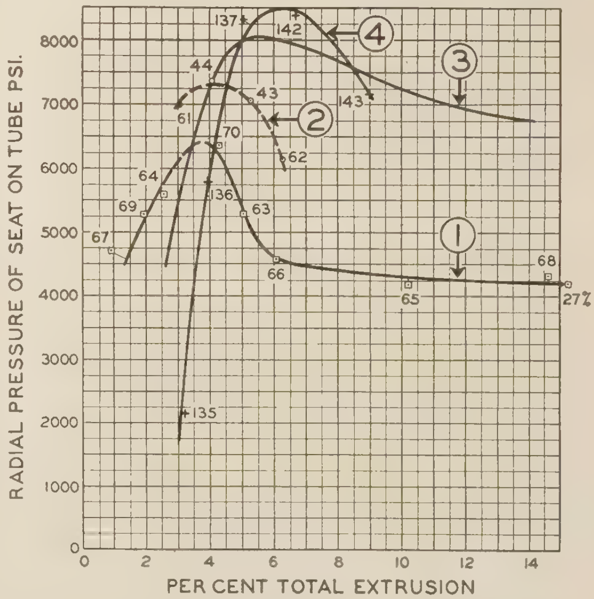

Fig. 5 Influence of Total Extrusion on Seat Pressure on Tube

($3^1/_4$-in-OD tube; 0.32-in-wall thickness. ① Tubes 130 Bhn, seats 105 Bhn. ② Tubes 118 Bhn, seats prerolled to 130 Bhn from 105 Bhn as received. ③ Tubes 118 Bhn, seats 128 Bhn. ④ Tubes 118 Bhn, seats 413–578 Bhn.)

seat No. 70, Fig. 6(*c*), are materially flattened, but no slipping of the tube relative to the seat due to rolling has occurred. Joint No. 70 is apparently rolled to the proper degree for maximum tube-seat pressure. The seat of test No. 66, Fig. 6(*b*), the tube of which was rolled past the critical point, shows definite indications of slippage due to excessive rolling. From this observation, it is clear that the greatest tube-seat pressure for this combination of

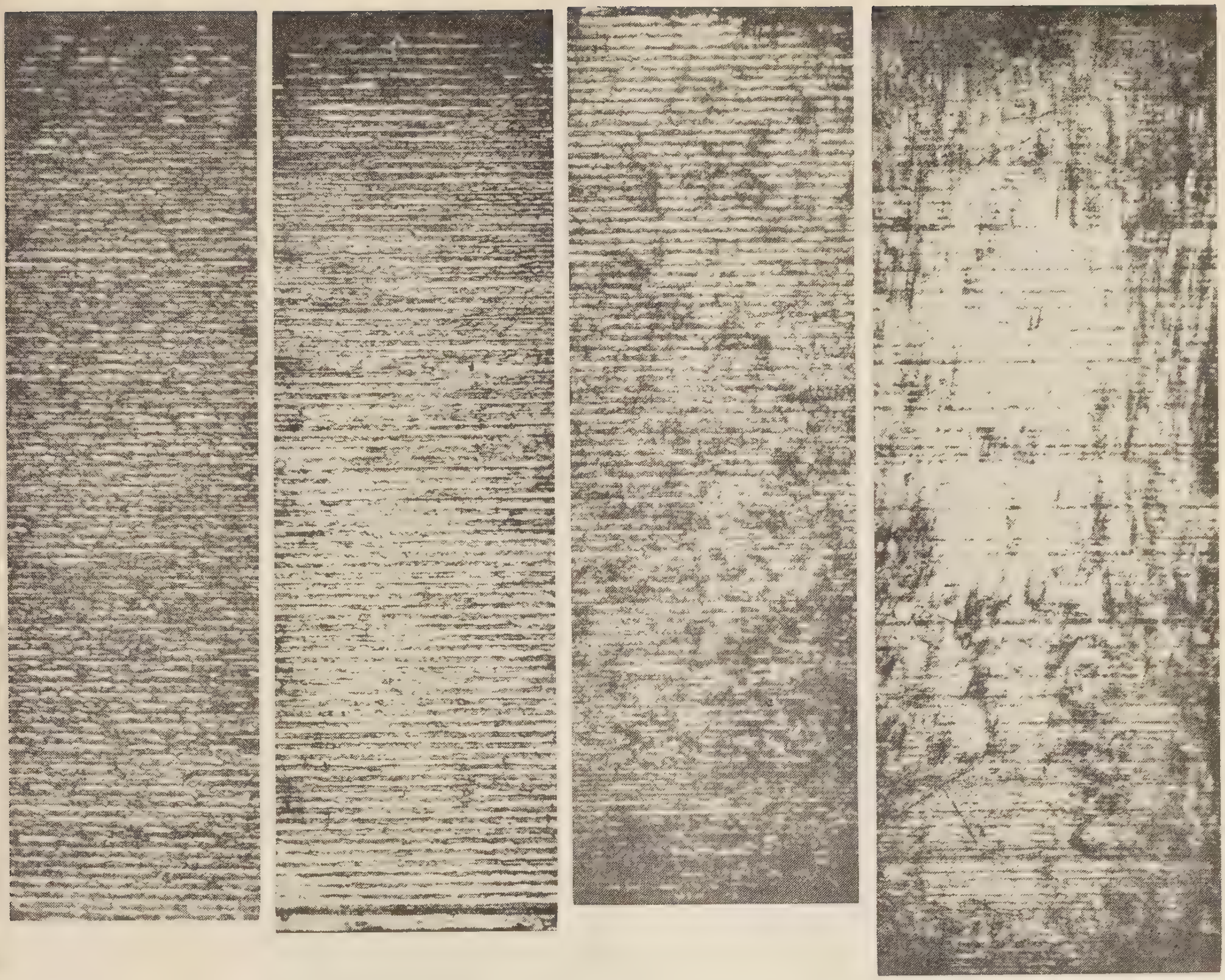

FIG. 6 TOP VIEWS OF BORE MARKS IN SEATS; $\times 6$
(*a*, Seat No. 67; *b*, seat No. 66; *c*, seat No. 70; *d*, seat No. 60. Top of each view is edge of seat. Bottom of each view is middle of seat.)

FIG. 7 PROFILE VIEWS OF BORE MARKS IN SEATS; $\times 50$
(*a*, Seat No. 67; *b*, seat No. 66; *d*, seat No. 60.)

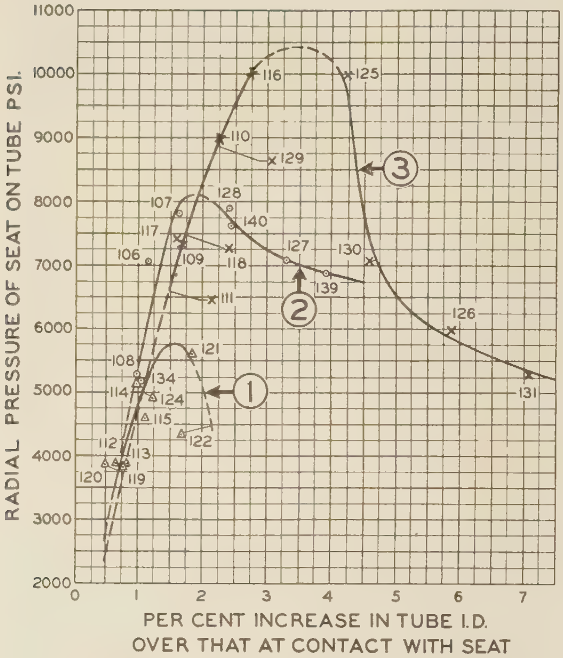

Fig. 8 Influence of Increase in Tube Inside Diameter, Over That at Contact, on Seat Pressure on Tube

($3\frac{1}{4}$-in-OD tubes, 118–122 Bhn; seats 128 Bhn. ① 0.17-in-thick tube. ② 0.32-in-thick tube. ③ 0.50-in-thick tube.)

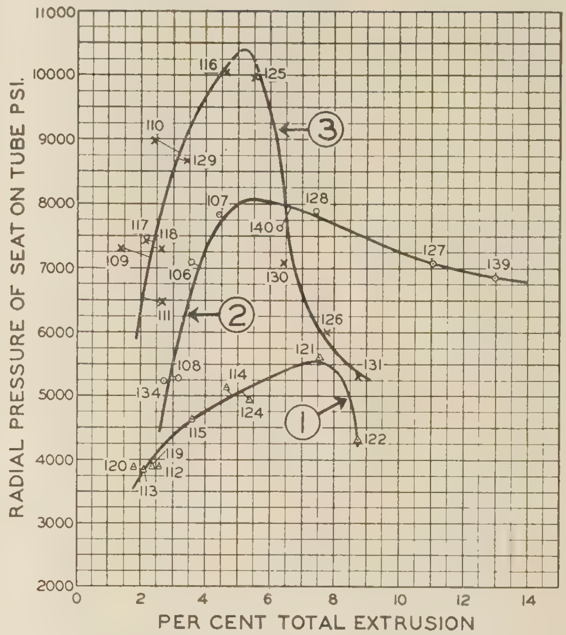

Fig. 9 Influence of Total Extrusion on Seat Pressure on Tube

($3\frac{1}{4}$-in-OD tubes, 118–122 Bhn; seats 128 Bhn. ① 0.17-in-thick tube. ② 0.32-in-thick tube. ③ 0.50-in-thick tube.)

tube and seat is obtained just before slipping, due to rolling, which occurs between the tube and the seat. The slipping that occurs beyond the critical degree of rolling breaks down the resistance to extrusion of the tube. The top view of the section of seat No. 60, shown in Fig. 6(*d*), illustrates the effect on the seat when the tube is rolled excessively.

Profile views (magnified 50 times) of the bore marks of sections of the seats taken from test Nos. 67, 66, and 60, respectively, are shown in Fig. 7 and are taken at the center of the sections shown in Fig. 6. These profile views bear out the statements just made concerning the top views of the same seats.

It is concluded that, to obtain the highest possible tube-seat pressure, it is necessary to prevent in some manner the easy extrusion of the tube from the seat.

Study of Tube-and-Plate-Hardness Combinations

A series of three groups of tests was run, in which the tube-and-plate-hardness combinations were maintained practically constant, but the tube-wall thickness was varied. The wall thicknesses used were 0.17, 0.32, and 0.50 in. The hardnesses are as noted in Figs. 8 and 9. These results do not entirely support Goodier and Schoessow's conclusion[3] that the maximum pressure obtainable by the expansion of a tube into a seat is approximately proportional to the wall thickness of the tube. The maximum pressures obtained with the various thicknesses of tube are approximately linear with wall thickness but do not follow a simple proportionality law.

Another series of tests included tube-seat-hardness combinations which ranged from an extreme of plate hardness to the other extreme of tube hardness. The results for all but one group of these tests are presented in Figs. 4 and 5. The hardness for tube and seat are noted with the figures. One group of plates was, as noted, of 105 Bhn as received; these plates were prerolled, a process to be described, raising their hardness at the seat to approximately 135 Bhn.

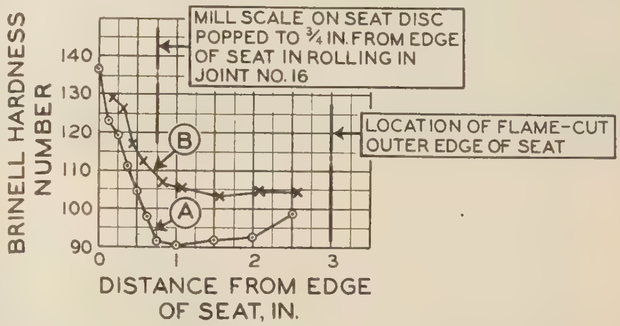

Fig. 10 Strain-Hardening of Tube Seats Due to Expansion

(Curve *A*, hardness of prerolled seat. Curve *B*, hardness of seat No. 16 after normal expansion of 0.17-in-thick tube.)

The process by which the hardness of the plates just mentioned was increased by prerolling was as follows: The plates were bored undersize by about $\frac{3}{16}$ in. in diam. A four-roll tube expander was then used and the hole rolled to a diameter about $\frac{1}{8}$ in. undersize. The hole was then finish-bored to a diameter of $3\frac{1}{32}$ in. This process of prerolling raised the hardness of the plate, in the vicinity of the seat, from 105 Bhn to about 135 Bhn. The maximum hardness obtainable by roll-hardening was in the neighborhood of 135 Bhn. A curve of the plate hardness, plotted against the distance from the edge of the seat is given in Fig. 10, curve *A*. This may be compared with curve *B*, a plot of the plate hardness arising from the expansion of the tube versus distance from seat.

Reference to Fig. 4 shows that the prerolling process gives most of the benefits of a hard plate. It appears that with this process the maximum seat pressure is reached with a slightly smaller increase in tube inside diameter, therefore, presumably with less cold-working and consequent thinning of the tube.

The expanding of the soft tube into the hard seat is relatively easy. Tack rolling is readily obtained and the proper

degree of rolling is a matter of only a few turns of the expanding tool. It will be noted in Figs. 4 and 5 that the tube-seat pressures obtained for the relatively soft tube in the hard seat were somewhat higher than those obtained using a somewhat softer plate, roughly 30 per cent greater. There is a variation in the character of the curves with increasing plate hardness, but a maximum of tube-seat pressure is present in all instances. The rate at which the tube-seat pressure drops off with increased rolling is dependent upon the relative hardness of tube and plate. There are not sufficient data to clearly define curve 4, Figs. 4 and 5; the drop-off being entirely governed by point 143 which may be at fault. It will be noted that, aside from curve 4, there is a tendency toward a less rapid drop-off in tube-seat pressure with increased hardness of plate relative to tube.

It may be noted that increasing the seat hardness from 105 to 130 Bhn produced a very definite increase in seat pressure, but that a further increase of seat hardness to an average of about 500 Bhn produced very little additional increase in seat pressure. This is in line with the conclusion reached theoretically by Goodier and Schoessow,[3] that increase in seat hardness relative to the tube will produce increased seat pressures only up to some maximum limit.

A final group of this series of tests was made in which the tube was approximately 200 Bhn, while the plate was of the approximate hardness of 128 Bhn. The joints using these combinations of hardness were rather difficult to form. Tack rolling was obtained only after numerous resettings of the expanding tool. Final rolling was made with not too great difficulty, but the plate was rather badly deformed. The results of these tests are listed in Table 1. Sufficient data were not obtained for a graphical representation. It will be noted, however, that rather high tube-seat pressures were obtained, extending over a wide range of rolling.

Results of Structural-Strength Tests

Little or no design data are available on the expanded joint. One of the aims of this investigation was to make a few strength tests as a start on a more comprehensive investigation of strength values of the expanded joint. The A.S.M.E. Boiler Code working pressure for the size of tubing tested is approximately 1800 psi; this hydrostatic pressure was used in all the samples tested. The three general types of tests run were tension, torsion, and transverse bending. The transverse bending, as will be noted on the sketch of this sample, Fig. 2, was a combination of pure bending and transverse shear. This combination of bending and transverse shear is such as would be encountered in actual practice; instances of pure bending or only transverse shear are very rare.

Torsion Tests. The torsional strengths obtained for the various samples tested are listed in Table 2 together with information on degree of rolling, hardnesses, etc. Sufficient data have not been taken to warrant the plotting of curves, but the results listed in the table indicate a rather constant strength with degree of rolling, for constant hardness of tube-and-plate combinations. The joint in which the plate was prerolled gave a much higher strength than the average of the other tests. By reference to Figs. 4 and 5, it will be noted that the prerolled joint was expanded beyond the critical degree. Whether this would affect the torsional strength of the joint or not is questionable.

Several repeated-loading tests were run on joints expanded to approximately the critical degree. The loading cycles and the number of cycles run are listed along with the strengths and other pertinent information in Table 2. Although the maximum load to which the samples were taken during the cyclic testing approached the ultimate strength of the joints, no failure was obtained. Experience has shown that in some instances downcomers and risers develop leaks after periods of use, and an attempt was made here to explain this failure of the joint as due to a progessive breakdown of the joint caused by repeated loadings. Because of the low frequency with which the loading on the tubes in a boiler is appreciably altered, 1000 cycles were assumed to be equivalent to about 20 years of normal service. It is obvious that the data taken are such as to prohibit any definite conclusions, but there is an indication that repeated loading in torsion to a maximum load of 75 per cent of the ultimate will not cause failure within a reasonable number of cycles.

Coefficients of friction, calculated from the ultimate strengths given, are included in Table 2. These are fairly constant below the critical degree of rolling, but rise appreciably beyond it. This rise is thought to be due to distortion of the tube and seat from circularity under excessive rolling.

Tension Tests. Table 3 lists the results of the tension tests on joints of various degrees of rolling. It will be noted that the strengths follow the tube-seat pressure, as indicated by either tube extrusion or increase in tube inside diameter. This correlation between the tensile strengths of the joints and tube-seat pressures is expected due to high frictional resistance to slip resulting from the direction of the bore marks in the seats. This is further emphasized by the generally higher values of coefficient of friction than were had from torsion tests of tubes expanded to less than the critical degree.

Transverse Bending Tests. Results of transverse bending tests are not at all consistent, as indicated in Table 4. The reason for the scattering, most notably in the harder tube–softer seat combinations, is not known. Several of the samples were loaded cyclically 1000 times in attempts to obtain leaks at loads somewhat below the ultimate strengths of the joints. No leaks were had until the samples were carried to failure after completion of the 1000 cycles.

In bending, it is possible that failure does not occur through slipping of the tube in the seat. Rather, the tube may distort in bending and allow leakage to take place through the distortions. In this case, the width of the seat will have an important bearing on the strength of the joint, with results not predictable except by direct tests. An increase in strength in transverse bending by widening the seat may be expected.

Indicators of Degree of Expanding

Discussion of the advantages and disadvantages of the indicators of degree of rolling is significant from the standpoint of their application to field erection of boilers and other pressure vessels. The present discussion is from the rather limited viewpoint of the experimentalist and is supplemented by a discussion in a companion paper by Maxwell,[6] from the broader viewpoint of the practical erector.

"Popping" of mill scale on the seat plate is of limited utility in the hands of inexperienced men. However, field experience with it proves its utility as an indicator of attainment of an optimum degree of expanding, corresponding to reaching a maximum seat pressure. It is an indicator distinctly lacking in sensitivity to differences in degree of expansion, but it was noted repeatedly that the beginning of mill-scale popping coincided with the attainment of optimum seat pressure. The size of scale flakes varies greatly, apparently due to inconstant bond between scale and plate. For this reason, the "beginning" of popping might mean the descaling of a band, varying from $^1/_4$ to $^3/_4$ in. around the seat. This indicator is useless, of course, in cases where the plate has no mill scale.

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[6] "Practical Aspects of Making Expanded Joints," by C. A. Maxwell, presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers. Published in this issue, p. 507.

TABLE 1 TUBE-SEAT-PRESSURE MEASUREMENTS

Test No.	% increase in tube I.D. over tack rolling Measured	Calculated	Extrusion of tube in inches open end	tube end	total	% total extrusion (ΔL/L)	Radial displ. gage pts. (x 10⁴) inches	Tube seat press. psi.	Hardnesses tube	plate	Tube wall thick. inches
60	5.56	5.01	.229	.178	.407	27.1		4 210	130	105	0.32
63	2.35	1.88	.044	.032	.076	5.07		5 290	130	105	0.32
64	0.83	0.94	.014	.024	.038	2.53		5 560	130	105	0.32
65	4.27	4.00	.083	.070	.153	10.2		4 200	130	105	0.32
66	2.60	2.68	.046	.045	.091	6.07		4 560	130	105	0.32
67	0.113	0.23	.003	.011	.014	0.93		4 680	130	105	0.32
68	5.43	5.71	.115	.104	.219	14.6		4 360	130	105	0.32
69	0.79	0.94	.012	.017	.029	1.93		5 260	130	105	0.32
70	1.62	1.66	.031	.031	.062	4.13		6 360	130	105	0.32
106	2.07	1.16	.026	.033	.059	3.59	46.0	7 070	118	128	0.32
107	2.71	1.61	.033	.038	.071	4.41	59.0	7 810	118	128	0.32
108	1.53	0.98	.021	.030	.051	3.12	43.0	5 280	118	128	0.32
127	3.41	3.30	.090	.089	.179	11.1	111.0	7 070	118	128	0.32
128	2.67	2.40	.061	.059	.120	7.41	97.0	7 890	118	128	0.32
134	0.86	1.02	.022	.022	.044	2.71	52.0	5 190	118	128	0.32
139	3.83	3.91	.095	.113	.208	13.0	111.0	6 870	118	128	0.32
140	2.24	2.43	.043	.059	.102	6.36	84.0	7 620	118	128	0.32
135	0.97	0.60	.016	.029	.045	3.09		2 220	118	578	0.32
136	1.30	1.72	.029	.033	.062	4.01		6 010	118	413	0.32
137	1.60	1.71	.032	.042	.074	5.01		8 310	118	418	0.32
142	1.90	2.01	.048	.049	.097	6.65		8 400	118	578	0.32
143	2.09	2.21	.075	.064	.139	8.99		7 170	118	413	0.32
133	1.76	2.03	.023	.033	.056	3.45	155.0	6 020	213	128	0.32
138	2.40	2.09	.031	.050	.081	5.00	201.0	7 710	210	128	0.32
141	4.58	4.69	.090	.071	.161	9.94	352(est)	8 270	207	128	0.32
43	1.25	2.17	.048	.038	.086	5.26		6 320	130	135	0.32
44	1.20	1.66	.038	.026	.064	4.01		7 350	130	135	0.32
61	1.06	0.45	-	-	-	3.01		7 000	130	135	0.32
62	2.30	2.46	.036	.058	.094	6.26		6 130	130	135	0.32
112	0.72	0.65	.022	.019	.041	2.54	58.0	3 890	122	128	0.17
113	0.68	0.78	.021	.013	.034	2.10	82.0	3 840	122	128	0.17
114	0.92	0.99	.045	.031	.076	4.62	106.0	5 120	122	128	0.17
115	1.06	1.09	.033	.026	.059	3.59	119.0	4 620	122	128	0.17
119	1.05	0.82	.017	.022	.039	2.40	78.0	3 880	122	128	0.17
121	1.56	1.84	.076	.046	.122	7.53	141.0	5 600	122	128	0.17
120	0.65	0.48	.016	.013	.029	1.79	60.0	3 860	122	128	0.17
122	1.56	1.67	.089	.052	.141	8.74	150.0	4 330	122	128	0.17
124	1.23	1.22	.046	.040	.086	5.31	83.0	4 920	122	128	0.17
109	1.23	1.68	.005	.017	.022	1.36	33.5	7 300	122	128	0.50
110	2.81	2.24	.0115	.028	.0395	2.45	65.0	8 980	122	128	0.50
111	2.52	2.14	.012	.030	.042	2.64	61.5	6 450	122	128	0.50
116	3.73	2.75	.017	.0585	.0755	4.62	98.0	10 040	122	128	0.50
117	1.49	1.62	.015	.0185	.0335	2.09	61.0	7 410	122	128	0.50
118	2.13	2.44	.013	.0305	.0435	2.69	86.0	7 260	122	128	0.50
125	4.07	4.21	.031	.061	.092	5.56	111.0	9 960	122	128	0.50
126	5.40	5.86	.061	.065	.126	7.78	115.0	5 980	122	128	0.50
129	3.00	3.05	.019	.037	.056	3.43	101.0	8 650	122	128	0.50
130	4.39	4.61	.047	.059	.106	6.46	119.0	7 080	122	128	0.50
131	7.56	7.08	.067	.075	.142	8.79	134.0	5 320	122	128	0.50

Tests 43, 44, 61, and 62 were made with pre-rolled plates.

TABLE 2 TORSION TESTS

Test No.	% increase in tube I.D. over tack rolling Meas.	Calc.	Extrusion of tube in inches open end	tube end	total	% total extrusion (ΔL/L)	Ultimate strength in.-lbs.	Initial leak in.-lbs.	Est. seat press. psi.	Load for 1000 cyc. tests in.-lbs.	Calc. Coef. of friction
47(1)	1.83	1.62	.050	.033	.083	5.50	104 320	104 320	7 000	- -	.503
71	1.77	2.00	.014	.017	.031	2.06	72 500	68 000(2)	5 300	- -	.444
72	1.80	2.03	.034	.034	.068	4.50	80 000	80 000	6 000	- -	.433
73	1.65	2.18	.032	.034	.066	4.40	83 440	83 440	6 100	- -	.446
74	1.43	1.51	.029	.032	.061	4.07	84 000	84 000	6 400	- -	.436
75	1.76	2.42	.040	.038	.078	5.30	82 040	82 040	5 100	64 000	.507
76	2.89	3.24	.066	.055	.121	8.00	85 260	85 260(2)	4 400	- -	.596
92	-	-	-	-	.060	4.00	81 200	68 000(2)	6 400	- -	-
93	-	-	-	-	.060	4.00	101 000	101 000	6 400	72 000	-

TABLE 3 TENSION TESTS

Test No.	% increase in tube I.D. over tack rolling Meas.	Calc.	Extrusion of tube in inches open end	tube end	total	% total extrusion (ΔL/L)	Ultimate strength (4) lbs.	(5) lbs.	Initial leak lbs.	Est. seat press. psi.	Calc. coef. of friction
48(1)	2.00	2.00	.060	.034	.094	5.86	42 000	52 300	3000(2)	6600	.412
83	1.13	0.98	.017	.016	.033	2.20	41 500	51 800	17000(2)	5400	.521
84	1.96	2.31	.033	.030	.063	4.19	52 000	62 300	14000(2)	6300	.584
85	2.71	2.79	.047	.058	.105	7.01	31 500	41 800	(3)	4500	.500

TABLE 4 BENDING TESTS

Test No.	% increase in tube I.D. over tack rolling Meas.	Calc.	Extrusion of tube in inches open end	tube end	total	% total extrusion (ΔL/L)	Initial leak in.-lbs.	Est. seat press. psi.	Load for 1000 cyc. tests in.-lbs.	Hardness BHN tube	seat
49(1)	1.88	1.77	.052	.038	.090	5.50	28 500	6 900	- -	130	135
51(1)	1.53	1.62	.038	.042	.080	5.00	97 500	7 200	- -	130	135
79	2.07	2.30	.038	.041	.079	5.27	48 000	5 100	- -	130	105
80	2.48	2.71	.037	.035	.072	4.80	19 200	5 600	- -	130	105
81	3.50	3.32	.061	.060	.120	8.08	96 400	4 400	- -	130	105
82	3.54	3.77	.062	.060	.122	8.20	72 000	4 400	- -	130	105
86	2.36	2.90	.038	.037	.075	5.00	57 000	5 300	- -	130	105
87	2.14	2.41	.038	.043	.081	5.40	61 700	5 000	- -	130	105
88	2.59	2.71	.049	.041	.090	6.00	85 500	4 600	- -	130	105
89	1.45	2.74	.037	.041	.078	5.19	85 500	5 300	- -	130	105
90	1.95	2.22	.026	.037	.063	4.20	26 100	6 300	- -	130	105
91	2.04	2.11	.034	.033	.067	4.47	38 000	6 100	- -	130	105
94	1.42	1.82	.041	.035	.076	5.06	67 800	8 000	34 900	118	128
95	1.50	1.31	.035	.034	.069	4.60	75 300	7 800	35 100	118	128
96	1.46	1.46	.024	.030	.054	3.60	54 600	7 300	21 300	118	128
97	1.31	1.20	.018	.032	.050	3.34	67 900	7 300	21 600	118	128
98	1.20	1.31	.017	.034	.051	3.40	64 400	6 400	31 200	118	128
99	1.23	1.20	.028	.031	.059	3.94	74 100	7 400	31 200	118	128

Notes for tables 2, 3, and 4.

(1) Tube seat pre-rolled.
(2) Leak occurred after ultimate strength had been reached.
(3) Leak occurred at a very low load after ultimate strength had been reached.
(4) The push-out effect of the internal pressure not included.
(5) The push-out effect of the internal pressure included.

Note: In all tests, 1800 psi hydrostatic pressure was maintained in the sample. Wall thickness, 0.32"; tube hardness, 130 BHN; plate hardness, 105 BHN.

The partial extrusion of the tube from either side of the seat has been used as an indicator of the degree of rolling. Neither of these partial extrusions has proved to be a satisfactory indicator of the degree of expanding, as shown by Table 1 which contains partial extrusion measurements for each tube. It had been hoped that plots could be made correlating these partial extrusions with the total extrusions and increase in the inside diameter of the tube after tack rolling, but the points were so scattered as to make the plots useless.

The total extrusion of the tube out of the seat is given by the sum of the partial-extrusion measurements. Although the partial extrusions of the tube are very poor indicators of degree of rolling, the sum of the partial extrusions is an excellent indicator, as shown by Figs. 5 and 9.

The sole objection to this indicator for use in the field is the complexity of its application. It is necessary to measure the extrusion of the open end of the tube at the tack-rolled condition and again at any time later when a measure of degree of expanding is wanted. Each time, it is necessary to add the extrusions on the two sides of the plate to get the total, requiring communication between observers on the two sides of the plate and the presence of a reliable man to add the two. As the expander must be removed to measure the extrusion of the open end it is obvious that the method is not practical for field use on production rolling.

The increase in the inside diameter of the tube over that at contact of tube with seat was found to be quite a satisfactory indicator of degree of rolling. This is shown by the plots of Figs. 4 and 8. In general, the method of operation is to indicate the degree of expansion by the mandrel travel from the "contact" condition, which for most precise results will require an empiric correlation of mandrel travel and increase in inside diameter of tube, measured after the withdrawal of mandrel and expander, i.e., after the elastic recovery of the tube. In the present investigation, the travel of the mandrel was used as an indicator during the rolling operation, but more precise measurements were taken of the inside diameter of the tube after withdrawing the tool at final rolling. For purposes of correlation, improvement was had in the indicated curves by calculation of the inside diameter of the tube at time of contact with the seat rather than by use of measured values of inside diameter at contact. The inside diameter at time of contact of tube with seat was calculated on the basis of a constant cross-sectional area of tube up to the time of contact.

The use of the radial displacements of gage points placed on the plate adjacent to the tube seat was not used as an indicator until late in the investigation. The gage points were placed at diametrically opposite positions at equal radii of 2 in. The results obtained were not entirely satisfactory but sufficient time was not available for a complete exploration of the variables affecting the indicator. Positioning of the points for the present investigation was entirely arbitrary. It is quite possible that moving the points away from the tube will bring about more consistent results. The measurements made are listed in Table 1.

Table 5 contains the physical test data of materials used in this investigation. The tests for which each material was used are noted in the table.

TABLE 5 PHYSICAL TEST DATA OF MATERIALS USED

Test Nos.	Remarks	Mod. of Elast. psi ($\times 10^{-6}$)	Yield Point psi	Yield Strength (off.=.1%) psi	Ultimate Strength psi	Elong. % in 2"	Red. of area %
18-19	plate		31500		62750	33.5	58.3
20-91	plate		28000		59000	34.5	58.6
92-On(1)	plate	25		31500	71250	33.0	51.7
	0.17" tube	32.7		53500	66600	25.0	61.5
92-On	0.32" tubing	27.6	40600	40000	59800	38.0	42.8
(2)	0.32" tubing	27.8		52000	110000	16.0	31.8
	0.50" tubing	30.5	37250	37800	62000	31.0	61.1
43-91	0.32" tubing		40750		65830	39.0	51.7

Notes for Table 5.

(1) Plate used in all tests subsequent to test No. 92.
(2) Tubing bored from tool steel stock.

Conclusions

A characteristic variation in seat pressure on the tube, with degree of expansion, has been found. The seat pressure rises rapidly with increase in degree of expanding to a sharply peaked maximum, then falls off equally rapidly to a limiting value which is nearly constant.

Seat pressure increases with thickness of tube but is not related to it by a simple law of proportionality. Increase in relative hardness of either seat or tube increases the radial pressure within limits which appear to be at about 150 Bhn for the seat plate. Beyond this hardness there is no appreciable increase in seat pressure. From the standpoint of ease of expanding and damage to the plate, a soft tube in a hard seat is much to be preferred to the opposite condition.

The sharply peaked maximum in seat pressure appears to be clearly due to restraining extrusion of the tube. It does not follow that any particular type of grooving will produce the required restraint.

Within the limits of a comparatively few tests, structural strengths of expanded joints have been determined and are given in the paper. Repeated-loading tests to 1000 cycles and to 70 per cent of the ultimate load resulted in no reduction of ultimate strength in either torsion or bending.

Acknowledgments

The authors are indebted to Mr. Paul Rekettye, of the erection department, The Babcock & Wilcox Company, for his many helpful suggestions during the course of this investigation and for the use of the mechanical strain gage designed and constructed by him. Dr. F. Eberle, Mr. W. E. Leyda, and Mr. E. F. Wilson of the research laboratory, The Babcock & Wilcox Company, have been of invaluable aid in their co-operation in obtaining physical test data and photomicrographs. The authors are also indebted to Messrs. Perry Cassidy, J. P. Craven, and J. C. Quinn for their many suggestions and co-operation throughout the investigation.

Practical Aspects of Making Expanded Joints

By C. A. MAXWELL,[1] BARBERTON, OHIO

As one of three current papers on the general subject of rolling-in boiler, heat-exchanger, and condenser tubes, this presentation deals more particularly with the practical application of recent tube-expanding equipment and methods largely developed by The Babcock & Wilcox Company. Progress in the solution of tube-rolling problems from 1924, when axial movement of tubes in their seats became particularly acute, down to the present time is noted by the author; accomplishments in this direction being the work of comparatively few engineers who have devoted their attention to the various aspects. Only since 1940, when a new expander was developed, have the weaknesses of the standard expander been overcome, making possible the expanding of tubes in seats of unlimited width. There are three basic tools for producing expanded joints from which all others are derived, the roll, the prosser, and the ball-drift expander. Details of each are given. The procedures followed are explained comprehensively, as is also the function of the ball-drift expander used for relatively small tubes in thick seats.

A SIMPLE procedure safely and economically to attach tubes to drums, tube sheets, and headers is so fundamental to boiler, heat-exchanger, and condenser design, that none of the power development of the past century could have been made without it. The method developed and almost universally used all these years, still constitutes the practice in more than 95 per cent of all designs and, where working pressures do not exceed 1750 psi, or temperatures of 750 F, this will probably continue to be the most practical and economical one. The method is the well-known expanded joint, a procedure apparently so simple, with satisfactory results so easily produced, that many engineers have given little thought to its development or the development of tools necessary for its use.

The simplicity is deceiving, for actually such a complexity of problems enters into the proper design of joints and tools, as almost to defy complete analysis. The apparent simplicity and good results with which joints have been made, even with unskilled labor and simple tools, have not encouraged research into the methods of making the joint, analysis of the results obtained, or a comprehensive study of the theoretical possibilities. There is a notable absence of any American literature on the subject, the principal exception being two papers by Cope and Fisher.[2] These papers were largely concerned with methods for the determination of a properly expanded joint and were real contributions to the art. The three current papers are largely for the purpose of setting forth the theoretical aspects and possibilities inherent in the design of the joint; the experimental values attained; and, in this paper, something of the various designs of joint, the tools used to produce them, and the technique of their use.

Elements of Joints

From its earliest use, the expanded joint has always consisted of a hole either in the drum plate, tube sheet, or header wall, into which a tube was inserted and expanded. A tool was used which, by cold-working, increased the diameter and length of the tube. The increase of tube size deforms the metal around the tube hole, with a resultant elastic reaction against the tube, holding it in place with great strength and resistance to leakage. As long as pressures were low and did not involve thick plates, tube seats were formed simply by drilling a plain hole in the material of the pressure vessel into which the tube was inserted and expanded.

As pressures increased, plates became thicker, and the seats thereby wider. It is this increased width of seat that has greatly complicated the expanded tube joint, as used in most high-pressure vessels. The cold-working of the tube wall results in an increased diameter of the tube, a definite increase in its length directly proportional to the seat width, and a reduction in thickness. The increase in length of the tube within its seat has both advantages and disadvantages. The advantage is that, with a properly designed seat, deformations on the tube can be forced axially against opposing deformations in the tube seat and thereby greatly reduce the tendency to leak. The deformations may be made by grooving the tube seat or rough-machining it, the tube being expanded into these.

The disadvantages are twofold:

1 During the expanding operation, as the tube moves axially along the seat, all projections on both tube and seat are placed in high shear; should this movement, which is maximum at the outer ends of the seat, exceed safe design factors, which will be set forth later, the keys produced purposely by machining grooves into the seat, and the lesser ones resulting from machining the tube and seat normally, may be either sheared completely or so damaged that leakage occurs at much reduced pressures.

2 As the tube moves axially, it is extruded from the seat with a force which may approach, in some cases, the ultimate strength of the tube section itself. It is obvious, then, that structural stresses are imposed not only upon the extruded tube and its joints, but on adjacent tubes and joints as well. These stresses are at a maximum when straight short tubes are expanded into rigid thick heads.

The problems involving the axial movement of tubes in their seats became acute about 1924, with the great increase of steam pressures and the consequent increased thickness of tube sheets. A considerable research program, conducted by A. W. Lienau,[3] the results of which are unpublished, was undertaken at that time to establish definite design data and expanding technique to satisfy the new conditions. Lienau found that with a standard roller expander (the tool in common use for producing expanded tube joints), optimum holding-power values were obtained when the seat width was between $1^1/_4$ and $1^1/_2$ in.

The reason for an optimum width of seat existing somewhere between $1^1/_4$ and $1^1/_2$ in. is that in this range the axial move-

[1] Superintendent of Erection, The Babcock & Wilcox Company. Mem. A.S.M.E.

[2] "Uniform Automatic Rolling-In of Small Tubes," by F. F. Fisher and E. T. Cope, presented at the A.S.M.E. Semi-Annual Meeting, Cleveland, O., June 8–10, 1942 (to be published early in 1943).

"Rolling-In of Boiler Tubes," by F. F. Fisher and E. T. Cope, Trans. A.S.M.E., vol. 57, 1935, pp. 145–152.

Contributed by the Power Division, jointly with the Research Committee and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[3] The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

ment of the tube reaches a point at which the keys formed during the tube-expanding process begin shearing off at the outer limits of the tube seat or at the points of greatest movement. Any width beyond these values contributed nothing to holding power and, in most cases, actually reduced resistance to leakage. He further found that the axial movement of the tube within its seat could be used advantageously if grooves were machined in the tube seat into which the metal of the tube could be forced; the axial movement then brought these projections on the tube sharply against the outside edges of the grooves, effectively keying and sealing the joint against leakage and, at the same time, greatly increasing its holding power. With the addition of the grooves, again all experiments indicated an optimum seat width of $1^1/_4$ to $1^1/_2$ in.

Research was conducted into types of grooving, including single, multiple, narrow, broad, and combinations of these. He found that two grooves about $^3/_{32}$ in. in width, $^1/_{32}$ in. in depth, and separated $^7/_{16}$ in. on centers, located in the center of the seat, gave the best all-round results. This width of groove developed sufficient shear values to insure adequate holding power. The depth of groove was kept shallow because excessive rolling was necessary to fill a groove deeper than $^1/_{32}$ in., in fact, it is seldom necessary to roll more than 0.015 to 0.020-in. projection into the grooves.

The two grooves separated by a land were found superior to a wide groove, because the intervening land backed up the tube during the expanding operation and thereby produced a desirable axial movement in the tube between the outside edges of the seat grooves, so necessary for tightness. The holding power of this type joint was not essentially different from one where the land was omitted and a single wide groove used, but its resistance to leakage was materially increased.

It was developed in these experiments that a groove or grooves definitely controlled the direction and amount of extrusion of the tube from the seat during the expanding operation. When placed in the center of the width of the seat, the extrusion is practically uniform and at a minimum in each direction and, therefore, predictable from the standpoint of design. Without a groove or grooves, the tube anchors during expanding at the point of greatest friction in the length of the seat. This results in greater extrusion from one side of the tube sheet than the other. The uncontrolled axial travel is then sufficient to shear away locking keys from the tube and its seat at the point of greatest travel.

The plain tube seat, from $1^1/_4$ to $1^1/_2$ in. in width into which a tube is properly expanded, can be made and maintained hydrostatically tight for any commercial pressure only so long as the forces operating on the joint are due wholly to the pressures within the vessel of which the joint may be a part. Pressures in excess of 10,000 psi are normally used to test such joints for tightness in the development of expanding equipment, and with adequate testing equipment could be carried much higher. However, in the modern high-pressure high-capacity boiler, simple conditions satisfied by plain seats do not exist. Forces due to structural design operate on the joint cumulatively with those produced by hydrostatic pressure. Temperature differentials during operation add further complications to the problem of securing adequate holding power and resistance to leakage. Lienau's investigation of these problems indicated that all joints withstanding pressures in excess of 450 psi, and on which any structural stress was imposed, as well as those subjected to temperatures above 750 F, should be grooved. Cassidy[4] gives values for smooth seats, one-groove seats, and two-groove seats of the type described previously in his paper. Assigning a holding-power value of 100 to the plain drilled seat reamed to size, the same seat with one groove becomes 139 and the double-grooved seat 153. These same relative values are assigned to leakage resistance.

[4] "The Trend of Modern Boiler Design," by P. R. Cassidy, presented at a meeting of the Engineers' Society of Western Pennsylvania, Oct. 15, 1935.

The holding power of any expanded joint is due to the unit pressure existing between tube and seat resulting from the expanding operation, the character of the surfaces in contact, and the area of those surfaces. Due to expander design, as Lienau discovered, tube-joint areas were limited by the maximum $1^1/_2$-in. width of seat, making necessary the counterboring of tube sheets in various ways to limit this width. The counterboring always appreciably reduced the ligament section and added other undesirable complications, among which was the difficulty of expanding typical counterbored joints such as that shown in Fig. 1, and the unbalanced stresses in the tube sheet, resulting from the tube seat being an uncentered fractional part of the total thickness of this sheet.

New Expander Developed

In 1940 an expander was developed by The Babcock & Wilcox Company which overcame these weaknesses of the standard expander and made possible the expanding of tubes in seats of unlimited width. This greatly simplifies the over-all problem, since the seat becomes a plain drilled hole having normally two grooves located near the outer or fire side of the sheet. The tube is expanded the full width of the seat whatever that width may be. There is practically no extrusion ($^1/_{64}$ in. average) from the outer surface of the tube sheet, hence no structural stresses due to extrusion are incurred. The remainder of any extrusion is carried to the inside of the drum; all grooves either those of design or those due to normal machining are adequately filled regardless of location, making possible a joint capable of developing, if desirable, the ultimate strength of whatever tube section may be used. Since the full thickness of the sheet is used as a seat and the holding power of the joint is approximately a direct function of the surfaces in contact, such a joint need not be cold-worked to the degree necessary with former joints to produce equivalent strengths. This joint is shown in Fig. 3. Modifications and combinations of all these joints have been used to meet specific conditions but basically they react according to the values herein set forth.

To produce the expanded tube joint there are three basic tools from which all others are derived, the roll, the prosser, and the ball-drift expander. Of these, the roll expander is by far the most useful and widely used. The prosser has but little use except with low-pressure vessels having very thin tubes and narrow seats. It need not be considered in this paper.

The roll expander has now been used for more than 75 years. It consists essentially of three or more rolls mounted in and separated by a cage. Through the center of this cage and between the rolls a tapered mandrel is forced, thereby moving the rolls radially against the restraint offered by tube walls. By rotating the mandrel, equivalent motion is given to the rolls. In all of these years the roll expander has progressed from the original "taper-rolling" form, through "parallel-rolling" and "self-feeding" types. The self-feeding feature made possible the application of power to the tool, and improvements in the steel of which it is made and in its heat-treatment gave the rolls and mandrels the ability to withstand the great pressures necessary to deform not only low-carbon-steel tubes with wall thicknesses occasionally in excess of $^3/_4$ in., but many of the alloy tubes of equal weight. The roll expander is now an exceedingly rugged and versatile tool, capable of expanding any weight of tube in commercial use.

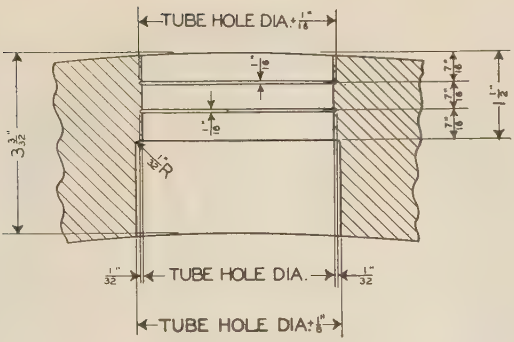


FIG. 1 TYPICAL COUNTERBORED TUBE SEAT

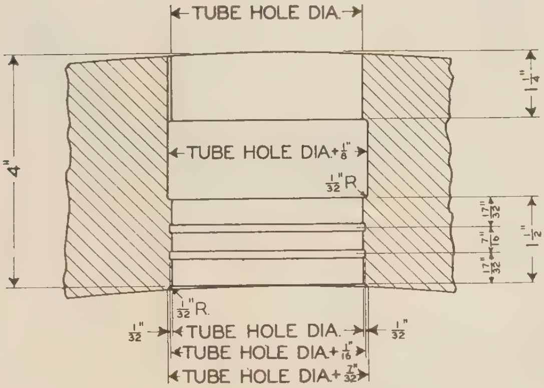


FIG. 2 TYPICAL RECESSED TUBE SEAT

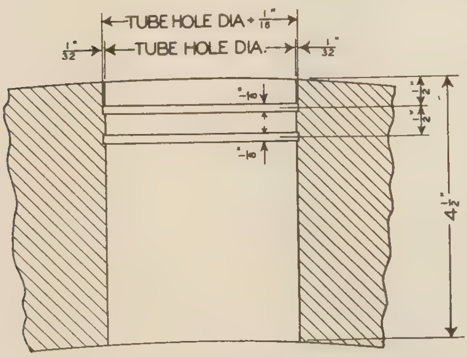


FIG. 3 TYPICAL TUBE SEAT THE FULL THICKNESS OF PLATE, FOR USE WITH RETRACTIVE-TYPE EXPANDER

Goodier and Schoessow[5] bring out an important and significant difference between the action of expanders having a small number of rolls and one which exerts a uniform pressure on the inside of the tube. As shown by their Fig. 4, the zone of influence of the roll expander is concentrated close to the hole as compared with a wide zone of influence (as shown by slip lines) in uniform-pressure expanding.

Prior to 1938, it was the general practice to use four-and five-roll expanders for the majority of applications. With rolls of this number, it was found possible to balance completely the action of the belling rolls. It was found, however, that this balance was unimportant in producing a sound joint, but that the effect in the ligament of the use of the greater number of rolls was distinctly harmful and detracted from optimum tube-joint values. Extensive experiments were conducted by J. P. Craven[6] who measured the plastic flow produced in tube seats of equivalent holding power using expanders of three-, four-, and five-roll designs. The results were so decidedly favorable to the three-roll expander that it was adopted for all except very special work.

It is a well-known fact that roll expanders with a greater number of rolls than three have a tendency to roll any hole in which they may be used to a circle, the degree of exactitude being a function of the number of rolls. Bearing manufacturers have utilized this principle by the design and use of expanders with a great number of rolls to bring journals to exact size and circularity. These multiroll expanders are very useful to boiler-maintenance men in truing up eccentric tube holes, making them as nearly circular as possible without removing ligament strength by reaming. In commercial practice, a drilled tube hole is never exactly round. It is also true that no hot-finished seamless tube is either exactly round or has walls of exactly uniform thickness completely around its periphery. It is also true that the inside circle of the tube is never exactly concentric with the outside. From this it is obvious that the tube hole must be reamed at great expense to make it exactly round, and the tube upset and machined exactly round externally and internally, if an expander having more than three rolls is to give completely satisfactory joints wherein all parts of the tube and its seat are uniformly stressed.

This is not the case with the three-roll expander, since the loading and consequently the stresses produced are exactly the same for each roll, and opposite each roll. With the three-roll expander, it becomes possible to obtain, without any special machining, an optimum expanded tube joint, using ordinary drilled holes as tube seats and the standard hot-drawn seamless tube without any special preparation except that all mill scale must be removed from tube surfaces in contact with tube seats. The removal of all mill scale, oil, water, or foreign material of any kind is vital to the success of any expanded joint by whatever means produced.

MAKING EXPANDER JOINTS WITH THE THREE-ROLL EXPANDER

To illustrate the ease with which expanded joints may be made with a three-roll expander under difficult conditions of "out of roundness" three experiments are submitted. The first, illustrated by Fig. 4, is of a round tube with a wall thickness of 0.387 in., the inside circle of which was bored oval-shaped, the major axis of this oval being $^1/_8$ in. greater than the minor. This tube was expanded into a round tube seat made in boiler plate $1^1/_4$ in. thick. After expanding, the usual tests for tightness were made. The joint did not leak at 10,000 psi, which was the highest pressure that could be applied with the equipment available. In Fig. 4, a white line has been drawn around the tube at its outside and inside peripheries. This white line in each case is a true circle. The discrepancy between it and the tube circle represents the "out of roundness."

In Fig. 5 is illustrated another condition where the hole in the plate forming the tube seat is machined to an oval, again having a major axis $^1/_8$ in. greater than the minor. Into this a round, commercially concentric tube was expanded. This joint was tested to 10,000 psi with no leakage. Again the white line inside and outside the tube indicates true circularity.

A third experiment is illustrated in Fig. 6, in which the tube hole was machined oval with a difference between major and

[5] "The Holding Power and Hydraulic Tightness of Expanded-Tube Joints," by J. N. Goodier and A. J. Schoessow, presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Published in this issue, p. 489.

[6] Assistant to Superintendent of Erection, The Babcock & Wilcox Company, Barberton, Ohio.

Fig. 4 Oval Inside Tube Expanded Into Circular Seat With Three-Roll Expander

Fig. 5 Tube With Concentric Circular Surfaces Expanded Into Oval Seat With Three-Roll Expander

Fig. 6 Tube With Oval Inside Surface Expanded Into Oval Seat With Three-Roll Expander

minor axes of $^1/_8$ in., and the inside of the tube likewise machined to an oval with a difference in major and minor axes of $^1/_8$ in. This tube was then expanded into the plate with the major axes of the ovals at 90 deg to each other. In this experiment, the hydrostatic test still indicated an absolutely tight joint at 10,000 psi.

During the past 4 years the three-roll expander has been used, to our knowledge, to erect 1492 stationary boilers of various sizes, from a small heating boiler to the 900,000-lb per hr 1500-psi public-utility type. A larger number of marine boilers have been erected with it. In 91.3 per cent of the boilers erected, the preliminary hydrostatic tests indicated a few "weeps" in each boiler needing light rerolling. They were passed as "bone" dry on the second test. In 8.6 per cent of the boilers erected, two preliminary tests and one final test were necessary before a bone-dry condition was obtained.

Surface Speeds of Expander Rollers

There are two other serious problems connected with the use of roll expanders. To secure parallel rolling it is necessary that the rolls be tapered. It is quite obvious then that the surface speed of such rolls must differ from end to end in their rotation,

being greater at the large end than the small. Necessarily then, the roll must slip on the tube surfaces being expanded. This slipping occurs at pressures greatly beyond the yield point of the tube, and any surface irregularity in either tube or roll will as a result produce flaking of the tube surface. The remedy for this is to reduce the friction between the roll and tube to an absolute minimum. This can be accomplished by keeping the roll to minimum length and taper, by removing all mill scale and other foreign matter from the surfaces to be rolled, by removing any outstanding metallic irregularities, and by supplying a lubricant whose film characteristics closely approach the lubricant used in the production of cold-drawn tubes, the expanding operation in this respect being quite similarly related to the cold-drawing operation. In selecting such a lubricant, care must be taken that it can be completely removed by boiling out before the vessel goes into operation. No mineral oil or grease has been found that will satisfy the requirements. Castor and other vegetable oils were tried without success. Castor oil makes a reasonably satisfactory lubricant, but after standing a short time on the tube, oxidizes to the point where it is almost impossible to remove by any boiling-out procedure. Most other vegetable oils react in a similar manner. The lubricant found to give best results was an adaptation of that used in the manufacture of cold-drawn tubes, which not only gave satisfactory lubrication, but being water-soluble, was easily removed by the boiling-out process.

Problem of Temperature Differential

The second problem is brought about by a temperature differential between the tube and its seat as a result of the expanding operation. The work done in cold-working the tube plus friction, due to the tapered roll slipping, creates a considerable difference in temperature between the tube and its seat which, depending upon the thickness of the tube, the width of the seat, and the speed at which the expander is operated may range from 10 F to as much as 50 F. Fig. 7 shows graphically what happens to the holding power of the tube as a result of such differentials. Obviously if a temperature differential occurs as a result of rolling, a reduction in holding power is indicated when the tube cools to equilibrium with the seat. Since the only factor, which can be readily and practically controlled in the elimination of this temperature differential, is the rate of rolling; a procedure involving this controlled rate, which will give to the joint its final expansion without an appreciable temperature differential, is required.

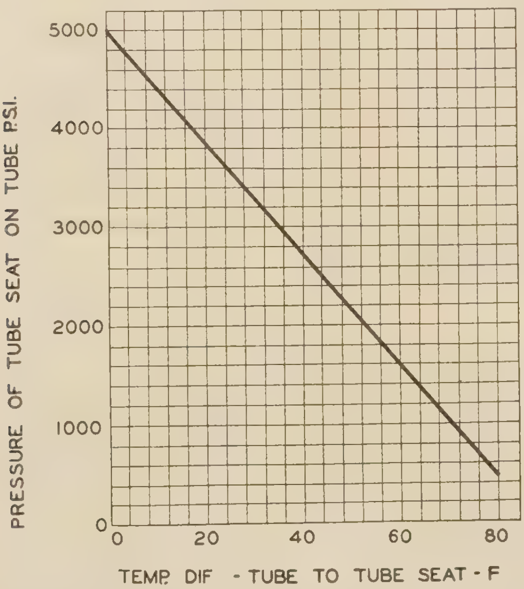


Fig. 7 Effect of Temperature Difference Between Tube and Seat on Seat Pressure

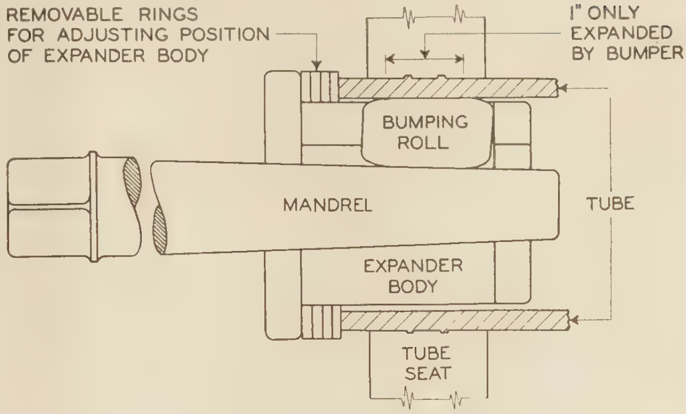


Fig. 8 "Bumping-Roll" Expander

The problem seems to be best solved by doing the expanding of heavy tubes in two or more stages which is found not only to correct the temperature-differential problem, but has other advantages as well. The rough heavy expanding is done by using a so-called "bumping roll" in all tubes in excess of $^5/_{16}$ in. wall thickness. The bumping-roll expander, shown in Fig. 8, is a conventional three-roll self-feeding expander with rolls tapered on each end, in order to leave an effective roll length of about 1 in. which operates exclusively and immediately over the grooved portion of the tube seat. After standing until heat equilibrium is reached, but preferably for a considerable time beyond, a standard three-roll expander covering the full seat width is used to complete the joint, and is manipulated in such manner that no traces of the "bumping" operation remain.

Use of bumping rolls in this way enables full utilization of the principle developed by Grimison and Lee[7] in a companion paper, that, if extrusion of the tube from the seat can be effectively prevented or limited, maximum radial pressure of seat on tube can be developed. It also promotes use of the principle developed by Lienau and referred to earlier, that, if an axial compression can be set up between the shoulders of two grooves in the seat, a tighter joint will result.

The "bumping" expander also materially assists in reducing flaking. It has been pointed out that flaking is caused by the slipping of the expander roll on the tube wall. The longer the roll, the greater difference in surface speed between each end of the roll, and the greater the tendency to flake. By first using a "bumper," the zone of expanding is divided into shorter increments in contact with the rolls, a desirable condition which continues until the finishing standard expander engages its full length.

By the use of the multistage method of expanding, the occurrence of leakage after a boiler is put into operation has been largely eliminated. In three instances, bad leakage was had, as

[7] "Experimental Investigation of Tube Expanding," by E. D. Grimison and G. H. Lee, presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers. Published in this issue, p. 497.

detailed later, which was definitely the result of faulty joint design. Otherwise it has not been necessary in the past 4 years to reroll a tube after it was placed in service on any boiler, within the knowledge of the author.

Retractive Expanding

All that has been set forth in this brief description of the expanded joint and the tools to produce it has been with the premise that such expanded joints are made in seats not exceeding $1^1/_2$ in. in length. With the standard expander, as has been repeatedly set forth, this represents the extreme. This limitation, however, is had with the standard propulsive-type self-feeding roll expander, the only tool available, until 1940, for making roll-expanded tube joints. With this standard tool it is necessary for the expander rolls to be in contact with the tube the entire width of t e seat. The extrusion of the tube beneath such long rolls was so great that it sheared and abraded the outer surfaces of the tube seat, making greater widths than $1^1/_2$ in. impractical.

Every theoretical consideration and cumulative experience with ball-drift expanding of tubes (to be discussed later) in thick plates indicated that, if a joint could be made in which small increments of the tube were expanded progressively from one side of the seat to the other, and where the axial movement of each increment is correspondingly small, the resulting extrusion will largely take place before the remainder of the expanded part of the tube is in heavy contact with the seat. The total extrusion of such a joint would not be changed, but if allowed to occur on the outside of a thick drum would greatly increase the structural loading, a problem which has been discussed previously.

If travel of the roll expander could be reversed, and the expanding operation started from inside the tube progressing toward the open end, the extrusion resulting from such rolling would take place on the open end of the tube inside the drum, increasing the tube projection, but imposing no structural stresses on either the expanded tube or upon its previously expanded neighbors. As the standard expander self-feeds both axially and radially, it could not be reversed, and a new design was necessary, as shown in Figs. 9 and 10. It is essentially a three-roll axially self-feeding expander, in which the radial pressure of the rolls against the tube is produced by a tapered mandrel retracted by a fine screw. The expander is inserted in the tube, as illustrated in Fig. 9, to a point where the rolls engage the tube opposite the outer edge of the tube seat. The screw retracts the mandrel to a point where the tube inside diameter has been increased by a predetermined amount. The mandrel is then rotated in the same manner as with a standard expander, the rolls traveling toward the open end of the tube and extruding the tube material ahead of them into the drum.

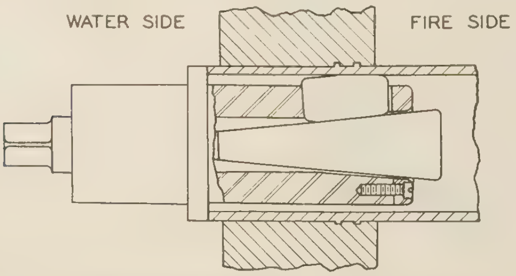


Fig. 9 Retractive-Type Expander; Starting Position

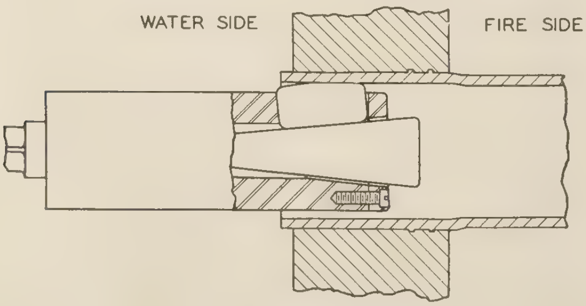


Fig. 10 Retractive-Type Expander; Finishing Position

The mandrel position does not change with relation to the rolls after the initial screw adjustment, thus maintaining the same increase in tube inside diameter and therefore the same radial pressure throughout the seat width. The initial mandrel position can be set to increase the tube inside diameter to a value corresponding to the optimum seat pressure on the tube as determined by Grimison and Lee's experiments.[7] Where variations in clearance and tube thickness are not held to an acceptable minimum for this method, it may have to be supplemented by another indicator such as mill scale cracking on the plate surrounding the seat or by extrusion as measured at the open end of the tube.

The rolls of the retractive-type expander are the absolute minimum of length required for self-feeding. The extrusion of the part of the tube directly under the rolls therefore is small and, if the increase in the tube inside diameter is set by positioning the mandrel to give optimum radial pressure, the effect will not be lost by excessive extrusion shearing off bore marks in the seat. The retractive-type expander develops maximum radial pressures in thick seats, greatly in excess of what would be possible with a propulsive type, as has been indicated by earlier experience.

The first three jobs erected using this type of expander were not tight on initial rolling and had to be rerolled fairly generally to make them tight. It was developed that this failure to achieve initial tightness was due to faulty location of the grooves. At that time the grooves in the seats were located near the inside edge of the seat; i.e., in that portion last rolled by the retractive expander. This meant that the extrusion which took place after a shoulder was formed against the edge of the groove, was sufficient to shear off the tongues of tube metal previously formed in the tool marks in the whole width of seat rolled-in previously.

On all units rolled in after these first three, the grooves were located on the fire side of the seat. This is the portion of the seat width rolled first by the retractive expander. This change in groove location allowed the new type expander to develop fully its fundamental principle. During the last 2 years the new tube seat and the retractive expander have been used on more than 100 very thick high-pressure drums. There has been no difficulty with leakage in this time other than the three cases first experienced.

This tool is limited in application and is essentially for use in drums where plate thicknesses exceed $2^1/_2$ in. It is also not adaptable to tubes under $2^1/_2$ in. in diameter.

Ball-Drift Expander

The expanding of relatively small tubes into thick seats with roll expanders presents several serious problems. As in the larger tubes, the first concern is that of axial movement within the seat. The shearing action between tube and seat materially reduces the holding power of the joint. The axial movement in a large number of cases, where small-diameter straight tubes are used to connect fixed heads, results in deflection or buckling of those tubes.

A second problem encountered in roller-expanding small tubes in thick seats is flaking of the rolled surfaces previously discussed in this paper. A third is the very small size of rolls and mandrels made necessary by the small space in which they must be used. Consequently, breakage and other grave difficulties arise.

If a ball slightly larger than the theoretical diameter of the expanded tube is forced through the portion of the tube to be expanded, the resultant effect is an increase in the size of the tube which approximates the results obtained by use of the roller expander. Obviously, if the tube is to be left at the theoretical expanded diameter for proper holding power and tightness values, the ball must be larger than this diameter because of the resilience of the tube and tube-seat ligament which is being worked elastically. In passing this ball through the tube the effect is that of expanding increments of the tube of infinitesimal length successively from one side of the tube seat to the other. For this reason, all axial movement of the tube takes place in these infinitesimal increments as they are being expanded and the cumulative effect is passed on to that portion of the tube which has not yet been expanded and is, therefore, not yet in contact with the seat. This eliminates the shearing action between tube and seat at the outer edges of the seat.

The ball-drift expander is shown in Fig. 11. The drift may be

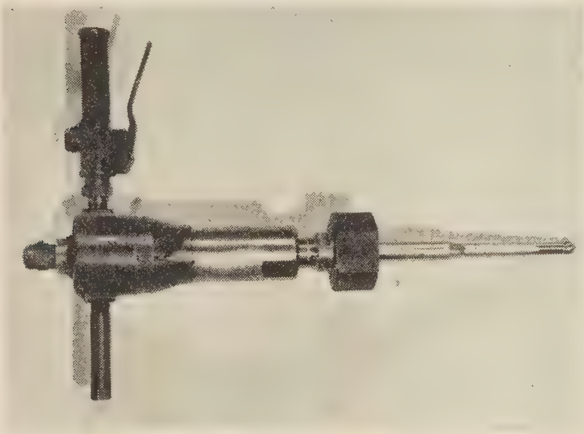


Fig. 11 Ball-Drift Expander and Air Hammer

driven from the open end of the tube, in which case a solid ball may be used. On the other hand, it may be operated retractively, by using a ball modified to a ringlike band, the outer surface being split to allow expansion by a conical mandrel. The entire assembly is placed inside the tube to the proper distance, the ball held rigidly in place and the mandrel pulled through it until the ball section expands to a predetermined size. The entire assembly is then pulled through the tube, expanding it in a manner identical with driving the solid ball into the tube. The important difference is that all extrusion is toward the free or open end of the tube in using the retractive-ball-drift expander. There are therefore no longitudinal forces developed in the tubes. Straight tubes of any length may thereby be expanded into heads of any thickness. This latter property is more important in heat exchangers than in most boiler work.

Since a ball may be considered an expander with an infinite number of rolls, it would be expected to produce a certain amount of plastic flow in the tube seat, if made large enough. These imaginary rolls are infinitesimal in length, and there exists an extremely high reaction against the tube seat around the entire perimeter of the tube hole. The width of this band at any one time is so small with relation to the width of the total tube seat that there is ample ligament to support the reaction and minimize plastic movement of the ligament. Although the ball makes line contact around the inside of the tube, the zone of influence on the tube seat is not a line, but a band of some indefinite width. Due to the variation of strain in the tube walls, the forces exerted in this band vary from a maximum opposite the diameter of the ball, to a minimum some unknown distance away from this diameter. As the thickness of the tube wall increases, the band of reactions higher than yield-point value increases in width on the tube seat. If the ratio of the width of this band to the width of the tube seat becomes sufficiently high, plastic movement of the ligament takes place in the tube seat, and the ball fails to produce a satisfactory expanded joint. This definitely limits the scope of this expanding method to small thin-walled tubes with medium to infinitely wide seats.

Within the scope of its use, this expanding operation which is known as the ball-drift method overcomes the most serious objection to roller expanding by producing an expanded joint wherein the tube is equally well keyed to the tube seat across its entire width. A ball of relatively large diameter which makes line contact only, with the comparatively soft metal of the tube, is used. The ball retains its original surface polish throughout a considerable number of expanding operations. As this ball makes only one or two passes along the tube surface, any tendency toward flaking is not cumulative.

If the surface of the ball is properly hardened and polished, and a suitable type of lubricant used, no flaking of the tube surface is detectable. Actually, the surface of a tube which has been expanded with a ball drift, properly made and used, is left with a mirrorlike polish.

A problem of paramount importance in the use of the ball-drift expander is the selection of the proper size of ball to make a properly expanded joint. This must be determined experimentally, by methods such as those reported by Grimison and Lee.[7] In the small tubes to which this type expander is adapted manufacturing tolerances are sufficiently close so that all tubes of any nominal size and gage can be expanded with a very limited range of ball sizes.

Optimum Expanding

Probably the most controversial issue connected with tube expanding of all types is the practical determination of the point in the expanding operation at which optimum results are obtained. In other words, at what point in the expanding operation shall it be stopped? That point is reached when the seat metal surrounding the tube exerts an elastic reaction slightly below the elastic limit of the metal. Goodier and Schoessow[6] have shown that to have this result after release of expanding pressure and withdrawal of the mandrel, it is necessary to deform the seat plastically by application of the expanding pressure. This is often undesirable because of its effect on adjacent tube holes and expanded joints, and a better result is attained by expanding somewhat short of the maximum radial pressure and using a wider seat to compensate for the lower seat pressure.

The practical measurement of this reaction in the tube seat is a very difficult problem for production expanding, as distinct from experimental work. It is therefore necessary to use empirical methods which will give results within the limits of commercial practice and tolerances. Fisher and Cope[2] have stated that the accurate measurement of the extrusion of the tube is a sufficient indicator. With this the author is in entire agreement, but only so long as the conditions under which each individual tube is rolled are identical and the total extrusions of the tubes in both directions are measured. Hardnesses of tubes and seats must be identical. The point of anchorage in expanding each tube, which determines the direction and amount of extrusion, must be the same. Each tube must be free of restraint to an equal degree outside each tube seat rolled, a condition seldom approached in any boiler design. The tube diameters and tube holes should be approximately the same size for each joint and the manipulation of the extensometer must be accurately done. Within a labora-

tory where all of these conditions are controllable, the measurement of extrusion is a tool giving almost an exact measure of the holding power of the tube. In the field, where any one or all of these conditions may vary, widely divergent results occur, which make necessary the rolling of all joints to meet the most unfavorable conditions.

Grimison and Lee[7] advocate the use of an increase in tube internal diameter (after contact) a device represented practically by mandrel travel. This is also recommended by Braun and Fleischmann.[8] This method is satisfactory when the tubes and seats are accurately machined or drawn to size. This is the case in naval boilers, using cold-drawn tubes in accurately drilled and double-reamed holes. In roll-expanding such tubes, mandrel travel is used, and, in drift-expanding its equivalent, judicious selection of size of the ball drift.

Lienau, Seibel, Oppenheimer, and other experimenters have tried measuring power input to an expander, correctly assuming that if conditions could be maintained uniform, the holding power of the joint was a function of the power input to an expander. It was evident to all of them that every variable between joints was reflected in a change in power requirement, making it probably the least accurate of all indicators. If every condition of surface, size, hardness, and friction can be held uniform, this method can be used as shown by Fisher and Cope[9] in their work with small, nonferrous tubes in heaters.

From the earliest use of the parallel-roll expander, the experienced skilled mechanic has watched with careful attention the appearance of scale-cracking at the edge of the tube hole during the expanding operation. This invariably indicated, in an accurate way the beginning of plastic flow at that point. Grimison and Lee's observations confirm the belief that the beginning of cracking of the mill scale around the seat indicates closely the attainment of the optimum degree of expansion. Goodier and Schoessow indicate the necessity of carrying plastic deformation into the seat to get maximum seat pressure. These theoretical and experimental observations confirm our belief in the cracking of mill scale as a reliable, but approximate, indicator of attainment of the optimum degree of expanding. Duplicate boilers, in the same location, erected by the same men, but using scale-cracking on one as opposed to different methods of indicating proper expanding, have shown the superior results obtained by observation of this simple phenomenon.

[8] "The Rolling-In of Upset and Close Tolerance Machined Tubes for Cracking Furnace Installations," by F. C. Braun and M. Fleischmann, *Refiner and Natural Gasoline Manufacturer*, vol. 19, July, 1940, pp. 53–59.

[9] Reference (2), first item.

Fig. 12 Mill Scale Cracking Around a Normally Expanded Tube

Fig. 12 shows the appearance of the metal around a normally expanded tube. This tube joint did not leak at 10,000 psi and failed in tension at 69,000 lb axial pull.

As pointed out in a companion paper, this method is applicable only when mill scale is present, which is not always the case. Considerable experimentation has been carried on, under the author's direction, in the use of brittle varnishes and other brittle coatings to take the place of mill scale. The difficulty encountered has been the variable effect of the temperature produced by rolling, on the coatings. Mandrel travel or equivalent measures must be accepted as the most accurate indicator in such cases.

Discussion—Expanded Tube Joints[1]

P. R. Cassidy.[2] Most experienced men who expand tubes judge the extent of proper expanding by what they call the "feel" of the expander. What they really mean is an integration of experience which indicates probably sound results. At times such men expand a tube which leaks on the hydrostatic test. Their reason will be that there was something wrong with the tube, the seat, or the expander.

Mr. Maxwell has stated in his paper [1a] that with the retractive expander and a properly designed seat, more than 100 drums for high pressure have been expanded in the last 2 years without a leak. While the record on lower-pressure drums below the range of retractive expanding is not perfect, Mr. Maxwell states there has been no leak in initial service in such boilers for 4 years.

The research covered in Messrs. Grimison and Lee's paper[1b] has been in progress for about a year. As in any basic research, they followed a few false scents before they found the right trail. They have indicated the need for additional work and have shown the direction to follow.

Messrs. Goodier and Schoessow[1c] have clarified the theory of stress-and-strain distribution in tubes and seats. The writer has not found practicable conclusions or useful correlation of facts in the bibliography referred to, particularly the foreign.

It was desired to tabulate the probable distribution of yield point and hardness of hot-finished and cold-drawn boiler tubes, grade A and medium carbon. It was expected that the upper and lower limits, and the mean, as affected by the range in chemistry and history of production would be indicated. We found that the amount of work involved would interfere with tube production and therefore could not be tolerated under war conditions. It is hoped that a return to normal conditions will permit correlation of such data with radial pressure for a given geometry of tube and seat and a given method of expanding. It should be unnecessary to expand a tube to predict the probable radial pressure, and the range between limits.

Please note that these three papers are signed and presented by the men who have done the work. They have had advice and guidance only where needed or asked for. The control of the research and direction of action was by a committee consisting of the authors and a few others. The results confirm the soundness of the method.

E. T. Cope.[3] The practical aspects of tube rolling of necessity take precedence over the theoretical because the heat exchangers in which the tubes operate are a joy or a grief to plant men in direct proportion to freedom from trouble. Often the rolled joints are the seat of such trouble. The writer is glad to discuss Mr. Maxwell's paper[1a] because most of his experience has been with the purely practical side of tube rolling.

The author states that the maximum useful length of a rolled boiler-tube joint is $1^1/_2$ in. This conclusion does not agree with the conclusions of Oppenheimer,[4] of Seibel,[5] and of the writer and his associates.[6,7] Oppenheimer states:

"It was found by tests.....that the holding force depends on the following factors:

"2 It is proportional to the thickness of the plate...."

Siebel concludes:

"It was found that the pressure at which the joint leaked or loosened increased with the thickness of the plate."

Results of the work done by the writer and his associates are shown in Table 1 of this discussion.

TABLE 1 EFFECT OF TUBE-SHEET THICKNESS ON HOLDING STRENGTH OF JOINT

Tube diam, in.	Tube wall thickness, in.	Tube sheet[a] thickness, in.	Holding strength, psi
$3^1/_4$	0.180	1	2800
$3^1/_4$	0.180	$2^1/_4$	4250
4	0.180	1	1100
4	0.180	$2^1/_4$	3800

[a] The joint extended entirely through the plate.

The writer agrees in principle with the author's statements regarding the effect of grooves in the tube-sheet hole. He has reported his findings elsewhere.[8] The double V-groove with a narrow land between the V's, placed at the center of the length of the tube seat, was most effective.

The "new expander" described by the author finds its counterpart in one illustrated in a German catalog[9] bearing the date 1934. A similar tool was made in 1939, by a prominent American manufacturer of power-station equipment. Neither of these tools appears to have had wide use in this country.

Advocacy of the three-roller expander, voiced strongly by the author, does not find support in the writings of European engineers. Two of these are quoted. Thum and Jantscha[10] arrive at the following conclusion:

"With the rolling methods used today (1930), however, the best degree of rolling cannot always be produced with certainty ....an increase in the number of rollers with consequent more uniform distribution of the roller pressure is advantageous."

And further:

"The attempt to reduce the permanent deformation of the holes in the header (tube sheet) has been successful with the five-roller expander. The five-roller expander is especially to be recommended where, on account of a thin sheet, and on account of a material that does not work-harden much, the seat would give too much; it is also to be recommended wherever the hole has already been considerably expanded through frequent changing of boiler tubes."

Lieberherr[11] arrives at the following conclusion:

[1] Papers to which these discussions apply are as follows:
(a) "Practical Aspects of Making Expanded Joints," published on page 507 of this issue of the Transactions.
(b) "Experimental Investigation of Tube Expanding," published on page 497 of this issue of the Transactions.
(c) "The Holding Power and Hydraulic Tightness of Expanded Tube Joints: Analysis of the Stress and Deformation," published on page 489 of this issue of the Transactions.

[2] Lieut. Colonel, Corps of Engineers. Mem. A.S.M.E.

[3] Research Department, The Detroit Edison Company, Detroit Mich. Mem. A.S.M.E.

[4] "Rolling Tubes in Boiler Plates," by P. A. Oppenheimer, *Power*, vol. 65, Feb. 22, 1927, pp. 300–303.

[5] "Walzerbindungen," by Erich Siebel, *Stahl und Eisen*, vol. 53, Nov. 23, 1933, pp. 1205–1215.

[6] "Rolling-In of Boiler Tubes," by F. F. Fisher and E. T. Cope, Trans. A.S.M.E., vol. 57, 1935, pp. 145–152.

[7] "The Latest Method of Rolling Boiler Tubes," by F. F. Fisher and E. T. Cope, Proceedings of the 13th General Meeting, National Board of Boiler and Pressure Vessel Inspectors, 1941, pp. 98–110.

[8] Ibid., Fig. 6.

[9] "Apparate und Maschinen für die Rohrbearbeitung," Kotthaus and Bush, Remscheid, Rheinland, Germany, 1934.

[10] "Einwalzen und Einpressen von Kessel- und Überhitzerrohren bei Verwendung verschiedener Werkstoffe," by A. Thum and R. Jantscha, *Archiv für Wärmewirtschaft und Dampfkesselwesen*, vol. 11, 1930, pp. 397–401.

[11] "Die Beanspruchung der Trommeln eines Wasserrohrkessels," by A. Lieberherr, *Schweizerische Bauzeitung*, vol. 102, Winterthur, Switzerland, 1933, pp. 87–91.

"It will also be of advantage in such cases (boilers for higher pressures) to use tube expanders with 6 instead of 3 rollers, and to use an arbor with small taper in order to avoid overstressing the sheet (tube sheet)."

The writer has made no tests on expanders having different numbers of rollers; however, the five-roller boiler-tube expander was adopted several years ago as standard in the company with which the writer is associated, and very satisfactory results have been obtained by the use of it. On the other hand the three-roller expander is used for small tubes such as are found in condensers and feedwater heaters. Dimensional limitations dictate the choice of this tool.

The writer does not recognize any difference in principle between the rolling of boiler tubes and tubes in condensers, feedwater heaters, or other types of heat exchangers. In consequence, he sees no reason for a tool such as the ball-drift expander for use in installing the small tubes in condensers and feedwater heaters. This feeling is premised on the fact that condenser tubes $^3/_4$ in. in diam and 0.050 in. wall thickness have been rolled-in in great numbers in the plants of the company by which the writer is employed by the use of a three-roller parallel self-feeding expander tool. The rolled portion in each case was $^3/_4$ in. long and the load necessary to push out a rolled tube was of the order of 1400 lb. This was equivalent to a holding strength of about 3200 psi. The rolling-in of these condenser tubes is described in a recent paper.[12] By using the method of rolling-in small tubes described therein, in which the maximum current input controls the degree of rolling, tubes of any commercial hardness and of normal variations in diameter may be expanded uniformly and automatically. In the light of these results obtained on rolled-in small tube joints, the writer questions the value of a procedure that necessitates the production of joints of such length as are indicated by the author's description.

The author has endorsed the Fisher-Cope method for the control of tube rolling, the "elongation method," described in references 6,7 and mentioned in reference12. However, he names limitations that he says restrict the use of it. The author is in error in his choice of limitations at least in so far as $^3/_4$-in. condenser tubes are concerned. In the closure to their paper,[12] the authors included the results of tests on four groups of condenser tubes. These results are given in Table 2 of this discussion.

Subcommittee No. 6 of the Joint Research Committee on Boiler Feedwater Studies.[13] Mr. Walker states:

"....but there seems to be complete agreement on the idea that caustic embrittlement can occur only when the following three conditions exist simultaneously:

(*a*) Certain chemical conditions of the boiler water.

(*b*) Physical conditions which permit high concentration of the dissolved solids.

(*c*) Contact of the concentrated solution with highly stressed boiler metal."

And further:

"Contact of the concentrated solution with highly stressed metal is a requirement which has been thoroughly demonstrated. Cracking has not been observed unless the stress is near or above the yield point. .... Previous cold working of the steel renders it more susceptible. .... The rolled-in tube ends, however, remain a vulnerable point."

The truth of these statements, regarding the susceptibility of a severely cold-worked steel tube has been shown elsewhere.[14] The ability to reproduce the conditions shown in these illustrations is also shown.[15]

Tube rolling, to be effective and lasting, should be done with due attention to the cold working of the metals involved. This feature the author has neglected to mention but it is to be hoped that he takes it into account in his new method of rolling.

The cracking of the scale around the tube hole, mentioned by the author, has been observed and used as an indicator of degree of rolling since early in tube-rolling history. It is hardly a good indicator even when it can be used, and it fails completely in the case of pickled drums and tube sheets, and in heat exchangers having brass or bronze tube sheets.

Considering now the paper by Messrs. Grimison and Lee,[1b] the writer does not agree with the major premise of the paper, viz., that seat pressure is the most important variable involved in a rolled joint. He will discuss this feature and others of this paper from the point of view of individuals who own and operate boilers and other heat exchangers. He believes with the authors that the rolling of joints will be improved as more knowledge is acquired by testing actual joints under controlled conditions and by abandoning the hunches, opinions, and guesses often employed.

TABLE 2 RESULTS OF ROLLING-IN $^3/_4$-IN. CONDENSER TUBES

Item	Tube diam, in.	Hole diam, in.	Hardness of tubes before rolling, Rockwell E	Elongation, in.	Push-out load, lb	Hardness of inside of rolled section, Rockwell E	Current to expander: Idling, amp	Current to expander: Maximum, amp
1	0.752	0.755	68	0.008	1728	90	0.82	1.25
2	0.748	0.760	67	0.008	1630	92	0.82	1.25
3	0.751	0.761	58	0.009	1670	88	0.82	1.25
4	0.750	0.760	77	0.007	1960	95	0.82	1.25

Item 1 of Table 2 was a combination of the largest tube of average hardness in the smallest hole; item 2, the smallest tube of average hardness in the largest hole; item 3, the softest available tube in the average hole; item 4, the hardest available tube in the average hole. An automatic current-limiting relay was used to terminate rolling. The results show that in these cases the measured elongations were in close uniformity; also the strengths of these joints agreed well. There appears to be no reason why similar results could not be obtained with feedwater-heater tubes or boiler tubes.

In his paper the author has not described a means for controlling the degree of rolling, nor has he mentioned the effect of the rolling on the tube and sheet metal. That this is a most important matter is pointed out very clearly by J. H. Walker in his report of

The selection of seat pressure as a criterion of joint strength was the natural one to make because the authors undertook to verify experimentally the assumptions made by Goodier and Schoessow.[1c] These were purely theoretical assumptions and required the second theoretical assumption that the surfaces involved were perfectly smooth and uniform. In the joints of an actual boiler, the seating surfaces are neither smooth nor uniform. In their tests, the authors made joints between surfaces that were neither smooth nor uniform. It is regrettable that they did not make the surfaces on these first joints smooth, perhaps even semipolished. Had they done this, their results would not have been clouded by the presence of at least two dominating

[12] "Automatic Uniform Rolling-In of Small Tubes, by F. F. Fisher and E. T. Cope, Trans. A.S.M.E., January, 1943, pp. 53-60.

[13] "Caustic Embrittlement Research Brings Results," by J. H. Walker, *Mechanical Engineering*, vol. 64, 1942, pp. 891–893.

[14] Closure ref. 6, Figs. 3 and 4, Trans. A.S.M.E., vol. 58, 1936, p. 69.

[15] Ibid., Figs. 5 and 6.

variables, namely, seat pressure and condition of the seating surfaces. The latter variable markedly affects the resistance of the surfaces to sliding and in consequence greatly influences the resistance of the joint to hydrostatic pressure.

The influence of the condition of the seating surfaces is emphasized by Oppenheimer[4] in these words:

"It was found by tests and also by mathematical analysis that the holding force (strength) of a shrunk fastening, the tube being simply rolled and not extending beyond the plate, depends on the following factors:

"1 It is directly proportional to the coefficient of slipping friction between tube and hole wall; that is, the holding force increases as the surfaces of tube and hole wall are rougher."

The writer has verified this fact quantitatively in a few tests that he made on 4-in-diam tubes, 0.208 in. wall, 70 Rockwell B hardness (124 Brinell) rolled into 1-in. plate. The tube holes were smoothed, bored to commercial tolerance for 4-in. tubes, and the tube ends were all cleaned with a commercial mechanical cleaner which removed all mill scale and smoothed out the pits. The ends of some of the tubes were further smoothed to a semi-polish, then all were rolled-in to optimum elongation, about 0.025 in. All of the joints were then broken by hydrostatic pressure, with the following results:

Tube ends semipolished, joints failed at 2033 psi.

Tube ends not semipolished, joints failed at 3700 psi.

From the practical point of view, seat pressure is a purely academic question but an interesting one. However, the holding strength of a joint, the strength to resist hydrostatic pressure, is the most important characteristic. It has real meaning to the engineers who design the boiler and also to those who operate and maintain it. Holding strength, expressed in pounds per square inch, as the writer and his associates have used the term, is used directly in determining the factor of safety of a joint.

The values of seat pressure, shown in Table 3 of the paper,[15] were not as consistent as the authors probably hoped for. Some of these results are in poor agreement; others are in satisfactory agreement. If one inspects Table 1 of the paper, he finds the results shown in Table 3 of this discussion.

TABLE 3 TOTAL EXTRUSION VERSUS SEAT PRESSURE

Joint no.	Total extrusion, in.	Estimated seat pressure, psi
134	0.044	5190
135	0.045	2220
63	0.076	5290
107	0.071	7810
68	0.219	4360
139	0.208	6870

The lack of agreement in these pairs of results is serious. On the other hand, the results shown in Table 4 of the paper[15] are in good agreement. Could the joints reported in Table 4 have been rolled at a later date than those reported in Table 1 after experience had been acquired? For comparison with the authors' tables of test and calculated results, the writer submits the observed results, Table 4 herewith, of tests made in the course of his work.

TABLE 4 JOINTS ROLLED-IN BY ELONGATION METHOD

Assembly no.	Elongation, in.	Holding strength, psi	Reduction in tube-wall thickness, per cent
F 1	0.010	1550	3.2
F 2	0.010	1625	4.8
F 3	0.010	1400	5.3
F 4	0.020	1475	3.7
F 5	0.020	1550	5.9
F 6	0.020	1250	3.2
F 7	0.030	2000	7.9
F 8	0.030	2150	7.5
F 9	0.030	1875	9.5
F 10	0.040	2250	13.9[a]
F 11	0.040	2200	8.6
F 12	0.040	2000	9.7

[a] Apparently some feature of this rolling was incorrect. It was especially noticed that the tube end protruded slightly beyond the tube sheet.

The writer has inspected the authors' results to ascertain where in the relationship of partial extrusion on the hot-gas side of the tube sheet is "not in good agreement" with the total extrusion. He finds that in 75 per cent of all the joints reported upon, the agreement is acceptable. If the joints reported upon in the authors' Table 4 are considered alone, 88 per cent are in acceptable agreement.

Some of the total extrusions reported appear to be very great; so great indeed that the metal of tube and sheet must have been left in a very severely stressed condition. Among these are joints listed in Table 5 of this discussion.

TABLE 5 EXCESSIVELY ROLLED JOINTS TAKEN FROM AUTHORS' TABLES

Joint no.	Total extrusion
60	0.407
65	0.153
68	0.219
127	0.179
139	0.208
141	0.161

The effect of excessive rolling is pointed out in a recent paper.[7] This paper shows[16] that excessive rolling results in an increase in hardness of the tube wall to a degree that it is believed will impair its usefulness and possibly render it liable to corrosion attack.

Other writers mention the effect of overrolling. The following quotation is taken from a translation of the paper by Thum and Jantscha:[10]

"The holding strength of the rolled joint reaches a maximum for a certain degree of rolling, and it decreases again after that more or less but always very definitely. From this the conclusion must be drawn that too much rolling has a distinctly unfavorable effect upon holding strength."

Lieberherr[11] observed the same fact and wrote the following: "It was found that the moderately rolled tubes, ... , remained tight or sweated only slightly, while those which had been rolled excessively showed leaks."

It is apparent that excessive rolling can lead only to unsatisfactory joint conditions. Furthermore, uncontrolled rolling leads to an unknown degree of rolling.

The authors seem to be somewhat confused in the following statements:

"The maximum seat pressure was believed to coincide with the maximum structural strength of the joint."

"The correlation between the tube-seat pressure and the per cent increase in the tube inside diameter over that measured at tack rolling is very poor . . ."

"The increase in the inside diameter of the tube over that at contact of tube with seat was found to be quite a satisfactory indicator of degree of rolling."

From these three statements the reader is led to conclude, (1) that seat pressure is a good indicator of the strength of a joint; (2) that the increase in the inside diameter of the tube over that at contact is quite a satisfactory indicator of degree of rolling; but (3) "the correlation between seat pressure and increase in the inside diameter of the tube of that at tack rolling is poor." These three statements are very confusing to the reader and need modification in order to clarify them.

The changes which occur when a tube is rolled-in, whether it is in a condenser, a feedwater heater, or a boiler tube, are very complex and the laws governing them are not too well under-

[16] Reference 7, Fig. 7.

stood. These changes depend upon the following physical characteristics of the tube, the tube sheet, and the tool:

(a) Initial hardness of tube and of tube sheet.
(b) Work-hardening characteristics of both metals.
(c) Outside diameter of tube.
(d) Thickness of tube wall.
(e) Diameter of tube hole.
(f) Thickness of tube sheet.
(g) Length of rolled joint.
(h) Angle of taper mandrel of tool.
(i) Lead angle of tool rollers.
(j) Revolutions per minute of expander tool.
(k) Degree of rolling.

The problem of tube rolling can best be simplified by a systematic attempt to treat one of these characteristics at a time holding all others as nearly constant as possible. Only by following this procedure can we hope for an eventual solution of this complex problem.

The writer in all of his work on tube rolling has considered the ability of the joint to withstand pressure as the most significant characteristic of the joint. Repeated tests and field applications have strengthened this point of view. On the other hand, the authors have made an experimental study of an academic question, selecting seat pressure as the important item. As an academic question, the work is worth while, but it is regrettable that they apparently did not hold other variables under close control. Had they done this, the writer believes that their contribution to the science of tube rolling would have been more valuable.

F. F. Fisher.[17] It is indeed gratifying to learn that the expanded joints in the boilers and heat exchangers of power plants along with the tools and procedures involved are finally receiving the attention they justly deserve. Mr. Maxwell's paper[16] is a welcome and creditable presentation of the complex problems involved and of necessity covers only the important phases of the subjects. The writer, however, does not agree with some of the views expressed and will therefore undertake to discuss the ones questioned.

According to the author, Lienau's investigation made in 1924, the results of which were not published, caused a redesign of the groove positions in the joints. There is no disagreement with the statements regarding the function and placing of grooves, but the citing of Lienau's paper as a source of information concerning the anchoring of tubes and the control of extrusion does seem to have been somewhat delayed.

In the author's review of the history of tube expanding, no advance in the art is recorded for the years from 1924 to 1938; however, others were contributing during this period, and a short review of this work done will be of interest. Among others is The Detroit Edison Company which in 1926, initiated a number of improvements in tools and procedures in order to alleviate an unsatisfactory condition existing in boiler erecting at that time. This condition may well be described as "trying to assemble high-pressure boilers, using tools and methods developed for low pressure."

This work began with a search for a steel of the correct composition and heat-treatment, capable of withstanding the more severe usage imposed by the stronger and thicker materials involved. The result of this effort was a suitable steel, which was recommended to the boiler and expander manufacturers in 1929, and which has been in general use since.

Another problem encountered was the removal of scale from and the cleaning of the tube ends preparatory to expanding. This was a job of considerable magnitude and, as usually done, resulted in many leaky joints, causing much rerolling. An efficient tube cleaner was developed by the writer to do this work. This tool, called the Rotary Tube-End Cleaner, removes all the mill scale and many irregularities on the tube ends and produces a uniform surface finish at a fraction of the cost of former methods.

In June, 1935, a paper was presented by F. F. Fisher and E. T. Cope,[6] which represented 5 years of investigation and observation of tube rolling and called attention to the axial elongation of the tube metal during expanding. It demonstrated that elongation can be used as an indication of the degree of expanding. The paper also evaluated the forces developed by the elongation and described means and procedures for their compensation or elimination in the boiler structure. Improvements in expanders are also suggested, such as a curving of the entrance ends of the expander rollers in order to eliminate a common cause for tube failures, namely, the excessive cold working of the metals forming the joint.

The design of the expander is not necessarily a limiting factor in the length of a joint, as stated by the author, but the joint may be of any length if the expander used for oil-still tubes is applied. This expander is the forerunner and the counterpart of the retractive expander described by the author, but with the tapers reversed. It is used in the manner as shown in Figs. 9 and 10, of the paper.[16] Even the ordinary standard self-feed expander is capable of rolling a satisfactory joint at least $2^1/_4$ in. in length. This has been especially true since most manufacturers have substituted precision-ground rollers for the rough and unground ones previously used, which cut into the tube metal and caused serious flaking. Still longer joints may be rolled by substituting rollers made in segments for the one-piece kind. However, as the optimum strength of the joint is reached at $1^1/_2$ in., as stated (Thum and Mielentz,[18] arrive at a length of $1^7/_8$ in.), and as this length also offers a resistance to bending equal to a solid connection, as demonstrated by Ruttmann,[19] it seems hardly necessary to go beyond this joint length except in special cases.

The writer holds no brief for either the three- or the five-roller tool, but does not believe that a three-roller expander is superior to one having a greater number of rollers. The author's citation of Fig. 4 in the Goodier and Schoessow paper[20] showing slip lines in expanded tubes, as evidence supporting the three-roller tool, appears to be somewhat confused. The upper view depicts the action of any expander whether having three, four, or five rollers, whereas the lower view shows the slip lines obtained by expanding with hydraulic pressure. In fact, the authors (Thum and Jantscha) of the paper[10] from which this illustration was taken are very much in favor of expanders with more than three rollers. This is very significant, inasmuch as they recommend a change-over from the three-roller expander commonly used in Germany to one having five rollers.

The writer cannot accept the author's reasoning regarding the use of a single belling roller, as is used with the three-roller expander. This tool is not nearly as stable as one having two belling rollers which are balanced or nearly balanced as found in the four- and five-roller expanders. His experience in the use of these tools runs contrary to that of the author.

[17] General Foreman of Mechanical Erection, The Detroit Edison Company, Detroit, Mich.

[18] "Das Verhalten von Einwalzstellen bei verschiedenen Ausbildungsformen und Betriebsbedingungen," by A. Thum and J. N. Mielentz, *Materialprüfungsanstalt* Darmstadt, Sonderaus gabe der *Mitteilungen, 55 der Vereinigung der Grosskesselbesitzer*, E. V., Dec., 1935, pp. 258–290.

[19] Über das Einwalzen von Rohren und der besonderer, Berucksichtiegung der Frage der Rundrisse in den Einwalzstellen von Siederohren," by W. Ruttmann, Dissertation for Dr. Ing., Technische Hochschule, Darmstadt, 1933.

[20] Ref. 1 (c), Fig. 4.

The tendency of three-roller expanders to correct the "out of roundness," as shown in Figs. 4, 5, 6 of the paper, is well known. In order to get the results shown, it is evident that the metal must be cold-worked to a high degree. This is not a desirable feature and is the reason why this tool is not favored by the writer. Although the surfaces involved in a joint rolled with a four- or five- roller expander may not be concentric, what difference does it make? The A.S.M.E. Boiler Code specifies that the minimum thickness of boiler tubes shall be not less than the nominal gage specified. Consequently, there is but little likelihood that the holding strength of any joint, even if eccentric, will be less than that of concentric joints of nominal gage.

Regarding the use of unreamed tube holes as recommended by the author, a drilled rough tube hole necessitates increased rolling, in order to produce tight joints, whereas the small additional cost of reaming the tube holes is more than offset by a reduction in the cost of rolling. Further evidence of the advantage of the use of smoother contact surfaces are found in other references. Siebel[5] found it difficult to produce tight joints when using tube-hole surfaces consisting of fine threads. Thum and Mielentz[18] found that the stability of a joint was increased about 35 per cent when using smoothly turned tube ends instead of tube ends that were cleaned by sandblasting.

The unpublished conclusions reached by J. P. Craven, as stated by the author, leading to the adoption of the three-roller expander by the author's company, are of great interest because they are contrary to the views of Thum and Jantscha[10] and Lieberherr.[11] Therefore, because of the fact that these authors disagree radically with the conclusions of J. P. Craven, it is highly desirable that the author explain what characteristics of the joint were adversely affected. Also what measurements were made and what tools were used to prove the detrimental effect on ligaments resulting from the use of rolling tools having more than three rollers.

The ball-drift expander described by the author is an adaptation of a process called "auto-frettage," once used to harden the bore of guns. Of the two types listed, the solid-ball type is the better because it produces a clean smooth hole in the tube end. The retractive type must have of necessity means for adjustment, as it must enter a small tube hole and then expand it to a larger tube inside diameter. Because of this the ball is split. On expanding, the two halves move apart, leaving a gap on both sides of the ball.

If and when the expanded ball is pulled through the tube, that portion of the tube located between the two halves of the ball is not expanded by one pass of the expander. However, a portion of the tube metal displaced by the ball flows into this unworked section and hardens this metal to an appreciable extent. Thus, when the ball is rotated 90 deg, and the second draw is made to finish the joint, two strips of metal harder than the other expanded tube metal will be produced equal in width to the gap between the ball halves.

A famous scientist once made a statement to the effect that any problem however difficult ceased to be a problem and became exact knowledge when suitable means of measuring were applied. According to the author, a measure is now available which is applicable to the problem under discussion, namely, optimum tube expanding. It is set forth by the author as follows: "The measurement of extrusion is a tool giving almost an exact measure of the holding power of the tube." The same view is held by Grimison and Lee,[1b] and Fisher and Cope.[6,7,12] This fact being established, there seems to be no valid reason why tube rolling cannot be put on a scientific basis. Since the extrusion has such definite value as a measure of the degree of expanding, its potentialities should be examined and developed to the fullest extent before adopting empirical, arbitrary indicators which have but limited applications and are often of little value in improving the art of tube rolling.

Since the introduction of "extrusion or elongation" as a measurement of the degree of expanding, considerable progress, based on this measurement, has been made in tube rolling, and much definite knowledge has been acquired. Advantage of this knowledge is taken in numerous cases as is demonstrated in the author's paper.[1a] For instance, a retractive expander has been developed to eliminate troublesome elongation incident to rolling tubes into long tube holes.

Some of the limitations cited are not real, are overemphasized, or could, if found necessary, be corrected by changes in the rolling technique or in the tools. A study of the test data in the Fisher and Cope papers[6,7,12] discloses the fact that the partial extrusion is applied successfully to tubes and tube-sheet combinations of various hardness, of differences in size of tube hole and tube or tube wall thickness, provided they are within commercial limits of tolerance or are as specified by the A.S.M.E. Boiler Code and are of commercial finish.

A value of the force transmitted by the extrusion to restraining members is evaluated by Fisher and Cope,[6] and there seems to be no reason to believe that the extrusion is affected by any restraining members, excepting possibly in the case of very short and stiff tubes.

A discussion of other means of indicating the degree of rolling mentioned in this paper[1a] is contained in the writer's discussion of the companion paper of Grimison and Lee,[1b] which follows:

It is unfortunate that the investigation undertaken by Messrs. Grimison and Lee was not carried through to completion because the conclusions drawn from a partial investigation may be at variance with the results of a more extensive test. German technical literature contains several reports by authorities paralleling the proposed investigation and covering every phase of the objectives, excepting the idea of using "extrusion" as the measure of the degree of expanding. The use of "extrusion" as a measure of rolling is an American development and was explained in 1935, in a paper by Fisher and Cope.[6] In this paper the term "elongation" was used instead of "extrusion" to describe the axial flow of the tube metal during the expanding procedure, a procedure also known as the rolling-in of tubes.

The authors do not divulge information which is not otherwise readily available, nor do they disclose a better or more practical solution of the tube-rolling problem. However, there is one conclusion to be drawn from its contents, namely, that the making of a tube joint causes a series of changes of such complex nature that the strength of the joint cannot be calculated with any degree of accuracy. Would it not be simpler and less confusing if the criterion of joint strength consisted of the actual joint strength as measured by a destructive hydraulic test? This, together with the stability of a joint, is of paramount interest to the designer and boiler owner.

In their investigation, the authors encountered and were surprised by a sudden reduction in tube-seat pressure which is analyzed as due to a smoothing of the contact surfaces. This behavior should be no surprise. It is explained by Maxwell,[1a] who quotes Lienau's unpublished investigation for the Babcock & Wilcox Company in 1924, a source of information also accessible to the authors. Erich Siebel[5] also depicts and comments on the phenomenon.

The behavior described also demonstrates the influence of the surface conditions on the strength of the joint and raises a question regarding the authors' assumption that the coefficient of friction is independent of the degree of rolling. Because of this, the writer questions the value of the authors' tests. It is obvious that joints cannot be compared unless they are tested under uniform conditions, best exemplified by smooth contact surfaces.

This condition is not fulfilled by rough-boring the test hole in a lathe or by buffing the tube end free of scale. Such buffing may produce anything from a smooth surface to a rough surface full of cuts and flats.

In their search for a reliable indication of the degree of expanding, the authors prove that the total extrusion is an excellent indication. It cannot be otherwise, as the extrusion is the visible and measurable indication of the work performed on a joint. As the measurement of total extrusion is difficult, it is evident that it cannot be applied in actual field work. The partial extrusion, however, as used in practice, is easily measured on the gas side of the boiler and is equally suited for the field or laboratory. The authors, however, claim that the partial extrusion is not a reliable indication of the degree of expanding. This statement is contrary to the findings of other investigators as will be shown.

It seems logical to assume that a parallel expander which produces a cylindrical hole should cold-work all tube ends in similar joints in the same manner. That this assumption is tenable and that the use of partial extrusion as an indication of the degree of expanding is entirely feasible can be proved conclusively (1) by actual tests, and (2) it is supported by the data submitted by the authors. To prove the first claim, the writer submits the data, Table 6 of this discussion, from a series of tests available in which the partial extrusion was used as a measure, but employing a standard parallel five-roll expander instead of a three-roll expander of unstated character such as the authors mention, and using the holding strength as a criterion instead of the author's seat pressure. This test is comparable to the results shown for tests Nos. 112 to 124, inclusive, of Table 1 of the paper.[16] The test shown is one of the first and was made before the significance of smooth surfaces was fully recognized.

TABLE 6 TEST DATA[a]

Elongation, in.	Holding strength, joint only, psi
0.005	2290
0.005	2480
0.005	2980
0.010	2510
0.010	3075
0.010	3285
0.015	2690
0.015	2885
0.015	2960
0.020	2480
0.020	2610
0.020	2775
0.025	1610
0.025	1840
0.025	1890
0.030	1860
0.030	1980
0.030	2375

[a] Results of tests made by the writer on sample tubes rolled into flat tube sheets to determine relationship between elongation and holding strength for $3^1/_4$-in-OD, 7-gage (0.18-in. wall) tube rolled into 1-in. tube sheet; tubes and tube holes selected at random.

A glance at the test data, Table 6, is convincing evidence that the partial extrusion is not only feasible but quite practical as a means of controlling the degree of rolling. Of course, it is not claimed that the actual joints will be of like uniformity, due to the difference in surface finish, but they will be uniformly cold-worked and still within acceptable limits.[21]

An inspection of the test data submitted by the authors discloses the fact that, in a majority of tests, the ratio of the partial and total extrusion are in acceptable agreement. By making allowance for mistakes that doubtless occurred in the intricate measurements necessary, the agreement in the ratios may be considered as good. The results listed in the authors' Table 4, are especially consistent, whereas joints Nos. 109 to 131, inclusive, seem to have been rolled in an unorthodox manner. Since one of the major differences in test procedure is in the type of expander used, it is quite possible that the use of a parallel five-roll expander would have produced a better agreement in the ratio between the partial and total extrusion.

Concerning one other method listed as suitable as an indicator, namely, the increase in tube inside diameter after the tube touches the tube hole, the authors seem to be quite confused, inasmuch as it is stated at one point that this is not a good indication of the degree of tube expanding, whereas another reference describes it as a good indicator. The test results submitted to support this are, however, open to criticism inasmuch as the joints were rolled by the "uniform entrance method," that is, to a uniform inside diameter. This method is successfully applied only to joints with tubes and tube holes of uniform dimensions and not to the commercial product found on the job. Calculations made by the authors for purpose of correlation may improve the curves but do not reflect the actual result.

It may not be amiss to state that this indicator, namely, the increase in the inside diameter of the tube over that at contact of tube with seat, is the well-known "Haftaufweitung," an arbitrary measure of the degree of expanding used for many years in Germany for field and laboratory work. To make this technique more practical, rolling machines have been developed that automatically roll to any desired Haftaufweitung after contact of the tube with the seat.

Ruttmann[19] after demonstrating that under like test conditions and with the same Haftaufweitung, a variation in the expander revolutions caused the joints to assume entirely different character (for instance, the tube-hole deformation of a joint finished with a few revolutions of the expander may be 8 times as great as that of a joint rolled with more revolutions) has this to say: "In view of the findings described, it is not at all certain that the Haftaufweitung can be accepted as a criterion of rolled joints, unless the kind of rolling and the number of revolutions of the expander are given." The same view is held by A. Thum and J. N. Mielentz,[18] and by Erich Siebel.[5] There are not many tubes in boilers with both ends of the tube rolled with the same number of expander revolutions.

The value of the other indicator of the degree of expanding treated in this paper, namely, the "popping of the mill scale on the seat plate," may be judged by the foregoing also. Its application is very limited, as it cannot be used on drums or headers without scale, recessed tube holes, tube sheets other than steel or iron, and machined tube sheets.

The writer fully appreciates the difficult and thorough work done by the authors and their frank way of stating the facts as they see them. Without going into the merit of the result of their tests, which have been treated in this discussion, it becomes evident that what is needed more than anything, to eliminate confusion and co-ordinate these and other test results to come, is a standard uniform testing method, uniform nomenclature, and a standard criterion of the strength of the joint.

It is felt that such standard test procedure would be of great benefit to all concerned and the writer would consider it a privilege to assist in such standardization.

T. McLean Jasper.[22] The writer's company has had a vast amount of experience in the cold working of ordinary carbon steels. Most of the steel used for pipe lines is cold-stretched, and much of the steel we fabricate for oil-well casing is cold-compressed. We have established the fact that such cold forming up to about $6^1/_2$ per cent stretch or compression causes no reduction in the corrosion or endurance resistance of the structures so made,

[21] Shown in ref. 6, Figs. 3, 4, 8, 9.

[22] Engineer, A. O. Smith Corporation, Milwaukee, Wis. Mem. A.S.M.E.

providing the subsequent operating stresses are normal to the service-stress allowances. Above the $6^1/_2$ per cent deformations, we have very little information.

FRANCIS HODGKINSON.[23] In the Grimison and Lee paper,[1b] the narrow zone between too little and too much rolling seems to indicate some uncertainty in the tube-expanding process. It is not clear how the operator knows when he has reached the optimum degree of tube rolling.

Tube expanding was adopted in the early days of boiler construction when tubes were employed. Since then various forms of welding have been developed. Why is not electric-arc welding to be preferred?

CLOSURE TO PAPER 1*a* BY E. D. GRIMISON AND G. H. LEE[24]

Despite Mr. Cope's wide experience in tube expanding, he has failed to grasp the fundamental mechanics of the expanded tube joint. The foregoing statement is borne out by the fact that he divorces entirely the contact pressure between tube and seat from the strength of the joint, which, by his own statement, is dependent upon the frictional resistance to slip of the tube relative to the seat. The coefficients of friction, listed in the paper,[1a] indicate that, due to their constancy, the tube-seat pressure is an important variable in determining the strength of the joint. The qualitative effects of roughness are well understood from the theory, but are generally submerged by the effects of grooves. The authors primarily were interested in the question of optimum degree of expanding and proceeded correctly in holding the roughness constant.

In pointing out what he claimed were poor agreements obtained for tube-seat pressures, Mr. Cope failed to note the variations in materials. Tests 63 and 68 had a tube hardness of 130 Bhn and a plate hardness of 105 Bhn; Tests 107, 134, and 139 had a tube hardness of 118 Bhn and plate hardness of 128 Bhn; Test 135 had a tube hardness of 118 Bhn and a plate hardness of 578 Bhn. Agreement between these tests was not expected. A little consideration of the plastic and elastic behavior of the material during the expanding operation will lead to the conclusion that agreement would be impossible. Reference is made to Table 1 of the paper[1a] and to Figs. 4, 5, 8, and 9, for some indication of the agreement between results obtained. The tube-seat pressures listed in Table 4 of the paper are estimated pressures, as noted, and were read from the curves of Figs. 4 and 5. The agreement indicated by Mr. Cope in these tests is no better than the agreements which were had in the other tests.

The authors are well aware of the undesirable aspects of excessive expansion and have specifically noted that better joints are formed for moderate degrees of expansion which are indicated to correspond to increases in inside diameters of the tubes lying in the range 1 to 3 per cent, depending upon the particular combinations of tube and seat conditions.

Mr. Cope quotes several statements as indicating confusion on the part of the authors. Careful reading of these statements with their context will show them to be merely qualifying statements, such as any careful investigator makes, which apply only within stated limits.

The paper was presented as a progress report on the authors' investigations in a field which they consider by no means exclusively their own. Their findings conclusively point to the validity of their assumptions of a constant coefficient of friction and dependence of joint strength upon tube-seat pressure, within limits and with effects of departures from these assumptions indicated. Simple hydrostatic tests are not sufficient indicators of the strength of expanded joints for reasons given by Mr. Maxwell.

Most of Mr. Fisher's significant comments have been answered in the replies to Mr. Cope's discussions. Both Mr. Cope and Mr. Fisher have overlooked the authors' desire to demonstrate the dependence of joint structural strength upon tube-seat pressure. In an orderly development, proof was necessary before the tube-seat pressure could be used as a measure of joint quality. Determinations of tube-seat pressures are much more economical of man power and equipment than the use of direct measurements of structural strengths. The data presented in the paper give complete proof of the utility of tube-seat pressure as an indicator of joint quality in an analytical procedure.

CLOSURE TO PAPER[1b] BY C. A. MAXWELL

There are many differences of opinion with that of the author, expressed in the criticism of the papers by Messrs. Cope and Fisher, which will undoubtedly disappear when by standardized testing of joints and proper nomenclature we all consider the same thing and conduct our tests under the same rules and defined conditions. This condition is thought to be particularly true when testing directly for the determination of the strength of a joint. Some experimenters have pushed the tube through the seat, overlooking the Poisson effect which actually increases the seat pressure and, hence, the holding power of such joints. By pulling the tube through the seat in the direction of most normal loads, the Poisson effect is reversed, thereby detracting from the strength of the joint.

Cope feels that, for proper testing, a joint should have hydraulic pressure applied until failure occurs. This does not result in true values, for the pressure has the effect of tightening the tubes in their seats, thereby giving more favorable values than would be realized in actual service. The maximum hydrostatic pressure which should be applied in combination with mechanical loading should be the Boiler Code pressure for the tube thickness. In attempting to force tubes out of expanded joints by hydrostatic pressure, we have often caused failure of the tube before any failure of the joint had occurred. In the modern boiler, structural and thermal loads on joints are of greater concern than those produced by hydrostatic pressure. Grimison and Lee[1a] have gone far toward real standardization, and their method, or an acceptable equivalent, must be used if we are to depend upon experimental values.

Fisher states that in oil-still practice an expander, similar in principle to the retractive type, is used, but which self-feeds into the tube and accomplishes the same short-roll effect, permitting rolling of any length of seat. The author is not familiar with the tool in question, but this principle cannot be used in boiler practice because of the large extrusion from the fireside of the tube sheet producing heavy structural loading on adjacent tubes and seats. Up to the moment and to the best of the author's knowledge, all the conditions for the successful use of a long seat are met only with a "retractive" expander of either the roll or ball-drift types.

As to both Cope's and Fisher's feelings that the length of tube seat is not of special importance so long as sufficient strength is developed to satisfy the hydrostatic requirements, again, it must be pointed out that every analysis of the problem indicates that the strength of the joint and its hydraulic tightness are directly measured by the area of the surfaces in contact, provided always that controlled extrusion is used in making the joint. By using the full thickness of the tube sheet as a seat, all

[23] Consulting Engineer, New York, N. Y. Former Vice-President, and Fellow, A.S.M.E.

[24] In the absence of any discussion of the companion paper by Goodier and Schoessow,[1c] general acceptance of their theoretical developments must be assumed. A paper which has since been presented by Dr. A. Nadai comes to substantially the same conclusions as Goodier and Schoessow's.

counterboring and its weakening effects on ligaments are obviated, the unbalanced stresses in the tube sheet by the uncentered position of the seat are eliminated, a stronger joint is obtained, and a much simpler, more economical construction is obtained. The importance of controlled extrusion is further emphasized by analysis of Grimison and Lee's experimental observations.[1a]

There are special cases, most often found in naval boilers and in heat exchangers with very heavy sheets, where ligament value can only be preserved by using a straight hole as a seat rather than one that has been counterbored.

The author is in complete agreement with Cope that with very small tubes expanded into seats of $^3/_4$ in. or even 1 in. length, the ball-drift expander is not necessary. Under these circumstances, the roll expander will perform satisfactorily, but where extrusion from the seat would produce structural loadings, the retractive ball-drift expander is definitely indicated, as in the case with straight tubes between rigid heads.

With tubes under 2 in. diam expanded in long seats, the only wholly satisfactory expander is the ball drift; space limitations prohibit any other type. More than 45 per cent of all Navy tubes have been expanded with ball drifts since 1934, with results which are astonishingly good. Fisher's objection to the ridge left by the split in the retractive ball ring is completely overcome in two ways; by a diagonal slit and by tandem rings with the slits opposed in position. It is not necessary to make a double pass.

For many years the author, and the company by whom he is employed, were heartily in favor of multiple-roll expanders—the more rolls the better. During those years we experienced, with others, much trouble in getting joints tight and keeping them so. The work of Craven which has been cited, and which we hope may some day be amplified and published, completely changed our concept of the problem. This work indicated that, if an expanding operation could be done with only one roll, the ideal would be attained. Since this was not practical, and three rolls were the minimum possible, we changed our practice accordingly with immediately beneficial results. It is indeed gratifying that, in a purely theoretical analysis of the problem Goodier and Schoessow[1c] should find complete confirmation of the practice.

Good work can and is being done by expanders having four and five rolls, but, as indicated by Mr. Fisher, it is necessary that tube seat holes be round, tubes upset, and machined both inside and out. After all this is done, however, we still find that greater plastic flow has occurred in ligaments opposite joints made with such tools, an obviously undesirable condition. Goodier and Schoessow very clearly point out that plastic flow immediately adjacent to the tube is a necessary condition, but that this flow must be confined to a very narrow zone to produce best results. Craven's experiments were conducted with very accurate strain gages and confirmed by Grimison and Lee.[1a] The foregoing is quite apart from the greatly increased expense incident to such accuracy in the treatment of tube ends.

None of the methods described by the author, Cope, or Fisher for the determination of an optimum expanded joint is new. Scale "cracking," or "popping," has been known and observed for many years, and, because it indicates very clearly that desired reactions have been reached in the ligament, it comes more nearly being a correct indicator than most. It has definite limitations, as Fisher points out and as was also indicated by Lee and Grimison.

Measurement of elongation or extrusion is also very old. The experienced boilermaker always scribed a line on the tube where it entered the seat and measured by eye its movement away from the seat during the expanding operation. Measuring this with a dial indicator was a distinct improvement for which thanks are due to Cope and Fisher. The "elongation" method has the limitations for field use described by the author, and it is found not usable with either the stage type of expanding or with retractive expanders where there is virtually no fireside extrusion.

The author is fully aware of all that has been said and written about cold working and the possible corrosion hazards involved. If his and the other two papers of this group presentation are studied carefully, the conclusion must be reached that means are being advanced whereby optimum results are obtained which call for minimum cold working. The greatly increased areas of tube seat possible with the retractive short-roll principle or the retractive ball drift, the utilization of as many grooves as desired, the increased friction of rough seats, all permit joints of equivalent strength and tightness to be made with very much less cold working of the metals involved than with some of the older methods.

With reference to caustic embrittlement, Cope points out we must have "physical conditions which permit high concentration of the dissolved solids." What more potent physical condition than tube leakage would build up this concentration? How important then that all our work be to the end that tube joints are made tight and remain that way? This is the object the author hopes to accomplish and welcomes the help and criticisms of Messrs. Cope and Fisher, or any others interested in the problem. Mr. Cope can help measurably by allowing us the use of the German catalog of 1934, wherein the retractive roll expander is described. We have not seen it, nor have we been able to find it through any of our sources of reference or information.

The Testing of Volute Springs

By BERNHARD STERNE,[1] DETROIT, MICH.

During the past year test work on volute springs has been undertaken on a scale probably never equaled before. The results of this work have necessarily influenced the theoretical thinking. Tests have been conducted both on suspension springs, which are under continuous static load, and on bumper springs which are normally unloaded. Laboratory tests on suspension springs took place for an extended period at constant stroke and predetermined heights with the help of a converted press. More lately, the springs have been tested at constant stroke and predetermined maximum load on a new constant-stress test machine. Field tests have been conducted at the proving grounds, partly over extremely rough terrain. The paper gives some instances of the redesigning of volute springs which was prompted by test results. It also deals with static loading tests, both axial and radial, and with differences in results caused by different manufacturing methods.

TEST work on volute springs has been undertaken during the past year on a scale which was probably never equaled before. Most of these springs had some function in vehicle suspensions, and several of the outstanding laboratories and proving grounds of the country which are concerned with suspension developments took part in the various test programs.

A large volume of the work was concerned with load-carrying suspension springs but another part dealt with bumper springs. The progressive rate increase of the volute spring is the feature which in the past has mostly furnished its appeal for bumper applications. When used as a suspension spring, it made possible the combination of the resilient load-carrying member and of the final shock-cushioning device into one piece of steel. Thus the volute spring obtained an early foothold in the suspension field while the difficulties of its computation and manufacture were sufficient to prevent it from becoming established in other lines of engineering endeavor.

This type of usage has probably impeded rather than furthered the progress on the volute spring for a long time. Suspensions in general comprise some of the most difficult spring applications because of the various freedoms of motion which must be maintained. The loading conditions in actual service can rarely be identified with the commonly accepted laboratory loading conditions, and consequently the spring trouble experienced on a suspension may be very difficult to verify or analyze in the usual test setups. This, together with the fact that the volute spring by its very nature is a member operating under a rather complex stress pattern, has led to many discouraging results in the past and has prevented a more widespread use of the volute spring for many decades.

Thus when the more recent work got under way it had to be from a fresh start because what little background existed was not available in published form. When the enormous amount of work, for instance, on helical springs, during the past twenty or thirty years is weighed against the relatively modest start which has now been made on volute springs, it will be realized that the information gathered during the past year must necessarily be very sketchy.

The life-test work on volute springs has brought one of the major problems of spring testing into focus, i.e., how to maintain from beginning to end the stress level at which the test is supposed to be conducted. The majority of endurance tests on springs are run at constant stroke and constant height and do not compensate for the losses in load and stress which occur during the test in consequence of spring settling. This is normally not a serious matter because the bulk of endurance tests takes place under stresses which are far below the yield point, and which therefore cause little if any settling in the spring.

However, suspension springs in many modern vehicles have habitually been stressed very highly, and tests on them have consequently been conducted at unusually high stress levels with attendant rapid spring settling. Moreover, the stresses in most volute springs are so unevenly distributed that in practically any test some parts of the blade will be very highly stressed. Finally, the temper in the volute spring is often sufficiently uneven so that the yield point will be reached much sooner in some

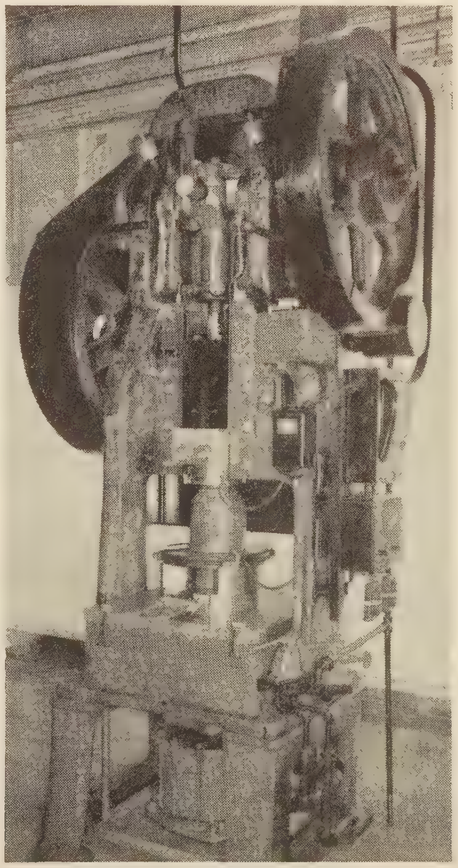


Fig. 1 Life Test at Constant Height on Converted Punch Press

[1] Experimental Engineer, Chrysler Corporation.

Contributed by the Special Research Committee on Mechanical Springs and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

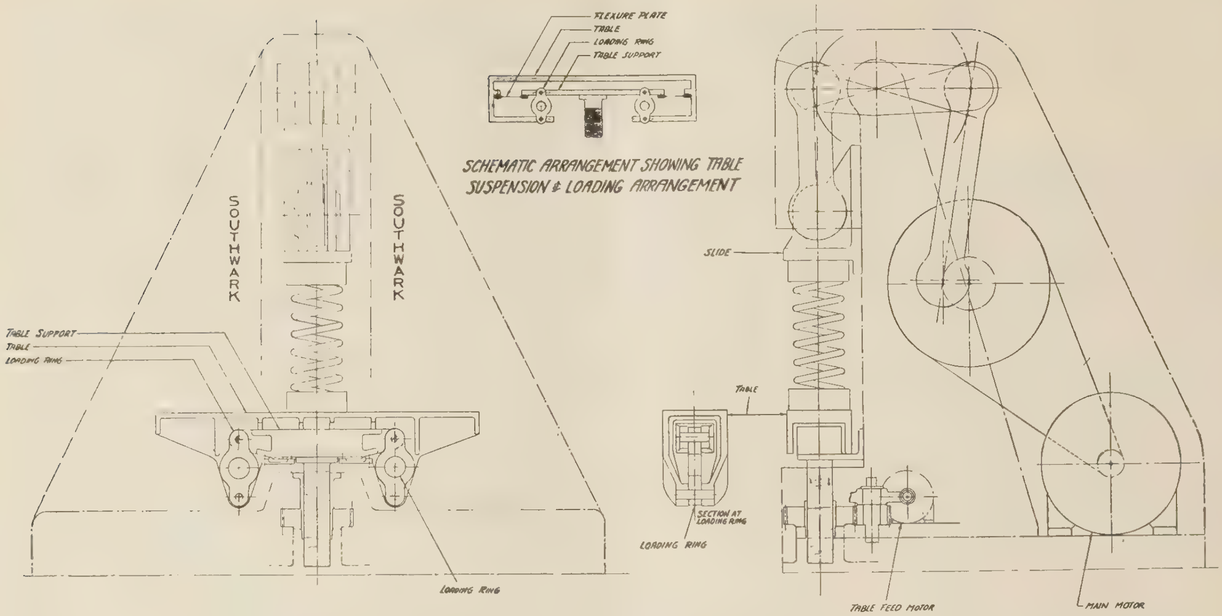


Fig. 2 Life Test at Constant Load on Special Machine

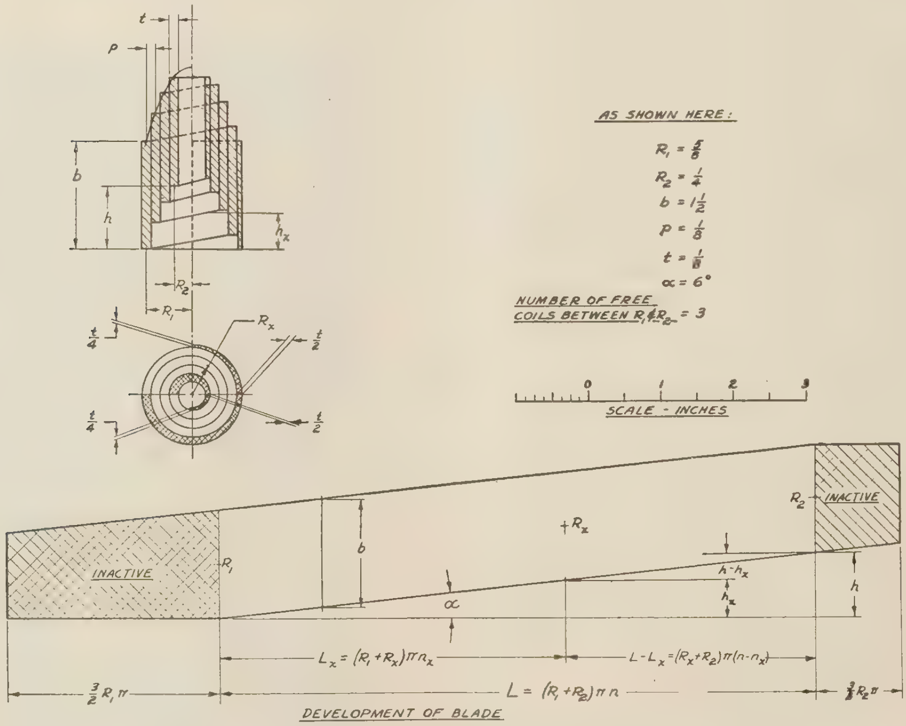


Fig. 3 Volute Spring With Constant Pitch Angle

parts of the spring than in others. Consequently a settling of 5 to 10 per cent of the total spring deflection in the course of a volute-spring life test is not a rare occurrence and will strongly influence the result.

Types of Machines Used for Testing

On the old-fashioned punch press there is no way of compensating for this lowering of the stress level. In our laboratory testing which has been carried on for nearly 2 years on such a machine, Fig. 1, we have made it a point at least to keep a record of the spring settling by periodic load checks. On the new constant-stress test machine which we have recently put in operation, Fig. 2, the maximum load at the start of the test is maintained until failure through automatic adjustment of the table height; and a paper strip provides a record of the table movement and thereby of the spring settling. The machine

has been in service only a few weeks. Less than 20 springs have been tested on it so far, and it is too early to draw general conclusions from these initial results. The available data show that the failures occur in reasonably close vicinity to the point of maximum torsional stress reached during the test. They also indicate that spring life in the tests is in no way proportional to the stress level at the point of failure. The life tends to be much shorter in a test which subjects a long part of the blade to a wide stress range, even though the maximum stress may be much lower than in another test where only a short part of the blade is subjected to a wide stress range.

When a common helical round-wire spring with constant pitch is fatigue-tested between two fixed heights, the maximum stresses at the surface of the wire are substantially proportional to the spring deflection. Also, for any given relative position between the wire surface and the spring axis, the stresses are substantially identical throughout the length of the wire, and failures are therefore apt to occur in any one of the active coils of the spring.

The stress pattern in the volute spring is much more complicated (1) because of the variable rate of the spring and (2) because with increasing load the maximum stress shifts from the outer heel point to some of the inner coils and keeps on shifting until the entire spring has gone solid. This discussion and the diagrams accompanying it are based on the methods of computation detailed in a previous paper by the author.[2] They are worked out for springs with constant pitch angle, Fig. 3, a feature which has generally been considered as a main characteristic of the volute spring.

Actually, of course, most finished volute springs do not possess a constant pitch angle. Their pitch angle is smaller at the inner heel point than at the outer, Fig. 4; it varies in the opposite direction from the pitch angle of the conical spring which is largest at the inner heel point. At present, the changes in volute-spring pitch angle vary considerably with design and manufacturing practices. Fig. 5 shows that a difference in the presetting operation is sufficient to change the pitch angle in the large and small coils appreciably, even in springs which have otherwise been produced identically and by the same manufacturer. When the presetting operation is performed with only the largest coils supported on a ring, and the smaller coils pushed down below the solid spring height, the outer coil is apt to retain its original high pitch angle, while a rather sudden "kink" in the profile line may develop at some point not supported by the ring. When the spring is mounted on a fixture which, at least theoretically, permits the presetting to be carried to a reverse parabolic envelope, the overstressing will still be greater in the smaller coils and will cause greater yielding there with greater reduction of the pitch angle. However, the transition from the larger pitch angle at the outer heel point to the smaller pitch angle at the inner heel point will now be more gradual.

When a generally satisfactory basis for uniform manufacturing practices can be reached, the computation methods will be changed to conform. In the meantime, the computation with constant pitch angle continues in widespread use, although certain allowances are made at least mentally regarding the peak stress values and the shape of the load-deflection curve. It is quite obvious, for instance, that even when the torsional stresses are computed with the Wahl correction factor (as in the diagrams, Figs. 6 to 10), values in excess of 200,000 psi could not be reached without exceeding the yield point of the best alloy steels in use at a hardness range which avoids brittleness. On the other hand if, in a given spring design with given total deflection, too much free reign were given in the variation of the pitch angle, the amount of energy to be stored by the spring would suffer appreciably, even though the stresses would benefit.

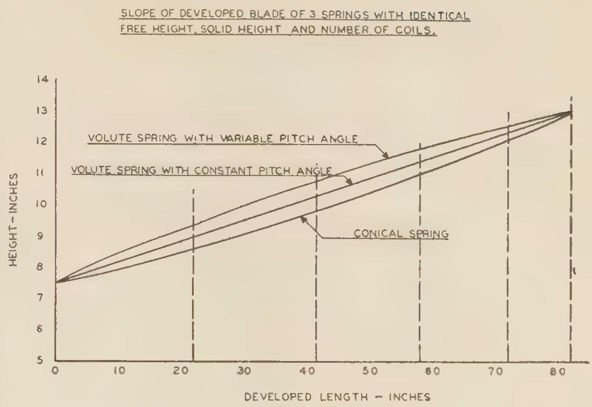


FIG. 4 SLOPE OF DEVELOPED BLADE OF CONICAL AND VOLUTE SPRINGS

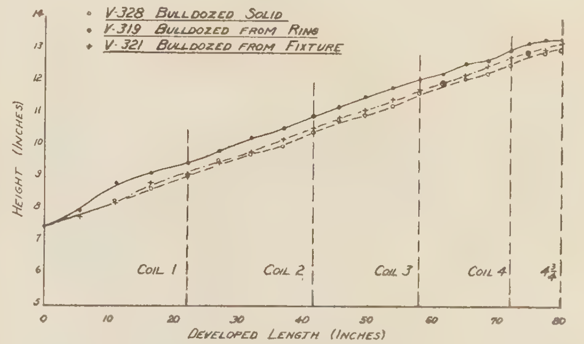


FIG. 5 SLOPE OF DEVELOPED BLADE OF VOLUTE SPRINGS AFTER PRESETTING

STRESS DIAGRAMS FOR TYPICAL VOLUTE SPRINGS

With these reservations in mind, the stress diagrams of two typical springs may now be considered. They have been chosen because they are comparable in diameter and in blade thickness. However, they vary in blade width and in total deflection, and one of them has a slight amount of taper in the active part of the blade. Diagrams have been drawn for five different test cycles. In all five cycles, the energy input equals approximately 22,500 lb-in. Two of them operate from a minimum load of approximately 1500 lb, two of them from 5000 lb, and one from 8500 lb. (Table 1 and Figs. 6 to 10.)

The lower left diagram in each illustration shows the load-deflection curve of the spring, with the region of the particular life-test cycle blocked out in heavy outline. The upper left diagram shows the maximum torsional stress in some part of the spring corresponding to each point in the load-deflection curve. The lower right diagram shows the load at which each part of the blade bottoms. The abscissa does not give the developed blade length, but more conveniently the coil radii from the outer to the inner heel point, thus providing the same length of the abscissa for each coil, whether it is a large outer or a small inner coil. The upper right diagram shows the stresses in each coil at every successive $^1/_4$ in. of deflection. The region of the particular life-test cycle is again blocked out in heavy outline. The lower border indicates the stresses under the minimum test load, while the upper border indicates the stresses under the maximum

[2] "Characteristics of the Volute Spring," by Bernhard Sterne, *S.A.E. Journal*, vol. 50, 1942.

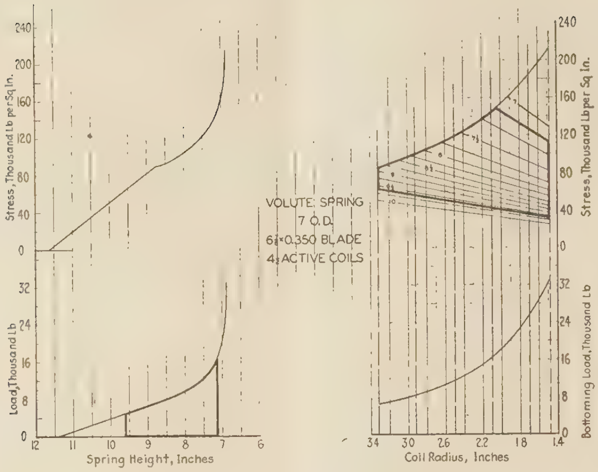


FIG. 6 LOAD AND STRESS DIAGRAMS FOR A VOLUTE SPRING OF $6^7/_8$ IN. BLADE WIDTH AND A TEST CYCLE BETWEEN $9^5/_8$ AND $7^1/_8$ IN. HEIGHT

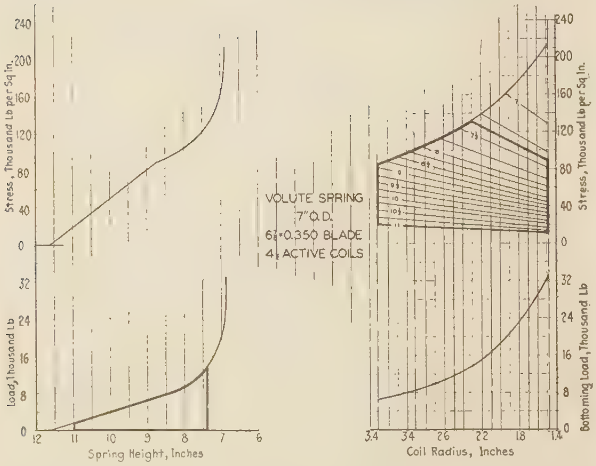


FIG. 7 LOAD AND STRESS DIAGRAMS FOR A VOLUTE SPRING OF $6^7/_8$ IN. BLADE WIDTH AND A TEST CYCLE BETWEEN 11 AND $7^3/_8$ IN. HEIGHT

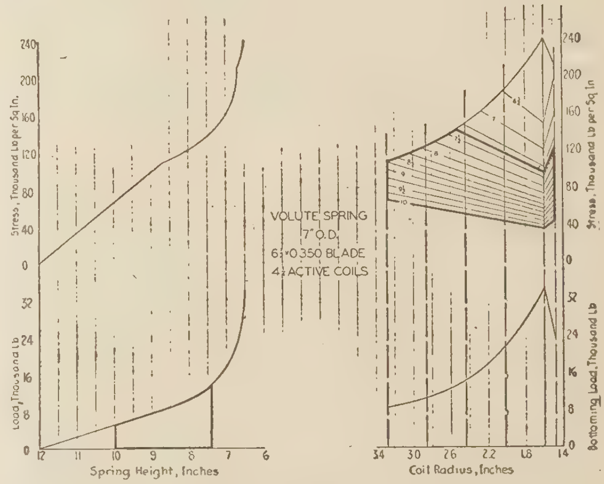


FIG. 9 LOAD AND STRESS DIAGRAMS FOR A VOLUTE SPRING OF $6^1/_2$ IN. BLADE WIDTH AND A TEST CYCLE BETWEEN 10 AND $7^3/_8$ IN. HEIGHT

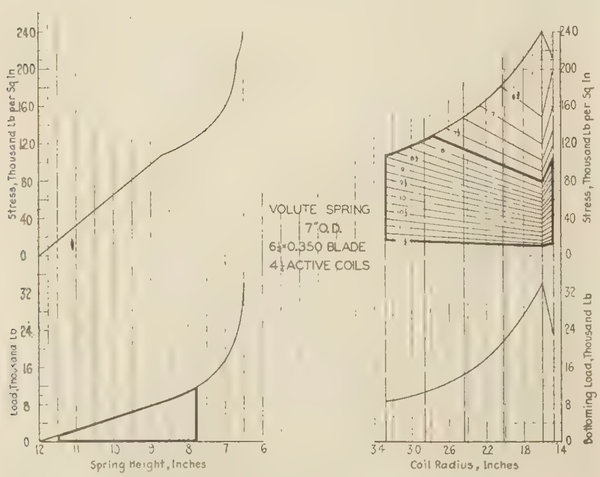


FIG. 8 LOAD AND STRESS DIAGRAMS FOR A VOLUTE SPRING OF $6^1/_2$ IN. BLADE WIDTH AND A TEST CYCLE BETWEEN $11^1/_2$ AND $7^3/_4$ IN. HEIGHT

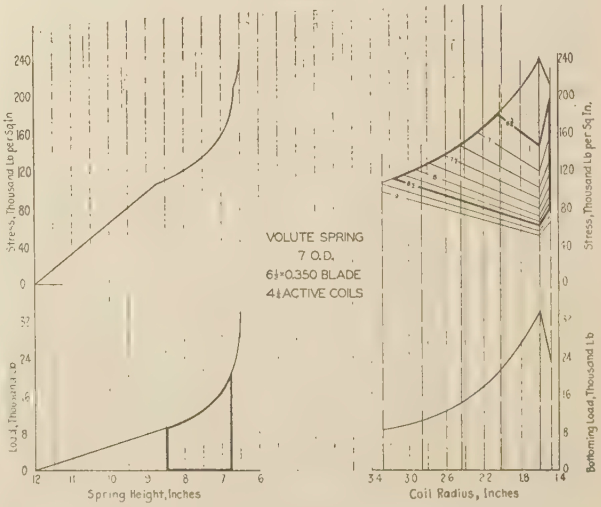


FIG. 10 LOAD AND STRESS DIAGRAMS FOR A VOLUTE SPRING OF $6^1/_2$ IN. BLADE WIDTH AND A TEST CYCLE BETWEEN $8^1/_2$ AND $6^3/_4$ IN. HEIGHT

TABLE 1 DATA ON VARIOUS LIFE TESTS CONDUCTED WITH TWO VOLUTE-SPRING TYPES OF SIMILAR DESIGN

Springs with Blade Width	Test Stroke	Max. Height at Start of Test	Load Range during Test (Constant)	Stress Diagram	Location of Max. Stress	Max. Stress	Max. Stress Range	Stress Range at Outer Heel	No. of Coils with Stress Range in excess of 60,000	Min. Life	Max. Settling
in.	in.	in.	lbs.	Fig.	Coil (from outer heel)	lbs/in^2	lbs/in^2	lbs/in^2	Coils	Cycles	in.
6-7/8	2-1/2	9-5/8	5000-1700	6	3-1/4	147,000	105,000	21,000	2-3/4	111,000	.08
"	3-5/8	11	1500-13500	7	2-1/2	150,000	114,000	64,000	4-1/2	63,000	.41
6-1/2	3-3/4	11-1/2	1500-11000	8	1-1/4	129,000	114,000	90,000	4-1/4	64,000	.36
"	2-5/8	10	5000-13500	9	1-3/4	143,000	90,000	42,000	3-1/2	140,000	.17
"	1-3/4	8-1/2	8500-21000	10	3 (see Text)	162,000 (see Text)	106,000	0	2-1/4	197,000	.11

test load. The vertical distance between both is a measure of the stress range in any particular part of the spring.

The upper right diagram in Fig. 10 shows that theoretically in this test the maximum stress is 198,000 psi, and the maximum stress range 118,000 psi, and that both prevail at the inner heel point. But here the previously discussed deviations from the computation for constant pitch angle come into play. It can be regarded as certain that the stresses at the inner heel point

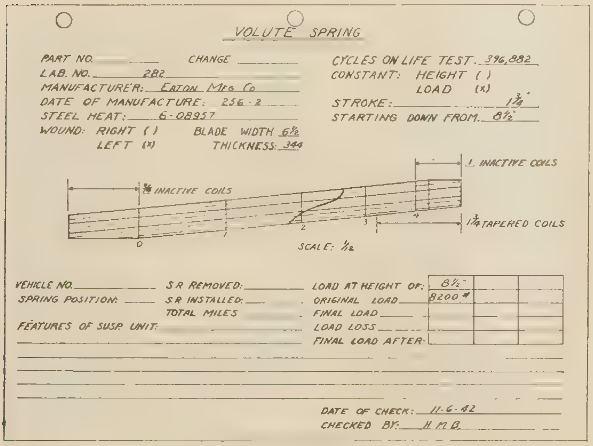


Fig. 11 Record Sheet for Volute-Spring Failure

in this test do not represent peak values; for this particular test with springs of this design has been conducted in both machines with more than twenty springs, and in all these tests no failure has ever taken place within the innermost active coil. Instead, the springs have broken within half a coil on either side of the stress peak at the junction of the third and fourth coils. In fact, the majority of all the breaks in all the different test cycles shown in the diagrams have started within half a coil of the location of the peak stress reached during the test, and the rest of the breaks have started within one coil of the peak-stress location. Fig. 11 shows a record sheet with entries on a failed spring. Figs. 12 and 13 illustrate a broken and a cracked coil, respectively, and represent typical failures as they occur in the life testing.

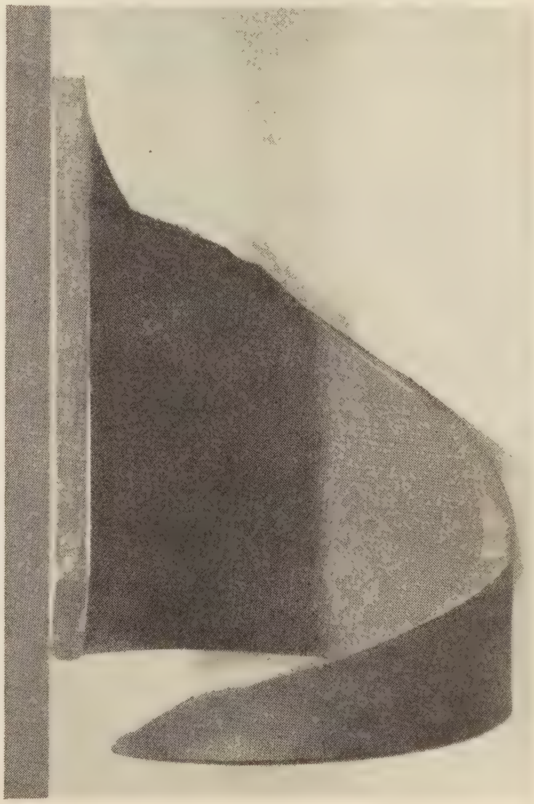


Fig. 12 Typical Volute-Spring Failure

Results of Life Tests on Volute Springs

The information in Table 1, on the results of the life tests, has been confined to the minimum number of cycles before failure in any one spring of that group, and to the maximum settling in any one spring of that group during the test. No importance should be attached to these figures themselves, but only to their relative magnitude. Also, no comparison should be attempted

Fig. 13 Partial Failure Near Blade Edge

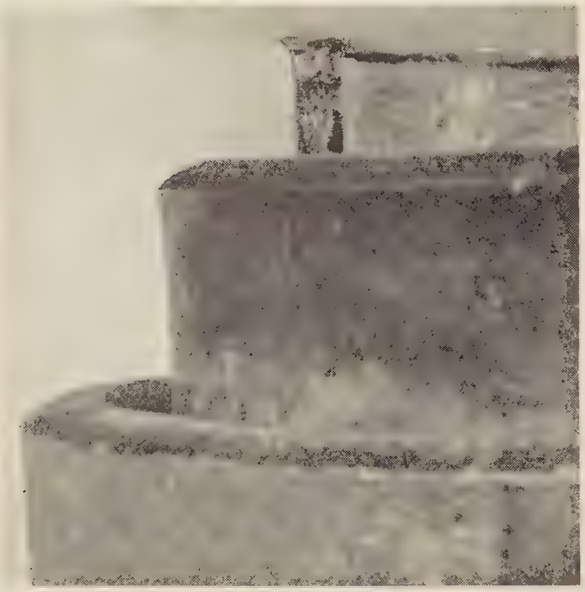


Fig. 14 Failure Due to Thin Blade End

between the two types of springs since they were manufactured from different materials. In order to bring out the apparent connection between short life and large settling, on the one hand, and a wide stress range in a long part of the blade (even at a comparatively low stress level), on the other hand, a column has been inserted with the number of coils in which the stress range during the test exceeds the (arbitrarily chosen) value of 60,000 psi.

It is particularly instructive to observe that the results of the test represented in Fig. 10 are about 3 times as good as the results of the test represented in Fig. 8. The test represented in Fig. 10 subjects $2^1/_4$ coils to a stress range in excess of 60,000 psi, and each point in these $2^1/_4$ coils reaches a maximum stress in excess of 140,000 psi. In the test represented in Fig. 8, none of the points subjected to a stress range in excess of 60,000 psi reaches a maximum stress of even 130,000 psi. However, there are as many as $4^1/_4$ coils (i.e., the entire spring) under this wide stress range, and the test results are correspondingly poor.

While caution has been expressed about taking the stress diagram in Fig. 10 too literally, still it is instructive in pointing toward one of the inherent difficulties encountered with the tapering of an active part of the blade. The purpose of this tapering is a stress reduction. Actually it succeeds only in reducing the bottoming stresses in the tapered section, and this at the cost of raising the stress level under lower loads, because the weakened section is brought closer to the bottoming point than it would be without tapering under the same loads. It is therefore well to consider the relative frequency with which the spring in actual service may be subjected to loads short of the final bottoming load, and to weigh the stresses repeatedly reached under these loads against the stresses under the maximum load which may be reached only in rare instances, and which may therefore not be as detrimental to the life of the spring.

Another word of caution regarding the tapering of the active part of the blade is in order. It reduces the blade thickness not only in the active section but also in the adjacent inactive end coil, and is apt to impair the bearing area at the small end to such an extent that failure may start due to the overloading of the blade edge, Fig. 14. In cases where very high stresses invite a very long taper, the loss in bearing area is quite pronounced and is accompanied by increased difficulties in the manufacturing operations to produce the taper.

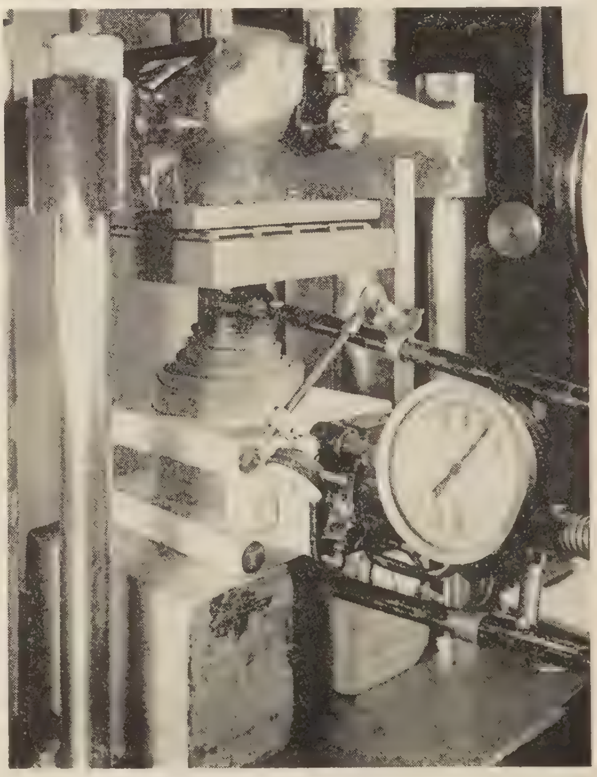


Fig. 15 Lateral Loading of a Volute Spring

Where all these considerations come together it seems logical to heed the conclusion to which H. O. Fuchs has come from his study of secondary stresses, namely, to omit one or more of the inner coils. So far, there has been no opportunity to verify the benefits of this step by tests, but they are to be conducted in the near future.

The shaping of the inner end of the spring gains particular significance when it is realized that in some of the known field applications a certain percentage of failures has originated in the inner active coil, in a region which has remained free from breakage in the laboratory testing. This is of course due to the sort of loading to which the spring is subjected in service and which cannot readily be duplicated in the laboratory. The regular loading and life testing in the laboratory take place in a purely axial direction. The more complex loading on the job may be the result of a combination of forces. The only application of a force other than axial which we have so far attempted in the laboratory was in the lateral direction. In a setup which held the inner and outer coils parallel with each other, Fig. 15, the lateral load was applied under various vertical loads. The diagram plotted from these tests, Fig. 16, indicated an initial lateral rate which was approximately twice the initial axial rate of the spring.

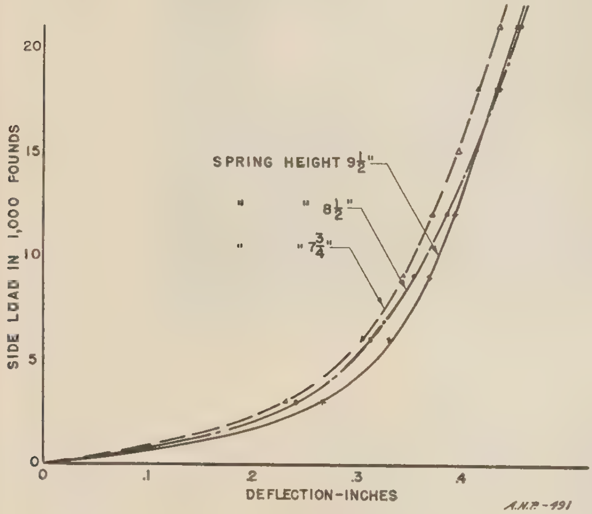


Fig. 16 Deflection Curves Under Lateral Loading

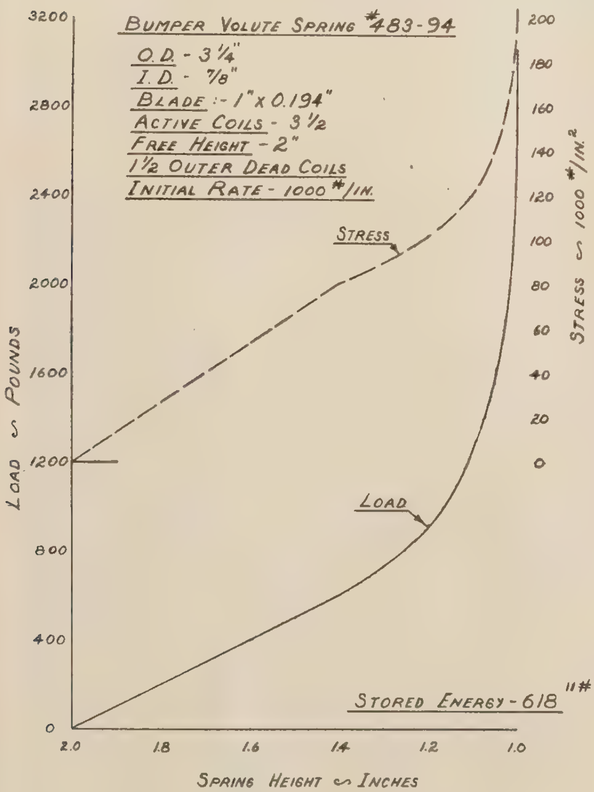


Fig. 17 Curves on a Bumper Volute Spring With 1 In. Total Deflection

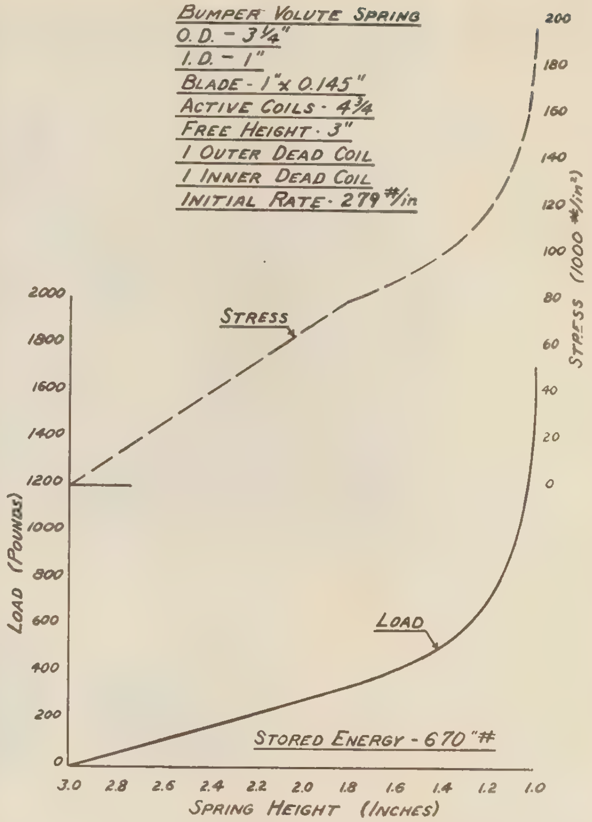


Fig. 18 Curves on a Bumper Volute Spring With 2 In. Total Deflection

Bumper volute springs have replaced rubber bumpers in several cases. Their load-deflection characteristics can be approximated to a certain extent, though frequently the available space limits the quantity of steel which can be employed and thus the energy which can be stored (Figs. 17, 18). Since these springs are usually under no load, they offer a special mounting problem. One method is to weld lugs to the outer inactive coil. For those who object to the welding of spring steel, a bracket has been worked out which fits over the outer inactive coil when it has been given a single trim operation (Figs. 19, 20). In some cases, the fitting of a soft-steel plug is required where the bumper spring engages a structural member which may suffer damage from the scraping of the hardened blade edge. Such springs have been tested on the punch press and also on the drop-weight machine under the periodic impact of a falling weight which represents more nearly the service application of this group of springs.

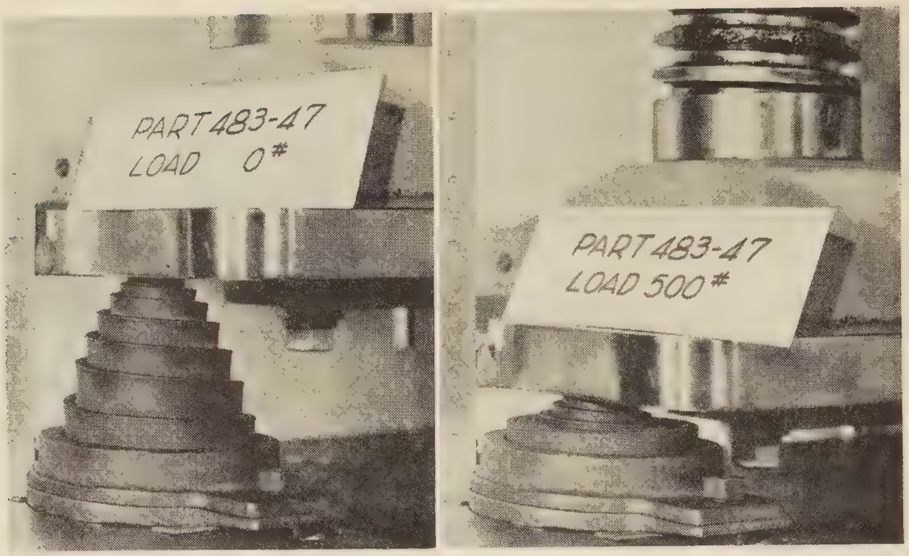


Fig. 19 Two Bumper Volute Springs and Mounting Cages

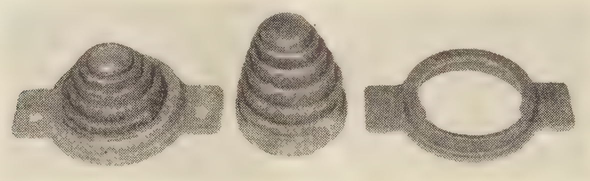


Fig. 20 Static Loading of a Bumper Volute Spring

Spring Research Sponsored by S.A.E. War Engineering Board

The study of the entire volute spring complex has been greatly stimulated by the activities of a special subcommittee which has been formed under the Spring Committee of the S.A.E. War Engineering Board. In this group, representatives of the makers and of the users of volute springs have united in a co-operative effort to find ways and means for the designing and building of better volute springs. Theoretical training and production experiences are being pooled in a wholehearted effort to serve the national interest. Research investigations of considerable scope have been started, and a manufacturing specification is being worked out for the larger suspension springs which require the use of a high-grade alloy steel such as NE 9262.

If any further proof were needed, the test work of the past year has confirmed the conviction of the volute-spring proponents that this is a resilient member of unique values which, however, can be realized only when the manufacturing technique has been more fully mastered. As stated in the draft of the manufacturing specification, the aim is to obtain best fatigue life with maintenance of loaded height. Minimum decarburization, close control of heating cycles, uniform quenching and drawing are some of the recognized difficult subjects for which the best possible practices are to be established. Continued test work will be an indispensable accompaniment to this development. It goes without saying that many of the lessons which we are learning in the course of this program, sometimes the hard way, will ultimately benefit the engineering knowledge on other types of springs. But, regardless of this, it appears an easy prediction to make that with so much talent and effort now concentrated upon the subject, the volute spring will emerge as a better understood and, therefore, a more valuable engineering unit than it has been in the past.

Discussion

H. O. Fuchs.[3] The author has provided some interesting test results on volute springs, as summarized in Table 1 of the paper. The writer was particularly interested in the apparent discrepancy between spring life and calculated stresses. Therefore an attempt was made to express in figures the "mental allowances" which the author mentions in connection with his calculation methods. Author and writer recognize that all stress calculations are approximations and the following reasoning is given as a second approximation.

We must consider both load stresses, imposed by the applied test load, and residual or trapped stresses, which result from manufacturing practices such as quenching, shotblasting, and in the case of volute springs particularly from presettling.

The trapped stresses can be calculated if the shape to which the spring was originally coiled, the shape to which it was set down, and either the yield point of the material or the shape of the spring after settling are known.

The load stresses can be calculated most conveniently from blade thickness, coil radius, and helix angle. It must be recognized that it is not safe to make any assumption about the helix angle. The author shows in his Fig. 5 how the helix angle is affected by various manufacturing practices. The shape of the coiling fixture and the shape of the presettling fixture determine the distribution of helix angles on the finished springs. This is a problem in springback which may be solved approximately by

[3] Product Study, General Motors Corporation, Detroit, Mich.

the use of Table 2 of this discussion, which was calculated by a method like that quoted by Timoshenko[4] for round bars.

We cannot expect consistent results from volute springs if we leave the coiling and presettling practice to the whim of the toolmaker. They must be known and specified as closely as hardness and chemistry of the steel. Table 2 of this discussion

TABLE 2 PRESETTLING OF RECTANGULAR STRIP IN TORSION

Nominal stress in first deflection (calculated from deflection)	Nominal stress in later deflection, after losing free height	Ratio of springback to original deflection
∞	1.5 Y	0
2.5 Y[a]	1.42 Y	0.57
2.0 Y	1.37 Y	0.685
1.8 Y	1.35 Y	0.75
1.6 Y	1.31 Y	0.82
1.4 Y	1.24 Y	0.885
1.2 Y	1.15 Y	0.96
Y	Y	1
$f<Y$		1

[a] (Y = yield stress).

results from an attempt to design a coiling fixture and presettling bowl for a definite distribution of helix angles and trapped stresses.

The load stress minus the trapped stress cannot exceed the yield stress. If the spring were presettled on a flat plate, bottoming load stress minus trapped stress may equal the yield stress. If the spring were presettled in a bowl, their difference would be less than the yield stress.

In applying this line of reasoning to the springs with $6^1/_2$-in. blade width mentioned in Table 1 of the paper it was assumed that the springs were originally coiled to a uniform helix angle of 0.112 radians, presettled on a flat plate, and that their yield stress was 115,000 psi. The resulting helix angles of such a spring can be scaled from Fig. 8 of the author's older paper.[5] The value of the yield stress was derived from comparison of the measured springback with springback calculated by means of Table 2 of this discussion.

Curves of bottoming load and bottoming-load stress, plotted over coil radius, were calculated, and the stress ranges obtained from the proportion

Stress range/bottoming stress = load range/bottoming load.

In Table 3 of this discussion, the result is summarized and com-

TABLE 3 COMPARISON OF DISCUSSER'S AND AUTHOR'S STRESS CALCULATIONS

Measured: Test stroke, in.	Measured: Maximum height, in.	Measured: Minimum life, cycles	Discusser's values[a]: Stress range, psi	Discusser's values[a]: Trapped stress, psi	Author's values: Maximum stress, psi	Author's values: Maximum stress range, psi
$3^1/_4$	$11^1/_2$	64000	115000	18000	129000	114000
$2^5/_8$	10	140000	87000	23000	143000	90000
$1^3/_4$	$8^1/_2$	197000	91000	38000	182000	106000

[a] Maximum total stress 115,000 psi in all three cases.

pared with the author's calculations. It is believed that stresses calculated in this manner give a much better correlation with test life; the test result seems to indicate that high trapped stresses produce an improvement beyond the reduction of maximum total stress. This seems reasonable but will require further experimental investigation.

H. C. Keysor.[6] Testing volute springs is a subject both broad and new. There is need for extensive work and we are therefore indebted to the author for blazing the trail so well.

When we seek to compare calculated stresses with endurance tests, we must recognize that volute calculations, as now usually made, are based upon premises which are fictitious in several respects.

Axial loading is assumed, which is contrary to fact. A series of tests now being conducted by the writer indicates that load eccentricity exists in a magnitude sufficient to affect fatigue tests.

A constant helix angle is assumed. This assumption is fairly accurate for some springs, depending upon design, method of coiling, and presetting, but in many cases it is far from the truth. For example, some test measurements are shown in Fig. 21 of this discussion. The developed bar contour was obtained by measuring with a surface gage the height at regular intervals along the bar. These heights are plotted against developed bar length in Fig. 21, for two cases, (1) before springs were cold-

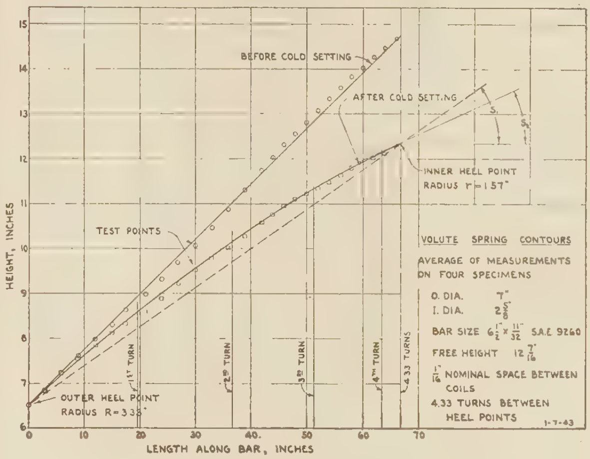


Fig. 21 Test Measurements of Volute-Spring Contours

set, (2) after cold-setting. For the first case, the test graph is approximately a straight line, as we should expect, since the bar in coiling naturally follows a constant helix angle if the mandrel is cylindrical. After cold-setting, the contour becomes curved, the helix angle decreasing as the coil radius decreases. The change is very marked. According to conventional methods, assuming a constant helix angle, we would calculate for the contour shown by the dotted line. At the inner heel point we would, on this basis, obtain our solid stress from tan S_1, which is about 50 per cent greater than the correct slope tan S_2, consequently the stress would be about 50 per cent in error.

The conventional method assumes that, at the outer heel, the bar is absolutely fixed against rotation but is free to angle in a plane tangent to the coil circumference. As a consequence, the elastic curve would be parabolic. Actually, the end conditions are different, resulting in an elastic curve with sinusoidal waves; hence closure on the base plane is not continuous, as indicated by the usual method of calculation, but proceeds in a series of jumps. This fact can be readily demonstrated by inserting a piece of paper between the base of the spring and the loading plate and applying a load. The pressure pattern obtained shows definitely that closure is discontinuous. It follows that stresses calculated on the assumption of continuous closure will be more or less fictitious.

The conventional method deals with load stresses only, taking no account of the residual stresses which must inevitably be present after cold-setting.

A consideration of these factors makes it clear that a good correlation between fatigue life and calculated stresses can hardly be expected until we can put our methods of calculation

[4] "Strength of Materials," by S. Timoshenko, vol. 2, second edition. D. Van Nostrand Company, Inc., New York, N. Y., 1940, p. 229.
[5] Author's ref. (2), p. 229.
[6] American Steel Foundries, Chicago, Ill.

on a more logical basis. Thus turning to the author's Table 1, we have the following:

Maximum stress, psi	Maximum stress range, psi	Minimum life, cycles
129000	114000	64000
143000	90000	140000
182000	106000	197000

We might expect an increase in life in the second item due to the stress range decreasing; yet comparing the second item with the third, we see that increasing the stress range and the maximum stress gives an increased life. This result is so contrary to principles of fatigue testing, that one is forced to conclude the stresses (which are calculated values) are in error due to the various deficiencies in calculation methods which have been discussed in the foregoing paragraph. A partial explanation, as given by the author, may be the decrease in the number of coils subjected to stress range in excess of 60,000. Nevertheless one would like to see what the actual or measured stresses would be, if it were possible to determine them.

The foregoing statements are made to emphasize the fact that theoretical analysis of volute spring is sadly deficient and far behind laboratory test work. The experimental engineer has a right to demand that the theorists furnish him with logical calculation methods which shall serve as a reliable guide in his investigations.

E. H. Lindeman.[7] Laboratory fatigue tests can provide much useful information for improving the durability of volute springs, and it is to be hoped that the program initiated by the author can be continued without interruption.

Data accumulated from laboratory tests, however, will have the greatest value only when it is possible to correlate results obtained in the laboratory and in the field. At the present time, volute springs are being used in suspension systems where the complex loading causes secondary stresses which are highly detrimental to fatigue life. In these instances, laboratory tests, such as those described, will be of limited value in determining the life expectancy of volute springs in field service. If tests were conducted on complete suspension units, the results obtained would be much more informative.

[7] Eaton Manufacturing Company, Detroit, Mich.

It is hoped that designers will recognize the fact that, in order to obtain maximum fatigue life, volute springs should be subjected to axial loading only. When these springs are properly used in suspension systems, data obtained from laboratory tests on axially loaded volute springs will acquire greater value.

Author's Closure

Both Mr. Fuchs and Mr. Keysor have correctly emphasized the importance of a sharper distinction between "calculated" stresses and "actual" or "net" stresses, the latter taking into account the residual stresses which are incorporated in the spring by various manufacturing operations. If we can succeed in determining more accurately the magnitude of the residual stresses and of their influence on fatigue life this knowledge will benefit not only the volute spring but also other types of springs. This study and the eventual control of the manufacturing operations by which the residual stresses are set up could be assisted enormously if a reliable method were found for checking residual stresses in a production part.

The discrepancy in spring loading between test setup and field usage mentioned by Mr. Lindeman is peculiar to the majority of suspension applications. The need for supplementing the spring laboratory tests by tests with the complete suspension unit is well recognized by suspension engineers. Still such tests are the exception rather than the rule because they require bulky equipment. One test installation of this type is at present in preparation, and it seems possible that the hope for achieving an entirely satisfactory correlation between theory, laboratory data, and service results may be realized within a reasonable period of time.

Volute-Spring Formulas

By C. J. HOLLAND,[1] CHICAGO, ILL.

Previous formulas for determining stresses, loads, and deflections in volute springs are very few, such as there are being unwieldy and difficult to apply. To correct this situation, the author has developed simple formulas for volute springs based on tensile stress which include factors covering the proper consideration of bar curvature, pitch angle, and bending stress. By geometry he shows that a line, drawn through the mean diameters of the turns of a volute spring, is a parabola, but that the line differs very little from a straight line forming the side of a triangle; that using the triangle as a base for the derivation of his formulas does not introduce any appreciable error and does permit of extreme simplification. He also points out that volute-spring formulas should be based on diameters, because the turns of a volute spring are not circles, and therefore using the radius will not give accurate results. The author carries through typical design calculations to show the application of the formulas, including the developed bar length and the weight of the finished spring.

The volute spring may be defined as a coil spring made from a relatively wide, relatively thin bar or blade, wound so that each adjacent turn or coil partially overlaps its adjacent turn or coil and so that a line representing the mean diameter of its turns or coils is a conical spiral line.

Volute springs are probably as old as any form of coil spring. Locomotives built by the Vienna works of the Privileged Austrian-Hungarian State Railway Company, during the years 1845–1857, were suspended on volute springs throughout.[2]

Although virtually discontinued as a suspension spring, Europeans have continued to use it in railroad cars as a buffer spring as well as in machine design. During the first world war, American spring manufacturers made a considerable number of heavy hot-wound volute springs for use as buffer springs in European-designed railroad cars, built during that period in the United States for export to France, Russia, etc. All of the volute springs referred to were designed to have considerable clearance between the turns. With the exceptions mentioned, heavy hot-wound volute springs were practically unknown in the United States until the late twenties.

It seems to the author that the main reason for this lack of interest in the volute spring in the United States was because of the fact that the formulas available for figuring loads, deflections, and stresses were either very long and tedious, or they did not give satisfactory results. Whenever a new design was required, it took altogether too much time to make the calculations, and the engineer turned to helical or elliptical springs, which he could calculate with good accuracy in a comparatively short time. This paper will develop and set forth formulas whereby the engineer can calculate the characteristics of volute springs with approximately the same ease and accuracy as he can calculate helical springs.

Characteristics of the Volute Spring

In a volute spring, formed from a bar of uniform cross section, the largest-diameter coil is the weakest and, conversely, the smallest-diameter coil is the strongest. Also, in a volute spring, the relation between load and deflection is proportional until the largest-diameter coil goes solid, or bottoms, after which the load increases faster than the deflection increases. In other words, when the largest active coil bottoms, the spring rate increases. As vehicle speed is increased, the intensity of occasional heavy shocks becomes greater. The volute spring with its increasing spring rate provides a greater factor of safety against oversolid blows for such cases. Add to this elastic increasing spring rate frictional damping, and the shock capacity is further increased, which is just the characteristic required for many modern vehicles.

To function as a load-supporting spring, a volute spring should be designed so that the largest coil, which is the weakest coil, will not bottom under its normal load. Assuming the bar section to be constant, the stress will vary from maximum in the largest or weakest coil to a minimum in the smallest or strongest coil. If any coil bottoms under the normal working load, the material in that coil is useless.

The author became interested in volute springs about 20 years ago. He made an effort to use the then known formulas but soon discovered that none of them was reliable. Accordingly, he reached the conclusion that such volute springs as had been made up to that time must have been on a cut-and-try basis, probably with a subsequent effort to fit a formula to the spring as produced. Because of the lack of reliable information, the author was forced to design his volute springs, coil by coil (and where greater accuracy was required, half coil by half coil), treating each individual coil of the volute spring as a single-turn rectangular-bar-cross-section helical spring of equivalent diameter. This proved to be so long and tedious a task, especially when a variety of designs were required to take care of different loads, deflections, and space conditions, that the conclusion was reached about 4 years ago that simple formulas could be developed. The formulas given later in this paper are the result, and they have been used in the author's organization for over 3 years with very satisfactory results.

Analysis of Volute-Spring Design Problems

The analysis which follows deals with volute springs which are hot-wound under pressure, so that the adjacent coils touch each other. Such springs resist closure, not only by elastic force but, in addition, by the friction force in the spring. These frictional forces vary with the pressure between coils, and with the hardness and roughness of the rubbing surfaces. For these reasons, the coefficient of friction will be a variable; therefore we will neglect friction and consider elastic forces only. Incidentally, friction will support a certain static load but will not support any dynamic load. The elastic force must do that job.

We will assume the bar section to be a true rectangle, that it is not distorted by coiling; and we will also assume that the resultant load on the spring is axial. The fact that the coils are guided by each other as they telescope one within the other during compression, thereby preventing the tipping action implied by a non-axial load, justifies this assumption. Also, hot-wound heavy-

[1] President, Holland Company. Mem. A.S.M.E.

[2] "Springs and Suspension," by T. H. Sanders, The Locomotive Publishing Company, Ltd., London, 1930, p. 121.

Contributed by the Special Research Committee on Mechanical Springs and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

duty springs, as manufactured in production quantities, are invariably coiled to an excess free height. After heat-treatment, when the springs are cold, this excess free height is removed, i.e., the springs are given a permanent set by compressing them one or more times under a load in excess of the rated capacity of the springs. Therefore, the spring will adjust itself to a condition of uniform stress in the solid state. To satisfy that condition the vertical pitch will vary from a maximum in the largest active coil to a minimum in the smallest active coil. Further, note that the stresses used in the formulas to be given are tensile stresses. Also, the diagram shows that volute-spring formulas should be based on coil diameters because the radius of any individual coil in a volute spring is different on one side of the center line from what it is for that same coil on the other side of the center line.

Based on these restrictions, the following formulas were devised, which permit of quick and accurate determination of the loads, deflections, and stresses in such volute springs. Their derivation is given in the Appendix, together with the nomenclature used. Also, two tables of constants used in these formulas are included. If space limitations or other conditions require maximum accuracy in any formula, the Appendix gives the basic derived formula, together with tables covering the variables included in the formulas in symbol form, thereby enabling the engineer to give effect to the correct values required to take care of Saint Venant's values for torsion in rectangular bars, bar curvature, pitch angle, and bending stress. The formulas are as follows:

Deflection (straight-line portion) in terms of load P_x, the value P_x not exceeding the capacity of the weakest coil D_1, equivalent to the capacity of a helical spring of rectangular-cross-section bar, having a mean diameter equal to D_1 and a cross section of the bar equal to tb, is given by

$$f_x = \frac{P_x(D_1^4 - D_2^4)}{3.28Gbt^3p} \qquad [A]$$

The formula for deflection (straight-line portion) in terms of tensile stress gives the total deflection in terms of tensile stress for any stress up to that which will close the weakest coil D_1 solid.

$$f_{vs} = \frac{S_c(D_1^4 - D_2^4)}{7.2GtpD_1} \qquad [B]$$

The condition represented by Equations $[A]$ and $[B]$ is one in which there is a variable stress in the different coils of the volute spring; in other words, any load on a volute spring deflects the weaker coils more than it does the stronger coils, and therefore the weaker coils operate under a higher stress and through a greater stress range than do the stronger coils, for any load under which the entire spring remains active. The various coils are uniformly stressed only when the entire spring is solid.

Tensile stress in terms of deflection, such that when all of the coils are closed solid each coil is uniformly stressed, is given by

$$S_c = \frac{f_{us}\,5.40Gtp}{(D_1^3 - D_2^3)} \qquad [C]$$

The author hopes that information contained in this paper will contribute to the sum total of volute-spring-engineering knowledge, and that spring engineers will find these new tools useful in solving some of their problems which have heretofore been very difficult.

Appendix

Nomenclature

The following nomenclature (dimensions given in inches and weights in pounds) is used in the Appendix:

$a = t$

b = axial width, or height of cross section, which also equals actual solid height of spring

$c = b/t$

f = deflection: f_{vs} = deflection under variable stress; f_{us} = deflection under uniform stress, viz., when entire spring is solid or bottomed

f_x = deflection under load P_x

h = solid slant height; assumed to equal $tg \times n$

n = number of active turns $= \dfrac{\text{OD} - (\text{ID} + 2tg)}{2tg}$

p = average horizontal pitch of turns $= \dfrac{(D_1 - D_2)tg}{2h}$ or $\dfrac{D_1 - D_2}{2n}$

t = thickness of cross section tb, rectangular shapes, assumed to equal $0.85tg$, which value will vary with coiling temperatures and pressures and/or clearance between turns

tg = thickness of cross section plus gap or clearance between turns

w = weight of 1 cu in. of steel = 0.2833

A = apex height of triangle or half cone

C = coefficient based on Vogt's term $\left(\dfrac{1.284}{1 - \dfrac{t}{D}}\right)$ which includes bar curvature, pitch angle, bending stress, and factor (1.3) which changes torsional stress to tensile stress (see Table 2)

D_1 = mean diameter of largest active turn = OD — 1.5 tg

D_2 = mean diameter of smallest active turn = ID + 1.5 tg

D_x = mean diameter of any convenient active turn $= \dfrac{D_2P_2}{P_x}$

G = torsional modulus of elasticity = 10,600,000

H = free height = b plus total deflection

L = developed length of bar $= \pi N\left(\dfrac{\text{OD} + \text{ID}}{2}\right)$

N = total number of turns $= n + 1.5$

P = load on spring

P_1 = load to make largest active turn solid

P_2 = load to make smallest active turn solid

P_x = any convenient load

S = torsional stress in compression springs, psi

S_c = tensile stress in compression springs, psi

W = weight of finished spring = 0.383 Nbt (OD + ID)

$\eta = t/D$

η_2, η_3 = Saint Venant's coefficients for torsion in rectangular bars (see Table 1)

TABLE 1 VALUES OF CONSTANTS FOR TORSION IN RECTANGULAR BARS

D/t	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
η_2 =	0.2985	0.3033	0.3070	0.3103	0.3125	0.3134	0.3158	0.3171	0.3182	0.3192	0.3200	0.3207	0.3214	0.3220	0.3226
η_3 =	0.2985	0.3033	0.3070	0.3103	0.3125	0.3134	0.3158	0.3171	0.3182	0.3192	0.3200	0.3207	0.3214	0.3220	0.3226

TABLE 2 VALUES FOR CONSTANTS TO USE IN CALCULATING VOLUTE SPRINGS

For D/t =	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$C = \dfrac{1.284}{1 - (t/D)}$ =	1.5409	1.4980	1.4674	1.4445	1.4267	1.4124	1.4007	1.3910	1.3828	1.3757	1.3696	1.3642	1.3595	1.3553	1.3516

Stress, Deflection, and Load Formulas for Hot-Wound Heavy Volute Springs in Which Vertical Pitch Angle Is Not Constant in Finished Spring[3]

In searching for some means of developing simple formulas, a volute spring was laid out in cross section with the mean diameters of its turns indicated by points. Drawing a smooth curve through those points gave a figure having the form of a parabola. Investigating the application of the mathematical formulas relating to parabolas indicated that the method of approach would be complicated. The cone offered a very close approximation and was chosen, and the formulas were developed as follows:

A volute spring is composed of a number of elementary spiral turns, each turn being in itself nearly the equivalent of a single-turn helical spring of rectangular-bar cross section, and each turn differing from its adjacent turn in one respect, viz., that each turn has a mean diameter less than the turn outside of it and greater than the turn inside of it. These increments of change in the mean diameter result in corresponding increments of change in the deflection of the successive turns. These values, being determined by the one general expression for deflection of helical springs, may be added together by resorting to calculus. Because the increments of change in the mean diameter are, in this case, in proportion to the increments of change in the assumed solid height, it follows that the increments of deflection also follow those of the assumed solid height, and that we may expect to arrive at the summation of the deflections through a summation of the increments of change in the assumed varying solid height, which, for successive individual coils, increases from 0 to its maximum, h (see Fig. 1).

R. F. Vogt[3] developed stress and deflection formulas, which include the influence of the curvature of the bar, pitch angle, and bending stress, as well as Saint Venant's coefficients for torsion and the factor (1.3) which converts the equivalent torsional shearing stress into tensile stress, for rectangular-bar helical springs. Note particularly that allowable working stresses must be increased to correspond, if the formulas based on torsional shearing stresses and tensile stresses are to give concordant results. His formulas for deflection and load are as follows:

$$f = \frac{\pi P n}{4\eta_3\eta^3 caG}(1 + 0.3\eta^2) \qquad [1]$$

$$P = \frac{2\eta\eta_2 S_c ca^2}{\dfrac{1.284}{1-\eta}} \qquad [2]$$

Converting these formulas by substituting symbols more generally recognized, the following is obtained for deflection

$$f = \frac{\pi P D^3 n}{4\eta_3 G b t^3}(1 + 0.3\eta^2) \qquad [3]$$

and as Vogt says,[4] "The direct shear correction factor $(1 + 0.3\eta^2)$ for curved bars varies so little from unity that in general practice it may be justifiably replaced by unity," we will eliminate it, and the formula becomes

$$f = \frac{\pi P D^3 n}{4\eta_3 G b t^3} \qquad [4]$$

and for load

$$P = \frac{2\eta_2 S_c b t^2}{D\dfrac{1.284}{1-\eta}} \qquad [5]$$

In order to avoid complications, as we proceed to develop simple formulas for a spring composed of variable diameters, we compile Table 2, giving values for the term $\left(\dfrac{1.284}{1-\eta}\right)$, and substitute in the formula for this term the symbol C, so that this formula becomes

$$P = \frac{2\eta_2 S_c b t^2}{CD} \qquad [6]$$

and for stress

$$S_c = \frac{PCD}{2\eta_2 b t^2} \qquad [7]$$

[3] Based on: "Stress and Deflection of Helical Springs," by R. F. Vogt, Trans. A.S.M.E., vol. 58, 1936, pp. 467–475.

[4] "Stress and Deflection of Helical Springs," by R. F. Vogt, Trans. A.S.M.E., vol. 58, 1936, p. 475.

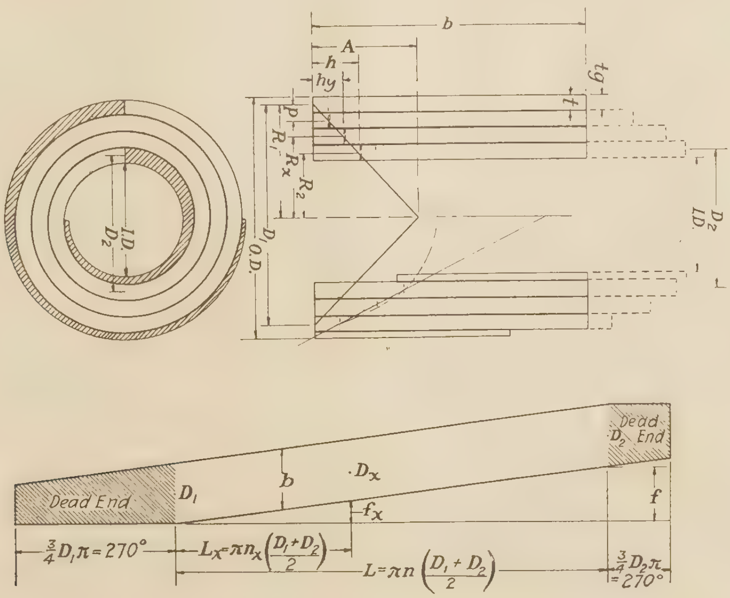


Fig. 1 Volute-Spring Formulas, Derivation Diagrams

Equations [4], [6], and [7] may be considered fundamental formulas for deflection, load, and stress in terms of tensile stress for single-turn ($n = 1$) helical springs of rectangular-bar cross section which include the consideration of bar curvature, pitch angle, and bending stress.

Referring to the diagram Fig. 1, it will be noted that A = apex height of the triangle or half cone; h = assumed solid slant height, and tg = bar thickness plus gap or clearance between turns. Therefore the expression for deflection, Equation [4], becomes

$$f = \frac{\pi P D^3 h}{4\eta_3 G b t^3 tg} \qquad [8]$$

In a volute spring, the value D is not a constant; it is a variable which we will represent by $(2R_x)$. We choose D and not R, because the coils of a volute spring are conical spirals; they are not cylindrical. Therefore in each individual turn

$$\frac{f_x}{f} = \frac{P_x}{P} \text{ or } f_x = \frac{P_x f}{P}$$

Substituting this value of f with the proper value of the variable D, we have

$$f_x = \frac{\pi P_x (2R_x)^3 h}{4\eta_3 G b t^3 tg}, \text{ or } f_x = \frac{8\pi P_x R_x{}^3 h}{4\eta_3 G b t^3 tg}, \text{ and}$$

$$df_x = \frac{8\pi P_x}{4\eta_3 G b t^3 tg} R_x{}^3\, dh_y$$

$$f_x = \int_0^h \frac{8\pi P_x}{4\eta_3 G b t^3 tg} R_x{}^3\, dh_y = \frac{8\pi P_x}{4\eta_3 G b t^3 tg} \int_0^h R_x{}^3\, dh_y$$

but, as shown in the diagram

$$\frac{A}{h_y} = \frac{R_1}{R_1 - R_x}, \quad \text{so} \quad R_x = R_1 - \frac{R_1 h_y}{A}$$

hence

$$R_x{}^3 = R_1{}^3 \left(1 - \frac{3hy}{A} + \frac{3h^2_y}{A^2} - \frac{h^3_y}{A^3}\right)$$

and

$$f_x = \frac{8\pi P_x R_1{}^3}{4\eta_3 G b t^3 tg} \int_0^h \left(1 - \frac{3h_y}{A} + \frac{3h^2_y}{A^2} - \frac{h^3_y}{A^3}\right) dh_y$$

$$f_x = \frac{8\pi P_x R_1{}^3}{4\eta_3 G b t^3 tg} \left(h - \frac{3h^2}{2A} + \frac{h^3}{A^2} - \frac{h^4}{4A^3}\right)$$

Also, from the diagram $\dfrac{A}{R_1} = \dfrac{h}{R_1 - R_2}$, or $A = \dfrac{hR_1}{R_1 - R_2}$

so

$$f_x = \frac{8\pi P_x R_1{}^3}{4\eta_3 G b t^3 tg}\left[h - \frac{3h^2(R_1 - R_2)}{2hR_1} + \frac{h^3(R_1 - R_2)^2}{h^2 R_1{}^2} - \frac{h^4(R_1 - R_2)^3}{4h^3 R_1{}^3}\right]$$

$$= \frac{8\pi P_x R_1{}^3}{4\eta_3 G b t^3 tg}\left[h - \frac{3h(R_1 - R_2)}{2R_1} + \frac{h(R_1 - R_2)^2}{R_1{}^2} - \frac{h(R_1 - R_2)^3}{4R_1{}^3}\right]$$

but $R_1 = \dfrac{D_1}{2}$ and $R_2 = \dfrac{D_2}{2}$, hence

$$f_x = \frac{\pi P_x D_1{}^3 h}{4\eta_3 G b t^3 tg}\left[1 - \frac{3(D_1 - D_2)}{2D_1} + \frac{(D_1 - D_2)^2}{D_1{}^2} - \frac{(D_1 - D_2)^3}{4D_1{}^3}\right]$$

and

$$f_x = \frac{\pi P_x D_1{}^3 h}{4\eta_3 G b t^3 tg}\left(\frac{D_1{}^3 + D_1{}^2 D_2 + D_1 D_2{}^2 + D_2{}^3}{4D_1{}^3}\right)$$

$$= \frac{\pi P_x h}{16\eta_3 G b t^3 tg}\left(D_1{}^3 + D_1{}^2 D_2 + D_1 D_2{}^2 + D_2{}^3\right)$$

therefore

$$f_x = \frac{\pi P_x h(D_1{}^4 - D_2{}^4)}{16\eta_3 G b t^3 tg(D_1 - D_2)}$$

Also, from the diagram, notice that the average horizontal pitch of the whole spring is not twice the value for one half the spring. As previously stated, the turns of a volute spring are conical spiral. They are not true concentric cylinders.

The average horizontal pitch

$$p = \frac{D_1 - D_2}{2n} \qquad [9]$$

or

$$p = \frac{(D_1 - D_2)tg}{2h} \qquad [10]$$

therefore

$$f_x = \frac{\pi P_x (D_1{}^4 - D_2{}^4)}{32\eta_3 G b t^3 p} \qquad [11]$$

If the value P_x equals the capacity of D_1, the largest active diameter or weakest turn of the spring, we have

$$P_x = P_1 = \frac{2\eta_2 S_c b t^2}{C D_1} \qquad [12]$$

similar to Equation [6].

Substituting this value of P_x, we have, in terms of tensile stress

$$f_{vs} = \frac{\pi S_c (D_1{}^4 - D_2{}^4)}{16\, G t p C D_1} \qquad [13]$$

which is the final formula for the total deflection of a volute spring under a load that will just close the weakest or largest-diameter turn in the spring.

The two formulas so far developed give the deflection in a volute spring under the condition of uniform load and varying stresses in the individual turns of which the spring is composed. Equation [13] gives the straight-line deflection, viz., that portion of the deflection, in a volute spring, in which the relation between load and deflection is proportional, in other words, the total deflection to just close or bottom the largest-diameter active turn.

To determine the total deflection in the spring for uniform stress at solid height, we have a summation of increments of deflection when each active turn is stressed to a maximum, viz., the whole spring is uniformly stressed when the whole spring is solid, at which point the spring becomes practically a solid block of steel.

As shown previously

$$f = \frac{\pi P D^3 h}{4\eta_3 G b t^3 tg} \qquad [14]$$

the same as Equation [8], and

$$P = \frac{2\eta_2 S_c b t^2}{CD} \qquad [15]$$

the same as Equation [6], and

$$f = \frac{\pi S_c D^2 h}{2CGttg} \qquad [16]$$

But, as in the previous case, D is a variable, $D = 2R_x$. Then

$$f = \frac{4\pi S_c R_x^2 h}{2CGttg}$$

and

$$df = \frac{4\pi S_c}{2CGttg} R_x^2 d\, h_y$$

From the diagram

$$R_x = R_1\left(1 - \frac{h_y}{A}\right)$$

and

$$R_x^2 = R_1^2\left(1 - \frac{2h_y}{A} + \frac{h^2_y}{A^2}\right)$$

substituting

$$f = \frac{4\pi S_c R_1^2}{2CGttg}\int_0^h \left(1 - \frac{2h_y}{A} + \frac{h^2_y}{A^2}\right) d\, h_y$$

and integrating and evaluating between limits of 0 and h

$$f = \frac{4\pi S_c R_1^2}{2CGttg}\left(h - \frac{h^2}{A} + \frac{h^3}{3A^2}\right)$$

but

$$A = \frac{hR_1}{R_1 - R_2}$$

so

$$f = \frac{4\pi S_c R_1^2}{2CGttg}\left[h - \frac{h^2(R_1 - R_2)}{hR_1} + \frac{h^3(R_1 - R_2)^2}{3h^2R_1^2}\right]$$

or

$$f = \frac{4\pi S_c h}{6CGttg}(R_1^2 + R_1R_2 + R_2^2)$$

Since $R_1 = \frac{D_1}{2}$, and $R_2 = \frac{D_2}{2}$

$$f = \frac{\pi S_c h}{6CGttg}(D_1^2 + D_1D_2 + D_2^2)$$

hence

$$f = \frac{\pi S_c h(D_1^3 - D_2^3)}{6CGttg(D_1 - D_2)}$$

but

$$p = \frac{(D_1 - D_2)tg}{2h}$$

therefore

$$f_{us} = \frac{\pi S_c(D_1^3 - D_2^3)}{12CGtp} \quad [17]$$

and

$$S_c = \frac{f_{us}12CGtp}{\pi(D_1^3 - D_2^3)} \quad [18]$$

the same as [C], which are the final formulas for total deflection and uniform tensile stress in a volute spring when all the turns are bottomed or solid.

For deflections per individual turn ($n = 1$) based on mean diameter and a given stress at solid, the Equations [4] and [6] are combined to put the equations in terms of stress, viz.

$$f = \frac{\pi S_c D^2}{2CGt} \quad [19]$$

The deflection for any value of D between D_1 and $D_2 = D_x$ may be found in the following manner: If a given load exceeds P_1, a portion of the volute spring bottoms or becomes solid. The solid point may be found because

$$P_x = \frac{2\eta_2 S_c bt^2}{CD_x}$$

(from Equation [6]) and as the load in the various turns varies inversely, as D

$$\frac{D_x}{D_2} = \frac{P_2}{P_x}$$

therefore

$$D_x = \frac{D_2P_2}{P_x} \quad [20]$$

The deflection for the whole spring is the sum of the two portions, viz., that portion bottomed or closed plus that portion, as yet, unclosed

$$f = \frac{\pi S_c(D_1^3 - D_x^3)}{12CGtp} + \frac{\pi S_c(D_x^4 - D_2^4)}{16CGtpD_x} = f = \frac{\pi S_c}{Gtp}\left[\frac{(D_1^3 - D_x^3)}{12C} + \frac{(D_x^4 - D_2^4)}{16CD_x}\right] \quad [21]$$

Simplifying the Formulas

In any formula, the fewer the terms, the easier it is to use and the less the time required to obtain the result. We can further simplify some of these formulas by eliminating terms representing constants, numeral and symbol, where no appreciable error is introduced. An inspection of Table 1 shows that the symbols η_2 and η_3, which are Saint Venant's coefficients for stress and deflection of rectangular bars in torsion, shows that they become equal for a ratio of $b/t = 6$, for which the numerical value of these coefficients is (0.2985), and remain so to infinity, for which the numerical value is (0.333), a total difference of (0.0348), or 9.6 per cent. (Most heavy hot-wound volute springs will have ratios of b/t nearer to 20 than 10.) If a formula contains any known approximation or error, it should be on the safe side. Therefore we choose a ratio of $b/t = 19 = 0.322$ and using that value we eliminate the symbols η_2 and η_3.

Likewise, an examination of Table 2 discloses that the numerical difference between ratios of D/t between 6 and 20 is 0.1893, or 12.29 per cent. Here we must needs use greater care. However, in heavy hot-wound volute springs, the D/t ratio of the smallest active coil will rarely be less than 8, and of the largest active coil in the same volute spring, more than 20. Between the ratio of 8 and 20, the difference is 0.1158, or 8 per cent. Choosing the more conservative figure, we select a ratio D/t of 11 or 1.4124. With these alterations, which the author's organization has used since 1939 with very satisfactory results, the important formulas become

$$[11] \quad f_x = \frac{\pi P_x(D_1^4 - D_2^4)}{32\eta_3 Gbt^3p} = f_x = \frac{P_x(D_1^4 - D_2^4)}{3.28Gbt^3p} \quad [22]$$

same as Equation [A] of paper.

$$[13] \quad f_{vs} = \frac{\pi S_c(D_1^4 - D_2^4)}{16GtpCD_1} = f_{vs} = \frac{S_c(D_1^4 - D_2^4)}{7.2GtpD_1} \quad [23]$$

same as Equation [*B*] of paper.

$$[17]\quad f_{us} = \frac{\pi S_e(D_1^3 - D_2^3)}{12CGtp} = f_{us} = \frac{S_e(D_1^3 - D_2^3)}{5.4Gtp} \ldots [24]$$

$$[18]\quad S_e = \frac{f_{us}12CGtp}{\pi(D_1^3 - D_2^3)} = S_e = \frac{f_{us}5.4Gtp}{(D_1^3 - D_2^3)} \ldots\ldots\ldots [25]$$

same as Equation [*C*] of paper.

$$[21]\quad f = \frac{\pi S_e}{Gtp}\left[\frac{(D_1^3 - D_x^3)}{12C} + \frac{(D_x^4 - D_2^4)}{16CD_x}\right] = f = \frac{S_e}{Gtp}$$

$$\left[\frac{(D_1^3 - D_x^3)}{5.4} + \frac{(D_x^4 - D_2^4)}{7.2D_x}\right] \ldots [26]$$

It is to be noted that taking diameters to the nearest 1/16 in. will not make the calculations for any particular spring exact, but they will be close enough for all practical purposes if the tensile stress does not border on the maximum.

Developed Bar Length

Fundamentally, the bar length in a spring is $L = 2\pi RN$ and $2R = D;\ N = n + 1.5$ = total number of turns in entire spring.

$$n = h/tg, \text{ and } N = h/tg + 1.5$$

so

$$L = \pi D\frac{h}{tg} + 1.5$$

but in a volute spring D is not a constant but is a variable, so for any increment of length where $D = 2R_x$, we have

$$dL = \frac{\pi}{tg}\,2R_x dh_y$$

or

$$L = \frac{2\pi}{tg}\int_0^h R_x dh_y$$

From the diagram

$$R_x = R_1\left(1 - \frac{h_y}{A}\right)$$

therefore

$$L = \frac{2\pi R_1}{tg}\int_0^h \left(1 - \frac{h_y}{A}\right) dh_y$$

and integrating

$$L = \frac{2\pi R_1}{tg}\left(h - \frac{h^2}{2A^2}\right)$$

Also, from the diagram, $A = \dfrac{hR_1}{R_1 - R_2}$

In this case, $R_1 = \dfrac{\text{OD}}{2}$, and $R_2 = \dfrac{\text{ID}}{2}$. Substituting these values

$$L = \frac{\pi\overline{\text{OD}}h}{tg}\left[1 - \frac{h(\overline{\text{OD}} - \overline{\text{ID}})}{2h\overline{\text{OD}}}\right]$$

or

$$L = \frac{\pi\overline{\text{OD}}h}{tg}\left(\frac{2h\overline{\text{OD}} - h\overline{\text{OD}} + h\overline{\text{ID}}}{2h\overline{\text{OD}}}\right)$$

and

$$L = \frac{\pi h}{tg}\left(\frac{\overline{\text{OD}} + \overline{\text{ID}}}{2}\right)$$

but $\dfrac{h}{tg} = n$ and $n + 1.5 = N$

therefore

$$L = \pi N\left(\frac{\overline{\text{OD}} + \overline{\text{ID}}}{2}\right) \ldots\ldots\ldots\ldots [27]$$

Weight of a Volute Spring

Basically, the weight of any spring is W = length × area × weight of 1 cu in. of steel. So for a volute spring $W = \pi Nbtw$, where w = 0.2833 lb = weight of 1 cu in. of steel.

Actually, due to tapered ends, scarfing, punching, grinding, etc., as performed in the manufacture of springs used in the author's organization, approximately 0.86 of the weight of the bar as calculated from the developed length = W = the actual weight of the finished spring.

Therefore

$$W = \frac{0.86 \times 3.1416 \times 0.2833}{2} Nbt(\overline{\text{OD}} + \overline{\text{ID}}) \ldots\ldots [28]$$

or

$$W = 0.383Nbt(\overline{\text{OD}} + \overline{\text{ID}}) \ldots\ldots\ldots\ldots [29]$$

An example of the use of these formulas is given on the following page.

Calculations—Volute Spring

Bar $5\frac{1}{2}$ in; OD = $4\frac{7}{8}$ in.; ID = $2\frac{1}{8}$ in.; F.H. = $7\frac{1}{2}$ in.; S.H. = $5\frac{1}{2}$ in.; Deflection = 2.00 in. Load @ R.H. of $1\frac{1}{4}$ in. = 7500 lb

The solid height sets the bar width, and by trial we decide on $\frac{5}{16}$ in. for thickness

$t = 0.85\,tg;\ 0.3125/0.85 = tg = 0.3677;\ 1.5 \times 0.3677 = 0.5516;\ 2 \times 0.3677 = 0.7354;\quad t^2 = 0.09766,\ t^3 = 0.03052$

		Cubed	4th power
$D_1 = \text{OD} - 1.5tg = 4.875 - 0.5516 = 4.3234$, which to nearest $\frac{1}{16}$ in. =	4.3125	80.2024	345.8728
$D_2 = \text{ID} + 1.5tg = 2.125 + 0.5516 = 2.6766$, which to nearest $\frac{1}{16}$ in. =	2.6875	19.4109	52.1668
	Difference	60.7915	293.7060

$$n = \frac{\text{OD} - (\text{ID} + 2tg)}{2tg} = \frac{4.875 - (2.125 + 0.7354)}{0.7354} = 2.74;\ 2 \times 2.74 = 5.48;\ n + 1.5 = N;\ N = 2.74 + 1.5 = 4.24$$

$$p = \frac{D_1 - D_2}{2n} = 4.3125 - 2.6875/5.48 = 0.2955:\ D/t \text{ for } P_1 = 4.3125/0.3125 = 13.8 \text{ and } C = 1.3843;\ D/t \text{ for } P_2 = 2.6875/0.3125 = 8.6 \text{ and } C = 1.453$$

$$S_c = \frac{f_{us}5.40Gtp}{(D_1^3 - D_2^3)} = \frac{2 \times 5.40 \times 10{,}600{,}000 \times 0.3125 \times 0.2955}{60.79} = 173{,}902 \text{ psi tensile stress}$$

$$P_1 = \frac{2\eta_2 S_c bt^2}{CD_1} = \frac{2 \times 0.32 \times 173{,}902 \times 5.5 \times 0.09766}{1.3843 \times 4.3125} = 10{,}014 \text{ lb};\ P_2 = \frac{2 \times 0.32 \times 173{,}902 \times 5.5 \times 0.09766}{1.453 \times 2.6875} = 15{,}309 \text{ lb}$$

$$f_x = \frac{P_x(D_1^4 - D_2^4)}{3.28Gbt^3p} = \frac{7500 \times 293.706}{3.28 \times 10{,}600{,}000 \times 5.5 \times 0.03052 \times 0.2955} = 1.27 \text{ in.}$$

Total deflection in terms of load when D_1 is just solid $= f_x = \dfrac{P_1(D_1^4 - D_2^4)}{3.28Gbt^3p} = \dfrac{10{,}114 \times 293.706}{3.28 \times 10{,}600{,}000 \times 5.5 \times 0.03052 \times 0.2955} = 1.706$ in.

Total deflection in terms of tensile stress when D_1 is just solid $= f_{vs} = \dfrac{S_c(D_1^4 - D_2^4)}{7.2GtpD_1} = \dfrac{173{,}902 \times 293.706}{7.2 \times 10{,}600{,}000 \times 0.3125 \times 0.2955 \times 4.3125} = 1.684$ in.

Total deflection in terms of tensile stress when all turns are solid $= f_{us} = \dfrac{S_c(D_1^3 - D_1^3)}{5.40Gtp} = \dfrac{173{,}902 \times 60.7915}{5.40 \times 10{,}600{,}000 \times 0.3125 \times 0.2955} = 2.00$ in.

$$\text{Developed bar length} = L = \pi N\left(\frac{\overline{\text{OD}} + \overline{\text{ID}}}{2}\right) = 3.1416 \times 4.24 \times \left(\frac{4.3125 + 2.6875}{2}\right) = 46.62 \text{ in.}$$

$$\text{Weight of spring} = W = 0.383Nbt\,(\overline{\text{OD}} + \overline{\text{ID}}) = 0.383 \times 4.24 \times 5.5 \times 0.3125 \times (4.875 + 2.125) = 19.54 \text{ lb}$$

Discussion

H. O. Fuchs.[5] Efforts to simplify the calculation of volute springs are very timely. To accomplish this purpose, the author introduces some well-justified approximations such as his fixed values of Saint Venant's and Vogt's coefficients and achieves some worth-while short cuts with small sacrifice of accuracy.

As to the fundamental assumption that the maximum bottoming stresses are uniform along the blade, the writer believes that it needs considerable qualification. The bottoming stress and load-deflection formulas which are given may be correct for the springs used in the author's organization. For the springs with which the writer is familiar, they would not even nearly apply. Examination of load-deflection curves is sufficient to prove the difference; life-test records further confirm it. The stress ranges from free to bottomed are much higher for the inner than for the outer coils. Assuming uniform maximum bottoming stress is therefore unsafe unless nonuniform trapped negative stresses are considered. Including these would spoil the simplicity of the formulas.

The writer agrees with the author that the assumption of uniform helix angle is usually not correct either and leads to more complicated formulas than the assumption of uniform bottoming stress. But he would be reluctant to use formulas which fail to lead to a correct load-deflection curve, and which err on the low side for the stress ranges.

All volute springs which the writer has seen are intermediate between uniform helix angle and uniform bottoming stress. Formulas can be constructed which give correct load-deflection curves and the correct bottoming-stress ranges at the inner heel and at a point near the outer heel, as well as much improved approximations for intermediate points. These formulas must include a factor to account for the different distribution of helix angles (or pitches) which may exist in two otherwise identical volute springs. Limiting values of this factor correspond to uniform bottoming stress and to uniform helix angle.

A calculating procedure which allows for these factors was set up. Dimensionless stress, load, and deflection coefficients were calculated and plotted for a wide range of possible volute springs.[6] With these precalculated coefficients volute-spring computations become very simple. It is hoped that apparently conflicting aims of accuracy and simplicity can both be satisfied in this manner.

[5] Product Study, General Motors Corporation, Detroit, Mich.

F. P. Gooch.[7] The author several times makes the statement that the coils which are closed solid are uniformly stressed. According to the writer's development (using the author's nomenclature), the load P_x, required to close solid a coil whose minor radius is R_x, will be Equation [33] of this discussion, which is:

[6] The methods and charts, developed in co-operation with several members of the Volute Spring Subcommittee of the S.A.E. War Engineering Board, are not yet published; copies are available on request.

[7] Ordnance Design Sub-Office, Philadelphia Ordnance District, The Franklin Institute, Philadelphia, Pa. Jun. A.S.M.E.

$$P_x = \frac{f}{L} \cdot \frac{c_1}{R_x^2}$$

where $c_1 = G\eta_3 bt^3$

and since the moment on the coil

$$M_x = P_x R_x$$

$$M_x = \frac{f}{L} \cdot \frac{c_1}{R_x}$$

Thus the moment on the solid coils varies inversely as the radius of the coil, and since for any stress under the elastic limit the moment is directly proportional to the maximum stress in the bar cross section, it follows that the smallest coils are most highly stressed and vice versa.

Actually, the cold-setting referred to by the author deforms the coils near the smallest radius, so that even under zero load the angle whose tangent is $\frac{f_x}{L_x}$ is not constant but decreases as L_x approaches L.

This plastic deformation has little effect on the lower part of the load-versus-deflection curve but does materially reduce (from the calculated value) the maximum load at which the entire spring will go solid.

Under Equation [A], the author apparently states that the straight-line portion of the deflection curve ends where the largest active coil goes solid. The result of the writer's investigation indicates that the straight-line portion of the deflection curve ends when the largest coil begins to go solid. The bottomed portion starts at R_1 and travels around in a spiral as the load is increased until the entire spring is solid.

The formulas for deflection may be derived very simply as follows:

For all coils free (before bottoming occurs) where ρ is a variable radius[8] and if U is potential energy of deformation

$$U = \frac{Pf_{vs}}{2} = \int_0^{2\pi n} \frac{M^2 ds}{2c_1} = \int_0^{2\pi n} \frac{P^2}{2c_1}\rho^3\, d\theta = \int_0^{2\pi n} \frac{P^2}{2c_1} \left[R_2 + \frac{(R_1 - R_2)}{2\pi n}\,\theta\right]^3 d\theta$$

then

$$f_{vs} = \frac{P\pi n}{2c_1}(R_1^2 + R_2^2)(R_1 + R_2)\ldots\ldots\ldots [30]$$

but Equation [35] of this discussion is

$$\pi n(R_1 + R_2) = L = \frac{\pi}{p}(R_1^2 - R_2^2)$$

so

$$f_{vs} = \frac{P\pi}{2c_1 p}(R_1^4 - R_2^4)\ldots\ldots\ldots [31]$$

When part of the coils are solid, the deflection of the free portion is

$$f_1 = \frac{P_x\pi}{2c_1 p}(R_x^4 - R_2^4)\ldots\ldots\ldots [32]$$

The load P_x required to close solid a coil of radius R_x and axial pitch h_x may be determined by equating internal and external potential energies

$$\frac{P_x h_x}{2} = \frac{M_x\phi}{2} = \frac{M_x^2 2\pi R_x}{2c_1} = \frac{P_x^2 R_x^2 \pi R_x}{c_1}$$

where ϕ is the angular deflection over the length of the coil; but

$$\frac{h_x}{2\pi R_x} = \frac{f}{L}$$

so

$$P_x = \frac{fc_1}{LR_x^2}\ldots\ldots\ldots [33]$$

then

$$P_1 = \frac{fc_1}{LR_1^2} \quad\text{and}\quad P_2 = \frac{fc_1}{LR_2^2}$$

Under load P_x, the length L_x will go solid and the deflection of the solid portion will be f_x. Now

$$f_x = f\frac{L_x}{L}\ldots\ldots\ldots [34]$$

then again letting ρ be a variable radius, and considering the spring as a spiral

$$\frac{d\rho}{dL} \approx \frac{p}{2\pi\rho}$$

and

$$L = \int dL = \frac{1}{p}\int_{R_2}^{R_1} 2\pi\rho\, d\rho = \frac{\pi}{p}(R_1^2 - R_2^2)\ldots\ [35]$$

From Equations [34] and [35]

$$f_x = f\frac{L_x}{L} = f\frac{\frac{\pi}{P}(R_1^2 - R_x^2)}{\frac{\pi}{P}(R_1^2 - R_2^2)}\ldots\ldots\ldots [36]$$

then adding the deflection of the free portion

$$f_t = f_x + f_1 = f\frac{(R_1^2 - R_x^2)}{(R_1^2 - R_2^2)} + \frac{P_x\pi}{2c_1 p}(R_x^4 - R_2^4)\ldots [37]$$

A. M. Wahl.[9] It should be noted that the formulas suggested by the author for volute springs include a factor of 1.3 (proposed by R. F. Vogt) which converts torsion stress to equivalent tensile stress on the basis of the maximum-strain theory of strength. In this connection, however, the writer would like to point out that the shear-energy theory is in better agreement with available fatigue-test data than the maximum-strain theory. Thus, for example, fatigue tests on high-carbon and alloy steels, such as those used in springs[10] show ratios of 1.7 to 2 between the endurance limits in bending and in torsion. Ludwik[11] reports values of 1.74 for this ratio for a chrome-nickel steel. These values differ markedly from the figure of 1.3, required on the basis of the maximum-strain theory used by the author, but are not far from the value of 1.73 predicted on the basis of the shear-energy theory. The latter theory also yields good results in predicting the yield points of ductile materials for static loading. For these reasons, it is the writer's opinion that, until further test

[8] "Applied Elasticity," by S. Timoshenko and J. M. Lessells, Westinghouse Technical Night School Press, East Pittsburgh, Pa., 1925, p. 32.

[9] Mechanics Department, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Mem. A.S.M.E.

[10] "Fatigue of Metals," by H. F. Moore and J. B. Kommers, McGraw-Hill Book Company, Inc., New York, N. Y., 1927, p. 147.

[11] "Kerb-und Korrosionsdauerfestigkeit," by P. Ludwik, *Metallwirtschaft*, vol. 10, 1931, pp. 705–710.

data are brought forth in support of the maximum-strain theory as applied to springs, a more logical basis would be the use of the shear-energy theory.

It might be argued that, if a ratio of 1.73 between equivalent tension and torsion stress (as required by the shear-energy theory) is used, the calculated equivalent stress in the spring at maximum load may be above the tensile strength of the material. The reason for this, however, lies in the fact that the calculated stresses (which are based on elastic conditions) are in error when the yield stress is exceeded and a redistribution of stresses occurs. Thus, if it be assumed that yielding occurs at constant stress, it may be shown that, in the case of a narrow rectangular blade under torsion (such as is used in volute springs), the load required to cause complete yielding over the section will be about 1.5 times the calculated value figures by Saint Venant's formulas and neglecting curvature effects. (If curvature effects are considered, this ratio of 1.5 will be further increased.) This means that appreciable yielding of the spring will not occur until the calculated torsion stress (figured by Saint Venant's formulas) reaches a value about 1.5 times the actual torsional yield stress of the material. Since, according to the shear-energy theory, the torsional yield stress is 0.577 times the tension yield stress, this means that yielding in the spring should occur at 0.577 (1.5) = 0.86 times the tension yield point. Although for most spring materials yielding does not occur exactly at constant stress, and hence these figures may have to be modified somewhat, this illustration does explain why statically loaded springs may under certain conditions carry much higher calculated stresses than those indicated on the basis of the tensile strength of the material. The reason for this, however, is not that the shear-energy theory is inapplicable, but rather that the formulas are in error after yielding occurs.

Author's Closure

Mr. Fuchs states that the bottoming stress and load-deflection formulas given by the author may be correct for the volute springs used by the author's organization, but for the volute springs with which he is familiar they will not even nearly apply. The difference is due to the following causes:

1 The springs with which he is familiar are designed to have a uniform helix angle, regardless of the fact that the diameter or radius of the coils is not uniform. By such a design if the largest active coil, which contains the largest amount of steel, is designed to make the most efficient use of that steel, the smaller coils will be overstressed by varying amounts, the smallest active coil being overstressed the most. Such a design will undoubtedly take a set in service, the smallest coil taking the greatest set. If such a design is preset, the smallest active coil will take an excessive set, and in either case the set is very apt to produce a minute rupture or ruptures which probably will result in early failure.

2 The formula he uses contains an error in the derivation of the radius, namely, R_1 equals the outside diameter minus p or tg divided by 2, both p and tg being defined as bar thickness plus gap. Looking at a section of one half of a volute spring, the derivation he uses appears to be correct. However, a plan view of the whole spring shows that R_1 or D_1 divided by 2 equals outside diameter minus 1.5 p or tg divided by 2 is very much more nearly correct. It should be pointed out that both of these derivations are based on the conventional form of volute spring, which has three quarters of a turn at both the outer and inner ends for the spring seats. If the design of the volute spring deviates from that assumption, correction should be made for such deviation.

Incidentally, the calculation procedure and charts referred to in the last paragraph of his discussion contain the error in deriving the radius just mentioned.

Mr. Gooch questions the author's statement that the straight-line portion of the load-deflection curve ends where the largest active coil goes solid. The author grants that Mr. Gooch's statement is more accurate. The volute spring, being a spiral, is theoretically composed of an infinite number of arcs of varying radii. Practically, in manufacture, the spring is probably composed of arcs which are more nearly half or quarter circles. If consisting of half circles, it appears that the straight-line portion of the load-deflection curve should end when the first half turn of the largest active coil bottoms, or, if of quarter circles, then when the first quarter turn of the largest active coil bottoms.

Dr. Wahl states that the factor used by the author for converting torsion stress to equivalent tensile stress should be based on the shear-energy theory rather than the maximum-strain theory. According to Dr. Wahl's discussion these two theories do not agree. Therefore, until such time as these theories are brought into agreement, or one is proved in error, a spring engineer who might use these formulas may prefer one theory, and another spring engineer the other theory. However, any spring engineer who desires to use these formulas is, by Dr. Wahl's discussion, put on notice that the factor used is based on the maximum-strain theory, and he will, no doubt, convert to whichever theory he believes is correct, or to neither by converting to torsion stress.

The author is grateful to the discussers for their very interesting comments. Admittedly, the analysis set forth by the author is only one approach. There comes to mind an expression credited to Mr. Rolls, of Rolls-Royce: "There may be a number of ways of doing a thing, but there is only one right way." The author would like to find that one right way. However, even that one right way will not produce a spring which will perform according to design if conditions, either chemical analysis or production methods, or both, vary irrespective of whether the variations are caused by necessity convenience or carelessness.

If may also be mentioned that the volute spring will, no doubt, be more widely used in the future than heretofore, but it will not be the answer to all of the spring-engineer's difficulties. It does not make as efficient use of the material as does the round wire helical spring, but it does use spring material more efficiently than the elliptical spring. However, that is only one of many characteristics which the spring engineer will consider before determining which form of spring or design of spring will be best suited to perform the required functions. It is the author's belief that volute springs are very interesting for designs in which an increasing spring rate for the latter portion of the travel is a desirable characteristic.

Notes on Secondary Stresses in Volute Springs

By H. O. FUCHS,[1] DETROIT, MICH.

Besides the torsion stress which can be calculated from twisting moment and cross section, other, secondary, stresses appear in volute springs; these are caused chiefly by the variation of twist from section to section or by bending of the spring. They may reach high values in the small-diameter coils. An approximation theory is given for the calculation of secondary stresses; strain-gage measurements and distortions of springs are shown. It is proposed to reduce the stresses on the smallest coils by three means, all of which would facilitate manufacture: (*a*) By increasing the inner diameter of the spring; (*b*) by supporting the ends more firmly with full-thickness stub tails; (*c*) by winding the straight blade on a slightly conical mandrel to produce a variable helix angle and more desirable stress distribution.

Secondary Stresses

The torsional stress produced by a central load will be called "primary" or "main stress," and the deflection of the blade center line, produced by a central load, "main deflection." All other stresses, particularly those produced by other distortions or warping of the blade, will be called "secondary stresses."

The main stress does almost all of the work of the spring. The load-deflection curve of the spring can be calculated with sufficient accuracy from the main stress alone. The secondary stresses contribute almost nothing to the useful work, but they may contribute to the failure of the blade.

Secondary stresses might be classed with stress concentrations, with which they share the feature that they cannot be found by simple considerations of equilibrium between the applied forces and the internal forces in a cross section. Secondary stresses are produced by variation of the twisting torque from section to section, by flexibility of the dead coils and by misalignment of the spring seats.

It is possible that a more thorough analysis than has been made so far of the curved bar under nonuniform twisting and bending moments would show still other types of secondary stresses.

Coning and Cone Stress

Conventional volute springs are wound with a uniform helix angle. Under load, the helix angle decreases; if the blade section is constant, the decrease is proportional to the square of the coil radius R, according to the theory of the main stress and deformation (1).[2] The decrease continues until the helix angle becomes zero at some point. Then bottoming occurs at that point, and the deflections continue only on the remaining active coils.

Fig. 1 shows the height plotted over the length of the developed blade, according to this theory. The blade was originally wound on a volute mandrel with axial generating lines. When bottomed, it can be considered as a flat spiral with axial generating lines. If the spring were required to conform at intermediate loads to a volute with variable helix angle and axial generating lines, the strip would have to bend about a radial axis. A round-wire volute may do this; a blade with high bending stiffness about a radial axis will produce the variable helix angle by conforming itself to a cone instead of a cylinder.

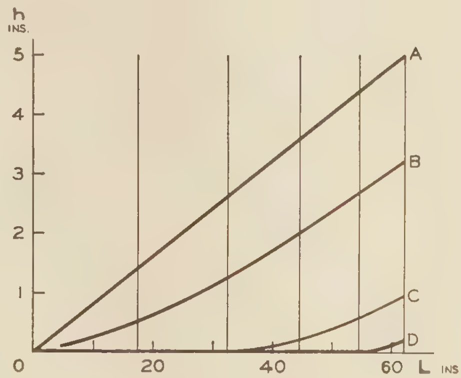


Fig. 1 Height Plotted Over Developed Blade Length at Various Loads

(*A*, No load, uniform helix angle; *B*, no coil bottomed; *C*, 2 coils bottomed; *D*, 4 coils bottomed.)

A thin broad strip coiled around a cylinder maintains a uniform helix angle; when coiled around a cone it will vary the helix angle at the rate of π times the cone angle per turn.

A volute blade will behave much like a developable surface. Since under load the helix angle increases toward the small end, the cones will open toward the small end. The cone angle is $\varphi = 2R\,\dfrac{d\alpha}{dL}$ where R is the coil radius, α the helix angle, and L the length measured along the blade.

Coning increases the radius of curvature of one small side of the blade and decreases the radius of curvature of the other small side. This introduces a bending stress, which in first approximation is zero along the center lines of the cross section and on the faces increases uniformly from zero at the middle to a maximum at the edges. On the edges of a conventional volute spring the stress will be, as shown in the Appendix

$$\sigma_c = \pm \frac{4}{5}\,\frac{P}{tR}$$

A volute spring with a blade section b = 6 in. and $t = {}^5/_{16}$ in. will show at the small coil with R = 1.2 in., under a load P = 30,000 lb, a cone angle φ = 0.73 deg and a cone stress σ_c = 64,000 psi.

Cold setting or bulldozing will change this picture by producing a variable helix angle and trapped cone stresses in the unloaded spring. The stress range will not be changed.

It might be noted that the main stress is calculated on the assumption that "an infinitesimal coil element will have the same

[1] Product Study, General Motors Corporation.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Special Research Committee on Mechanical Springs and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

action as a similar element of a straight helical spring," and the cone stress is introduced by considering the action of neighboring elements which are loaded by a different twisting torque.

End Restraints—Eccentric Loading

The theory of the main stress and of the main deflections is based on Saint Venant's results for the twisting of a straight bar under uniform torque, rigidly restrained against torsion at the end, but free to distort its cross section. The resulting twist angles are then applied to a volute-shaped center line, assuming that the end or heel section is restrained against torsion, but free to change its helix angle and warp its cross section.

The effect of nonuniform torque has been touched upon in the preceding section. The assumption that the end section is free to change its helix angle is probably nearly correct. Observation shows that the end section rocks about an axis through the heel point and a point about 180 deg from the heel. The assumption that the end section is rigidly restrained against torsion is only a rough approximation. The tail of the volute is far from rigid; as a result, the end section leans heavily inward and the active section at 180 deg from the heel bottoms long before the helix angle near the heel has become zero. This step-by-step bottoming has been pointed out before by Keysor. As a result, the largest active coil assumes a cone shape open toward the large end with its attendant cone stresses.

Furthermore, the load becomes eccentric, shifting from the center toward the heel until the point across from the heel has been bottomed, then away toward the first bottomed point, back again to the second bottomed point, and so on.

For a coil spring, where conditions are somewhat similar, the amount of eccentricity has been calculated by Keysor (2) and confirmed experimentally by Pletta and Maher (3).

For a volute, the calculations become much more difficult. It is certain, however, that even with perfect alignment of the springs seats the load will become eccentric. The effect of an eccentric load can be replaced by that of a central load, combined with a moment applied to the end. An additional end moment may be introduced in certain volute-spring applications by misalignment of the spring seats, which can be very pronounced.

The effect of an end moment will be:

1 To increase the torque in sections on one side of the spring.
2 To decrease the torque in sections on the opposed side.
3 To introduce arch stresses. These are most pronounced in sections where the end moment produces zero twisting torque and maximum bending torque.

Arch Stress

The action of a bending moment on one of the coils of the volute, approximated by a split circular ring, is shown in Figs. 2 and 3. Fig. 3 shows (much exaggerated) the deformations produced by the moment.

The stresses produced by the moment M will vary from section to section. If the ring has a pitch radius R and its cross section is a narrow rectangle of width b and thickness t, the stresses in sections A-A and C-C will be torsion stresses $\tau_E = \pm \dfrac{3M}{bt^2}$ which add to or subtract from the main torsion stress.

In section B-B there will be a bending stress necessary to furnish internal reactions to the externally applied moment; this is

$$\sigma_B = \pm \frac{6M}{b^2t}$$

on the upper and the lower face.

In addition to this direct bending stress there is a stress produced by the change of curvature of the originally circular ring. On the upper face, the curvature is flattened out, on the lower face it is increased. These stresses were first observed by M. Olley in 1936, on the bent parts of torsion rods, and called "arch stresses" by him. By a calculation derived in the Appendix the arch stress on the edges of a steel blade of narrow rectangular section, coiled as shown, is found to be

$$\sigma_A = \pm \frac{2M}{Rt^2}$$

In the split ring, shown in Fig. 3, there is tension on the inner

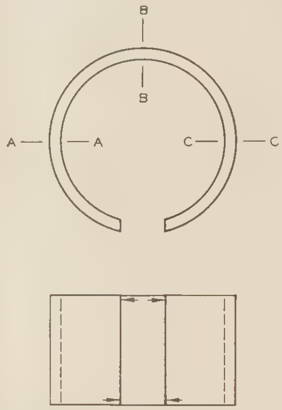


Fig. 2 Moments Applied to a Split Ring

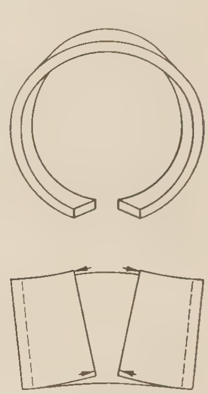


Fig. 3 Distortion of Ring Shown in Fig. 2
(Greatly exaggerated.)

upper and outer lower edge, compression on the outer upper and inner lower edge.

Values of $t = {}^5/_{16}$ in., $b = 6$ in., which correspond to a typical blade, give the following stresses produced by the moment M

$$\tau_E = 5.13M \qquad \sigma_B = 0.534M \qquad \sigma_A = 20.6M/R$$

A load of 30,000 lb, applied ${}^1/_4$ in. off center, will produce in the small coil at radius $R = 1.2$ in., an arch stress

$$\sigma_A = 130{,}000 \text{ psi}$$

It follows that the small coils of a volute spring are subject to very high stresses when the load becomes eccentric, or a bending moment is applied to the spring.

In a volute, the same value σ_A would have been found as cone stress from the variation of twisting torque at section B-B. However, the same type of stress, with an additional term for bending variation, appears also in round sections where no coning is visible, whenever the applied moment produces a twisting torque which varies from section to section. It may lead to fractures in such applications as hooks on the ends of extension springs or in bent ends of torsion rods, where it was first observed.

Combination of Main Stress and Secondary Stresses

Cone and arch stresses are maximum at the edges of the blade and decrease to zero at the middle of the sides. Torsion stresses are maximum at the middle of the sides and decrease to zero at the edges, according to a law which is given in infinite-series form by Timoshenko (4).

Fig. 4 shows some curves of torsional-stress distribution over the long sides of rectangles, plotted from the infinite-series law. For volute-spring blades with a ratio $b \div t = 20$, this means that 85 per cent of the edge stress combines with the full torsion stress. With $b = 6$ in., failures would be likely to occur about ${}^1/_2$ in. from the edge, which corresponds to observation.

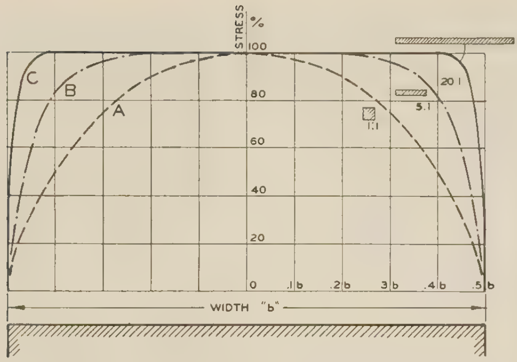


Fig. 4 Distribution of Torsion Stress Over Long Side of Rectangular Section of Width b, Thickness t
(Curve A: $b = t$. Curve B: $b = 5t$. Curve C: $b = 20t$.)

Table 1 shows, for comparison, the various stresses produced by a load P, applied e inches from the center, on a volute coil of radius R and section 6 in. $\times$ $^5/_{16}$ in. The stresses are also expressed in percentage of the main torsion stress for R = 3 in. and R = 1.2 in.

The secondary stresses become more dangerous as the coil radius decreases.

TABLE 1 MAIN STRESS AND SECONDARY STRESSES PRODUCED BY A LOAD P, APPLIED e IN. FROM CENTER, ON A VOLUTE COIL OF RADIUS R AND SECTION 6 $\times$ $^5/_{16}$ IN.

Type of stress	Symbol	Stress, psi	at R = 3 in., per cent	at R = 1.2 in., per cent
Main torsion stress	τ	$5.1\,PR$	100	100
Direct shear	τ_D	$0.8\,P$	5	13
Torsion from eccentricity	τ_E	$5.1\,Pe$	$e \times 33$	$e \times 83$
Cone stress	σ_C	$2.6\,P/R$	5.7	35
Arch stress	σ_A	$20.6\,Pe/R$	$e \times 45$	$e \times 280$
Direct bending	σ_B	$0.5\,Pe$	$e \times 3$	$e \times 8$

Arch stress may add to or deduct from the cone stress, as the case may be. Direct shear adds to torsion on the inside of the coil; deducts on the outside. Eccentricity torsion will add on one side, deduct on the other.

Maximum arch stress occurs at a point where the eccentricity torsion is zero and varies around the coil according to a sine law. Eccentricity torsion varies according to a cosine law; the combination of the two will be most dangerous at some intermediate point between their respective maxima.

Evidence of Distortions

Figs. 5 to 8, inclusive, show the changes in helix angles, the rocking about the heel, the coning, and the bending by eccentricity. The wires attached to the springs were used to measure cone and helix angles.

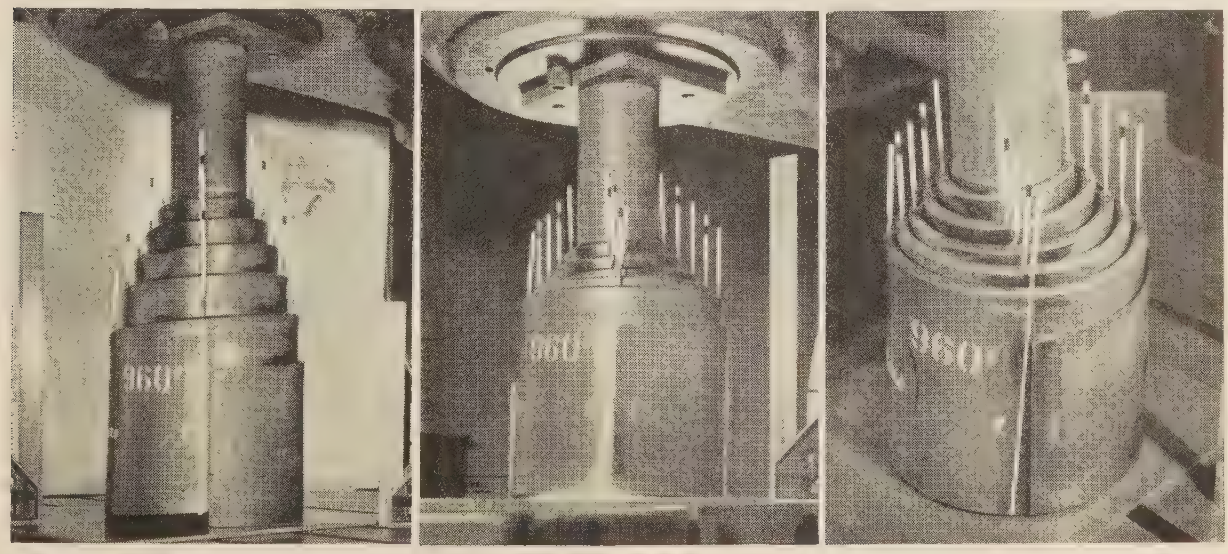
Figs. 5(a) to (c) show three views of the same side of a spring. Fig. 5(a) is the spring under a small preload. In Figs. 5(b) and (c), normal load is applied and the point across from the heel is bottomed while the intermediate points are free and the tail lifted off. Fig. 5(b) shows how little is gained by grinding the seat square with the axis when the spring is free. Fig. 5(c) is the same condition as Fig. 5(b), seen from higher up. It indicates bending of the whole spring, concave toward the heel. Fig. 6 is the rear view of the same condition, showing how the tail is lifted at this intermediate point and giving further evidence of coning and bending.

Figs. 7(a) to (c) show the heel side of another spring under no load, normal load, and twice normal load. Coning, rocking, and variable leaning can be observed. Fig. 8 shows a view with the heel at the left side of the illustration. Coning of the heel is very pronounced. In this spring, coning was so strong that the tail section marked 2 was able to make contact with the seat.

A test which was rigged up to check the arch-stress theory is sketched in Fig. 9, and in Table 2 a comparison between stresses measured with a photocell strain gage and calculated stresses is given for this check test. The theory is definitely confirmed, except that both distortion angle and measured stresses are about 8 per cent lower than calculated. This may be caused by the fact that the internal strain energy of the arch stresses themselves is neglected in the calculations.

A further confirmation of these theories is furnished by failures in service and tests, which frequently occur somewhere near $^3/_4$ in. from the edge, where the combination of torsion stress and edge stresses is most dangerous.

Strain-gage measurements on complete volute springs showed edge stresses of up to 60,000 psi at the accessible outer edge, where



(a) (b) (c)

Fig. 5 Views of a Volute Spring
(a, No load; b, normal load; c, normal load. In all cases heel may be seen at left side.)

Fig. 6 Volute Spring Under Normal Load
(Heel seen at right side.)

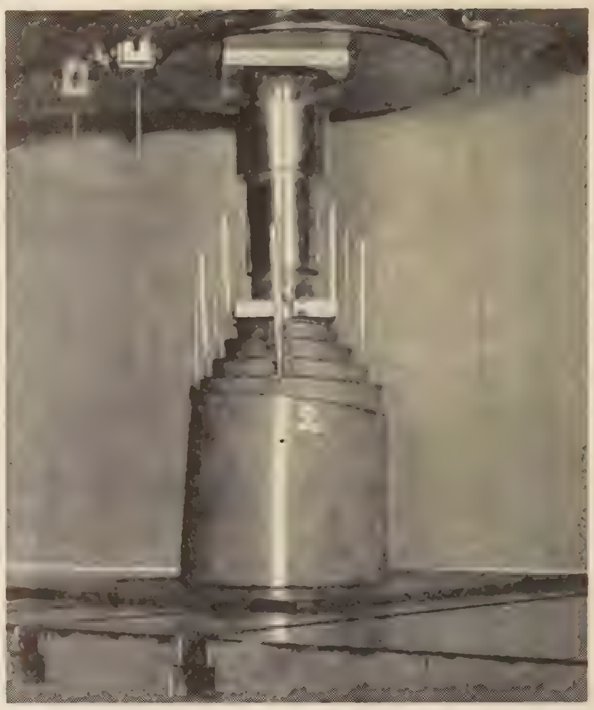
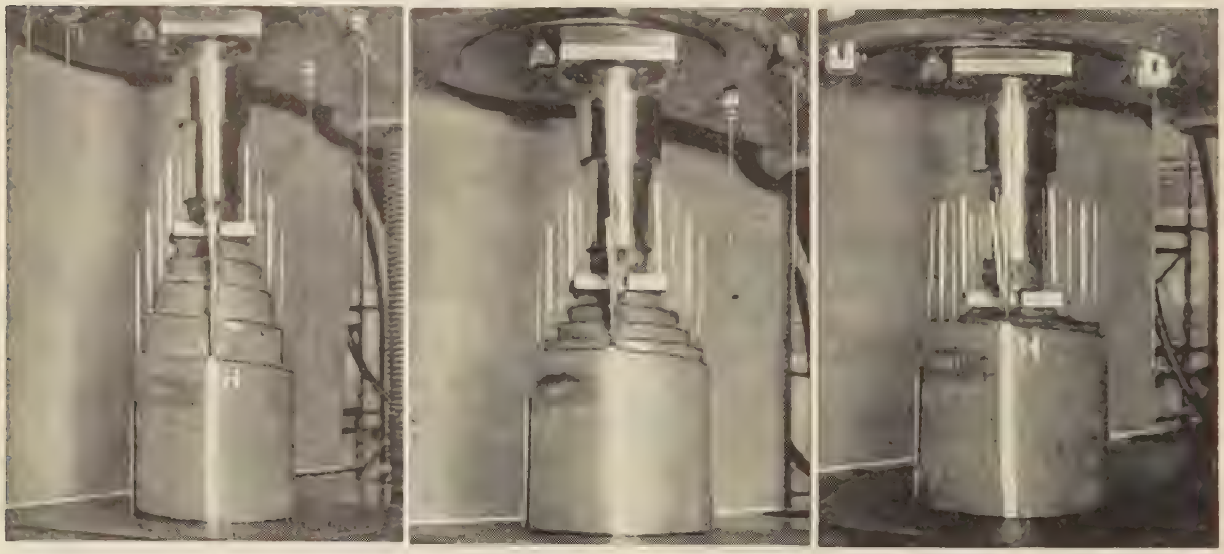


Fig. 8 Volute Spring Under Normal Load
(Heel seen at left side.)



(a) (b) (c)

Fig. 7 Views of a Volute Spring
a, No load; b, normal load; c, double normal load. In all cases heel may be seen in front.)

torsion stresses are zero; at the inaccessible inner edge higher stresses would be expected. Coning has also been observed and described by Reynal (5).

Means to Reduce Stresses

1 Secondary stresses are largest on the smallest coils. For this reason, it seems attractive to design springs with fewer coils and larger inside diameters. Contrary to what might be expected, it is sometimes possible to produce a spring with equal work capacity, equal normal rate and equal maximum stress by using fewer coils and less steel, because the smaller volume of steel can then be used more efficiently. No definite rule can be given, but an investigation of current designs shows that it is not necessarily best to pack as much steel as possible into a given space.

2 Load eccentricity can be reduced by supporting the heel

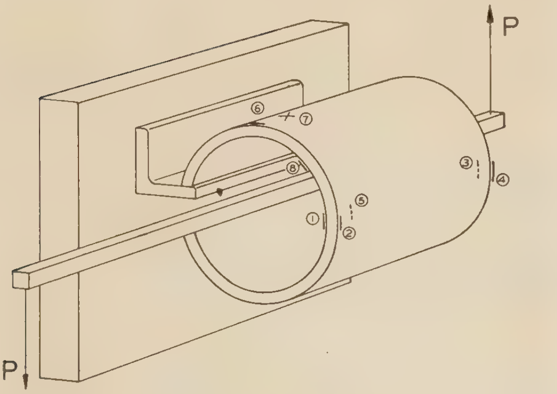


FIG. 9 CHECK TEST FOR ARCH STRESSES
(Dimensions of ring: $6^3/_4$ in. OD; $6^1/_8$ in. ID; width, $4^1/_4$ in.; torque applied, 1500 in-lb.)

TABLE 2 COMPARISON OF MEASUREMENTS AND CALCULATIONS FOR CHECK TEST FIG. 9[a]

	Stress, psi		
		Calculated	
Point	Measured	With arch stress	Without arch stress
1	+11000	+11950	+ 1650
2	— 5000	— 8150	+ 1550
3	—12000	—11950	— 1650
4	+ 7500	+ 8150	— 1550
5	+ 6500	+ 6400	+ 860
6	— 1000	0	0
7	11500	12000	12000
8	+ 4000	...	...
Angle of slot, deg	1.75	1.9	1.9

[a] Stress at point 7 is pure shear.

section more firmly. This can be done by using the full blade thickness for the tail. The additional weight and space which are required can be reduced by using only $^9/_{16}$ turns for the tail instead of $^3/_4$ turns; the last quarter of the tail contributes nothing to the support of the heel. Incidentally, this design would save hot-forging of the tapered tail and thereby decrease the cost and also the extent of surface deterioration by decarburization. A modified spring seat might be needed.

3 By coning the coiling mandrel in toward the small coils, it is possible to wind a volute with a desirable variable helix angle, providing at the same time a small draft angle to facilitate removal of the spring from the mandrel.

A mandrel with a constant cone angle will decrease the helix angle from the large toward the small coils by an equal amount for each turn. The coil radius also decreases by an equal amount per turn. The bottoming stress in a volute spring is proportional to $\alpha t/R$ where α is the helix angle, t blade thickness, R coil radius.

In Fig. 10, the product αt is plotted over the coil radius R. The bottoming stress at any radius is proportional to the slope of a line drawn from the origin to the curve at that radius.

Line *A-A* represents a conventional spring with uniform helix angle and thickness. The efficiency is very low because the large volume of steel in the outer coils must work at less than one half the maximum bottoming stress. The spring will fail at the small end. Line *B-B* represents a spring with small coils tapered in thickness (1). The maximum stress is reduced, the efficiency increased at the cost of a slight sacrifice in final rate and load. Line *C-C* represents a spring of uniform bottoming stress, obtained with constant blade thickness by a variable helix angle. This spring also sacrifices some of the rate build-up but would be very efficient if all strokes would deflect the spring an equal amount.

In service, small deflections may occur more often than large deflections, and the spring *C-C* would therefore fail near the large end, though the stress range is higher at the small end. Reducing the stress on the frequently bottomed large coils and increasing it on the seldom bottomed small coils will increase the service efficiency above that of a spring with uniform bottoming stress and give a higher-rate build-up. Such a spring is indicated by line *D-D*.

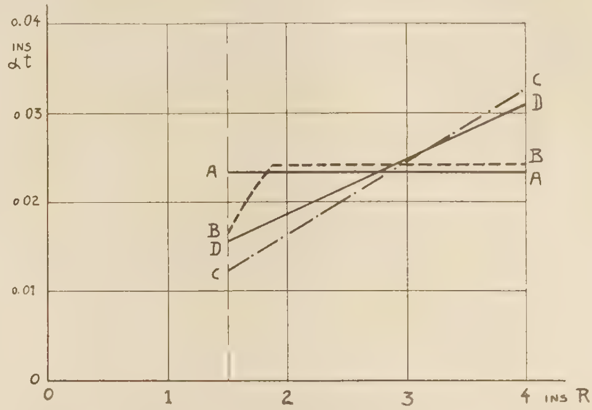


FIG. 10 BOTTOMING STRESS DISTRIBUTION ALONG BLADE OF VOLUTE SPRINGS $\left(\tau = \text{CONSTANT} \times \dfrac{\alpha t}{R}\right)$

(Bottoming stress at any point is proportional to the slope of a line drawn from the origin to the curve at that point. *A-A* conventional; *B-B* tapered in thickness on small coil; *C-C* uniformly stressed, variable helix; *D-D* compromise, variable helix.)

When comparing efficiencies on the basis of Fig. 10, it must be kept in mind that the amount of steel which works between $3^1/_2$ and 4 in. coil radius is more than twice as great as the amount between $1^1/_2$ and 2 in. radius. Raising the stress on the outer coil until failures occur equally often at all points of the spring in service will, therefore, increase the efficiency rapidly. The cone angle of the unloaded spring would have to be of the same order as the cone angle which the spring now assumes under load, i.e., approximately $^1/_2$ deg.

The bottoming loads of the various coils increase in inverse proportion to the square of the coil radius in ordinary volute springs; in inverse proportion to the coil radius in uniformly stressed volute springs of constant blade section. The final bottoming load is of no importance in itself, because any volute spring will safely carry loads much above the final bottoming load, and the flexibility just before bottoming the last coil is so low that brackets and levers may be more flexible than the spring itself.

THEORETICAL BASE

The secondary stresses were found by a method of successive approximations. The main deflections were calculated from the published theory (1) which was considered as a first approximation. Rubber and paper strips were then used to find what secondary distortions would be induced by the main deflections; the distortion strains were calculated with simple assumptions based on symmetry, change of curvature, etc., and the stresses computed from the strains. This method is relatively rapid and believed to be approximately correct as long as the results check with strain measurements and location of failures, although it lacks a good theoretical foundation.

A more thorough theoretical study of the general case of a curved bar under arbitrary load or nonuniform torsion and bending would be of great value if the results can be expressed in practicable form. It might be mentioned that studies of a straight bar under a special nonuniform torsion have been published (6, 7, 8) and show stresses similar to arch stresses.

Appendix

Cone Angle and Variation of Helix Angle. Fig. 11 shows a blade element, developed; length of element dL; helix angle at one end α, at other end $\alpha + d\alpha$.

Approximating: Cosine $\alpha, \approx 1$, sine $\alpha,\varphi \approx$ tangent $\alpha,\varphi \approx \alpha,\varphi$ gives $a = \frac{dL}{d\alpha}$.

Coiling the blade center line with a radius R will produce an element of cone surface.

Angle included at the cone apex is "cone angle" $\varphi = \frac{2R}{a}$

$$\varphi = 2R\,\frac{d\alpha}{dL} \qquad [1]$$

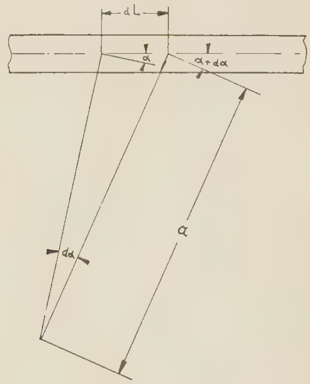


Fig. 11 Developed Blade Element

Cone Stress. A cylindrical-coil element of thickness t, width b, length dL, coil radius R is forced to become an element of a cone with cone angle φ.

At center line radius remains unchanged, for symmetry

At upper face radius becomes $R - \frac{1}{4}b\varphi$

At lower face radius becomes $R + \frac{1}{4}b\varphi$

Stress at edge $= \sigma_c = E \times \frac{t}{2}\left(\frac{1}{R} - \frac{1}{R + 0.25b\varphi}\right)$

$$\frac{1}{R} - \frac{1}{R + 0.25b\varphi} \approx \frac{b\varphi}{4R^2}$$

$$\sigma_c = \frac{\varphi}{8}\,\frac{tb}{R^2}\,E \qquad [2]$$

Variation of Helix Angle ($d\alpha/dL$) Along Blade of Volute Spring With Uniform Blade Thickness. Fig. 12 shows a top view of a

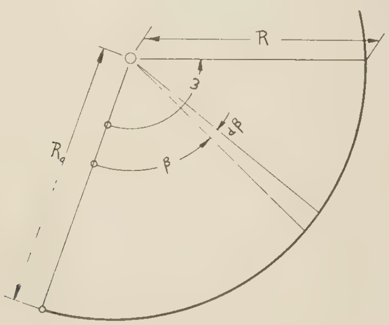


Fig. 12 Top View of Volute Coil Near Heel

volute coil near the heel ($\omega = 0$). Radius R decreases from R_0 at the rate of p units per turn, so that at section β

$$R_\beta = R_0 - \frac{p}{2\pi}\beta$$

Under a central load P, the twist angle $d\phi$ in the element $d\beta$ is

$$d\phi = \frac{P}{GT}\left(R_0 - \frac{p}{2\pi}\beta\right)^2 d\beta$$

This will change the helix angle at ω by

$$d\alpha = -\sin(\omega - \beta)\,d\phi$$

The heel itself will change its helix angle by $-\alpha_0$ and lean inward by ψ_0 which will change the helix angle at ω by

$$-\alpha_0 \cos\omega - \psi_0 \sin\omega$$

Under load, the helix angle at ω will, therefore, decrease by

$$\Delta\alpha = \alpha_0\cos\omega + \psi_0\sin\omega + \frac{P}{GT}\int_{\beta=\sigma}^{\beta=\omega}\left(R_0 - \frac{p}{2\pi}\beta\right)^2 \sin(\omega - \beta)\,d\beta$$

$$\Delta\alpha = \alpha_0\cos\omega + \psi_0\sin\omega + \frac{P}{GT}\left[\left(R_0 - \frac{p}{2\pi}\beta\right)^2 - \frac{p^2}{\pi^2} + \frac{p}{\pi}R_0\sin\omega - \left(R_0^2 - \frac{p^2}{2\pi^2}\right)\cos\omega\right]$$

and

$$\alpha = \alpha_{\text{no load}} - \Delta\alpha \qquad \frac{d\alpha}{d\omega} = -\frac{d\Delta\alpha}{d\omega}$$

$$\frac{d\alpha}{d\omega} = \left[\alpha_0 - \frac{P}{GT}\left(R_0^2 - \frac{p^2}{2\pi^2}\right)\right]\sin\omega - \left[\psi_0 + \frac{P}{GT}\frac{p}{\pi}R_0\right]\cos\omega + \frac{P}{GT}\frac{p}{\pi}R$$

The first two terms describe a cyclic variation which can be completely canceled out by tilting the spring seat in the direction of $-\alpha_0$ by the amount of the first square bracket and in the direction of $-\psi_0$ by the amount of the second square bracket. The last term describes how the helix angle of the tilted spring increases with increasing distance from the heel and is the only one which is of interest here.

$$\frac{d\alpha}{d\omega} = -\frac{d\Delta\alpha}{d\omega} = \frac{PRp}{\pi GT}$$

$$\frac{d\alpha}{dL} = \frac{d\alpha}{d\omega}\,\frac{d\omega}{dL}$$

$$\frac{d\alpha}{dL} = \frac{Pp}{\pi GT} \qquad [3]$$

Cone Stress in Volute Springs. Introducing Equation [3] in Equation [1] gives

$$\varphi = \frac{2}{\pi}\,\frac{RPp}{GT} \qquad [4]$$

Equation [4] in Equation [2] gives

$$\sigma_c = \frac{1}{4\pi}\,\frac{E}{G}\,\frac{Pptb}{RT} \qquad [5]$$

with $\frac{E}{G} = \frac{8}{3}$, $\frac{p}{t} = \frac{5}{4}$, $T = \frac{bt^3}{3}$, this becomes

$$\sigma_c = \pm\frac{4}{5}\,\frac{P}{tR} \qquad [5a]$$

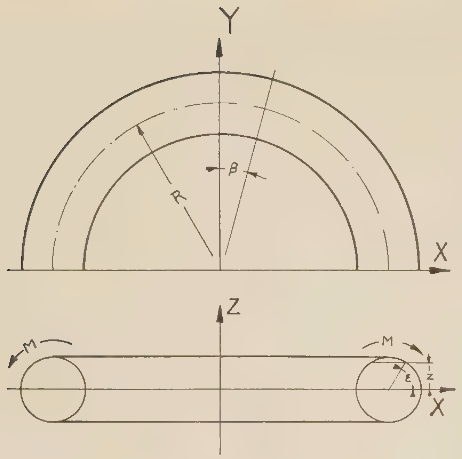


FIG. 13 HALF RING, WITH ENDS TWISTED AGAINST EACH OTHER BY TORQUE M

Arch Stress. Fig. 13 shows a half ring the ends of which are twisted against each other by a torque M. At a section β this will produce a bending moment $M_B = M \cos \beta$ and a twisting moment $M_T = M \sin \beta$ producing an increment of bend angle

$$d\theta = \frac{M}{EI} \cos \beta dl$$

and an increment of twist angle

$$d\phi = \frac{M}{GT} \sin \beta dl$$

in an element of length dl.

In section β a line which at no load was perpendicular to the x, y-plane will now lean by an angle γ in the x, z-plane and by an angle ϑ in the y, z-plane. The increments caused by the element dl at β will be

$$d\gamma = \left(\frac{M}{EI} \cos^2\beta + \frac{M}{GT} \sin^2 \beta\right) dl$$

$$d\vartheta = \left(\frac{M}{GT} - \frac{M}{EI}\right) \sin \beta \cos \beta dl$$

and with

$$\frac{M}{EI} = N, \quad \frac{M}{GT} = K, \quad dl = Rd\beta$$

$$\gamma = \frac{N + K}{2} R\beta + \frac{N - K}{2} R \sin \beta \cos \beta$$

$$\vartheta = \frac{N - K}{2} R \cos^2 \beta$$

The center line of a slab at height z above the center plane had, before load application, a curvature $-\frac{1}{R}$ and co-ordinates

$$x = R \sin \beta \qquad y = R \cos \beta$$

After load application its co-ordinates will be

$$x = R \sin \beta + \gamma z \qquad y = R \cos \beta + \vartheta z$$

and its new curvature at $\beta = 0$ will be

$$\frac{d^2y}{dx^2} = \frac{d}{d\beta} \frac{dy}{dx} \times \frac{d\beta}{dx}$$

where

$$\frac{dy}{dx} = \frac{dy}{d\beta} \times \frac{d\beta}{dx}$$

This works out to be at $\beta = 0$

$$\frac{d^2y}{dx^2} = -\frac{1}{R} \frac{1 + (N - K)z}{(1 + Nz)^2} \approx -\frac{1}{R} [1 - (N + K)z]$$

so that the change in curvature of this slab becomes

$$\frac{z}{R} (N + K)$$

and if the width of the slab is t, the arch stress becomes

$$\sigma_A = \frac{t}{2} \frac{z}{R} (N + K)E$$

$$\sigma_A = \frac{tz}{2R} \left(\frac{1}{I} + \frac{E}{G} \frac{1}{T}\right)M \qquad [6]$$

For a volute-spring section, the maximum stress will occur at the edge $z = \frac{b}{2}$ and $\frac{1}{I}$ is negligible against $\frac{1}{T}$ so that with $\frac{E}{G} = \frac{8}{3}$ and $T = \frac{bt^3}{3}$

$$\sigma_A = \frac{2M}{Rt^2} \qquad [6a]$$

For a circular wire of diameter d, a slab at z will have the width $d \cos \epsilon$ where $\sin \epsilon = 2z/d$.

The direct bending stress will be

$$\sigma_B = \frac{Md}{2I} \sin \epsilon$$

The arch stress

$$\sigma_A = \frac{M}{I} \frac{d^2 \sin \epsilon \cos \epsilon}{4R} \left(1 + \frac{E}{G} \frac{I}{T}\right)$$

with $\frac{E}{G} = \frac{8}{3}$ and $\frac{I}{T} = \frac{1}{2}$ the sum of the two stresses becomes

$$\sigma = \sigma_B + \sigma_A = \frac{32M}{\pi d^3} \left(\sin \epsilon + 1.17 \frac{d}{R} \sin \epsilon \cos \epsilon\right)$$

The location of the highest stressed point will be at the angle where the expression in brackets is a maximum and this depends upon the ratio $\frac{d}{R}$. For this point, the expression in brackets varies from 1 for $d/R = 0$ to 1.2 for $d/R = 0.6$.

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Discussion

H. H. Clark[3] and E. H. Lindeman.[3] The existence of secondary stresses in volute springs becomes apparent to anyone who has carefully observed the distortions which accompany deflection. In describing the nature of the most important of the secondary stresses and in evaluating them with respect to the main torsion stresses, the author has made a highly significant contribution to the fund of knowledge concerning the volute spring.

The author explains that the variable-pitch angle produced by cold-settling a volute spring is in consequence to coning, because the blade of the spring is inherently too stiff to permit edge-bending. This concept must be borne in mind to understand adequately the distortions resulting from deflection of the spring.

The suggestion that a controlled variable helix angle be used to promote more uniform bottoming stresses is good and should be developed with respect to specific applications. Similar practice is in effect where small conical round-wire springs are concerned. These springs are generally produced on an automatic coiler, and the helix-angle variation is obtained by bending the wire about a radial axis, but the springs none the less are volute springs by definition.

In practice, large volute springs coiled to a constant helix angle are cold-set and a degree of helix-angle variation is attained. By using properly shaped fixtures in this procedure it should be possible to control the amount of overstress applied to each coil and thereby obtain the maximum benefit from the cold-settling operation. In obtaining a variable helix angle by winding a volute spring on a conical arbor, allowances must be made for the further variations in helix angle which will be produced by the cold-setting operation. Another factor to be considered in the design of a conical arbor is the anticlastic curvature of the blade produced in the coiling operation.

A spring with a variable helix angle will not permit the extreme build-up of load in the final stages as afforded by the conventional constant-helix-angle spring. This condition is recognized by the author, who very aptly also points out that the conventional spring is so stiff just before the last coil bottoms that the flexibility of other elements in the system may be more flexible than the spring itself.

What the author states in regard to using fewer coils and larger inside diameters and to eliminating a portion of the tail seems entirely reasonable. The more uniform stress distribution obtained would tend to correct one of the inherent defects of most volute-spring designs, which require too little of the steel to do too much of the work. Current difficulties, where steel procurement is concerned, should prompt further investigation of these design considerations.

For the successful application of volute springs, there are three fundamental aspects to be considered:

1 The springs must be loaded with minimum eccentricity during all stages of deflection.

2 The spring design itself must be an optimum.

3 The metallurgical phases of the fabrication of the springs must be satisfactory.

So far, the third item has received the lion's share of attention. It is to be hoped that the author's remarks lead to further consideration of the first and second items.

H. C. Keysor.[4] In this paper the author has considered factors, hitherto neglected, or misinterpreted, which help to shed light on the apparently erratic stress distribution in volute springs. Thus, we have applied Saint Venant's theory of torsion to the case of a variable torque, while one of the basic assumptions of this theory is that each element of length of the rod is strained in the same way, which obviously limits the theory to a constant torque.

The paragraph on end restraints is well stated and of timely interest. This subject has received scant attention from spring engineers, although it has a vitally important bearing on fatigue tests of springs. The combination of the spiral form of the center line with a bar cross section widely different from the circle creates end conditions worthy of careful analysis. The partial fixation of the heel points against radial or tangential movement is a condition which will affect the secondary stresses and must be considered in any analytical treatment.

Referring to the subject of reducing load eccentricity, the stub tail proposed by the author would undoubtedly be helpful. However, there is another possibility; to determine the total angle between heel points (or the number of turns), and the ratio between coil radii at the heel points, which will be required to satisfy the condition of axial loading, subject to the various end restraints.

The use of a slightly coned mandrel, as suggested by the author, would give a spring with the helix angle increasing with increase of radius, and the tendency would be toward the condition of uniform solid stress. Carried to an extreme, the effect would be as shown in Fig. 14 of this discussion.

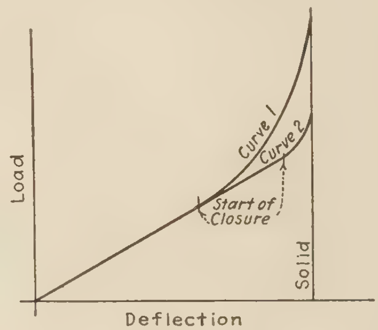

Fig. 14

In this figure, curve 1 is for a volute with a constant helix angle; curve 2 is for the coned spring referred to previously, or a spring which has been cold-set to obtain the same variation in helix angle. If the travel from free to solid remains the same, the spring of curve 2 will have the start of closure delayed because the helix angle at the outer heel point has been increased. The capacity of curve 2 is considerably diminished; moreover, the closure range, over which the load rate is increasing, has been reduced to such a narrow space as to be practically outside the useful range of movement. We have, therefore, arrived at a volute design which has virtually the same load-deflection characteristics as a helical spring. The spring of curve 1 is inefficient from a weight standpoint, as stated by the author, because the relatively large volume of steel in the outer coils is understressed as compared to the inner coil. Apparently, this is the price which must be paid for the characteristic of increasing load rate.

Maurice Olley.[5] Thanks to this paper we know that, in the cyclic operation of the spring, the stress ranges will be such as to concentrate initial fatigue failures toward the inside of the

[3] Eaton Manufacturing Company, Detroit, Mich.

[4] American Steel Foundries, Chicago, Ill.

[5] British Ministry of Supply Mission, Detroit, Mich. Mem. A.S.M.E.

coils and within an inch or so of the edge of the strip toward the small end of the spring. This appears to agree in general with the experience of fatigue testing.

But with regard to the actual quantitative evaluation of maximum stresses, we see more clearly every year that we have not produced true figures even on relatively simple things like round-wire coil springs. We are still less likely to do so on a complicated structure like a volute spring.

All such structures depend upon the cold-setting operation for satisfactory working life. It is remarkable that this operation, though its importance is increasingly realized, still remains in its original crude form of setting the spring down solid a certain number of times. We do not know the actual residual stresses which occur after cold setting, or their distribution in the thickness of the material.

Recent work on fatigue testing of volute springs shows the comparative ease with which, under short strokes, failures can be produced in the large coils, close to the heel of the spring. This would never be expected from the calculated shear stresses, but it would be expected from the fact that the large ends are not at present overstressed (or cold-set) as effectively as the small ends.

The result of cold setting, as always, is that the metal of the spring works far more uniformly all over than would be expected from initial calculations. In other words, the spring is more efficient than we had expected.

We know, from analogy with other types of springs, that we cannot get full benefit from the cold-setting operations until we improve the surface either by grinding off or by recarburizing the outer surface of the strip. We also have every reason to believe that the details of the cold-setting operation require close study and experiment to produce optimum results.

Finally, perhaps it should be pointed out that life of the volute spring in the vehicle is dependent upon its mounting. Springs made out of flat strips are particularly sensitive to bending loads such as tend to bend the spring as a whole. Therefore the mounting in the vehicle should be contrived so as to compress the spring axially.

Author's Closure

On several of the points mentioned in the discussion, we now have some more information than was available in December, 1942.

1 Inefficient stress distribution is the price which must be paid for increasing load rate. This statement of Mr. Keysor can be amplified by a schedule of prices. Taking the uniform bottoming-stress condition (Fig. 14, curve 2) as one extreme and the uniform helix angle (Fig. 14, curve 1) as the other extreme, it will be found that in going from curve 2 toward curve 1, the price is at first very moderate but increases very steeply as curve 1 is approached. The most interesting and most widely used volute springs are intermediate between curve 1 and curve 2. A method for designing such springs has been worked out.[6]

2 Eccentric loading: From measurements made available by Mr. Keysor it appears that to design a volute spring in which the elastic axis coincides with the geometric axis would be possible, if at all, only if bottoming proceeds from both ends toward the middle. On all known volute springs, a load in the geometric axis will produce a tilt and lateral shift of the ends, and axial compression of the spring will produce an oblique eccentric load. The elastic axis does not coincide with the geometric axis and shifts as bottoming proceeds.

3 Cold setting: Mr. Olley, a pioneer in this field, points out the importance of the subject and the dearth of information about it. Cold setting works in two ways:

(*a*) It traps residual stresses.

(*b*) It may strain-harden the material.

Strain hardening can be investigated by the shape of the torque twist curve from torsion tests.[7,8] The mathematical tools for calculating residual stresses are available and have been applied in the analysis of volute-spring test results.[6,9] The advantage of residual stresses results from the fact that a stress range of 150,000 psi (from 0 to +150,000) may correspond to a life expectancy of 60,000 cycles. The same stress range but (from —50,000 to +100,000) may correspond to a life expectancy of 600,000 cycles. The difference seems to be much more pronounced in the region of finite life, where most machine elements operate, than in the region of the endurance limit, which usually is the only object of research. Research into the interrelation of stress range, maximum stress, and life in the finite-life region promises to be rewarded by great savings of materials, space, and weight.

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Studies of Heat Transmission Through Boiler Tubing at Pressures From 500 to 3300 Pounds

By W. F. DAVIDSON,[1] P. H. HARDIE,[2] C. G. R. HUMPHREYS,[3] A. A. MARKSON,[4] A. R. MUMFORD,[5] and T. RAVESE[3]

This paper is a report on studies of heat transfer and pressure drops in steam-generating tubes at pressures from 500 to 3300 psi and under exposure to furnace heat in large steam-generating units. Most of the test surfaces were in the form of flat spirally coiled tubes, but for comparison one straight tube 50 ft long was tested. Specific problems investigated include the influence of variations of the steam-water ratio and of variations in tube dimensions (scale factor) on tube-metal temperatures and on pressure drops. Some data relate to tubes operating under conditions approaching those often associated with operating failures. The test conditions were extended in several instances to include water below the saturation temperature.

Some of the possible engineering analyses of the data have been made. Such an analysis of the relation of metal temperature to heat absorption and internal fluid conditions led to the proposal of a function ϕ which seems to offer some possibilities of development as a function correlating the factors influencing an abnormal rise in metal temperature. Similarly an analysis of the pressure drop has led to the proposal of a modification of the usual pressure-drop correlation to include the thermodynamic effect of transverse-momentum changes during evaporation.

Supplementary investigations reported upon include pressure drop of water at saturation temperature for the pressure range 250 to 2500 psi, through flow-distributing equipment designed for forced-circulation boilers, and heat-transfer coefficients for the specific auxiliary equipment used.

1—OBJECTIVES, APPARATUS, INSTRUMENTATION, OPERATION, AND RECORDS

Two general problems are usually in the mind of a purchaser of large steam-generating units, i.e., that of making an engineering determination of the relative merits of different boiler designs submitted for the intended service by manufacturers in response to requests for proposals; and that of analyzing projects contemplating the use of higher steam pressures. In either case, emphasis is on those elements which determine operating limits or points where troubles may develop, although in the second case the possibilities of lower-cost designs are also of importance.

In the first narrowing of the problem when setting up the research program, the water-heating (economizer) and steam-heating (superheater) sections of the steam-generating circuit were omitted from consideration, attention being directed to the evaporating surfaces. There were several reasons for this, chief of which was that acceptable engineering data were available on heat transfer and pressure drop for the first two types of surface and these would not change greatly with pressure; while published data on the performance of the evaporating surfaces were less reliable and confined almost wholly to pressures below about 1500 psi. It is scarcely necessary to point out that as the pressure approaches the critical, the physical properties of the vapor and the liquid converge; and that this is certain to have important effects both on heat transfer and circulation and to introduce disturbing uncertainties in extrapolating from experience with lower pressures.

Several efforts were made to plan a laboratory scale study which would give the desired data on heat transfer and hydraulic constants, but none of these was successful because of an almost complete lack of data with which to establish the conditions under which the experiments should be conducted if they were to be of value in establishing the abnormal limiting conditions which lead to operating difficulties or which establish design limits, especially for units of the contemplated size.

Boiler design is essentially an extension from past practice, from small to large, from low pressure to high pressure, from low temperature to high temperature (in this it is not greatly different from much other engineering design). Hence available data were largely based on averages for entire elements such as an entire waterwall. Further, there appeared to be no established basis for comparing data obtained with one size of tube with those for another size of tube, and to test all possible sizes of tubes would be impossible. The initial work then would have to be done in actual boiler furnaces working with test surfaces made from tubes at least approximating in size those used in large-sized boilers. If some limits could be established and the scale effect determined, then laboratory experiments could be set up to provide the refinements of detail.

Three major specific problems were selected for study, each for a range of pressures from 500 to 3300 psi and heat-transfer rates of from 30,000 to about 200,000 Btu per hr per sq ft of projected area.

These problems were as follows:

1 To determine the influence on the temperature of the tube metal of:

- (*a*) The steam-water ratio.
- (*b*) Liquid under conditions of rising temperature in the absence of evaporation.

2 To determine the resistance to flow of:

- (*a*) Steam-water mixtures.

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[4] Formerly with Consolidated Edison Company of New York, Inc.; now with Hagan Corporation, Pittsburgh, Pa. Mem. A.S.M.E.

[5] Formerly with Consolidated Edison Company of New York, Inc.; now with Combustion Engineering Company, Inc., New York, N. Y. Fellow A.S.M.E.

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(b) Liquid under conditions of rising temperature in the absence of evaporation.

3 To determine the influence (scale factor) of variations in dimensions of the tube.

In each of these problems, special attention was to be given to conditions approaching those under which tube-metal temperatures showed signs of abnormal increase or instability.

Incidental problems which were examined because not much additional time or equipment was involved were:

4 To determine the pressure drop through flow-distributing equipment designed for use in forced-circulation boilers.

5 To determine the heat-transfer coefficients for the specific types of auxiliary heat-exchanging equipment used in these investigations.

Apparatus

General. The stated objectives of the investigation required the use of test elements of sufficient size so that extension of the results to production units would not entail too many uncertainties. In order more nearly to approximate the conditions of ultimate application, available full-sized boiler furnaces were used.

Initially, a recirculating system made up of evaporating surface, a steam and water drum, and a circulating pump was considered but the difficulties of securing a suitable circulating pump for the high pressures and of accounting for gland leakage finally led to the adoption of a once-through arrangement. Such an arrangement necessitated apparatus to preheat the water to the saturation temperature for any test pressure and later to cool it before dissipating the pressure. Consequently the auxiliary equipment constituted the major physical bulk of the installation.

The equipment, for convenience in discussion, may be divided into pumps, preheaters, test surfaces, calorimeter (to determine the heat absorbed by the test surfaces), coolers, and equipment for the control and release of the pressure. The instrumentation included provision for determining quantity of fluid flowing, fluid pressures, fluid pressure drops, fluid temperatures, and metal temperatures in addition to several other items needed for control.

Test Surfaces: First Series. The test surfaces used in the first series of tests were flat pancake coils of tubing installed on the rear wall of an oil-fired boiler in the Sherman Creek Station. The boiler, of the straight-tube type, had a rectangular refractory-walled furnace with three oil burners in the front wall opposite the test coil.[6] The furnace size, approximately 12 ft 5 in. wide × 15 ft 1 in. long (in the direction of flame travel) was great enough, together with the location of the first-pass opening, to prevent direct impingement of the flame on the test surface. A longitudinal section through the boiler and furnace is shown in Fig. 1. The position of the test surface in relation to the oil burners and the first-pass opening is clearly indicated.

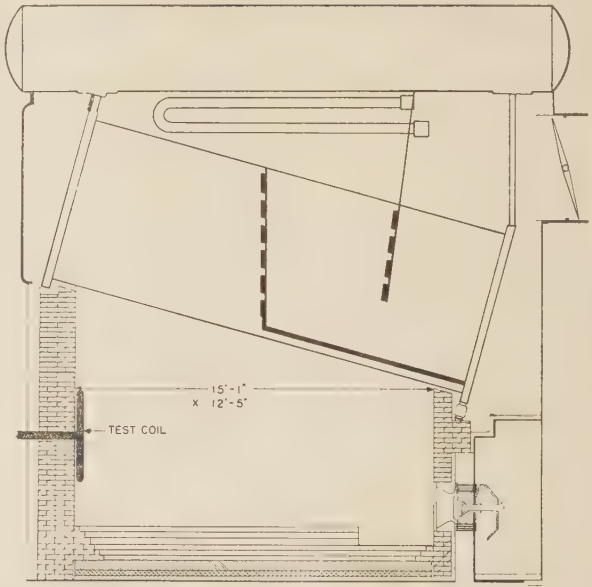


Fig. 1 Test Coil in Oil-Fired Boiler at Sherman Creek Station

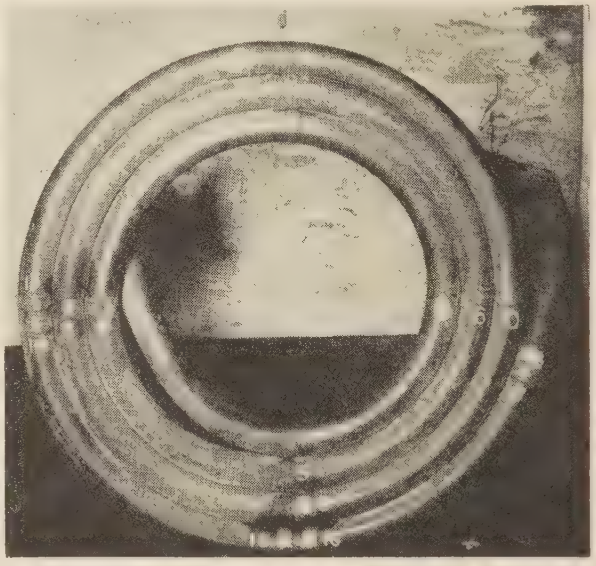


Fig. 2 Test Coil Showing Thermocouple Positions

The choice of the flat spiral or pancake form for the tubes under test in the first series was not made without realization of the probable effect of the form on the segregation of steam and water and on the pressure drop, but the pancake form offered certain important advantages which have been well established as a result of experience. (A straight tube was used for comparison in the second series of tests.) Most important of these advantages was that only by the use of a coil could the test surface be arranged within a sufficiently small area of the furnace wall to insure reasonably uniform heating. Among other advantages were greater ease of interchanging test surfaces and fewer or smaller corrections to pressure-drop measurements.

Five of the six coils were selected so as to form two series, one of uniform bore and wall thickness but varying length, and the other of uniform length but varying bore, and with a wall thickness proportional to the bore. The tubing was Shelby seamless (0.25 carbon) with a wall thickness in each case meeting Boiler Code requirements for 3000 psi working pressure. In order to escape from the fixed proportion of wall thickness to bore established by a given working-pressure design, and in order to make possible a later direct comparison between a pancake coil and a straight tube (installed vertically), a sixth coil 50 ft long was made of tubing designed for a working pressure of 2000 psi. Similar tubing was used for the straight-tube tests to be described.

All coils, with the exception of the 50-ft coil, were normalized after fabrication and the inner surfaces were cleaned and bonderized. Examination of samples after testing showed that there

[6] The maximum heat release from these three burners during test conditions was roughly 73 × 10^6 Btu per hr.

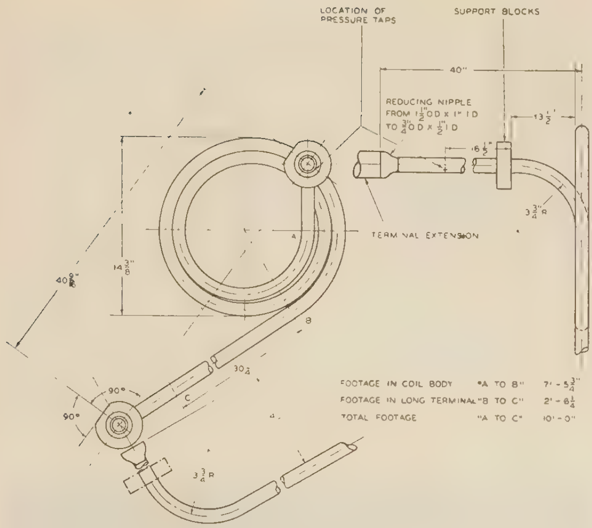


FIG. 3 DETAILS OF TEST COIL, $^3/_4$ IN. OD × $^1/_2$ IN. ID × 10 FT LONG

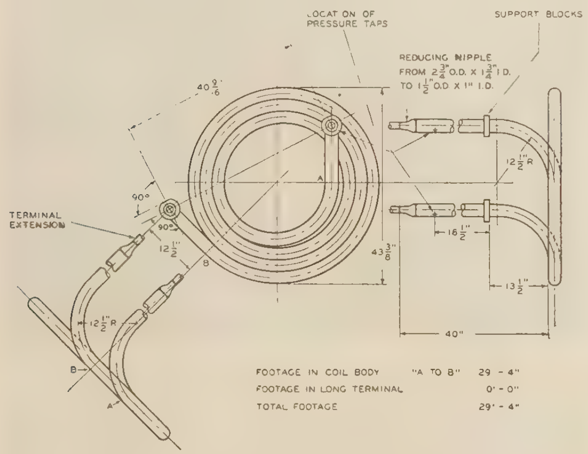


FIG. 4 DETAILS OF TEST COIL, $2^3/_4$ IN. OD × $1^3/_4$ IN. ID × 30 FT LONG

was only an insignificant amount of flattening caused by coiling.

In Fig. 2 is shown one of the coils ready for installation in the furnace, while Figs. 3 and 4, respectively, show the smallest and largest coils of the series. Fig. 5 and Table 1 list the essential dimensions of all test surfaces.

To insure ready interchangeability of the coils each was provided with tails and end fittings which were the male ends of packed joints on the permanent piping. A tapped connection for a Parker compression fitting was provided in a suitable welded boss on each tube for use in pressure and pressure-drop measurements. A guide collar on each tube tail was arranged to fit into thimbles installed in the boiler wall to hold the coil in proper position for connection to the permanent piping. The liberal use of jigs gave uniformity of dimensions with the result that no serious field-assembly problems were encountered.

Fig. 6 shows one of the tails of one of the coils of a test surface. At the right, the tubing was bent to enter the wall thimble in which it was centered by the supporting collar. Just outside the furnace wall was the coil anchor and the differential-pressure connection with its compression fitting. From this typical drawing, it is possible to estimate the amount of tubing over and above

TABLE 1 PRINCIPAL DIMENSIONS OF TEST SURFACES

Coil number	1	2	3	4	5	6	Straight
Inside diam, in.	0.5	0.5	0.5	1.0	1.75	0.92	0.92
Outside diam, in.	0.75	0.75	0.75	1.5	2.75	1.25	1.25
Internal cross section, sq ft/1000	1.36	1.36	1.36	5.46	16.7	4.61	4.61
Length (A to E^a on coils), ft	10.2	20.3	30.1	30.1	30.7	53.3	52
Length inner convolution, (A to B), ft	3.0	2.5	2.5	5.0	6.7	3.7	..
Length fully shielded (B to C on coils), ft	1.1	11.0	19.8	14.7	13.8	38.5	52
Length outer convolution (C to D), ft	3.6	4.2	5.0	7.8	10.2	8.8	..
Length straight section (D to E on coils), ft	2.5	2.6	2.8	2.6	0	2.3	..
Number of turns	2.3	5.3	7.3	4.3	3.5	8.3	..
Projected external area, sq ft	0.63	1.27	1.89	3.76	7.03	5.55	5.42
Number of surface-temperature - measuring points	14	17	26	11	14	26	23
Length between pressure taps, ft	15.4	25.6	35.2	35.6	36.2	58.4	..
Inner diam of coils, in.	11.2	8.5	8.2	19.5	26.5	14	..
Outer diam of coils, in.	14.3	16.5	19.6	31.5	43.3	33.5	..

[a] Refer to Fig. 5.

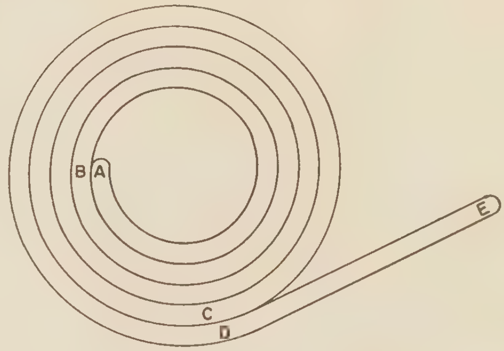


FIG. 5 DIAGRAM OF COIL
(To be used in conjunction with Table 1.)

that directly heated which was included in the pressure-drop figures. Further along was the fitting used to connect the test surface to the permanent piping. This packed pressure joint was completely successful from an operating and construction standpoint.

The coils, except the smallest, were moved into and out of the furnace through a T-shaped opening in the rear wall.

To give an approximate check on furnace conditions, a small control coil made from $^3/_4$-in-OD and $^5/_8$-in-ID tubing 20 ft long was installed permanently on the rear furnace wall slightly above and to one side of the test coil. This coil and a test coil are shown in position in Fig. 7.

Test Surfaces; Second Series. In order to test the performance of a straight tube 50 ft long, a twin-set boiler (at Kips Bay Station) of more than 50 ft furnace height, fired by pulverized coal, was selected for the installation. The tube used for the evaporating runs was installed with a water-heating tube on either side, acting in the manner of a guard tube. Except for a slight offset to compensate for the change of spacing of the end-wall water tubes above and below a junction header the tubes were straight and vertical. The water-heating tubes were looped together at the top to provide sufficient continuous water-heating surface. All tubes were made from the same tube stock as was used for coil No. 6.

The necessity of providing about twice as much water-heating surface as evaporating surface, even when all surface was exposed to furnace conditions, led to the plan of placing a water-heating tube on either side of the evaporating tube. This arrangement had a further advantage of providing a side shield for the middle tube, making the exposure to the furnace generally similar to that existing in the coil.

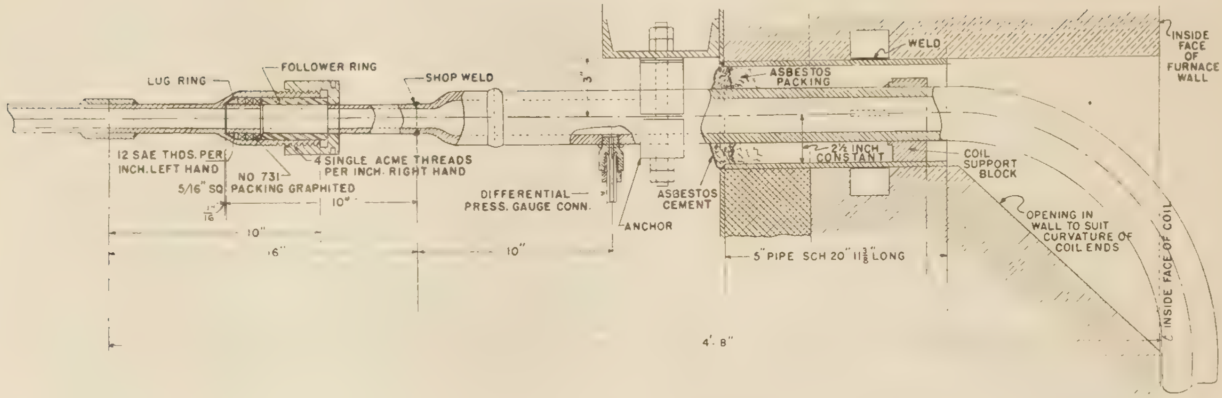


Fig. 6 Connection Detail of Test Coil

Auxiliary Apparatus. Fig. 8 shows a flow diagram of the equipment used for the tests of the coils. Water from the station-feed system was supplied at station-feed pressure to the suction of the triplex pumps. From the pumps the water flowed through a high-pressure heat exchanger, in which it was heated by the water discharged from the calorimeter. The water was then raised to within a few degrees of saturation temperature in a convection heater which was supplied with heat by withdrawing hot gas from the furnace. This water was then fed through the tube comprising the test surface. From the test surface, the water flowed through a calorimeter which withdrew sufficient heat to cool it to a temperature below saturation (approximately that at which it entered the test surface) and, therefore, afforded a measure of the heat picked up in the furnace. From the calorimeter, the water flowed through the high-pressure heat exchanger to a subcooler. In the subcooler, the temperature was lowered to below 150 F to prevent flashing in the throttling valves when water pressure was reduced to atmospheric pressure in the hand-operated throttling valves and discharged either to waste or to a weigh tank.

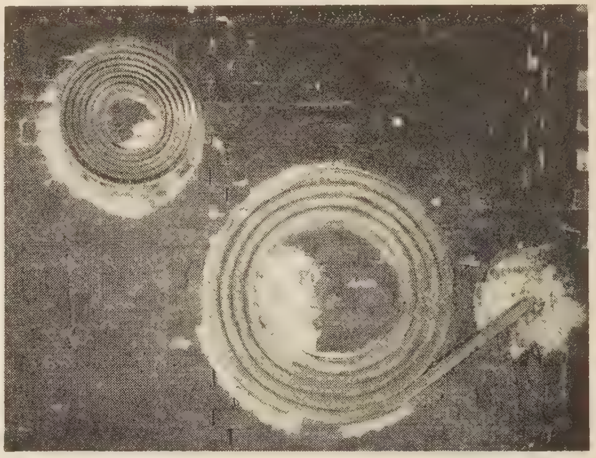


Fig. 7 Control Coil and Test Coil in Position

The auxiliary equipment had to be capable of adding 3,500,000 Btu per hr to the liquid under conditions of maximum flow and test pressure and of disposing of the same amount plus the 1,000,000 Btu per hr absorbed by the test surface. The heat exchanger supplied 500,000 Btu per hr of the added heat, and the convection heater 3,000,000 Btu per hr. Under the same conditions of flow and pressure 1,000,000 Btu were transferred in the calorimeter and 3,000,000 Btu per hr in the subcooler. With low pressures and minimum flow, about 300,000 Btu per hr were added to the liquid (3000 in the heat exchanger and 297,000 in the convection heater), and 10,000 were absorbed by the test surface and transferred in the calorimeter.

Fig. 9 shows the flow diagram for testing the straight tube at the Kips Bay Station. Fig. 10 shows the lower extremities of the water-heating and test tubes installed in the furnace of the boiler at Kips Bay Station. Water from the station feed system was pumped by the triplex pump to the water-heating tube which was installed as a vertical loop in the furnace and of which each leg was adjacent to the steaming tube. The hot water, in some cases steaming, that was discharged from the water-heating loop passed through the heat exchanger, in which it was cooled to a few degrees below its saturation temperature, and in which of course any steam formed was condensed. This slightly subcooled water then passed through the steaming tube to the calorimeter in which the heat picked up in the steaming tube was measured. The water then passed through the subcooler to the throttling valves and the receiver as during the tests of the coils. Temperature control of the discharge from the water-heating surface was accomplished using the proper combination of cooling sections on the heat exchanger. This operation was manual and was guided by a Leeds and Northrup Micromax thermocouple recorder showing the water temperature leaving the heat exchanger and entering the steaming tube. The calorimeter installed at the outlet of the steaming tube was used to measure the furnace steam tube heat absorption. The differential pressure manometer was connected across the water-heating tube for measurement of the pressure drop of the fluid flowing in the tube.

Pumps. The pumps, of which there were two, were Aldrich-Groff controllable-capacity pumps and are shown in Fig. 11. Each was driven by a three-phase induction motor and had a capacity of 5000 lb per hr against a discharge pressure of 4000 psi with a feedwater temperature of 210 F. These pumps proved to be unusually well adapted to the work because the adjustable-stroke feature with special hand control provided a delivery controllable independently of the discharge pressure, and after calibration the stroke setting could have been used to determine flow with an error not over 0.5 per cent. Further the relatively high speed (crankshaft speed 300 rpm) and triplex arrangement reduced pulsation to a point where most was absorbed by the auxiliary equipment, and it was not necessary to use special metallic-hose pulsation absorbers as originally planned.

Since the program contemplated the investigation of the separate effects of flow and pressure on heat transfer in the test sur-

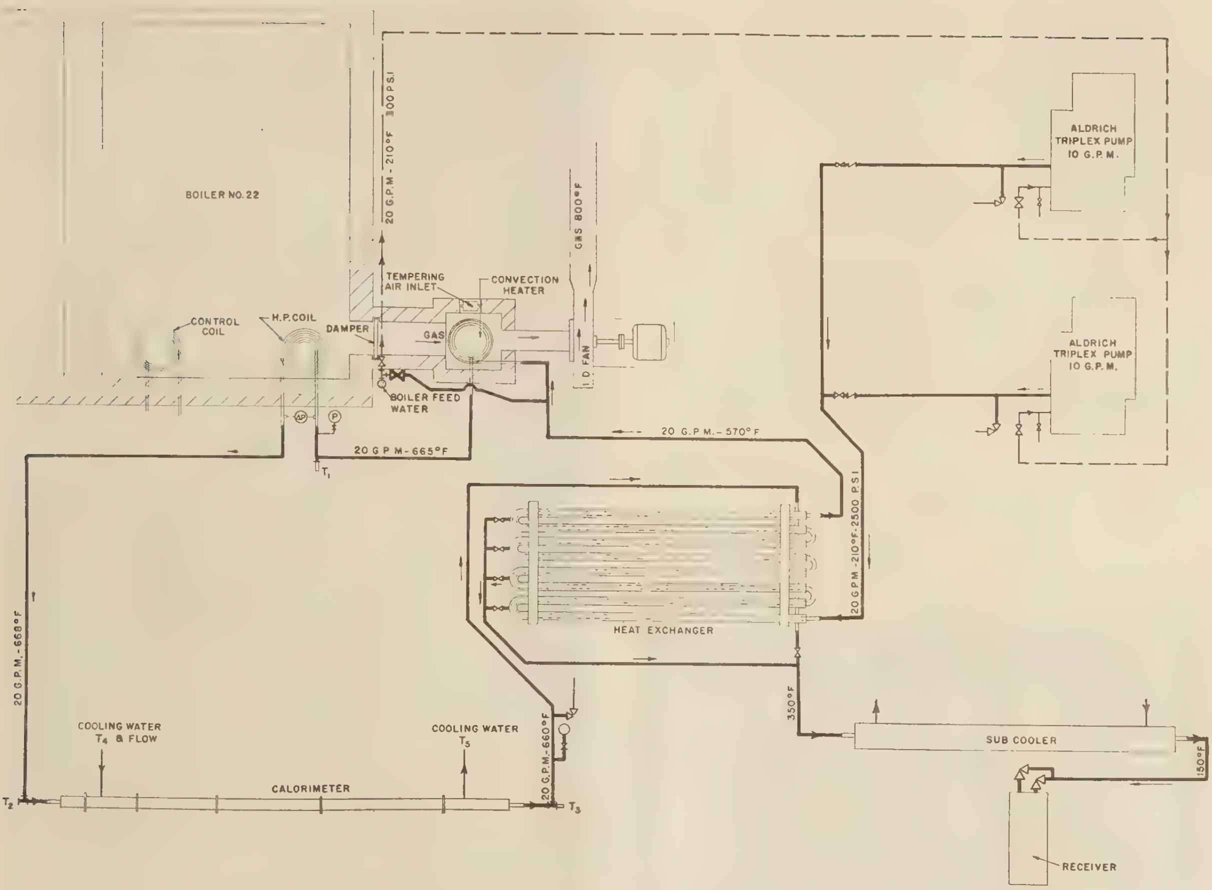


FIG. 8 FLOW DIAGRAM FOR TEST EQUIPMENT AT SHERMAN CREEK STATION

faces, certain features of the apparatus which facilitated this will be discussed. The pumps operated with constant speed and with constant volumetric efficiency for a given pump stroke and discharge pressure so that once the stroke had been set to give the desired flow, the flow remained constant. The pressure in the system was controlled by the adjustment of the needle valves on the discharge from the system. Because the flow was fixed by any given stroke setting of the pumps, the valve setting influenced only the pressure. Such independence of pressure and flow adjustments could not have been readily obtained with a centrifugal pump.

Preheaters. To preheat the water for the coil experiments, two heaters were installed, one a liquid-to-liquid exchanger which recovered heat from the water leaving the calorimeter, and the other a gas-to-liquid convection heater receiving the heat from hot gases withdrawn from the furnace.

In the low-level exchanger, both surfaces were designed for a working pressure of 3000 psi and were arranged as a tube within a tube in four sections. The two tubes of each section were bent as one while one was inside the other, thus making possible extreme simplification of the construction. The four elements were mounted on a structural-steel rack with the units arranged in a vertical plane. The outer tubes, through which the water passed on its return circuit, were connected by piping and valves in such a manner that any number of sections could be used. By selecting the proper number of sections in this exchanger, the load on the convection heater could be adjusted so that its control equipment functioned at a satisfactory point on its range. Of course, any decrease in the work done by the exchanger had to be absorbed, in the downstream direction, by the subcooler. The insulated high-pressure heat exchanger is shown in Fig. 12, which also shows the weigh scale used for determining rates of flow. Dimension and areas of the unit are given in part 5 of this paper.

The water passed from the inner coil of the high-pressure heat exchanger to the convection heater which is shown at the right in Fig. 11. By means of hot furnace gases induced to flow through this heater by a fan the water was brought to the saturation temperature corresponding to the pressure in use. The furnace gases were confined by a cylindrical metallic baffle so that they flowed up through the center coils to the top and thence downward over the outer coils to the fan connection. The convection heater was designed for the absorption of 3,000,000 Btu per hr, or sufficient to heat the water to 695 F (the saturation temperature corresponding to 3000 psi) at a flow of 10,000 lb per hr when fed with furnace gases from the oil-fired test boiler.

A firebrick damper suspended in the duct from the furnace is to the right of the convection heater in Fig. 11. At the bottom of the insulated cylinder of coils is a hand-controlled damper controlling the temperature of the incoming furnace gases by tempering them and aiding in keeping the automatic-control damper in its best range. There is another hand-controlled damper in the inlet box to the fan to keep the gas temperature entering the fan within safe limits. In the discharge duct, there is a damper controlled by a Leeds and Northrup Micromax temperature controller, actuated by a thermocouple on the water-discharge line from the heater. After the hand-controlled dampers had been suitably adjusted,

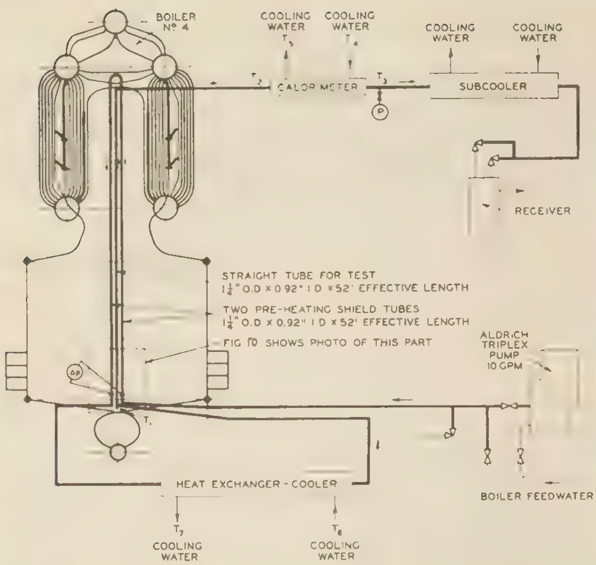


Fig. 9 Flow Diagram for Test Equipment at Kips Bay Station

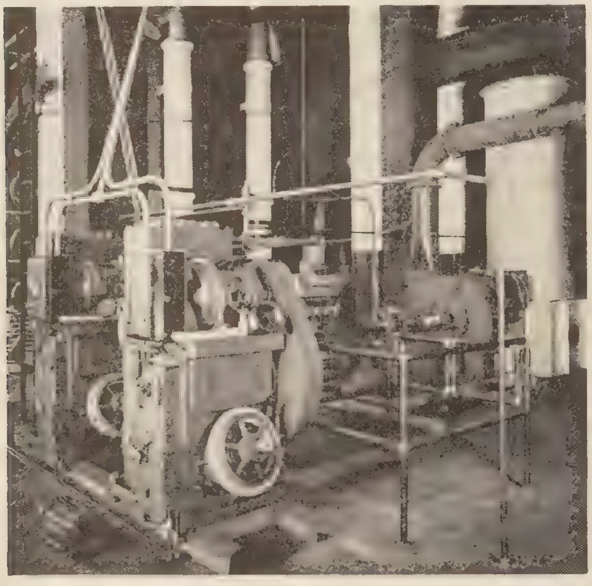


Fig. 11 Aldrich-Groff Controllable-Capacity Pumps at Sherman Creek Station

Fig. 10 Lower Extremities of Water Heater and Test Tubes Installed in Furnace

the controller maintained the water temperature constant within a range of less than 2 F.

Calorimeter. From the test surface, the mixture of steam and water passed to the calorimeter which was designed to determine the heat absorbed by the test surface and the quality of the mixture discharged from the surface. The calorimeter, for the coil tests, consisted of a single straight tube covered by a series of five water jackets of different lengths, as shown in Fig. 13. The water outlet from each jacket was manifolded with that from the other jackets in such a manner that any combination of available cooling areas could be assembled to suit the several test surface areas and range of furnace conditions. Detailed dimensions of the calorimeter are given in part 5.

Coolers. From the calorimeter, the water passed through the outside tube of the high-pressure heat exchanger to the subcooler, shown in Fig. 14. The subcooler consists of a shell surrounding a coil. The cooling water passed around the coil while the high-pressure discharge from the exchanger passed through the tube. The jacket water-cooled the discharge water to a temperature of about 150 F to permit throttling to atmospheric pressure without flashing.

Piping. The connecting piping was $1^5/_8$-in-OD and 1-in-ID seamless-steel tubing, and was shop-normalized after forming. All field welds were sleeve welds. Insulation was by Fiberglas and Carey high-temperature pipe covering. All valves were of the screwed type 5000 psi standard.

It may be of interest to note that the few screwed joints, over which some concern had been felt initially, gave no trouble, proba-

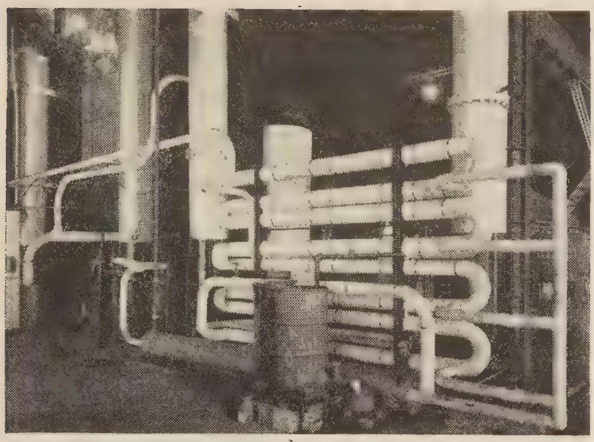


FIG. 12 HIGH-PRESSURE HEAT EXCHANGER AT SHERMAN CREEK STATION

bly because all field threads were lathe-cut or made with new tools and set up with Copalite.

Instrumentation. All temperature measurements for test purposes were made with thermocouples, although numerous Weston dial-type thermometers were installed and found to be very helpful in making preliminary adjustments and during periods between tests. Thermocouples were made from glass-fiber insulated wire with welded junctions. No. 24 Awg iron constantan was used for liquid and steam temperatures and No. 22 Awg chromel-alumel for metal temperatures.

For liquid-temperature measurements, from two to four thermocouples were installed in a well arranged for insertion into the liquid stream through packed stuffing-box fittings, as shown in Fig. 15. Fiberglas sleeving was used to give added electrical insulation where needed. In the case of the thermocouple actuating the damper control on the convection heater, it was found satisfactory to peen this into the piping at a point well protected by insulation.

The temperature of the metal at the outside surface of the tubes

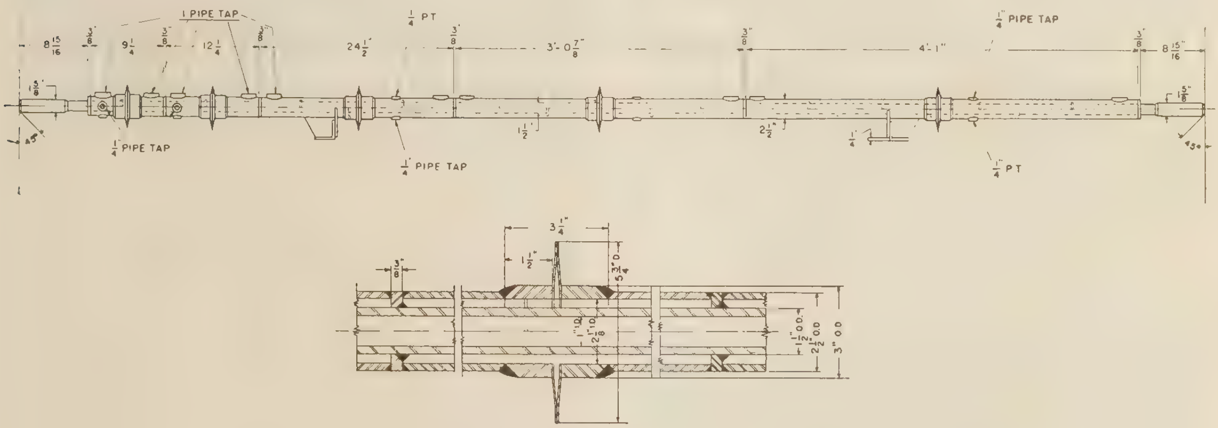


FIG. 13 DETAILS OF CALORIMETER, CALORIMETER JOINTS, AND COOLING-WATER BAFFLES

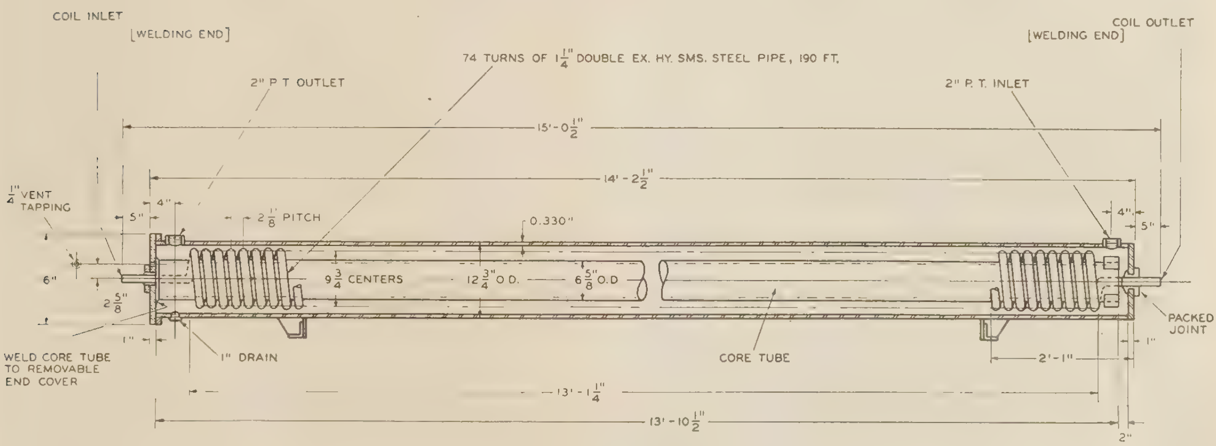
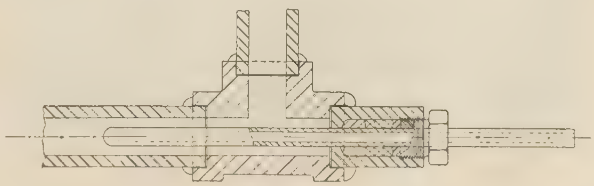


FIG. 14 DETAILS OF SUBCOOLER

FIG 15 HIGH-PRESSURE THERMOCOUPLE WELL AND PACKED (STUFFING-BOX) CONNECTION



nearest the source of heat was an important source of information in this phase of the investigation. For this reason and because we believe the method of installation is novel, it will be described rather fully.

The coils made from tubing less than $2^3/_4$ in. OD were sufficiently flexible so that adjacent convolutions could be sprung far enough apart to permit working on the surfaces normally separated by about $^1/_{16}$ in. A lip of metal was carved on the furnace side of the coil by a round-nosed chisel, and two grooves were cut leading from the lip toward the wall side of the coil. A sketch of the grooves and couple is shown in Fig. 16. The grooves were

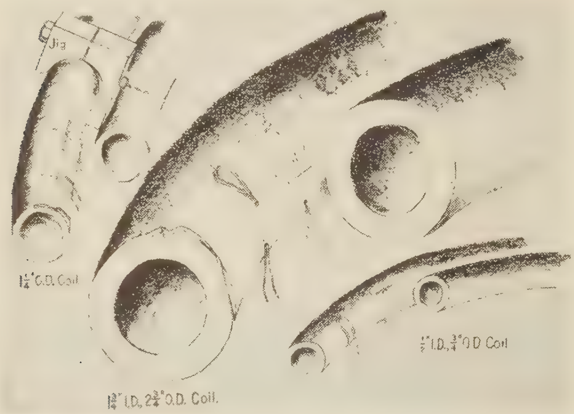


Fig. 16 Method of Installing Thermocouples for Tube Surface Temperature Measurements

made large enough to accommodate the thermocouple wire. The junction was inserted in the drilled hole and the lip peened over to the normal contour of the tube. The peening held the couple firmly with the weld about $^1/_{32}$ in. below the outer surface. To protect the couple wires a cover plate, about 1 in. square and previously formed to fit the tube surface, was placed over the grooves and tack-welded in place, with one edge over the peened lip. The cover plates were $^1/_{32}$ in. thick for the $^1/_2$-in-OD tubes, but this thickness proved too small for the heavier-walled tubes of larger diameter because of welding difficulties and $^1/_{16}$-in-thick plates were used on the larger sizes.

From 11 to 26 thermocouples were placed on each coil (exact number shown in Table 1) at predetermined positions so that the entire length of the coiled tube was served. The thermocouples were located on the vertical and horizontal diameters of the coil in most cases, but in a few cases diagonal diameters were also used for some of the measuring points. On the straight section of tubing leading from the coil to the exit tailpiece duplicate couples were installed, because at this exposed location the couple life was somewhat uncertain. The couple wires from cover plates to terminal box were protected by Fiberglas sleeving and were carried out of the furnace along the inlet or outlet tails of the coil.

On the coils of the more rigid tubing, a No. 50 (0.07 in.) hole was drilled tangentially through the metal of the wall to permit the thermocouple wire to pass from the furnace side to the wall side without the necessity of springing the convolutions apart to cut grooves.

After the original five coils had been tested and before the test on the sixth coil, the possible improvements on the method of installing these thermocouples were discussed as thoroughly as possible. The 50-ft tube was designed for 2000 psi, and therefore the wall thickness was less in proportion to the projected area. It was agreed that the peening operation might have the effect of relocating the junction by damaging insulation without concluding that the effect of such relocation would alter the findings fundamentally. However, to eliminate the question in further work, a jig was devised and applied satisfactorily.

The jig guided the drill by means of a case-hardened bushing but was not used for drilling to full depth. In order to judge the depth properly, the jig was removed and the drilling continued until the surface of the tube was disturbed over the point of the drill. The wires were protected as before but the cover plate was moved away from the junction and set so that it would cover the open drill hole and grooves.

In the test of the 50-ft straight tube, the couples were inserted in the same manner as in the 50-ft coil. The wires, however, were brought to the back of the tube on alternate sides to reduce the number of failures which might be caused if one or the other side were exposed. In the straight-tube setup, the evaporating tube was guarded on each side by a water-heating tube, on the riser of which couples were placed but located on the back to show temperature trends only; hence they required no special protection.

The thermocouple potentials were measured with a Leeds and Northrup No. 8662 double-range portable precision potentiometer giving a sensitivity of 0.01 mv or better. Direct temperature measurements on the liquid were made using an ice-junction reference, while differential temperature measurements were made between selected thermocouples. Temperature measurements on the metal were made using a cold-junction correction determined by reference to an ice junction. Rotary selector switches provided a ready means for selecting the desired thermocouple or combination.

Sample thermocouples were made from each lot of wire and were calibrated against a platinum-resistance thermometer and Mueller bridge. In no case within the range used was the departure from the Leeds and Northrup standard curve greater than 1 deg F.

The use of the differential temperature measurement proved particularly useful in determining the degree of subcooling and the temperature rise of cooling water in the calorimeter. In the first case it established the departure from saturation temperature by direct measurement, and in the second case it resulted in compensation for any change in inlet-water temperature. In other cases the saving in computation, if not in actual measurements, was of value.

Flow rates in the test circuit were determined as previously mentioned from the weighed water. Flows of circulating water through the calorimeter were measured by rotameters. Two instruments with an extra float provided three ranges. Fig. 17 shows two of the rotameters one of which was used to check the flow through the pilot coil. All were checked by weighed water measurements.

For the indication of pressures two Crosby 12-in. gages 0–4000 psi and one Baldwin-Southwark 12-in. pressure gage 0–4000 psi were used. The latter was marked in 10-psi divisions and had an accuracy of ± 10 psi from 800 to 1000 psi and an accuracy of ± 1 per cent from 1000 to 4000 psi. Two of these gages are shown in the upper right of Fig. 18.

In this figure also are shown the Foxboro indicating differential pressure gages with autosyn transmitters used to measure the pressure drop through the test coils. The manometer chambers of these instruments were designed for 5000 psi. The differential gages and an extra chamber gave ranges of 20, 100, and 500 in. of water, and suitable valves permitted the selection of the appropriate range.

The pressure drops were taken across two taps on the straight tail sections of the coils, and the measurements therefore include the straight portions of the tube. The tapped holes were carefully reamed free of burrs. On the inlet side, the pressure tap was placed as far as possible from the transition piece. For the $^1/_2$-in-

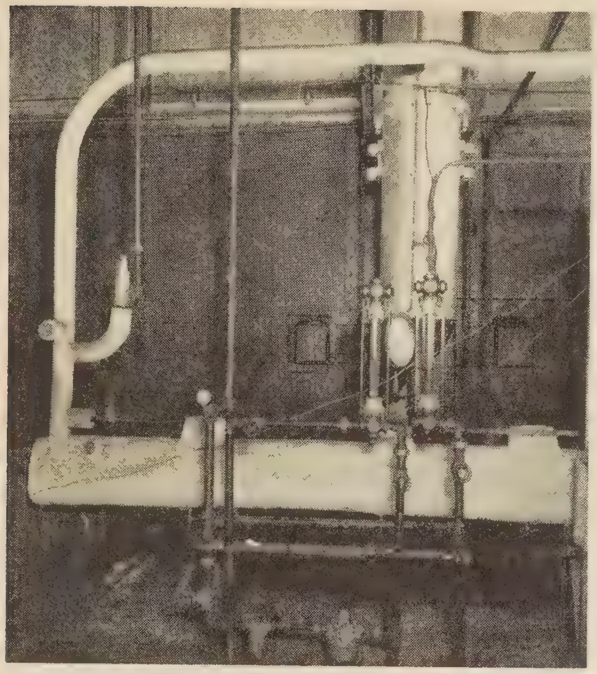


Fig. 17 Rotameter Installation on Test Equipment

ID coils, the transition piece constituted a smooth enlargement and the downstream distance of the tap was about 10 pipe diam.

The effect of pump pulsation on the pressure measurement was gratifyingly small, as judged by the agreement between the indicating-gage pressure and the thermocouple indication of saturation temperature which was as close as the limits of observational error permitted. There was no observable pulsation on the differential pressure gage.

Operation. Because boiler operation had to be independent of the testing program, provision was made to circulate water from the station feedwater system through the high-pressure test equipment at station feed pressure during periods between tests.

The process of raising the pressure during tests involved simply closing the throttling valves to the proper point after the pumps had been adjusted to the desired flow. One of the throttling valves was used as a coarse control and the second for fine control. The pressure in the system was first raised to 1000 psi; then the furnace-gas convection heater was brought into operation by opening the slide damper to the furnace and by setting the tempering damper. As the temperature of the discharged water from the convection heater rose the system pressure was increased and was always kept above that corresponding to the saturation pressure of the convection-heater discharge until the conditions desired for the test were attained.

The compact arrangement of the test equipment made possible easy and constant communication between the observers at the temperature- and pressure-control stations. The temperature of the water entering the test coil was normally kept from 3 to 7 deg F below that corresponding to saturation at the test pressure.

The proper amounts of calorimeter cooling surface and cooling-water flow were set up in accordance with the degree of subcooling desired and the limiting temperature of the cooling-water discharge which was at atmospheric pressure. An indicating rotameter metering the cooling-water flow made it possible to hold this quantity constant during any test by the operation of a hand-control valve.

The test surface temperatures were observed and when the temperatures and the differential pressures indicated that stable conditions had been attained, observations were started and continued until at least two sets of observations had been recorded for each run. The data were quickly calculated in a preliminary manner by one observer, in order to permit early decision as to the necessity for any additional check runs and to determine the next step in flow rate, heat-absorption rate, or both.

Each run was completed in from 20 to 30 min. Tests were planned for the use of one pressure during each testing day, whenever possible. Those runs called "liquid heating," during which the water temperatures leaving the test surface were below saturation, were usually made preceding those conducted at saturation temperature.

At the close of the testing period each day the heat supply to the convection heater was closed off. Usually after about 20 min, the temperature had fallen sufficiently to permit transfer to the station feed system and the pumps could be shut down. As long as there was heat in the furnace water was kept flowing through the system but the cooling-water supply to the subcooler was sufficient to care for the heat pickup and the calorimeter cooling water could be shut down.

In the event of bursting a coil, the convection heater was cooled off while flow was maintained until the metal temperatures dropped below the danger point.

The starting-up procedure for the high pressure at Kips Bay was similar to the Sherman Creek installation. The pressure on the high-pressure system was gradually increased to the desired value starting at 1000 psi as the first increment. The water-temperature-control calorimeter was cut in with the proper cooling section and water flow in order to maintain the water temperature entering the steaming tube constant. Some regulation of the

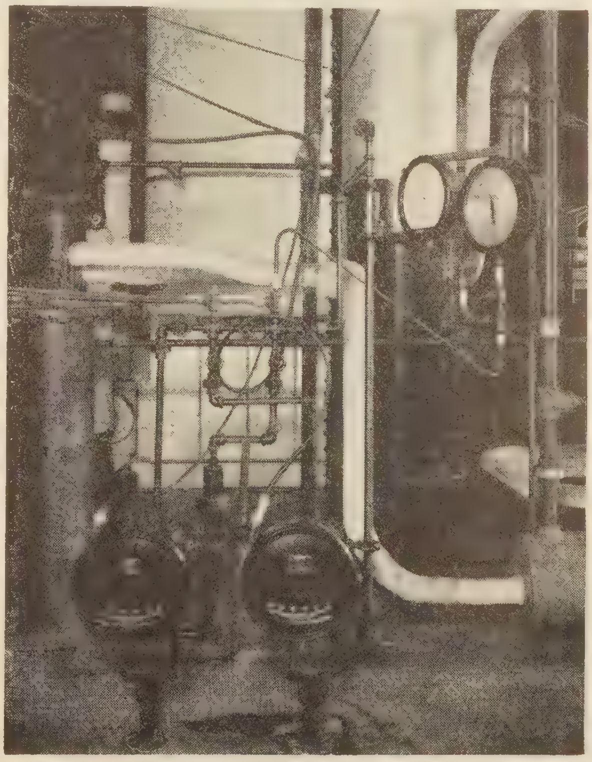


Fig. 18 Pressure and Differential Pressure Gages

water flow through the calorimeter was necessary during the run because of the variation in water temperature entering the calorimeter and slight variations in furnace heat absorption. Cooling water to the second calorimeter, measuring furnace heat absorption was adjusted so that it would subcool the water sufficiently to be sure that the fluid leaving the calorimeter was entirely liquid and measured by the rotameter. The heat absorption at Kips Bay proved constant enough to allow the admission of a mixture of estimated composition of steam and water to the evaporating tube. This was done on a number of runs.

Records. A summary of the test data for the seven tubes tested is given in Table 2. As will be noted, this table gives the absolute pressure p, the temperature of the fluid entering T_1 and T_2 leaving the test tube, the flow rate w, the mass flow G, the velocity of the fluid leaving the tube V, the over-all heat-absorption rate Q, the quality of the fluid leaving the tube X, i.e., ratio (weight of steam to weight of water and steam), the average temperature drop between the outside wall of the tube and the fluid (Δt) and the fluid pressure drop in its passage through the tube (Δp).

The data include about 99 per cent of the test runs. The runs for which data are omitted are those in which an unavoidable change in furnace conditions made the observations of doubtful value and a few cases in which there were obvious errors of recorded data. Although the data are presented in the order of coil numbers arbitrarily assigned, the tests or runs were chronologically numbered. Therefore, the run with the largest identifying number was the last one made, no matter what its position in the table. The run numbers used to identify the separate runs each constituting a test are made up of one or two digits followed by one or two letters which, in turn, are followed, in some cases, by an additional digit. The first one or two digits constitute an identifying number which was changed only when the date changed or when the operating pressure was changed. Thus a series of tests in which the first one or two digits remain the same was made on the same date and at the same pressure. The first letter in the identifying number indicates that a change, in some factor other than pressure such as flow or heat absorption, was made over the factors existing in the previous runs. The second letter (always H) is used to indicate those runs in which the temperature of the liquid entering the test coil was sufficiently lower than the saturation temperature corresponding to the pressure in use so that no evaporation took place and the runs could be only liquid heating. Where the letters are followed by an additional digit, the presence of these single digits (1 and 2) indicates two sets of readings taken with no intentional change in conditions.

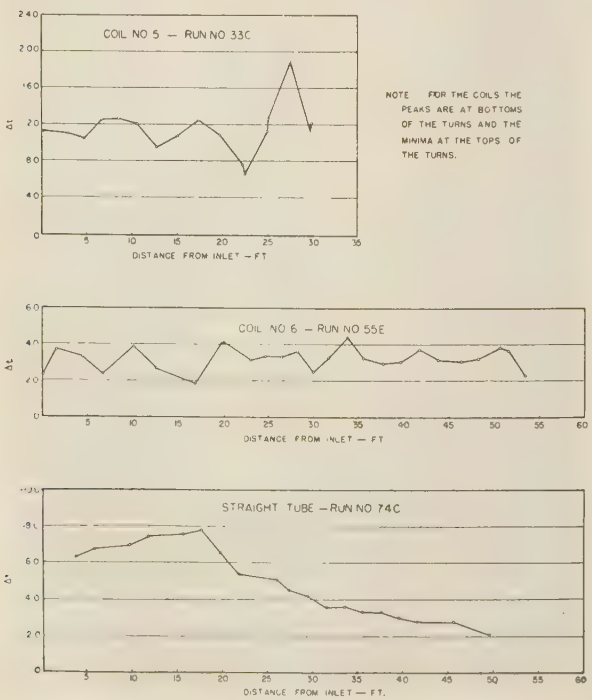


Fig. 19 Typical Tube-Temperature Data

The physical dimensions of each tube are given for convenience in Fig. 5 and Table 1. The test data presented include the following ranges of operating conditions:

(*a*) Heat-absorption rate, 20 to 220 M Btu per hr per sq ft of projected outside tube area
(*b*) Fluid pressure, 290 to 3300 psi abs
(*c*) Quality, 0–100 per cent steam leaving tube
(*d*) Fluid velocity, 2 to 90 fps leaving tube.

Fig. 19 shows graphically three typical sets of tube-surface temperature-elevation readings for steaming runs. The data for the liquid-heating runs showed similar characteristic variations from point to point on the tubes. Individual temperature differences were determined by assuming a constant fluid temperature at the saturation point, for all evaporating runs in which the final quality was less than 1, and by assuming a uniform rise in temperature along the tube length for all nonevaporating runs. Curves similar to those shown in Fig. 19 were constructed for each run and the average Δt, shown in the tables, was determined from such curves.

2—TEMPERATURE DROP THROUGH TUBE WALL AND INSIDE FILM

A rigorous analysis of the heat transfer in terms of the generalized resistance concept is made difficult by the self-imposed limitations of the investigation. The test surfaces were tubes, both straight and spiral-coiled, exposed to radiant heat on one side only. Therefore, the transverse heat-flux pattern was unsymmetrical with respect to the front and back of the tubes. However, this is the usual condition for most of the radiant-heat-absorbing surface in large steam generators. Engineering analyses of the experimental data may be made of which the following do not exhaust the possibilities of the data by any means.

The test data themselves are given in Part 1, Table 2. Most of the headings represent direct observations or arithmetically derived quantities. However, the meaning of the term Δt, defined as the average temperature drop between the outside of the coil and the fluid, should be understood.

Although considerable care was used in placing the thermocouple junctions uniformly close to the outside of the tube surface, there were small variations of location of essentially unknown value. In any case, the junction cannot be considered as giving more than an "average" outer tube temperature. The effect of junction location presumably is proportionately more uncertain on the thin-walled tubes.

Another uncertainty arises from the fact that, as in Fig. 19, the variations in Δt for a single test are sometimes more than 2 to 1. Detailed analyses, too voluminous for pertinent inclusion, show that the average value of all tube temperatures taken convolution by convolution remains constant along the tube length for each liquid-heating run and for each evaporating run where overheating (as defined later) did not take place. This leads to the conclusion that the average of all the values along the tube is the reliable value for use in engineering analyses.

However, this is merely another way of stating the first important conclusion of the experiments. One of the first observed

facts on the steaming runs was that the mean rise of temperature of the outer tube surface above that of the fluid did not change from inlet to outlet of the coils, even though the quality, or steam content, of the fluid increased continuously along the tube. For a constant mass flow G, pressure p, and heat-absorption rate Q, the tube temperature was virtually constant along the tube. Further, when mass flow was changed with constant pressure and heat absorption, the tube thermocouple temperatures likewise were virtually unchanged. Therefore the first conclusion offered states that within wide experimental limits the heat transfer to high-pressure boiling water is approximately independent of the quality of the fluid in the tube. Fig. 19 (coil No. 6) exemplifies the mass of data leading to this conclusion by the constancy of Δt along the length of the tube in which quality was increasing uniformly.

Following this, the next conclusion advanced for discussion is that the interface (film) coefficients are so high that the problem of dealing with tube temperatures is one of metallic conduction in a thick tube under many conditions of operation at high pressure. The essential reasons for concluding this and some corollaries thereto follow.

The experiments show that, as a first approximation for the majority of the tests on all coils and the straight tube, Δt when plotted against Q may be represented by a straight line passing through the origin. This is shown in Fig. 20(a) for the spiral coils. In fact Δt is approximately proportional to the tube diameter and, therefore, to the tube thickness, since tubes 2–5, inclusive, were designed according to the A.S.M.E. formulas for fixed pressure. This initial approximation states that for geometrically similar tubes the rise of metal temperature is directly proportional to the tube thickness at the same rate of heat absorption—a fact of considerable practical importance. It is based on the following experimental conditions and circumstances.

The radiant-heat transfer to furnace tubes is essentially a two-resistance problem. The transfer of heat from the furnace to the tube obeys the fourth-power law of radiation. When the tube-surface temperature changes, the heat transferred by the furnace to the tube other things being the same is virtually unchanged. With a furnace temperature of 2500 F, the heat transferred to a tube with a temperature below 1000 F is not greatly affected by the tube temperature. Under consideration then are the metal resistance and the inner resistance at the tube and fluid interface which may be likened to film resistance.

The experiments show that the interface resistance is so low for most of the experiments that, as an approximation, it may be neglected. This means that when tubes are run under conditions giving negligible interface resistance, the actual metal temperature involves a study of conduction through tube walls for tubes of the particular shape involved whether covered by blocks, studs, or other surface extensions. This study is not in any wise a part of the present paper.

Fig. 20(b) shows the plot of Δt versus Q for coil No. 6 and for the straight tube. The straight tube considered as a first approximation has more metal resistance than the curved tube. It is still, however, principally a matter of metal conduction to be accounted for by the installation differences and other local factors connected with the reception of radiant heat.

Thus the useful conclusion is established that when a tube is operating "normally" the expected approximate temperature of geometrically similar tubes may be established by experiments on the one tube for technical purposes.

As a matter of everyday parlance, the term "overheating" has a clear but loosely defined meaning. It is necessary to define it as it will be used in the paper.

When the over-all coefficients show that the interface resistance is becoming a crucial element in the over-all resistance, the term "overheating" is applied in this paper to the condition. Thus an "overheated" tube in the sense the term is here used will not necessarily fail immediately.

Several attempts were made to calculate the interface coefficients on the basis of resistance methods, but these failed to give worthy results for obvious reasons. However, rough estimates based on extrapolating the over-all coefficients of different tubes to a tube of zero thickness indicate that the coefficients from metal to boiling fluid can exceed 5000 Btu per hr per sq ft of inside projected area per deg. Such results are mentioned only in passing as supplying illumination rather than data.

So far only those steaming runs in which the tube temperatures

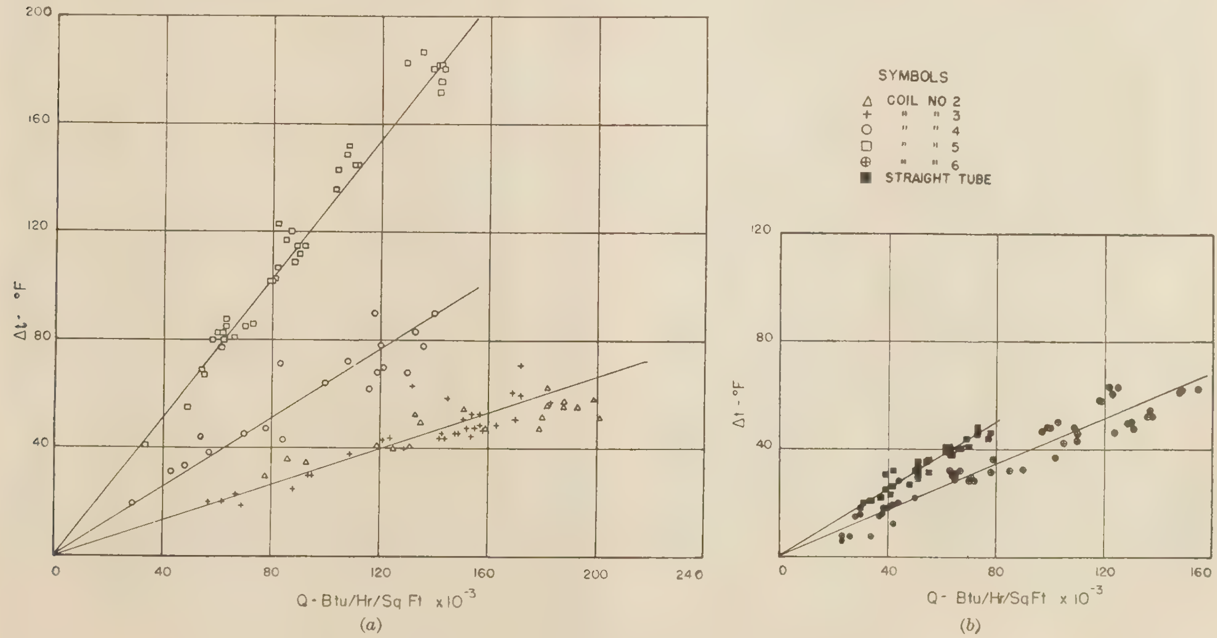


Fig. 20 Temperature Difference Versus Heat Absorption

TABLE 2 SUMMARY OF DATA

Nomenclature

p = nominal pressure, psi
T_1 = inlet temperature, F
T_2 = outlet temperature, F
w = flow rate, lb per hr
G = mass flow, lb per sec per sq ft of cross section
V = velocity at outlet, fps
Q = heat absorption, M Btu per hr per sq ft of projected area
X = quality of fluid at outlet
Δt = average temperature drop, outside tube wall to fluid, deg F
Δp = pressure drop through tube, in. water at 68 F

Run no.	p	T_1	T_2	w	G	V	Q	X	Δt	Δp
Coil No. 1 0.5 in. × 0.75 in. × 10.2 ft										
14BH	1500	522	583	1150	240	6	142	0	97	42
14C	1500	589	596	1230	250	12	142	0.10	63	89
14DH	1500	495	582	906	180	4	142	0	107	19
15AH	2000	675	622	1220	250	6	130	0	75	37
15B	2000	633	636	1250	260	11	133	0.13	51	81
15C	2000	630	636	764	160	10	136	0.22	52	42
15D	2000	626	636	404	82	7	137	0.43	56	22
16AH	2800	640	676	1112	230	7	126	0	69	54
16B	2800	683	686	1090	220	11	139	0.25	58	55
16C	2800	680	685	683	140	7	140	0.29	56	28
16D	2800	678	685	512	100	7	140	0.53	56	20
16E	2800	677	685	685	74	8	154	0.86	72	14
Coil No. 2 0.5 in. × 0.75 in. × 20.3 ft										
17AH	1500	531	564	2510	510	11	81	0	49	234
17B	1500	586	596	2520	510	20	93	0.06	35	350
18AH	2000	586	617	2430	500	12	88	0	46	238
18B	2000	629	636	2460	500	19	86	0.07	36	331
19AH	2500	629	652	2375	480	13	77	0	43	240
19B	2500	662	669	2410	490	19	78	0.09	30	303
20AH	2000	336	426	2450	500	10	183	0	88	192
20BH	2000	337	445	2000	410	8	181	0	85	134
20CH	2000	336	472	1570	320	6	180	0	83	90
20DH	2000	338	522	1080	220	5	171	0	104	45
20E	2000	627	636	1100	220	20	182	0.42	63	160
21AH	1500	508	568	2520	510	11	149	0	70	220
21B	1500	593	596	2520	510	29	151	0.13	55	482
22AH	2000	559	617	2460	500	12	159	0	63	240
22B	2000	634	636	2450	500	24	135	0.14	50	410
23A	1500	590	596	2470	500	26	133	0.11	53	490
24AH	2500	528	588	2390	490	11	141	0	62	235
24B	2500	665	668	2420	490	20	125	0.16	40	431
24CH	2500	597	644	2380	490	13	139	0	55	275
25AH	2800	617	661	2350	480	13	135	0	54	282
25B	2800	681	685	2350	480	22	119	0.18	41	432
25C	2800	681	685	1360	280	17	131	0.39	41	180
26AH	1500	467	571	1740	360	8	174	0	81	125
26B	1500	587	596	737	150	24	179	0.53	48	148
26C	1500	586	596	509	100	24	188	0.81	56	120
26D	1500	586	597	480	100	24	193	0.88	56	100
27AH	2500	588	656	1780	360	10	151	0	63	155
27B	2500	662	669	1200	240	14	159	0.26	48	156
28A	2500	664	669	1030	210	14	199	0.29	59	...
28B	2500	663	669	1240	250	19	182	0.37	57	172
28C	2500	663	669	1100	230	20	188	0.44	58	148
29AH	3000	655	690	3200	650	21	206	0	63	510
29B	3000	691	696	1770	360	23	201	0.58	52	250
29C	3000	693	697	1060	220	17	180	0.92	52	111
30AH	1500	488	567	1940	400	9	150	0	60	149
Coil No. 3 0.5 in. × 0.75 in. × 30.1 ft										
42AH	1500	501	552	5270	1080	24	165	0	66	...
42B	1500	592	596	5270	1080	52	163	0.10	49	...
42C	1500	582	596	2700	550	32	155	0.18	48	...
42DH	1500	394	483	2700	550	11	138	0	68	364
43AH	2000	518	592	2660	540	12	132	0	58	372
43B	2000	630	636	2660	540	28	124	0.18	44	...
43C	2000	627	636	1490	300	23	121	0.31	43	333
43D	2000	628	637	1470	300	27	132	0.34	63	361
43EH	2000	447	603	1420	290	6	143	0	76	...
44AH	2500	529	617	2605	530	13	160	0	70	390
44B	2500	662	668	2605	530	30	154	0.26	53	...
44D	2500	660	668	2590	530	33	183	0.32	57	...
44F	2500	659	668	1840	380	19	94	0.21	30	465
44GH	2500	543	621	1830	370	9	102	0	50	370
45AH	3000	615	673	2560	520	16	131	0	52	420
45B	3000	690	695	2560	520	24	129	0.30	40	...
45C	3000	691	695	2560	520	23	144	0.37	44	...
45D	3000	689	695	2560	520	21	143	0.35	46	...
46A	3300	689	709	3080	630	32	151	gas	53	...
46B	3300	691	706	4310	880	37	128	gas	47	...
46C	3300	706	709	4440	900	45	67	gas	29	...
46D	3300	706	709	3130	640	32	61	gas	25	...
47A	500	461	468	2090	430	71	145	0.16	59	...
47B	500	461	468	2090	430	86	172	0.20	71	...
48A	1500	590	596	1950	400	39	169	0.28	61	...
48B	1500	587	596	1270	260	35	172	0.44	60	442
48C	1500	590	596	755	150	32	170	0.75	51	277
48D	1500	588	596	750	150	18	88	0.38	25	155
48E	1500	585	595	740	150	15	69	0.29	19	15
49A	2000	633	636	1450	300	15	57	0.15	20	230
49B	2000	633	636	1460	300	16	67	0.17	23	254
49CH	2000	537	611	1450	300	7	76	0	39	127
49D	2000	631	636	1450	300	19	109	0.29	38	318
49E	2000	631	636	1450	300	25	151	0.40	51	384
49F	2000	628	636	970	200	26	157	0.63	53	252
49G	2000	628	636	710	140	22	154	0.86	45	185
50A	2500	628	668	910	180	17	158	0.70	47	150
51A	2500	661	668	890	180	11	62	0.32	20	116
51BH	2500	575	664	905	180	5	63	0	42	63
51C	2500	661	668	930	190	14	95	0.49	30	150
51D	2500	664	668	1150	230	21	142	0.62	44	250
51E	2500	660	668	930	190	20	148	0.78	46	180
51F	2500	658	668	820	170	20	149	0.89	46	152
52A	3000	687	695	1350	280	20	152	0.80	48	251
52B	3000	687	695	1080	220	18	157	1	49	184
Coil No. 4 1.0 in. × 1.5 in. × 30.1 ft										
5E	2000	633	636	5110	260	16	136	0.21	78	118
6A	2000	629	636	4030	203	13	120	0.22	78	80
6B	2000	632	636	2700	140	12	133	0.39	83	62
6C	2000	632	636	1510	76	11	140	0.74	90	36
7A	1500	592	596	1440	73	11	108	0.50	72	41
7B	1500	588	596	722	36	5	54	0.50	44	30
7C	1500	591	596	578	29	3	29	0.33	19	32
8A	2000	631	636	1710	86	9	118	0.54	90	40
8B	2000	632	636	1300	66	7	83	0.50	71	28
9A	2500	663	669	2110	110	5	100	0.16	64	29
9BH	2500	467	643	2110	110	3	129	0	147	6
10AH	2800	518	649	2590	130	4	122	0.00	122	10
10B	2800	680	685	2120	110	9	119	0.68	68	45
10C	2800	680	685	2490	130	9	117	0.56	68	38
10D	2800	682	685	1990	100	7	78	0.48	47	22
10E	2800	678	685	850	43	3	48	0.66	33	12
11A	2500	661	669	1080	55	4	43	0.37	31	14
11B	2500	663	669	1580	80	7	70	0.43	45	21
12AH	3000	559	669	3040	150	5	132	0	110	5
12B	3000	691	696	3060	160	9	116	0.48	62	39
12C	3000	690	696	2300	120	8	121	0.76	70	26
12D	3000	689	696	2320	120	7	84	0.45	43	31
12E	3000	692	696	1540	78	6	57	0.64	38	12
13A	3200	678	706	1500	77	33	70	Crit.	80	8
13B	3200	680	706	1520	77	3	74	Crit.	80	8
13C	3200	685	706	1520	77	3	69	Crit.	72	8
13D	3200	685	707	1510	77	3	69	Crit.	72	8
13F	3200	694	707	1510	77	3	65	Crit.	62	10
Coil No. 5 1.75 in. × 2.75 in. × 30.7 ft										
31AH	1500	405	520	5170	86	2	96	0	215	2
31B	1500	591	596	5300	88	6	80	0.18	102	30
32AH1	2000	535	602	5260	87	2	66	0	140	4
32AH2	2000	535	603	5260	87	2	67	0	140	4
32B1	2000	629	636	5200	87	5	62	0.16	80	19
32B2	2000	629	635	5200	87	5	66	0.17	81	19
32C1	2000	630	636	3240	54	4	63	0.27	85	20
32C2	2000	631	636	3240	54	4	61	0.27	77	20
33AH1	2500	523	617	5100	85	2	89	0	190	3
33AH2	2500	522	613	5100	85	2	86	0	192	3
33B1	2500	658	668	5080	85	5	90	0.28	112	14
33B2	2500	659	667	5080	85	5	92	0.30	115	14
33C	2500	662	668	4550	76	5	89	0.34	115	13
33D1	2500	658	668	4540	76	5	73	0.25	86	12
33D2	2500	659	668	4540	76	5	70	0.25	85	12
33E1	2500	658	669	4540	76	4	62	0.20	73	11
33E2	2500	659	668	4540	76	4	63	0.21	78	11
33F1	2500	660	668	3450	57	3	58	0.28	70	12
33F2	2500	660	668	3450	57	3	60	0.29	73	12
34AH1	3000	552	641	5000	83	2	85	0	194	2
34AH2	3000	552	642	5000	83	2	87	0	194	2
34BH1	3000	578	654	5010	83	2	78	0	174	2
34BH2	3000	579	655	5010	83	2	79	0	173	2
34C	3000	690	695	5060	84	5	81	0.39	103	6
34D1	3000	689	695	4200	70	4	82	0.49	107	6
34D2	3000	689	695	4200	70	4	88	0.53	109	6
34E	3000	688	695	4130	69	4	49	0.22	55	6
34F	3000	688	695	4130	69	3	33	0.10	41	4
34G	3000	688	695	4240	71	4	55	0.26	67	4
34H	3000	689	695	2540	42	3	54	0.54	69	5
35AH1	1500	457	579	3980	66	1.5	84	0	176	1
35AH2	1500	458	581	3980	66	1.5	86	0	176	1
35B1	1500	590	596	4010	67	6	87	0.26	120	30
35B2	1500	590	596	4010	67	6	85	0.25	117	29
35C	1500	590	596	3050	51	5	82	0.32	123	32
36AH1	500	322	451	5440	91	2	107	0	202	3
36AH2	500	322	449	5440	91	2	105	0	203	3
36B1	500	461	467	5490	91	17	110	0.18	145	51
36B2	500	460	467	5490	91	17	111	0.18	145	51
37AH1	2000	461	579	5250	87	2	106	0	233	3
37AH2	2000	463	580	5250	87	2	105	0	231	3
37B1	2000	629	635	5120	87	6	103	0.28	136	21
37B2	2000	631	635	5120	87	6	104	0.28	143	22
37C1	2000	632	636	4710	78	6	107	0.33	149	22
37C2	2000	632	635	4710	78	6	108	0.34	152	22
38AH1	1500	406	495	10560	176	4	147	0	280	10
38AH2	1500	407	500	10560	176	4	155	0	277	10
38BH1	1500	412	575	5280	88	2	145	0	273	3
38BH2	1500	410	572	5280	88	2	142	0	276	3
38C1	1500	470	596	4790	80	4	143	0.09	181	8
38C2	1500	468	597	4790	80	4	139	0.07	181	7
38D1	1500	588	596	10560	176	11	142	0.15	176	47
38D2	1500	589	596	10560	176	11	142	0.15	182	47
38E	1500	589	595	5310	88	9	135	0.31	187	32
38F	1500	590	597	4550	76	8	129	0.34	183	31

Run no.	p	T_1	T_2	w	G	V	Q	X	Δt	Δp
39A	2000	629	636	10280	171	10	141	0.19	172	34
40A	2500	661	668	10150	169	9	141	0.23	182	27
Coil No. 6 0.92 in. × 1.25 in. × 53.3 ft										
53AH1	1500	460	519	6730	405	8	82	0	44.2	197
53AH2	1500	457	517	6730	405	8	85	0	46.7	200
53B	1500	591	596	6760	407	20	79	0.10	36.0	380
53C	1500	588	596	4450	268	15	65	0.12	30.9	212
54AH1	1500	535	625	5170	312	8	115	0	72.1	140
54AH2	2000	536	625	5170	312	8	114	0	70.6	140
54B	2000	631	636	5170	312	19	109	0.23	48.3	320
54C	2000	630	636	3090	186	7	103	0.38	50.2	180
54D	2000	631	636	1900	114	14	99	0.61	48.3	113
54E	2000	624	636	1680	101	14	100	0.67	47.9	100
54F	2000	628	636	3940	237	18	110	0.31	45.9	228
54G	2000	628	635	2350	141	14	97	0.47	46.6	140
55AH1	2500	565	600	10330	623	15	88	0	39.4	...
55AH2	2500	564	595	10330	623	15	77	0	38.0	...
55BH	2500	571	629	5130	309	8	76	0	47.5	155
55C1	2500	664	668	5130	309	15	78	0.21	31.4	280
55C2	2500	666	668	5130	309	15	71	0.20	29.6	283
55D1	2500	663	668	2600	156	10	67	0.36	31.7	112
55D2	2500	665	668	2600	156	10	64	0.36	30.1	110
55E	2500	663	668	1810	109	7	63	0.51	32.0	80
56A1	500	455	467	5110	308	23	50	0.06	21.7	68
56A2	500	456	468	5110	308	20	44	0.05	20.1	53
56C1	500	455	464	1600	100	8	30	0.12	15.5	110
56C2	500	456	467	1600	100	7	28	0.11	14.7	100
57AH1	3000	529	638	5100	307	8	132	0	81.8	130
57AH2	3000	531	639	5100	307	8	131	0	78.2	130
57B	3000	693	695	5100	307	20	129	0.57	49.5	285
57C	3000	692	695	4100	247	18	131	0.73	47.5	190
57D	3000	688	695	3230	195	15	124	0.83	46.2	135
57E	3000	688	Super	2730	164	15	130	Super	57.6	103
57F	3000	687	695	3530	213	9	42	0.14	12.2	100
57G	3000	688	695	2170	131	7	42	0.36	13.1	47
57H1	3000	684	695	1770	107	5	34	0.28	7.2	31
57H2	3000	685	695	1770	107	5	23	0.14	5.8	28
58A	2500	659	668	1780	107	5	26	0.17	7.3	36
60AH1	1500	475	589	1970	119	3	50	0	66.2	28
60AH2	1500	478	588	1970	119	3	49	0	67.9	29
60B1	1500	584	596	1970	119	12	64	0.30	27.9	91
60B2	1500	587	596	1970	119	12	61	0.29	25.9	89
60D	1500	580	596	1030	62	11	65	0.58	29.4	62
60E	1500	575	600	770	46	10	65	0.79	29.0	55
61A	2000	627	636	1690	102	10	70	0.47	28.2	72
61B	2000	619	636	1210	74	10	72	0.65	28.2	56
61C	2000	618	635	980	59	9	72	0.83	28.3	49
62AH1	2500	507	574	8800	530	12	130	0	71.0	332
62AH2	2500	509	579	8800	530	12	136	0	67.5	332
62BH	2500	508	638	4400	265	7	138	0	88.1	103
62C	2500	659	668	4400	265	19	130	0.40	49.9	228
62D1	2500	659	668	3010	181	17	130	0.61	50.6	150
62D2	2500	662	668	3010	181	17	136	0.65	52.2	150
62E	2500	659	668	2240	135	16	138	0.89	52.4	134
62F	2500	659	668	2040	123	16	137	0.98	54.4	120
62G	2500	655	668	2010	121	13	110	0.76	43.1	105
62H	2500	657	668	2030	122	12	102	0.71	37.1	100
62I	2500	661	668	2030	122	11	85	0.60	32.0	89
62K	2500	656	668	1400	85	10	90	0.91	32.5	60
63A	500	456	466	4700	283	52	123	0.18	60.6	...
63B	500	454	467	2520	152	51	125	0.35	63.2	415
63C	500	448	467	1430	86	49	122	0.60	63.1	265
63D	500	445	466	1090	66	47	119	0.77	58.3	210
63E	500	445	467	960	58	47	118	0.87	58.0	198
64AH1	2000	448	598	3900	235	6	128	0	87.1	60
64AH2	2000	447	596	3900	235	6	126	0	89.3	60
64BH	2000	455	618	4220	254	6	155	0	105.3	105
64C	2000	632	636	4420	266	24	155	0.41	62.5	342
64D	2000	627	636	2350	141	20	149	0.73	62.1	182
64E	2000	627	637	1900	114	19	148	0.90	63.7	145
64F	2000	625	636	1530	92	14	105	0.79	42.4	100
65A	1500	585	596	2150	130	8	39	0.15	18.1	72
65B	1500	589	596	790	48	7	38	0.46	18.0	52
65C	1500	587	596	530	32	7	40	0.73	18.4	52
65D1	1500	577	Super	400	24	7	41	Super	21.4	48
65D2	1500	574	598	400	24	7	42	1.00	18.9	51
66A	2500	662	668	1430	86	6	37	0.36	14.8	38
66B	2500	657	668	790	48	5	38	0.68	15.6	31
66C	2500	655	668	590	36	4	38	0.92	15.2	28
66D1	2500	657	Super	480	29	4	36	Super	20.8	29
66D2	2500	657	Super	480	29	4	38	Super	24.2	28
66E	2500	653	668	480	29	3	23	0.65	7.7	30
66F1	2500	651	Super	310	19	3	22	Super	10.7	31
66F2	2500	651	Super	310	19	3	23	Super	14.6	32

Run no.	p	T_1	T_2	w	G	V	Q	X_{in}	X_{out}	Δt
Straight Tube 0.92 in. × 1.25 in. × 52 ft										
68AH	500	403	454	1640	99	2	17	0	0	36
69AH1	500	372	431	3530	213	4	42	0	0	63
69AH2	500	384	441	3530	213	4	40	0	0	61
69B1	500	459	467	2500	151	18	42	0	0.11	320
69B2	500	452	467	2500	151	15	39	0	0.09	30.6
70AH1	1500	469	540	2300	139	3	36	0	0	54
70AH2	1500	483	557	2300	139	3	38	0	0	55
70B1	1500	595	596	1550	94	8	37	0	0.23	22
70B2	1500	591	596	1550	94	8	41	0	0.25	23.0
70C	1500	580	596	1680	101	7	40	0	0.18	24.7
71AH1	2000	503	582	2250	135	3	41	0	0	73
71AH2	2000	511	589	2250	135	3	41	0	0	69
71B	2000	623	636	1530	92	6	39	0	0.25	24.9
72AH1	2500	454	523	3750	226	5	55	0	0	67
72AH2	2500	438	507	3750	226	5	53	0	0	65
72BH1	2500	567	651	2270	143	4	53	0	0	77
72BH2	2500	564	648	2270	143	4	52	0	0	80
72C1	2500	641	668	1440	87	6	46	0	0.33	26.2
72C2	2500	653	668	1440	87	7	51	0	0.44	28.8
72D	2500	654	668	1550	94	7	48	0	0.38	26.8
73A1	500	444	467	3250	196	18	54	0	0.08	35.6
73A2	500	460	467	3180	192	25	55	0	0.12	36.2
74AH1	2000	443	524	5160	311	6	89	0	0	136
74AH2	2000	436	513	5160	311	6	84	0	0	127
74BH1	2000	541	626	3450	208	5	76	0	0	79
74BH2	2000	532	623	3450	208	5	80	0	0	94
74C1	2000	624	636	2510	151	12	78	0	0.32	46.3
74C2	2000	624	636	2510	151	12	77	0	0.31	43.9
75A1	2500	642	668	950	57	4	31	0	0.35	19.9
75A2	2500	664	668	950	57	5	33	0	0.50	20.8
76A	2000	623	636	1100	66	5	34	0	0.31	19.9
77A1	1500	588	596	1100	66	6	31	0	0.25	19.6
77A2	1500	586	596	1100	66	6	30	0	0.24	18.4
78A	2500	640	668	1770	107	6	50	0	0.26	32.2
78B	2500	656	668	1560	94	7	51	0	0.42	30.9
78C1	2500	668	668	1300	78	7	51[a]	0.01	0.58	35.8
78C2	2500	658	669	1460	88	7	55	0	0.50	31.4
78D	2500	638	668	1260	76	6	51	0	0.44	29.8
78E	2500	613	668	1110	67	5	50	0	0.41	32.2
78F1	2500	665	668	1110	67	7	50	0	0.67	Note[b]
78F2	2500	668	668	1110	67	7	50[a]	0.03	0.70	Note[b]
79A	2000	620	636	1450	87	7	51	0	0.36	32.0
79B	2000	636	636	1330	80	9	51[a]	0.10	0.55	35.6
79C	2000	636	636	1170	71	9	51[a]	0.11	0.64	33.2
80A	1500	595	596	2220	124	12	61	0	0.26	41.2
80B1	1500	573	596	2450	148	11	62	0	0.19	41.0
80B2	1500	584	596	2250	136	11	62	0	0.24	39.0
80C	1500	581	596	2250	136	11	62	0	0.23	40.0
80D1	1500	591	597	1860	112	12	66	0	0.33	40.0
80D2	1500	584	596	1860	112	11	61	0	0.29	39.3
80E	1500	584	596	1680	101	11	63	0	0.33	39.7
80F	1500	585	596	1380	83	11	63	0	0.42	38.2
80G	1500	578	596	1220	74	11	64	0	0.47	38.0
80H	1500	597	597	990	60	12	63[a]	0.06	0.69	40.0
80J	1500	596	596	990	60	13	63[a]	0.13	0.75	38.5
80K	1500	596	596	990	60	14	63[a]	0.20	0.81	43.4
80L	1500	596	596	990	60	14	63[a]	0.22	0.84	40.6
81A1	2000	589	636	2670	161	8	67	0	0.13	40.1
81A2	2000	595	636	2670	161	9	70	0	0.17	41.3
81B1	2000	625	636	2330	140	10	69	0	0.30	44.0
81B2	2000	599	636	2330	140	9	73	0	0.24	46.7
81C	2000	589	636	1640	99	8	73	0	0.36	44.8
81D	2000	636	636	1640	99	12	73[a]	0.05	0.57	47.8
81E	2000	636	636	1640	99	13	73[a]	0.15	0.68	46.3
81F	2000	636	636	1640	99	14	73[a]	0.18	0.70	Note[c]
82A	2500	626	668	1840	111	7	68	0	0.34	54.1[d]
82B	2500	639	668	1640	99	8	72	0	0.50	55.5[d]
82C	2500	668	668	1640	99	10	72[a]	0.07	0.73	51.9[d]
82D	2500	668	668	1480	89	11	72[a]	0.14	0.89	57.5[d]
82E	2500	668	Super	1310	79	....	72[a]	0.24	Super	Note[f]
83A	500	444	467	4190	252	23	68	0	0.08	54.7[d]
83B	500	446	468	3170	191	25	68	0	0.12	55.0[d]
83C	500	461	468	1900	114	28	67	0	0.25	50.4[d]
83D	500	468	468	1900	114	44	68[a]	0.14	0.40	55.1[d]
83E	500	468	468	1900	114	69	68[a]	0.38	0.64	57.2[d]
84A	1500	577	596	1950	117	11	64	0	0.27	48.0[d]
84B	1500	577	596	1450	87	10	64	0	0.33	51.3[d]
84C	1500	596	596	1450	87	21	64[a]	0.41	0.84	48.2[d]
84D	1500	596	Super	1130	68	....	64[a]	0.75	Super	Note[e]
84E	1500	587	596	1130	68	6	31	0	0.24	28.2
84F	1500	579	596	740	45	5	32	0	0.38	28.8
84G	1500	557	Super	330	20	....	36	0	Super	Note[f]
85A	2000	636	Super	350	21	....	34[a]	0.63	Super	Note[g]
86A	2500	621	Super	350	21	....	34	0	Super	Note[h]

[a] Assumed to be the same as preceding runs (see text).

[b] Tube temperatures 18, 20, and 22 ft from inlet were much higher than before and were fluctuating (see text).

[c] Tube temperatures 16 and 18 ft from inlet were much higher than before and were fluctuating (see text).

[d] Tube temperatures on first quarter of tube higher, remainder of tube temperature about normal (see text).

[e] Virtually all tube temperatures (except those near the center portion of tube) were very high and fluctuating (see text).

[f] Tube temperatures (i.t.-o.t.) about the same as preceding runs over first half of tube but very high and fluctuating over remainder (see text).

[g] Tube temperatures (i.t.-o.t.) about the same as preceding runs over first third of tube but very high and fluctuating over remainder (see text).

[h] Tube temperatures (i.t.-o.t.) about the same as preceding over first 6 ft of tube but very high and fluctuating over remainder (see text).

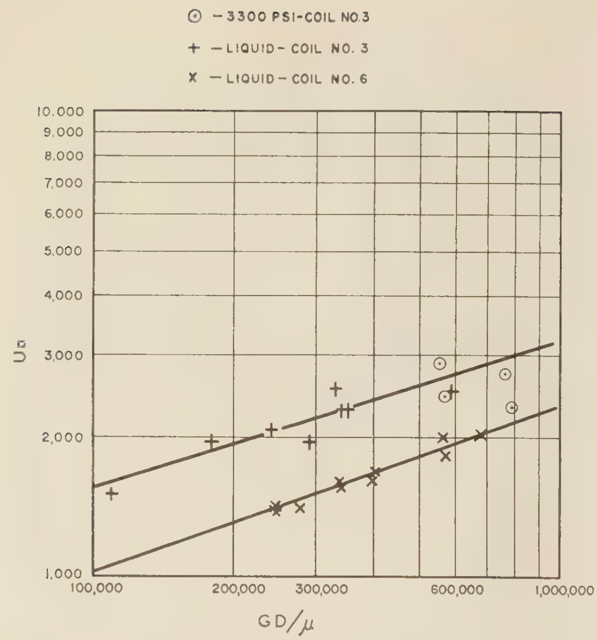


FIG. 21 LIQUID-HEATING OVER-ALL TRANSFER COEFFICIENT VERSUS REYNOLDS NUMBER

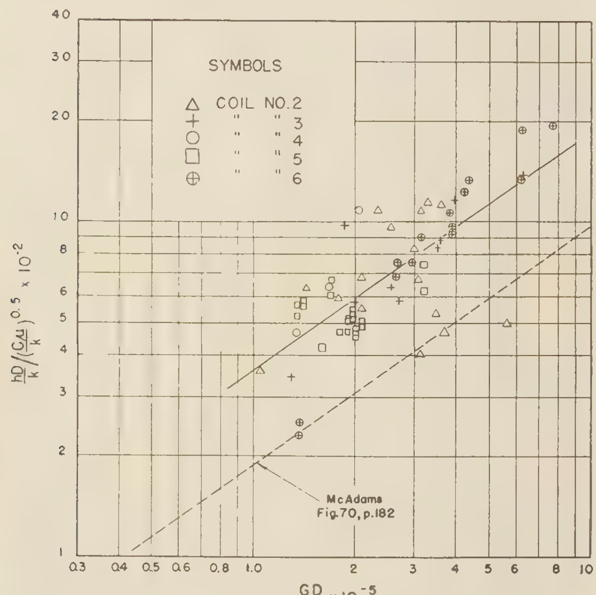


FIG. 22 NUSSELT-PRANDTL FUNCTION VERSUS REYNOLDS NUMBER

were considered normal have been discussed. Most of the runs fell into this class because it was not until the quality approached certain values that any noticeable overheating was normally observed. As shown in Fig. 19, the metal temperatures along the tube were high and low, the highest being found at the bottom of the convolutions and the lowest at the top, indicating quite a vertical variation in radiant heat. Because of this condition the beginning of overheating of the coils was first manifested by abnormally higher temperatures at the bottom of the turns, the top and side temperatures showing no appreciable change. In some cases as the flow was reduced this increase would appear on only the last convolution, while in other cases most of the bottom temperatures would be high. Further decrease in flow would usually cause these peak temperatures to increase still further and then fluctuate. Finally, all the temperatures near the outlet end of the coil would be high and fluctuating, especially if the steam was superheated. The average Δt terms, reported in Table 2, have the increased peaks included (except for coil No. 5), but the averages do not include the temperatures on the end portion of the tube if all temperatures had gone up. In the case of coil No. 5 the peaks and valleys were so great on virtually all runs, probably because of the large diameter of the spiral and corresponding variation in furnace heat, that only the temperatures taken on the horizontal center line of the coil could be used to obtain consistent values of Δt even for low-quality runs.

The final analyses here presented represent an attempt to generalize the experimental results on a systematic basis.

The studies discussed thus far have been concerned with those runs in which evaporation took place. Although interest in liquid-heating or nonevaporating studies was not primary, several useful purposes were served and data of general interest were provided by making some runs in which the fluid temperature was kept below the saturation temperature corresponding to the pressure. For example, when the fluid entered the calorimeter at a temperature below the saturation temperature, it was possible to compare the heat picked up by the cooling water with that given up by the fluid flowing from the test surface and thus provide a combined check on the temperature and flow measurements. In addition, the coil temperatures during the liquid-heating runs were useful in indicating that the cause of the differences in the metal temperatures was not steam blanketing but differences in heat absorption along the tube, because the same differences existed when the fluid was entirely liquid. The pressure-drop data collected during the liquid-heating runs were helpful in the general correlation of pressure-drop information (refer to Part 3).

The Δt terms observed during the liquid-heating runs were higher than for the evaporating runs at the same rates of heat absorption and therefore the over-all heat-transfer coefficients U_0 were lower. When plotted against Reynolds number on logarithmic paper, U_0 increased rather gradually with increase in Reynolds number, as can be seen in Fig. 21, which is typical.

A better picture of the performance of liquid-heating tubes, exposed to radiant heat on one side, would be available if the constant metal resistance for each size tube could be determined and then subtracted from the over-all resistance, thereby providing the film resistance for correlation. To compute the metal resistance from the tube thickness entails too many assumptions under these conditions of exposure, but another method of approximating the inside film resistance has been tried and is briefly described. If virtually all of the resistance when steaming is assumed to be metal resistance, as has been indicated to be the case, then an approximate value of the metal resistance can be obtained. Fig. 22 (refer to Appendix 1, page 26) shows the correlation of the coefficients obtained in this manner and plotted as the product of the Nusselt number and the Prandtl number to the 0.5 power against Reynolds number. Although the points scatter considerably and are higher than other data for water heated in pipes, they show a similar trend. The dotted line is reproduced from McAdams[7] extended to higher values of Reynolds number.

The final explanation is based on what the test results indicate was happening in an over-all manner, bearing in mind that it is not possible to present the complete physical picture which will ultimately furnish a full detailed explanation.

[7] "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, Inc., New York, N. Y., 1933, p. 109.

Therefore the parameters applied in this study are to be regarded as engineering conveniences for explanation and bounding purposes and do not exclude others which might be even more convenient or which to a student of heat transfer may seem to have been neglected.

In a tube in which a fluid is being heated without a change of state such as an economizer or a superheater tube, the rates of heat transfer are systematically predictable. This is well known, needs no elaboration, and is verified by Fig. 21 for coil No. 3, on which the liquid-heating over-all coefficients and the critical-pressure over-all coefficients are shown.

In the steaming tubes of these experiments the fluid usually entered the tube at a temperature slightly below the saturation temperature corresponding to the pressure. Therefore for the first small portion up to the point where evaporation began, the conditions were those of an economizer and are not of immediate interest, since well-known types of heat-transfer correlations apply. However, when evaporation or change of state begins, the heat-transfer coefficients change, usually to higher values. The fluid at the inception of evaporation consists of a two-phase mixture of steam bubbles and water, the water constituting the continuous phase. At some unknown value of the quality, or perhaps because of the separation of the mixture, the steam becomes the only continuous phase with drops or slugs of water in it, or there coexist two continuous phases such as a layer of steam and a layer of water. Finally all the water disappears and the heat transfer becomes predictable again by the laws of convective heat transfer as in a superheater.

To represent the experiments from this point of view, it is best to depart from the Δt relationship to a study of the over-all coefficients of heat transfer. From the data on free boiling it may be reasoned that the stirring effect of the bubbles is an important factor in the maintenance of heat transfer somewhat independent of quality. When, however, the bubbles have disappeared and wetting is by droplets (when vapor constitutes the continuous phase) the experiments still indicate the maintenance of cooling. One simple hypothesis is offered for both cases, the reasoning being purely deductive. The hypothesis is that the turbulence which produces cooling can be measured by the amount of thermal energy transformed into mechanical energy when the water changes into steam. This idea is expressible in terms of the volume rate of steam generation per unit area of heating surface, or as $\frac{q' \cdot v_g}{h_{fg}}$. If this is written as $\frac{q' \cdot v_g}{h_{fg} \cdot w}$ where w is the product of bubble frequency and diameter, it is Jakob's parameter for free boiling.

The steam made at the heating surface must be replaced by fresh liquid and, by analogy, from the principle of mass action, the available liquid (coolant) may be proportional to the concentration of liquid or, to hypothesize, to the volume rate of water flow per unit cross section of the tube. This equals

$$G(1 - x_s)v_f$$

where x_s is the quality at the section considered.

The use of the ratio of the volume of steam made per second per unit of heating surface to the rate at which the volume of water is flowing per unit cross section of the tube offers a convenient parameter for study as the single elements are dimensionless. It is

$$\left(\frac{q'}{h_{fg}G}\right)^x \left(\frac{1}{1 - x_s}\right)^y \left(\frac{v_g}{v_f}\right)^z$$

The parameter of principal interest is the first term $\left(\frac{q'}{h_{fg}G}\right)^x$ or the ratio of the rate at which steam is generated by weight at the inside surface of the tube to the mass flow in the tube. The second parameter $\left(\frac{1}{1 - x_s}\right)^y$ represents a measure of the water content in the tube. Terms v_g and v_f are tabular values.

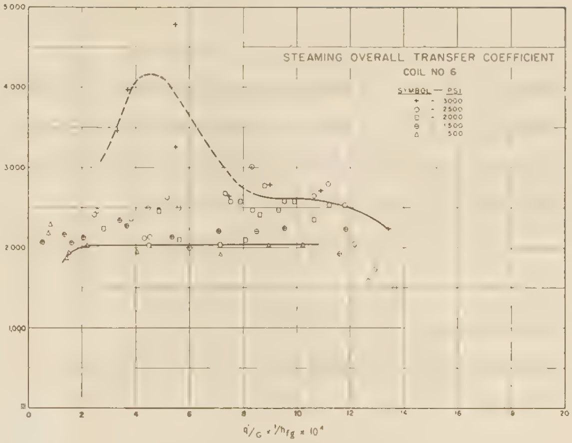


FIG. 23 COIL NO. 6: STEAMING OVER-ALL TRANSFER COEFFICIENT VERSUS PARAMETER

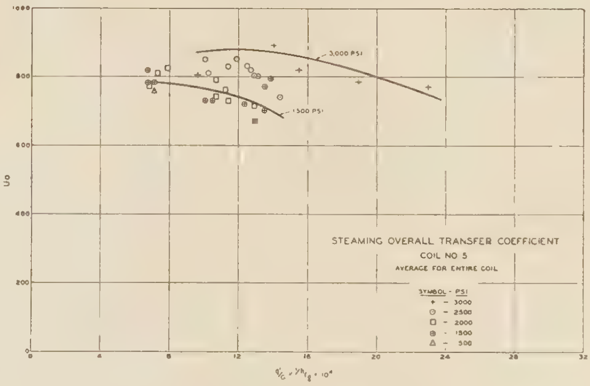


FIG. 24 COIL NO. 5: STEAMING OVER-ALL TRANSFER COEFFICIENT VERSUS PARAMETER
(Average for entire coil.)

For simplicity of reference to the first parameter put

$$\phi = \frac{q'}{h_{fg}G} 10^4$$

in which q' = Btu per sec per sq ft of inside tube surface, on a projected basis

G = pounds of fluid per sec per sq ft of inside tube cross section

h_{fg} = tabular latent heat at pressure of test

Fig. 23 shows the over-all coefficients plotted by pressures for coil No. 6. There is a distinct distribution according to pressure. The coefficients rise at initial values of the parameter and then fall. The values of the parameter which indicate the tendency of the coefficients to decrease is of the order of 7 to 10. Fig. 24 shows the over-all coefficients for coil No. 5, the largest-inside-diameter tube tested. This is a good tube to consider for the parametric boundary is again of the order of 7 to 10, but has an added significance in that when a preliminary test of this tube showed a tendency toward overheating the mass flow was increased slightly in order to protect the tube. Therefore the tests approached the limit of highly conservative operation. Coils

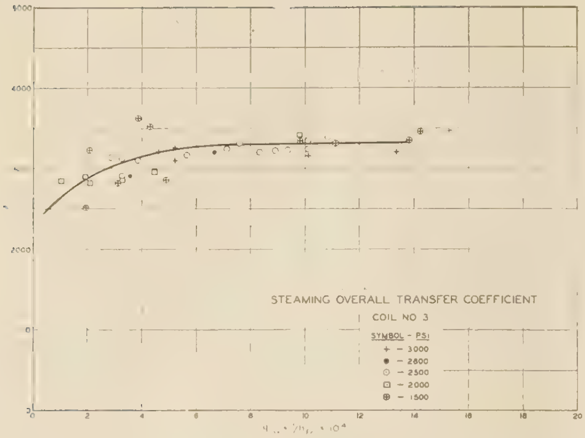


FIG. 25 COIL NO. 3: STEAMING OVER-ALL TRANSFER COEFFICIENT VERSUS PARAMETER

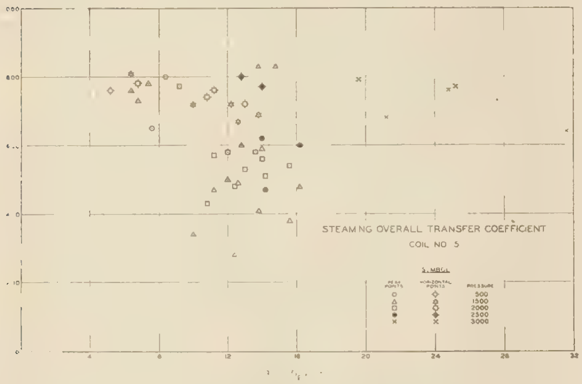


FIG. 26 COIL NO. 5: STEAMING OVER-ALL TRANSFER COEFFICIENT VERSUS PARAMETER

No. 5 and No. 6 may also be said to represent the preferable experimental precision. Coil No. 3 (0.5 in. ID) also was tested a sufficient number of times to warrant the inclusion of the results in Fig. 25. Here the coefficients rise as with the other larger tubes but at the maximum values of the parameter show no signs of decrease.

Coil No. 5 is of special interest to those who seem irresistibly drawn toward the association of overheating with complete disappearance of the water phase. None of the tests on this coil it will be noted exceeded $X = 0.35$ except at 3000 psi.

The analysis was carried into the individual turns of the coil with the interesting results of Fig. 26 which gives the point values of the over-all heat-transfer coefficient for coil No. 5 against the parameter. Since there was no direct measurement made of the heat-transfer rate q' along the tube, it was necessary to make certain assumptions which can be briefly described as follows: From those runs which were considered normal, i.e., those with no indication of steam binding at the bottom of the convolutions, the q' at the bottom was assumed to be proportionally higher than the average q' by the same percentage as the Δt at the bottom was higher than the average Δt. These values of q' at the bottom of each convolution were further modified for differences in total heat transferred during the abnormal runs, as compared with the normal runs, by assuming that the q' terms at these points changed directly with the change in total heat transferred. The values of U' were computed by dividing the estimated point values of q' by the measured values of Δt at the same points. The values of U' for the horizontal center-line points on the coil are the same as the average values for the coils. The Δt terms at these points appear to be normal for all runs made on this coil.

The following tentative conclusions may be drawn:

1 For tubes up to 1.75 in. ID overheating is unlikely for values of ϕ less than 7.

2 Small-inside-diameter tubes are safer than large-inside-diameter tubes for values of ϕ greater than 10.

3 While large values of the parameter, other things being equal, are reached more quickly at high pressures, the superior cooling property of the steam and water tends to mitigate this and suggests that at pressures over 2800 psi the heat transfer should be examined from the point of view of the usual convective correlation.

4 When the value of the parameter is of the order of 10 or more the phenomenon of steam blanketing may be expected. Steam blanketing is more likely to occur on the large-inside-diameter tubes than on the small for the same values of ϕ.

THE STRAIGHT TUBE

The straight tube 50 ft long, which presents the clearest examples of normal and abnormal operating conditions, will be examined from the foregoing points of view. To permit a detailed examination of the possible application of the parameter ϕ several plots of Δt along the straight tube are presented with derived results at points corresponding to those at which Δt was measured. The curves presented to illustrate the behavior of the straight tube were constructed as follows:

(*a*) Term Δt is plotted in every case as an observed value.

(*b*) For the high-mass-flow tests, average heat absorption and average Δt, as previously demonstrated, plot on a straight line. Thus for these tests the point heat absorption is given by Δt times U_0 (average) to a sufficient accuracy for our purposes.

(*c*) Term ϕ is calculated from G and the heat absorption at the point.

(*d*) Term X is calculated from average values of Q and a smooth curve drawn. Term X, of course, is not saw-toothed but must increase continuously.

(*e*) In the curves which show superheating and steam blanketing, Q, ϕ, and X are always derived from the preceding normal tests at the same pressure and furnace-heat absorption.

In Fig. 27, Δt shows no abnormal condition. The values are moderate for a mass flow of 117 lb per sec per sq ft, an average heat-absorption rate of 64,000 Btu per hr per sq ft, a final quality of $X = 0.27$, and a pressure of 1500 psi. The derived heat absorption parallels the Δt curve and the quality X rises uniformly from the point of inception of evaporation to the end of the tube. The value of the parameter never rises above about 7.

In Fig. 28 at the same pressure, heat absorption, and somewhat lower mass flow (68 lb per sec per sq ft) and with an initial quality X of 0.75 abnormal Δt measurements are shown. Up to measuring point 13, stable conditions prevailed, but immediately thereafter Δt rose to excessive values. The computed quality reached a value of $X = 1$ at point 13 and thereafter the fluid must have been steam increasingly superheated as it approached the end of the tube. The parameter ϕ is higher than the normal values of Fig. 27 and reaches a value of 12 at two points. It is obvious that a value of the parameter of about 12 is an indication of a condition of operation which resulted in overheating.

Fig. 29 shows the results obtained on a test at 2000 psi under conditions which can be described as normal. The mass flow was 140 lb per sec per sq ft, the heat absorption 73,000 Btu per hr per sq ft, and the final quality was 30 per cent. The highest value

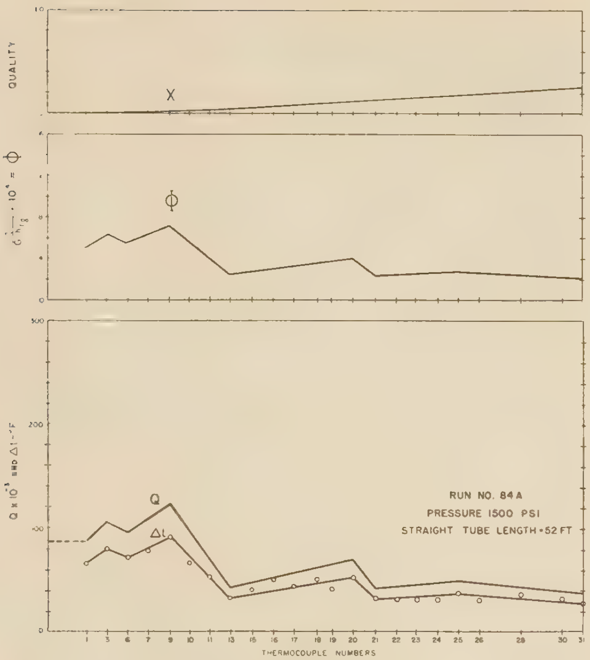


Fig. 27 Normal Behavior of Straight Tube, 1500 Psi

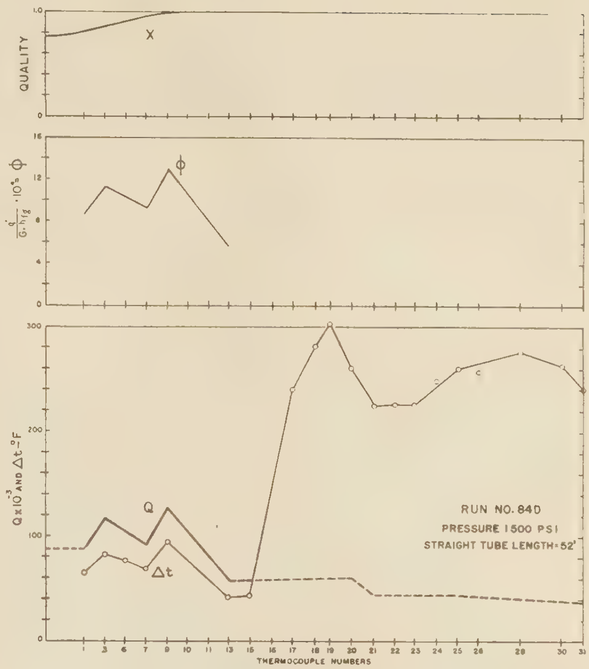


Fig. 28 Abnormal Behavior of Straight Tube, 1500 Psi

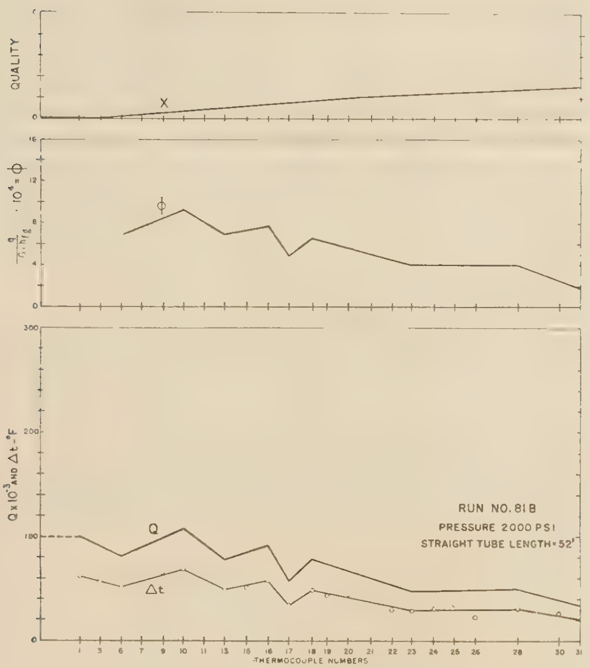


Fig. 29 Normal Behavior of Straight Tube, 2000 Psi

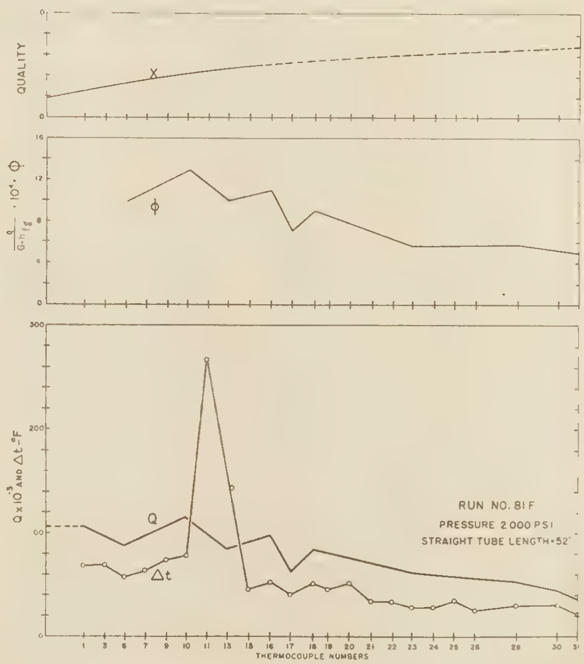


Fig. 30 Behavior Indicative of Steam-Blanketing in Straight Tube, 2000 Psi

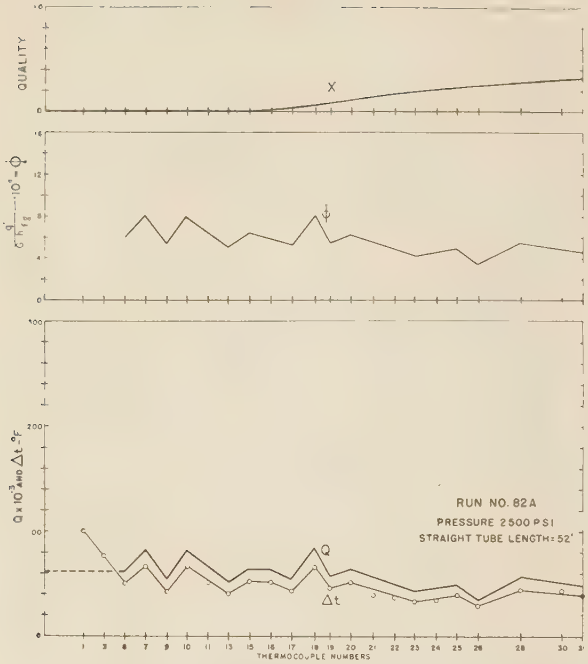


Fig. 31 Normal Behavior of Straight Tube, 2500 Psi

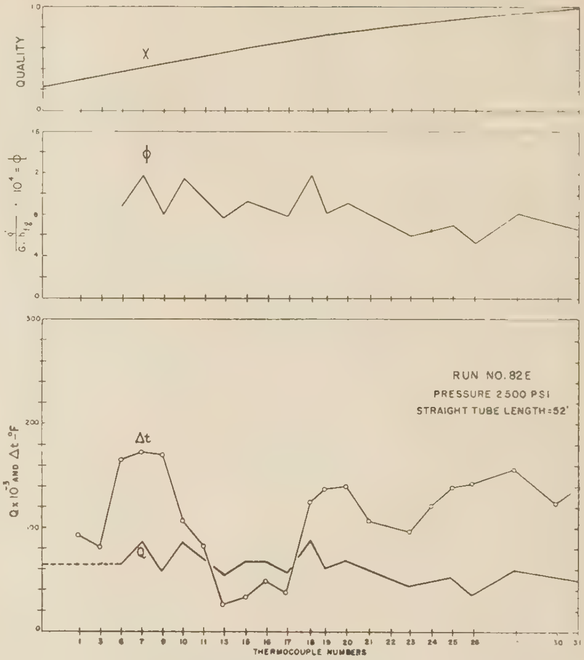


Fig. 32 Abnormal Behavior of Straight Tube, 2500 Psi

reached by the parameter was about 9.5. With Δt indicating no conditions of excessive surface temperature, it can be concluded that the value of the parameter which will serve as a warning is now between the values of 9.5 and 12.

Fig. 30 is interesting because it shows what is believed to be a condition of steam-blanketing due to high local heat absorption possibly caused by flame impingement. Fig. 30 should be compared with Fig. 29, because both were studies at the same pressure and approximately the same heat absorptions. For Fig. 30, the mass flow was reduced from 140 to 99 lb per sec per sq ft, and the initial quality was raised from 0 to 16 per cent. The value of Δt rose above the limits of the chart shortly after point 10 and returned to normal values at point 16. It is in this length of the tube that steam-blanketing or the coexistence of continuous phases of vapor and liquid is assumed. At point 16 and thereafter the fluid reverted to a mixture and regained its ability to cool the tube. The computed values of the quality were low and ranged from 35 to 45 per cent in this location. The value of the parameter reached 13.0 at point 10 and this value may be associated with the evidence of relative overheating probably caused by steam-blanketing.

Fig. 31 shows conditions which can be described as normal at 2500 psi. With the fluid temperature below saturation at the tube inlet evaporation did not start until about point 14–15, and thereafter the quality rose to a final value of 34 per cent. The values of Δt and q' are parallel and not excessively high at any point. The parameter rises to a value of about 8 at three points but the rise was not sustained so that no indications of distress appeared.

Fig. 32 shows another test at 2500 psi in which the heat absorption was approximately the same as for the test shown in Fig. 31, but in which the mass flow was reduced from 111 to 79 lb per sec per sq ft, and instead of a liquid at a temperature below the saturation entering the tube, Fig. 32 shows a mixture quality of 23 per cent entering the tube. Steam-blanketing probably occurred over three sections of the tube as indicated by the high values reached by Δt. It is reasonable to assume that continuous phases of vapor and liquid phases coexisted in these areas with the vapor phase in contact with the tube and superheated although the final quality of the mixture was computed as 100 per cent. It is interesting to note that this tube operating under these conditions of mass flow, heat absorption, and quality tended toward undesirable conditions which probably would have resulted in permanent damage if sustained. The parameter reaches values of above 12 at two of the high Δt regions but not the third. It is reasonable to assume that the average quality in the first two sections was low enough to permit restoration of cooling when the "hot-spot" area was passed but that after the quality passed 80 per cent, restoration of cooling was not accomplished even though the heat transfer had decreased.

Six interesting values of ϕ have been presented, three under normal conditions (6, 9.5, and 8) and three under abnormal conditions (12, 13.0, and 12). It can be said, therefore, that when the value of the parameter exceeds 9.5 there exists some danger of steam-blanketing, and a condition of steam-blanketing will exist when the value of the parameter reaches 12.

The parameter can be easily applied to any tube. If the parameter is expressed partly in its elements, the scale factor arising from the physical dimensions of the tube appears very clearly and simply.

Thus

$$\phi = \frac{q'}{h_{fg}G} = \frac{q'}{h_{fg}(w/a)} = \frac{\pi}{4}\,\frac{q'd^2}{wh_{fg}}$$

where
w = flow rate, lb per sec
a = internal tube area, sq ft
d = inside tube diameter, ft

Similarly the final quality in a uniformly heated tube where the water enters at saturation is

$$X = \frac{q'ld}{h_{fg}w} \quad \text{or} \quad X = \frac{4}{\pi}\,\phi\,\frac{l}{d}$$

Thus ϕ can be directly transformed to the final quality by the application of a constant (for any given tube) which concerns only the physical dimensions of the tube and the distribution of heat along the tube. In q' and G the physical dimensions of the tube are included so that this parameter may be found on examination to answer one of the objectives of this investigation which was to determine the influence on the performance of a tube of variations in its dimensions, i.e., a scale factor.

Time was not available to study the application of the other suggested parameters $1/(1 - X_s)$ and (v_g/v_f). One of the conclusions of the present experiments is that $1/(1 - X_s)$ will have a form giving it a very minor role until high values of X_s are reached. For high pressures, it is believed that the effect will remain minor even for these. However, within the limits of the experimental procedure, the objectives of the investigation have been met. Further study of the data may broaden the basis of the correlations and expand the application of the parameters to an extent permitting definite conclusions with increased assurance

3—PRESSURE DROP THROUGH THE TUBE

As discussed in Part 1 of this paper, under "Objectives," it was the intention of these studies to examine the resistance to flow of water, under conditions of rising temperature, through the tubes and of steam-water mixtures, under evaporating conditions, through the tubes. An exhaustive analysis of fluid friction was not one of the objectives of the study and the experiments were not planned to include data of sufficient refinement to make such an analysis possible. However, in the work of correlating the available pressure-drop information and the examination of the effect of the curvature of the tubes, certain interesting indications were developed concerned with the dependence of the friction factor on the energy changes of boiling and these are presented. Such dependence is sufficiently indicated at least to encourage further investigation under more favorable experimental conditions.

The information derived from the observed pressure drops across the spiral coils, both liquid-heating and steaming runs, is presented in the form of friction coefficients. These coefficients are defined by the familiar formulation

$$dp_F = \frac{4fV^2dl}{2gDv} \quad \ldots\ldots\ldots\ldots [1]$$

where
dp_F = differential frictional pressure drop
f = friction factor
V = velocity
v = specific volume
dl = differential length
D = inside diameter of tube

The units are feet, pounds, and seconds.

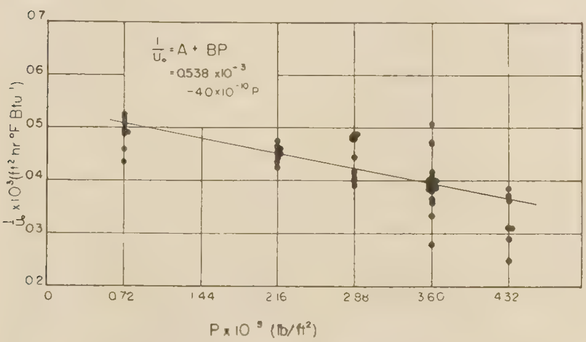


Fig. 33 Coil No. 6, Over-All Thermal Resistance Versus Pressure

Liquid Friction. For the experiments in which the fluid did not change state (liquid-heating runs), the pressure drop Δp is taken as entirely frictional without engineering error. The friction factor f was calculated from the usual equation derived from Equation [1] for the condition of no change of state

$$f = \frac{C\,\Delta p}{G^2 v} \quad \ldots\ldots\ldots\ldots [2]$$

where

$$C = \frac{2gD}{4l}$$

Δp = pressure drop
G = mass flow
v = arithmetic-average specific volume

The values of f computed from Equation [2] are plotted against the average Reynolds numbers (GD/μ) in Fig. 35, for the six coils tested. The curve marked "McAdams"[7] is given to permit a comparison of the correlation of these results with an accepted correlation for isothermal friction in commercial tubes up to 2 in. ID. It may be concluded that within the precision of these determinations which is of the order of 5 per cent for the liquid-heating runs, the liquid friction, while slightly higher than the standard, presents no very unusual aspects.

Curvature of the Tubes. Previous work[8] indicates that curved tubes will exhibit considerably higher friction than straight tubes in the viscous-flow region, and that the friction approaches that of a straight tube of the same length in the highly turbulent zone when the zones are defined by the Reynolds criterion. The parameter usually employed to relate curved- and straight-tube friction is

$$A = r/R$$

where r = inside tube radius
R = radius of curvature

A consistent form of this parameter is e^A as suggested by W. J. Wohlenberg. Mean R for a spiral coil form may be defined by the equation

$$R_m = \int R\,dl / \int dl$$

Integration of the foregoing expression for a spiral of Archimedes yields no significant difference for the coils tested from the arithmetic-average radius. For the six tubes A_m varies from 0.036 to 0.05 which is not a sufficient difference to afford a basis for experimental discrimination.

In order to answer the question as to the significance of any difference between the coils and a straight tube, the isothermal friction was determined for the straight tube and the results are shown in Fig. 34. The conclusion, that for value of A below 0.05 (or in tubing carefully bent to a mean radius more than 20 times its inside diameter) there is no significant difference in friction for Reynolds numbers of over 50,000, is definite enough.

Steaming Friction. The formulation of the factors for the steaming runs with the spiral coils presents unusual aspects and both accidental and systematic sources of error. The precision of these results for coils Nos. 1, 2, 3, 4, and 6 is about 10 per cent and for coil No. 5 about 20 per cent. A discussion of these experiments is necessary since the conclusion will be advanced that steaming friction is greater than the ordinary hydraulic friction of mixtures of steam and water.

The basic equation used for the reduction of the results is

$$-d(\Delta p) = \frac{VdV}{vg} + \frac{dF}{v} + \frac{dz}{v} \quad \ldots\ldots\ldots\ldots [3]$$

[8] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Great Britain Aeronautical Research Committee, Clarendon Press, Oxford, England, vol. 1, 1938, p. 312.

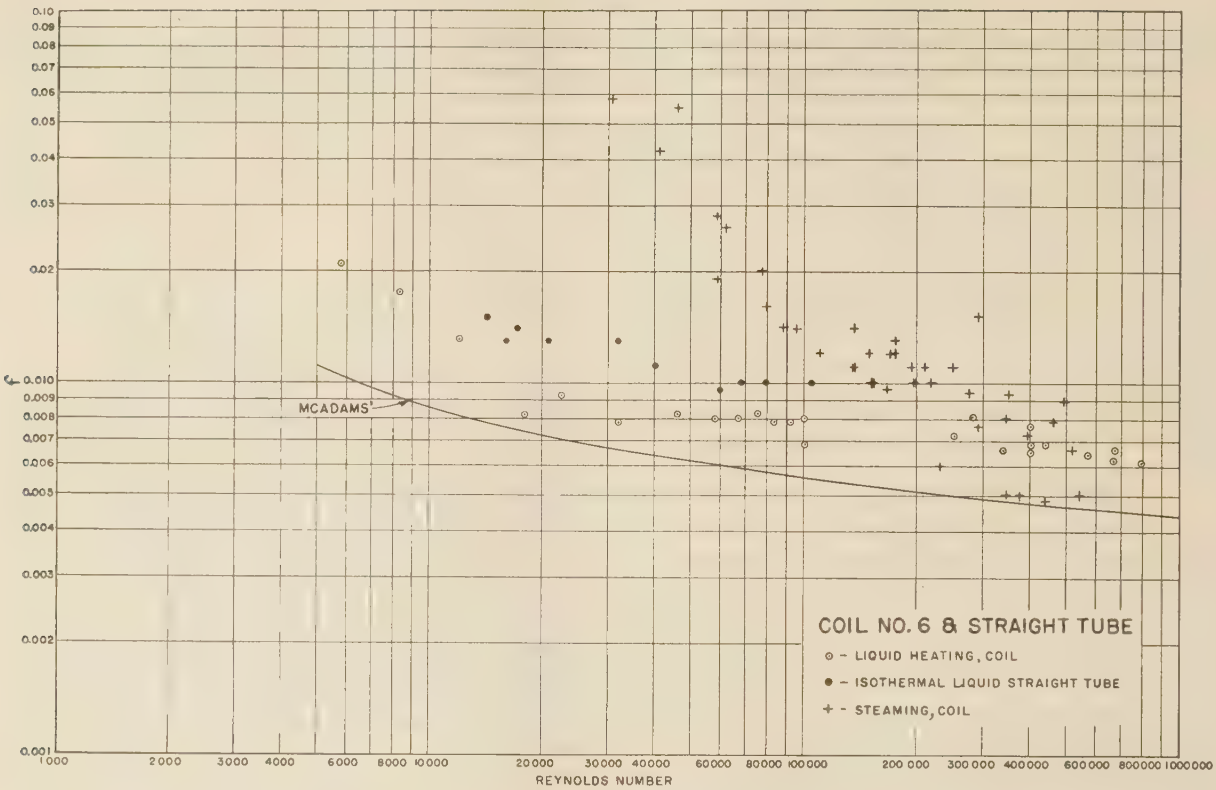


FIG. 34 FRICTION FACTOR VERSUS AVERAGE REYNOLDS NUMBER

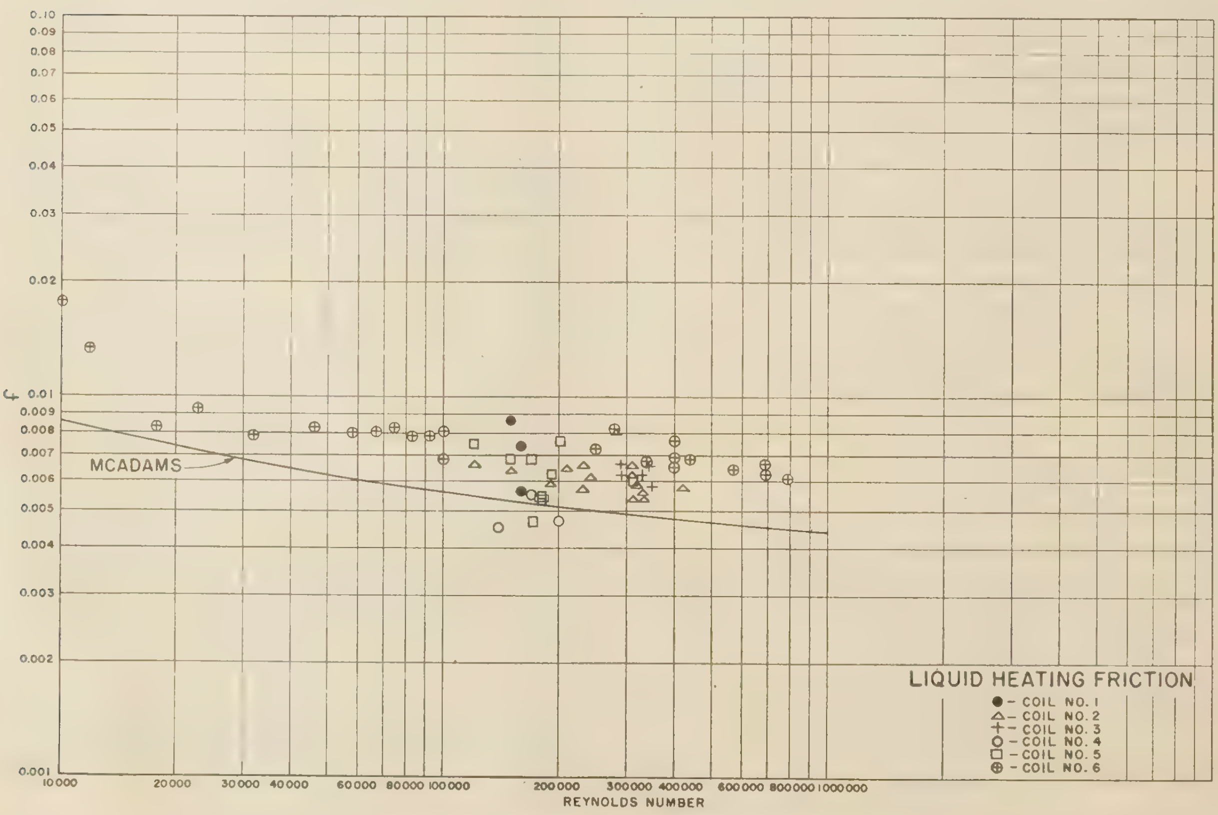


FIG. 35 ISOTHERMAL FRICTION FACTOR VERSUS REYNOLDS NUMBER

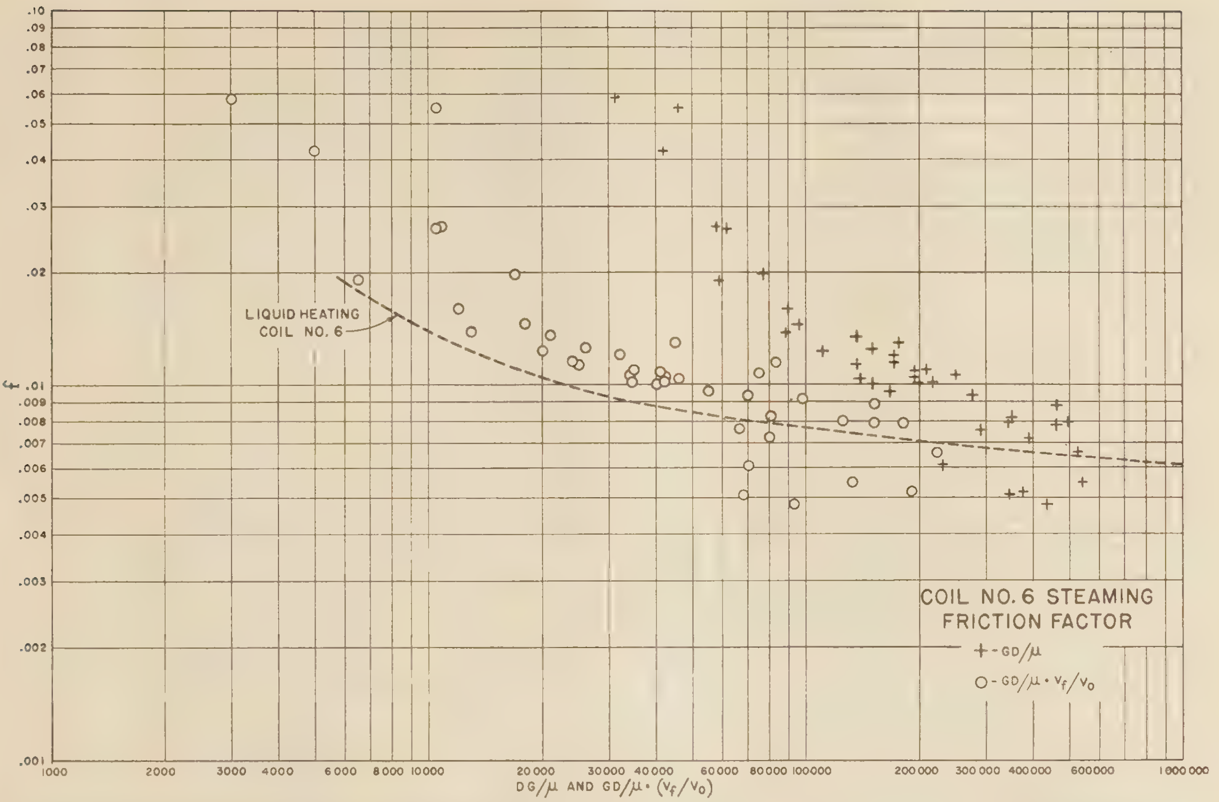


FIG. 36 COIL NO. 6, STEAMING FRICTION FACTOR VERSUS REYNOLDS NUMBER AND REYNOLDS NUMBER × DIMENSIONLESS PARAMETER V_f/V_0

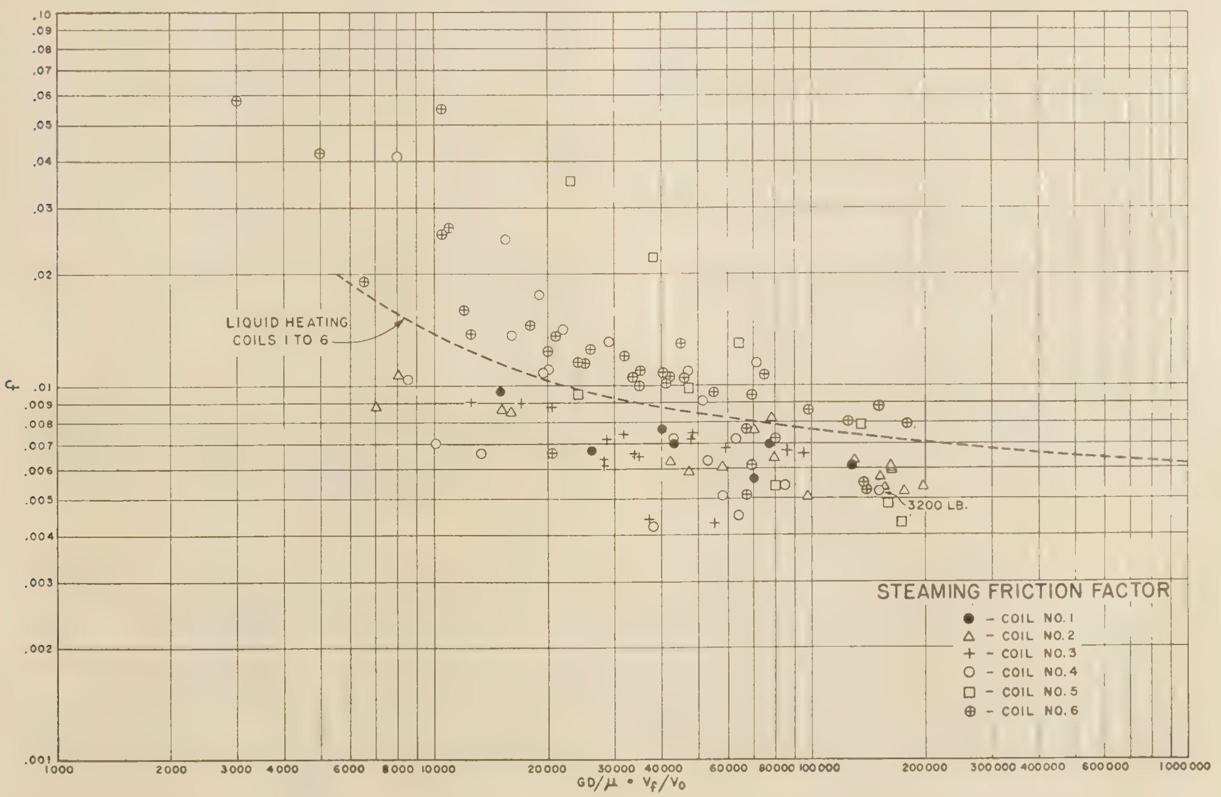


FIG. 37 ALL COILS STEAMING FRICTION FACTOR VERSUS REYNOLDS NUMBER × DIMENSIONLESS PARAMETER V_f/V_0

where Δp = pressure drop across tube
V = velocity
v = specific volume
F = friction loss (head)
z = gravity elevation

In order to calculate dF/v (that part of the observed pressure drop due to friction), each right-hand term must be put in the form of an exact differential and integrated.

For a tube of constant cross section VdV/vg is an exact differential readily integrated to give

$$\Delta p_a = \frac{G^2(v_0 - v_f)}{g} \quad [4]$$

where G = mass flow
v_0 = specific volume at tube outlet
v_f = specific volume at tube inlet

The units are feet, pounds, and seconds.

Δp_a is acceleration pressure and is independent of all error but that of outlet quality.

Expression dz/v represents that part of the observed pressure drop which is due to the summation of all the hydrostatic heads in the spiral and which would be zero if the spiral were full of cold water. This gage correction for a spiral tube in which the volume changes along the tube is easy to formulate analytically and correspondingly difficult to calculate. Its value is not far from that given by

$$\Delta p_g = \sum \frac{l_p}{v_{\text{avg}}} \quad [5]$$

where the indicated summation is the algebraic summation of all the vertical projections of the turns divided by the arithmetic-average specific volume in the turn.

This gage correction varies only with the outlet quality and pressure and has a range of values from 0 to 10 in. of water. On coil No. 5, which is the large tube and which had the lowest observed pressure drops, the magnitude of the calculated correction is in one or two cases equal to the observed pressure drop and therefore these results are less certain. The results used in the calculations of this tube are based on pressure drops at least twice that of the correction. On the smaller tubes, where the observed pressure drops are high, the effects of this correction are negligible but it remains as an inherent defect in the experiment.

Having evaluated Δp_a and Δp_g

$$\Delta p - (\Delta p_a + \Delta p_g) = \frac{dF}{v} = \Delta p_f \quad [6]$$

These observed differentials were thus reduced.

The change of specific volume with pressure at constant enthalpy is negligible for high pressures and small pressure drops. If f is taken as a constant in Equation [1], then for uniform heat absorption it is easily shown[9] that Equation [1] can be expressed as an exact differential which when integrated gives for f

$$f = \frac{2gD}{4l} \times \frac{\Delta p_f}{G^2 v_{\text{avg}}}$$

For the theoretical objections to this type of treatment see discussion by V. J. Skoglund of a paper by W. H. McAdams and others.[10]

[9] "A Method of Estimating the Circulation in Steam-Boiler-Furnace Circuits," by A. A. Markson, T. Ravese, and C. G. R. Humphreys, Trans. A.S.M.E., May, 1942, Appendix 2, pp. 281.

[10] "Vaporization Inside Horizontal Tubes," by W. H. McAdams, W. K. Woods, R. L. Bryan, Trans. A.S.M.E., vol. 63, 1941, pp. 545–551.

A plot of these coefficients against the inlet Reynolds numbers is given in Fig. 36 for coil No. 6. It is evident that for the lower values of Reynolds number the correlation departs considerably from the isothermal. Two possibilities remain, one of which is that the independence of the friction factor and the curvature of the tube, which was demonstrated for liquid flow, is not true when evaporation is taking place. This could not be demonstrated in the 50-ft straight tube because of the magnitude of the gage correction.

The other is that the friction in a tube heated on one side with a change of state occurring in the fluid would depend upon the transverse momentum changes thereby produced. Such a state of affairs possesses *a posteriori* reasonableness based on present general knowledge of friction and heat transfer in the relation to momentum changes.

The dimensionless parameter v/v_{outlet} is chosen to represent this thermodynamic effect. Fig. 36 shows the correlation of coil No. 6 against GD/μ and $(GD/\mu)(v_f/v)$. Finally, it will be noted that on this correlation the high values represent that interpretation of these results can be accomplished in terms of separation of the mixture, bubble slip, etc. This effect, as would be expected, disappears at high values of $(GD/\mu)(v_f/v)$.

Fig. 37 shows the steaming friction factor for all coils plotted against Reynolds number modified by the dimensionless parameter v/v_0. The curve of the friction factor for the liquid heating is superimposed on the data for comparison and indicates that, when the steaming friction factor is modified by the parameter, the correlation is comparable to the liquid-heating friction factor.

4—PRESSURE DROP THROUGH FLOW-DISTRIBUTING EQUIPMENT DESIGNED FOR USE IN FORCED-CIRCULATION BOILERS

During the progress of the tests at Sherman Creek Station, the design for a large forced-circulation boiler was progressing independently. The designers, aware of the high-pressure tests, brought to the authors' attention doubts which existed as to the reconcilability of pressure-drop and flow data from European and Canadian sources. The ease with which data could be obtained with the use of the equipment at Sherman Creek made it advisable to test the pressure drop and flow over a range of high pressures at temperatures only slightly below the corresponding saturation point.

A short section of a distribution header fitted with a removable

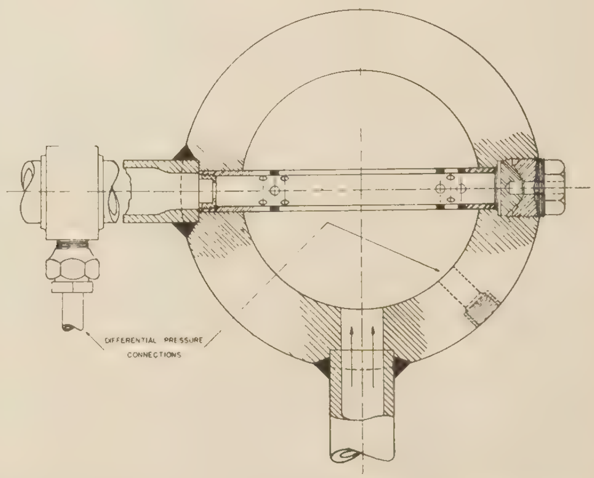


FIG. 38 STRAINER AND ORIFICE ASSEMBLY IN DISTRIBUTION HEADER

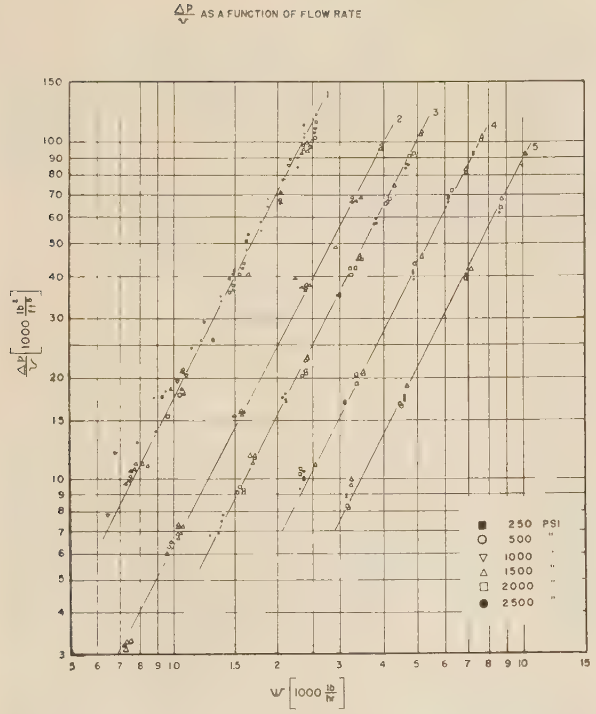


FIG. 39 FUNCTION OF PRESSURE DROP THROUGH ORIFICE VERSUS FLOW RATE

strainer-and-orifice assembly and provided with a side inlet arranged 90 deg circumferentially from the $^7/_8$-in-ID discharge connection, designed for bifurcation as shown in Fig. 38, was supplied by Combustion Engineering Company, Inc. The test section was mounted on the outside of the boiler between the existing connections provided for the interchangeable heat-transfer coils and was thoroughly insulated to maintain the temperature of the fluid at very low flows. Five orifices ($^1/_4$, $^5/_{16}$, $^3/_8$, $^7/_{16}$, and $^1/_2$ in. nominal size), installed at the outlet of the short section of the header, were tested.

The upstream pressure tap was in the header and the downstream pressure tap was several pipe diameters from the orifice. With these differential-pressure connections, the indicated pressure drop (maximum value 18 psi) included the pressure loss in the strainer and the partial recovery of the velocity pressure from the orifice. The data so obtained do not permit computing the coefficient of discharge for orifices, ample information on which is available in the literature. Furthermore, the values of pressure drop versus flow rate are applicable only to the particular arrangement and dimensions of the test section used. While there is no reason to suspect that the flow through this type of orifice assembly should not obey the simple relationship

$$w = k(\Delta p/v)^{0.5}$$

where w = flow rate, lb per sec
k = constant
Δp = pressure drop, psf
v = specific volume, cu ft per lb

in which the orifice is the prime resistance and the Reynolds numbers are high, it was thought worth checking at high pressures and at temperatures sufficiently below the corresponding saturation points so that flashing did not occur.

Fig. 39 shows that $\log_w$ plotted as abscissas against log $(\Delta p/v)$ as ordinates gives a single straight line of slope 2 for each orifice when pressures varied from 250 to 2500 psi and temperatures were from 3 to 15 deg below the corresponding saturation point. The results show that sufficient engineering accuracy may be obtained by testing identical assemblies with low-pressure cold water.

5 HEAT-TRANSFER COEFFICIENTS OF AUXILIARY EQUIPMENT

The readiness with which heat transfer over a wide range of pressure could be measured on the special heat-exchanging equipment used in these investigations led to the determinations reported in this section of the paper.

The heat-exchanging equipment was all designed to fit the conditions imposed by the investigation through the use of existing coefficients or extrapolations. The results of the principal investigation show that the coefficients used were satisfactory. Heat-transfer data were obtained over a pressure range from 500 to 3000 psi on the following heat-exchanging equipment: (1) The calorimeter; (2) the high-pressure heat exchanger; (3) the subcooler; (4) the convection heater.

Except for the temperatures of the cooling water used in the subcooler, which were measured with suitable mercury-in-glass thermometers, water temperatures on the high-pressure heat exchanger, the subcooler, and the convection heater were measured with iron-constantan thermocouples peened into the pipe surface. The accuracy of the peened thermocouples for heat-transfer data was established by comparing the temperatures obtained with the method described in Part 1. The majority of the measurements differed among themselves by 1 deg or were identical. Heat transfer in the calorimeter constituted part of the data necessary for the major investigation and no additional instrumentation was required.

The Calorimeter. The essential dimensions of the calorimeter are given in Table 3.

TABLE 3 ESSENTIAL DIMENSIONS OF CALORIMETER

High-pressure tube:						
Diameters, in.	1.5 OD × 1 ID					
Internal area, sq ft	0.00545					
Jacket:						
Diameters, in.	2.5 OD × 2.125 ID					
Area annular opening, sq ft	0.0124					
Jacket section:	1	2	3	4	5	Total
Length, ft	0.76	1.00	2.03	3.06	4.05	10.9
Surface (based on $1^1/_2$ in. OD), sq ft	0.3	0.4	0.8	1.2	1.6	4.3

During most of the investigation the fluid entering the calorimeter was a mixture of vapor and liquid, while in the remainder the fluid was entirely liquid. Those runs in which only liquid entered the calorimeter are segregated in group 1, Fig 40, and have been designated liquid-heating runs. All runs in which the entering fluid contained some vapor have been arranged in group 2 and have been designated steaming runs. In all cases the fluid discharged from the calorimeter had been cooled to at least 4 or 5 deg F below the corresponding saturation temperature and was, therefore, entirely liquid.

The total heat transferred was determined from the rise in temperature and the flow of the jacket water. The coefficients of heat transfer were based on the outer surface of the inner tube as given in Table 3. For the computation of Reynolds numbers of the liquid-heating runs, the inlet and outlet temperatures were averaged arithmetically and the absolute viscosities at this temperature are from the A.S.M.E. Fluid Meters Report Part 1 (1937). The viscosity of the saturated liquid was used for the steaming runs. For fluid within tubes the Reynolds number was based on the inside diameter; for fluid in the annular opening between the jacket and inner tube, it was based on an equivalent diameter

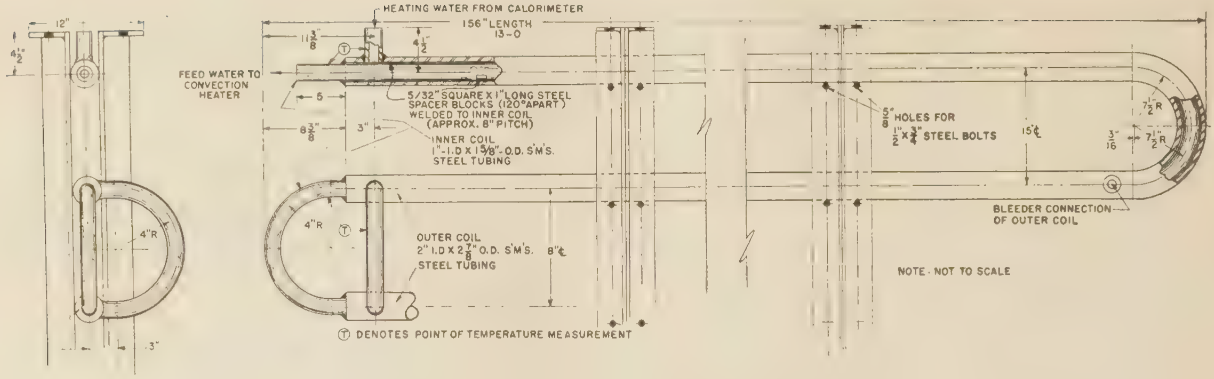


FIG. 41 DETAIL ON ONE LOOP OF HIGH-PRESSURE HEAT EXCHANGER

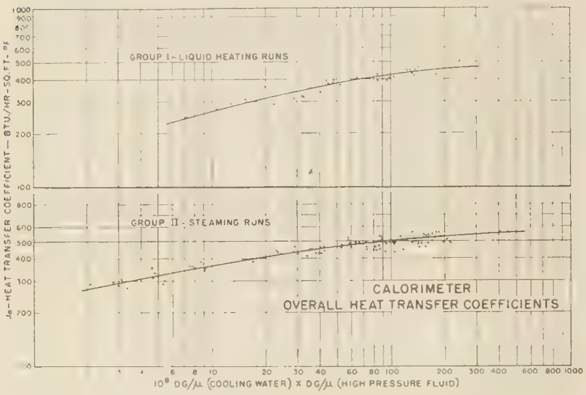


FIG. 40 OVER-ALL HEAT-TRANSFER COEFFICIENTS FOR CALORIMETER VERSUS PRODUCTS OF REYNOLDS NUMBERS

which was computed by means of a formula used by McAdams.[11]

$$D_e = (D_2^2 - D_1^2)/D_1$$

where D_e = equivalent diameter

D_2 = outer diameter of annular opening

D_1 = inner diameter of annular opening

In order to limit the application of the results expressly to the range of values actually used in these investigations of the

TABLE 4 CALORIMETER-PERFORMANCE DATA

		Group
Inlet temperature, high-pressure fluid, F	339-689	1
Inlet temperature, high-pressure fluid, F	467-695	2
Outlet temperature, high-pressure fluid, F	300-613	1
Outlet temperature, high-pressure fluid, F	462-690	2
Inlet temperature, jacket water, F	55-70	1 and 2
Outlet temperature, jacket water, F	100-205	1 and 2
Mass flow, high-pressure fluid, lb per sec per sq ft	82-370	1
Mass flow, high-pressure fluid, lb per sec per sq ft	39-370	2
Inlet quality of mixture	per cent	2
Mass flow, jacket water, lb per sec per sq ft	30-168	1 and 2
Heat pick-up, jacket water, M Btu per sq ft per hr	90-234	1
Heat pick-up, jacket water, M Btu per sq ft per hr	144-314	2
Reynolds number high-pressure fluid	58000-550000	1
Reynolds number high-pressure fluid	43000-512000	2
Reynolds number jacket water	5300-56000	1
Reynolds number jacket water	6500-56000	2

$$\text{Ratio} = \frac{R_D \text{ high-pressure fluid}}{R_D \text{ jacket water}} \text{ (for Group 1)} = 4.1\text{–}13.5$$

$$\text{Ratio} = \frac{R_D \text{ high-pressure fluid}}{R_D \text{ jacket water}} \text{ (for Group 2)} = 3.0\text{–}17.2$$

calorimeter, the upper and lower limits of temperatures, heat transfer, mass flow, and Reynolds numbers are given in Table 4 for both the high-pressure fluid and the jacket water.

[11] "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Co., Inc., New York, N. Y., 1933, p. 235.

In Fig. 40, the over-all heat-transfer coefficient is plotted against the product of the Reynolds numbers of the jacket water and the high-pressure fluid for data groups 1 and 2. While the correlation of the over-all heat-transfer coefficient against the product of the Reynolds numbers is highly empirical, it yields satisfactory results for the particular conditions of operation which prevailed. In Fig. 40, the heat transfer is based on the total outer surface of the inner tube, and the data have not been evaluated for heating surface covered by each jacket section.

It is not difficult to eliminate the conductivity of the metal from the over-all coefficients by use of the resistance concept and thus produce a combined film coefficient for correlation against more rational forms of dimensionless parameters or even against the same parameters. These expedients were not resorted to as they supply no new extension of existing data.

An examination of the complete data indicated that after 350 service hours the heat-transfer coefficient had fallen off about 15 per cent.

High-Pressure Heat Exchanger. This heat exchanger consisted of four identical loops arranged in series, one of which is shown in section with all necessary dimensions in Fig. 41. The temperature-measuring points and by-pass connections for arranging the circuit for one or more loops are also indicated. As stated in Part 1, the heated liquid flows through the inner tube and the heating liquid through the outer tube.

The total heat transferred was obtained from the flow and the heat absorbed by the inner tube although the heat drop in the outer tube checked to within 3 per cent in most cases. The heat-transfer coefficient reported is an over-all figure, based on the outer surface of the inner tube. Table 5 contains the data on computed surface, cross section, range of observed temperatures, mass flows, and Reynolds numbers.

The flow rate of the inner tube was the same at all times as the flow rate of the outer tube and therefore the Reynolds numbers of the inner and outer tubes were always proportional, assuming the absolute-viscosity variation to be small. The coefficients

TABLE 5 HIGH-PRESSURE HEAT-EXCHANGER PERFORMANCE DATA

Inner tube, internal area, sq ft	0.00545
Outer tube, annular area, sq ft	0.00742
Inner tube, outlet temperature, F	381 to 608
Outer tube, inlet temperature, F	400 to 686
Inner tube, mass flow temperature, lb per sq ft per sec	71 to 529

$$\text{Ratio} = \frac{\text{Mass flow — inner tube}}{\text{Mass flow — outer tube}} = 1.36$$

Average heat-transfer rate, Btu per sq ft per hr	12,500 to 70,300			
Reynolds number, inner tube	68,000 to 436,000			
Number of loops	1[a]	2	3	4
Cumulative surface (based on 1 5/8 in. OD), sq ft	10.45	20.90	31.35	41.80
Inner tube, inlet temperature, F	322-536	275-435	226-309	189-222
Outer tube, outlet temperature, F	358-644	310-569	266-466	232-404

[a] This loop is the one shown in Fig. 42 and contains the outlet water connection to the inner tube and the inlet water connection to the outer tube.

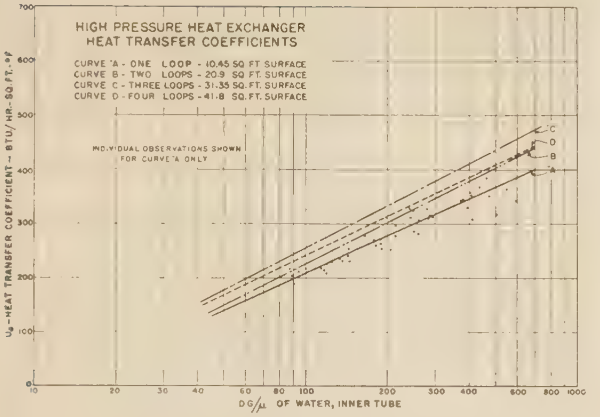


FIG. 42 HEAT-TRANSFER COEFFICIENTS FOR HIGH-PRESSURE HEAT EXCHANGER VERSUS REYNOLDS NUMBER

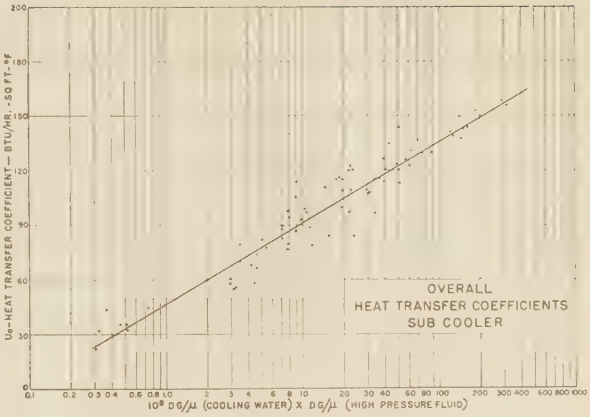


FIG. 43 HEAT-TRANSFER COEFFICIENTS FOR SUBCOOLER VERSUS PRODUCTS OF REYNOLDS NUMBERS

were therefore related to the Reynolds numbers of the fluid in the inner tube. In Fig. 42 is shown the plot of the over-all heat-transfer coefficients for the single upper loop (curve *A*), loops 1 and 2 (curve *B*), loops 1, 2, and 3 (curve *C*), and loops 1, 2, 3, and 4 (curve *D*). In order to avoid confusion, the actual points through which the curves were drawn are shown for curve *A* only.

Subcooler. The subcooler is illustrated in Fig. 14, Part 1, together with its dimensions. Table 6 gives the calculated values used in the heat-transfer calculations. The outer surface of the high-pressure coil was used as the basis for computation of the heat-transfer coefficients. The total heat transfer was obtained from the flow rate and temperature drop through the high-pressure coil. The flow rate of the cooling water was computed from the total heat transferred and the cooling-water temperature rise.

Reynolds numbers of the cooling water and high-pressure water were computed in the same manner mentioned in the calorimeter section. The inner diameter of the shell was assumed to be the diameter of the high-pressure coil at the center of the tubing for the purpose of computing the equivalent diameter.

TABLE 6 SUBCOOLER PERFORMANCE DATA

Surface (based on $1^5/_8$ in. OD), sq ft	80.8
Internal area, sq ft	0.00545
Shell, internal net area, sq ft	0.213
Inlet temperatures, high-pressure water, F	232–491
Outlet temperatures, high-pressure water, F	51–183
Inlet temperatures, cooling water, F	43–70
Outlet temperatures, cooling water, F	63–125
Mass flow, high-pressure water, lb per sq ft per sec	71–529
Mass flow, cooling water, lb per sq ft per sec	7.4–114
Heat transfer, Btu per sq ft per hr	1000–32000
Reynolds number, high-pressure water	7600–340000
Reynolds number, cooling water	4200–94500

$$\text{Ratio} = \frac{R_D \text{ high-pressure water}}{R_D \text{ cooling water}} = 0.48\text{–}5.9$$

Fig. 43 shows the plot of over-all heat-transfer coefficients using the product of the Reynolds numbers of the high-pressure water and the cooling water as the parameter.

Convection Heater. The convection heater contains two gas passes with four heating coils in the inner pass and three heating coils in the outer, concentrically arranged. In Fig. 44 will be seen a flow diagram of the water circuit. In such a setup parallel flow and counterflow occur within the same pass. It will also be noted that the outlet-water connection is at the end of the first pass, which makes evaluation of the inlet-temperature difference difficult without intermediate temperature measurements. Because of the limited scope of the work, it was assumed that the log-mean temperature-difference formulation could be applied.

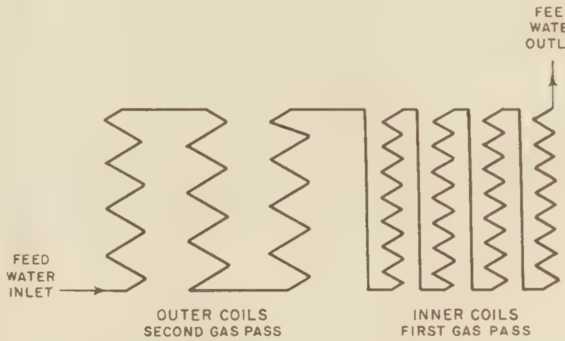


FIG. 44 WATER CIRCUIT OF CONVECTION HEATER

The temperatures measured were the inlet and the outlet water temperatures of the convection heater and the outlet flue-gas temperature. Because of the arrangement of the inlet first-pass duct work, which admitted tempering air into the heater as required, the inlet-gas temperature could not be measured satisfactorily. Therefore, this temperature was computed from the heat absorbed by the water, metered flue-gas flow, and outlet flue-gas temperature.

In Table 7 will be found the essential dimensions of the convection heater, observed data, and computed values.

TABLE 7 CONVECTION-HEATER-PERFORMANCE DATA

Number of gas passes	2
Inner pass, diameter, ft	2.4
Inner pass, length, ft	9.16
Inner pass (annular section), ft	3.75 OD × 2.4 ID
Inner pass, length, ft	8.16
Inner pass, net area, sq ft	2.42
Outer pass, net area, sq ft	4.15
Inner pass, 4 water coils, 12 in. diam., 10 in. pitch, 11 turns each, 2 in. OD × $1^1/_4$ in. ID	
Outer pass, 3 water coils 32,[a] 37, 42,[a] in. diam, 4 in. pitch, 24 turns each 2 in. OD × $1^1/_4$ in. ID	
Coil surface, based on 2 in. OD, sq ft	460
Flue-gas-temperature range, inlet, F	500–1600
Flue-gas-temperature range, outlet, F	380– 620
Water-temperature range, inlet, F	281– 592
Water-temperature range, outlet, F	456– 693
Heat absorbed by water, range Btu per sq ft of coil surface per hr	200–5500
Flue-gas velocity range, inlet, fps	20– 60
Mass flow range, lb per sq ft per sec	17– 325

[a] These coils on same center line.

The curve in Fig. 45 is a plot of the heat-transfer coefficient using the mass flow of the flue gas as the parameter.

The foregoing measurements of heat transfer provide a com-

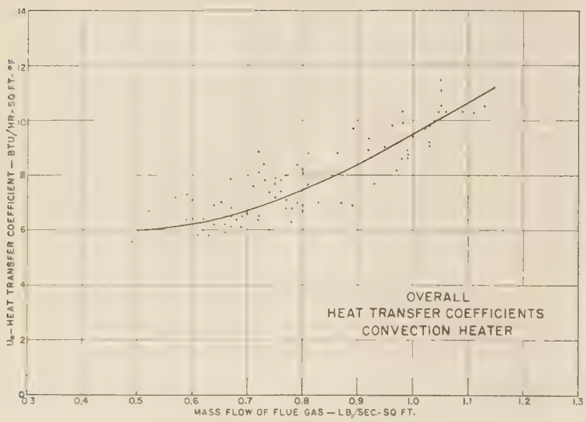


Fig. 45 Heat-Transfer Coefficients for Convection Heater Versus Mass Flow of Flue Gas

parison of three arrangements of liquid-to-liquid and liquid-to-vapor-liquid transfer surfaces. The straight tube, within a jacket constituting the calorimeter and using comparatively cold jacket water, can be compared with the tube within a tube of the four-section U-bend high-pressure heat exchanger using cooling water at higher temperatures. Such a comparison indicates the coefficients are of the same order of magnitude when the parameter is put on the same basis. The subcooler with its coiled inner tube shows low transfer rates due to the low mass flow around the coil. The convection-heater coefficients compare with the coefficients of new commercial economizers.

Acknowledgment

The authors cannot permit this opportunity to pass without expressing appreciation of the invaluable help given by the many individuals and groups who co-operated in the investigations and thereby made them possible. The engineering, construction, and operating department of the Consolidated Edison Company and the operating departments of the New York Steam Corporation were especially helpful and co-operative. Boiler manufacturers gave encouragement and assistance. The Whitlock Coil Pipe Company fabricated much of the special heat-transfer equipment. In addition to the authors, the following individuals took an active part in the collection of data or in its analysis: W. H. Dargan, E. C. Kehoe, H. B. Reed, Jr., and T. J. McGowan.

Appendix 1

DERIVATION OF FIG. 22

Fig. 22 in which the Nusselt-Prandtl function is plotted against the Reynolds number and compared with the correlation given by McAdams is obtained as follows:

Fig. 33 shows for coil No. 6 the over-all resistances $\frac{1}{U_0}$ plotted against pressure. A trend toward lower coefficients at lower pressures is represented by the straight line shown, $\frac{1}{U_0} = A + BP$, which is purely empirical in form and merely states that the resistance decreases with the pressure in approximately this form. By solving this equation for 3000 psi pressure and assuming that the resistance given is virtually that of the metal, the liquid-heating runs for coil No. 6 are then solved for a pseudo film coefficient, using this value of $\frac{1}{U_0}$. However, the value of $\frac{1}{U_0}$ used must include some interface resistance and therefore be high by some degree. The calculated Nusselt-Prandtl function is therefore high and the conclusion that Fig. 22 validates for high pressures and temperatures the generalized concepts of heat transfer is well substantiated. It is of interest to note that for these conditions the Prandtl number is of the order of one.

Appendix 2

CORRELATION OF Δt WITH HEAT TRANSFER, TUBE WALL THICKNESS, AND PRESSURE

In Appendix 1 it was shown, with the aid of Fig. 33 for coil No. 6, that the relation between the coefficient of heat transfer and the pressure could be represented by the empirical equation $1/U_0 = A + BP$. A similar relation was found to exist for all other coils and for the straight tube, although the spread of the points was somewhat greater. For all of the coils and the straight tube, the slope of the lines was essentially the same and corre-

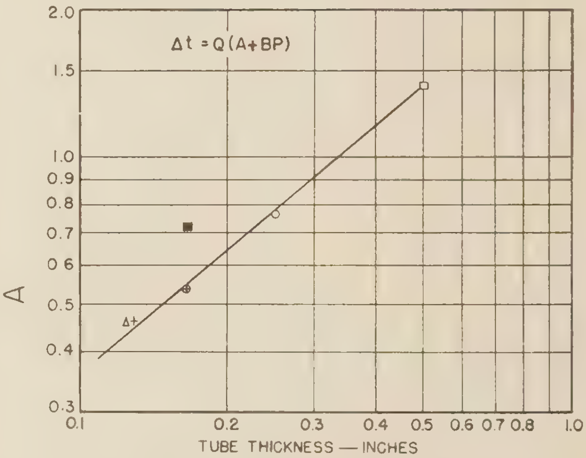


Fig. 46 Values of A Vs. Tube-Wall Thickness

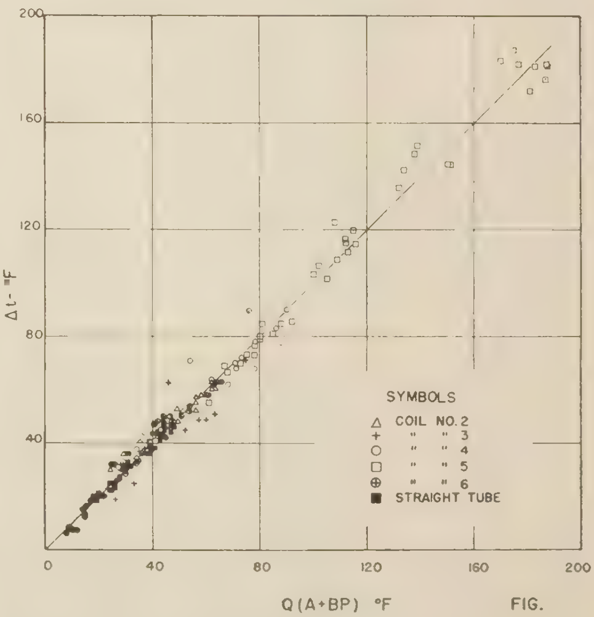


Fig. 47 Correlation of Experimentally Determined and Computed Values of Δt

sponded to $B = -4.0 \times 10^{-10}$. By determining the intercept at zero pressure, the value of A was determined for each coil and straight tube. These values are plotted in Fig. 46 as a function of tube wall thickness.

We have put $1/U_0 = A + BP$ .... [1]

By definition $U_0 = Q/\Delta t$ .... [2]

where Q = heat absorption, Btu per hr per sq ft of projected area (values in Table 2 × 1000)

Δt = average temperature drop (Table 2)

P = pressure, lb per sq ft absolute

Substituting $\Delta t = Q(A + BP)$ .... [3]

where A for a given location of tube is a function of tube thickness, as shown in Fig. 46, and B has the value just stated. For different locations the magnitude of A will change depending mainly on circumferential distribution of heat flow. Note that the value of A for the straight tube which was tested in a water-cooled furnace is above the line passing through the points for the coils which were tested in a refractory furnace.

As a measure of the correlation between values of Δt computed by the use of Equation [3] and observed values, all data have been plotted in Fig. 47. Cases for which the computed and observed values are identical will fall on the 45-deg diagonal line.

Discussion

John Blizard.[12] Two very important facts have been brought out in this paper. One is the wide range of intensity of heat absorption which occurred over a comparatively small area in the particular furnace in which the experiments were carried out, and the other is the high local heating of the wall of a tube. Since the cause of this local heating remains obscure, it is highly desirable that it be investigated further in order to determine the factors which caused it.

For the first time an attempt has been made to show the relation between the rate of radiation on the front of a tube and the temperature of the tube, and velocity and properties of a mixture of water and steam flowing in the tube.

Since over a small area near the front of the tube one may expect but a small variation in the rate of absorption of radiation, the direction of flow of heat through the tube near the front should be almost entirely radial. It is therefore somewhat disconcerting to find, from the experiments with the coils, that the calculated thermal resistance of the tube itself, based on radial flow, is greater than the measured over-all thermal resistance from the wall of the tube to the fluid inside the tube. The thermal resistance of the wall of the straight tube is less than this over-all resistance.

The friction factor for isothermal flow of water in the straight tube was found to be very high, which presumably was caused by some undue roughness of the tube. It is important to know whether this frictional resistance was determined before or after the experiments. If it were measured after the experiments, it is possible that the roughness was caused by the presence of matter deposited on the wall of the tube during the experiment.

G. M. Dusinberre,[13] W. S. Kimball,[14] and H. G. Elrod, Jr.[14] Section 2 of the paper treats of the measured tube-surface temperatures and gives data on variations along the length of the tube. The writers were interested in the circumferential and interior temperature distributions at any section of the tube. As they were unable to find a treatment of this particular heat-transfer problem, it was thought that the solutions obtained might be of interest.

An approximate solution was obtained experimentally, using an electrical analogy. An isothermal interior boundary of the tube and a distant source of radiation were represented by copper conductors. The tube metal and the region in front of it were represented by a sheet of gelatin made slightly conducting. The rear boundary of the tube, assumed nonconducting, was represented by trimming away the gelatin. Equipotential lines in the gelatin then represented isotherms in the tube metal. This apparatus has obvious defects and is mentioned only because, in spite of these, the curves obtained were closely similar to those of Fig. 48 of this discussion.

Two cases were then studied analytically; (1) a tube receiving radiant energy uniformly per unit of projected area, as from a distant source, and (2) a tube receiving radiant energy uniformly about a semicircumference. In both cases it was assumed that interface resistances and external convection and conduction were negligible. The first two assumptions are in line with the authors' conclusions. The third would hold if the refractory backing were practically at tube-surface temperature.

The corresponding mathematical conditions, and the Fourier series arrived at, are given later in this discussion. Making the calculations for radii proportional to those of tubes Nos. 1 to 4 of the original paper, and reducing temperatures to an arbitrary scale of 0 at the inner boundary and 100 at the hottest point of the outer boundary, we get the isotherms shown in Figs. 48 and 49 of this discussion. As symmetry is assumed, only half a tube section is shown.

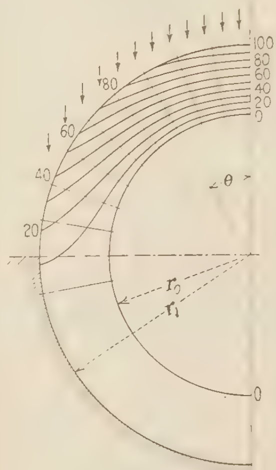


Fig. 48

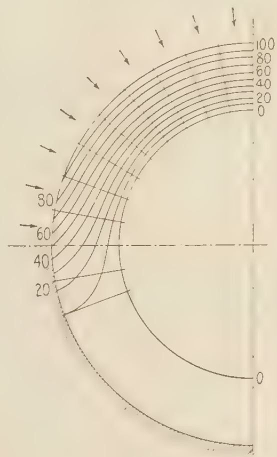


Fig. 49

Each of the cases studied is rather extreme. Any real furnace condition will probably lie between these and, in addition, will be unsymmetrical for most tubes in the furnace.

The significance of these figures, as regards the authors' data, is that asymmetry of the flame may introduce appreciable error, and this always in the direction of making the observed Δt too low. If furnace conditions approached those of Fig. 48, and if the thermocouple were 25 deg eccentric, the measured Δt would be 10 per cent less than the maximum at that section. With the thermocouple on the axis, eccentricity of the flame would introduce a similar error.

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The foregoing is no criticism of the authors' work, since they have explicitly pointed out the many factors entering into their averaged Δt figures and, at the same time, have justified the engineering utility of those figures. It is merely a point to be noted by those who undertake closer analyses of these valuable tests.

Mathematical Conditions of Analytical Studies

The following nomenclature is used:

T = temperature
r = radius
θ = angle measured from tube axis nearest fire
0 = subscript denoting inner boundary
1 = subscript denoting outer boundary
Q = maximum heat flux at r_1
k = thermal conductivity of tube metal

To express the condition of negligible interface resistance at the inner boundary, we have:

$$T_0 = \text{constant } (= 0, \text{ for convenience}) \qquad [7]$$

To express the condition of negligible conduction across the rear outer boundary, we have

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_1} = 0, \text{ for } \frac{\pi}{2} < \theta < \pi \qquad [8]$$

To express the conditions of radiant-energy absorption in Case 1, we have

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_1} = \frac{Q}{k}\cos\theta, \text{ for } 0 < \theta < \frac{\pi}{2} \qquad [9]$$

and in Case 2 we have

$$\left(\frac{\partial T}{\partial r}\right)_{r=r_1} = \frac{Q}{k}, \text{ for } 0 < \theta < \frac{\pi}{2} \qquad [10]$$

Writing Laplace's equation in cylindrical co-ordinates, assuming $T = f(r) \times \phi(\theta)$, and taking account of symmetry and of Equation [7] we get

$$T = (r^m - r_0^{2m} r^{-m})(B_m \cos m\theta) \text{ for } m \neq 0 \qquad [11]$$

$$T = C \log \frac{r}{r_0} \text{ for } m = 0 \qquad [12]$$

Taking account of Equations [8] and [9] and calculating Fourier coefficients we get for Case 1

$$T = \frac{Q}{k}\left[\frac{r_1}{\pi}\log\frac{r}{r_0} + \frac{r_1^2}{2}\left(\frac{r - r_0^2 r^{-1}}{r_0^2 + r_1^2}\right)\cos\theta + \frac{r_1^3}{3\pi}\left(\frac{r^2 - r_0^4 r^{-2}}{r_0^4 + r_1^4}\right)\cos 2\theta - \frac{r_1^5}{30\pi}\left(\frac{r^4 - r_0^8 r^{-4}}{r_0^8 + r_1^8}\right)\cos 4\theta + \frac{r_1^7}{105\pi}\left(\frac{r^6 - r_0^{12} r^{-6}}{r_0^{12} + r_1^{12}}\right)\cos 6\theta - \ldots\ldots\right] \qquad [13]$$

Taking account of Equations [8] and [10] we get for Case 2

$$T = \frac{2}{\pi}\,\frac{Q}{k}\left[\frac{\pi r_1}{4}\log\frac{r}{r_a} + r_1^2\left(\frac{r - r_0^2 r^{-1}}{r_0^2 + r_1^2}\right)\cos\theta - \frac{r_1^4}{9}\left(\frac{r^3 - r_0^6 r^{-3}}{r_0^6 + r_1^6}\right)\cos 3\theta + \frac{r_1^6}{25}\left(\frac{r^5 - r_0^{10} r^{-5}}{r_0^{10} + r_1^{10}}\right)\cos 5\theta - \frac{r_1^8}{49}\left(\frac{r^7 - r_0^{14} r^{-7}}{r_0^{14} + r_1^{14}}\right)\cos 7\theta + \ldots\ldots\right] \qquad [14]$$

F. G. Ely.[15] The authors' ingenious use of thermocouples in the experimental equipment has contributed very materially to analysis and correlation of the data, and to their comprehension of what was taking place within the tubes. Similar application of this tool to the problems of operating furnaces can further extend our knowledge of the many factors, not dealt with in the present tests, which influence the performance and safety of generating tubes.

During the past year, the writer's company has covered some very interesting ground in that direction. A technique of ther-

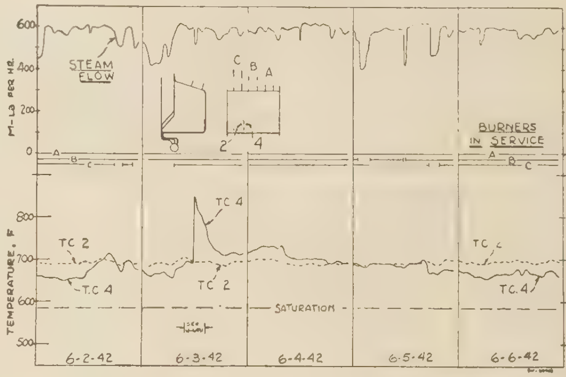

Fig. 50 Log Plot of Tube Temperatures Registered by Two Thermocouples Over Period of 4 Days

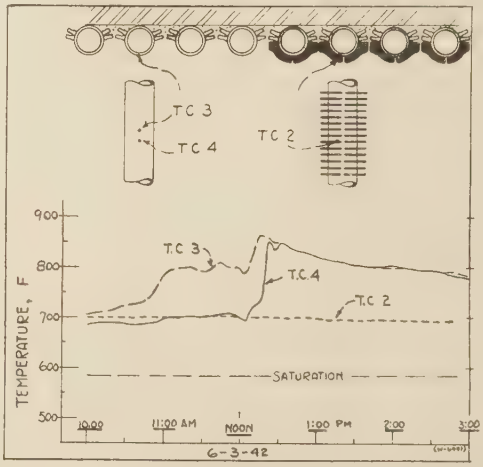

Fig. 51 Details of Wall Construction and Thermocouple Positions

mocouple installation, using chord-drilled holes in the tube wall for protection of lead wires, has resulted in a very satisfactory life of thermocouples, to the extent that there have been no casualties in a period of approximately 10 months to date. The couples referred to are planted in the surface of primary furnace-wall tubes of three slag-tap units and have been connected to recording potentiometers for the sake of obtaining a continuous record. Several excerpts from this record are shown in the accompanying illustrations.

Fig. 50 represents a log plot of tube temperatures as registered by two thermocouples over a period of four consecutive days early in June, 1942. Location in the furnace is indicated in the thumbnail sketch as being in the furnace front wall, approximately 3 ft above the floor, while the respective location of

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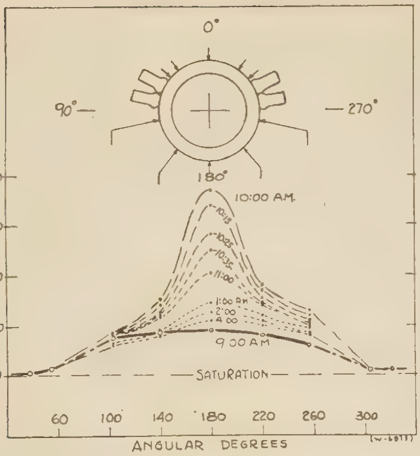

Fig. 52 Locations of Thermocouples in Surface of Tube Wall

burners is indicated by letters A, B, and C. The furnace-wall construction is principally of partial-stud type, having bare-tube surface exposed to radiant heat and gases. A small experimental area of stud plates and chrome ore had been applied to a portion of the front wall, as shown in dotted outline. Thermocouple No. 4 was located in the bare-tube surface of the partial-stud construction, and thermocouple No. 2 was located under the stud plates of a closely adjacent tube. (Wall construction and thermocouple positions are indicated more in detail in Fig. 51.)

The temperature record indicates an abrupt rise of 165 deg for thermocouple No. 4, shortly before noon on June 3, 1942, with a more or less rapid decline in the course of 5 or 6 hr, followed by a further gradual decline to lower temperatures in a period of several days. Throughout this time, thermocouple No. 2 recorded a fairly uniform temperature, at a substantial margin above the saturation temperature corresponding to steam pressure, and showed a general responsiveness to changes in burner operation and other furnace conditions. It will be noted that the Δt values for the bare tube (thermocouple No. 4) are frequently lower than those of the tube with stud-plate covering (thermocouple No. 2), and that its range of variation in Δt is much more extreme.

Explanation of these characteristics is found in the behavior of slag which is deposited upon the wall surfaces exposed to the furnace. On the bare tube surfaces, a beaded or lacy structure of fused-slag droplets and streamers, which may change in pattern or be completely sloughed off in local areas, has been repeatedly observed for the type of fuel and rates of operation applying to this furnace, while on the stud-plate construction the slag coating is consistently thin and adherent over the surface of stud plates and refractory.

Indication of the local effects of slag is shown in Fig. 51, where a repetition of the temperature record for June 3, 1942, has been plotted to an enlarged time scale. The record of an additional thermocouple, TC-3, is included in this plot, the couple having been installed in the same tube at a distance $2^1/_2$ in. above TC-4. A rise in temperature is noted during the hour preceding the abrupt rise of TC-4, and it may reasonably be inferred that creeping or peeling of the lacy slag coating had begun at the upper location, leaving that portion of the tube exposed to the full effects of furnace radiation, before the lower position was affected.

This record indicates the improbability that internal-film resistance played an important part in the rise of Δt for the tube. Occurrences of a similar nature have not been infrequent in the record, and cases have been noted where a rise of TC-4 preceded the rise of TC-3. This would seem to dispel any conjecture that, in the case of Fig. 51, a persistent internal steam film at TC-3 had gradually enlarged to the extent of influencing TC-4.

Further interesting data on the effects of slag, and the changing distribution of temperature around the circumference of a furnace wall tube, are shown in Fig. 52 of this discussion. The diagram indicates locations of ten thermocouples planted in the surface of the tube wall, and observed temperature values are plotted against a developed scale of position, for intervals of time during a 7-hr period in which slag coating of the bare-tube surface had been displaced and gradually restored by normal processes in the pulverized-coal-fired furnace. Beginning with a moderate and uniform temperature differential at 9:00 a.m., there is indicated a rapid increase of 285 deg in Δt value at the exposed face of the tube by 10:00 a.m., when it was observed that slag coating had disappeared from this locality of the tube surface. The succeeding record shows a gradual return to "normal" conditions by 4:00 p.m.

One may hopefully expect that the accumulation of knowledge of this sort, in conjunction with and supplementary to the valuable contribution that the authors have made by their investigations, will ultimately lead to a solution of our major problems in designing for safety and performance of steam-generating units.

Max Jakob.[16] One of the most interesting statements in this valuable paper is that in high-pressure boilers with forced circulation, the film resistance between the tubes and the boiling water is almost negligible compared with the resistance against thermal conduction through the tube wall. If this is true, then

[16] Research Professor of Mechanical Engineering, Illinois Institute of Technology and Armour Research Foundation, Chicago, Ill. Mem. A.S.M.E.

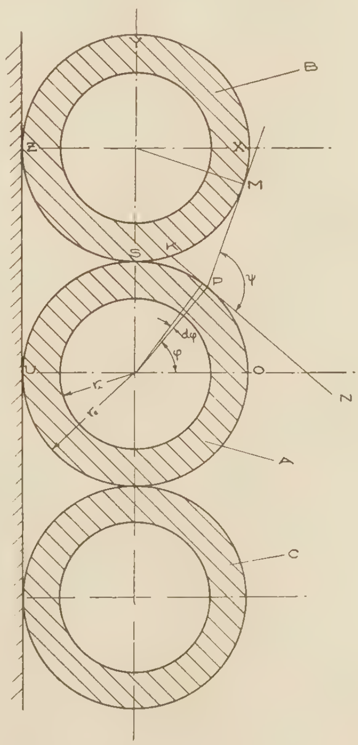

Fig. 53 Notations

the problem of heat transfer resolves into a study of heat conduction. This will be the subject of the present discussion.

Considering a cross section through the tubes (Fig. 53 of this discussion), the heat is assumed to arrive at a steady rate from the right side (furnace side). The left side (wall side) is supposed to be virtually insulated by the boiler wall. Only a moderate fraction of the heat input is due to convection. This may be distributed rather equally on the outer surface of the tube except close to S, where convection will be very small. The main part of the heat energy arrives as radiation which first will be assumed to come from all directions of the furnace space with equal intensity. Then, in unit time, the surface element $Lr_e \cdot d\varphi$ is met by an amount of radiation, $B\omega Lr_e \cdot d\varphi$, where B is a constant, L is the tube length, and ω is the solid angle which encompasses the incoming radiation. Neglecting the reflected radiation coming from the surface KM, it can be proved easily that ω is proportional to the angle ψ formed by the tangents MP and NP. So the distribution of the arriving radiation can be calculated by trigonometry. The graduate student S. P. Kezios did this for the writer, and obtained the following result

$$\psi = \frac{3\pi}{2} - \text{arc sin}\,\frac{2\cos\varphi}{(5 - 4\sin\varphi)^{1/2}} - \text{arc sin}\,\frac{1}{(5 - 4\sin\varphi)^{1/2}} \quad \ldots\ldots\ldots [15]$$

At $\varphi = 0$ the angle becomes $\psi_0 = \pi$. The ratio ψ/ψ_0 plotted versus φ (in Fig. 54, curve I) shows the distribution of the incoming radiation. It is peculiar that at $\varphi = 0$, the derivative $d\psi/d\varphi = 1$ and not zero as might have been expected.

If, instead from all sides, the radiation arrived parallel to OU, however, again homogeneous in intensity, then the time rate of heat, meeting the surface element mentioned, would be

$$dq_{e\varphi} = QLr_e \cos\varphi \cdot d\varphi \quad \ldots\ldots\ldots [16]$$

where Q is the heat energy arriving per hour and square foot of projected area. At $\varphi = 0$

$$dq_{e0} = QLr_e \cdot d\varphi \quad \ldots\ldots\ldots [17]$$

so that the distribution of radiation would be given by the ratio $dq_{e\varphi}/dq_{e0} = \cos\varphi$ (see Fig. 54, curve II).

The difference between the two kinds of distribution is not great. Considering that the influence of the convective heat input tends to flatten the peak of curve I, Fig. 54, and that in the experiments the test coils were placed just opposite to the oil burners, the subsequent calculations have been based on cosine distribution, according to curve II. It will further be assumed that the arriving heat $QL2r_e$ is absorbed at once. This would be exact if the surface were perfectly black. The change in distribution due to reflection will be neglected for the time being. The total amount of absorbed heat, however, will not be influenced by these assumptions because $QL2r_e$ will be taken as equal to the increase of enthalpy of the fluid per hour as measured calorimetrically by the authors. It is as though this energy rate were distributed to the tube wall by sources of heat decreasing in strength as $\cos\varphi$ from $\varphi = 0$ to $\varphi = \pi/2$.

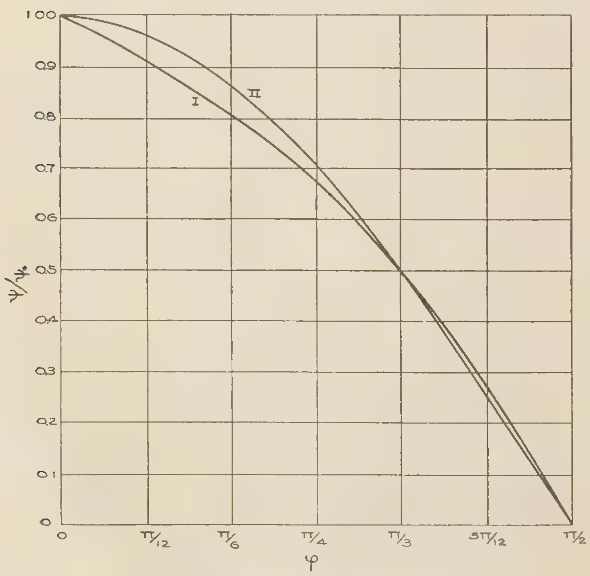

Fig. 54 Distribution of Radiation

The thermal conduction through the tube wall under the steady-state condition is governed by the partial differential equation

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \varphi^2} = 0 \quad \ldots\ldots\ldots [18]$$

where t is the temperature at the radial distance r from the center line and the angular distance φ from the direction UO (see Fig. 53). Prof. Hans Reissner kindly advised the writer that, for a complete circular ring by means of Fourier series in a relatively simple manner, the following solution of the differential equation is obtained

$$t - t_i = A_0 \ln r + B_0 + \sum_{n=1}^{n=\infty}(A_n r^n + B_n r^{-n})\cos n\varphi \quad \ldots [19]$$

The boundary conditions are

$$t = t_i \quad \text{for } r = r_i$$

and

$$\frac{\partial t}{\partial r} = \Phi \quad \text{for } r = r_e$$

where Φ is an arbitrary function of φ, which will be prescribed by the Equations [20] and [21], and t_i is the temperature of the inner surface of the tube. This is slightly above the saturation temperature t_s and is assumed to be constant. Actually t_i will change somewhat with φ, but this may be neglected in the calculation.

The coefficients of Equation [19] are

$$A_0 = \frac{r_e}{\pi}\int_0^{\pi/2} \Phi \cdot d\varphi$$

$$B_0 = -(\ln r_i)A_0$$

$$A_n = \frac{2}{\pi n r_e^{\,n-1}[1 + (r_i/r_e)^{2n}]}\int_0^{\pi/2} \Phi \cdot \cos n\varphi \cdot d\varphi$$

$$B_n = -r_i^{\,2n} A_n$$

Heat can enter the tube only in a direction perpendicular to the external surface; a heat flow in the tangential direction is here impossible because there is no cross-sectional area through which heat could flow under the effect of tangential temperature differences. Only when the heat has entered the solid body it may flow in any direction. For the boundary $r = r_e$ the basic law of heat conduction combined with Equation [16] yields

$$dq_{e\varphi} = QLr_e \cos\varphi \cdot d\varphi = kLr_e \cdot d\varphi \cdot \left(\frac{\partial t}{\partial r}\right)_{r=r_e}$$

or

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_e} = \frac{Q}{k}\cos\varphi \ldots\ldots\ldots\ldots\ldots [20]$$

This holds from $\varphi = 0$ to $\pi/2$, whereas in the range from $\varphi = \pi/2$ to π, according to our assumption

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_e} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [21]$$

Integration of Equation [16] leads to

$$q_{e\varphi} = QLr_e \sin\varphi \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

where $q_{e\varphi}$ is the time rate of heat entering the surface between $\varphi = 0$ and $\varphi = \varphi$. This equation holds only for the quadrant *OPS*.

In the range from $\varphi = \pi/2$ to π no heat is added to the maximum value obtained from Equation [22]. Hence for the quadrant *SU*

$$q_{e\varphi} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots [23]$$

If, as supposed, the temperature t_i of the inside surface of the tube is independent of φ the heat-flow lines will end in the radial direction at $r = r_i$. Therefore

$$dq_{i\varphi} = kLr_i \cdot d\varphi \cdot \left(\frac{\partial t}{\partial r}\right)_{r=r_i} \ldots\ldots\ldots\ldots [24]$$

Substituting t from Equation [19] and performing the differentiation

$$dq_{i\varphi} = kLr_i\left[\frac{A_0}{r_i} + \sum_{n=1}^{n=\infty} n(A_n r_i^{\,n-1} - B_n r_i^{\,-n-1})\cos n\varphi\right] d\varphi \ldots [25]$$

But from Equations [19] and [20]

$$A_0 = \frac{r_e}{\pi}\int_0^{\pi/2} \Phi \cdot d\varphi = \frac{r_e}{\pi}\int_0^{\pi/2} \frac{Q}{k}\cos\varphi \cdot d\varphi = \frac{Qr_e}{k\pi} \ldots [26]$$

Hence by substitution and integration

$$q_{i\varphi} = kLr_i\left[\frac{Qr_e}{kr_i}\cdot\frac{\varphi}{\pi} + \sum_{n=1}^{n=\infty}(A_n r_i^{\,n-1} - B_n r_i^{\,-n-1})\sin n\varphi\right] \ldots [27]$$

where $q_{i\varphi}$ is the rate of heat given up inside between $\varphi = 0$ and $\varphi = \varphi$.

In the limits $\varphi = 0$ and $\varphi = \pi$ Equation [27] yields $2q_i = 2QLr_e$ which is identical with the total heat delivered to the tube from outside.

Denoting by t_{e0} the temperature at the point $O(r = r_e, \varphi = 0)$, a temperature ratio for arbitrary co-ordinates r and φ may be defined by

$$\Psi = \frac{t - t_i}{t_{e0} - t_i} \ldots\ldots\ldots\ldots\ldots\ldots [28]$$

The values of Ψ vary from 0 (for $t = t_i$) to 1 (for $t = t_e$).

The numerical calculation showed that the series in Equation [19] converges quite well. It was sufficient to evaluate the terms for $n = 1, 2, 4$, and 6, those for $n = 3, 5$, and 7 becoming equal to zero.

For the case of $r_e/r_i = 1.5$, the calculation gave the temperature distribution shown in Fig. 55 of this discussion. This figure contains isothermal lines for $\Psi = 1, 0.8, 0.6, 0.4, 0.2, 0.1, 0.01$ and 0. Further, heat-flow lines are drawn from the points $a, b, c,$ d, e, f, g, and h on the outside surface to j, k, l, m, n, p, q, u on the inside surface, respectively. Each of the first four channels, beginning with the symmetry line $a\,j$, carries $1/5$ of the total heat flow, the channel between $e\,n$ and $f\,p$ carries $1/10$ of the heat flow, each of the two last channels carries $1/20$.

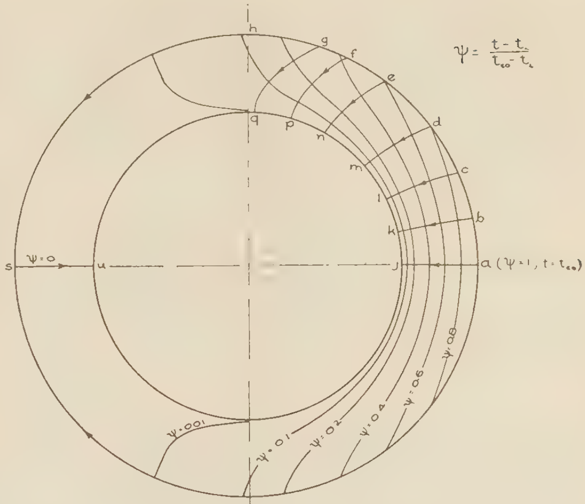

FIG. 55 ISOTHERMAL AND FLOW LINES

The points $a, b, c \ldots$ were determined by means of Equation [22] the points $j, k, l \ldots$ from Equation [27].

From Fig. 55, it is seen that, immediately below the external tube surface, the heat does not flow in a radial direction any more. Obviously, a bend occurs at an infinitely short radial distance from the surface. Since the circumference of the tube is not an isothermal line, the heat flow inside cannot proceed in a radial direction as mentioned.

It is further seen that about 96 per cent of the incoming heat is received in the range $\varphi = 0$ to $\varphi = 74$ deg at the outside and is given up inside in the first quadrant ($\varphi = 0$ to $\varphi = 90$ deg), and only 4 per cent in the rear quadrant. The temperature difference $(t_e - t_i)$ at $\varphi = \pi/2$ is about 12 per cent of $t_{e0} - t_i$ (at $\varphi = 0$).

The authors measured only values of the temperature t_{e0}, and represented the difference $\Delta t = (t_{e0} - t_s)$ for all tests in their Table 2 and Fig. 20 as dependent upon Q. By means of Equation [19] of this discussion, the writer has calculated the difference $\theta_{e0} = (t_{e0} - t_i)$ which should not differ much from Δt. Because all terms of this equation have Φ as a factor and Φ is proportional to Q/k, the result is directly proportional to the measured value Q and conversely to the thermal conductivity k of the steel.

J. W. Donaldson[17] has measured the conductivity of a similar steel as used by the authors, namely, a sample with 0.26 per cent carbon, forged and normalized at 1650 F. The result was $k = 32.4$ B hr^{-1} ft^{-1} F^{-1} at 212 F and $k = 31$ at 750 F. The average tube temperature in the authors' tests was between about 500 and 700 F; therefore, the value $k = 31.4$ B hr^{-1} ft^{-1} F^{-1} has been used for the calculation.

For $\varphi = 0$ Equation [19] becomes

$$\theta_{e0} = t_{e0} - t_i = C_1 Q \ldots\ldots\ldots\ldots\ldots [29]$$

where C_1 is an individual constant for each type of tubing, expressed by

[17] "The Thermal Conductivity of Wrought Iron, Steel, Malleable Cast Iron and Cast Iron," by J. W. Donaldson, *Journal of the Iron and Steel Institute*, vol. 128, 1933, pp. 255–274.

TABLE 8 OBSERVED AND CALCULATED FACTORS OF PROPORTIONALITY C IN B^{-1} HR FT² F

Coil no.	$2r_e$ in.	$2r_i$ in.	10^3C	10^3C_1	10^3C_2	10^3C_3	10^3C_4
2	0.75	0.5	0.333	0.385	0.366	0.302	0.286
3	0.75	0.5	0.333	0.385	0.366	0.314	0.301
4	1.5	1.0	0.64	0.77	0.73	0.60	0.56
5	2.75	1.75	1.29	1.55	1.47	1.22	1.15
6	1.25	0.92	0.427	0.494	0.469	0.418	0.404
Straight tube	1.25	0.92	0.626	0.494	...	...	...

$$C_1 = \frac{r_e}{k} X$$

and X is a function of r_i/r_e as the only parameter.

This corresponds to the authors' representation of Δt as a function of Q by straight lines (Fig. 20 of the paper), which may be expressed by the equation

$$\Delta t = C_0 Q \qquad [30]$$

In the paper, such straight lines are given for the coils Nos. 2 to 6 and for the straight tube. In Table 8 of this discussion, the values of C_0 for these cases are compared with the values of C_1 determined theoretically by means of Equations [29] and [19].

The theoretical values C_1 for the coils are 15 to 25 per cent, as an average 19 per cent, higher than the observed values C_0. Since the individual tests differ up to ±20 per cent and even more from the straight lines of Fig. 20 of the paper, the agreement between C_1 and C_0 might be considered as satisfactory. However, two items have been neglected so far, the consideration of which will change the picture considerably.

A part of the systematic deviation is due to the reflection of the radiation and can be found in the following way:

The tubes are not perfectly black, but have an absorptivity α. Hence, according to Equation [16] of this discussion, a surface element of tube A at angle φ (Fig. 53) reflects the amount

$$dq_A = (1 - \alpha)QLr_e \cos\varphi \,.\, d\varphi$$

The following part of this hits the surface of the neighboring tube convolution B

$$dq_B = \frac{\pi - \psi}{\pi}(1 - \alpha)\, QLr_e \cos\varphi \cdot d\varphi$$

But, according to Fig. 54, approximately

$$\frac{\psi}{\pi} = \cos\varphi$$

Substituting this

$$dq_B = (1 - \alpha)QLr_e(1 - \cos\varphi)\cos\varphi \cdot d\varphi \qquad [31]$$

Integrating in the limits $\varphi = 0$ and $\varphi = \pi/2$, the reflected radiation which hits the tube B becomes

$$q_B = \left(1 - \frac{\pi}{4}\right)(1 - \alpha)QLr_e \qquad [32]$$

The absorptivity of the tubes can be estimated to be $\alpha = 0.7$. Then $q_B = 0.0644\ QLr_e$ is obtained. This means that 6.4 per cent of the heat which arrives in the quadrant $\varphi = 0$ to $\varphi = \pi/2$ will be reflected. Of this, 30 per cent will be re-reflected, partly into the furnace space, and partly to tube A. So it can be estimated that 5 per cent of the primary radiation which meets A will eventually be absorbed by B, and the same amount will be reflected by B and absorbed by A. None of this reflected radiation meets the surface at O. Thus, about 5 per cent of the furnace radiation is selectively absorbed between the angles $\varphi = -\pi/2$ and $+\pi/2$, and only $0.95\,Q$ instead of Q should be used in calculating the temperature excess $\theta_{e0} = (t_{e0} - t_i)$ at the point O. Basing the comparison on the values Q, given by the authors, $C_2 = 0.95\ C_1$ would have to be used for C_1 in Equation [29]. The corrected values C_2, given in the sixth column of Table 8, are still 11 per cent, as an average, greater than the observed values of C_0.

The second item neglected in the calculation is the absorption of heat by the not-shielded parts of the coil surface. In our theory it has been assumed that each turn of the coil A is shielded against radiation from the sides, by neighboring turns B and C. However, the inner and outer convolutions of the coil were exposed to radiation at one side, and the straight end section of the coil was exposed on both sides.

Considering tube B in Fig. 53 of this discussion, as exposed along $SXYZ$, the section YZ may receive less heat than the section SX. Section XY, however, will receive more than SX, and at all events, two quadrants on each exposed side will receive considerably more radiation than supposed in the theory.

In order to take care of this lack of shielding, the writer estimates that to the angle of exposure π on which the theory is based at least $^3/_8\,\pi$ must be added for the lengths of the inner and outer turns, and $^3/_4\,\pi$ for the end lengths. Using the numerical values of these lengths given by the authors, the calorimetrically measured values of Q had to be reduced for the calculation of θ_{e0} from Equation [19] so that C_0 would become greater. Again, instead of this, the authors' values of Q have been retained as a basis of comparison, and therefore the values C_2 of Table 8 have been diminished correspondingly. In this way the values C_3 of Table 8 of this discussion have been found. By adding $\pi/2$ and π, respectively, to π, instead of $^3/_8\,\pi$ and $^3/_4\,\pi$, the values C_4 of the table were obtained. The fact that the observed value C_0 for the straight tube is much higher than that for the same type of tubing used for coil No. 6, and higher than the calculated values of C_1, is partly or entirely caused by the opposite of a shielding effect. Two colder tubes neighbored the test tube so that heat was lost from the section SU of tube A (see Fig. 53) to section SZ of tube B, and more heat was lost from OS to SX than in the case of a coiled and shielded tube.

Therefore in this case an addition to the value of C_1 would be necessary for calculating θ_{e0} from Equation [29], using the authors' values of Q.

The values of C_3 and C_4 for the coils are smaller than the observed values C_0, as an average 6 and 10 per cent, respectively. It will be shown that these differences are not due to a deficiency of the theory, but to the film resistance at the inner tube wall.

A surface element at the inside of the tube receives from the wall and gives up to the liquid the following rate of heat

$$dq_{i\varphi} = kLr_i \cdot d\varphi \left(\frac{\partial t}{\partial r}\right)_{r=r_i} = h_{l\varphi}Lr_i \cdot d\varphi \cdot (t_i - t_s)$$

where $h_{l\varphi}$ is the local film coefficient of heat transfer on the inside at the angle φ. Hence

$$h_{l\varphi} = \frac{k(\partial t/\partial r)_{r=r_i}}{t_i - t_s} \qquad [33]$$

According to Equation [25]

$$\left(\frac{\partial t}{\partial r}\right)_{r=r_i} = \frac{A_0}{r_i} + \sum_{n=1}^{n=\infty} n(A_n r_i^{\,n-1} - B_n r_i^{\,-n-1}) \cos n\varphi \qquad [34]$$

The temperature difference between the inner surface and the boiling liquid, on the other hand, can be expressed by

$$t_i - t_s = \frac{C_0 - C_4}{C_0} \cdot \Delta t \qquad [35]$$

Here it is assumed that C_0 is based on exact measurements and C_4 is the theoretical value, reduced exactly from C_2 in the indicated way.

For $r_e/r_i = 1.5$ and $\varphi = 0$, one obtains from Equations [33] to [35]

$$h_{i0} = \frac{Q}{\Delta t} \cdot \frac{1.5(2.906)}{\pi} \cdot \frac{C_0}{C_0 - C_4}$$

and, using Equation [30]

$$h_{i0} = \frac{1.5(2.906)}{\pi(C_0 - C_4)}$$

For coil No. 4, according to Table 8, $(C_0 - C_4) = 0.08\ (10^{-3})$ so that $h_{i0} = 17{,}300$ B hr^{-1} ft^{-2} F^{-1}.

This is an enormously high coefficient of heat transfer. If C_3 instead of C_4 were assumed as exact, one would obtain $h_{i0} =$ 34,600. Such high values have been observed so far only on the surface of steam bubbles at atmospheric pressure immediately after their formation[18] when they still adhere on the heating surface and are very small. The size at which bubbles leave the surface decreases with increasing pressure, and the wiping effect of forced convection acts in the same sense, namely, reducing the temperature difference $(t_i - t_s)$. Therefore, extremely high local coefficients may actually occur in forced-circulation high-pressure boilers.

It is more usual to consider mean coefficients. For any range, $\varphi = 0$ to $\varphi = \varphi$, a mean coefficient $h_{m\varphi}$ can be calculated by means of the equation

$$\left.\begin{aligned} q_{i\varphi} &= kLr_i \int_0^{\varphi} \left(\frac{\partial t}{\partial r}\right)_{r=r_i} d\varphi = h_{m\varphi} L r_i \varphi (t_i - t_s) \\ &\text{or} \\ h_{m\varphi} &= \frac{q_{i\varphi}}{Lr_i\varphi(t_i - t_s)} = \frac{k}{\varphi(t_i - t_s)} \int_0^{\varphi} \left(\frac{\partial t}{\partial r}\right)_{r=r_i} d\varphi \end{aligned}\right\} \quad \ldots\ldots [36]$$

$q_{i\varphi}$ may be calculated from Equation [27], and $(t_i - t_s)$ substituted from Equation [35].[19]

For $r_e/r_i = 1.5$ and $\varphi = \pi/2$, Equations [36] and [30] lead to

$$h_{m,\pi/2} = \frac{Q}{\Delta t} \cdot \frac{C_0}{C_0 - C_4} \frac{1.5(1.922)}{\pi} = \frac{1.5(1.922)}{\pi(C_0 - C_4)}$$

In the case of coil No. 4 this becomes $h_{m,\pi/2} = 11{,}450$ B hr^{-1} ft^{-2} F^{-1}, whereas using C_3, 22,900 B hr^{-1} ft^{-2} F^{-1} would be obtained.

Finally, relating the film coefficient to the entire inside surface $h_{m,\pi} = \dfrac{1.5}{\pi(C_0 - C_4)} = 5970$ B hr^{-1} ft^{-2} F^{-1} or adopting C_3, 11,940 B hr^{-1} ft^{-2} F^{-1} are obtained.

Values of this order of magnitude are in agreement with what the authors expected, saying: "Rough estimates indicate that the coefficients from metal to boiling fluid can exceed 5000 Btu per hr per sq ft of inside projected area per deg."

So, the present analysis has shown that the authors are correct in their claim that the heat conduction in the tube wall is essential for the heat transmission. However, it is seen that a temperature drop $(t_i - t_s)$ of about 10 per cent of Δt, as an average, occurs at the inside wall. The analysis has further revealed that almost the entire heat energy absorbed by the outer front side of the tube remains in the range of angles $\varphi = -\pi/2$ to $+\pi/2$ and is given up to the liquid in this range. So, the rear side of the tube is not engaged at all in the heat transfer; it acts solely as heat protection to the furnace wall. Therefore the local heat-transfer coefficient on the front side of the inner surface will be several times as great as the mean coefficient related to the entire inner surface of the tube. The exceedingly strong formation of steam bubbles at the front side will cause vehement radial and rotational movements of the mixture in the liquid cross section, and these, in combination with the axial and centrifugal motion in the coil, cause the surprisingly high local heat transfer which the analysis has shown.

Reasons for the lesser transfer in the straight tube are seen in the cooling by the adjacent tubes and in the missing of one of the factors, mentioned, namely, the centrifugal effect of the coils.

[18] "Heat Transfer in Evaporation and Condensation," by M. Jakob, *Mechanical Engineering*, vol. 58, 1936, pp. 643 and 729.

[19] Since $t_i > t_s$ the assumption that t_i is constant all around the tube may not hold any more. However, the vehement heat transfer on the front side goes along with a great transfer coefficient, and the small heat transfer on the rear with a low coefficient so that the assumption $(t_i - t_s) =$ const will not be far from reality.

C. F. Kayan.[20] With unequal distribution of radiant heat on the tube surface exposed within the furnace, the question of the temperature distribution through the tube material assumes an important aspect. Just how maximum values of heat rate are damped out in transmission through the metal, and how the effect is influenced by values of conductivity (itself a function of temperature) and of surface conductances, can be studied only with difficulty by direct thermal experimental means.

Some approach of course is possible through mathematical analysis. But perhaps more readily is a solution possible by means of various electrical analogies, and particularly so under unsteady or transient conditions, where thermal resistance and heat storage are replaced by electrical resistance and capacitance, respectively.

On the basis of some accepted heat distribution over the outside surface, we can visualize setting up the problem by analogy and producing isothermals for the tube metal, whereby the conditions throughout may be studied with considerable convenience. When transient conditions are particularly involved, then the full value of the resistance-capacitance method may be realized and temperatures studied as a function of time.

The same type of studies can be applied to other portions of the boiler structure, including, for example, the thermal behavior of heavy drum shells under changing conditions, as well as of the boiler refractory sections.

Henry Kreisinger.[21] This discussion relates to Fig. 30 of the paper, and particularly to the expression "indicative of steam-blanketing." It seems that the expression is not descriptive of the process taking place at the inside surface of the tube. The lower graph of the figure shows the temperature of the furnace-side surface of the tube above the temperature of the saturated liquid. At an elevation of 18 to 20 ft from the lower end of the tube, the temperature reached about 270 F, whereas at other elevations it was 30 to 70 F. The high temperature is attributed to steam-blanketing. The usual meaning of "steam-blanketing" seems inaccurate in this case. The term has been applied to the condition taking place inside of a nearly horizontal tube heated from above in natural-circulation boiler. It was reasoned that steam being much lighter than water segregates at the top portion of the surface inside of the tube and forms a layer or a blanket between the heated surface and the water.

On the test represented by Fig. 30, a mixture of water and steam was forced through a vertical furnace-wall tube at the rate of 99 psf of cross section per sec. Initially, the mixture contained

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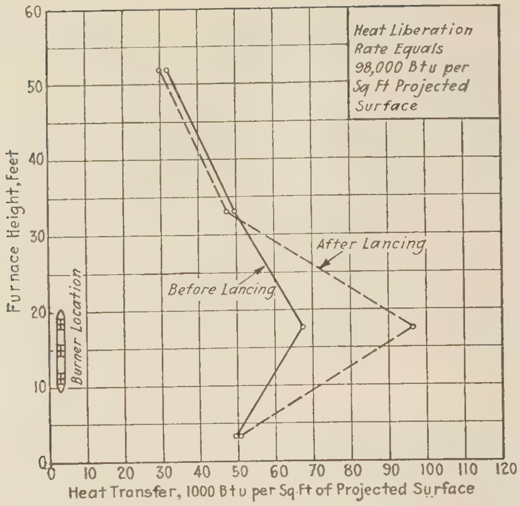

Fig. 56 Effect of Lancing on Heat Transfer

18 per cent of steam by weight which is nearly 62 per cent by volume. At the elevation where the high tube temperature was observed, the mixture contained about 40 per cent of steam by weight or about 83 per cent by volume. At the top end of the tube there was 70 per cent of steam by weight, which is close to 94 per cent by volume. These proportions of the mixture represent the average of the cross section of the tube; near the surface facing the furnace the proportion of steam was undoubtedly higher. The average velocity of the mixture at the three sections was approximately 5.5, 9, and 13.5 fps, respectively. If the term "steam-blanketing" can be applied to a mixture containing an average of 83 per cent of steam by volume, and moving with a velocity of 9 fps, it should be applicable to a section higher up where the steam is 90 per cent by volume and the velocity 12 fps, and where the tube temperature was not high.

The tests on the straight tube were made under conditions which differed in two important respects from the tests made on coils. The straight tube was located in a pulverized-coal-fired boiler and extended the full height of the furnace. The rate of heat transfer varied with the height, being highest 18 to 20 ft from the lower end, where the burners were located, and lowest at the top of the furnace farthest away from the burner zone. The heat transfer was also affected by the thickness of ash layer deposited on the tube by the burning of pulverized coal. On the other hand, the coiled tubes covered a comparatively small area on the rear wall of an oil-fired furnace, and the heat transfer was nearly uniform over the entire heating surface of the coil and was not affected by slag. The high tube temperature in the tests, shown in Fig. 30, was very likely due to a high rate of heat transfer caused by the close proximity of burners and temporary removal of the ash layer at that point.

Fig. 56 of this discussion shows the rate of heat transfer by furnace walls at four elevations in a dry-bottom tangentially-fired pulverized-coal furnace. The data for this figure were obtained by installing on one furnace wall four short elements, connected separately to the water supply, and measuring the quantity of water and its temperature rise. One set of measurements was made before and the other soon after lancing the furnace walls. The four points of each group are connected by straight lines in a similar way as the temperature points in Fig. 30 of the paper. Before lancing, the rate of heat transfer was about twice as high in the burner zone as it was in the top of the furnace. After lancing, the heat transfer in the burner zone was increased about 50 per cent and was about 3 times as high as it was at the top of the furnace.

Fig. 57 of this discussion shows the effect of lancing on the temperature of furnace-wall tubes. The data for this figure were obtained in a slagging-bottom tangentially-fired pulverized-coal furnace. The graph shows the temperature indicated by one of twelve thermocouples embedded in the furnace-side surface of a furnace wall tube 3 in. OD with $^{15}/_{16}$-in-wide fins. The couple was located $13^1/_2$ ft from the bottom of the furnace in line with the top of the burners. The walls were lanced through doors at an elevation of $14^1/_2$ ft from the bottom of the furnace so that the wall area where the couple was located received a thorough deslagging during each lancing period. The lancing was done with a mixture of compressed air and water.

Before lancing, the temperature of the furnace-side surface of the tube was 648 F, about 63 F above the temperature of saturated steam. The tube temperature dropped to 627 F during a part of the lancing period and rose to 781 F, or about 196 F above saturated-steam temperature, immediately after lancing. The temperature drop during the lancing was probably due to the cooling action of the mixture of air and water used for lancing. The temperature rise was due to the greatly increased rate of heat transfer of the deslagged tube. After lancing, the temperature gradually dropped due to deposition of ash and reached the original temperature about 6 hr after lancing. Similar temperature variation occurred with each lancing period. It should be noted that Fig. 57 is plotted on a time basis, whereas Fig. 30 of the paper is plotted on a length-of-tube basis.

The straight tube in Fig. 30 of the paper was in the furnace about a week and during that period was probably covered with ash and cleaned a number of times. The temperature rise indicated by couple 11 might have been due to increased heat transfer after lancing, or the ash might have dropped off by itself. The graph Q, Fig. 30, should have a shape similar to those of Fig. 56, instead of the gentle slope shown. There is no doubt that the rate of heat transfer in the burner zone was much higher than it was in the lower part and in the upper part of the furnace.

When the percentage of steam by volume in the mixture is high, as it was in the test, Fig. 30, an appreciable quantity of heat may be absorbed by the steam by temporary low superheating, because the specific heat of steam near the saturated condition is high. At the point of high tube temperature, the mixture contained an average of 84 per cent steam by volume. The percentage of steam next to the heated surface must have been considerably higher, because the steam made must be moved away from the surface into the stream of the mixture. This lateral movement of the steam is produced to some extent by the expansion of the steam, but mostly by the turbulence due to the axial velocity. As the superheated steam enters the body of the stream of the mixture, it imparts the heat to the water in the

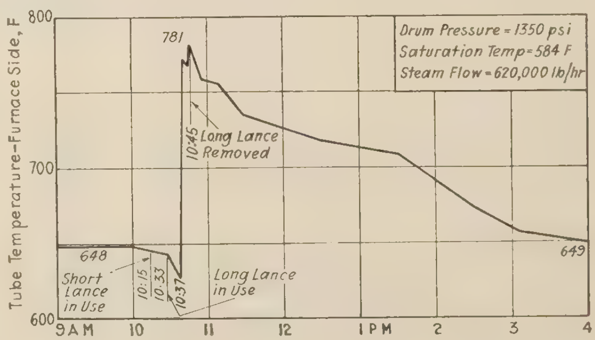

Fig. 57 Effect of Lancing on Tube Temperature

mixture and loses its superheat. The heat transfer constantly tends to increase the percentage of steam at the heating surface, and the turbulence constantly tends to equalize the mixture over the entire section of the tube. However, the equalization always lags behind; the lag is large with slow axial velocity and small with high velocity.

Furthermore, the mixture of high percentage of steam at the heated surface is not replaced with water but with a mixture of lower steam percentage. There is therefore always a considerable percentage of steam at the heated surface which is greater than the average of the cross section. At the point of high tube-metal temperature there was probably not less than 90 per cent of steam against the heated surface. Water will absorb heat from the heated surface with a comparatively small temperature difference; but the steam, due to its lower heat capacity, requires a much higher temperature difference to transmit the heat. There is therefore a tendency for the tube temperature to rise where steam is in contact with the tube. With an increased rate of heat transfer, this rise of tube temperature is more than proportional to the increase in rate, since more heat must be transmitted to a greater percentage of steam in contact with the tube, because of the lag in the equalization of the mixture.

In the case of the straight tube, the flow of the mixture through the tube was somewhat streamlined and there was less turbulence to remove the steam from the heated surface than was the case with the coiled tube where the continuous turning of the flow broke the streamlining. This less intense turbulence in the straight tube would account for the greater temperature difference (Δt), shown in Fig. 20(b) of the paper. For the same rate of heat transfer Δt is about 30 per cent greater with the straight tube than it is for the coil. This greater value of Δt is undoubtedly due to the higher resistance to the flow of heat from the metal into the body of the mixture. The paper refers to this resistance as the "interface resistance."

V. Paschkis.[22] Electrical models for the analysis of heat flow have been in use for a considerable length of time. Langmuir[23] was one of the first to describe and recommend them. Most models are based on geometrical similarity between a body subjected to a temperature field and a body subjected to an electrical potential.

A second type of analysis by means of electrical analogy is available. It is based on the replacement of differential equations by equations of finite differences, which are represented by "lumped" electrical circuits.

The heat (and mass) flow analyzer, which the writer wants to mention here, was first devised by a Dutch engineer, C. L. Beuken, and introduced to this country in extended and developed form by the writer.

In order to apply the method, the body which is to be investigated is visualized as cut into sections. A boiler tube might, e.g., be divided as in Fig. 58 of this discussion. The heat flow in each section is then represented by two resistors according to the two directions of heat flow. If transient conditions are to be considered, an electrical condenser is placed in the center of each section. The method has been previously described more fully.[24]

While the basic problem has been solved by analysis, and by the numerical method, a number of considerations seem to make the electrical-analogy method worth considering.

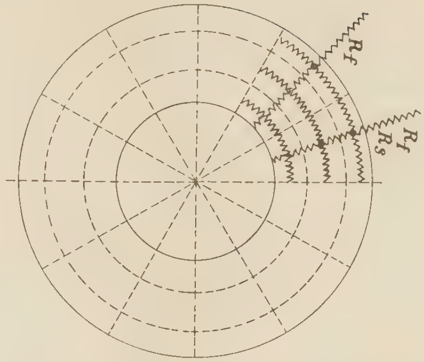

Fig. 58 Schematic Diagram of Tube and Sections

(The thickness of the tube is divided in a number of sections, namely, three in the example. The circumference is also divided into a number of sections, namely, twelve in the example. Two sets of resistors are used, namely, one for radial heat flow, and one for heat flow in the circumference. In the illustration only a few resistors are shown. At the intersection of radial and circumferential resistors, a condenser can be applied to represent the thermal capacity. [To be used only, if transient phenomena are to be studied.] On the outside surface, additional resistors, shown as R_f in the illustration, represent the outside-film resistance [including radiation and convection]. They can be changed during the experiment to represent faithfully the film resistance at the temperatures prevailing at the respective part. Where necessary, additional resistors R_s can be introduced to represent the thermal resistance S of a layer of scale. In case of local formation of scale, such resistors are only applied at the parts of the circuit, representing the places where scale occurs.)

In the discussion, the influence of slag deposit has been mentioned repeatedly. Local or uniform slag deposit can be readily represented on the analyzer by just adding a resistor (see Fig. 58).

It has been assumed that the inside temperature is constant all over the circumference. It is very easy to drop that assumption on the analyzer.

The formation of bubbles will result in a local difference of inside-film conductance.

The thermal conductivity of steel may change or be different at various parts of the wall.

Various loads are to be considered; possibly also, the sudden change of load would be of interest.

All these factors can be readily examined on the analyzer.

W. S. Patterson.[25] The heat absorption by the boiler tubing used in the experiments conducted by the authors was determined by measuring the initial and final conditions of the fluid circulated through the tubes. The heat quantity so determined was then divided by the heating surface of the tubes, in order to arrive at a heat-absorption rate per square foot of surface, which was then tabulated as Q and used in some of the charts and equations for the purpose of correlation. It is the method of evaluating this heating surface which the writer desires to discuss.

One of the most important determinations when dealing with radiant absorption, and especially when making comparisons involving different tube arrangements, is the effective radiant-heating surface. The authors used the projected outside surface of the tubes without any regard to the fact that arrangements were not identical. For example, those portions of the tube coil constituting the inner and outer convolutions and the straight section should be treated in an entirely different manner from that portion constituting the intermediate convolutions. It could be considered proper to evaluate the surface of the latter and of the "straight tube" in the manner used by the authors, but the other portions of the coils were exposed to radiation from the re-

[22] Research Associate, Department of Mechanical Engineering, Research Laboratories, Columbia University, New York, N. Y.

[23] "Flow of Heat Through Furnace Walls," by I. Langmuir, E. Q. Adams, and F. S. Meikle, Trans. American Electrochemical Society, vol. 24, 1913, pp. 53–84.

[24] "A Method for Determining Unsteady-State Heat Transfer by Means of an Electrical Analogy," by Victor Paschkis and H. D. Baker, Trans. A.S.M.E., vol. 64, 1942, pp. 105–110.

[25] Director, Calculating Division, Engineering Department, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

TABLE 9 EFFECT OF SPACING ON EFFECTIVE RADIANT SURFACE OF TUBE

Outside diameter of tube D, in.	1.25				
Total tube surface, sq in. per ft	47.1				
Projected tube surface, sq in. per ft	15				
Tube spacing S, in.	1.25	2.5	5.0	7.5	8.75
Ratio S/D	1	2	4	6	7
Wall area covered, sq in. per ft tube length	15	30	60	90	105
Wall-area evaluation factor (Hottel[26])	1.0	0.89	0.59	0.44	0.382
Effective radiant surface, sq in. per ft tube length	15	26.7	35.4	39.6	39.9
Effective radiant surface/projected tube surface	1.0	1.78	2.36	2.64	2.66
Effective radiant surface/total tube surface	0.319	0.567	0.752	0.841	0.848

TABLE 10 EFFECT OF REFLECTED RADIATION ON EFFECTIVE SURFACE OF COIL NO. 6[a]

Portion of coil	*A-B*	*B-C*		*C-D*	*D-E*
Length, ft	3.7	38.5		8.8	2.3
Projected surface, sq ft	0.385	4.01		0.917	0.240
Total projected surface, sq ft			5.55		
Shielded projected surface	0.1925	4.01		0.4585	none
Effective surface of unshielded portion, sq ft	0.5120	..		1.2200	0.639
Effective radiant surface, sq ft	0.7045	4.01		1.6785	0.639
Total effective radiant surface, sq ft			7.032		

[a] Refer to Table 1 and Fig. 5 of the paper.

fractory wall in which the coils were mounted, and this must be allowed for in evaluating the surface.

H. C. Hottel has shown[26] that, when a row of tubes is backed by a refractory wall, the effective surface of the tubes is not equal to the projected outside surface of the tubes except when the center-to-center spacing is equal to the tube diameter. When the tubes are spaced apart so that heat can pass through to the refractory and be reradiated to the back of the tubes, the effective surface of the arrangement is much greater than the projected tube surface and approaches the full tube surface as the tubes are spaced farther and farther apart. To illustrate these relationships, Table 9 of this discussion has been prepared. This is based on a $1^1/_4$-in-OD tube, because the writer proposes to show that by proper evaluation of heating surface a much better correlation for coil No. 6 and the "straight tube" could have been obtained.

It is shown in Table 9 that, when a $1^1/_4$-in. tube has as much as 3 or 4 in. of refractory to each side and is not imbedded in the refractory, the effective radiant surface will be 2.66 times the projected outside tube surface. Therefore in evaluating certain portions of the coil surface, the authors used too-low values for heating surface because a factor at least as high as the value 2.66 should have been applied to one half the projected surface of the inner and outer convolutions of the coils and to the entire projected surface of the straight section. Table 10 of this discussion shows these corrections applied to coil No. 6, with the final result that the effective heating surface is 27 per cent greater than the projected surface calculated by the authors.

The "straight tube," which is compared with coil No. 6 in Fig. 20 (*b*) of the paper, requires no such surface correction, and therefore correction of the coil surface to a higher value would decrease the value of Q, Btu per sq ft per hr, for the coil, while leaving the values for the straight tube as already calculated. The data points for the coil, if thus corrected, will therefore move to the left and fall in much better alignment with the data for the straight tube.

Even with the foregoing correction, the data for the No. 6 coil will not coincide exactly with the data for the straight tube. This might be attributed to the fact that, in addition to the reflected radiation which reaches the coil and is accounted for as noted, there is considerable radiation from the hot refractory to the back of even the shaded coils. This would not be the case if the coils had covered the entire wall area; but with the arrangement used, it would certainly seem logical to increase the effective radiant surface of the coils even more than indicated in Table 10, and this would bring the data for the No. 6 coil in very good agreement with the data for the straight tube.

Calculations similar to those outlined in Tables 9 and 10 of this discussion can be used for correcting the projected surface of the other coils to the "effective" radiant-surface basis. These calculations will show that the effective surface of the coils is greater than the projected surface used by the authors by the following amounts: coil No. 1, 94 per cent; coil No. 2, 46 per cent; coil No. 3, 36 per cent; coil No. 4, 50 per cent, and coil No. 5, 46 per cent. Each tabulated value of Q would therefore be decreased according to the magnitude of the increase in surface.

In the writer's opinion, the use by the authors of an arbitrary surface for each of the coils was unfortunate when a simple means was available for rational evaluation of the effective surface of the inner and outer convolutions of the coils. The use of effective surface would have helped to explain not only some of the differences noted between the "straight tube" and coil No. 6, but also would have had an effect on the correlation of data for the other coils, since it will be noted that the percentage correction on surface is different for each coil.

E. P. Partridge.[27] The writer wishes to consider particularly the evidence that a condition of "film-boiling" may be established in a vertical waterwall tube. Eighteen years ago, Pridgeon and Badger,[28] discussing some unexpected trends in data from an experimental evaporator, asked the question: "Does this indicate a film so thin and a rate of steam formation so high that the tube begins to be insulated by a film of steam?" Only 5 years ago, Drew and Mueller[29] demonstrated the existence of film-boiling in a series of experiments with seven organic liquids, also summarizing pertinent scattered data from as far back as 1888. Their photographs and those of Sauer and his associates[30] are visual proof that, when an effort is made to increase progressively the heat flow from a metal surface to a boiling liquid, the surface will ultimately become blanketed with a continuous film of vapor.

In the experiments of Drew and Mueller, Sauer and his associates, Rhodes and Bridges,[31] and Akin and McAdams,[32] on film-boiling, heat was supplied internally to a pipe wall from condensing steam or hot mercury. The liquid under investigation was outside the pipe, and the rate of heat transfer was studied as a function of the temperature difference from the tube wall to the boiling liquid. As this temperature difference was increased progressively, the rate of heat transfer first increased, then leveled off at a maximum, and finally decreased. In a typical experiment with a nickel-plated copper tube boiling water at atmospheric pressure, Akin and McAdams found a maximum rate of heat transfer of 370,000 Btu per sq ft per hr, when the temperature drop from the tube surface to the boiling water was 42 F. Doubling the temperature difference to 85 F cut the rate of heat trans-

[26] "Heat Transmission," by W. H. McAdams, first edition, sponsored by the Committee on Heat Transmission, National Research Council, McGraw-Hill Book Company, Inc., New York, N. Y., 1933, Fig. 17.

[27] Director of Research, Hall Laboratories, Inc., Pittsburgh, Pa.

[28] "Studies in Evaporator Design: V—Effect of Surface Conditions," by L. A. Pridgeon and W. L. Badger, *Industrial and Engineering Chemistry*, vol. 16, 1924, p. 477.

[29] "Boiling," by T. B. Drew and A. C. Mueller, Trans. American Institute of Chemical Engineering, vol. 33, 1937, pp. 449–473.

[30] "Heat Transfer to Boiling Liquids," by E. T. Sauer, H. B. H. Cooper, G. A. Akin, and W. H. McAdams, *Mechanical Engineering*, vol. 60, 1938, pp. 669–675.

[31] "Heat Transfer to Boiling Liquids," by F. H. Rhodes and C. H. Bridges, Trans. American Institute of Chemical Engineering, vol. 35, 1939, pp. 73–95.

[32] "Boiling: Heat Transfer in Natural Convection Evaporator," by G. A. Akin and W. H. McAdams, Trans. American Institute of Chemical Engineering, vol. 35, 1939, pp. 137–158.

fer to 52,000, in the neighborhood of one seventh of the maximum.

In these experiments, the temperature difference determined the heat transfer; in the waterwall of a boiler, however, the independent and dependent variables are reversed. As the furnace radiates heat to a tube, the temperature drop from the tube wall to the water boiling within the tube automatically must adjust itself to drive the heat on into the water at the rate it is being received. Any increase in the resistance to the flow of heat away from the metal into the water must accordingly be reflected in an increase in the temperature of the tube wall. This situation parallels the experimental arrangement of Mosciki and Broder, and of Nukiyama, quoted by Drew and Mueller,[29] who employed electrically heated wires immersed in the liquid. These earlier investigators found that, when the temperature drop from the wire to the boiling liquid had been progressively increased, a further relatively slight increase in power input would finally cause such an increase in the temperature of the wire that it frequently would melt.

The possibility that the temperature of a vertical boiler tube might reach excessive levels as a result of film boiling was considered during the study of steam-blanketing of inclined tubes reported 3 years ago by Partridge and Hall[33] but was dismissed because it seemed improbable that the local rate of heat input at the "hottest" portion of a wall tube would cause a continuous film of vapor to develop. Today, as a result of the full-scale studies reported in this paper, the improbable becomes real. Film-boiling must be considered as the probable cause of failure in those cases where wall tubes substantially free of scale and sludge show damage by overheating in the region exposed to the greatest radiation.

As a form of steam-blanketing, film-boiling may lead to any of the types of damage previously described with reference to inclined tubes.[33] The internal surface along the side toward the furnace may be grooved or deeply pitted by the reaction of the water and steam to oxidize the hot steel to black magnetic iron oxide and the removal of this oxide by mechanical action or chemical attack; the tube may suddenly blister and draw out to a thin-edged rupture; or, as a result of a repeated cycle, of establishment of a vapor film with consequent overheating followed by collapse of the film with resultant quenching by the relatively cool water, the tube may fail with a brittle thick-edged fracture of the type so well discussed by Calvert[34] in connection with marine boilers.

Only slight changes in the rate at which water was supplied to the vertical wall tube in the investigation by the authors sufficed to establish or to remove the "hot spot." How many boilers are operating today with wall tubes subjected to repeated quenching as a result of minor variations in circulation is a matter for speculation, but failures attributable to this action may be expected to occur as a result of overload conditions imposed by war necessity.

We should be particularly grateful to the authors for showing conclusively that it is not the relative amount of liquid and vapor in a tube which determines whether or not the tube will overheat but, instead, the mass flow of fluid in relation to the heat input.

W. T. Reid.[35] Even a casual reading of this most interesting paper indicates the extreme care taken by the authors in determining, measuring, and controlling the factors affecting the transfer of heat through boiler tubes; their obviously expensive and unusually complete test equipment indicates an eagerness to obtain data of maximum reliability and usefulness. A detailed study of the paper shows that their efforts to divorce mass effects, such as those occurring in an entire waterwall, from the action taking place in a single tube have yielded data of tremendous value for future design or investigational purposes. It is particularly fortunate that the authors saw fit to include in Table 2 a summary of the data obtained in this study; it is certain that these data will be utilized further in the future in related investigations.

The development of the parameter of the ratio of the rate at which steam is generated in a tube to the mass flow in the tube offers interesting possibilities. The authors' suggestion, that when this parameter exceeds 9.5 danger of steam-blanketing and resultant abnormal tube conditions exists, indicates that with foreknowledge of the rate of heat transfer to the tube, relatively simple relationships fix the mass flow necessary to prevent steam-blanketing and possible damage to tubes. Unfortunately, however, in existing boilers, measurement of the rate of heat transfer to individual tubes is accomplished with difficulty, although field investigations of heat absorbed in various zones of pulverized-coal furnaces have been made. Measurement of tube temperatures by specially installed thermocouples, such as described in this paper, will not serve to permit estimation of heat-transfer rates, because, as shown in Fig. 30, tube temperatures can increase 180 F or more at the higher values of the "safety" parameter without being accompanied by increased transfer of heat. Therefore, in actual installations, utilization of the parameter developed in this investigation may be useful under certain conditions in explaining actions leading to tube failures, without being able to predict the occurrence of failure unless essential information on mass flow and steam quality in the tube is available.

The use of an oil-fired "test" furnace for this investigation is obvious; the extremely low ash content of the fuel assured clean and reproducible heat-absorbing surfaces, and the authors' deductions on heat transfer by radiation to their tubes are correct. However, in all actual installations where coal is the fuel, ash or slag accumulations on the tube surface will generally prevent the transfer of heat from furnace to tube from obeying the fourth-power law of radiation. Rather, the transfer of heat from furnace to the outer slag surface will follow the radiation law, whereas further transfer from slag to tube will be fixed by the thermal conductivity of the slag. Because the thermal conductivity of slag is low, a large proportion of the thermal gradient from furnace to water will occur across the slag layer. It is suggested that the physical condition of the slag or of the external scale on the outer tube surface may be taken as a rough measure of the heat transfer through the tube-slag combination when studied in conjunction with the actual tube-metal temperature obtained by embedded thermocouples; the composition of the slag must be known, of course.

By means of data already available, it is possible to predict temperatures at which certain physical changes take place in slag deposits; thus under favorable conditions the slag may be a gage of the rate at which heat is being absorbed by the tube. Visual examination of the slag deposit on the tube, knowledge of slag properties, and temperatures of the outer tube surfaces possibly could be combined to indicate the distribution of heat along the tube, and therefore the parameter at various points along the tube if the mass flow within the tube were known. Conversely, knowing the distribution of heat along the tube and the tube temperature by embedded thermocouples, so that the parameter could be estimated possibly by experience with test installations,

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[33] "Attack on Steel in High-Capacity Boilers as a Result of Overheating Due to Steam Blanketing," by E. P. Partridge and R. E. Hall, Trans. A.S.M.E., vol. 61, 1939, pp. 597–621; discussion, vol. 61, 1939, pp. 621–622; vol. 62, 1940, pp. 711–717.

[34] "Factors Influencing the Failure of Naval Boiler Tubes," by A. P. Calvert, Journal, American Society of Naval Engineers, vol. 51, 1939, no. 1, pp. 1–34.

[35] Acting Supervising Fuel Engineer, Bureau of Mines, Central Experiment Station, Pittsburgh, Pa. Mem. A.S.M.E. This discussion is published by permission of the Director, Bureau of Mines, U. S. Department of the Interior.

the mass flow in the tube could be calculated, making unnecessary any pitot-tube or nozzle installations for direct measurement.

The authors do not explain satisfactorily the anomaly evidenced in Fig. 19, for coil No. 5, wherein the maximum metal temperatures were observed at the bottom of the turns and the minimum metal temperatures at the tops of the turns. It is difficult to conceive of a coil less than 4 ft diam receiving heat by radiation in an uneven manner, unless it is mounted near the top of a very small furnace or near the bottom of a furnace with an overheated hearth; no probing of the wall of the test furnace for transfer of radiant heat is reported. Had vertical variation in radiant heat been responsible, it could have been corrected by inclining the test coils at such an angle that the radiation-angle factors were equalized. If no appreciable vertical variation in radiant heat actually was present, then an unexplained action is occurring within the coiled tubes.

Authors' Closure

We are grateful to the discussers for the contributions they have made to a fuller interpretation of the significance of our data and for additional information they have provided. Several questions have been asked which would require further testing to answer. Such work must be left for the future.

Mr. Blizard's comment that the over-all measured thermal resistance of the coil is less than the calculated resistance of the tube wall need not be too disconcerting. Similar calculations were made by the authors and a few instances were found in the curved tube to which Mr. Blizard's comment would apply. On the straight tube the results were in closer agreement, it is true, but it is the difference between the curved tube and the straight tube which may be significant because on one tube the amount by which the calculated thermal resistance differs from the measured value could easily be accounted for in the assumption of the value of the thermal conductivity of the iron or the exact surface to use.

Mr. Blizard's question on the isothermal friction in the straight tube can be answered by stating that the tube was rough and slightly incrusted when examined. The experiments on friction were run during the heat-transfer experiments, and therefore it may be concluded that the straight tube tests were run on a tube of more than commercial roughness.

The discussions of Dusinberre, Kimball, and Elrod, of Jakob, and of Patterson, should be answered jointly as they bear on the same subject. Dusinberre analyzes the temperature distribution in a tube heated on one side under two radiant assumptions. The acceptance of the authors' conclusion, that for certain ranges of tube operation the film coefficients are virtually negligible, releases a troublesome boundary condition and enables them to arrive at the noteworthy and straightforward analysis of the distribution of temperature in the metal. This analysis brings out the important point that discrepancies in results based on tube temperature measurements, such as noted by Blizard, may be accounted for partially by the eccentricity of the flame, to use their expression. It happens that some preliminary measurements which were made on the side of the curved tube showed that the distribution of temperature in this particular tube tends to indicate a distribution such as predicted by Fig. 48 rather than Fig. 49 of their discussion.

Dr. Jakob's discussion deals with the effect of the shielded and unshielded portions of the tubes on the relationship of the Δt measured by the authors to a mean integrated value of Δt which was not measured. This relationship is affected by the particular radiation mechanisms which exist. His analysis leads initially to a distribution very similar to that given by Dusinberre within the assumptions of each discusser. Next he applies further analysis to determine the effect of reradiation and shielding. The analysis then leads to an estimation of the local coefficient of heat transfer at the point where Δt was measured (for coil No. 4) as of the order of 17,000 to 34,000 Btu per square foot, indeed a very high figure to be accounted for, not only by taking into account the factors of bubble size and turbulence, but also the rather remarkable change of physical properties of steam and water at high pressures. Finally, his calculated over-all mean coefficients (outside surface to fluid) of from 6000 to 12,000 may be taken as an indication that the results of the authors' investigation, when submitted to scientific scrutiny, are not at all invalidated as engineering data.

The discussion of Patterson deals with the effect of shielding upon the exposed tube surface. The statement that the authors used the projected surface without regard to the arrangement of surface is not well founded. The authors do not believe that Hottel's method of correction is applicable because all coils were embedded in fire clay up to the center line of the tube in the plane of the coil. Even the straight portion at the outlet end was similarly protected when each coil was installed although this part of the fire clay often broke off before the last test was completed as can be seen in Fig. 7. Furthermore, the data indicate that the effective surfaces for all coils were proportional to their projected areas without any such corrections to the different coils as Mr. Patterson has computed, namely, 27 to 94 per cent. Although his correction to coil No. 6 would make its Δt's more nearly comparable with those obtained for the straight tube, similar correction to the other coils would destroy the good correlation obtained for all coils as shown in Fig. 46. It is hard to believe that this was just a chance occurrence. Might not the difference between coil No. 6 and the straight tube be better explained by a difference in the heat-flow pattern between tubes in a water-cooled furnace and those in a refractory furnace?

Ely emphasized the important effects of slag and ash accumulations on the tubes and points out how localized these effects may be (Figs. 51 and 52, for example). Unfortunately, neither our methods nor his provide a direct measure of the influence on heat transfer. We can deduce certain values if we assume that the average film coefficient at the inner tube surface has not changed, but just here we reach the place where we are still without proof. The difference between our explanation of the performance shown in Fig. 30 and the explanation of Kreisinger and the discussion of Partridge is evidence of that. If in an actual boiler the radiation of heat is at a certain equilibrium value consonant with the amount of slag in the boiler and deslagging uncovers a small portion of bare tube, the rate of heat absorption to this surface will increase considerably because the local receiving surface may have had its temperature lowered by perhaps 1000 degrees below that of adjacent slag-covered surface. In that case Δt as observed by Ely may go up in correspondence with the rate of heat absorption which is a perfectly understandable thing. In fact, the paper affords a means of discrimination between such behavior and that of an overheated tube which is simple enough, for if the observed rise of Δt in a furnace tube is proportional to the readily calculated increase of radiant absorption everything can be assumed to be normal. Should, however, the rise be considerably higher than this, the limiting value of φ has been exceeded. The point is so important that furnace designers will have much to think about in this connection.

Kreisinger's discussion deals with the effect of slag on Δt. He points out that the high temperature point on Fig. 30 was probably caused by slag. This is actually a misconception for the abnormalities in Δt reported were always controllable by varying the flow through the tube. This is also true of Fig. 32 and is also shown on the coils in Figs. 23, 24, and 26 in terms of

over-all heat-transfer coefficients plotted against φ. His discussion of steam-blanketing is interesting and should be carefully noted. Thus, he does not see why, if a tube is overheated at a point where the velocity is high and the mixture is 83 per cent by volume of steam, it should not be "blanketed" at a point further up where the velocity is still higher, but the percentage of steam is also higher. The answer, of course, is that the term "steam-blanketing" is too loose and has no meaning as applied to a tube as a whole. We should not apply such a term in the sense of its being a physical property of the tube at some particular time. It is the particular relationship of mass flow and heat absorption existing at a point or zone which seems to govern. Reasoning which omits either of these two factors is naturally incomplete.

The discussion of Partridge is concerned with the question of whether a heat-transfer mechanism involving such tremendous coefficients as indicated by the authors and calculated by Jakob can break down. In the literature cited by Partridge from the field of chemical engineering, this possibility seems to be an established fact. Since, as Partridge points out, the present experiments produced similar results when the ratio of heat absorption to mass flow passed the parametric limits defined in the paper, it would be very important to check the parameter "φ" given in the paper with experiments made on other fluids to either confirm, restrict, or disprove the generality of this function. Here is an opportunity for theoretical workers in this field to test the validity of φ using the data which are cited by Partridge. Partridge calls film boiling a form of steam-blanketing.

Reid calls attention to the apparent anomaly in Fig. 19 for coil 5 in which the highest Δt values are observed at the bottom of the coil. This must be due to eccentricity of the radiation since as in Fig. 3 the bottom of the coil was closer to the refractory floor than the top. Fortunately, the tests run with only liquid heating in the tube show the same characteristic distribution of Δt as when steaming is taking place. Thus, the conclusion that it is distribution of radiation rather than separation, etc. of the mixture in the tube cannot be seriously questioned.

This brings us to a comment on the discussions of Kayan and Paschkis. The methods for the solution of transient-heat-flow problems are intriguing, but until we know far more about the details of what happens at the solid-fluid interface and rather more about the conditions on the outer surface of the tube, such methods seem rather restricted.

Finally, we should like to call attention to the third paragraph of part 1 of the paper. Laboratory-scale studies will be necessary to answer many of the questions raised by our discussers and in the minds of designing engineers; we believe that our "reconnaisance" has developed a map that will show the boundaries of the critical zones. We have shown good agreement between results with large and small tubes covering a wide range of experimental conditions; we have established a rough criterion for predicting the conditions under which tube "overheating" may be expected. With this background, new experimental work can be planned for the laboratory or the field with a much greater assurance of ultimate success. It is our earnest hope that others will soon be able to carry on from here. The paper has stimulated interest in and work on problems of furnace-heat transmission as evidenced by the extensive program of research work on this and related subjects being prepared for presentation before the Society.

Influence of Nonuniform Development of Heat Upon the Temperature Distribution in Electrical Coils and Similar Heat Sources of Simple Form

By MAX JAKOB,[1] CHICAGO, ILL.

The maximum temperature in an electrical coil of simple shape can be found by measuring the surface temperature and the increase of the electrical resistance under load which yields the mean temperature. This well-known method is based upon the assumption that a coil is a body in which heat is developed almost uniformly all over the volume. But, owing to the increase of the electrical resistance with temperature, more heat per unit volume is developed in the warmer sections of the coil than in the colder ones. The purpose of the present paper is to find the influence of this lack of uniformity in some simple cases. It is further shown how for some usual shapes of coils the maximum temperature can be found approximately from the equations derived for certain simple shapes, and the results for such a case are compared with experimental results taken from literature. The results of the paper refer, but are not restricted to electrical coils. They can be used also for exothermal chemical reactions in cases where steady-state and linear increase of the heat of reaction with temperature can be assumed, and the method can be extended to endothermal reactions. The application to the cases of plate, hollow cylinder, and hollow sphere with different temperature at the two free surfaces will form the subject of another paper.

1 INTRODUCTION

An electrical coil is a rather inhomogeneous body because it is built up from conducting and insulating materials. However, considering equal volumes of such size that several or many layers of these materials are included, it can be assumed that in each volume the same Joulean heat is developed if the resistance of equal lengths of the conductor is the same all through the coil.

The heat developed in such a body is conducted to the surface as in a homogeneous medium whose thermal conductivity is equivalent to that of the mixture of materials in the coil. In a steady state of heat development and heat flow (the only one to be considered in this paper) the temperature t decreases from a maximum value t_0 somewhere inside the coil to lowest values t_s at the surface. It will be assumed that t_s is uniform all over the coil surface. This holds closely for a coil in an oil bath or in a gas stream and can be used as an approximation in numerous other cases.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

The mean temperature of a coil is defined by

$$t_m = \frac{1}{V}\int_v t \cdot dV \qquad [1]$$

where the integral is to be taken over the total volume V of the coil.

Vidmar (1)[2] seems to have observed first that in practical cases the ratio

$$\varphi = (t_m - t_s)/(t_0 - t_s) \qquad [2]$$

varies only from $^2/_5$ to $^2/_3$. He suggested using $\varphi = 2/4$ for the calculation of t_0 from t_s and t_m. This proposal is important because t_0 is essential for the durability of a coil, but, unfortunately, cannot be measured in a simple way. The temperature t_s, on the other hand, can be determined directly by thermometers or thermocouples, and t_m by the usual measurement of the electrical resistance of the coil, cold and under load.

Rogowski (2) proved that uniform heat development in an infinitely wide plane plate, an infinitely long cylinder, or a sphere leads to $\varphi = 2/3$, $2/4$, and $2/5$, respectively.

For infinitely long coils of rectangular cross section (side lengths $2a$ and $2b$) the author (3) derived the values of φ in Table 1.

TABLE 1 TEMPERATURE RATIO φ FOR COILS OF RECTANGULAR CROSS SECTION

b/a =	∞	10	5	2.5	1.5	1
φ =	2/3	0.625	0.58	0.52	0.485	0.475

Only later the author became aware that Humburg (4) had already dealt with this case in a more general, however different, way.

Considering the temperature distribution in a coil, the temperature differences $\theta = t - t_s$; $\theta_0 = t_0 - t_s$, and $\theta_m = t_m - t_s$ will be used for convenience and simply called temperatures, as though t_s were the zero point of the temperature scale. Then according to Rogowski

$$\theta = \theta_0(1 - x^2/s^2) \qquad [3]$$

where θ is the temperature at a point inside the coil, and x and s are the perpendicular distances of this point and of the surface, respectively, from the median plane, center line, and center of the infinitely wide plane, infinitely long cylinder, or sphere. A common proof of this equation for the three kinds of bodies has been given by the author (3).

The values of φ, just given, as well as Equation [3], are for uniform development of heat in the coil. But heat is not developed uniformly in an ordinary coil. Consider a copper coil with t_s =

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

$25C$ and $t_0 = 125C$. The electrical resistance of unit length of the conductor at the temperature t is

$$R' = R_s'(1 + \epsilon\theta) \quad [4]$$

where R_s' is that resistance[3] at the temperature t_s, and ϵ is a temperature coefficient which for pure copper and $t_s = 25C$ equals $0.004C^{-1}$. Hence $R_0' = 1.4\ R_s'$ at the temperature $t_0 = 125C$. But according to the laws of Ohm and Joule, the heat developed by an electrical current is proportional to the resistance. Hence in the present case 40 per cent more heat per unit volume is produced at the warmest place inside the coil than just below the surface.

It seems that the influence of this inhomogeneity of heat development has never been investigated. It can be expected that t_0 will be much greater and φ will be smaller than at uniform heat production. The purpose of this paper is to determine this influence for the infinitely wide plane plate, the infinitely long cylinder, and the sphere. The results will allow a fair estimate for the practically most important case of a coil with rectangular cross section.

It will be seen that the influence on the temperature distribution is surprisingly small, and the influence on the maximum temperature t_0 is enormous; in fact, t_0 tends to become infinitely great at a rather moderate electrical current in the coil.

The derivations and their results are not restricted to electrical coils, but hold as well for the nonuniform heat development due to chemical reactions in such a range where the heat of reaction can be considered as a linear function of the temperature. They could also be extended to inhomogeneous cooling as occurring in endothermal reactions. However, steady-state conditions with negligible influence of convection exist only seldom in chemical processes. Another possible application concerns the temperature distribution in the oil film of bearings, the importance of which has recently been emphasized (5, 6).

2 THE PLANE PLATE

A long, hollow, cylindrical, electrical coil, whose inner radius is great compared with the radial thickness, behaves closely as an infinitely wide plane plate. If the length of such a coil is 10 times the thickness, then according to Table 1, φ is only about 4 per cent smaller than for infinite length. Therefore the case of the infinitely wide plane plate has considerable practical importance.

Let x = perpendicular distance of a point of the plate from its median plane
$\theta = (t - t_s)$, temperature at this point
k = equivalent (apparent) thermal conductivity of plate material
q''' = rate of heat energy developed in unit volume[3]

Then the temperature distribution under steady-state conditions will be found from the well-known differential equation

$$\frac{d^2\theta}{dx^2} = -\frac{q'''}{k} \quad [5]$$

k does not vary much with the temperature. The change of q''' can be considered as linear with temperature, at least in the temperature range occurring in electrical coils. Both changes can practically be represented by

$$\frac{q'''}{k} = m + n\theta = m\left(1 + \frac{n}{m}\theta\right) = m(1 + \epsilon\theta) \quad [6]$$

where m and n are constants, and $\epsilon = n/m$ is a temperature coefficient. Physically these constants are defined as $m = q_s'''/k_s$ and $n = \epsilon q_s'''/k_s$ where the subscript s refers to surface temperature $(\theta = 0)$.

Since $q''' \gtreqless 0$, $m \gtreqless 0$, and $n \gtreqless 0$; the latter can be assumed because in general the change of q''' with temperature exceeds that of k.

From Equations [5] and [6]

$$\frac{d^2\theta}{dx^2} + n\theta = -m \quad [7]$$

For $n > 0$, the reduced equation

$$\frac{d^2\theta}{dx^2} + n\theta = 0 \quad [8]$$

has the solution

$$\theta = M \cdot \cos(x\sqrt{n}) + N \cdot \sin(x\sqrt{n}) \quad [9]$$

where M and N are constants of integration.

A particular solution of Equation [7] is

$$\theta = -m/n \quad [10]$$

Hence its general solution is

$$\theta = -\frac{m}{n} + M \cdot \cos(x\sqrt{n}) + N \cdot \sin(x\sqrt{n}) \quad [11]$$

For reasons of symmetry, only the interval from $x = 0$ to $x = s$ needs consideration where $2s$ is the thickness of the plate.

From Equation [11] and the boundary conditions

$$d\theta/dx = 0 \quad [12]$$

when

$$x = 0$$

and

$$\theta = 0 \quad [13]$$

when

$$x = s$$

$$M = \frac{m}{n}\frac{1}{\cos(s\sqrt{n})} \quad \text{and } N = 0$$

and by substitution in Equation [11]

$$\theta = \frac{m}{n}\left[\frac{\cos(x\sqrt{n})}{\cos(s\sqrt{n})} - 1\right] = \frac{1}{\epsilon}\left[\frac{\cos(\xi\sigma)}{\cos\sigma} - 1\right] \quad [14]$$

where for convenience the dimensionless quantities $\xi = x/s$ and $\sigma = s\sqrt{n} = s\sqrt{\epsilon q_s'''/k_s}$ have been introduced. At $x = 0$ the temperature becomes

$$\theta_0 = \frac{m}{n}\left[\frac{1}{\cos(s\sqrt{n})} - 1\right] = \frac{1}{\epsilon}\left[\frac{1}{\cos\sigma} - 1\right] \quad [15]$$

From Equations [14] and [15]

$$\frac{\theta}{\theta_0} = \frac{\cos(x\sqrt{n}) - \cos(s\sqrt{n})}{1 - \cos(s\sqrt{n})} = \frac{\cos(\xi\sigma) - \cos\sigma}{1 - \cos\sigma} \quad [16]$$

Hence the temperature distribution across the plate is not parabolic as according to Equation [3], but cosinoidal.

The case of uniform heating $(n = 0)$ is not included in Equation [9]. For this case, Rogowski already has found

$$\theta_0 = \frac{q'''}{k} \cdot \frac{s^2}{2} \quad [17]$$

[3] The prime sign denotes per unit length; three primes are used for unit volume.

According to Equation [14], θ is proportional to m, but according to Equation [16], the temperature distribution is independent of the value of m.

In a heated coil or a plate under exothermic reaction, $\theta > 0$, according to definition. But for arbitrary values of $x < s$, Equation [14] yields positive values of θ only if

$$\left.\begin{aligned} &\cos(x\sqrt{n}) > \cos(s\sqrt{n}) \\ \text{and} \quad & \\ &s\sqrt{n} < \pi/2 \end{aligned}\right\} \quad [18]$$

At the limit

$$s\sqrt{n} = \pi/2 \quad [19]$$

θ becomes infinitely great for every value of x. Hence if the linear Equation [6] were true up to this limit, every point of the plate would be at infinitely high temperature.

So Equation [18] does not mean a physical restriction of the problem. It shows only that already at a finite value of $s\sqrt{n}$ the temperature at every point, except on the surface, would exceed any finite value.

Substituting n from Equation [19] in Equation [6]

$$q_s''' = \frac{\pi^2}{4}\frac{k_s}{\epsilon s^2} = 2.467\frac{k_s}{\epsilon s^2} \quad [20]$$

Consider a coil of volume V and electrical resistance R_s at temperature t_s. Then for an electrical current i by definition and Ohm's law

$$q_s''' = \frac{i^2 R_s}{V}$$

and by substitution from Equation 20

$$i = \sqrt{2.467\, k_s V/(\epsilon s^2 R_s)}$$

This is the current at which the temperature would become infinitely high. To understand this from the energy viewpoint, it must be kept in mind that the increase of the electrical resistance inside the coil which is due to the temperature increase will cause an enormous increase of q''' inside the coil, although q_s''' (on the very surface) is supposed to be held at a moderate value.

For $s\sqrt{n} > \pi/2$, Equation [14] yields negative values of θ for certain values of x which is physically impossible. The particular integral $\theta = -m/n$ which has been used for the solution of the differential equation belongs to the range of negative values of θ without physical equivalent. This is a striking example of finding physically real solutions by using a mathematical concept without physical meaning.

At the limits $\sigma = 0$ and $\sigma = \pi/2$ Equation [16] reduces as follows

For $\sigma = 0$

$$\frac{\theta}{\theta_0} = \frac{0}{0}$$

The simplest method of determining this value is to expand the cosines in Equation [16] in series and to cancel those terms which for $\sigma \to 0$ became small of higher order.

The result is

$$\frac{\theta}{\theta_0} = 1 - \xi^2 \quad [21]$$

which is identical with the parabolic distribution according to Equation [3].

For the other limiting value, $\sigma = \pi/2$, Equation [16] yields

$$\frac{\theta}{\theta_0} = \cos\left(\xi\frac{\pi}{2}\right) \quad [22]$$

Table 2 contains numerical values of θ/θ_0 as function of ξ, calculated by means of Equation [16], for different values of σ, and Fig. 1 is a graphical representation of this function for the limiting values $\sigma = 0$ and $\sigma = \pi/2$ and one parameter between ($\sigma = 1$). Fig. 1 shows the surprising result that from zero to infinitely high temperature ($\sigma = 0$ to $\sigma = \pi/2$), the temperature distribution changes only very little.

TABLE 2 TEMPERATURE DISTRIBUTION IN PLANE PLATES

$\sigma =$	0	$\pi/18$	$\pi/6$	$\pi/3$	$4\pi/9$	$\pi/2$
ξ	θ/θ_0					
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.990	0.990	0.990	0.989	0.989	0.988
0.2	0.960	0.960	0.959	0.956	0.954	0.951
0.3	0.910	0.910	0.909	0.902	0.895	0.891
0.4	0.840	0.840	0.838	0.827	0.817	0.809
0.5	0.750	0.750	0.745	0.732	0.717	0.707
0.6	0.640	0.640	0.635	0.618	0.600	0.588
0.7	0.510	0.510	0.505	0.486	0.466	0.454
0.8	0.360	0.360	0.356	0.338	0.320	0.309
0.9	0.190	0.189	0.186	0.176	0.164	0.156
1.0	0.000	0.000	0.000	0.000	0.000	0.000

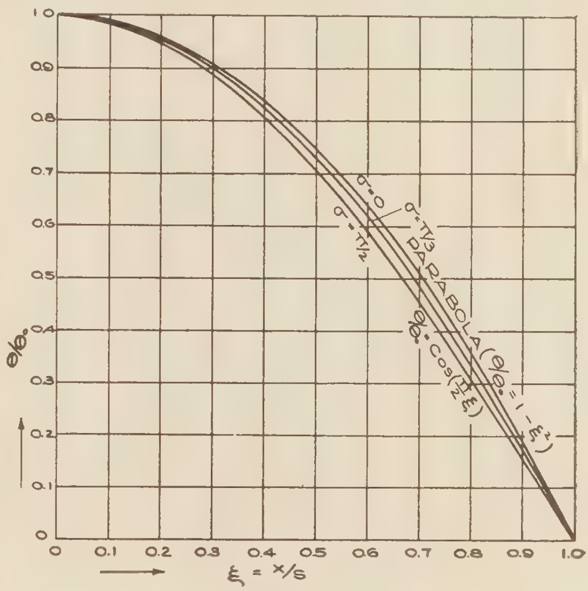

FIG. 1

This means that also the ratio $\varphi = \theta_m/\theta_0$ will vary only a little. Equation [1] in this case becomes

$$\theta_m = \frac{1}{s}\int_0^s \theta \cdot dx \quad [23]$$

Substituting θ from Equation [14] and integrating

$$\theta_m = \frac{1}{\epsilon}\frac{\sin\sigma - \sigma\cdot\cos\sigma}{\sigma\cdot\cos\sigma} = \frac{1}{\epsilon}\left(\frac{\tan\sigma}{\sigma} - 1\right) \quad [24]$$

From Equations [15] and [24]

$$\varphi = \frac{\theta_m}{\theta_0} = \frac{\sin\sigma - \sigma\cdot\cos\sigma}{\sigma\,(1 - \cos\sigma)} \quad [25]$$

For $\sigma = 0$ one obtains $\varphi = \frac{0}{0}$.

Again, the simplest way for determining this is using the trigonometric series. The result is $\varphi = 2/3 = 0.6667$, as given already in Section 1.

Substitution of $\sigma = \pi/2$ in Equation [25] yields $\varphi = 2/\pi = 0.6366$ as lower limit. This is only $4^1/_2$ per cent smaller than the value 2/3, independent of how thick the plate or how large the temperature coefficient ϵ may be.

The maximum temperature θ_0, however, according to Equation [15], varies from 0 to ∞ when φ changes from 0.6667 to 0.6366. Some values of $\epsilon\theta_0$ are given in Table 3, and plotted in Fig. 2 in logarithmic scale.

TABLE 3 MAXIMUM TEMPERATURE IN PLANE PLATES

$\frac{360}{2\pi}\sigma$	$\epsilon\theta_0$	$\frac{360}{2\pi}\sigma$	$\epsilon\theta_0$
0	0	80	4.76
3	0.00138	81	5.40
10	0.0154	88	27.7
30	0.155	89.75	229
45	0.416	90	∞
60	1.000		

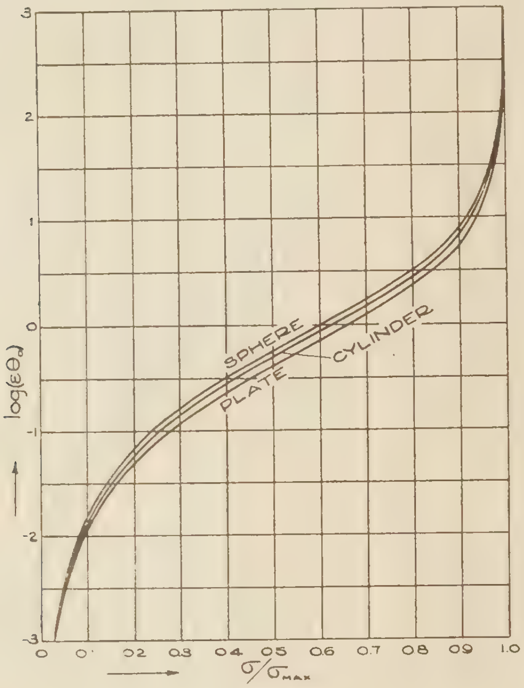

FIG. 2

3 THE CYLINDER

The case of an infinitely long full cylinder, although never realized completely, is practically important for the following reason: A hollow cylindrical coil of quadratic cross section behaves almost like an infinitely long cylindrical coil of equal cross-sectional area, provided the inner diameter of the hollow cylindrical coil is so great that the curvature can be neglected. This presents the possibility of calculating approximately the maximum temperature in the hollow cylindrical coil of quadratic cross section under consideration of the nonuniform development of heat, for the time being the only way to deal with this case (see Section 5).

If r is the perpendicular distance of a point of the cylinder from the center line, and $2s$ is the diameter of the cylinder, the other symbols having the same meaning as for the plate, then under steady-state conditions the temperature distribution will be found from the differential equation

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \cdot \frac{d\theta}{dr} + n\theta = -m \qquad [26]$$

The reduced equation is

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \cdot \frac{d\theta}{dr} + n\theta = 0 \qquad [27]$$

Introducing

$$y = r\sqrt{n} \qquad [28]$$

Equation [27] becomes

$$\frac{d^2\theta}{dy^2} + \frac{1}{y} \cdot \frac{d\theta}{dy} + \theta = 0 \qquad [29]$$

This is Bessel's differential equation of zero order. Its solution is

$$\theta = M \cdot J_0(y) + N \cdot Y_0(y) \qquad [30]$$

where J_0 and Y_0 are the Bessel functions of zero order and first and second kind, respectively, and M and N are constants of integration.

As for the plate, a particular solution of the complete differential equation is

$$\theta = \quad m/n \qquad [10]$$

Hence, the general solution is

$$\theta = -\frac{m}{n} + M \cdot J_0(r\sqrt{n}) + N \cdot Y_0(r\sqrt{n}) \qquad [31]$$

M and N are found from the boundary conditions

$$\frac{d\theta}{dr} = 0 \qquad [32]$$

when

$$r = 0$$

and

$$\theta = 0 \qquad [33]$$

when

$$r = s$$

From Equation [31]

$$\frac{d\theta}{dr} = M \cdot J_1(r\sqrt{n}) - N \cdot Y_1(r\sqrt{n}) \qquad [34]$$

where J_1 and Y_1 are the Bessel functions of first order and first and second kind, respectively.

From Equations [32] and [34]

$$N = 0$$

so that Equation [31] becomes

$$\theta = -\frac{m}{n} + M \cdot J_0(r\sqrt{n}) \qquad [35]$$

From Equations [33] and [35]

$$M = \frac{m}{n \cdot J_0(s\sqrt{n})}$$

Substituting this in Equation [35] gives

$$\theta = \frac{m}{n}\left[\frac{J_0(r\sqrt{n})}{J_0(s\sqrt{n})} - 1\right] = \frac{1}{\epsilon}\left[\frac{J_0(\rho\sigma)}{J_0(\sigma)} - 1\right] \qquad [36]$$

where $\rho = r/s$ and $\sigma = s\sqrt{n}$. This equation is analogous to Equation [14].

At $r = 0$, the temperature becomes

$$\theta_0 = \frac{m}{n}\left[\frac{1}{J_0(s\sqrt{n})} - 1\right] = \frac{1}{\epsilon}\left[\frac{1}{J_0(\sigma)} - 1\right] \ldots\ldots [37]$$

From Equations [36] and [37]

$$\frac{\theta}{\theta_0} = \frac{J_0(r\sqrt{n}) - J_0(s\sqrt{n})}{1 - J_0(s\sqrt{n})} = \frac{J_0(\rho\sigma) - J_0(\sigma)}{1 - J_0(\sigma)} \ldots [38]$$

Again the temperature distribution is not parabolic, as according to Equation [3]; this time it follows a Bessel function of zero order and first kind.

Also here the case of uniform heating ($n = 0$) is not covered by the foregoing equations, but according to Rogowski

$$\theta_0 = \frac{q'''}{k} \cdot \frac{s^2}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [39]$$

As in the case of the plane plate, the temperature θ is proportional to m and the temperature distribution is independent of m.

In analogy to the behavior of the plate, the condition $\theta > 0$ restricts the physical validity of Equation [38] to cases where

$$J_0(r\sqrt{n}) > J_0(s\sqrt{n})$$

For arbitrary values of $r < s$ this holds only between $s\sqrt{n} = 0$ and 2.4048, the first zero point of the function J_0, as Equation [16] holds only up to the first zero point of the cosine function in the case of the plane plate.

If

$$s\sqrt{n} = 2.4048 \ldots\ldots\ldots\ldots\ldots\ldots [40]$$

then, according to Equation [38], θ becomes infinitely great for every value of r. The rate of heat development at which this occurs can be found exactly as for the plate, the result being

$$q_s''' = (2.4048)^2 \frac{k_s}{\epsilon s^2} = 5.7832 \frac{k_s}{\epsilon s^2} \ldots\ldots\ldots [41]$$

Also Equation [38] simplifies at the limits $\sigma = 0$ and $\sigma = 2.4048$. For $\sigma = 0$

$$\frac{\theta}{\theta_0} = \frac{0}{0}$$

Expanding the Bessel functions in Equation [38]

$$\frac{\theta}{\theta_0} = \frac{\left(1 - \frac{\rho^2\sigma^2}{2^2} + \ldots\right) - \left(1 - \frac{\sigma^2}{2^2} + \ldots\right)}{1 - \left(1 - \frac{\sigma^2}{2^2} + \ldots\right)}$$

For $\sigma \to 0$ this becomes

$$\frac{\theta}{\theta_0} = 1 - \rho^2 \ldots\ldots\ldots\ldots\ldots\ldots [42]$$

which again gives the same parabola as Equation [3]. For $\sigma = 2.4048$, Equation [38] yields

$$\frac{\theta}{\theta_0} = J_0(2.4048\rho) \ldots\ldots\ldots\ldots\ldots [43]$$

Table 4 contains numerical values of θ/θ_0 as function of ρ, calculated by means of Equation [38] for different values of σ. In Fig. 3, the two limits, $\sigma = 0$ and $\sigma = 2.4048$, are represented, together with two intermediate curves ($\sigma = 1$ and $\sigma = 2$). Again the variation of the temperature distribution is minimal compared

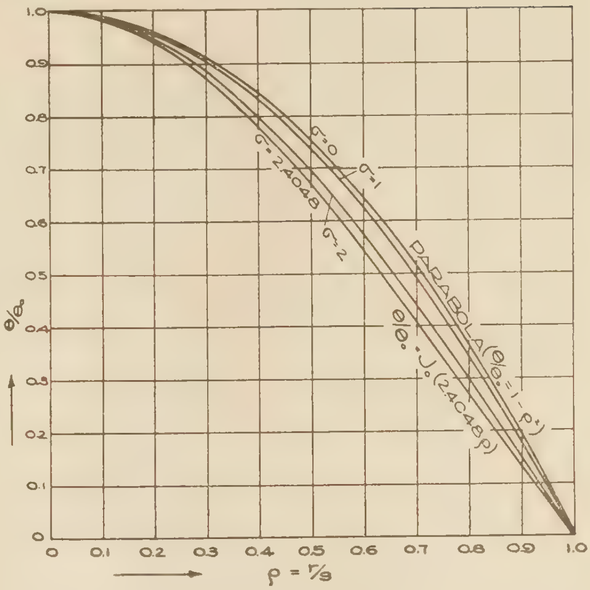

FIG. 3

with the change of the maximum temperature which can be seen from Table 5 and Fig. 2.

TABLE 4 TEMPERATURE DISTRIBUTION IN CYLINDERS

$\sigma =$	0	0.1	1	2	2.3	2.4048
ρ	θ/θ_0					
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.990	0.990	0.988	0.988	0.986	0.986
0.2	0.960	0.960	0.956	0.950	0.945	0.943
0.3	0.910	0.910	0.906	0.886	0.878	0.874
0.4	0.840	0.840	0.831	0.801	0.795	0.782
0.5	0.750	0.750	0.736	0.698	0.684	0.670
0.6	0.640	0.640	0.626	0.576	0.553	0.543
0.7	0.510	0.510	0.492	0.442	0.417	0.408
0.8	0.360	0.360	0.346	0.299	0.276	0.268
0.9	0.190	0.190	0.179	0.150	0.136	0.125
1.0	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 5 MAXIMUM TEMPERATURE IN CYLINDERS

σ	$\epsilon\theta_0$	σ	$\epsilon\theta_0$
0	0	2.3	17.0
0.1	0.0025	2.39	129
1	0.307	2.4	398
2	3.47	2.4048	∞

The mean temperature in this case is defined by

$$\theta_m = \frac{1}{\pi s^2}\int_0^s \theta \cdot 2\pi r \cdot dr \ldots\ldots\ldots\ldots [44]$$

Substituting θ from Equation [36] and performing a part of the integration

$$\theta_m = \frac{m}{n}\left[\frac{2}{s^2 J_0(s\sqrt{n})}\int_0^s r \cdot J_0(r\sqrt{n}) \cdot dr - 1\right]$$

Substituting $r\sqrt{n} = y$, $s\sqrt{n} = \sigma$, and $n/m = \epsilon$

$$\theta_m = \frac{1}{\epsilon}\left[\frac{2}{\sigma^2 J_0(\sigma)}\int_0^\sigma y \cdot J_0(y) \cdot dy - 1\right] \ldots\ldots [45]$$

The integral is found as follows

$$J_0(y) = 1 - \frac{y^2}{2^2} + \frac{y^4}{2^2 \cdot 4} - \ldots$$

$$y \cdot J_0(y) = y - \frac{y^3}{2^2} + \frac{y^5}{2^2 \cdot 4} - \ldots$$

$$\int_0^\sigma y\cdot J_0(y)\cdot dy = \left[y\left(\frac{y}{2} - \frac{y^3}{2^2\cdot 4} + \frac{y^5}{2^2\cdot 4^2\cdot 6} - \ldots\right)\right]_0^\sigma$$
$$= \left[y\cdot J_1(y)\right]_0^\sigma = \sigma\cdot J_1(\sigma)$$

Hence

$$\theta_m = \frac{1}{\epsilon}\left[\frac{2\cdot J_1(\sigma)}{\sigma\cdot J_0(\sigma)} - 1\right] \qquad [46]$$

and using Equation [37]

$$\varphi = \frac{\theta_m}{\theta_0} = \frac{1}{\sigma}\,\frac{2\cdot J_1(\sigma) - \sigma\cdot J_0(\sigma)}{1 - J_0(\sigma)} \qquad [47]$$

For $\sigma = 0$, again $\varphi = \frac{0}{0}$, but using the Bessel series

$$\varphi = \frac{1}{\sigma}\,\frac{2\left(\frac{\sigma}{2} - \frac{\sigma^3}{2^2\cdot 4} + \ldots\right) - \sigma\left(1 - \frac{\sigma^2}{2^2} + \ldots\right)}{1 - \left(1 - \frac{\sigma^2}{2^2} + \ldots\right)} = 0.5$$

as the parabolic distribution of temperature requires.

Substituting $\sigma = 2.4048$ in Equation [47] yields

$$\varphi = \frac{2(0.51915)}{2.4048} = 0.432$$

instead of the value of 0.5 for uniform heat development, still a rather moderate difference considering the range of temperature variation from $\theta = 0$ to $\theta = \infty$.

4 THE SPHERE

Spherical coils may be used only occasionally. However, the calculations for a sphere also have practical significance because they give approximations for cylinders of height equal to diameter, and for cubes of the same volume. Spherical shape is further sometimes used in devices for chemical reactions.

The procedure is the same as for plate and cylinder. Here

r = radial distance of any point of the sphere from its center
$2s$ = diameter of sphere

The complete differential equation is

$$\frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr} + n\theta = -m \qquad [48]$$

and the reduced equation is

$$\frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr} + n\theta = 0 \qquad [49]$$

Substituting $\theta = z/\sqrt{r}$ in Equation [49]

$$\frac{d^2z}{dr^2} = \frac{1}{r}\frac{dz}{dr} + \left(n - \frac{1}{4r^2}\right)z = 0$$

and substituting $r = y\sqrt{n}$ in this equation

$$\frac{d^2z}{dy^2} + \frac{1}{y}\cdot\frac{dz}{dy} + \left(1 - \frac{1}{4y^2}\right)z = 0$$

This is the Bessel equation of order $\nu = 1/2$. Its general solution is

$$z = \frac{1}{\sqrt{y}}\left(M\cdot\sin y + N\cdot\cos y\right)$$

or

$$\theta = \frac{1}{r\sqrt[4]{n}}\left[M\cdot\sin(r\sqrt{n}) + N\cdot\cos(r\sqrt{n})\right] \qquad [50]$$

where M and N are the constants of integration.

Again, a particular solution of the complete differential equation is

$$\theta = -m/n \qquad [10]$$

so that its general solution becomes

$$\theta = -\frac{m}{n} + \frac{1}{r\sqrt[4]{n}}\left[M\cdot\sin(r\sqrt{n}) + N\cdot\cos(r\cdot\sqrt{n})\right] \qquad [51]$$

From the boundary conditions

$$\frac{d\theta}{dr} = 0 \qquad [52]$$

for

$$r = 0$$

and

$$\theta = 0 \qquad [53]$$

for

$$r = s$$

$$M = \frac{m}{n}\,\frac{s\sqrt[4]{n}}{\sin(s\sqrt{n})} \quad \text{and } N = 0$$

are obtained. Substituting these in Equation [51]

$$\theta = \frac{m}{n}\left[\frac{s}{r}\,\frac{\sin(r\sqrt{n})}{\sin(s\sqrt{n})} - 1\right] = \frac{1}{\epsilon}\left[\frac{\sin(\rho\sigma)}{\rho\cdot\sin\sigma} - 1\right] \qquad [54]$$

Substituting $\rho = 0$, the maximum temperature is found as follows

$$\theta_0 = \frac{0}{0}$$

But

$$\frac{\sin(\rho\sigma)}{\rho\cdot\sin\sigma} = \frac{\sigma}{\sin\sigma}\cdot\frac{\sin(\rho\sigma)}{\rho\sigma}$$

and for $\rho \to 0$

$$\frac{\sin(\rho\sigma)}{\rho\sigma} = 1$$

Hence

$$\theta_0 = \frac{1}{\epsilon}\left(\frac{\sigma}{\sin\sigma} - 1\right) \qquad [55]$$

For $n = 0$, however, according to Rogowski

$$\theta_0 = \frac{q'''}{k}\cdot\frac{s^2}{6} \qquad [56]$$

From Equations [54] and [55]

$$\frac{\theta}{\theta_0} = \frac{\frac{1}{\rho}\sin(\rho\sigma) - \sin\sigma}{\sigma - \sin\sigma} \qquad [57]$$

As in the former cases, θ is proportional to m and θ/θ_0 is independent of m.

According to Equation [54], θ is positive only if

$$\sin(\rho\sigma) > \rho\sin\sigma$$

This is the case in the range from $\sigma = 0$ to $\sigma = \pi$ as can easily be checked from the behavior of the sine in a circle with unit radius or by looking up a table of sine values.

For $\sigma > \pi$ negative values of θ would occur, which is without physical equivalent.

For $\sigma = \pi$ the temperature would become infinitely great for every point inside the sphere. The rate of heat production q_s''' at which this occurs is found as in the former cases, the result being

$$q_s''' = \pi^2 \cdot \frac{k_s}{\epsilon s^2} = 9.870 \frac{k_s}{\epsilon s^2} \qquad [58]$$

For $\sigma = 0$, Equation [57] yields

$$\frac{\theta}{\theta_0} = \frac{0}{0}$$

By expansion of the sine series and neglecting the terms which become small of higher order

$$\frac{\theta}{\theta_0} = 1 - \rho^2 \qquad [59]$$

which again is the same parabola as described by Equation [3].

With the other limiting value, $\sigma = \pi$, Equation [57] yields

$$\frac{\theta}{\theta_0} = \frac{\sin(\rho\pi)}{\rho\pi} \qquad [60]$$

For

$$\rho = 0 \qquad \theta/\theta_0 = 1$$

For

$$\rho = 1 \qquad \theta/\theta_0 = 0$$

Table 6 contains numerical values of θ/θ_0 calculated by means of Equation [57] and in Fig. 4 curves for the limiting parameters $\sigma = 0$ and $\sigma = \pi$, and for $\sigma = \pi/2$ and $\sigma = 5\pi/6$ are presented. Again the variation of θ/θ_0 is small, although a little greater than in the former cases. The variation of the maximum temperature in this range is shown in Table 7 and Fig. 2. It is remarkable how close to one another come the curves for plane, cylinder, and sphere in this graph.

The mean temperature in a sphere is defined by

$$\theta_m = \frac{1}{(4/3)\pi s^3} \int_0^s \theta \cdot 4\pi r^2 \cdot dr \qquad [61]$$

Substitution of θ from Equation [54] and introduction of $y = \rho\sigma = r\sqrt{n}$ lead to

$$\theta_m = \frac{1}{\epsilon}\left(\frac{3}{\sigma^2 \sin\sigma}\int_0^\sigma y \sin y \cdot dy - 1\right) = \frac{1}{\epsilon}\left[\frac{3}{\sigma^2 \sin\sigma}(\sin\sigma - \sigma\cos\sigma) - 1\right] = \frac{1}{\epsilon}\,\frac{\frac{3}{\sigma^2}\left(\sin\sigma - \sigma\cos\sigma\right) - \sin\sigma}{\sin\sigma} \qquad [62]$$

TABLE 6 TEMPERATURE DISTRIBUTION IN SPHERES

$\sigma =$	0	$\pi/2$	$3\pi/4$	$5\pi/6$	$35\pi/36$	π
ρ	θ/θ_0					
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.990	0.990	0.986	0.986	0.983	0.983
0.2	0.960	0.958	0.948	0.943	0.938	0.935
0.3	0.910	0.902	0.884	0.882	0.862	0.858
0.4	0.840	0.824	0.798	0.785	0.763	0.757
0.5	0.750	0.727	0.691	0.676	0.644	0.636
0.6	0.640	0.612	0.570	0.550	0.514	0.505
0.7	0.510	0.479	0.436	0.415	0.377	0.368
0.8	0.360	0.332	0.292	0.275	0.242	0.234
0.9	0.190	0.171	0.146	0.135	0.114	0.109
1.0	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 7 MAXIMUM TEMPERATURE IN SPHERES

$\frac{360}{2\pi}\sigma$	$\epsilon\theta_0$
0	0
90	0.570
135	2.335
150	4.24
175	34.1
179.5	358
180	∞

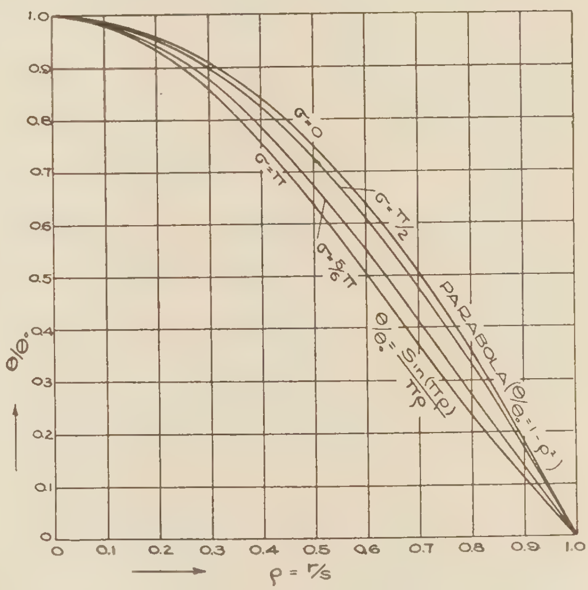

FIG. 4

From Equations [55] and [62]

$$\varphi = \frac{\theta_m}{\theta_0} = \frac{\frac{3}{\sigma^2}(\sin\sigma - \sigma\cos\sigma) - \sin\sigma}{\sigma - \sin\sigma} \qquad [63]$$

For $\sigma = 0$, again

$$\frac{\theta_m}{\theta_0} = \frac{0}{0}$$

Here, however, one must be careful with the expansion of the series. Terms up to σ^5 are partly needed. Then

$$\frac{\theta_m}{\theta_0} = \frac{\frac{3}{\sigma^2}\left[\left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \ldots\right) - \sigma\left(1 - \frac{\sigma^2}{2!} + \frac{\sigma^4}{4!} - \ldots\right)\right] - \left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \ldots\right)}{\sigma - \left(\sigma - \frac{\sigma^3}{3!} + \ldots\right)}$$

and for $\sigma \to 0$

$$\varphi = \theta_m/\theta_0 = 2/5$$

as given in Section 1.

The author has checked this and the former calculations of the maxima by the usual calculus of undeterminates. In the present case, one has to differentiate four times until the result $\varphi = 0.4$ is obtained.

The other limit, $\sigma = \pi$, leads to

$$\varphi = \frac{\theta_m}{\theta_0} = \frac{3}{\pi^2} = 0.304$$

This is 24 per cent less than under the assumption of uniform heating where, also at this limit, φ would be equal to 0.4.

5 A NUMERICAL EXAMPLE, CHECKED BY EXPERIENCE

In various firms of the electrical industry, the maximum temperature inside coils will have been measured; little, however, seems to have been published that could serve for checking the theory developed in this paper.

Rogowski and Vieweg (7) performed experiments on a hollow cylindrical coil under different conditions of load and cooling. Most suitable for comparison with the theory is their test No. 2 where the coil was cooled from all sides by streaming air so that the surface temperature will have been rather uniform. The inner diameter of the coil was 80 mm, the outer diameter was 168 mm, while the cross section was quadratic with 44 mm side length. The coil was made up of copper wire, 1 mm diam, insulated by silk, soaked with lacquer. The wire was wound in 40 layers, each including 35.4 windings as an average, the total number of windings being 1416.

For applying the theory, the equivalent thermal conductivity k of the coil material is needed. Very little seems to be known about this. The simplest way to find it would be by direct measurement on a hollow cylindrical coil or on plane plates, built up from the particular insulated wire, using one of the standard methods for testing plates or hollow cylinders of heat-insulating material.

The authors (7) give no values of k; so this had to be calculated. A theory of exact calculation of k from the geometrical configuration and the thermal conductivities of the constituents of a coil has not been developed as yet. For this reason, the following approximative method has been used:

The wire of the coil is assumed to be replaced by a conductor of rectangular cross section having the same cross-sectional area as the wire and insulation of uniform thickness. The dimensions of the cross section are chosen so that 40 layers of 35.4 windings (as an average) fill the cross-sectional area of the coil. This is performed by the system shown in Fig. 5, which has exactly the same number of windings, number of layers and areas of cross section of copper and of insulation as the original coil.

Now it is assumed that for heat flow in the vertical direction, the horizontal lines *AA*, *BB*, *CC*, etc., and, for heat flow in the horizontal direction, the vertical lines *aa*, *bb*, *cc*, etc., are isothermals. This then allows the calculation of k for vertical and horizontal heat flow (k_v and k_h), if the thermal conductivities k_m and k_i of the metal and the insulation are given.

The thermal conductivity of copper is well known; it has been assumed to be $k_m = 3.49$ watt cm^{-1} C^{-1}.

For varnished silk, values of k_i between 0.00159 and 0.00128 have been measured at temperatures from 11 to 128 C, with mean values of 17 to 80 C (8). The influence of temperature, if any, has not been revealed by these tests. A value $k_i = 0.00143$ watt cm^{-1} C^{-1} will be assumed. This is not only the average of the

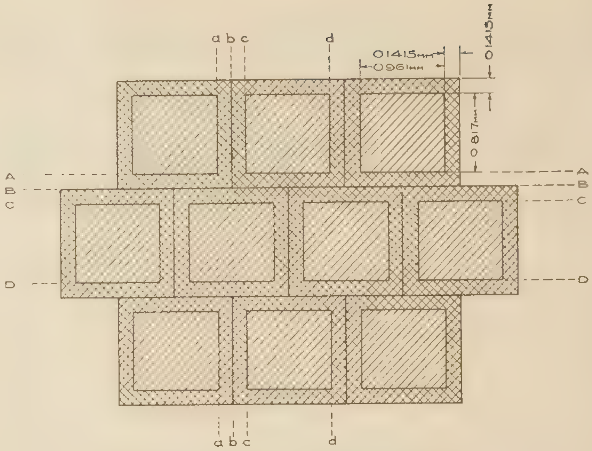

Fig. 5

values given, but also the value measured at temperatures approximating those in the coil experiment under consideration.

The calculation, based on Fig. 5, $k_m = 3.49$ and $k_i = 0.00143$, led to $k_v = 0.00556$ and $k_h = 0.00506$. It may be noticed that 57.5 per cent of the volume was taken by copper, having a 2400 times greater thermal conductivity than the insulating material. Nevertheless, the equivalent (apparent) thermal conductivity was only 3.7 times that of the insulating material.

It is further seen that the thermal conductivity is different by about 10 per cent in the two considered directions. An average value $k = 0.00531$ watt cm^{-1} C^{-1} will be assumed in the calculation of the coil temperature.

The authors indicate that the coil had a resistance $R \approx 11$ ohm in the cold state; it may be assumed that this was at about 20 C. Then, at $t_s = 76.1$ C (the measured surface temperature), according to Equation [4], the resistance will have been

$$R_s = 11[1 + 0.004\ (76.1 - 20)] = 13.46 \text{ ohm}$$

Since the volume of the coil was $V = 756$ cm^3,

$$R_s''' = R_s/V = 0.0178 \text{ ohm per cm}^3,$$

Using now $t_s = 76.1$ C as zero point, according to the definition $\theta = t - t_s$, Equation [4] becomes

$$R''' = R_s'''(1 + \epsilon\theta) \ldots\ldots\ldots\ldots\ldots\ldots [64]$$

It is not generally recognized that ϵ, as defined by Equation [4] or [64], is not always equal to 0.004 C^{-1} for copper, but depends upon t_s. From experiments of Holborn (9) it can be derived that for pure copper

$$\epsilon = 0.0043 \text{ when } t_s = 0 \text{ C}$$

and

$$\epsilon = 0.0030 \text{ when } t_s = 100 \text{ C}$$

both almost independent of the temperature t, up to 500 C or more. An interpolated value $\epsilon = 0.0033$ will be assumed for $t_s = 76.1$ C.

The current in the experiment was $i = 3$ amp. Hence, $q_s''' = i^2R_s''' = 0.1602$ watt per cm^3, $m = q_s'''/k_s = 0.1602/0.00531 = 30.2$ C per cm^2, and $n = \epsilon m = 0.0033\ (30.2) = 0.0997$ cm^{-2}.

As mentioned in Section 3 a hollow cylindrical coil of quadratic cross section will behave not much differently from an infinitely long full cylindrical coil of the same cross-sectional area. Since the cross-sectional area of the test coil was $4.4^2 = 17.6$ cm^2, the

equivalent radius of an infinitely long cylindrical coil is $s = \sqrt{17.6/\pi} = 2.486$ cm. Hence $\sigma = s\sqrt{n} = 0.774$.

Substituting this in Equation [37]

$$\theta_0 = \frac{1}{0.0033}\left[\frac{1}{J_0(0.774)} - 1\right]$$

and taking $J_0(0.774) = 0.8545$ from a table of the Bessel functions, $\theta_0 = 52.2$ C.

This would be for nonuniform heat development in an infinitely long cylinder.

For uniform heat development, on the other hand

$$\theta_0 = \frac{q'''}{k} \cdot \frac{s^2}{4} \qquad [39]$$

and for quadratic cross section with the cross-sectional area $4a^2 = \pi s^2$, according to Jakob (3)

$$\theta_0 = 0.59\,\frac{q'''}{k} \cdot \frac{a^2}{2} = \frac{0.59}{0.64}\left(\frac{q'''}{k}\,\frac{s^2}{4}\right) \qquad [65]$$

Assuming that the ratio 0.59/0.64 will approximately hold also for nonuniform heating, a reduced value $(\theta_0)_{\text{red}} = \frac{0.59}{0.64}\,52.2 = 48$ C is found for the maximum temperature in the quadratic cross section of the test coil, whereas Rogowski and Vieweg's electrical-resistance measurements led to 46.3 C. Without taking care of the inhomogeneity of heat development, the calculation would have given 42.9 C.

The mean temperature of the cylindrical body, according to Equation [46] is

$$\theta_m = \frac{1}{0.0033}\left[\frac{2}{0.774}\,\frac{J_1(0.774)}{J_0(0.774)} - 1\right]$$

and taking $J_1(0.774) = 0.3587$ from a table, $\theta_m = 26.1$ C.

Here, according to Jakob (3), the reduction ratio is 0.28/0.32, which gives $(\theta_m)_{\text{red}} = \frac{0.28}{0.32}\,26.1 = 22.8$ C, whereas 23.9 C has been measured and 20.4 would have been found by calculation without considering the inhomogeneity of heat production.

The excellent agreement between theory and experiment must not be overestimated. It is somewhat incidental, because the assumed value of k_i is uncertain and the experiments are none too good. The author would rather conclude from this coincidence that k_i has been chosen satisfactorily. Moreover, the reverse way, that is measuring θ_m and finding k by means of the theory, may be suggested as a simple and reliable method of determining the apparent thermal conductivity of coil materials.

Finally, it will be calculated which electrical current i would bring the coil to infinitely high temperature, according to the theory.

From Equation [41]

$$q_s''' = i^2 R_s''' = 5.783\,\frac{k_s}{\epsilon s^2}$$

and substituting $R_s''' = 0.0178$ ohms per cm³, $k_s = 0.00531$ watt cm^{-1}C^{-1}, $\epsilon = 0.0033$ C^{-1}, and $s = 2.486$ cm, the limiting value $i = 9.20$ amp is found. Since 3 amp as used in the experiment caused a very moderate temperature increase, it is surprising that 9.2 amp would be sufficient to bring the temperature beyond any finite value.

This will be compared with the result of the old theory by assuming a current of $i = 9.20$ amp in the coil. In that theory, a constant mean value $\frac{q'''}{k} = \frac{q_s'''}{k_s}(1 + \epsilon\theta_m)$ is to be assumed. Substituting this in the equation for an infinitely long cylinder, Equation [39]

$$\theta_0 = \frac{q_s'''}{k_s}\,\frac{s^2}{4}\,(1 + \epsilon\theta_m)$$

But, as mentioned in Section 1, $\theta_m = 0.5\,\theta_0$. Substituting this, θ_0 is the only unknown, and with $q_s''' = (9.20)^2\,0.0178 = 1.506$ watt per cm³, and k_s, ϵ, and s as given previously, $\theta_0 = 1566$ C is obtained, or reduced to square cross section, as before, $(\theta_0)_{\text{red}} = \frac{0.59}{0.64}\,1566 = 1443$ C, instead of ∞.

From these numerical calculations, it is seen that the influence of the nonuniform development of heat increases rapidly at higher load and should not be neglected even at moderate rates of heat production.

Conclusions

The theory of heat conduction at steady state in infinitely wide plane plates, infinitely long cylinders and spheres has been extended to the case of internal heat development at a rate which increases linearly with the temperature.

This always occurs in electrical coils under load. It may occur also in bodies which undergo chemical reactions in a temperature range where the heat of reaction can be assumed as increasing linearly with temperature. If, for instance, a reacting solid or crushed substance is steadily moved as the coal bed on the traveling grid of a furnace, or through a cylindrical pipe with negligible axial heat conduction or convection, then heat is transmitted only by conduction in a direction perpendicular to the movement of the reacting body, and, in a given length section, the temperatures are not changing in time. The temperature equilibrium in a potato silo or in a pile of stored coal belongs to the same group of problems. Inside, an exothermal reaction occurs, and it can be assumed that the heat is conducted to the surface, the equivalent conductivity being due partly to convection. In a coal pile a quasi-steady state sometimes exists for months; if the equilibrium is disturbed self-ignition may occur. The main difficulty for precalculation of the maximum temperature is the inhomogeneity and odd shape of the pile.

Another possible application concerns the temperature distribution across the oil film of a bearing. Here, however, the heat development depends not only upon the temperature distribution but also upon the velocity distribution of the rotating oil, which itself depends upon the viscosity and by this on the temperature distribution. This will require a special analysis. Moreover, the development of heat will probably become smaller with increasing temperature.

Neither this nor the case of heat sinks instead of heat sources, i.e., the case of endothermal reactions, has been dealt with in this paper, although the latter would not have presented particular difficulties.

The main results of the present paper are the following:

1 Owing to the increase of the electrical resistance inside a coil with temperature, the temperature tends to assume an infinitely high value already at a rather moderate electrical current.

2 The temperature distribution in a coil is not much different from that at uniform development of heat, even at infinitely high temperature.

3 By this also the ratio of the mean to the maximum temperature does not change much.

4 Therefore it is possible to deal approximately with cases for which the exact solutions have not been found as yet.

The differential equations and their solutions are not restricted to the simple cases dealt with in this paper. In particular, they hold as well for the practically important cases of plate, hollow

cylinder and hollow sphere with different temperatures at the two free surfaces, and for heating hollow cylinders and spheres from outside to inside, as well as in the opposite sense. These cases will be treated in a later paper on the subject.

Acknowledgment

The author is indebted to Dr. J. W. Calkin and to Dr. R. Oldenburger for some mathematical advice.

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Discussion

E. S. Davis.[5] The influence of nonuniform development of heat on the temperature distribution in bodies of simple shape is of rather great engineering interest.

The author has ably covered the case where the temperature coefficient of conductivity is linear. However, there are many materials whose coefficients are not linear. Let us consider the case where the coefficient $\frac{dk}{dT}$ may be expressed by the form

$$1 + \alpha\theta + B\theta^2 \qquad [66]$$

The basic equation governing the case of the flat plate is still

$$\frac{d^2\theta}{dx^2} = +\frac{q'''}{k} \qquad [67]$$

If we represent

$$q'''/k = m\,(1 + \alpha\theta + B\theta^2) \qquad [68]$$

then

$$\frac{d^2\theta}{dx^2} = m\,(1 + \alpha\theta + B\theta^2) \qquad [69]$$

Representing the temperature at the median line by T_m, at the surface by T_s, with x at the surface $= 0$, and $x = s$ at the median, we may express the boundary conditions as $\theta = (T - T_m) = 0$, and $\frac{d\theta}{dx} = 0$ at $x = s$, and $\theta = (T_s - T_m)$ at $x = 0$.

Multiplying Equation [68] by $d\theta$ and integrating

$$\int_0^{\frac{d\theta}{dx}} \frac{d\theta}{dx}\, d\left(\frac{d\theta}{dx}\right) = \int_0^{\theta} m(1 + \alpha\theta + B\theta^2)d\theta \qquad [70]$$

$$\frac{d\theta}{dx} = \sqrt{2m}\,\sqrt{\theta + \frac{\alpha\theta^2}{2} + \frac{B\theta^3}{3}} \qquad [71]$$

then integrating again

$$\int_0^{\theta} \frac{d\theta}{\sqrt{\theta + \frac{\alpha\theta^2}{x} + \frac{B\theta^3}{3}}} = \sqrt{2m}\int_s^x dx \qquad [72]$$

$$\frac{2}{\sqrt{\mu}}\, sn^{-1}\left(\sqrt{\theta/\nu},\ \sqrt{\nu/\mu}\right) = \sqrt{2m}\,(x - s) \qquad [73]$$

where μ and ν are roots of

$$\frac{B\theta^2}{3} + \frac{\alpha\theta}{2} + 1 = 0 \qquad [74]$$

or

$$(\mu, \nu) = -\frac{3\alpha}{4B} \pm \frac{3B}{2}\sqrt{\frac{\alpha^2}{4} - \frac{4B}{3}} \qquad [75]$$

Here, sn represents one of the Legendre, or more precisely, the Jacobi functions.[6]

This solution shows the proper procedure to take in case the thermal conductivity varies greatly from a linear function. The same type of analysis may be applied to the case of the cylinder and the sphere, if the necessity arises.

R. H. Norris.[7] The author has made a useful contribution in reassuring us that no serious error is introduced by the customary neglect of the effect of nonuniformity of heat generation on the calculated temperature rise of electrical coils.

His analysis provides a convenient basis for a discussion of the ratio of the temperature rise, calculated by his new "exact" method, to the rise calculated by the common, less accurate, method which assumes uniformly distributed heat generation. This ratio is of most practical significance when it is based on the same total heat dissipation for both methods. Since its value on this basis, however, does not seem directly evident from the author's curves, it is discussed more explicitly in the following paragraphs:

The total heat dissipation is proportional to the slope, at the coil surface, of the curve of temperature distribution, as plotted, for example, in Fig. 1 of the paper. The slopes of the highest and lowest curves shown in Fig. 1, differ by a ratio of 1.27 to 1, for the same maximum temperature.

For the extreme case here considered, this means (for reasons explained later) that a 27 per cent error would be introduced in the calculated maximum temperature rise, if the customary approximate method were used, instead of the "exact" method when assuming the same total heat dissipation for both methods. (This extreme case is the one for which the parameter σ has the value $\pi/2$.)

In the range of practical interest, however, the corresponding error becomes practically negligible. It is only 5 per cent for a case more unfavorable than any likely to be encountered in practice, namely, the case in which the temperature rise from

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[4] Title Translated.

[5] Development Engineer, American Locomotive Company, Alco Products Division, New York, N. Y. Deceased.

[6] "Functions of the Complex Variable," by J. Pierpont, Ginn & Company, Boston, Mass., 1914.

[7] General Engineering Laboratory, General Electric Company, Schenectady, N. Y. Jun. A.S.M.E.

surface to center is assumed 100 C, and the temperature coefficient ϵ, assumed as high as 0.00364 (corresponding to surface temperature of 40 C). The value of the parameter σ, corresponding to these conditions, can be shown to be 0.748.

For the general case, a convenient expression can be derived, from the author's results, to express the ratio of the temperature rise, by the "exact" method to the rise by the customary approximate methods. This ratio, for a given total heat generation, is given by the equation:

$$\frac{(\theta_0)_{\text{exact}}}{(\theta_0)_{\text{approx}}} = \frac{2\,(1 - \cos\sigma)}{\sigma \sin\sigma}$$

The figures of 27 per cent and 5 per cent discrepancy previously mentioned, were obtained from this expression, since it gives 1.27 and 1.05 for $\sigma = \pi/2$, and 0.748, respectively. This expression is obtainable by differentiation, with respect to ξ, of the author's Equation [16], and evaluation of the resulting expression for the value $\xi = 1$. The values thereby obtained correspond to the relative temperature gradients at the surface, and hence to the relative heat dissipation for a given maximum temperature or, conversely, to the relative maximum temperature for a given heat dissipation.

As regards effective, or average, thermal conductivity of an array of round electrical conductors embedded in homogeneous insulation, data have been made available by R. Richter.[8] It would seem that these data might be better than the author's approximations, for evaluation of the thermal conductivity for the test sample. These data of Richter are also available in a book by A. D. Moore.[9]

Rufus Oldenburger.[10] This paper fills a long existing need for an accurate understanding of the temperatures in electrical coils, which understanding will prove of value in preventing failures caused by the overheating of conductors. The author emphasizes a technique, so important in modern engineering, of treating problems from different branches of engineering by one method applicable to all of these problems, because the underlying mathematical laws are identical.

That the solution of the differential equation for the temperature difference θ yields, in general, positive values of θ for only certain values of x in the range $0 \leftrightarrow s$ is somewhat surprising since a value of θ has physical meaning only if θ is positive or zero. For the simplicity of the analysis let us restrict ourselves to the author's solution of the infinitely wide plane plate. Analogous remarks will hold for the other coils treated. With the differential Equation [7] of the paper, and the boundary conditions Equations [12], [13], the solution Equation [14] is obtained, which can be shown by mathematical methods to be the unique solution for the type of functions considered. Since the differential Equation [7] is fundamental, and has been thoroughly verified by theory and experiment, the negativeness of θ in Equation [14] can be due only to the choice of the boundary conditions. The value of θ is negative, for example, when

$$\cos\xi\sigma < \cos\sigma,\ \cos\sigma > 0,\ 0 \leq \xi \leq 1$$

These inequality relations can be satisfied for a proper choice of σ and $\xi\sigma$ in the ranges

$$2\pi \geq \sigma \geq \frac{3\pi}{2},\ \frac{\pi}{2} > \xi\sigma > 0$$

One of the boundary conditions, namely, Equation [13], is the mathematical equivalent of the property that the surface of the coil is maintained at a constant temperature. Clearly, Equation [13] is indispensable to a description of the physical situation, and cannot be discarded or modified. This brings us to Equation [12], which states that, at the median plane of the plate, where $\xi = 0$, $(x = 0)$, the temperature attains a critical value, which, when the solution Equation [14] is obtained, turns out to be the maximum value of θ. For θ to take on its maximum at $\xi = 0$, it is not, however, necessary to have

$$d\theta/dx = 0 \quad [d\theta/d\xi = 0]$$

It seems quite likely that the derivative with respect to x of the solution for θ in the problem in question does not vanish at $x = 0$, at least for the cases where Equation [14] yields negative values of θ.

If one allows x to be negative, the condition Equation [12] is not needed to obtain the author's solution Equation [14]. If we write θ as $\theta(x)$, the symmetry assumption of the author's paper makes Equation [12] superfluous since $\theta(-x) = \theta(x)$ implies that $N = 0$, which is all that one obtains from Equation [12]. If x is not allowed to take on negative values, some condition like Equation [12] is needed to determine N. The author's development is, however, valid for negative as well as positive values of x.

Since by physical considerations, the solution θ of Equation [7] and its derivatives should be continuous for $s \geq x \geq 0$, the solution of Equation [7] will in any case be given by Equation [11]. Also, by Equation [13] we have

$$M\cos\sigma + N\sin\sigma = m/n$$

If $\cos\sigma \neq 0$, solving for M we have

$$\theta = \frac{m}{n}\left(\frac{\cos\xi\sigma}{\cos\sigma} - 1\right) + N\,(\sin\xi\sigma - \cos\xi\sigma\tan\sigma)$$

From physical considerations (See Table 2 of the paper) it appears that N should be chosen so that for $s \geq x \geq 0$ (i. e. $1 \geq \xi \geq 0$) we have $\theta > 0$, and θ decreasing uniformly from its value at $x = 0$ to its value at $x = s$, that is, over the range $s \geq x \geq 0$, we must have $d\theta/dx < 0$. We remark that Equation [7], and $\theta > 0$, $n > 0$ imply that

$$d^2\theta/dx^2 < 0$$

whence the (x, θ) curve must be concave down over the range $s \geq x \geq 0$.

The relations $\theta > 0$ and $(d\theta/dx) < 0$ are equivalent to the pair of inequalities

$$N\,(\sin\xi\sigma - \cos\xi\sigma\tan\sigma) > \frac{m}{n}\left(1 - \frac{\cos\xi\sigma}{\cos\sigma}\right)$$

$$N\,(\cos\xi\sigma + \sin\xi\sigma\tan\sigma) < \frac{m\sin\xi\sigma}{n\cos\sigma}$$

which are linear in N. For each value of N which satisfies the foregoing inequalities, the solution obtained for θ in terms of N is one of the desired type. A mathematical study should be made of these inequalities in N to determine whether or not a solution N can always be found, and if so, among the possible solutions which value of N fits the actual physical situation. The value $N = 0$, obtained by the author in the cases he considered, satisfies the inequalities mentioned, and is uniquely determined by the property that it yields for $d\theta/dx$ the smallest possible numeri-

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[9] "Fundamentals of Electrical Design," by A. D. Moore, first edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1927, pp. 95, 96.

[10] Professor of Mathematics, Illinois Institute of Technology, Chicago, Ill.; and Mathematical Consultant, Woodward Governor Company, Askania Regulator Company, Le Roi Engine Company, and New York Life Insurance Company.

cal value at $x = 0$. It appears reasonable that, in any case, the desired value of N is that unique value which satisfies the inequalities stated and makes $d\theta/dx$ at $x = 0$ smallest in absolute value.

That the particular solution, Equation [10], of Equation [7] has no physical meaning is in a sense not surprising, inasmuch as any solution of a differential equation will serve as the particular solution used in its treatment. Even though this differential equation is used to explain a physical situation, one would not expect every solution to be physically realizable. That the simplest particular solution of Equation [7], given by Equation [10], is not realizable in an actual coil is, however, rather novel.

This paper is not only a significant forward step toward the understanding of heat in coils, but also emphasizes the need for a thorough mathematical and experimental analysis of boundary conditions. Such an analysis should yield the solution of coil heat problems for those cases considered in the paper, where present methods fail.

V. Paschkis.[11] The author states, that his deductions apply also to exotherm chemical reactions, if certain conditions are fulfilled (heat of reaction increasing in linear proportionality with temperature).

It appears to the writer, that this latter field of application (chemical reactions) is better represented in the paper than that of electric coils.

The entire deductions are based on the definition of the "equivalent thermal conductivity" of the coil. In view of the fact that the thickness of the insulation amounts only to 26 per cent of the thickness per layer in one direction and to 23 per cent in the other, the assumption of a fictitious homogeneous body appears to call for proof by experiment. This would seem especially so because of the very unexpected result of "infinite" temperature with current increased only 3.06 times.

V. N. Tramontini.[12] In connection with an investigation of the heat losses from small-diameter cylinders by free convection, the writer had occasion to make an analysis of a problem which, in many respects, is similar to the case of the temperature distribution in cylindrical coils which is so ably treated by the author. The problem, specifically, was the determination of the temperature distribution across a small-diameter platinum wire carrying electric current and losing heat uniformly from the surface. The variation of electrical resistance with temperature was considered and the thermal conductivity was assumed to be constant, whereas the author includes the variability of the thermal conductivity in his calculation. For platinum, the rate of change of thermal conductivity with temperature is less than one seventh as great as the rate of change of electrical resistivity. A further simplification was made in assuming the resistivity to be a linear function of the temperature.

Setting up a differential equation by means of a heat balance for a differential ring of unit length, the writer obtained a solution of the same form as Equation [31], that is, the temperature distribution as represented by Bessel functions of zero order. The author's results, had they been available, could have been applied directly to the solution of this problem.

The result of the numerical computation for a platinum wire of 0.01 in. diam, with a voltage drop of 0.76 v per in. and a surface temperature of 212 F, showed that the radial temperature variation was negligible.

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Author's Closure

It is gratifying that the present paper has been discussed from such different viewpoints. The author was pleased that even a mathematician such as Mr. Oldenburger found certain results of the paper to be somewhat surprising. In his analysis, the fact that finite temperatures are possible only for rather restricted values of the thickness of the coil is interpreted from the "physical situation," i.e. from the boundary conditions. He further states that, from a mathematical viewpoint, the practical case of the coil is only a special case of a wider mathematical complex covered by the differential equations used in the paper. The author, used to looking at a problem from a physical rather than from a mathematical viewpoint sees the solution of the puzzle in the fact that at any place of the coil an increase of temperature causes an increase of the electrical resistance, and an increase of the electrical resistance causes an increase of temperature if the electrical current is not changed. Hence only the relative strength of these two effects is decisive for the question whether the temperature increase is stopped at a finite temperature or never.

Mr. Davis deals with the case of nonlinear dependence of q'''/k on the temperature and gives a solution for a dependence of second order. On a former occasion[13] the author has mentioned that for the relation

$$\frac{d^2\theta}{dx^2} = \frac{-q'''}{k} = \psi(\theta)$$

where ψ is an arbitrary function of θ, the general solution is

$$x = \int \frac{d\theta}{\sqrt{C_1 + 2\int\psi(\theta)\cdot d\theta}} + C_2$$

with the constants of integration C_1 and C_2. This corresponds to Davis' Equation 72. However, from a practical standpoint it seems to be preferable to replace a nonlinear function ψ by two or more linear sections as an approximation and to apply to these the simpler solution for linear distribution as given in the paper.

The author had an opportunity to see Mr. Tramontini's unpublished manuscript. In fact, Equation [31] has been derived there for the case of constant thermal conductivity. Certainly the writer will not have expected any considerable temperature difference in the cross section of the fine platinum wire under consideration except for excessively strong heating.

Concerning Mr. Paschkis' discussion the author refers to the second sentence of Part 1 of his paper. According to that statement he is confident that Rogowski and Vieweg's coil with 1416 windings in a cross section of 3 sq in. can be considered as a quasi-homogeneous heat source and conductor and that application of the theory to this and similar cases gives satisfactory results. Therefore the author would be glad if Mr. Paschkis were right with his carefully formulated judgment that the method seems to be more fit for chemical reactions than for electrical coils. However, the author agrees with Mr. Paschkis in so far as a more exact experimental check of the theory than just by the tests of Rogowski and Vieweg would be desirable.

The author learned from Mr. Norris's discussion that Richter and Moore have determined the equivalent thermal conductivity using a graphical method for drawing the flow lines in the insulation. According to their curves the ratio of equivalent thermal conductivity to the conductivity of the insulation material of Rogowski and Vieweg's coil would be 3.8, instead of 3.7 as found by the author's less exact, but simpler method. This agreement is very satisfactory.

[13] "Allgemeine Grundlagen der Wärmeuebertragung," by M. Jakob in *Der Chemie-Ingenieur*, vol. 1, part 1, Akademische Verlagsgesellschaft, Berlin, 1933, p. 182.

Another item discussed by Mr. Norris is the case of a fixed heat dissipation which is essential for the precalculation of a coil. There is another practical case, namely, to find the maximum temperature of a given coil for a given amperage. Measurements of the electrical resistance of the coil, cold and under load, yield the mean temperature excess θ_m. The maximum temperature excess $\theta_0 = \theta_m/\varphi$. For the case of the plane plate, the old theory gave $\varphi = 2/3$, throughout. The new theory gives values of φ between 2/3 and $2/\pi$.

For the case, dealt with by Norris, Equation [24] yields $\theta_m = 66$ C. Then, according to the old theory, $\theta_0 = 1.5(66) = 99$ C instead of 100 C, according to the new theory. The difference in this procedure is so small because here $(d\theta/d\xi)_{\xi=1}$ is not the same for both theories. Applying the old theory, $d\theta/d\xi = -2\theta_0 = -2(99) = -198$ C; applying the new theory, $d\theta/d\xi = -\dfrac{\sigma \cdot \sin\sigma}{1 - \cos\sigma} = -1.91(100) = -191$ C. This shows that, using the old theory for calculating θ_0 from θ_m a $3^1/_2$ per cent too steep temperature gradient at the surface has been supposed; however, Fig. 1 shows that with a steeper gradient at $\xi = 1$ and parabolic temperature distribution the same point as in the new theory may be reached at $\xi = 0$, that is θ_0 may come out all right. Assuming a fixed value of θ_m namely, the one found by measurement, the temperature-distribution curves for the old and new theory in a θ,ξ-diagram will intersect between $\xi = 0$ and $\xi = 1$.

It is seen that engineers neglecting in their calculations the influence of nonuniformity of heat generation in a coil, had just luck; they scarcely will have foreseen that the two errors involved, in using the old theory, namely, too steep temperature drop at $\xi = 1$, and parabolic instead of cosinoidal temperature distribution, might cancel one another almost entirely. However, it is comforting that this happens, as Mr. Norris remarks at the beginning of his discussion.

The Numerical Solution of Heat-Conduction Problems

By H. W. Emmons,[1] Cambridge, Mass.

Two- and three-dimensional steady-flow heat-conduction problems are readily solved to any degree of approximation by application of the relaxation method introduced by Southwell. Except for the simplest geometric shapes and boundary conditions, the relaxation method is far superior to analytical methods of solution in point of time required to reach a given desired accuracy. The relaxation method has the further advantages of permitting the calculator to put into the calculation all the physical intuition he may have about the problem and, at the same time, to know at each step just how seriously his solution still differs from the correct answer. By an extension of the method, one-, two-, and three-dimensional transient-heat-flow problems are easily solved. For one-dimensional problems, the new method is identical to the graphical method of Schmidt.

Introduction

The steady-state one-dimensional transfer of heat is easily calculated by the use of Fourier's heat-conduction equation

$$Q = -kA\frac{dT}{dx} \qquad [1]$$

The symmetrical transfer of heat between concentric cylinders and spheres is essentially a one-dimensional problem and yields to Equation [1]. The result is most simply expressed in terms of mean areas; log mean for cylinders and geometric mean for spheres.

For all other problems of steady-state heat conduction and all problems of nonsteady heat conduction, the analytical solutions are not very simple and recourse is made, when possible, to experimental (1)[2] or graphically presented (2) results.

Many attempts have been made to derive solutions by numerical and graphical methods. Schmidt (3), in 1924, published his graphical method of solution of nonsteady conduction problems. This method has been used many times without essential change (4, 5).

The corresponding numerical methods for solving the LaPlace equation have generally not been concerned directly with the steady-state heat-conduction problem but are, of course, directly applicable. Some of the attempts have been described (6, 7, 8, 9). All these methods are laborious at best and leave much to be desired from a practical point of view.

It is only with the introduction of the idea of relaxation by R. V. Southwell (10) that the numerical method comes to life as a practical method of solution. In the following section, the relaxation method for the solution of steady-state heat-conduction problems in two and three dimensions is given. The method is identical to that of Southwell (10), except that the physical picture has been recast in terms of heat flow rather than forces on a soap film. In a final section, the generalization of the method of Schmidt (3) for solving nonsteady heat-flow problems is given and its connection with the relaxation method is noted.

Numerical Solution of Steady-State Heat-Conduction Problems

To illustrate the method of relaxation consider the two-dimensional problem of Fig. 1. Suppose the temperature is known on the boundary and that the temperature distribution and heat flow inside are desired. Instead of considering the body as a continuum, let it be replaced by a net of conducting rods. Each rod, between points 0–1 for example, represents the material in the small square drawn, in so far as the conduction of heat in the horizontal direction is concerned. Thus, the heat conducted to point 0 from point 1 along the rod 1–0 is

$${}_1Q_0 = kb(T_1 - T_0) \qquad [2]$$

where T_1, T_0 are the temperatures of points 1 and 0, respectively; k is the thermal conductivity of the material; b is the length, in the third dimension, of the body being considered.

Conduction of heat along the rods 2–0, 3–0, 4–0 yield similar expressions, the total being

$$Q_0 = kb(T_1 + T_2 + T_3 + T_4 - 4T_0) \qquad [3]$$

Since k, b are constants for a given solution, it is more convenient to use the heat flow

$$Q_0' = \frac{Q_0}{kb} = T_1 + T_2 + T_3 + T_4 - 4T_0 \qquad [4]$$

This can be considered as the heat flow in altered units, the unit being the amount of heat that would flow along a rod with unit temperature difference.

Term Q' represents the rate at which energy must be removed from point 0 if its temperature and that of the surrounding points are to be maintained fixed. Thus the term Q' is the strength

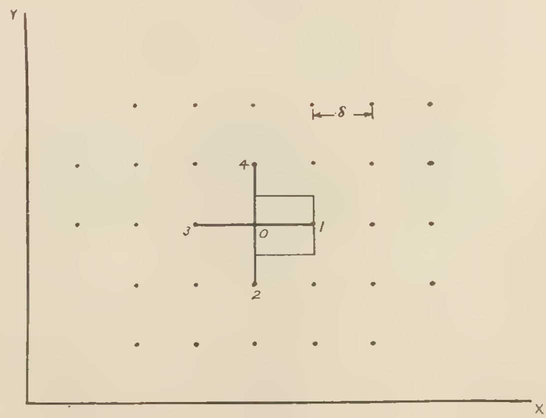

Fig. 1

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Heat Transfer Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

of the heat sink required at each of the lattice points. For steady-state heat conduction Q' must be zero at every interior point.[3] Boundary points may have any value of T or Q' consistent with the boundary conditions. Along a boundary of fixed temperature, Q' may have any value. Along an insulated boundary Q' must be zero as at interior points. Along a boundary where a heat-transfer coefficient is given, Q' must be related to the boundary temperature T_0 by use of the equation

$$Q = h\delta b(T_s - T_0) \qquad [5]$$

where T_s is the surrounding temperature from which heat is being transferred.

To solve a problem, lay out the two-dimensional region to scale and put on a square net of points. It is well to start with only a few points, that is, a large value of δ. Next, assume values of the temperature at each point and calculate the strength of sink Q', required to maintain it. At this point enters the idea of relaxation. Consider first the point of largest Q'. Make this value zero by changing the temperature of the point. According to Equation [4], a change of Q_0' equal to —4 times the change of T_0 will result while Q_1', Q_2', Q_3', Q_4' will change by the change of T_0, all other temperatures being held constant. Thus, Equation [4] describes the relaxation pattern of Fig. 2, which shows the way in which a sink originally at point 0 is distributed to points 1, 2, 3, and 4 by changing the temperature of point 0 by one unit.

The final condition that Q' equal zero at all interior points is met by a continued application of the relaxation process aided with whatever physical intuition the calculator is able to add. Thus, if the temperature appears to be rising in a certain region, overshoot the zero conditions of Q' so that the temperature will rise more rapidly toward its final value. No harm is done by overshooting the final value of temperature, as successive calculations will establish it again.

A simple problem is solved in Fig. 3. All the work is shown. Two successively more accurate solutions are shown. Each new solution starts from the results obtained in the previous one. The heat loss from the entire furnace is found by applying Equation [2] to all the rods which start from the inside wall. In the successive solutions, the thermal resistance takes successively more accurate values.

[3] In the Appendix, a more rigorous derivation of the finite-difference formulas from the differential equations of heat conduction is given. The derivation in the paper is preferred only because of the physical picture related to the calculations.

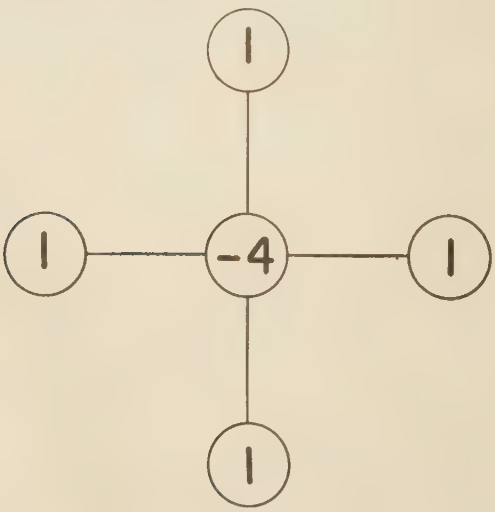

Fig. 2

The speed and accuracy of the relaxation method of solution can be judged from Table 1, in which the time for solution and the deviation of the results from experimental values are given. A computing machine is not required for the relaxation method. The same problem solved by the method of Shortley and Weller

TABLE 1 TIME FOR SOLUTION AND DEVIATION FROM EXPERIMENTAL RESULTS

Number of points used	Calculated thermal resistance, Btu per deg F per hr	Deviation from experimental solution, per cent	Hours required for calculation
Two-dimensional problem			
By arithmetic-mean area	$\frac{0.0735}{kb}$	15.3	0.05
By logarithmic mean area	$\frac{0.0758}{kb}$	11.9	0.08
12	$\frac{0.0806}{kb}$	5	0.75
19	$\frac{0.0825}{kb}$	2.8	1.75
Three-dimensional problem			
By arithmetic-mean area	$\frac{0.0167}{kL}$	36.3	0.05
By geometric mean area	$\frac{0.0204}{kL}$	22.0	0.05
31	$\frac{0.0229}{kL}$	12.6	1
145	$\frac{0.0252}{kL}$	3.9	10

(7) took 11 hr with the use of a calculating machine.

The great ease of the relaxation method prompted a student, Mr. Frank Lockard, to attempt the corresponding three-dimensional problem. His solution is given in Fig. 4. Each point is surrounded by six other points by using a square lattice of points. The corresponding formulas are: For six points surrounding each point Fig. 5

$$_1Q_0 = k\,\delta\,(T_1 - T_0) \qquad [6]$$

$$Q' = \frac{Q}{k\delta} = (T_1 + T_2 + T_3 + T_4 + T_5 + T_6 - 6T_0) \qquad [7]$$

An intermediate number of points between the solutions, Fig. 4 (a and b), could have been obtained by surrounding each point with eight points, by using a body-centered square lattice of points. For eight points surrounding each point, Fig. 6, the formulas are

$$_1Q_0 = \frac{1}{2}\,k\,\delta(T_1 - T_0) \qquad [8]$$

$$Q' = 2\,\frac{Q_0}{k\delta} = (T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 - 8T_0) \qquad [9]$$

Only the results are shown in Fig. 4. Note the check of the final value for the thermal resistance with the experimental value obtained by Langmuir (1). Table 1 summarizes both the two- and three-dimensional solutions and their checks with experimental values.

Numerical Solution of Nonsteady Heat-Conduction Problems

In the consideration of a steady two-dimensional heat flow in Fig. 1, sinks of heat were required at each lattice point to maintain constant point temperatures until the relaxation method removed the excess heat flow. Consider the five points of a two-dimensional lattice shown in Fig. 7. The excess heat flow to point 0, Q_0 of Equation [3], is physically used to heat the material in the surrounding square. The change of temperature of this material during time δ_t is related to Q_0 by

$$Q_0\delta_t = \rho\delta^2 bc\{T_0(t + \delta_t) - T_0\} \qquad [10]$$

By substituting Q_0 from Equation [3]

Start solution with linear temperature distribution

Relax Q' to get final temperature

Heat transferred is

$$Q = kb\left(232 + 208 + \frac{4.5}{5} \times 202\right)$$
$$= 622\ kb$$

Thermal resistance

$$R = \frac{\Delta T}{8Q} = \frac{400}{8 \times 622\ kb} = \frac{0.0806}{kb}$$

Superscripts indicate step of calculation

° = Original values

1, 2, 3 = Successive steps

Start with solution of course net Fig. 3a

Relax Q' to get final temperature

Heat transferred

$$Q = kb\left(\frac{195}{2} + 2 \times 111 + 2 \times 102 + \frac{4}{5}\,102\right)$$
$$= 606\ kb$$

Thermal resistance

$$R = \frac{\Delta T}{8Q} = \frac{400}{8 \times 606\ kb} = \frac{0.0825}{kb}$$

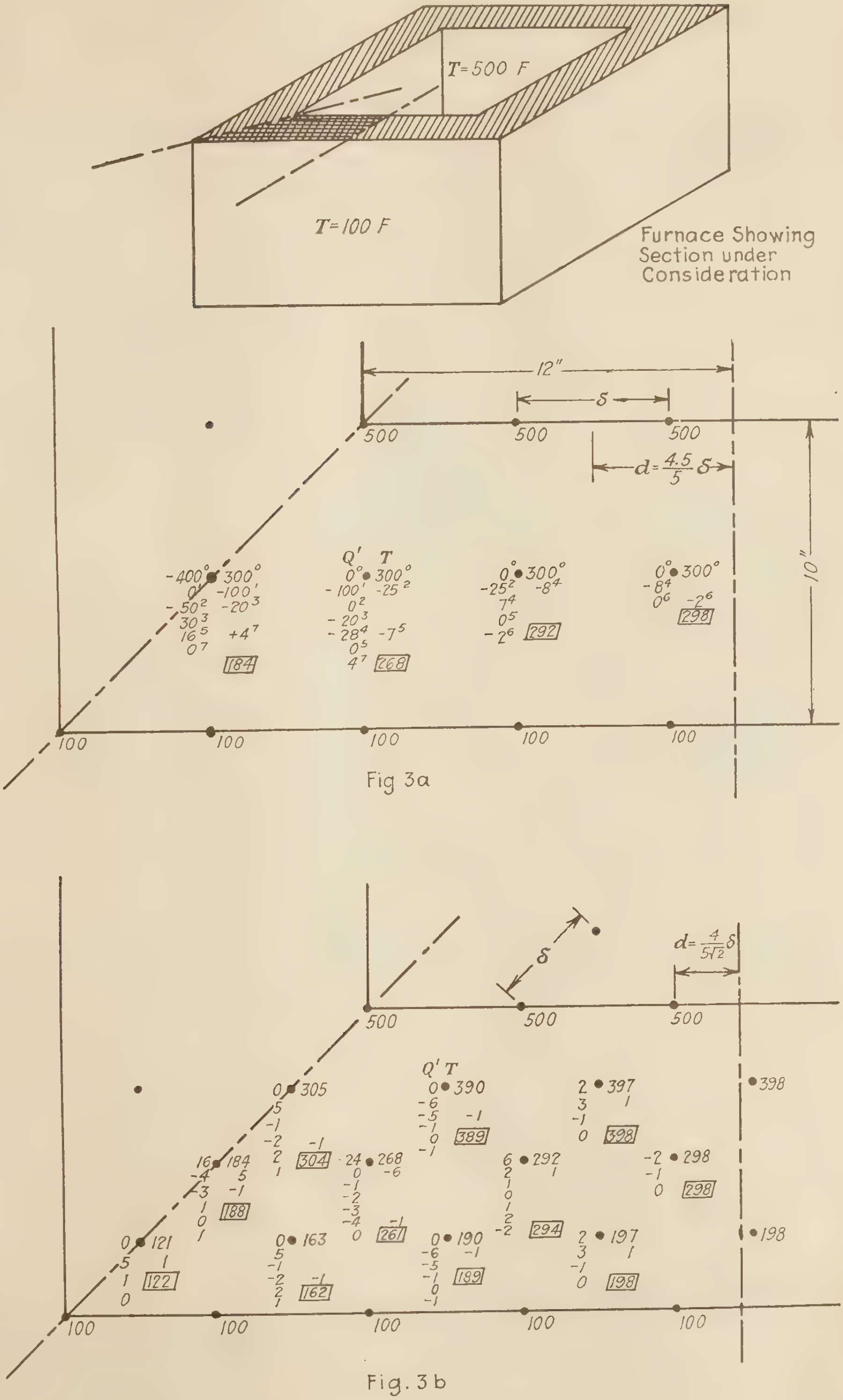

FIG. 3

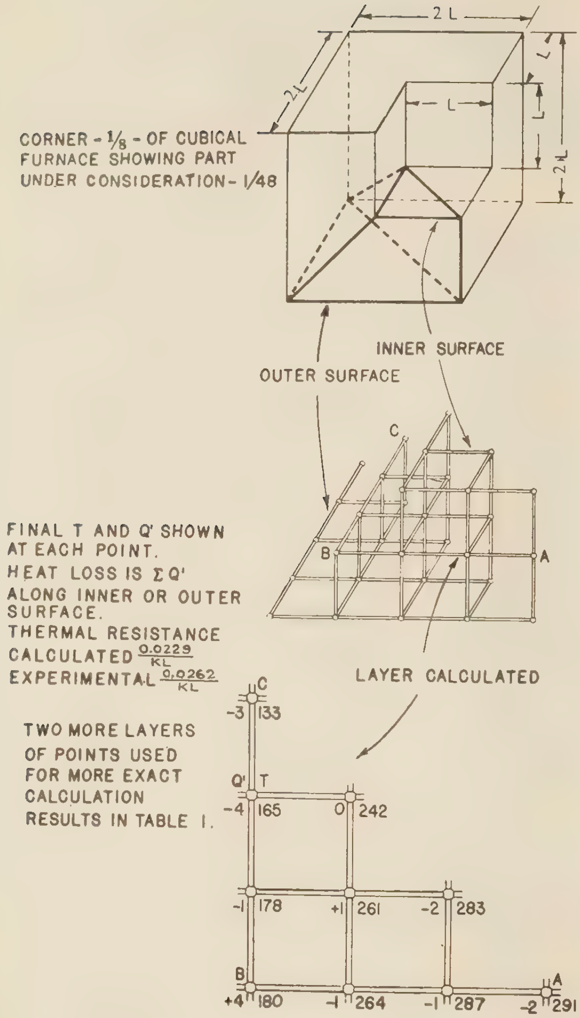

Fig. 4

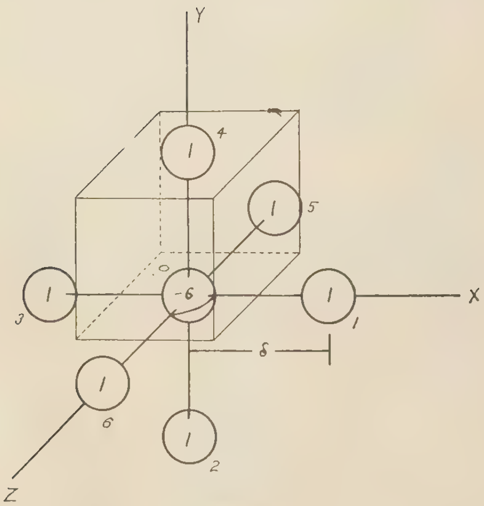

Fig. 5

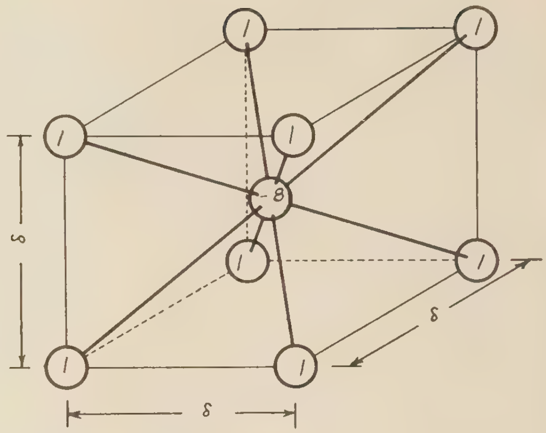

Fig. 6

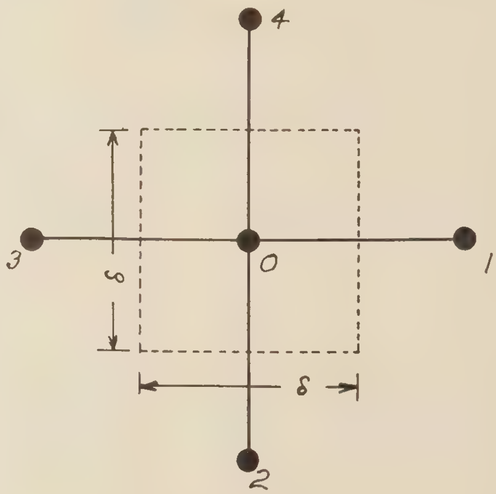

Fig. 7

$$(T_1 + T_2 + T_3 + T_4 - 4T_0)\delta_t = \frac{\delta^2}{a}\left\{T_0(t + \delta_t) - T_0\right\} \quad [11]$$

where $a = \dfrac{k}{\rho c}$ = thermal diffusivity.

Since δ_t is an arbitrary (small) time increment choose

$$\delta_t = \frac{\delta^2}{4a} \quad [12]$$

Then Equation [10] becomes

$$T_0(t + \delta_t) = \frac{T_1 + T_2 + T_3 + T_4}{4} \quad [13]$$

This equation, derived in a more rigorous but less physical way, has been the basis of most of the previous numerical methods for solving LaPlace's equation (6, 7, 8, 9). These methods of solving the steady-state heat-conduction problem start from an arbitrary temperature distribution and proceed, essentially as nature proceeds, toward the steady-state values. Southwell's relaxation method is thus seen to make its big gain in ease of solution by cutting loose from the physical method of heat-flow equalization and by substituting an arbitrary process under the direct control of the calculator. The less convenient process of Equation [13]

Transient heating of furnace wall. At time 0 wall is at uniform temperature T = 100 F. Inner-surface temperature suddenly raised to 500 F.

a = 0.01 ft²/hr, δ_x = 0.208 ft, δ_t = 1.083 hr.

The six temperatures shown at each point are at intervals of 1.083 hr.

The heat loss is $\Sigma Q'$ along inner surface.

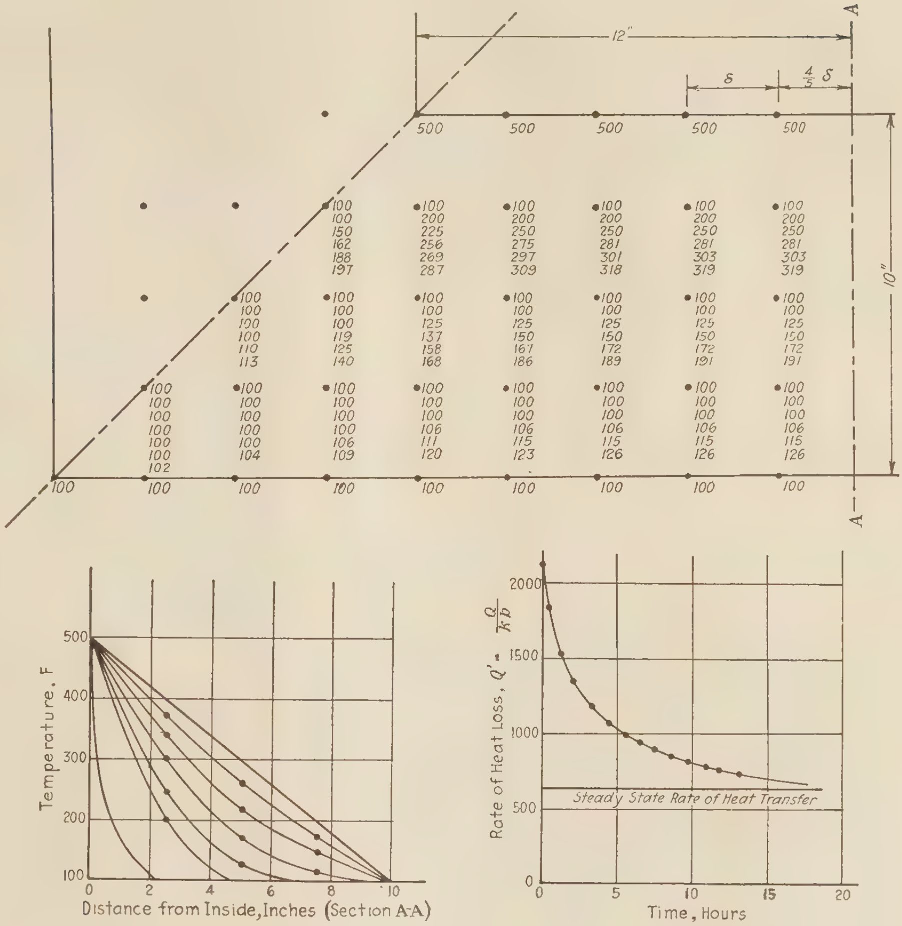

FIG. 8

is now reserved for the solution of the two-dimensional nonsteady heat-flow problems. Fig. 8 illustrates such a solution.

Problems of nonsteady heat flow in a rod can be similarly attacked and yield the equations

$$\delta_t = \frac{\delta^2}{2a}, \quad T_0(t + \delta_t) = \frac{T_1 + T_2}{2} \qquad [14]$$

corresponding to Equations [12] and [13]. The graphical method of E. Schmidt (3) is based on Equation [14], which indicates a simple averaging process accomplished by drawing straight lines. Thus, this well-known graphical method of solution of one-dimensional transient-heat-flow problems is closely related to the numerical methods of solution of two-dimensional heat-flow problems.

Appendix

Derivation of Difference Equations From Differential Equations

In this paper, the physical problem of one, two, or three dimensions has been replaced by an approximation in which a network of conducting rods replaces the physical continuum. Engineering problems always involve more or less inaccurately known boundaries or boundary conditions so that the rod approximation can always be chosen with "sufficient accuracy." The equations derived in the paper are exact for the physically approximate rod problem. The same equations can be derived for the physically more exact continuum by converting the exact differential equation to the approximate difference equation.

To illustrate, consider the two-dimensional nonsteady heat-conduction problem described by

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad [15]$$

By definition of the partial derivative

$$\frac{\partial T}{\partial t} = \int_{\delta \to 0} \frac{T(x, y, t + \delta_t) - T(x, y, t)}{\delta_t} \qquad [16]$$

As an approximation, the limit operation can be omitted provided δ_t is "sufficiently small." In the same way, referring to Fig. 1, the second derivatives can be approximated as follows

$$\left(\frac{\partial T}{\partial x}\right)_3 \sim \frac{T_0(t) - T_3(t)}{\delta}$$

$$\left(\frac{\partial T}{\partial x}\right)_0 \sim \frac{T_1(t) - T_0(t)}{\delta}$$

Therefore

$$\frac{\partial^2 T}{\partial x^2} \sim \frac{\left(\frac{\partial T}{\partial x}\right)_0 - \left(\frac{\partial T}{\partial x}\right)_3}{\delta} = \frac{T_1(t) - 2T_0(t) + T_3(t)}{\delta^2} \quad [17]$$

By substitution of such expressions into the differential Equation [15], there results the approximate finite-difference equation

$$\frac{T_0(t + \delta_t) - T_0}{\delta_t} = a\frac{T_1 + T_2 + T_3 + T_4 - 4T_0}{\delta^2} \quad [18]$$

For steady-state conduction, the left side of Equation [18] must be zero for all interior points, and thus follows Equation [4] of the paper with $Q' = 0$.

For nonsteady conduction put

$$\delta_t = \frac{\delta^2}{4a} \quad [19]$$

which is possible since δ_t and δ are arbitrary small quantities. Equation [18] then becomes

$$T_0(t + \delta_t) = \frac{T_1 + T_2 + T_3 + T_4}{4} \quad [20]$$

Equations [19] and [20] are identical to Equations [12] and [13] of the paper.

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Discussion

G. M. DUSINBERRE.[3] The writer and others had been working on a heat-transmission problem which presented some unusual features. About a week before the Annual Meeting (1942), this problem was offered to the author for solution by the numerical method. The fact that an analytical solution had already been obtained gives a further check on the accuracy of the method, but the impressive point is the speed with which the author obtained a solution.

[3] Assistant Professor of Mechanical Engineering, Virginia Polytechnic Institute, Blacksburg, Va.; now on duty at U. S. Naval Academy, Annapolis, Md. Mem. A.S.M.E.

The problem was to determine the surface and interior temperature distribution in a waterwall boiler tube exposed to the furnace on one side only. The following assumptions were considered justified by the physical situation:

(*a*) Conduction in the axial direction of the tube was negligible, giving a two-dimensional steady-state problem.

(*b*) There was negligible film resistance on the water side, making the inner boundary isothermal at the temperature of the evaporating liquid.

(*c*) Conduction was negligible across the outer semicircumference away from the fire.

(*d*) Convection was negligible on the exposed semicircumference, leaving only radiation to consider.

(*e*) In regard to the net radiation, variation in the temperature factor and emissivity factor was negligible, leaving only the configuration factor to consider.

Under these conditions the specification for the exposed boundary is the heat input per unit area or, mathematically, the normal derivative of the temperature.

The particular condition offered for analysis was that of parallel symmetrical radiation, resulting in uniform heat input per unit of projected area or a cosine distribution per unit of developed area.

Perhaps a brief account of the writer's experience with this problem will illustrate the merit of the author's numerical method.

An experimental solution was first attempted, using an electrical analogy. A copper disk represented the isothermal interior of the tube. A sheet of gelatin made slightly conducting represented the tube metal and the region exterior to the tube. A second copper terminal served to feed in current. Nonconducting boundaries were represented by trimming away the gelatin. Equipotential lines, representing isotherms, were determined with a pair of probes. Curves were obtained showing the general trend but the results were not considered satisfactory, because of the failure to represent the discontinuity at the tube surface. (This, however, could have been corrected by subdividing the electrolyte outside the region representing the tube.)

A study was then made of numerical methods but the literature did not reveal a satisfactory method of dealing with a boundary where a partial derivative rather than the function itself was given.

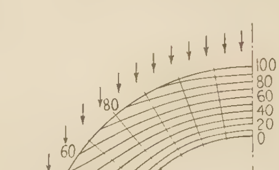

FIG. 9

Finally the analytical method was used and solutions obtained[4] for this case and some others. This involved painstaking evaluation of Fourier coefficients followed by laborious computation of a large number of points, each requiring five or six terms of the series. The only check against errors of computation was the fairness of the plotted curves, and this check was available only at the last stage of the work.

The results of the analytical solution are shown as Fig. 9 of this discussion and may be compared with the author's results.

[4] Only with the assistance of the writer's associates, Messrs. W. S. Kimball and H. G. Elrod.

The author can state the time which was required for his work.

M. Jakob.[5] The author called the method of finite differences for determining the temperature change in transient heat conduction "Schmidt's method" as usual. Recently, however, the writer discovered that almost the same method was used and published long before Schmidt's first publication[6] in a doctors' dissertation of Binder which has appeared in book form.[7] Almost six pages of this booklet are devoted to that method. It is not generally known either that the first application of the method to cylindrical and spherical bodies is due to two French engineers, Nessi and Nisolle.[8]

C. F. Kayan.[9] As one who has tackled the problem of heat flow from the viewpoint of the electrical-conduction analogy, the writer wishes to compliment the author on this new development. That it should prove a very useful tool in solving heat-conduction problems must be readily appreciated, although it would appear to have better applicability to certain problems in the steady state rather than to problems in the transient state.

Is it possible to start with only a few points and later to add more, or must the points be used immediately from the very start? The small amount of time required is indeed attractive, and the question arises as to whether on detailed refinement the time is greatly increased, or just merely increased proportionally to the number of points involved. Thus, it might be possible to start with a rough "survey" first, and then follow up on detailed analysis only where needed.

V. Paschkis.[10] In closing his presentation, the author stated that each of the various methods of solving heat-flow problems has its field of useful application. This statement is as true as it is important.

He mentioned two methods based on the application of finite differences, i.e., the graphical method and the numerical. There is, however, a third method, that of electrical analogy. It is especially well adapted to the treatment of transient phenomena.

It is remarkable how close this method is related to the numerical method, presented by the author. Instead of "rods" take resistors, representing the thermal resistance, and add at the intersection of the rods, electrical condensers (representing the thermal capacity), and you have the complete scheme of the electrical network used for the solution of heat-flow problems.

The network previously described has been arranged at Columbia University so as to be very flexible capable of solving a great number of problems.

In weighing the speed of work of numerical solution as against the electrical method, the question of periodic heat flow, possibly covering long periods, or very slow transient phenomena should be mentioned. In this case, the electrical method seems advantageous.

In his work with the electrical analyzer, the writer has encountered some interesting questions, which probably the numerical and the electrical method have in common. Refer to Fig. 10 of this discussion, representing two plates. Heat is generated by friction at the interface. How should the section be taken (for transient or periodic heat flow)?

Taking nonsquare sections (e.g. thin and long) will yield wrong results (obscuring lateral heat flow).

Taking thicker sections is inaccurate, because most of the phenomena occur very close to the surface.

Taking many small square sections increases the amount of work considerably, especially with the numerical solution.

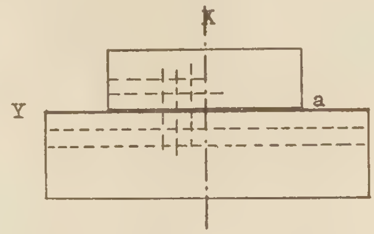

Fig. 10

Suggestions of how to proceed with subdividing for the numerical solution would be very interesting.

L. T. Wright, Jr.[11] Conduction is the mode of heat transfer which lends itself most readily to analytical study, but it is interesting to notice that engineers are really just beginning to consider any but the simplest cases of steady conduction.

Several comments on the numerical solution of unsteady-heat-conduction problems by the approximation method first introduced by Schmidt may prove useful. For materials with high thermal diffusivity (such as concrete), the simplification introduced by taking $\delta_t = \delta_x^2/2a$ leads to an inconveniently large number of time increments when small space subdivisions are used. For example, in the case of a 3-in. layer of concrete with $a = 0.045$ sq ft per hr, the first approximation might well start with three subdivisions of the layer. Thus with $\delta_x = {}^1/_{12} = 0.0833$ ft, $\delta_t = 0.077$ hr, or 4.6 min. If the calculations are to be carried for a period of 24 hr, there will be some 1250 entries in a tabular solution for just the first approximation. If the next approximation uses a value of δ_x one half as large as the preceding one, the number of entries in the tabular solution will be approximately 8750. The writer attempted to overcome this difficulty by using a different relationship between δ_t and δ_x, but it was found that convergence was poor.

In some cases a relatively small value of δ_x must be used to obtain a good approximation of the accurate solution. It was found that a good approximation to the accurate solution at any particular time t' could be obtained from three or four rough approximations using large values of δ_x by plotting the temperature at t' found from these approximations against δ_x and extrapolating to $\delta_x = 0$.

One important type of unsteady-conduction problem is that in which the condition of periodicity in time replaces an assumed initial-temperature distribution. The unsteady heat flow in a building wall, due to the variable outdoor-air temperature and the variable solar radiation, is of this type. While an initial distribution of temperature may be assumed in this case, it will require the results of the calculations of at least two complete daily cycles to remove the effects of a wrong initial assumption.

The Schmidt method may be extended to the case of two or more materials in series. In this extension, it is not always

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[6] "Über die Anwendung der Differenzenrechnung auf technische Anheiz—und Abkuehlungs—probleme." Festschrift zum siebzigsten Geburtstag August Foeppls, by E. Schmidt, Berlin, 1924, p. 179.

[7] "Über aeussere Wärmeleitung und Erwärmung elektrischer Maschinen," Dissertation, Techn. Hochschule Muenchen, by L. Binder, Wilhelm Knapp, Halle, 1911, pp. 20–26.

[8] "Méthodes graphiques pour l'étude des installations de chauffage et de réfrigération en régime discontinu," by A. Nessi and L. Nisolle, Dunod, Paris, 1929.

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possible to take $\delta_t = \delta_x^2/2a$ in each of the materials, for δ_t must be the same for each material. This requires that the ratio of the thicknesses of the materials must be some integral multiple of the square root of the ratio of their thermal diffussivities; i.e., L_1/L_2 must be an integral multiple of $\sqrt{(a_1/a_2)}$.

There are available, in general, three methods of solving a heat-conduction problem; an analytical solution, an approximate numerical solution, or a solution by electrical analogy. The solution by electrical analogy can usually be ruled out from the start because of the very limited availability of the expensive and complicated equipment required. The choice in the usual case lies then between an analytical solution and an approximate numerical solution. If the problem under study is a specific problem dealing with given materials and conditions, the numerical solution is probably the better. If, however, the problem is a general study, such as the general effects of various thermal properties of the materials, the writer believes that an analytical solution with its attendant calculations will prove more useful than a number of approximate solutions. For example, in the study of the general effects of the thermal properties of building materials on the unsteady heat conduction through building walls, the use of numerical solutions requires more time than does the analytical solution.

Author's Closure

The relaxation method makes generally available to engineers accurate solutions of complex steady heat-conduction problems which would not be attempted by conventional analytical or numerical methods. The water-tube problem suggested by Mr. Dusinberre is typical. The numerical solution of this problem using the new method was obtained in about 10 hours. The analytical and numerical solutions were identical when drawn to a scale about three times the size of Fig. 9 of Mr. Dusinberre's discussion. During the 10 hours spent on the problem, three successively more accurate solutions were obtained; the first one of which would have been adequate for most practical purposes. The first approximation was obtained in 2 hours, the second at the end of three hours. Of the two hours spent on the first approximation, only 15 minutes was required for the relaxation process, the other $1^3/_4$ hours was required for the transformation and boundary-condition calculation.

Several features of this water-tube problem are different from those described in the paper and make it worth a few words of explanation. The concentric cylindrical form makes it difficult to replace the conducting continuum by rods in the manner desired, and the variable heat-flow boundary condition needs some special treatment. When solving a problem by numerical methods, facts disclosed by the conventional analytical methods should not be forgotten. Instead of solving Laplace's equation for the temperature in the x, y (real) plane, it is better to transform conformally by $z = \ln r$ and solve the problem in the z, θ plane. It is well known that Laplace's equation is invariant on conformal transformation so that the net of points is placed in the rectangular region in the z, θ plane.

To treat the boundary condition, it is first transformed to the z, θ plane by the foregoing defintion of z. Then a special formula is set up for the boundary point

$$Q' = \frac{1}{2}T_1 + T_2 + \frac{1}{2}T_3 - 2T_0 + \frac{\partial T}{\partial z} \qquad [21]$$

Here Q' is to be relaxed to zero by this special relaxation pattern. Since the boundary heat flow term $\dfrac{\partial T}{\partial z}$ is constant it enters in calculating Q' at the start of the problem but never enters the relaxation pattern, and thus does not affect the ease of solution.

The addition of fins to the watertube can be handled rather easily in the z, θ plane by the addition of the appropriate conducting rods. In this case, it might well be desirable to add extra points on a finer net in the vicinity of the fins in order to improve the accuracy in the region of more rapidly changing temperature. At any time during a solution, extra points can be added in regions that require them.

Mr. Kayan's idea of using a few points for a rough survey followed by more points where needed is exactly what should be done. The work of relaxation is proportional to the number of points used, so that a fine net should only be used where temperature gradients change rapidly.

The remainder of the discussions were directed to the solution of transient-heat-flow problems. These problems are not solved by the use of the relaxation idea. Their solution is a slight modification of the standard finite-difference methods and is related to the relaxation method, as noted in the paper. As Mr. Wright specifically notes, the simplest relation between δ_t and δ_x sometimes yields awkward increments requiring many points. This is perfectly correct and no development of numerical methods to date can offer much help. It is in this field of transients that the electrical networks offer promise.

It is not clear to the author exactly what problem is intended in Fig. 10 of Mr. Paschkis's discussion, but in general the numerical method of treating any problem with some sort of discontinuity is a matter of piecing together known facts. It is believed to be correct to say that no method of solution makes greater use of all that is known *a priori* about the problem. In this case, for example, as far as the discontinuity and its immediate neighborhood are concerned, the bodies are semi-infinite solids. Analytical solutions for such discontinuities are known and generally simple. Thus, the numerical solution would proceed from the external boundaries and overlap the known solution (for the discontinuity in a semi-infinite solid). The numerical solution would thus alter the known solution in all regions which are affected by the real external boundaries. Fig. 10 could well be treated in this way.

Several individuals have privately requested that a list of things to be considered in solving a problem numerically be added to the discussion as a guide to others attempting such solutions. It is impossible to give any rigid set of instructions for a method the beauty of which lies in its flexibility. However, operating instructions would run about as follows:

1 Study the problem, mentally, until all conditions are clearly understood, especially the discontinuities, and all "obvious" facts are discovered. The "obvious" of course varies with the computer, but the more the better.

2 Transform the problem into one with as nearly rectangular boundaries as possible using only "simple" transformations. By simple is meant something considered simple to the operator.

3 In the transformed problem (or original if no simple transformation is known) note all discontinuities (if any), and replace them by known solutions good in their immediate neighborhood. For steady-state-conduction problems, the most common discontinuity is the point source whose solution is the logarithmic radial temperature distribution in the immediate vicinity of the source.

4 Divide the conducting material into a conducting-rod network with a few points only; about a dozen is usually enough as a starter.

5 Derive any special formulas needed for the boundary conditions.

6 In an auxiliary freehand sketch make a "first-approxima-

tion" solution by the field-mapping method. This is useful but not an essential step.

7 By the result of item 6 plus all known facts, guess values of the temperatures at all net points.

8 Calculate Q' for all points, using the special formulas of item 5 for the boundary points.

9 Relax Q' beginning at points where Q' is largest. If all points surrounding a given point 0 have Q''s with the same sign as Q'_0 be sure to overshoot the zero of Q'_0 by a considerable margin, perhaps leaving a Q'_0 of almost the same magnitude but opposite sign. If a group of adjacent points have nearly the same Q''s, change them as a group; i.e., change all the temperatures by the same amount and use Equation [2] of the paper for each rod individually, only rods going out of the group being counted.

10 After Q' has been reduced to zero for every point, add more points on a finer net and follow steps 8 and 9, if additional accuracy is desired.

11 After the desired number of points has been calculated check Q's to discover any errors introduced during the work. If any Q's remain relax them as before.

12 From the solution retransform to the real plane or calculate the desired heat flows from the temperature gradients found in the solution.

Effectiveness of Shear-Stressed Rubber Compounds in Isolating Machinery Vibration[1]

By BAXTER C. MADDEN, Jr.,[2] DAYTON, OHIO

This paper is concerned with determining the effectiveness of rubber compounds in resilient mountings in the isolation of vibrating machinery where the rubber compound is stressed principally in shear. Seven rubber compounds were investigated. Certain considerations to be taken into account when appraising the effectiveness of a resilient mounting are stated. A dimensionless criterion is developed for the effectiveness of a rubber compound in terms of physical properties measurable by testing apparatus. The criterion is evaluated for each of the seven compounds. The results indicate that the criterion is worthy of consideration.

RUBBER has been widely used as the elastic medium in machinery mountings because of its resiliency, inherent damping, lack of static friction, excellent sound-absorbing quality, strong adhesion to metal, and because it requires no lubrication. While gum-rubber compounds may not be available for some time, the principles set forth may be applied equally well to rubberlike materials. The general problem of the mitigation of machinery vibration by means of elastic mountings is well covered by many authors (1, 2, 3, 4, 5).[3]

The investigation (6), upon which this paper is based was concerned with the determination of the effectiveness of various rubber compounds, stressed principally in shear, in the isolation of vibrating machinery. This refers to the mechanical and dynamic properties of unaged rubber compounds, without reference to qualities such as drift, oxidation, or continuation of vulcanization.

In this investigation, the general case of a machine supported by an elastic mounting, in which 6 deg of freedom exist, was reduced to an idealized system having only 1 deg of freedom, namely, linear deflection along one axis. The experimental equipment was so constructed that the test specimens were loaded dynamically and subjected to shear in one direction. In most machinery installations it is possible to design a rubber mounting in which all troublesome modes of vibration cause shear deformation of the rubber element. Thus the results of this investigation are applicable to installations in which more complex vibrations exist.

Study of Rubber Machinery Mountings

The first phase of the investigation was to learn the behavior of typical rubber compounds when subjected to vibration simulating service conditions of machinery mountings. This involved the use of a typical mounting, the style and important dimensions of which are indicated in Fig. 1. Seven such mountings were made, each employing a different composition of rubber. These compositions, together with the cure and hardness, are shown in Table 1. The chemical components and the hardness values agree generally with those recommended for motor mountings by well-known suppliers of rubber-compounding materials (7, 8). Hardness tests, using a type A Durometer, of 22 mountings, used in 12 makes of 1941 model automobiles, revealed values for the rubber ranging from 33 to 57, the average being 41. The rubber compounds used in the investigation were therefore considered as approximately representing the range of chemical composition and hardness encountered in successful applications of rubber machinery mountings.

TABLE 1 COMPOSITIONS OF COMPOUNDS TESTED

Designation of compound	*A*	*B*	*D*	*F*	*G*	*J*	*K*
Parts by weight:							
Smoked sheet (rubber)	100	100	100	100	100	100	100
Channel carbon black	..	..	..	20	20	..	50
Gastex carbon black	..	..	..	20	20	..	..
P-33 carbon black	..	25	25	..	..	..	..
Stearic acid	1.5	1.5	1.5	2.0	2.0	1.0	3.0
Pine tar	..	..	..	2.0	2.0	..	3.0
Neozone A (phenyl-α-naphthyl-amine)	..	..	..	..	..	1.0	1.0
Neozone D (phenyl-β-naphthyl-amine)	1.0	1.0	1.0	1.5	1.5	..	..
Ureka C (benzothiazyl-thio-benzoate)	0.6	0.6	0.6	..		..	
Captax (mercapto-benzothiazole)	..	..	..	1.0	1.0	1.0	1.0
Zimate (zinc dimethyl-dithio-carbamate)	..	..	..	0.1	0.1	..	
Zinc oxide	5	5	30	5	5	5	3
Diphenyl-guanidine	..	..	..	..	..	0.1	
Guantal (diphenyl-guanidine-phthalate)	0.4	0.4	0.4	..	..	..	..
Sulphur	3.0	3.0	3.0	2.5	2.5	2.0	3.0
Cure:							
Time, min	25	25	25	20	40	40	40
Temperature, F	307	307	307	307	307	307	307
Hardness: Durometer "A"	40	47	52	43	45	30	60

NOTE: Compound *K* contained 1 part Heliozone (a wax).

The laboratory testing apparatus consisted essentially of a motor-driven eccentric with a connecting rod which imparted a practically simple harmonic vertical motion to *B* (see Fig. 1), the amplitude of which was approximately 0.0015 in., the frequency being variable from zero to 50 cycles per sec. A dead-weight load on *A* was variable, approximately from 10 to 100 lb. The motions of both *A* and *B* were restricted substantially to vertical displacements by means of radius rods approximately 4 ft long. Two modified Geiger torsiographs, one connected to each member of the mounting, were so arranged that the vibrations of *A* and *B* were recorded simultaneously on the same paper tape. Thus the frequency of the vibrations at any instant could be measured, as well as the amplitude of *A* and the amplitude of *B*. With *B* stationary, it was possible to displace and suddenly release *A*, recording the ensuing free vibration of *A*. In this manner the natural frequency of the free vibration of the loaded mounting was determined. Deflections over a range of static loads were found by applying known weights and measuring the deflections recorded by the torsiographs.

Behavior of Shear-Stressed Rubber Mounting

It was theorized that a shear-stressed rubber mounting might behave similarly to a steel spring with a dashpot connected to its

[1] This paper was prepared from the author's thesis entitled, "The Effectiveness of Shear-Stressed Rubber Compounds in the Isolation of Vibrating Machinery," submitted in partial fulfillment of the requirements for the degree of Master of Science at the University of California.

[2] Major, Air Corps, Army Air Forces Matériel Center, Wright Field.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Aviation Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

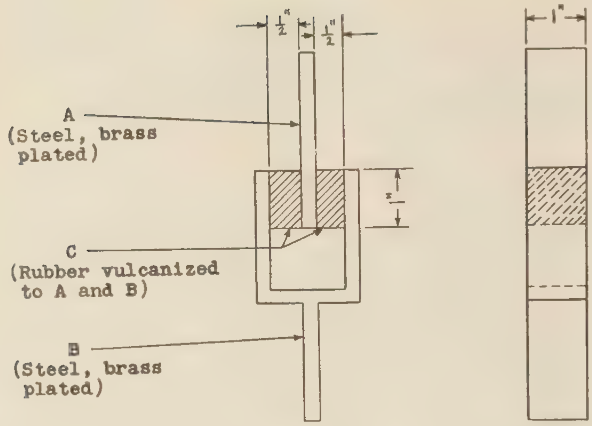

FIG. 1 DETAILS OF TYPICAL RUBBER MACHINERY MOUNTING USED EXPERIMENTALLY

terminals to provide viscous damping. In any resiliently mounted machine having vibratory forces set up within it, the ratio of the periodic force, transmitted to the foundation structure by the mounting, to the periodic force impressed upon the machine is defined as the transmissibility of the mounting, under the given conditions of imposed static load and frequency of impressed periodic force. The transmissibility of a spring and dashpot system, such as that described, supporting a mass upon which is impressed a periodic force, is expressed mathematically by the following equation[4]

[4] See Bibliography reference (1), pp. 74–84.

$$\epsilon = \sqrt{\frac{1 + \left(2\dfrac{c}{c_c}\dfrac{\omega}{\omega_n}\right)^2}{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2 + \left(2\dfrac{c}{c_c}\dfrac{\omega}{\omega_n}\right)^2}} \qquad [1]$$

where

ϵ = transmissibility
c = damping force per unit velocity, lb per in. per sec
c_c = critical damping force per unit velocity,[5] lb per in. per sec
ω = frequency of impressed periodic force, radians per sec
ω_n = natural frequency of free vibration of machine when secured to resilient mounting, radians per sec

It may be shown[6] that for any given vibration frequency the ratio of the amplitude of A to that of B (Fig. 1) is a direct measure of the transmissibility of the mounting.

On each of the seven experimental mountings, runs were made covering the frequency range and the load range of the apparatus. For each load, ω_n was determined experimentally. At each of several operating speeds the value of ϵ was computed from the transmitted amplitude and the impressed amplitude recorded on the tape. Experimentally obtained values of ϵ were plotted against corresponding values of ω/ω_n. Comparing the resulting curves with similarly plotted curves derived from Equation [1], using trial values of c/c_c, it was found that very close agreement existed between experimental and theoretical curves. The apparent value of c/c_c remained practically constant for each rubber compound, being affected but little by changes in load. The compound with least damping indicated a value of c/c_c of 0.023, while that with most damping indicated 0.071.

[5] See Bibliography reference (1), Equation [22], p. 44.
[6] See Bibliography reference (2), chapter 11.

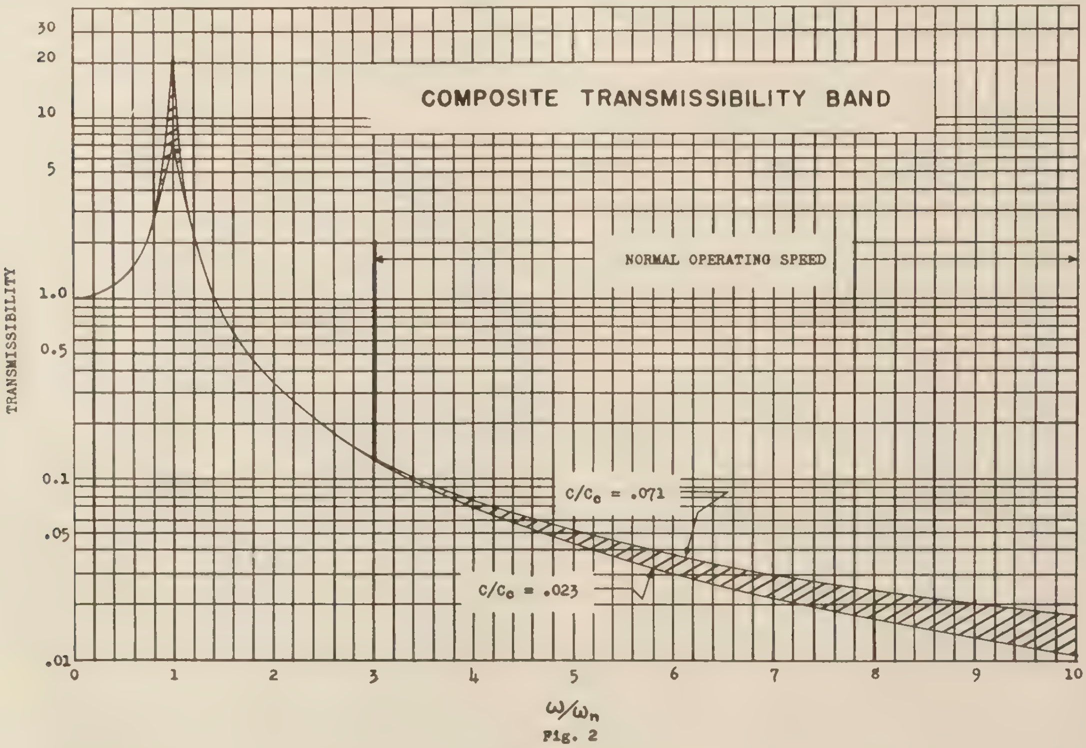

FIG. 2 COMPOSITE TRANSMISSIBILITY BAND

Resilient mountings for machinery should be designed so that ω_n lies somewhere between one third and one tenth of the normal operating speed of the machine. If ω/ω_n is less than 3, not much advantage is to be gained from the employment of a resilient mounting (see Fig. 2); if ω/ω_n is greater than 10, the machine usually lacks the necessary stability because of the deflection of the mounting per unit applied load becoming excessive, which follows from the necessity of making ω_n small. Observation[7] indicates that the normal operating speed of a resiliently mounted machine is usually such that the value of ω/ω_n is well within the range of 3 to 10.

Results of First Phase of Investigation

The results of the first phase of the investigation pointed to the following conclusions regarding shear-stressed rubber machinery mountings:

1 They behave substantially the same as the theoretical spring-dashpot system.

2 They have a value of c/c_c lying between 0.023 and 0.071.

3 The resiliently mounted unit of machinery, with a properly designed mounting, usually operates at a speed such that ω/ω_n lies between 3 and 10.

These conclusions are presented graphically in Fig. 2, where the "composite transmissibility band" covers the range of values of transmissibility likely to be encountered in shear-stressed rubber machinery mountings. The boundary curves were obtained by substituting values of 0.023 and 0.071 for c/c_c in Equation [1].

Criterion for Measuring Effectiveness of Compounds Studied

The second phase of the investigation was the development of a criterion for the effectiveness of shear-stressed rubber compounds in isolating machinery vibration. In general there are four considerations to be taken into account:

1 Upon the magnitude of the periodic force transmitted to the foundation structure, at the normal operating speed of the unit, depends the amplitude of vibration of the foundation structure, which is of primary interest when considering the comfort of personnel in the vicinity of the unit while it is operating at normal speeds.

2 The magnitude of the maximum periodic force transmitted to the foundation structure at resonant frequencies is many times greater than the impressed periodic force and may be severe enough to cause failure of the mounting or some portion of the structure adjacent to it. While this transmitted force may be sufficient to cause a vibration amplitude of the foundation structure which is annoying to personnel, it is usually of such short duration that this annoyance need not be taken into account, except where the machine involved is continually starting and stopping. This consideration therefore is of interest in regard to the structural stresses set up in the mounting and in the adjacent structure.

3 The maximum amplitude of vibration of the unit at resonant frequencies is of interest with respect to the alignment of shafts between the unit and other machinery; the amount of flexing of connections to the unit, such as ducts for water, steam, oil, gasoline, or exhaust gases; and the effect of vibration upon instruments which may be mounted on the unit itself such as pressure gages, thermometers, electrical relays, governors, or other mechanisms.

4 The same considerations pertain to the amplitude of vibration of the unit at its normal operating speed as in the case of item 3. This amplitude is much less than that at resonance, but it exists continuously while the unit is operating at normal speed.

[7] See Bibliography references (3, 4, 9).

Mathematical Relationships

Approximate mathematical relationships will be developed for each of the four considerations defined in order to analyze the properties of rubber compounds which affect their performance in resilient mountings. For simplicity, all measurements will be based upon foot-pound-second units. The following nomenclature will be used:

m = mass of unit of machinery supported by resilient mounting (slugs)

P_0 = value of periodic force impressed upon mounting due to unbalanced force in unit of machinery, lb

P_1 = value of periodic force transmitted to foundation structure through resilient mounting, lb

x_0 = vibration amplitude of unit of machinery at given value of ω/ω_n, ft

k_{st} = static spring constant (load per unit deflection) of resilient mounting, determined from slope of "static load-deflection" curve, lb per ft

k_{dy} = dynamic spring constant (load per unit deflection) of resilient mounting (lb per ft), computed from relationship $\omega_n^2 = k_{dy}/m$, at any given value of load.[8] Term ω_n is determined experimentally with a known value of m. Rubber compounds do not exhibit the same spring constant dynamically and statically as do metal springs. Rubber compounds appear to be stiffer under an oscillating load than under a static load, i.e., $k_{dy} > k_{st}$

γ = ratio of dynamic stiffness to static stiffness k_{dy}/k_{st} of resilient mounting

Q = breaking strength of resilient mounting, lb

ϵ_{max} = maximum value of transmissibility (at resonance)

ϵ, ω, ω_n have been previously defined. Note that $\epsilon = P_1/P_0$.

In a unit of machinery embracing a rotating member, the amplitude of the periodic force impressed upon the mounting is usually proportional to the second power of the circular frequency. Then

$$P_0 = C\omega^2 \qquad [2]$$

where C is the mass of the rotating element (slugs), multiplied by its eccentricity in feet. The eccentricity may increase somewhat with speed but, in general, this increase is negligible and for the purpose at hand C may be considered a constant. By definition

$$P_1 = P_0\epsilon \qquad [3]$$

Substituting Equation [2] in Equation [3]

$$P_1 = C\omega^2\epsilon \qquad [4]$$

The vibration amplitude of the unit of machinery is expressed by the following equation[9]

$$x_0 = \frac{P_0}{k_{dy}} \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\frac{c}{c_c}\frac{\omega}{\omega_n}\right)^2}} \qquad [5]$$

Equations [4] and [5] are considered exact and are the basic equations upon which the criterion is developed.

In the first consideration (i.e., the magnitude of the periodic force transmitted to the foundation structure at the normal operating speed of the unit), as an approximation, let

$$\epsilon \approx \frac{1}{1 - (\omega/\omega_n)^2} \qquad [6]$$

[8] See Bibliography reference (1), p. 57.

[9] See Bibliography reference (1), p. 56.

TABLE 2 VALUES OF P_1

ω/ω_n	Exact value of P_1 from Equation [4], $\frac{c}{c_c} = 0.023$	Exact value of P_1 from Equation [4], $\frac{c}{c_c} = 0.071$	Approximate value of P_1 from Equation [8]
3	$1.13C\omega_n^2$	$1.21C\omega_n^2$	$C\omega_n^2$
4	$1.09C\omega_n^2$	$1.22C\omega_n^2$	$C\omega_n^2$
5	$1.07C\omega_n^2$	$1.28C\omega_n^2$	$C\omega_n^2$
6	$1.07C\omega_n^2$	$1.36C\omega_n^2$	$C\omega_n^2$
7	$1.07C\omega_n^2$	$1.44C\omega_n^2$	$C\omega_n^2$
8	$1.08C\omega_n^2$	$1.54C\omega_n^2$	$C\omega_n^2$
9	$1.09C\omega_n^2$	$1.64C\omega_n^2$	$C\omega_n^2$
10	$1.11C\omega_n^2$	$1.75C\omega_n^2$	$C\omega_n^2$
Average	$1.26C\omega_n^2$		$C\omega n^2$

(This is true for zero damping.) For $\omega/\omega_n > 3$, a further approximation may be made

$$\epsilon \approx (\omega_n/\omega)^2 \quad [7]$$

Substituting Equation [7] in Equation [4]

$$P_1 \approx C\omega_n^2 \quad [8]$$

To show that the approximation in Equation [8] is reasonable, Table 2 compares the exact values of P_1, as determined by Equation [4], with the approximate values of P_1, as determined by Equation [8].

From Table 2, it is evident that an improvement of Equation [8] would be

$$P_1 \approx 1.26C\omega_n^2 \quad [9]$$

For low values of damping[10]

$$\omega_n^2 = k_{dy}/m \quad [10]$$

or

$$\omega_n^2 = \gamma k_{st}/m \quad [11]$$

Substituting Equation [11] in Equation [9]

$$P_1 \approx (1.26C/m)\gamma k_{st} \quad [12]$$

Since C and m are constants for any given unit of machinery, let $(1.26C/m) = C_1$, another constant. Then

$$P_1 \approx C_1\gamma k_{st} \quad [13]$$

Thus it is evident that the properties of the rubber compound which influence the effectiveness of the mounting in reducing the force transmitted to the foundation structure, at the normal operating speed of the unit, are γ and k_{st}.

For the second consideration (i.e., the magnitude of the maximum periodic force transmitted to the foundation structure), Equation [4] becomes

$$P_{1\max} = C\omega_n^2\epsilon_{\max} \quad [14]$$

Substituting as in Equation [12]

$$P_{1\max} = (C/m)\gamma k_{st}\epsilon_{\max} \quad [15]$$

Let $C/m = C_2$, another constant. Then

$$P_{1\max} = C_2\gamma k_{st}\epsilon_{\max} \quad [16]$$

It is to be noted that Equations [14] to [16], inclusive, are exact and not approximate. It is seen that the maximum periodic force transmitted to the foundation structure at resonant frequencies is influenced by γ, k_{st}, and $\epsilon_{\max}$.

In the third consideration (i.e., the maximum amplitude of vibration of the unit of machinery), the exact relationship, Equation [5], is modified by multiplying the right-hand member by $\sqrt{1 + \left(2\frac{c}{c_c}\frac{\omega}{\omega_n}\right)^2}$. This results in the following approximate equation (see Equation [1])

$$x_{0\max} \approx (P_0/k_{dy})\epsilon_{\max} \quad [17]$$

The maximum error introduced by this modification occurs with $\frac{c}{c_c} = 0.071$. Since ω/ω_n is unity when $\epsilon_{\max}$ occurs, the maximum value of the foregoing modifying factor is as follows $\sqrt{1 + (2 \times 0.071)^2} = 1.010$. Thus this modification introduces a maximum error of 1 per cent, which is negligible. Substituting Equation [2] in Equation [17], and recalling that at resonance $\omega = \omega_n$

$$x_{0\max} \approx (C\omega_n^2/k_{dy})\epsilon_{\max} \quad [18]$$

Substituting Equation [10] in Equation [18]

$$x_{0\max} \approx (C/m)\epsilon_{\max} \quad [19]$$

But $C/m = C_2$; then

$$x_{0\max} \approx C_2\epsilon_{\max} \quad [20]$$

From Equation [20], it is apparent that the only property of the rubber compound influencing the maximum amplitude of vibration of the unit of machinery is $\epsilon_{\max}$.

In the fourth, and final, consideration (i.e., the amplitude of vibration of the unit of machinery at its normal operating speed), an approximate relationship, deduced from Equation [18], is as follows

$$x_0 \approx (C\omega^2/k_{dy})\epsilon \quad [21]$$

Substituting Equation [7] in Equation [21]

$$x_0 \approx C\omega_n^2/k_{dy} \quad [22]$$

Substituting Equation [10] in Equation [22]

$$x_0 \approx C/m \quad [23]$$

But $C/m = C_2$, therefore

$$x_0 \approx C_2 \quad [24]$$

To check the validity of Equation [24], Table 3 is presented. The values of x_0 in this table are determined from Equation [5], which is exact, in the following manner:

Substituting Equation [2] in Equation [5]

$$x_0 = \frac{C\omega^2}{k_{dy}} \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\frac{c}{c_c}\frac{\omega}{\omega_n}\right)^2}} \quad [25]$$

Equation [25] may be written

$$x_0 = \frac{C(\omega/\omega_n)^2\omega_n^2}{k_{dy}} \cdot \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\frac{c}{c_c}\frac{\omega}{\omega_n}\right)^2}} \quad [25a]$$

Substituting Equation [10] in Equation [25a] and recalling that $C/m = C_2$

$$x_0 = C_2(\omega/\omega_n)^2 \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\frac{c}{c_c}\frac{\omega}{\omega_n}\right)^2}} \quad [26]$$

TABLE 3 CHECK ON EQUATION [24]

ω/ω_n	x_0 as determined by Equation [26], $\frac{c}{c_c} = 0.023$	x_0 as determined by Equation [26], $\frac{c}{c_c} = 0.071$
3	$1.125C_2$	$1.115C_2$
4	$1.067C_2$	$1.065C_2$
5	$1.042C_2$	$1.040C_2$
6	$1.029C_2$	$1.028C_2$
7	$1.021C_2$	$1.020C_2$
8	$1.016C_2$	$1.016C_2$
9	$1.013C_2$	$1.012C_2$
10	$1.010C_2$	$1.010C_2$

[10] See Bibliography reference (1), pp. 46 and 57.

From Table 3, it may be seen that an approximate value of x_0 within 6 per cent of the exact value, at any part of the band designated "normal operating speed" in Fig. 2, is given by

$$x_0 \approx 1.07C_2 \qquad [27]$$

Thus, the amplitude of vibration of the unit of machinery at its normal operating speed is seen to be substantially independent of the properties of the rubber compound.

To summarize, the effectiveness of the shear-stressed rubber compounds in the typical mounting is indicated by the following

$$P_1 \approx C_1\gamma k_{st} \qquad [13]$$

$$P_{1\max} = C_2\gamma k_{st}\epsilon_{\max} \qquad [16]$$

$$x_{0\max} \approx C_2\epsilon_{\max} \qquad [20]$$

Factors Governing Selection of Rubber Mountings

In the ideal resilient mounting, P_1, $P_{1\max}$, and $x_{0\max}$ should each have the lowest possible value, that is, the values of γ, $\epsilon_{\max}$, and k_{st} should each be a minimum. The ratios γ and $\epsilon_{\max}$ are measurable only when the rubber compound is subjected to an oscillating load, while k_{st} is measurable when the compound is loaded statically. In general, the selection of a rubber compound and the design of the mounting should be based upon as low a value of k_{st} as is compatible with the allowable static deflection, drift, and strength requirements. The determination of drift depends upon measurements over a long period of time and is beyond the scope of the present investigation. Good design of a resilient mounting would dictate that the working load imposed upon the rubber compound should not exceed its endurance limit. Since the determination of the endurance limit of each compound is also beyond the scope of this investigation, the breaking strengths of the mountings, under gradually applied loads, are used as an indication of the relative endurance limits of the rubber compounds. By reducing the shear-stressed area of the rubber compound (i.e., keeping the volume of the rubber blocks constant and increasing their thickness, thus increasing the over-all width of the mounting), the value of k_{st} would be reduced; at the same time, however, for a given load upon the mounting, the value of the unit shear stress in the compound would be increased. Therefore it is apparent that a compound having a high strength (large value of Q) is desirable.

This leads to the suggestion of a criterion β', for the determination of the relative effectiveness of different rubber compounds used in the typical mounting

$$\beta' = \frac{\gamma\epsilon_{\max}k_{st}}{Q} \qquad [28]$$

In general, the compound exhibiting the lowest value of β' would be the most effective.

Since this criterion was derived for a mounting of specific dimensions, it is desirable to define the component factors in a more general manner:

As previously defined, γ and $\epsilon_{\max}$ are dimensionless ratios.

Define G as the applied load W (lb) per unit of shear-stressed rubber area A (sq in.) necessary to produce unit deflection of the mounting Δ (in.) per unit thickness of the rubber stressed in shear t (in.)

$$G = \frac{W/A}{\Delta/t} = \frac{Wt}{\Delta A}, \text{ psi} \qquad [29]$$

When G is thus defined, it represents the shear modulus of elasticity of the rubber compound (10, 11, 12). In the typical mounting, $t = 0.5$ in., $A = 2$ sq in., and $W/\Delta = k_{st}/12$ lb per in. Then if k_{st} is in lb per ft

$$G = k_{st}/48 \text{ psi} \qquad [30]$$

Define S as the breaking strength Q (lb) per unit of shear-stressed area A (sq in.) of the rubber compound

$$S = Q/A \text{ psi} \qquad [31]$$

In the typical mounting, $A = 2$ sq in. Then

$$S = Q/2 \qquad [32]$$

Substituting Equations [30] and [32] in Equation [28]

$$\beta' = \frac{24\gamma\epsilon_{\max}G}{S} \qquad [33]$$

Since G and S have like dimensions, β' is dimensionless.

Let

$$\beta = \beta'/24 \qquad [34]$$

For comparing the effectiveness of rubber compounds, β may be used instead of β'. Therefore, in a generalized dimensionless form, the suggested criterion is

$$\beta = \frac{\gamma\epsilon_{\max}G}{S} \qquad [35]$$

Thus, in general, the most effective rubber compound for use in a shear-stressed resilient mounting is the one with the lowest value of β.

TABLE 4 SUMMARY OF DATA AND RESULTS OF CALCULATIONS

Compound	Load, lb	f_n Cycles per sec	k_{dy} Lb per in.	k_{st} Lb per in.	γ **	G psi	ϵ_{max}* **	S psi	β **	β* **
A	27.65	10.5	312	287	1.09	71.8	20.0	90	17.4	17.2
	47.65	8.0	312	277	1.13	69.3	...	..	17.4	...
	67.65	6.6	302	273	1.11	68.3	...	..	16.8	...
B	47.65	9.5	440	387	1.14	96.8	19.6	410	5.28	5.19
	67.65	7.8	421	362	1.16	90.5	...	..	5.02	...
	87.65	7.0	439	355	1.24	88.8	...	..	5.26	...
D	27.65	14.5	593	555	1.07	139	22.2	465	7.11	6.98
	47.65	11.0	589	514	1.15	129	...	..	7.09	...
	67.65	9.1	572	490	1.17	123	...	..	6.89	...
	87.65	7.95	567	490	1.16	123	...	..	6.82	...
F	27.65	14.9	627	422	1.49	106	11.1	390	4.48	4.30
	47.65	11.0	589	362	1.63	90.5	...	..	4.18	...
	67.65	9.3	598	353	1.69	88.3	...	..	4.23	...
G	27.65	15.1	644	369	1.75	92.3	9.4	413	3.68	3.43
	47.65	10.9	578	340	1.70	85.0	...	..	3.29	...
	67.65	9.2	585	318	1.84	79.5	...	..	3.33	...
J	47.65	6.8	225	187	1.20	46.8	17.0	380	2.51	2.51
K	47.65	17.1	1430	698	2.05	175	7.05	503	5.03	5.09
	87.65	12.8	1470	445	3.30	111	...	..	5.14	...

* Average value.
** Dimensionless ratio.

$f_n = \omega n/2\pi$.
Testing temperature 75 F ± 3 deg.

A summary of the data and results of calculations leading to the evaluation of β for each of the compounds tested is presented in Table 4. It is apparent from column 10 of this table that β is nearly a constant for each compound, regardless of applied load. Therefore it is allowable to use the average, as given in column 11, as the representative value of β for each compound.

Neglecting the high value of β obtained for compound A, which was obviously due to poor strength, it is seen that for compounds which have a satisfactory strength and shear modulus of elasticity, the value of β varies approximately from 2.5 to 7. With this variation in the value of β, it was justifiable to neglect errors occasioned by using approximations in its development.

It is believed that the suggested criterion is useful as a general measurement of the effectiveness of rubber and rubberlike compounds employed in resilient machinery mountings. However, the characteristics of the machine and the type of service involved may affect the choice of magnitudes of certain factors comprising the criterion. Thus, two rubber compounds may have nearly the

same value of β and nearly the same value of the ratio G/S; the first compound may exhibit a relatively large value of γ and a low value of ϵ_{max}, while the second compound exhibits a relatively small value of γ and a large value of ϵ_{max}, the product, $\gamma\epsilon_{max}$, being approximately the same in both cases. If the machine which is to be mounted on rubber supports is one which operates at full speed during most of its running time and passes quickly through the resonant frequency as it is started and stopped, the second compound, with the lower value of γ, would be more satisfactory, since from Equation [13], γ influences the force transmitted to the foundation structure at the normal operating speed. Since the resonant speed would occur only briefly, sufficient time would not be allowed for large amplitudes of vibration to build up, and the larger value of ϵ_{max} would not be detrimental (see Equations [16] and [20]). On the other hand, if the machine is one which operates at, or near, the resonant frequency for extended periods of time, or if it passes slowly through this frequency as it is started or stopped, the first compound, with the lower value of ϵ_{max}, would be more satisfactory.

Conclusions

It has been shown in this paper that the common practice of using only hardness, or shear modulus of elasticity, as the criterion is not satisfactory for the rational design of rubber mountings for machinery.

There is sufficient variation indicated in the value of the derived criterion for different rubber compounds to justify its consideration.

It is suggested that rubber compounds showing good effectiveness by the criterion be tested further to find those having highest endurance limits, lowest values of rate of drift, and good aging properties.

In conclusion, it should be emphasized that the characteristics of the machine and the service for which it is to be used must be known to choose a rubber compound well suited for use in a resilient mounting for that machine.

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Discussion

W. N. Findley.[11] There are two questions which the writer would like to raise in connection with this paper; (*a*) the use of β as a criterion of the effectiveness of rubber compounds for the isolation of vibration, and (*b*) the "dynamic spring constant."

[11] Associate in Theoretical and Applied Mechanics, College of Engineering, University of Illinois, Urbana, Ill. Jun. A.S.M.E.

The proposed criterion is composed of terms representing three different properties of the rubber compound; that is, γ and G are a measure of the rigidity of the rubber, ϵ_{max} is a measure of the hysteresis damping of the rubber, and S is a measure of the strength of the rubber.

A material of low rigidity is desirable from the standpoint of designing small-size mountings, whereas, under certain speeds of operation, large damping is desirable to reduce the transmissibility. The strength of the material, on the other hand, does not in any way enter into the design of a mounting from the standpoint of its service in isolating vibration. It is, however, necessary to have sufficient strength to prevent failure of the mounting in service. Thus it would seem to the writer that the three factors, stiffness, damping, and strength should be used as separate criteria rather than lumped together as a single criterion. For example, two compounds, A and B, might have equal damping property and equal stiffness, but A might have markedly greater strength and greater cost than B. Under a certain application, the strength of both materials might be adequate. In such a case the criterion β would indicate the more costly material A to be markedly superior, whereas both materials would perform with equal satisfaction.

The writer is puzzled by the fact that the author's "dynamic spring constant" k_{dy} was as much as 3 times the spring constant k_{st}, obtained from static tests. Two possible explanations present themselves:

1 Creep (drift) may take place at such a rate as to affect the value of k obtained from "static" tests, or the "static" tests may possibly be affected by other factors, such as the use of a secant rather than a tangent line in measuring the spring constant.

2 The equation from which k_{dy} was computed may not represent the actual conditions of the problem with sufficient accuracy. The force equation from which the equation for k_{dy} was derived is

$$m\ddot{x} + C\dot{x} + k_{dy}x = 0$$

This equation says that the forces producing the acceleration $\ddot{x}$ during the vibration are made up of two parts, a damping force $C\dot{x}$, which varies linearly with velocity $\dot{x}$, and a spring force $k_{dy}x$, which varies linearly with displacement x; that is, C and k_{dy} are constants and do not vary with acceleration, velocity, or displacement. Thus, k_{dy} must equal k_{st} if the foregoing equation accurately describes the motion.

It is entirely possible that the forces involved in the problem considered by the author cannot be represented accurately by this expression. It may be necessary to use higher powers of the velocity or displacement in the force equation. In such a case Equations [4], [5], and $\omega_n^2 = \dfrac{k_{dy}}{m}$ would not be the correct solutions to the problem, and the use of these equations in such a case would be of questionable value.

H. O. Fuchs.[12] Data on the dynamic properties of rubber and rubberlike compounds are highly welcome and the author's paper is a valuable contribution to the subject.

In tests conducted in a roughly similar manner, we found in general considerably lower damping values for natural-rubber compounds. Perhaps the difference can be explained by the fact that we deducted the damping provided by machine joints, air resistance, etc., from the gross measured values. It is not clear whether the author used this precaution.

The author follows established precedent in interpreting and

[12] General Motors Corporation, Detroit, Mich.

expressing his data with a simple mechanical system such as is shown in Fig. 3 of this discussion. In the range with which he deals (low c/c_c and relatively low ω/ω_n), he checked test results against the assumptions and found reasonable agreement, provided c/c_c is held constant, which means that the viscosity in the dashpot must vary in inverse proportion to the frequency of vibration.

Other investigators[13] have used the same analog as a basis for expressing the test results and have found that apparent viscosity varies in inverse proportion to a fractional power (about 4/5) of the vibration frequency.

The writer would like to point out that the simple mechanical system used by the author and others can only be used with extreme caution as a basis of comparison for rubberlike compounds:

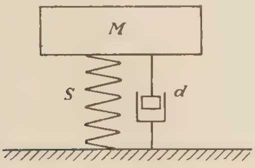

Fig. 3 Ordinary Damping

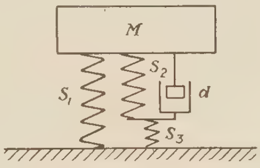

Fig. 4 Indirect Damping

S, S_1, S_2, S_3 represent springs
M represents mass
d represents dashpot

1 The apparent variation of dashpot viscosity with frequency cannot be explained by this spring-dashpot system. Note that viscosity appears to vary with frequency, not with velocity or work done per second.

2 The transition from static rate to dynamic rate cannot be explained by the analog.

3 The analog invites to wrong conclusions, such as the belief expressed in the literature that damping spoils isolation at high frequency ratios ω/ω_n. This is true for the analog but not true for rubberlike compounds.

While the analog, shown in Fig. 3 of this discussion, may safely be handled by careful investigators, such as the author, it is apt to create confusion when an attempt is made to design rubber insulators or predict their performance. The author's statements that mountings behave substantially the same as the theoretical system, and that certain equations are exact, should be restricted to the range which he investigated and to a system with variable viscosity. With higher frequency ratios, such as occur on multicylinder internal-combustion engines, and higher damping values, such as are found in neoprene and similar compounds, the differences become very important.

The writer proposes to consider rubberlike compounds as analogous to the system of springs and damper, shown in Fig. 4 of this discussion, similar to systems previously shown in the literature.[14]

Without recourse to some mechanism which adjusts dashpots and springs in response to frequency, the system, shown in Fig. 4, displays the properties characteristic of a rubberlike substance, namely:

1 The dynamic spring rate is higher than the static spring rate because spring S_2 is practically blocked out.

2 The "apparent" damping, deduced from amplitude at resonance, will decrease with frequency, as it should, because spring S_3 will work more and more and the dashpot less and less.

[13] "Some Dynamic Properties of Rubber," by C. O. Harris, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 64, 1942, pp. A-132, A-134.

[14] "Elasticity, Plasticity, and Structure of Matter," by R. Houwink, Cambridge, University Press, London, England, 1939.

3 Isolation at high frequency ratios depends only upon the rates of the springs S_1 and S_3 and not upon the damping as measured at resonance.

A set of equations for undamped systems, for the ordinary damped system of Fig. 3 of this discussion, and for the system with indirect damping, Fig. 4, are given in Table 5 to facilitate comparison and application.

The most striking difference between ordinary and indirect damping appears in the equations for the force ratio F. With

TABLE 5 COMPARISON OF EQUATIONS FOR DAMPED AND UNDAMPED SYSTEMS

	Amplitude ratio A	Force ratio F
Undamped..........	$\dfrac{1}{1-Q}$	1
Ordinary damping (Fig. 3)...........	$\sqrt{\dfrac{1}{(1-Q)^2+DQ}}$	$\sqrt{1+DQ}$
Indirect damping (Fig. 4)...........	$\sqrt{\dfrac{1+KDQ}{(1-Q)^2+KDQ(R-Q)^2}}$	$\sqrt{\dfrac{1+R^2KDQ}{1+KDQ}}$

A = amplitude produced by harmonic force on mass, divided by static amplitude produced by that force
F = force on support produced by harmonic displacement of mass, divided by force produced by static displacement of same amount
Q = square of ratio, forced frequency/natural frequency, calculated from mass and static rate. ($Q = \omega^2/[k_{st}/m]$ in the author's notation)
D = square of twice relative damping, calculated from dashpot-resistance coefficient, mass, and static rate ($D = c^2/k_{st}m$)
R = ratio of static rate to rate measured if dashpot is frozen solid
K = square of static rate divided by square of rate measured across dashpot terminals if mass is held fixed

ordinary damping, F increases without limit with increasing frequency ratio Q. With indirect damping, F approaches the fixed value R asymptotically with increasing Q. Transmissibility is the product $A \times F$.

The author shows excellent judgment in expressing the force transmitted at operating speed as a function of k_{dy}/k_{st}. But whereas the system used by him leads to this conclusion only for low values of damping and then only if the damping is allowed for by a factor taken as 1.26, to average values between 1.07 and 1.75, the proposed system with indirect damping leads directly to this correct conclusion without regard to damping at resonance. Indirect damping is a better representation of the behavior of rubber than ordinary damping. It also shows the connection between measured damping and measured ratio k_{dy}/k_{st}. For zero damping, this ratio obviously becomes unity. It might be noted that indirect damping includes ordinary damping as a special case: If the spring S_3 is made very stiff, K approaches zero, R^2K approaches unity, and the equations reduce to those for ordinary damping.

Author's Closure

The author does not agree with Mr. Findley's statement that the strength of the material does not in any way enter into the design of a mounting from the standpoint of its service in isolating vibration. It is the author's belief that full utilization should be made of the strength of the material or, better, as pointed out in the paper, its endurance limit. With reference to k_{dy} being as much as 3 times k_{st}, Mr. Findley's first explanation is definitely pertinent. The tangent of the "static load-deflection" curve was used to determine the static spring constant. It was observed, however, that when the deflection was measured approximately 1 sec after increasing the shear stress by 10 psi, the increment in deflection was about 88 per cent of that measured

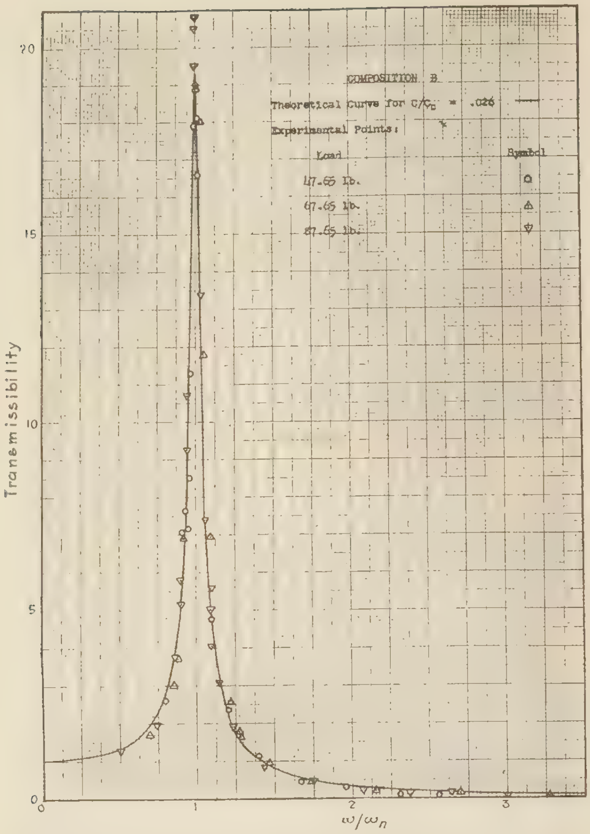

Fig. 5 Curve Demonstrating Close Agreement Between Experimental and Theoretical Data Obtained in Vicinity of Resonance

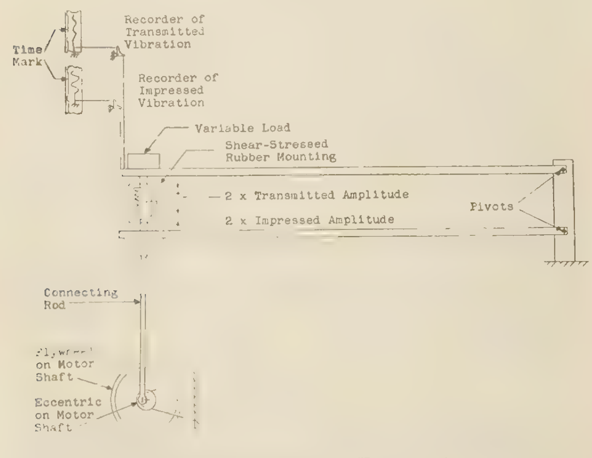

Fig. 6 Schematic Diagram of Testing Apparatus

after 30 min; 2 min after the increase in stress, the increment in deflection was approximately 99 per cent of that measured after 30 min. Therefore, in establishing the "static load-deflection" curves, all measurements of deflection were made 2 min after any change in load. For practical purposes such data represented stable static conditions, since deflections occurring over a greater period of time would be in the province of creep or drift phenomena. It is readily apparent that values of k_{st} thus obtained may be very much less than values for k_{dy}, where the time for plastic flow to occur was of the order of 0.03 to 0.08 sec. Regarding the accuracy of representation of the forces involved by the simple force equation quoted by Mr. Findley, the author reiterates the statement in the paper that very close agreement existed between experimental and theoretical curves. A typical example of such agreement is presented in Fig. 5 of this closure, which is a plot of data obtained in the vicinity of resonance, together with the theoretical curve obtained by substituting a value for c/c_c of 0.026 in Equation [1] of the paper.

A schematic diagram of the testing apparatus is shown in Fig. 6 of this closure, to supplement the verbal description in the paper.

The author did not take into account damping provided by machine joints and air resistance as did Mr. Fuchs. Lower values of damping would tend to lessen errors due to approximations in the development of the criterion, since more serious deviations appeared toward the upper limit of observed damping. Furthermore, if the relatively constant damping chargeable to machine joints and air resistance were to be subtracted from the reported values of damping, the values of the criterion β would cover an even greater range, thus making more apparent its usefulness.

In the literature, there appears to be a divergence of conclusions with respect to the variation of apparent viscosity with frequency. Harris,[13] as stated by Mr. Fuchs, concluded that it varies inversely to about the 4/5 power of the frequency. This was based upon the decaying free vibrations of a pendulum connected to the rubber specimen. Gehman[15] stated that the product, apparent viscosity times frequency, is a constant to a first approximation. This would mean that the apparent viscosity varies approximately inversely to the first power of the frequency. Stambaugh[16] concluded that the internal friction is approximately inversely proportional to the frequency. Both of the latter investigators calculated damping from the maximum amplitude of displacement observed at resonance when the rubber specimen was subjected to a forced vibration. The author observed in his work that values of damping calculated from successive amplitudes of a decaying free vibration were from 7 to 58 per cent greater, depending upon the compound, than values of damping calculated from the maximum transmissibility at resonance due to a forced vibration. The latter method was used because it simulated service conditions of resiliently mounted machinery. It is possible that the methods of determining the damping used by different investigators may have led to divergent conclusions regarding the variation of apparent viscosity with frequency.

Mr. Fuchs presents a refinement of the mechanical analog and accompanying mathematical relationships which may become necessary when considering higher frequencies than considered by the author. The author feels that Mr. Fuchs has added valuable information on the subject of the paper.

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Aerodynamic Center, Control and Stability of Airplanes

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In this paper the author proposes a representation of the longitudinal control, and of the longitudinal static-stability theory of the airplane: (1) By a consistent and exclusive use of the aerodynamic centers of all carrying airfoil bodies and the moment coefficients about these centers; (2) by the reference of all states of flight and of the location of the center of gravity to a basic state of flight with noncarrying tail plane and neutral fuselage orientation, with the characterization of other states of flight by the lift force and an effective angle of attack of the tail plane; (3) by the value of the ratio of static moments of wing and tail about the center of gravity as the measure of stability, which must overcome mainly the destabilizing moment of the fuselage and of the propeller. The analysis shows a relatively great effect of the fuselage and propeller moments on the control and stability. Both effects necessitate a greater force of the elevator and appear as the second important destabilizing terms in the stability condition.

Introduction

Among the five general conditions of the longitudinal stability of the airplane the one which does not contain the inertia forces is the most important. It is also the simplest condition and was known before G. H. Bryan developed the general theory of airplane stability in 1903, derived from E. I. Routh's theory of small oscillations of a system about a state of steady motion.

This "static longitudinal stability," considered from the basis of the general (dynamic) stability theory, appears as a rather elementary subject. However, it has seemed to the author that the problem is capable of a yet simpler and more comprehensive treatment with respect to different states of flight, location of center of gravity, action of elevator, and influence of fuselage and propeller.

By means of a consistent use of the aerodynamic centers of the wing and of the tail plane, the author proposes to derive control and stability conditions from a basic state of flight with non-carrying tail plane and neutral position of the fuselage, and from the ratio of the static moments of the lift forces concentrated in their aerodynamic centers.

It is well known that both a wing section (airfoil) or an entire wing, as well as the tail plane wing, and all bodies having a shape similar to that of the airfoil sections, always contain a point about which the moment of the aerodynamic forces is independent of the angle of attack. Furthermore this point, the "aerodynamic center," is always located adjacent to the forward quarter point of the airfoil chord. The amount of the dimensionless moment coefficient depends upon the camber of the airfoil and is zero for symmetric airfoils. This property of the aerodynamic center will be used consistently to determine the action of the stabilizer and elevator of the tail plane and the stability condition.

Equilibrium of Moments Acting on Airplane and Location of Center of Gravity

We start with a condition of horizontal flight which, in general, may correspond to the desired condition at maximum speed. It may be assumed that there is a flight condition in which the tail plane does not experience any upward or downward aerodynamic force (no positive or negative lift), and in which the fuselage axis is lying in the direction of the relative air flow, Fig. 1. For

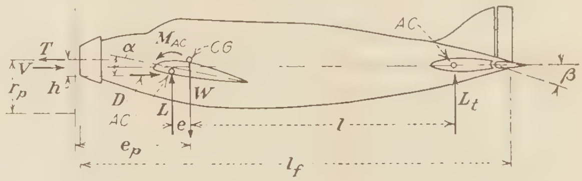

Fig. 1 Relation of Surfaces, With Airplane in Horizontal Flight

the tail plane this means that, on account of the downwash angle behind the main wing, the tail plane must be inclined about a very small angle relative to the direction of flight.

Using the notations given in Fig. 1 the equilibrium of moments about the center of gravity, requiring that the resultant moment must be zero, furnishes the following equation for the location of the center of gravity relative to the aerodynamic center of the main wing

$$\Sigma M = Le - M_{ac} - Dh = 0 \quad [1]$$

or

$$C_L qSe - C_{M_{ac}} qSc - C_D qSh = 0 \quad [1a]$$

where in the usual nomenclature

$q = \frac{\rho}{2}V^2$ = dynamic pressure
V = velocity of flight
C_L = lift coefficient $= 2\pi\eta\alpha \dfrac{1}{1 + 2\eta/A}$
$C_{M_{ac}}$ = moment coefficient referred to aerodynamic center
$C_D = C_{Di} + C_{Dp}$ = drag coefficient of whole structure referred to wing area
A = aspect ratio $= \dfrac{b^2}{S}$
b = wing span
S = wing area
η = a drag coefficient of wing of the order of magnitude 0.9
l = distance of aerodynamic center (called ac) of tail plane from center of gravity of whole structure (called cg)
e = distance of ac of main wing from cg
α = angle of attack referred to line of zero lift of airfoil
c = mean chord of main wing

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Explicitly, therefore

$$\frac{e}{c} = \frac{C_{Mac}}{C_L} + \frac{h}{c}\left(\frac{C_L}{\pi A} + \frac{C_{Dp}}{C_L}\right)\ldots\ldots\ldots\ldots [1b]$$

The second term gives the influence of the eccentricity of thrust and drag centers on the location of the center of gravity; the first term gives the influence of the moment coefficient.

Assuming, for instance

$$\frac{h}{c} = 0.08,\ A = 6,\ C_{Mac} = 0.04,\ C_L = 0.3,\ C_{Dp} = 0.03$$

the relative distance of cg from ac of the wing becomes

$$\frac{e}{c} = 0.133 + 0.0093$$

The Stability Condition

For the flight condition, considered as basic for the normal location of the center of gravity, the static-stability condition must be derived by varying the angle of attack α to $\alpha + \delta\alpha$, and by figuring the variation of the resultant moment.

Several new terms then appear in the summation of the moment variations, of which the decisive term is the lift force on the tail plane as a stabilizing term, while the inclination of the fuselage axis and of the propeller axis exerts a destabilizing influence. On the other hand the moment coefficient C_{Mac} which does not depend upon the angle α disappears, and so the moment about the aerodynamic center has only an indirect influence on the stability condition through the value of the distance between cg and ac.

The moment contribution δM_F of the *fuselage*[2] can be expressed by using the analogy with the moment of a prolate ellipsoid

$$M_{ell} = C_{ell}'qQ \sin 2\alpha\ldots\ldots\ldots\ldots [2]$$

where Q signifies the volume of the body and C_{ell} a coefficient depending upon the eccentricity value of the maximum cross-section diameter to over-all length. Term C_{ell} changes from zero for a sphere, to unity for an infinitely thin elliptical rod.[3]

In the special case considered here, Liepmann in the range of usual relative dimensions of airplane fuselages and following some experimental data suggests

$$\delta M_f = 2C_f'qS_fl_f\delta\alpha\ldots\ldots\ldots\ldots [2a]$$

where S_f is the maximum cross-sectional area, l_f is the length over-all, and C_f' is a constant coefficient of the order of magnitude 0.4 ($\delta\alpha$ in radians).

The moment contribution of the *propeller* consists of two parts, (1) caused by a transverse force acting vertically in the plane of rotation, and (2) caused by a couple in the vertical plane containing the propeller axis.

Item (1) can be written in the form

$$\delta M_{p1} = C_{p1}'q_pS_pe_p\delta\alpha\ldots\ldots\ldots\ldots [3a]$$

where

$q_p = \frac{\rho}{2}V_p^2$ = dynamic pressure at propeller tip

$S_p = \pi r_p^2$ = circumscribed propeller area

e_p = distance between propeller hub and center of gravity (see Fig. 1)

C'_{p1} = coefficient constant for small angles up to about 15 deg

Item 2 has a similar form

$$\delta M_{p2} = C_{p2}'q_pS_pr_p\delta\alpha\ldots\ldots\ldots\ldots [3b]$$

Flachsbarth and Kröber[4] give diagrams for angles up to 90 deg, while here they are needed only for small angles. Furthermore, they give diagrams for values of pitch to diameter 0.2 and 0.5, which are smaller than are used in modern propellers.

Lesley, Worley, and Moy[5] have made extensive measurements for a greater range of pitch-diameter ratios, starting from a pitch ratio of 0.7 and running to 1.28 for advance values up to $V/nD = 1.3$ ($V/\omega r = 0.42$).

Taking into account the fact that the forces and moments also depend upon the shape of the blade, the results of the two papers, comparing $P/D = 0.5$ in the first paper[4] with $P/D = 0.7$ in the second paper[5] at the same $V/\omega r = 0.268$ ($V/nD = 0.840$), agree fairly well; for instance

$\delta\alpha = 10$ deg $V/\omega r = 0.268$

P/D	$C_{p1}\delta\alpha\cdot 10^2$	$C_{p2}\delta\alpha\cdot 10^2$
0.5	0.18	0.07
0.7	0.1134	0.0805

Choosing an example from the Lesley paper[5]

$V/\omega r = 0.416$ ($V/nD = 1.306$)

$P/D = 1.284$, $\delta\alpha = 10$ deg. $= 0.1746$ radian

$10^2C_{p1}\delta\alpha = 0.41$, $C_{p1}' = 2.35\cdot 10^{-2}$

$10^2C_{p2}'\delta\alpha = 0.0877$, $C_{p2}' = 0.502\cdot 10^{-2}$

The moment contribution of the main wing and the tail plane with suffix t for tail plane are

$$\delta M = \delta\alpha(C_{Lt}'q_tS_tl - C_L'qSe)\ldots\ldots\ldots\ldots [4]$$

The resultant moment change, completed by the terms of Equations [2a], [3a], [3b], and [4], follows now from Equation [1a] in the form

$$\delta M = \delta\alpha[C_{Lt}'q_tS_tl - C_L'qSe - 2C_f'qS_fl_f - q_pS_p(C_{p1}'e_p + C_{p2}'r_p)]\ldots\ldots [5]$$

where $q_t = \frac{\rho}{2}V_t^2$ is the dynamic air-flow pressure following from the velocity V_t, retarded by the screening effect of the airplane parts in front of the tail plane, and where the + sign signifies the stabilizing sense of rotation.

The stability condition in preliminary form only for the basic state of flight can be stated by means of static moments as follows

$$S_tl > Se\frac{C_L'}{C_{Lt}'}\cdot\frac{q}{q_t}\left[1 + \frac{S_f}{S}\cdot\frac{l_f}{e}\cdot\frac{2C_f'}{C_L'} + \frac{S_p}{S}\cdot\frac{q_p}{q}\left(\frac{e_p}{e}\cdot\frac{C_{p1}'}{C_L'} + \frac{r_p}{e}\cdot\frac{C_{p2}'}{C_L'}\right)\right]\ldots\ldots [5a]$$

It may be interesting to estimate the order of magnitude of the second and third terms relative to the first term

$$C_L' = \frac{2\pi\eta}{1 + 2\eta/A} = \frac{1.8\pi}{1.3},\qquad C_{Lt}' = \frac{2\pi\eta}{1 + 2\eta/A_t} = \frac{1.8\pi}{1.6}$$

$$C_f' = 0.4,\ \frac{S_f}{S} = 0.02,\ \frac{S_p}{S} = 0.24,\ \frac{l_f}{e} = 40,\ \frac{r_p}{e} = 5.5,\ \frac{e_p}{e} = 10,\ \frac{q_p}{q} = 9$$

We get from Equation [5a]

[2] First introduced in the paper, "An Improved Longitudinal Stability Calculation," by H. P. Liepmann, *Journal of the Aeronautical Sciences*, vol. 9, March, 1942, pp. 181–184.

[3] See also, "Application of Practical Hydrodynamics to Airship Design," by R. A. Upson and W. A. Klikoff, U. S. N.A.C.A., Technical Report No. 405, 1932.

[4] "Experimentelle Untersuchungen an schräg angeblasenen Schraubenpropellern," by O. Flachsbart and G. Kröber, *Zeitschrift für Flügtechnik und Motorluftschiffahrt*, vol. 40, 1929, pp. 605–614.

[5] "Air Propellers in Yaw," by E. P. Lesley, G. F. Worley, and Stanley Moy, Technical Report No. 597 U. S. N.A.C.A., 1937.

$$S_t l \gtrless Se \frac{C_L'}{C_{Lt}'} \cdot \frac{q}{q_t} (1 + 0.147 + 0.117 + 0.014)$$

The result shows, for the relative dimensions thus chosen as practical values, that the destabilizing effect of the hull is about 15 per cent, and of the propeller due to the pitching force, 12 per cent and due to the pitching moment, 1.4 per cent, of the destabilizing effect of the main wing.

The inequality Equation [5a] gives the stability condition in terms of the static moment of the tail surface in such a way that the moment coefficients of the main wing or tail surface do not appear, but have only an indirect influence by locating the center of gravity or the steady positive or negative lift on the tail plane for other than the basic state of flight.

In the literature, the stability condition is usually expressed by the slope of the moment function with a moment coefficient referred to the leading edge of the wing. This alternative condition could be expressed by means of Equation [5] if, in Fig. 1, the lift force is moved parallel to itself from the aerodynamic center to the leading edge and, in counteraction thereof, the moment coefficient is increased so that

$$C_{M\text{le}}c = C_{M\text{ac}}c + C_L \frac{c}{4}$$

where the moment coefficient is taken positive in the diving sense, and the aerodynamic center, to fix the ideas, is assumed to be at the quarter-point.

Equation [5] then takes the form

$$\delta M = \delta\alpha \left[C_{Lt}'q_tS_tl - C_L'qS\left(e + \frac{c}{4}\right) + C_{M\text{le}}'qSc - 2C_f'qS_fl_f - q_pS_p (C_{p1}'e_p + C_{p2}'r_p)\right] \ldots\ldots\ldots\ldots [5b]$$

The expression in brackets is then called the slope of the moment function, which must be inclined in the opposite direction to the angle of deviation of the airplane.

Other States of Flight

The stability condition in the foregoing section has only been derived for the basic condition of noncarrying tail plane and the fuselage (or hull) and the propeller in such a position as not to influence the location of the center of gravity by their up- or down-turning moments. Yet it can be shown that the condition, Equation [5a], does not change for other states of flight except in the value of q_t/q and for loose elevator control.

The first correction q_t/q which concerns the change in retardation of the airflow across the tail plane caused by the change either of the angle of the airplane axis relative to the direction of flight, or of the power given off by the propeller, must be left to a special treatment, preferably experimental.

The second correction depends upon the special inertia and aerodynamic balance of the elevator control, and therefore also requires a separate investigation.

In fact, the change for other states of flight, apart from the corrections just mentioned, concerns not the stability but only the upward or downward lift on the tail plane. This can be seen in Equation [1a] by observing that the moment coefficient $C_{M\text{ac}}$ does not change with the angle of inclination, while the lift coefficient C_L of the main wing and of the tail plane changes with the inclination of the airplane axis relative to the direction of flight. In addition, a hull-and-propeller moment appears as a function of this inclination. Therefore, an additional moment is necessary for equilibrium in Equation [1a], which must be furnished by the angular position β of the elevator in co-operation with the fixed part of the tail plane, Fig. 1.

Designating $\Delta\alpha$ as the change of inclination of the reference axis of the airplane axis with $\Delta C_L = C_L'\Delta\alpha$, the corresponding changes of the lift coefficients, Equation [1a] changes into

$$\frac{\Delta\alpha_t}{\Delta\alpha} C_{Lt}'q_tS_tl - C_L'Seq\left[1 + \frac{2C_f'}{C_L'} \cdot \frac{S_f}{S} \cdot \frac{l_f}{e} + \frac{q_p}{q} \cdot \frac{S_p}{S} \left(\frac{C_{p1}'}{C_L'} \cdot \frac{e_p}{e} + \frac{C_{p2}'}{C_L'} \cdot \frac{r_p}{e}\right)\right] = 0 \ldots\ldots [6]$$

where it must be noted that the angle change $\Delta\alpha_t$ of the zero-lift line of the tail surface is different from the main angle change $\Delta\alpha$ because, due to the angular motion β of the elevator control surface the zero-lift line, which originally coincided with the axis of symmetry of the tail plane airfoil, moves with the control in a certain relation but less than the rotation $\Delta\alpha$ so that

$$\frac{\Delta\alpha_t}{\Delta\alpha} = 1 + k\frac{\beta}{\Delta\alpha} \ldots\ldots\ldots\ldots\ldots\ldots [6a]$$

where in first approximation k is a constant dependent upon the relation between the total area S_t and the movable elevator part S_{el} of the horizontal tail surface.[6]

Equations [6] and [6a] give the amount of upward and downward lift of the tail plane and with it the amount of elevator angle β to change the inclination of the reference axis of the airplane relative to the direction of flight. By "reference axis" is meant the axis corresponding to a noncarrying horizontal tail plane. The state of flight for which the tail plane shall not exert a positive or negative lift force can be chosen arbitrarily, but preferably for a state of high speed. The reason for this recommendation is twofold, (1) the parasitic drag forces, important at high speed, become smallest for noncarrying tail plane (and noncarrying fuselage), (2) also the control forces on the elevator can be kept in moderate limits for steep diving if the reference state of flight is nearest to the state of diving.

Validity of Stability Condition for All States of Flight

In Equation [6], adding the additional tailplane term which makes the total moment in some other than the basic steady state of flight zero, and assuming besides a transient change $\delta\alpha$ of the inclination of the airplane axis, the expression for the change of total moment does not differ from the expression in Equation [5]. Therefore, the stability condition, Equation [5a], must remain virtually the same for cruising as well as for climbing and diving if the eventual changes of the relative retardation q_t/q of the flow behind the main wing and the eventual change in the propeller terms are taken into account.

Effect of Change of Center of Gravity

The location e of the center of gravity behind the aerodynamic center of the main wing relative to the distance l of the aerodynamic center of the tail plane from the center of gravity follows from the completed Equation [1a]. It must be completed for the inclination $\Delta\alpha$ of a new reference axis against the hull axis and against the plane of rotation of the propeller under the supposition of noncarrying tail plane. It can then be shown that a change of the distance of the center of gravity, originally derived from a state of flight of noncarrying tail plane and not inclined fuselage, is equivalent to a transition of the

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[6] (a) "Model Experiments on the Pitching and Hinge Moment Due to Elevators of Various Sizes on B.E. 2C. Tailplanes," by H. B. Irving and A. S. Batson, R. and M. no. 679, Great Britain Aeronautical Research Committee Technical Reports, vol. 1, 1920–1921, pp. 317–326 ($k \sim 0.21 \frac{S_{el}}{S_t}$, when $\frac{S_{el}}{S_t} < 0.3$).

(b) "General Theory of Thin-Wing Sections," by M. Munk U. S. N.A.C.A. Technical Report 142, 1922, p. 245.

reference axis to a state of flight also with noncarrying tailplane, the reference axis being inclined about an additional angle $\Delta\alpha$ relative to the hull axis. In fact Equation [1a] is completed to

$$C_L qSe - C_{M_{ac}} qSc - C_D qSh + C_f qS_f l_f + q_p S_p (C_{p1} e_p + C_{p2} r_p) + C_{Dt} S_t q_t l \, \Delta\alpha = 0$$

Arranging the terms analogous to Equation [1b], one obtains, for the distance e of the center of gravity behind the aerodynamic center, the relation

$$\frac{e}{c} = \frac{C_{M_{ac}}}{C_L} + \frac{C_D}{C_L} \cdot \frac{h}{c} - \frac{C_f}{C_L} \cdot \frac{S_f}{S} \cdot \frac{l_f}{c} - \frac{q_p}{q} \cdot \frac{S_p}{S} \left(\frac{C_{p1}}{C_L} \cdot \frac{e_p}{c} + \frac{C_{p2}}{C_L} \cdot \frac{r_p}{c} \right) + \frac{C_{Dt}}{C_p} \cdot \frac{q_t}{q} \cdot \frac{S_t}{S} \cdot \frac{l}{c} \Delta\alpha$$

The last three terms give the additional influence of the inclined axis of the fuselage and the propeller, and of the eccentricity of the tailplane, which latter had been assumed to be in the fuselage axis.

Expressed by the inclination $\Delta\alpha$ of the last-named axis, the last equation becomes

$$\frac{e}{c} = \frac{C_{M_{ac}}}{C_L'\alpha} + \frac{C_L'\alpha}{\pi A} + \frac{C_{Dp}}{C_L'\alpha} - \frac{\Delta\alpha}{\alpha} \left[\frac{C_f'}{C_L'} \cdot \frac{S_f}{S} \cdot \frac{l_f}{c} + \frac{q_p}{q} \cdot \frac{S_p}{S} \left(\frac{C_{p1}'}{C_L'} \cdot \frac{e_p}{c} - \frac{C_{p2}'}{C_L'} \cdot \frac{r_p}{c} \right) - \frac{C_{Dt}}{C_L'} \cdot \frac{q_t}{q} \cdot \frac{S_t}{S} \cdot \frac{l}{c} \right] \quad \text{[1c]}$$

For an angle of attack α equal to the angle assumed for noninclined fuselage axis, Equation [1c] gives a smaller value of the distance e with fuselage axis turned up ($\Delta\alpha$ positive) and viceversa for $\Delta\alpha$ negative.

The stability condition $\delta M \gtrless 0$, pertaining to Equation [1c], is derived just as in Equation [5a] by varying the angle α to $\alpha + \delta\alpha$, which shows that its expression does not differ from Equation [5a]. Only in the value of q_t/q, that is, in the relative retardation of the airflow behind the main wing V and in the value of the propeller term, a numerical difference may arise.

Taking into account the fact that the stability condition, apart from the numerical value of q_t/q_i does not change with the state of flight and that the distance becomes smaller for hull and propeller axis inclined upward, it is seen that the required minimum distance of the aerodynamic centers becomes smaller for a combination of upward-inclined fuselage axis and noncarrying tailplane and larger for downward-inclined fuselage axis.

Nevertheless, for reasons of small drag at maximum speed and of moderate control forces in a steep dive, a small angle of attack α, combined with noncarrying tailplane and noninclined fuselage axis is advisable. This, as shown previously, means a backward location of the center of gravity as the basis for the stability condition. It is also in agreement with the requirement that a shifting of the center of gravity backward, due to the unavoidable shifting of load, must be taken into account for the determination of the necessary excess stability.

Centrifugal-Pump Performance as a Function of Specific Speed

By A. J. STEPANOFF,[1] PHILLIPSBURG, N. J.

The object of this paper is to examine the performance of centrifugal pumps as a function of specific speed. Based on established gross pump efficiencies, hydraulic efficiency is evaluated by eliminating mechanical and leakage losses. General laws of variation of disk friction and leakage losses with the specific speed are established. A special study is made of leakage losses, and coefficients necessary for calculation of flow through small annular clearances are given as function of Reynolds number. Knowing hydraulic efficiency, hydraulic losses are calculated, and these in turn permit determination of the necessary experimental coefficients for improved formulas for the theoretical head developed by impellers of centrifugal pumps.

Introduction

For a study of centrifugal-pump performance as it is affected by specific speed, it is convenient to consider double-suction single-stage pumps because they cover a wider range of specific speeds and their design is standardized to a high degree for the whole industry. General conclusions drawn from such a study will apply to any type of centrifugal pump irrespective of the differences in mechanical or hydraulic arrangements.

Fig. 1 shows gross efficiencies of double-suction pumps for different specific speeds which can be considered as commercial efficiencies well established for sizes of pumps indicated on the curve. Efficiencies as shown were obtained with pumps having impellers of approximately 15 in. diam and a capacity of 5000 gpm or greater, at specific speeds above 1500. For lower specific speeds pump sizes and capacities were correspondingly smaller. With larger pumps better efficiencies are possible at lower specific speeds, but as a rule pumps of low specific speed are built in small sizes only. This paper is more concerned with the trend of changes of efficiencies and various losses with the specific speed than with their absolute values.

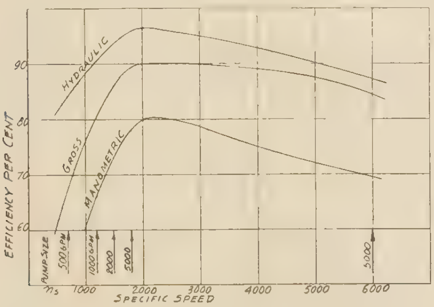
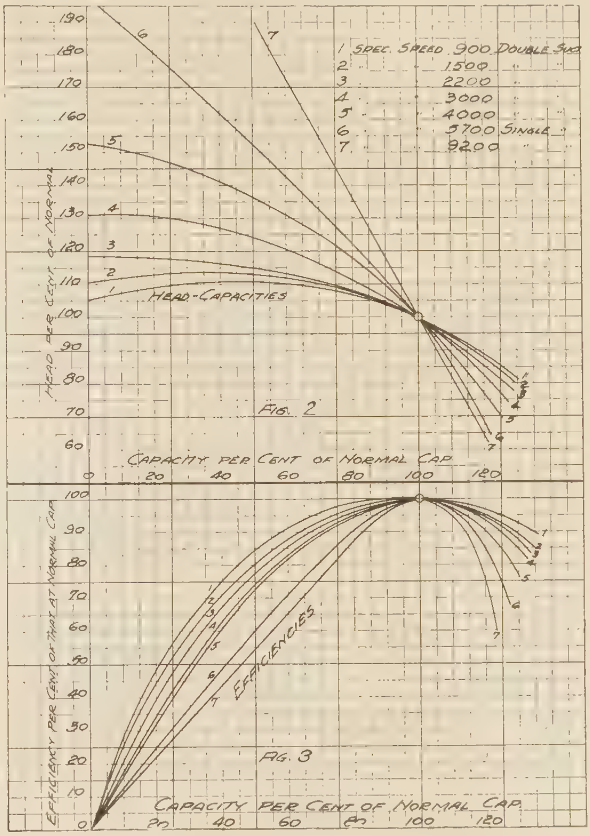


Fig. 1 Double-Suction-Pump Efficiencies



Figs. 2 and 3 *Q-H* and Efficiency Curves for Different Specific Speeds

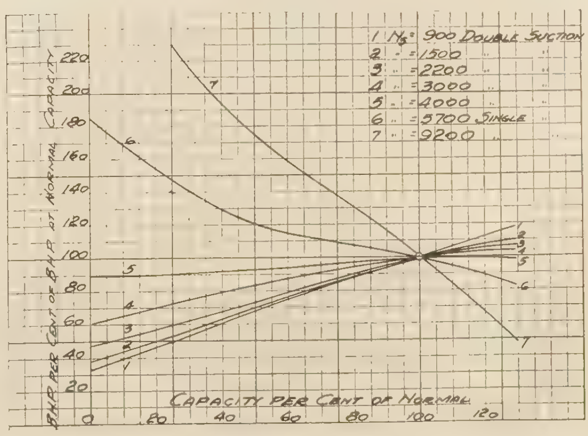


Fig. 4 Brake-Horsepower Curves for Different Specific Speeds

[1] Development Engineer, Ingersoll-Rand Company. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

Figs. 2, 3, and 4 show typical head-capacity, efficiency, and brake-horsepower curves for pumps of different specific speeds.[2] Only slight variation from average curves for a given specific speed is possible without appreciable sacrifice in efficiency. The head-capacity, efficiency, and brake-horsepower curves are interrelated so that change in form of one will be followed by change in form of the other two (1).[3] Since the theoretical head-capacity curve is a straight line, irrespective of the specific speed, the variation in the form of the head-capacity, efficiency, and brake-horsepower curves with specific speed is caused by the losses. To investigate the trend of variation of losses with specific speed and evaluate their relative magnitude is one of the objects of this paper.

Leakage Loss

The gross efficiency of a centrifugal pump can be expressed as a product of three efficiencies

$$e = e_m e_h e_v \qquad [1]$$

where

e_m = mechanical efficiency, which accounts for bearing and stuffing-box friction losses, and the disk friction loss

e_h = hydraulic efficiency, which accounts for hydraulic losses of head between the two points where the suction and discharge heads are measured

e_v = volumetric efficiency = $\dfrac{Q}{Q + Q_L}$, where Q is the measured pump capacity and $(Q + Q_L)$ is the capacity going through the impeller. Term Q_L is the leakage through the close clearance between the rotating element and the stationary casing. Development of Equation [1] can be found elsewhere (2).

Leakage loss in a centrifugal pump can take place in one or more of the following places: (a) Between the casing and impeller at the impeller eye; (b) between the two adjacent stages in multistage pumps; (c) at the stuffing box; (d) through balancing devices of multistage or single-stage pumps; (e) at bleed-off from the stuffing box through a close-fitted bushing to reduce the pressure on the packed stuffing box; (f) leakage past vanes and the casing in open-impeller pumps; and (g) any water used for bearing or stuffing-box cooling.

Evidently volumetric efficiency is lower for small-capacity and high-head pumps than for high-capacity and low-head pumps. The author has published (2) a considerable amount of the experimental data on leakage through the impeller and casing wearing rings of various designs, i.e., plain cylindrical, stepped, grooved, and labyrinth. The data were presented as coefficients of discharge C in the equation

$$Q_L = CA\sqrt{2gH_L} \qquad [2]$$

where

Q_L = amount of leakage
A = free area of clearance
H_L = head across clearance

It has been found by these tests that the coefficient of discharge through the plain cylindrical clearances depends upon the width of the ring, the pressure H_L, the ring peripheral velocity or pump speed, and the clearance. Therefore the selection of a proper coefficient for calculation of the leakage loss is complicated and may require extrapolation or interpolation if the conditions fall beyond the available range of clearances, head H_L, or ring width. To a great extent, the variation of the coefficient of discharge C in Equation [2] is caused by the imperfection of the formula itself which really expresses the discharge through an orifice, and coefficient of discharge C accounts for the entrance loss, friction loss through the ring, in addition to the velocity head at discharge from the ring. More consistent results are obtained if formula [2] is modified to account for several items of loss of head through the clearance, and the coefficient of friction through the ring is plotted as a function of the Reynolds numbers as is done for pipe flow.

The author has made a study of some available test data on the flow through annular clearances, with one or both members forming the clearance stationary, and has prepared a chart, Fig. 5,

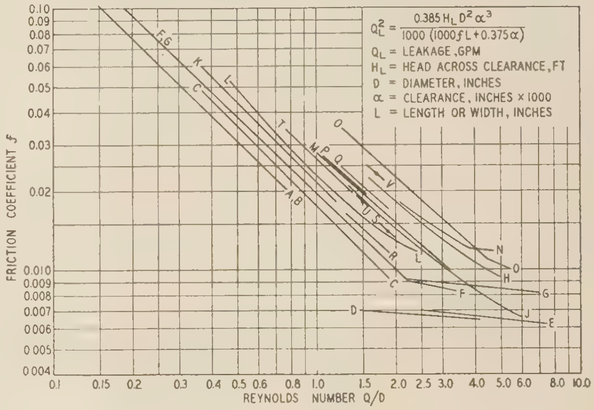

Fig. 5 Flow Through Annular Clearances

showing the coefficients of friction f plotted against Reynolds numbers. The following equation was used for calculation of the amount of leakage

$$Q_L^2 = \frac{0.385\, H_L D^2 \alpha^3}{1000(1000fL + 0.375\alpha)} \qquad [3]$$

where

Q_L = leakage, gpm
H_L = head across clearance, ft
D = average diameter of annular clearance, in.
α = clearance on diameter in thousandths of an inch; $\alpha = 8$ where clearance $a = 0.008$ in.
f = coefficient of friction along throttling annular surfaces
L = length (width of ring) of clearance, in.

This equation is developed from the fundamental formula for the head loss by the flow through the annular clearance (3).

$$H = f\frac{LV^2}{2gm} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g} \qquad [4]$$

where

m = mean hydraulic radius of section normal to flow in inches; equals $a/4$ in this case
V = velocity through clearance, fps
a = $\alpha/1000$ clearance on diameter, in.

The first term on the right-hand side of Equation [4] is loss due to friction; the second term represents the entrance loss of rings with square sharp corners, Fig. 6(a), and the last term is velocity head at discharge through the ring.

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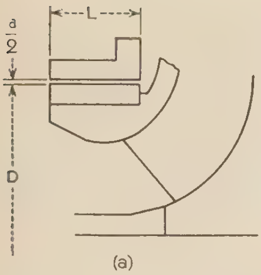
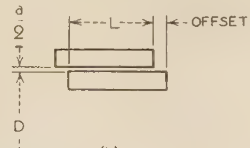
[2] Curves very similar to those in Figs. 2, 3, and 4 appeared in *Escher-Wyss News*, 1934. See Bibliography reference (1).

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.



(a)



(b)

FIG. 6(a) RING WITH SQUARE SHARP CORNERS

FIG. 6(b) IMPELLER AND CASING WEARING RINGS OFFSET

The value of the coefficient f is plotted against Reynolds number as abscissa, which for water reduces to

$$R = \frac{Q \text{ (gpm)}}{D \text{ (in.)}} = \text{const} \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

The more usual form of expression for Reynolds number is

$$R = \frac{Vd}{\nu} = \text{const} \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

where V is velocity $= \dfrac{Q}{\pi D(a/2)}$ in this case

d = pipe diameter or mean hydraulic radius, $m = \dfrac{a}{4}$

ν = kinematic viscosity = 0.01 stokes for water

Thus in the present case

$$R = \frac{Q \times 2 \times a}{D\pi a \times 4 \times 0.01} = \frac{Q}{D} = \text{const}$$

When selecting coefficient f for liquids other than water, divide $\dfrac{Q \text{ (gpm)}}{D \text{ (in.)}}$ by the liquid kinematic viscosity in centistokes. To convert Reynolds number gpm/in. into fundamental units (make it dimensionless), it should be multiplied by 792.

To use Equation [3], it is necessary first to assume an arbitrary value of coefficient f, then calculate Q; after that Q/D can be determined, and the value of f can be taken from the chart Fig. 5. The value of Q is refigured again and, if materially different from the first value, a new ratio of Q/D is calculated and a new coefficient f is taken from the chart. It is seldom that more than two approximations are necessary.

Scattering of points on chart, Fig. 5, can be explained in several ways, as follows: (*a*) It is difficult to measure accurately the true pressure across the clearance in a centrifugal pump. (*b*) The clearances are more or less eccentric, or elliptic, resulting in error of clearance measurement. (*c*) The impeller and casing wearing rings may be offset one with respect to the other, Fig. 6(*b*), resulting in suppressed contraction on one side of the clearance. (*d*) The effect of rotation speed on coefficient of discharge. (*e*) Balancing drums are sometimes provided with additional throttling surfaces which are adjustable axially, while only radial clearances were considered in these calculations. (*f*) The ring may have been scored during tests and not inspected after the test.

It should be noted that in the case of pipe flow on charts similar to that shown in Fig. 5, the scattering of points is generally accounted for by the relative roughness of the pipe walls. This factor is of secondary importance in the case of wearing rings or balancing drums, as these parts are usually given good machine finish.

Sources of the data in Fig. 5 are indicated in Table 1, where size and length of throttling surfaces are given together with the range of pressures under which the amount of leakage was measured. These may be helpful in selecting a proper coefficient. The throttling surfaces were, in all cases, plain cylindrical smoothly finished. The effect of labyrinth, circular, or spiral grooves on the amount of leakage was reported by the author in a previous paper (2).

LEAKAGE LOSS VERSUS SPECIFIC SPEED

Using coefficients from Fig. 5, the author has calculated the leakage loss for a number of double-suction horizontally split pumps of different specific speeds. In all cases actual clearances

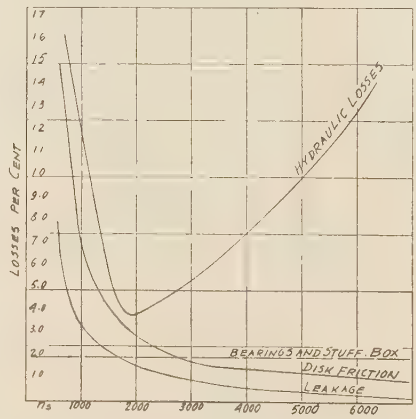


FIG. 7 LOSSES VERSUS SPECIFIC SPEED OF DOUBLE-SUCTION PUMPS

TABLE 1 LEAKAGE DATA PLOTTED IN FIG. 5

Curve symbol	Data by	Clearance a, in.	Diameter D, in.	Length, in.	Speed, rpm	Peripheral velocity, fps	Maximum head H_L, ft	Bibliography ref.
A	Schneckenberg	0.008	3.18	$7^1/_8$	0	0	82	9
B	Schneckenberg	0.014	3.18	$7^1/_8$	0	0	82	9
C	Schneckenberg	0.020	3.18	$7^1/_8$	0	0	82	9
D	Schneckenberg	0.032	3.18	$7^1/_8$	0	0	82	9
E	Schneckenberg	0.046	3.18	$7^1/_8$	0	0	82	9
F	Becker	0.033	0.865	6.9	0	0	23.5	10
G	Becker	0.049	0.865	6.9	0	0	12.0	10
H	Aisenstein	0.014	$4^1/_2$	$4^5/_8$	1750	35.0	1000	11
J	Aisenstein	0.019	$5^3/_4$	$8^1/_2$	1750	43.8	1000	11
K	Stepanoff	0.012	$4^1/_8$	$^7/_8$	1400	25.3	40	2
L	Stepanoff	0.012	$4^1/_8$	$1^1/_{16}$	2500	45.2	140	2
M	Stepanoff	0.012	$4^1/_8$	$^1/_2$	2500	45.2	140	2
N	Stepanoff	0.017	$4^1/_8$	$^{11}/_{16}$	2500	45.2	140	2
O	Stepanoff	0.0203	$4^1/_8$	$^{11}/_{16}$	2500	45.2	140	2
P	Yendo	0.020	11.8	0.788	1200	62.0	16.4	4
Q	Yendo	0.020	11.8	0.394	1200	62.0	16.4	4
R	Stepanoff	0.008	$2^3/_4$	5	3550	42.5	1085	New
S	Stepanoff	0.008	$2^3/_4$	$5^1/_2$	3550	42.5	1430	New
T	Stepanoff	0.006	$2^1/_4$	$1^3/_4$	3550	34.7	800	New
U	Stepanoff	0.015	7.427	$5^{13}/_{16}$	1450	47.3	311	New
V	Stepanoff	0.013	7.427	$5^{12}/_{16}$	1450	47.3	309	New

and wearing-ring widths were used for plain flat rings. The results are shown in Fig. 7. It will be observed that leakage loss is rapidly decreasing with increasing specific speed.

For the purpose of further discussion it is important to establish some general relationships between leakage loss and specific speed.

It will be shown that the power loss due to leakage is constant for pumps of the same specific speed, irrespective of pump size and speed. To demonstrate the truth of this statement, Equation [2] will be used

$$Q_L = CA\sqrt{2gH_L} \qquad [8]$$

Substitute for the clearance area its equivalent $A = \dfrac{a\pi D}{2}$ and express H_L as K_rH, where K_r is a factor less than 1, indicating the pressure across rings as a fraction of the pump total head. It will be assumed that this factor is constant for similar pumps. Also, it will be understood that the coefficient of discharge C for the clearance is constant for similar pumps. Then, the power lost because of the backflow through the clearance is

$$(hp_L) = (Q_L\gamma H)/550 \qquad [9]$$

where γ is the weight of 1 cu ft of water. The ratio of power lost due to leakage of two pumps is equal

$$\frac{(hp_L)_1}{(hp_L)_2} = \frac{a_1D_1(H_1)^{3/2}}{a_2D_2(H_2)^{3/2}} \qquad [10]$$

If similarity of the two pumps is extended to the wearing-ring diameters and clearances, then

$$\frac{a_1D_1}{a_2D_2} = \frac{(D_{10})^2}{(D_{20})^2} \qquad [11]$$

where D_{10} and D_{20} are the outside impeller diameters of two pumps. Substituting this into Equation [10] we obtain

$$\frac{(hp_L)_1}{(hp_L)_2} = \frac{D_{10}^2}{D_{20}^2}\,\frac{(H_1)^{3/2}}{(H_2)^{3/2}} \qquad [12]$$

Making use of Equation [13] for the type-unit-capacity relationships for similar pumps, we change Equation [12] to [14]

$$\frac{Q_1}{\sqrt{H_1}\times D_{10}^2} = \frac{Q_2}{\sqrt{H_2}\times D_{20}^2} = \text{const} \qquad [13]$$

$$\frac{(hp_L)_1}{(hp_L)_2} = \frac{Q_1H_1}{Q_2H_2} \qquad [14]$$

that is, the power lost in leakage is proportional to the pump output. Hence if expressed as a percentage of the pump output, this loss is constant for all similar pumps. In practice, however, the leakage loss is greater for smaller pumps, as the clearances cannot be reduced below a certain minimum; also because the wearing rings of larger pumps are wider, and the coefficient of discharge C is smaller.

Comparing the leakage-loss horsepower for pumps of the same output and different specific speed, rpm, and size, we may write Equation [10] as before

$$\frac{(hp_L)_1}{(hp_L)_2} = \frac{a_1D_1H_1^{3/2}}{a_2D_2H_2^{3/2}} = \frac{D_1^2H_1^{3/2}}{D_2^2H_2^{3/2}} \qquad [15]$$

assuming that the width of the wearing rings is the same (coefficient C is the same for two cases), and that the clearances vary in the same ratio as the impeller wearing-ring diameters.

The radial velocity at the impeller eye varies with the impeller type and speed, and the ratio may be expressed in terms of the head as follows

$$\frac{c_{m1}}{c_{m2}} = \frac{K_{m1}}{K_{m2}}\,\frac{H_1^{1/2}}{H_2^{1/2}} \qquad [16]$$

Assuming that the impeller-eye area $= \pi D^2/4$ is available for the flow, or that the pump is of the overhung-impeller construction, or that the effect of the presence of the shaft in the impeller eye upon the ratio of the impeller-eye areas may be neglected, we can write

$$\frac{Q_1}{Q_2} = \frac{c_{m1}}{c_{m2}}\,\frac{D_1^2}{D_2^2} = \frac{K_{m1}H_1^{1/2}D_1^2}{K_{m2}H_2^{1/2}D_2^2} \qquad [17]$$

Combining this equation with Equation [15] we obtain

$$\frac{(hp_L)_1}{(hp_L)_2} = \frac{Q_1H_1K_{m2}}{Q_2H_2K_{m1}} = \frac{K_{m2}}{K_{m1}} \qquad [18]$$

Since water horsepower (whp) is the same for both pumps, the ratio of the leakage-loss horsepower, expressed in per cent of whp will be given by Equation [18]. The value of the factor K_m is increasing with the increase of the specific speed, hence for the same pump output the leakage loss is higher for the lower-specific-speed pump. For example, for $n_s = 1000$ the factor $K_{m1} = 0.12$; for $n_s = 2000$ the factor $K_{m2} = 0.155$, and the ratio of the leakage-loss hp is equal to $0.155/0.12 = 1.3$.

Disk-Friction Loss

Having established a connection between the specific speed and leakage loss, use will be made of Equation [1] to determine hydraulic efficiency for pumps of different specific speed, knowing the gross pump efficiency and evaluating the mechanical efficiency. It will be shown that variation of mechanical efficiency e_m is mostly affected by the disk-friction loss, which varies greatly with the specific speed, while the remaining mechanical losses, i.e., bearing and stuffing-box friction, are essentially independent of specific speed.

General relationships will be established governing variation of disk-friction loss for pumps of different types.

Disk Friction of Pumps of Same Specific Speed but of Different Size and Speed (12). Consider two pumps of the same specific speed and impeller diameters D_1 and D_2, heads H_1 and H_2, capacities Q_1 and Q_2, and speeds n_1 and n_2, respectively. The power consumed by disk friction is

$$hp_f = KD^5\,n^3 \qquad [19]$$

where K is a numerical constant, depending on the units used. The ratio of the disk-friction horsepower of the two pumps is

$$\frac{(hp_f)_1}{(hp_f)_2} = \frac{(D_1n_1)^3\,(D_1)^2}{(D_2n_2)^3\,(D_2)^2} \qquad [20]$$

Combining this equation with those for the type unit capacities Equation [13] and type unit speed Equation [21]

$$\frac{n_1D_{10}}{\sqrt{H_1}} = \frac{n_2D_{20}}{\sqrt{H_2}} = \text{const} \qquad [21]$$

we obtain Equation [22]

$$\frac{(hp_f)_1}{(hp_f)_2} = \frac{Q_1H_1}{Q_2H_2} \qquad [22]$$

or that the disk-friction horsepower varies as the pump output. This could be expected from an inspection of the formula for the disk-friction loss as both the disk-friction power loss (hp_f) and the pump output, whp, vary as the cube of the speed and fifth

power of the impeller diameter. From Equation [22], it follows that the ratio of the disk-friction horsepower to the water horsepower is constant for the same specific speed irrespective of the size and speed of the pump.

Disk Friction of Pumps of Same Output but of Different Specific Speed, Size, and Rpm. For two pumps of the same water horsepower, the ratio of specific speeds is equal to

$$\frac{n_{s1}}{n_{s2}} = \frac{n_1 Q_1^{1/2} H_2^{3/4}}{n_2 Q_2^{1/2} H_1^{3/4}} = \frac{n_1 H_2^{5/4}}{n_2 H_1^{5/4}} \qquad [23]$$

Ratio of the heads may be obtained from

$$\frac{u_1}{u_2} = \frac{\phi_1 \sqrt{H_1}}{\phi_2 \sqrt{H_2}} = \frac{D_1 n_1}{D_2 n_2} \qquad [24]$$

where u_1 and u_2 are the peripheral velocities of the two impellers at the outlet and ϕ_1 and ϕ_2 are the speed factors from

$$u = \phi \sqrt{2gH} \qquad [25]$$

Combining Equations [23] and [24] with Equation [19], we get for the ratio of the horsepower consumed by the disk friction of the two pumps

$$\frac{(hp_f)_1}{(hp_f)_2} = \frac{n_{s2}^2 \cdot \phi_1^5}{n_{s1}^2 \cdot \phi_2^5} \qquad [26]$$

and if $\phi_1 = \phi_2$ approximately, the expression simplifies to

$$\frac{(hp_f)_1}{(hp_f)_2} = \frac{n_{s2}^2}{n_{s1}^2} \qquad [27]$$

that is, for the same pump output, the disk-friction horsepower varies inversely as the square of the specific speed. The effect of the factor ϕ can be seen from the following example:

For $n_s = 1000$, the factor $\phi_1 = 0.95$. For $n_{s2} = 2000$ for the same vane angle $\phi_2 = 1$. Then, considering the variation of ϕ the ratio of the disk-friction horsepower is equal to

$$\frac{(hp_f)_1}{(hp_f)_2} = \frac{2000^2 \times 0.95^5}{1000^2} = 4 \times 0.77 = 3.08$$

It is evident that the ratio of disk-friction horsepower, expressed in per cent of water horsepower will vary also inversely as the square of the specific speed (for $\phi_1 = \phi_2$) because water horsepower is equal for the two pumps.

Actual Disk-Friction Loss Versus Specific Speed. Using Pfleiderer's formula (3), where D is in feet

$$hp_f = \frac{0.38 n^3 D^5}{10^9} \qquad [28]$$

disk-friction loss was computed for double-suction pumps of different specific speeds taking actual impeller diameters. This loss expressed as percentage of pump brake horsepower (not whp) is plotted in Fig. 7. Attention is called to the rapid rise of disk-friction loss at specific speeds below 2000. It is interesting to note that almost throughout the entire range of specific speeds, leakage loss is approximately equal to one half of the disk-friction loss.

Comparing the values for disk friction from the curve, Fig. 7, with that given by Equation [26], it is found that the latter gives a greater rate of variation of disk-friction loss with the specific speed than that represented by the curve. However, for a small variation of specific speed, the ratio of the disk-friction losses given by Equation [26] agrees approximately with that from the curve, Fig. 7.

A study of test data by Gibson and Ryan (6) and also Le Conte (7) shows that (*a*) disk-friction loss increases with the clearance between the disk and the stationary wall 4 per cent, (*b*) painting of a rough cast-iron casing reduces the disk-friction loss 16 to 20 per cent; (*c*) polishing the disk reduces the loss 13 to 20 per cent; (*d*) badly rusted cast-iron casing walls increase disk-friction loss 30 per cent, as compared with a clean bronze casing.

Mechanical Losses. Although the nature of mechanical losses in the bearings and stuffing boxes is well understood, there are very little actual data available on the value of these losses. The difficulty lies in the fact that these losses are small and difficult to measure with ordinary equipment of pump-manufacturing testing facilities. On the other hand, it is felt that such tests will be of slight value to the pump manufacturers in so far as improvement of pump performance is concerned. Both stuffing-box and bearing designs are governed by requirements for satisfactory mechanical performance. The matter of friction loss in both is of secondary importance. Besides, friction loss in the stuffing boxes is affected by a number of factors such as size and depth of stuffing box, pump speed, method of packing and lubrication, so that any actual figures of losses would be illustrative of a certain type of stuffing-box application only.

With ball-bearing sizes well standardized, and the high degree of accuracy of manufacturing attained, the friction loss in the ball bearings varies for the same size and load for different makes of bearings. Also, the method of lubrication of ball bearings affects the losses in the ball bearing, as is evidenced by the bearing running temperatures. It has been found that the type of coupling between the pump and driver affects the bearing behavior, and hence losses, as some couplings impose more or less axial load on the thrust-type ball bearings. All these points are illustrated in Fig. 8(*a*, *b*, *c*), where higher ultimate bearing temperatures indicate higher friction losses.

Masanao Yendo (4) has found that stuffing-box and bearing friction losses are from 2 to 3 per cent of the total brake horsepower for 5- and 8-in. single-suction pumps. Within the speed limits of Yendo's tests (5-in. pump, 1400 rpm), it appears that the friction losses have a linear relation to the speed. Earlier tests by Daugherty (5) show that friction in bearings increases at a higher rate than the first power of the speed. Since the brake horsepower increases as the cube of the speed, the mechanical efficiency

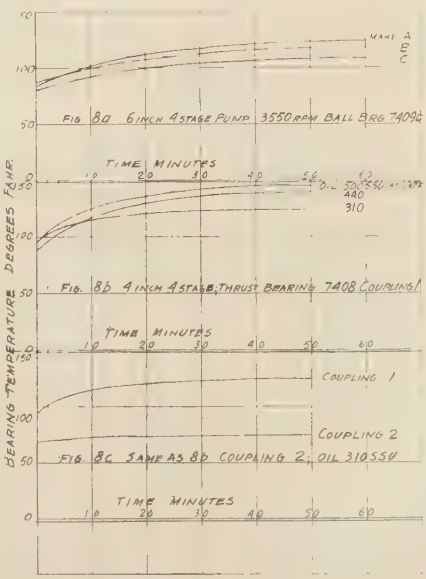


Fig. 8 Effect on Bearing Behavior of Type of Coupling Between Pump and Driver

including disk-friction loss should improve with the speed. This is offset partially by the fact that high-speed pumps usually have larger shafts and bearings and have higher stuffing-box pressures.

For the purpose of this paper it will be assumed that the stuffing-box and bearing friction loss is 2 per cent and is independent of specific speed. The mechanical efficiency, as defined, is

$$e_m = 1 - \frac{\text{Mechanical losses}}{\text{Bhp}} \qquad [29]$$

Mechanical losses include the disk friction and stuffing-box and bearing friction. By adding 2 per cent bearing and stuffing-box friction to the disk-friction loss, mechanical efficiency can be calculated for different specific speeds. Using Equation [1], hydraulic efficiencies are calculated for different specific speeds from known gross efficiencies. These are also plotted in Fig. 1. By inspecting this curve, it should be noted that the hydraulic efficiency reaches its maximum at a specific speed of 2000, decreasing on both sides of this point. The decrease of hydraulic efficiency at lower speeds is caused mainly by increased hydraulic friction in the impeller and volute as all hydraulic passages are small in comparison with the impeller diameter. This is further accentuated by the fact that low-specific-speed pumps are built of smaller sizes than the medium- and high-specific-speed pumps.

Decrease of hydraulic efficiency at higher specific speeds is caused by lack of proper guidance of water by the impeller vanes, due to short vane length and small lap of vanes. An increase in the number of vanes introduces additional friction, which again will reduce the hydraulic efficiency. The fact that better gross efficiencies are obtained at high specific speeds with single-suction vertical pumps with diffusion casings, Fig. 9, indicates that the 90-

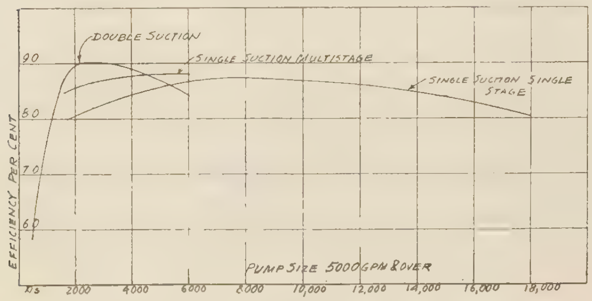


Fig. 9 Single-Suction Pump Gross Efficiency Versus Specific Speed
(Pump size 5000 gpm and over.)

deg turn in the double-suction impeller results in losses which are not existent in vertical single-suction pumps of the propeller type.

However, there are other factors contributing to the higher efficiency of vertical pumps of high specific speeds, such as (*a*) no stuffing boxes and no load on radial guide bearings; (*b*) no disk friction, as pumps are generally built with open impellers; (*c*) no suction-nozzle losses and entrance loss reduced to a minimum; (*d*) axial approach and discharge from the impeller; (*e*) vertical pumps are usually submerged, while horizontal pumps are tested under suction lift.

The gross efficiency of vertical pumps is yet further improved when two or more stages are used. In that case such losses as entrance, discharge-elbow loss (usually charged against the pump), and bearing friction, are not increasing in proportion to the number of stages; hence their value as percentage of the pump power input becomes smaller.

The hydraulic efficiency as figured from Equation [1] and plotted in Fig. 1, for different specific speeds, might be called "actual" hydraulic efficiency to distinguish it from the "theoretical" hydraulic efficiency based on the theoretical head as given by Euler's equation.

$$e_{ht} = \frac{Hg}{u_2c_{2u} - u_1c_{1u}} \qquad [30]$$

This value, sometimes called "manometric efficiency," is considerably lower than the "actual" hydraulic efficiency. In fact, it is lower than the gross pump efficiency, indicating that the theoretical head as given by Euler's equation is too high. The "manometric" hydraulic efficiency was calculated for the double-suction pumps and plotted in Fig. 1, for different specific speeds. The inconsistency of Euler's equation for the theoretical head, as evidenced by the fact that hydraulic efficiencies are obtained which are lower than the pump gross efficiencies, has led several investigators to discard this equation as incorrect and to offer new theories which are supposed to agree better with the experimental results. These have always been followed by bitter discussions by the followers of the classical elementary theory.

Another group of investigators (notably Pfleiderer) endeavored to apply corrective factors to the tangential component of the absolute velocity c_{2u} to make the value of the theoretical head and hydraulic efficiency more reasonable. These corrected formulas for the theoretical head have always included some experimental factor. To determine such corrective factors, two methods are possible, i.e., (*a*), as suggested in this paper, is to evaluate mechanical and volumetric efficiency and then use Equation [1] to calculate the actual hydraulic efficiency from gross pump efficiency found experimentally; (*b*) hydraulic losses are calculated and added to pump total head to obtain the "actual" theoretical head. While it is comparatively simple to set up formulas for hydraulic losses through the pump, determination of the constants for such formulas presents insurmountable difficulties, particularly if it is intended to extend the same laws for all points on the head-capacity curve.

To make the problem still more difficult, there is no way to separate the several items constituting hydraulic losses such as friction, shock at entrance and discharge from the impeller, and diffusion loss in the impeller and casing, even if the sum of hydraulic losses is determined by some such means as outlined in this paper.

Losses Versus Capacity at Constant Speed. This paper has dealt with pump performance and specific speed at the best efficiency point. Variation of several losses for the same pump at various capacities at constant speed can be determined in the following ways:

1 All mechanical losses, including the disk-friction, stuffing-box, and bearing losses, are constant at constant speed. However, expressed as a percentage of the brake horsepower, the values representing these losses at several capacities will vary, depending upon the shape of the brake-horsepower curve, Fig. 4. Thus, for low-specific-speed pumps, the brake horsepower is decreasing at partial capacities, hence mechanical losses as percentages of brake horsepower will increase or mechanical efficiency will decrease. In Fig. 10, mechanical losses are plotted against capacity for a 12-in. double-suction pump of 1900 specific speed.

2 The leakage loss (gpm) increases slightly as capacity decreases, as a result of higher heads. However, expressed as percentage of the total impeller capacity ($Q + Q_L$), the leakage loss is rapidly increasing toward zero capacity, where this loss reaches 100 per cent value. This has been plotted in Fig. 10, for the same pump.

3 The determination of hydraulic losses for different capacities presents very serious difficulties. If Equation [31]

$$e = e_m e_v e_h \qquad [31]$$

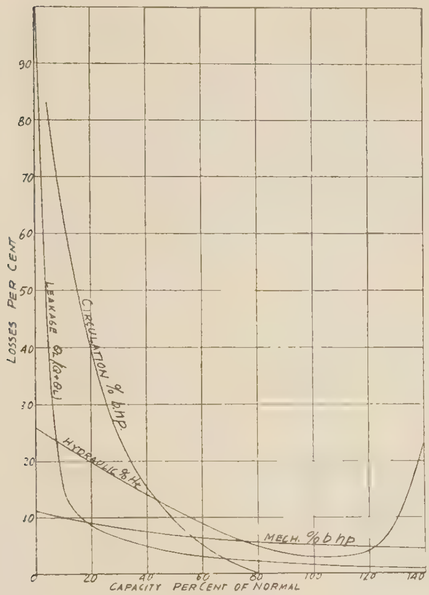


FIG. 10 LOSSES VERSUS CAPACITY
(n_s = 1900 constant rpm.)

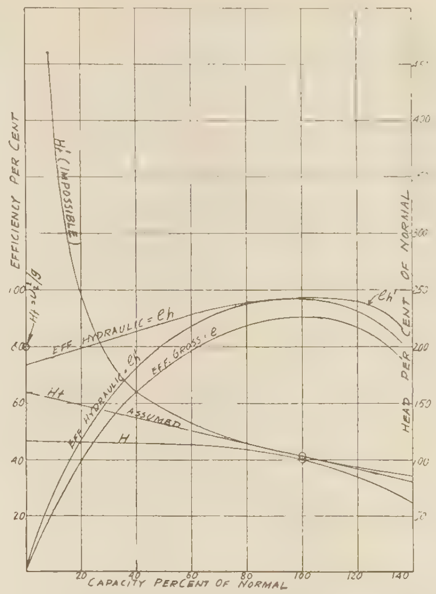


FIG. 11 PERFORMANCE OF 12-IN. DOUBLE-SUCTION PUMP
(n_s = 1900.)

is used for calculating hydraulic efficiency e_h (as has been done for the best efficiency point), absurdly low values are obtained at low capacities. Calculating the theoretical heads from these values of hydraulic efficiencies gives heads much higher than

$$H_t = \frac{u_2^2}{g} \qquad [32]$$

which is a maximum possible theoretical head at zero capacity with infinite number of vanes and vane angles smaller than 90 deg. Fig. 11 shows these hydraulic efficiencies e_h' calculated from

$$e_h' = \frac{e}{e_v e_m} \qquad [33]$$

and also the theoretical heads H_t' as given by the hydraulic efficiency e_h' or

$$H_t' = \frac{H}{e_h'} \qquad [34]$$

The discrepancy cannot be explained by the inaccuracy of determination of leakage and mechanical losses. This led Pfleiderer to assume the existence of a different kind of loss, which is zero at the best efficiency point and a maximum at zero capacity. This loss of power (not head, because theoretical head is too high already) is caused by the exchange of momentum of water particles in the impeller passages at the periphery with particles in the volute moving with much lower velocities. It is somewhat similar to the disk-friction loss.

Evidently the smaller the capacity the greater are the shearing forces between the liquid in the impeller and the volute, and the higher is the power absorbed by these shearing forces. Daugherty (5) realized the existence of such a loss and called it "churning loss." It is believed that a major part of this loss is caused by the relative circulation of water within the impeller channels at low rates of flow through the impeller (backflow). Such circulation is higher with high-specific-speed pumps (less vane surface) and is responsible for the power increase at reduced capacities. No formulas have yet been offered for expressing this "circulation" loss.

The following procedure provides means to estimate "circulation" loss. A reasonable value of the "actual" theoretical head is assumed from the following considerations: (a) It is expected that the circulation loss is zero near best efficiency point; (b) the theoretical head-capacity curve is a straight line; and (c) the theoretical head is lower than u_2^2/g at shutoff. To satisfy all these conditions, a straight line was drawn, in Fig. 11, tangent to the theoretical head curve, (H_t'), as calculated, disregarding the circulation loss and touching it at the best efficiency point. Using this theoretical head a true hydraulic efficiency is found (Fig. 11, e_h)

$$e_h = \frac{H}{H_t} \qquad [35]$$

From this a new gross efficiency is found excluding the circulation loss, from

$$e' = e_v e_m e_h \qquad [36]$$

and finally, by dividing (whp) by a new gross efficiency, e', the (bhp)$'$ is obtained, which excludes the circulation loss

$$(\text{bhp})' = \frac{\text{whp}}{e'} \qquad [37]$$

The latter is found as a difference between the actual (bhp) and (bhp)$'$. Expressed as percentage of (bhp), circulation loss is plotted in Fig. 10.

From the foregoing outline, it is seen that a study of losses for the entire head-capacity range of pumps of different specific speeds is really a difficult task, considering variation of head, capacity, efficiency, and brake-horsepower curve form with specific speed, illustrated in Figs. 2, 3, and 4.

CONCLUSIONS

1 The optimum gross pump efficiency of centrifugal pumps varies with the specific speed of pumps, reaching its maximum at specific speeds of 2000 to 3000 (Fig. 1) for double-suction pumps.

2 Variation of losses with specific speed at best efficiency points is shown in Fig. 7. Hydraulic losses were found by elimi-

nating mechanical and leakage losses, knowing gross efficiency of pumps.

3 It has been shown that disk-friction loss and leakage loss expressed in percentage of pump output do not depend upon pump size and speed.

4 A general relationship has been established between disk-friction and leakage losses and specific speed.

5 Friction coefficients for the calculation of the flow through the annular clearances has been compiled and plotted against Reynolds numbers. These are independent of the length of the throttling surfaces, their diameter, and the clearance.

6 "Actual" hydraulic efficiencies were determined for pumps of different specific speed and plotted in Fig. 1. Also "theoretical" hydraulic efficiencies are given in the same figure for a comparison.

7 A study of losses at partial capacities at constant speed reveals the existence of "circulation" loss. This is a loss of power caused by relative internal circulation within impeller channels and also from channel to channel. This loss is zero near the best efficiency point and is a maximum at zero capacity. This loss is greater with high-specific-speed pumps.

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Discussion

J. W. Daily.[4] In his analysis, the author makes use of two conceptions which, as he indicates, are invitations to an argument. It is little wonder, since both are used loosely and badly in the literature. The author states that the "theoretical head-capacity curve is a straight line irrespective of specific speed," and in another paragraph mentions the so-called "monometric efficiency." The two are related in that they appear from the consideration of an idealized pump which gives us the so-called classical elementary theory. This theory results from a conception of a flow pattern which conforms exactly to the path prescribed by the impeller-vane shape, but which does not actually exist in any pump. As a consequence, the head-capacity curve for this idealized flow is entirely divorced from physical facts. Unfortunately, the relations of the classical theory to a theoretical head curve based on the true state of affairs is usually not made clear. The basis of the pump theory; i.e., the principle that the rate of change of angular momentum which the fluid undergoes as it passes through the machine is proportional to the torque applied, is a general theorem which must hold for the actual as well as the ideal machine. The theoretical equation for head

$$h = \frac{u_2C_{u2} - u_1C_{u1}}{g} = \frac{u_2C_2\cos\alpha_2 - u_1C_1\cos\alpha_1}{g}$$

where

h = head

u = tangential velocity of impeller

C_u = $C\cos\alpha$ = tangential component of absolute velocity of flow

is merely an expression of the theorem and can be used for any pump, if the true values of the velocities and angles are inserted. It will represent then the theoretically possible head for the actual machine under consideration.[5] Note that this is still a theoretical head since losses have not been deducted, but that it is something entirely different from the "idealized" head curve obtained by assuming no deviation of the flow from the ideal path prescribed by the impeller-vane shape. The hydraulic losses are the difference between this head and the measured net head.

It is true that determination of the actual velocities and angles of flow is difficult, but some measurements[6] have been made, indicating not only that the flow deviates from the ideal path, as is generally recognized, but that the amount of deviation changes with the discharge. These observations mean that the theoretical head for the actual machine, as calculated from the foregoing equation, will not only be less than the ideal head, but that the head-capacity curve will not be linear. From this viewpoint it is clear that the use of a linear-head relationship is permissible only on the assumption that the curvature effects are small. This may or may not be true. Similarly, it follows that the comparison of the "ideal" head and measured head curves to give the "manometric efficiency" is of value only in that the latter quantity gives some indication of the amount of deviation of the actual flow from the idealized flow.

In the discussion of the losses versus capacity at constant speed, the author makes use of the conception of a "circulation loss" which varies continuously from zero at the best efficiency to a maximum at shutoff. The writer agrees that the major part of this loss is probably caused by "backflow" circulation. Measurements in the hydraulic machinery laboratory at the California Institute of Technology have definitely established the existence of backflow through the impeller, accompanied by prerotation at the impeller inlet, for lower than normal rates of flow.[6] This should absorb a large share of the horsepower expended as the capacity is reduced to zero.

The assumption that the "circulation losses" are a minimum at the best efficiency point is reasonable where no information to the contrary is available. A study of the cavitation behavior of a given pump, operating at constant speed over a wide range of discharges, has indicated that the optimum operating point for the

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[5] This quantity is called "head input" by some authors, e.g., "Hydraulics," by R. L. Daugherty, McGraw-Hill Book Co., Inc., New York, N. Y., 1937.

[6] "Experimental Determinations of the Flow Characteristics in the Volutes of Centrifugal Pumps," by R. C. Binder and R. T. Knapp, Trans. A.S.M.E., vol. 58, 1936, pp. 649–661.

Also, unpublished results of additional measurements of the velocities at the impeller inlet and discharge made in the Hydraulic Machinery Laboratory of the California Institute of Technology.

impeller is sometimes quite far from the best efficiency point.[7] This probably also indicates that the so-called "shock" losses are a minimum at some other point than the best efficiency.

In closing, the writer would like to point out that a starting point for the evaluation of losses in a centrifugal pump can very well be the torque (brake-horsepower) curve, after elimination of the mechanical- and disk-friction losses. In addition to its relation to the proper theoretical head, as just indicated, the torque or brake-horsepower is very often the most reliable measurement obtained in laboratory tests. This should contribute to the accuracy of the evaluation.

O. H. Dorer.[8] Gross or net efficiency may be considered, as in this paper, as a product of three other efficiencies. However, it is disputed that bearing and stuffing-box loss is as high as given. It is further difficult to determine from a pump test the extent of H_1 referred to total generated head, and thus be able to calculate leakage loss exactly.

Working an example from Fig. 1 of the paper; 6000 gpm, 90 per cent net efficiency, $ns = 2000$, there results a pump of 225 ft head, 1500 rpm, 378 hp, total losses = 37 hp.

From Fig. 7 of the paper, losses figure about as follows:

(*a*) Stuffing box and bearings, 7 hp. This appears very high and must require water-cooled boxes and bearings.

(*b*) Leakage horsepower = 5.6 which is equivalent to about 120 gpm, 175 ft head, roughly in agreement with Fig. 5, and also with $C = 0.75$ in $Q = CAV$.

(*c*) Disk horsepower $= 11.2 = \dfrac{D^5\,N^3}{6.5 \times (10)^{14}}$ assume $\phi = 1$.

(*d*) This leaves 13.2 hp to be charged as hydraulic loss, giving a hydraulic efficiency of 96.5 per cent, which is disputed.

It is very probable that bearing losses are considerably less and this leaves a larger amount of horsepower chargeable as hydraulic loss.

Our practice is to charge leakage against hydraulic loss, and all other losses against disk horsepower. Tests of a dummy impeller indicate $\dfrac{D^5\,N^3}{4.9}$ covers these latter losses. Our tests involved 10 hp as against 18.2 of the foregoing example, and the proportionate bearing loss (using percentage values of Fig. 7) would be 3.85 hp. We know such an amount of horsepower could not have been used; the ball bearings operated entirely normally without cooling, etc.

Accordingly, the high hydraulic efficiency shown (viz. 96.5 per cent) would not obtain, and this figure reduces to 93.5 per cent. Following through for other specific speeds, the hydraulic efficiency will be a flatter curve, and this nullifies the apparent peak of hydraulic efficiency shown in Fig. 1 at 2000 n_s.

We do not credit a 6000-gpm pump with more than 92.5 per cent hydraulic efficiency (about 89 per cent net) and would calculate 14.5 hp chargeable to bearings, stuffing boxes, and disk loss, which leaves 21 hp chargeable to hydraulic loss, still leaving 5.6 hp as leakage loss. This 21 hp equals 13.8 ft of head at 6000 gpm flow.

Now, when viscous oils are handled, there is a breakdown in the head and an increase in horsepower due mainly to disk-friction effects. Assuming an oil 1000 times more viscous than water, the spiral friction plus impeller loss increases $3^1/_3$ times, since the Reynolds number has decreased 1000 times. This works out to approximately 46-ft total loss, or a net developed head of $225 + 13.8 - 46 = 192.8$ ft. From other data, the expected developed head is approximately 187 ft (about 6 ft less than is obtained by means of this calculation). With 96.5 per cent hydraulic efficiency, the breakdown in head is about 28 ft, resulting in a developed head of 205 ft, which is too high a figure.

On the other hand the disk horsepower increases approximately as 1000^{36} or to 176 hp which makes a net efficiency of 52 per cent. (Note that the coefficient 0.36 is not constant with viscosity changes.)

Using the author's hydraulic efficiency and disk-horsepower determination results in much less head loss and much higher net efficiency than is known to be obtained on oils.

In conclusion, we believe it to be necessary to check performance on viscous fluids to develop the facts as to the relative losses caused by disk friction, leakage, and stuffing-box-bearing loss. This paper stresses certain of these losses so we can really visualize them and aids in confirming the water analysis of centrifugal pumps.

Karl Eklund.[9] The author's final statement that "... a study of losses for the entire head-capacity range of pumps of different specific speeds is really a difficult task...," certainly cannot be denied in view of the multitude of contributing variables. The author's approach to the problem is interesting and instructive, to say the least; although it is believed that his conclusions in all cases have not considered all pertinent variables to the question. The main difficulty to the use of specific speed (N_s), speed factor ϕ, and all other definitions to the technical practice, such as unit speed and unit discharge, is that the relations expressing these functions hold between two machines only at the same efficiency and only if the influence of Reynolds number is neglected.

The writer has made no attempt to derive the author's Equation [3] from the basic Equation [4], but suggests that if Q_L entered Equation [3] through the relation that V (Equation [4]) equals Q_L divided by the clearance area, which is noted to be $\pi D(a/2)$, then a recheck is in order, because the clearance area can be more closely approximated by (πDa) rather than one half that value. This can be seen by the fact that the clearance area is $(\pi/4)\ [(D + 2a)^2 - D^2]$.

It is believed the recheck will show that the author's friction coefficient (Fig. 5, and Equation [3]) is one fourth the true friction coefficient. If Fig. 5 were then amended, by multiplying by 4 all friction coefficients shown thereon, it would become evident that the author's curves (Fig. 5) could be substantially represented by $F = (64/R)$, to an upper limit of $R = 1584$, (i.e., $Q/D = 2$ in the author's dimensions), where R is the true Reynolds number rather than one in terms of gallons per minute per inch. The author will then have shown that the friction factor for laminar flow, as derived from the classical theory, is applicable in this specific instance of laminar flow.

The author states that "disk-friction horsepower varies as the pump output," and in accordance with his Equation [19]. These statements are not to be denied. Their completeness, however, is questionable. Dimensional analysis indicates that the author's Equation [19] might be written

$$hp_f = KD^5N^3\rho f(R)\ldots\ldots\ldots\ldots\ldots\ldots[38]$$

where ρ is the mass density of the fluid pumped. Test data placed on file by the writer at Cornell University in 1939, indicate that there is a relationship between disk-friction horsepower and the Reynolds number. In this instance data were obtained by rotating a disk in a closely confined space occupied consecutively by ten different fluids of known kinematic viscosity. The test

[7] "A Theory of Cavitation Flow in Centrifugal-Pump Impellers," by C. A. Gongwer, Trans. A.S.M.E., vol. 63, 1941, pp. 29–40.

[8] Assistant Manager, Centrifugal Pump Engineering Division, Worthington Pump and Machinery Corporation, Harrison, N. J. Mem. A.S.M.E.

[9] Lieutenant Colonel, Corps of Engineers, U. S. Army. Fort Belvoir, Va.

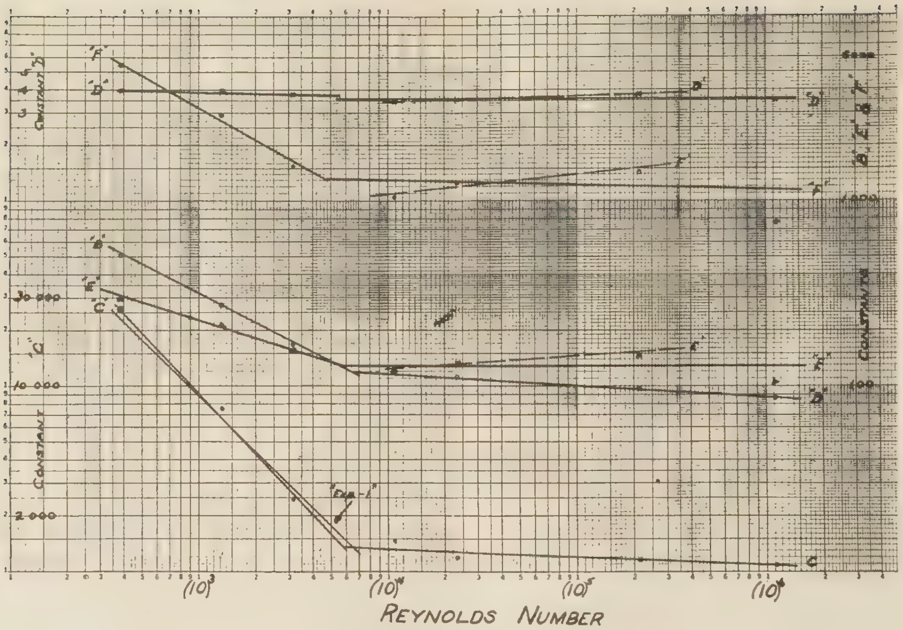

Fig. 12 Specific Functions of R for Double-Suction Volute Pump

data were rather meager, in that (D^5N^3) was a constant throughout, but did indicate the following relationship

$$Hp = 380(R^{-0.48}) \quad [39]$$

The writer had occasion (1939) to examine the performance of a centrifugal pump in terms of the Reynolds number. Some very interesting relationships were found. That Reynolds number must be considered is evident from the fact that a complete study of the forces acting must consider pressure forces, gravity forces, inertia forces, viscous forces, and elastic forces. The consideration of all of these forces leads to the conclusion that the head produced by a pump is a function of Reynolds number, Froude number, and an operating characteristic.

The form of this function can be ascertained by dimensional analysis. Neglecting gravity forces (cavitation) and elastic forces, the function can be put in the following form by straightforward reasoning

$$\frac{gH}{N^2d^2} = [\pi^2 A - D] - [B - E]\left(\frac{Q}{d^3N}\right) - [C + F]\left(\frac{Q}{d^3N}\right)^2 \quad [40]$$

where d is the impeller diameter, constant A is a function of pump size, and constants B to F (inclusive) are specific functions of the Reynolds number, all other terms being in the author's terminology. It will be noted that this is a dimensionless equation in terms of R and an operating characteristic. An application of the foregoing equation to a specific double-suction volute pump leads to the determination of the specific functions of R, which are illustrated in Fig. 12 of this discussion.

Curves D', E', and F' (Fig. 12) should be disregarded in so far as this discussion is concerned. The figure shows that a definite discontinuity exists in all curves at the same point, which can be taken as the critical $R(R_c)$ below which laminar or viscous flow predominates. Note that the friction losses (constant C) are a function of the reciprocal of R in the viscous region (Hagen-Poiseuille Law) and indicative of a rough equivalent circular pipe in the turbulent region. Note also that shock losses (constants D, E, and F) are virtually independent of R in the turbulent region, as might be anticipated from the fact that these losses are akin to a form resistance dependent upon pump geometry.

Since Fig. 12 expresses losses in dimensionless terms, it can be used as a basis for the prediction of the performance of similar pumps under any conditions of head, discharge, size, or fluid pumped, with results accurate to the extent that the Froude number (cavitation) can be neglected. Points of maximum efficiency can also be predicted with reasonable accuracy from these curves, as well as can the complete curves for flow efficiency (i.e., hydraulic efficiency), shock loss, and friction loss for any operating condition or pump size desired.

R. G. Folsom.[10] Problems dealing with the relative magnitudes of the small losses taking place in centrifugal pumps are difficult of analysis, and it is encouraging to find that the author has attempted to develop logical methods of approach.

Leakage Loss. It should be noted that the clearance effects e_v = volumetric efficiency and e_m = mechanical efficiency. The latter is due to energy required to rotate one wearing ring with respect to the stationary one, when the clearance space is filled with a real fluid. It would be interesting to have the author's estimate of the approximate order of magnitude of this power loss for normal commercial-pump construction.

If the basis of Equation [4] is correct, the magnitude of values of f in Fig. 5 of the paper should correspond approximately to experimental values obtained in the usual pipe-friction studies by use of the Weisbach equation. For sections other than circular, this equation is written as

$$H = f' \cdot \frac{L}{4m} \cdot \frac{V^2}{2g}$$

since $D = 4m$ for circular pipes. From the definition of the hydraulic radius (the cross-sectional area divided by the wetted perimeter), m for the space between the wearing rings is $a/2$ in-

[10] Associate Professor of Mechanical Engineering, University of California, Berkeley, Calif. Mem. A.S.M.E.

stead of $a/4$ as used by the author. Using the correct value of the hydraulic radius and the foregoing definition of f', the author's value of f should be multiplied by 8 to have the magnitude for direct comparison to the normal pipe-friction coefficients. The multiplication to obtain the dimensionless Reynolds number then becomes about 1600.

Changing the magnitude of the co-ordinates of Fig. 5, in order to compare the experimental values of f with the usual pipe-friction values, it is found that the author's values are in the order of twice the normal pipe experimental coefficients. Since the actual length of fluid travel through the clearance space is greater than the length L, due to the average peripheral-velocity component caused by the rotating ring, the magnitude of the values in Fig. 5 should be greater than the corresponding smooth-pipe coefficients. In order to have a satisfactory correlation of coefficients on the basis of Reynolds number, some consideration of the influence of the rotating wearing ring must come into the basic equation (Equation [4]). Without this added correction, the general applicability of Fig. 5 is not much more than that of the coefficients defined in Equation [2].

Disk Friction. It is possible to obtain directly an approximate relationship between the magnitude of the disk-friction power loss and the specific speed, through consideration of the efficiencies expressed in Fig. 1 and Equation [28] for the disk-friction loss.

It will be assumed (following the author's procedure) that the equation for disk-friction loss applies throughout the specific-speed range being considered. This equation is the author's Equation [28]

$$hp_f = 0.38\, n^3D^5 \times 10^{-9}$$

Taking the ratio of disk-friction power loss to the water horsepower

$$\delta = \frac{0.38\, n^3D^5 \times 10^{-9}}{\dfrac{QwH}{550 \times 448.8}}$$

From the definition of specific speed

$$QH = \left(\frac{n_s}{n}\right)^2 \cdot H^{5/2}$$

Then

$$\delta = \frac{1503 \times 10^{-9}\, (nD)^5}{n_s^{\,2}\, H^{5/2}}$$

Using the design factor

$$\varphi = \frac{u_2}{\sqrt{2gH}} \cong 1$$

and the definition

$$u_2 = \frac{\pi D_n}{60}$$

Then

$$\delta = \frac{12.8 \times 10^4}{n_s^{\,2}}$$

The disk friction may be expressed as a percentage of the pump brake-horsepower through

$$\delta' = 100\, \delta e = \frac{12.8 \times 10^6}{n_s^{\,2}} \cdot e$$

From this equation with values of e taken from Fig. 1, it is possible to obtain a disk-friction loss versus specific-speed curve similar to the one presented in Fig. 7 of the paper.

I. J. Karassik.[11] The nomenclature used in the discussion which follows differs slightly from the author's, because it was desired to conform as nearly as possible to the recently published standards for hydraulics and hydraulic machinery.[12] In order to avoid any misunderstanding, the variations are as follows:

η = gross pump efficiency
η_h = hydraulic efficiency
η_v = volumetric efficiency
η_m = mechanical efficiency
P_b = brake horsepower
P_d = disk horsepower
P_m = mechanical horsepower

The gross efficiency of a centrifugal pump is expressed by the author as the product of three efficiencies in the equation.

$$\eta = \eta_m \times \eta_h \times \eta_v \qquad [41]$$

This expression, rarely used in the past, is of great interest because it presents a simpler method of evaluating the component parts of the pump efficiency than the more complicated relation used by Pfleiderer.

However, it is obvious from this equation that disk-friction losses and the mechanical losses proper, that is, stuffing-box and bearing losses, are combined into a single term. The mechanical efficiency based on this conception is then defined as

$$\eta_m = \frac{P_b - (P_d + P_m)}{P_b} \qquad [42]$$

When dealing with water, this procedure is more erroneous than it is dangerous. If, however, the effect of viscosity is to be taken into consideration, as it needs to be when a centrifugal pump is handling viscous fluids, the fallacy of multiplying mechanical losses by a viscosity correction factor is so obvious that no further reason need be given for discarding this practice. Instead, it is suggested that the mechanical efficiency of a centrifugal pump be henceforth defined as

$$\eta_m = \frac{P_b - P_m}{P_b} \qquad [43]$$

while a new conception, the "disk-friction efficiency," is introduced and defined as

$$\eta_d = \frac{P_b - P_d}{P_b} \qquad [44]$$

To be absolutely exact, it would have been necessary to define it as

$$\eta_d = \frac{(P_b - P_m) - P_d}{(P_b - P_m)} \qquad [45]$$

However, such a definition would lead to unnecessary complications without adding appreciably to the accuracy of the computations. It must be considered that the average mechanical losses in a centrifugal pump do not exceed 2 per cent of the brake horsepower and that the error introduced by substituting Equation [44] of this discussion for Equation [45] is of a minor nature. This error can be easily approximated. If it is assumed that in a given pump the following relations prevail

$$P_b = 100 \qquad P_d = 10 \qquad P_m = 2$$

[11] Application Engineer, Worthington Pump & Machinery Corporation, Harrison, N. J. Mem. A.S.M.E.

[12] "American Standard Letter Symbols for Hydraulics," American Standards Association, New York, N. Y., 1942.

the disk-friction efficiency as determined by Equation [45] is

$$\eta_d = \frac{(100 - 2) - 10}{100 - 2} = 0.898$$

while on the basis of Equation [44] it becomes

$$\eta_d = \frac{100 - 10}{100} = 0.90$$

and the error is of the order of 0.2 per cent. Since the evaluation of the component parts of the pump gross efficiency cannot hope to be accurate within this order of magnitude, it is entirely proper to deviate from the exact to the practical. It is hoped, therefore, that the practice of separating mechanical losses proper from disk-horsepower losses will become generally accepted.

It is unfortunate that hydraulic engineers have not followed more widely the lead of their aeronautical colleagues in the habit of dealing with consistent units. The use of mongrel units reaches a high degree of inconsistency in the definition of the Reynolds number of a centrifugal pump in terms of gallons per minute, inches, and centistokes. The use of gallons per minute instead of cubic feet per second, and inches instead of feet, is unfortunate enough without the substitution of centistokes borrowed from the c.g.s. system for the perfectly natural feet square per second English unit for kinematic viscosity. The danger of using mixed units arises from the fact that no guarantee exists which will insure that all hydraulic engineers will use the same mixture, with the consequent result that comparisons of recorded test data become unnecessarily difficult and unreliable. On the other hand, the use of consistent units renders the Reynolds number truly dimensionless, and test data can be compared easily even though one engineer has been using the c.g.s. system and another the English-units system.

It is interesting to note that the author makes reference to a conversion factor which can be used to express the Reynolds number in fundamental (dimensionless) units. Hydraulic engineers should, however, go further than this and avoid the use of inconsistent units altogether.

A relation is developed by the author for the ratio between the disk horsepower of pumps of the same output but of different specific speeds and is given as

$$\frac{P_{d1}}{P_{d2}} = \frac{n_{s2}^2 \cdot \phi_1^5}{n_{s1}^2 \cdot \phi_2^5} \qquad [46]$$

It is also stated that if $\phi_1 = \phi_2$, the disk-friction horsepower will vary inversely as the square of the specific speed. The assumption that ϕ will remain constant and independent of the specific speed is erroneous and the gravity of the error is magnified in this particular case since the ratio of the ϕ factors appears in the fifth power. Having plotted hydraulic, gross, and manometric efficiencies along with numerous other pump variables against specific speeds, the author could have gone a step further and included a curve of ϕ versus n_s for average designs. Such a curve would have shown that the relation between these two variables can be expressed in the form

$$\phi = a \cdot n_s^b \qquad [47]$$

where a is a constant, depending upon whether the value of n_s is based on gallons per minute or cubic feet per second, while the exponent b is approximately 0.114.

Replacing for ϕ in the author's Equation [26], we get

$$\frac{P_{d1}}{P_{d2}} = \left[\frac{n_{s2}}{n_{s1}}\right]^{1.43} \qquad [48]$$

a relation possibly more difficult to memorize but, at least, more accurate than Equation [27] of the paper.

Although disk-friction formulas are very frequently cited by pump engineers, it is to be suspected that there is more divergence between the various accepted formulas than there is reason to be. This divergence is of the order of 25 to 30 per cent between the lowest and highest formulas in use, and it is hoped that a testing technique may soon be developed which will settle the matter once and for all, a reliable and accurate relation being developed which would be acceptable to all pump designers.

In connection with these disk-horsepower formulas, it may be advisable to mention that they are generally developed for water as the fluid handled by the pump, and therefore hide the specific weight in the constant itself. It is safer to make the formula include a specific-gravity multiplier, lest the user forget to do so when a fluid other than water comes into consideration.

Elsewhere, it has been mentioned that Fig. 4 of the paper indicated that pumps of lower specific speed have a brake-horsepower curve which falls off faster toward shutoff, and therefore this type of pump was more suitable for boiler feed service since advantage may be derived at light loads from a faster decrease in horsepower consumption in an 1800-rpm pump when compared to a 3600-rpm pump. Such a conclusion neglects taking a few important facts into consideration: To begin with, it is well known that a pump designer has a certain leeway in his selection of impeller angles and fluid waterways so that the shape of the horsepower curve may be varied somewhat within limits. In addition, whatever advantage may have been obtained by the use of the lower specific speeds is more than used up because of the higher disk-friction horsepower, which varies as the fifth power of the impeller diameter, and only the third power of the speed. These facts are so well known that, at the present time, countless boiler feed pumps, operating at speeds in excess of 5000 rpm, are in use aboard ship.

R. T. Knapp.[13] Dr. Stepanoff is to be complimented upon this paper as it is a distinct contribution to the literature on centrifugal-pump design. The author's analysis of the various factors affecting pump performance, and the discussion of the effect of changing specific speed upon these individual factors, are items of particular value in this paper. It must be emphasized, however, that the trends the author points out are just trends and that there may be other design factors which are of greater magnitude than the change of specific speed. For example, in Figs. 2 and 3 the author shows typical *Q-H* and efficiency curves for different specific speeds. These are reproduced in Fig. 13 of this discussion. Superimposed upon them will be seen the head-capacity and efficiency curves for a series of pumps tested in the laboratory of the California Institute of Technology. It will be observed that these latter curves do not always fall in their proper places in the series presented by the author; and, moreover, that the variation in the steepness of the head-capacity curves is a great deal wider than would be expected on the basis of the author's series. Similar discrepancies will be noted in the efficiency curves.

The writer feels that the general value of some of the equations in the paper has been lessened by the use of a nonconsistent series of dimensions or by nonstandard definitions of well-accepted terms. For example, Equation [3], which is called an equation of leakage loss, is solved instead for the square of the leakage loss. In it the leakage is measured in gallons per minute, the head in feet, the diameter in inches, and the clearance in thousandths of an inch. Term f is defined as the coefficient of friction, but an examination of Equation [4] shows that it is

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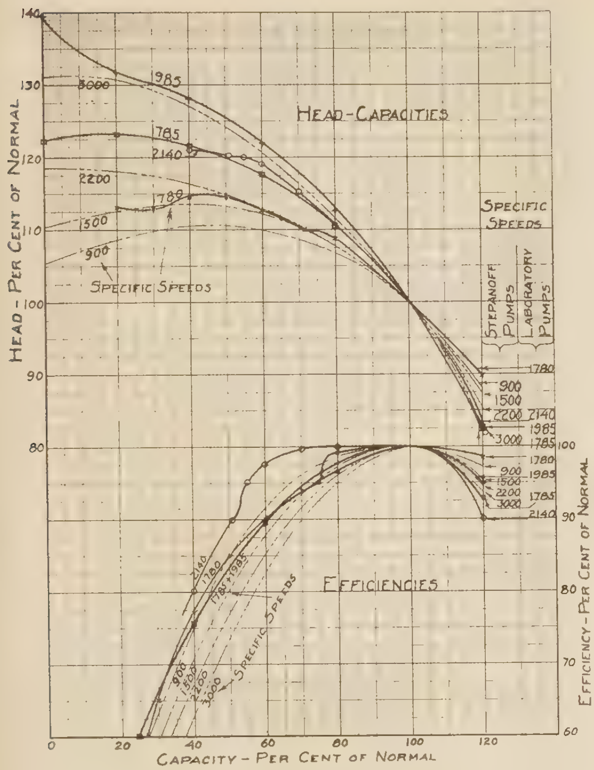

FIG. 13 *Q-H* AND EFFICIENCY CURVES WITH SUPERIMPOSED HEAD-CAPACITY CURVES FOR DIFFERENT SPECIFIC SPEEDS

actually the drag coefficient C_d, which is numerically equal to one fourth of the coefficient of friction, as normally defined in the usual pipe-friction formula. Again, in Equation [6], Reynolds number is defined as the flow in gallons per minute divided by the diameter in inches. Unfortunately, this is not Reynolds number, but only something that is roughly proportional to it for cold water. Reynolds number is properly defined in Equation [7]. However, here the author has made another error by defining the velocity v as the capacity in gallons per minute divided by the area in square inches. Furthermore, Equation [6] has no physical significance in respect to the leakage loss, because the diameter D has nothing to do with the head loss through the clearance space. This loss depends upon a and L. The only possible connection with D would be through the unjustified assumption that a varies linearly with D.

The writer fails to understand the argument of the author in the paragraph on leakage loss versus specific speed, in which the statement is made that the leakage loss is greater for smaller pumps as the clearances cannot be reduced below a certain minimum; also because the wearing rings of larger pumps are wider and the coefficient of discharge C is smaller. The coefficient C is basically a function of the ratio of the wearing-ring width to the clearance. In most cases, the wearing rings are proportionately wider in smaller pumps than in larger ones. Thus, it seems that only the first half of the author's statement is correct.

The writer always has a vague feeling of uneasiness when he observes a normal disk-friction equation being applied to the runner of a hydraulic machine. Practically all experimental work that has been done on the subject merely used disks revolving in cases. Such experiments failed markedly to simulate actual pump conditions. It is possible, however, that these equations give a fair estimate of the power transmitted to the fluid in the clearance spaces from the impeller disk. However, there is no adequate justification for the further assumption that this power is all lost. The writer has previously pointed out,[14] as quite possible, the fact that a substantial but unknown fraction of this power may be recovered in the volute. This fraction may also vary with specific speed. Such a power regain may explain a seeming paradox which exists in present pump design.

As the author indicates, test data by Gibson, Ryan, and Leconte show that disk friction increases as the clearance between the disk and the stationary wall is increased. However, it has been found in general American practice that higher over-all efficiencies are obtained with pumps having large clearances between the disk and the case than with pumps having similar design but smaller clearances. The tacit assumption has been that in some miraculous way the larger clearances permitted better flow in the volute, which resulted in increased efficiency of energy transformation in the fluid discharged from the impeller. The increase in efficiency from this source has been assumed to more than balance the extra loss from the increased disk friction. However, it is extremely difficult to see physically why the flow conditions in the volute should not become worse rather than better as the clearance increases. May it not, therefore, be equally probable that the true explanation of the increased efficiency is the exact reverse of this, i.e., that the increased clearance made it possible to deliver to the volute part of the energy that was transmitted to the fluid through disk friction, and the increased efficiency due to this added energy may more than counterbalance the poor flow conditions in the volute introduced by the added clearance.

In discussing the factors which contribute "to the higher efficiency of vertical pumps at high specific speeds," the author states that "such pumps have no disk friction as they are generally built with open impellers." It is believed that this is a mistaken idea. The concept of disk friction involves the slippage between a rotating body of fluid and a solid surface. In a pump having a shrouded impeller, such loss can occur (*a*) between the fluid in the clearance space and the impeller disk, and (*b*) between this same fluid and the case. The fluid is assumed to rotate at some speed between that of the case, i.e., zero, and the impeller. In the open impeller with no shrouds, the main slippage is between the rotating fluid and the case, but now the fluid velocity is compelled by the open vanes to be nearly that of the impeller so that the rubbing velocity between the fluid and the case is higher than it is with the shrouded impeller. As the author points out, the loss is generally assumed to vary as the cube of the slip speed. Thus, this doubling of the actual rate of slip indicates that the energy lost in friction on the stationary surface in the open-impeller pump may be more than the combined disk friction in the comparable closed-impeller unit.

The author, in his discussion of manometric efficiency versus actual hydraulic efficiency, states that one way to obtain the "actual" theoretical head is to calculate the hydraulic losses and add them to the pump total head. In discussing the application of this technique, he states that the determination of the constants in the expressions for the hydraulic losses present "insurmountable difficulties," and in the following sentence states, "there is no way to separate the several items constituting hydraulic losses such as friction, shock at entrance, and discharge at the impeller and diffusion loss in the impeller and casing." The writer is forced to disagree with these statements on the basis that trial attempts to determine experimentally these different sources of loss have met with very fair success in the hydraulic machinery laboratory of the California Institute of Tech-

[14] "Experimental Determinations of the Flow Characteristics in the Volutes of Centrifugal Pumps," by R. C. Binder and R. T. Knapp, Trans. A.S.M.E., vol. 58, 1936 pp. 649–661.

nology. Furthermore, it is the writer's conviction that a large fraction of significant progress in centrifugal-pump and turbine design in the future is going to come as the result of carefully controlled experimental laboratory investigations, designed specifically for the purpose of ascertaining what actually happens in an operating hydraulic machine, not only at the "design point," but throughout the entire possible range of operation.

C. R. Mockridge.[15] An investigation of stuffing-box friction losses was under way at the Harrison plant of the Worthington Pump and Machinery Corporation before the beginning of the present war. Its completion was of necessity postponed until normal business conditions return. The results so far have shown that the loss is relatively small in large pumps. In small pumps its proportion is liable to be higher, necessitating greater care in the selection of the packing and sleeve materials, and in the adjustment of the gland.

The frictional horsepower absorbed in stuffing boxes in cold-water service depends upon the following factors:

1 Kind of packing
2 Finish, concentricity, and material of the shaft sleeve
3 Length of the stuffing box
4 Diameter of the shaft sleeve
5 Revolutions per minute of the shaft sleeve
6 Fluid pressure sealed against
7 Leakage from the stuffing box.

The apparatus, used in the tests which were made, consisted of a freely supported housing or body containing two packed stuffing boxes under an adjustable water pressure, and an independently supported and driven shaft. The frictional resistance of the packing tended to make the housing rotate, and the torque necessary to hold it stationary, in conjunction with the speed, measured the friction. Provision was also made to collect the leakage and measure the degradation of pressure through the packing.

[15] Centrifugal Pump Engineer, Worthington Pump & Machinery Corporation, Harrison, N. J. Mem. A.S.M.E.

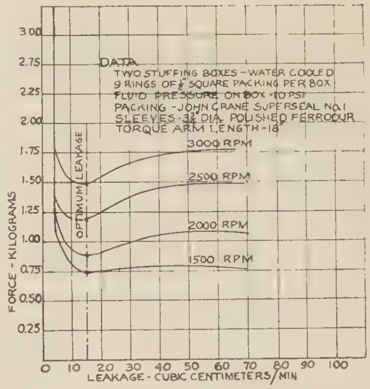

Fig. 14 Variation of Packing Friction Force With Leakage

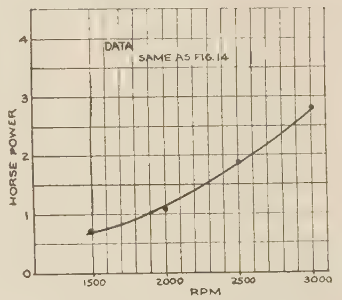

Fig. 15 Variation of Frictional Loss With Speed at Optimum Gland Leakage

Fig. 14 of this discussion shows a set of curves obtained from one of the test runs made. It should be noted that the frictional torque is very high with a tight gland and small leakage, but that

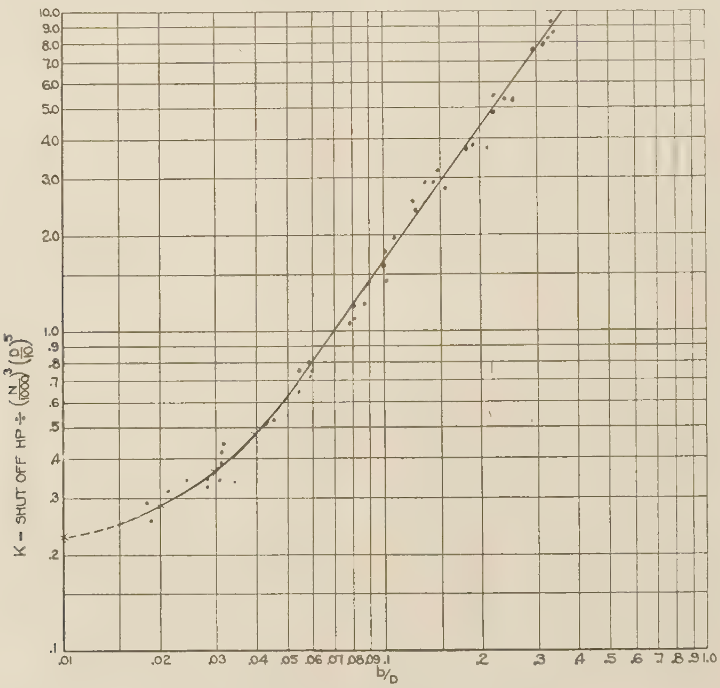

Fig. 16 Variation of Factor K With b/d

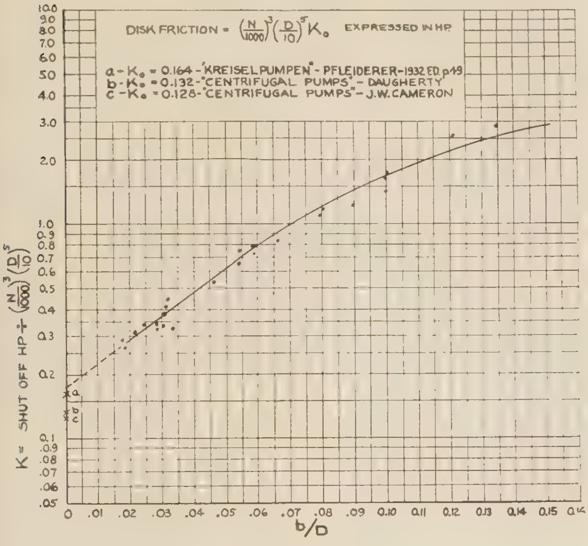

Fig. 17 Variation of Factor K With b/d Showing Extrapolation to $b/d = 0$ at Which Point $K = K_0$

it decreases rapidly as the gland is loosened and the leakage increased. The torque increases to some extent when the packing pulls away from the bottom of the box on further loosening of the gland, but this effect is not as marked with higher fluid pressures.

Fig. 15, which is derived from Fig. 14, shows the frictional horsepower plotted against speed for the optimum leakage. The condition of optimum leakage normally exists in service when the gland is initially drawn up tight, then loosened and drawn up finger-tight, this being followed by a run-in period.

In normal service with double-suction pumps, the fluid pressure against the packing is generally less than atmospheric pressure. In order to prevent air being drawn into the pump on suction lift, water under pressure is introduced into sealing rings located in the packing space. This sealing pressure breaks down to the suction pressure at the bottom of the box, and to atmospheric pressure at the gland. As most of the friction is developed near the final breakdown points, and two breakdown points exist in each box, the actual friction with water seal will be approximately twice the amount shown in Fig. 15.

The discussion of circulation loss in this paper is of considerable interest to the pump designer. If the circulation loss at the 20 per cent capacity point is calculated, as outlined in this paper, it will be seen that its percentage increases with n_s. The writer recently plotted a curve from more than fifty tests on Worthington double- and single-suction volute pumps, where the fundamental proportions of the impellers were not unduly distorted. This curve, given in Fig. 16 of the present discussion, shows the effect of circulation loss on the shutoff horsepower and is a plot of b/D against K, where

$$K = \frac{\text{Shutoff horsepower}}{\left(\frac{n}{1000}\right)^3\left(\frac{D}{10}\right)^5}$$

b = impeller water way width, in.
D = impeller diameter, in.
n = speed, rpm

An average of the points plotted could be represented by a smooth curve for double-suction and single-suction impellers up to $b/D = 0.38$.

Fig. 17 shows some of the points calculated in the lower range of the curve using a logarithmic scale for K only. Leakage and stuffing-box losses were neglected in the computation since the original purpose of the curve was to provide a means of estimating the shutoff brake horsepower on an impeller before test. It undoubtedly would have been better to include these losses, although their values could only be roughly estimated. It is also possible that the scatter of points could be reduced by taking into account the relation of casing-throat width to impeller width, and by some classification of impeller design.

The curve, in Fig. 17 of this discussion, has been extrapolated to show the limiting value of K as b/D approaches zero, since at this point, brake horsepower at shutoff = disk friction (neglecting leakage and stuffiing-box losses). The K value at $b/D = 0$ is called K_0, and the values of K_0 used by several recognized authorities in computing disk friction have been indicated on the curve. The writer believes that a refinement of this method of determining the disk-friction constant K_0 is more correct for centrifugal pumps than one which is based on experiments with flat disks.

Term K was first plotted against n_s but since there was considerable scatter, the ratio b/D was tried and found to be better. The ratio b/D may be related to n_s in the following manner

$$\phi = \frac{u_2}{\sqrt{2gH}} = \frac{D \times n}{1840\sqrt{H}}$$

where u_2 = peripheral speed, fps

$$K_{vr} = \frac{V_r}{\sqrt{2gH}} = \frac{Q(gpm)}{\sqrt{H} \times 70 \times bD}$$

allowing for area occupied by vane tips; where V_r = radial velocity at impeller exit.

If the values of n and Q are substituted in

$$n_s = \frac{n\sqrt{Q}}{H^{3/4}}$$

the following relationship is obtained:

$$n_s = 15{,}400\left(\frac{b}{D}\right)^{1/2}\phi\sqrt{K_{vr}}$$

in normal design ϕ varies from 0.90 to 1.4 and K_{vr} varies from 0.1 to 0.2, hence $\phi\sqrt{K_{vr}}$ varies from 0.28 to 0.63. It is possible to draw lines of constant $\phi\sqrt{K_{vr}}$ and an n_s scale in Fig. 16 of this discussion, thus making it possible to locate the n_s values for a given ratio of width to diameter when ϕ and K_{vr} are known.[16]

The following typographical errors were found in the text of the paper:

1 In Equation [14] "=const" should be eliminated.

2 Equation [29] should read $e_m = 1 - \frac{\text{mechanical loss}}{\text{brake horsepower}}$

The statement below Equation [19] of the paper, that K is a numerical constant, depending upon the units used is not correct, since K is also an empirical factor. In Equation [28], it would have been desirable to state that D is measured in feet. Also the term "radial clearance," in the writer's opinion, would be better than "diametral clearance," since the value used is one half of the clearance determined by the use of inside and outside micrometers.

[16] These lines have been omitted as they are not considered essential for the present discussion.

ARVID PETERSON.[17] While this paper does not disclose anything that is not known to engineers in the centrifugal-pump industry, it should, however, be of considerable interest to users of centrifugal pumps.

Curves showing pump efficiencies, as function of specific speed, similar to the one in Fig. 1 of the paper, have been published before, such as those by R. L. Daugherty[18] and by R. G. Folsom.[19]

The method described for calculating the hydraulic efficiency, i.e., from the gross efficiency, as determined by test and elimination of losses which can be determined with a fair degree of accuracy, such as leakage, disk, stuffing-box and bearing friction, is well known and has been used for many years by pump designers.

The curves, shown in Figs. 2, 3, and 4, of the paper, are very interesting. The writer agrees that only slight variation from average curves for a given specific speed is possible without appreciable sacrifice in efficiency.

It is interesting to note that the power required by the pump at partial capacities and constant speed is less for a lower-specific-speed pump than for one of higher specific speed. This is important when it comes to selection of pumps which must operate a great deal of the time at reduced capacities.

These curves should, therefore, be of great interest and help to users of centrifugal pumps. For instance, a high-specific-speed pump, such as a propeller pump, operating at one-half capacity requires about 75 per cent more power than it does at full capacity, while a lower-specific-speed pump, such as a mixed-flow pump, requires only about 20 per cent more power at one-half capacity than it does at full capacity. In the case of a straight centrifugal pump, the power consumption is lower at one-half capacity than it is at rated capacity.

In the case of boiler-feed and other high-pressure pumps, it will often be of advantage for a customer to buy an 1800-rpm pump instead of one operating at 3600 rpm because of the lower power consumption at reduced capacities.

One of the curves, in Fig. 7 of the paper, indicates that, for a definite specific speed, the leakage loss is a fixed percentage of the water horsepower. While this may be true theoretically and for strictly similar pumps, it does not apply in practice.

If we should consider two pumps both designed for 1000 gpm, one designed for 165-ft head at 1750 rpm, and the other one for 3 times as high head, or 495 ft at 4000 rpm, the specific speed of both would be the same, or 1200. With impeller-eye diameters and ring clearances according to usual commercial practice, the leakage through the rings would be considerably greater in the 495-ft-head pump and, therefore, also the percentage leakage would be greater, the capacity of the two pumps being the same. This would also mean that the gross efficiency would be reduced. It could be stated that in the case of a constant-capacity pump, at constant specific speed, the gross efficiency is a function of total head. This statement is true not only because of change in leakage rate, but also because of the fact that increased head results in higher velocity of the liquid leaving the impeller. In commercial pumps, the efficiency conversion of the velocity energy into pressure is reduced with increased head.

While it is true and well known that the optimum gross efficiency of centrifugal pumps varies with the specific speed, it is equally true that, at definite specific speeds, the gross efficiency varies with the total head, a fact which is not clearly brought out in the paper. In other words, leakage and hydraulic losses are not necessarily a function of specific speed.

[17] Chief Engineer, Centrifugal Pump and Compressor Department, De Laval Steam Turbine Company, Trenton, N. J. Mem. A.S.M.E.

[18] "Hydraulics," by R. L. Daugherty, McGraw-Hill Book Company, Inc., New York, N. Y., fourth edition, 1937.

[19] "Some Performance Characteristics of Deep-Well Turbine Pumps," by R. G. Folsom, Trans. A.S.M.E., vol. 63, 1941, pp. 245–250.

In view of the fact that impeller diameters usually, and as is also the case in this paper, are given in inches, it should be stated that in Equation [28], for disk friction, the impeller diameter must be taken in feet, or still better, the constants in the formula should be changed to correspond to impeller diameter in inches.

A. F. SHERZER.[20] Fig. 1 of the paper would be much more useful were it based on pumps of smaller than 5000 gpm capacity. Probably less than 1 per cent of all centrifugal pumps sold are in that range, that is, above 5000 gpm.

Expressing the efficiency of centrifugal pumps as a function of specific speed is not entirely logical and leads to some misunderstanding. Pumps can have the same specific speed and differ widely in efficiency. For example, a well-known pump builder lists a pump for 1300 gpm at a head of 100 ft and 1750 rpm with an efficiency of 76 per cent. He also lists a pump for 3700 gpm at a head of 200 ft and 1750 rpm with an efficiency of 84 per cent. Both pumps have a specific speed of 2000.

The author's curve, Fig. 1, shows a gross efficiency of 90 per cent, which is surely a maximum value and rather contradicts the last sentence on the first page of the paper.

In Fig. 1, the gross efficiencies are exceptionally high, but are no doubt a matter of record. Even so, they would be misleading to anyone not experienced in pump design unless some explanation were made. Specific speeds as high as 6000, even with double-suction pumps, are not entirely centrifugal pumps to the same extent that pumps of specific speed 2500 would be. This might cause some misunderstanding as the laws of action are not entirely common to the two. The manometric efficiencies shown might as well have been drawn in free-hand, since they are after all a matter of opinion and depend upon a number of assumptions which are not susceptible of proof and, worse yet, not equally agreeable to all experts. It would be interesting to see how the author would calculate the manometric efficiency for a pump of specific speed equal to 6000. Giving calculations in detail would bring out the rather uncertain nature of this estimation.

The hydraulic efficiency given is also somewhat open to question. To be exact, for a specific speed equal to 6000 in Fig. 1, there appears to be about 2.5 per cent difference between the gross and hydraulic efficiencies. That would require volumetric and mechanical efficiencies of about 99 per cent each, which surely are not to be expected in average practice.

With reference to Equation [1], it is no doubt possible to express any efficiency as a product of three other efficiencies, or any number in fact, if they are carefully assumed to agree with the facts. The writer feels that the author has made too hard work of the matter of pump efficiency. The efficiency of a pump is the output divided by the input, which is equal to the output divided by the output plus losses. These values are usually expressed in terms of horsepower. Now, in Equation [1], the author states that efficiency = $e_m \times e_h \times e_v$. The true efficiency is a matter of record, and any values of $e_m \times e_h \times e_v$ whose product gives the true efficiency could be used. That would not necessarily prove their correctness. The two items e_m and e_h appear to be expressed in terms of loss of power, while e_v is clearly a loss of capacity. It might by coincidence be true in some cases that the loss of capacity in per cent was the same as the loss of power due to leakage also in per cent. Still there is no logical relation between them. Due to the fact that they are both small, not much harm is done any way you look at it, but it would seem better if e_v were expressed in the same units as e_m and e_h.

The author states: "Since the theoretical head-capacity curve is a straight line, irrespective of the specific speed, the variation

[20] Professor of Mechanical Engineering, University of Michigan Ann Arbor, Mich.

in the form of the head-capacity, efficiency, and brake-horsepower curves with specific speed is caused by the losses." The writer does not agree at all with either statement, and the author seems to have some doubts himself in view of his later comments. The shapes of the various curves mentioned are due to features of design, such as angles, areas, velocities, etc. and are not necessarily due to faulty design or losses of power. A steep characteristic is not due to losses of energy. If it were, there would be a wider difference in efficiency between flat and steep characteristics. As a matter of fact, we can design satisfactory pumps with either flat or steep characteristics. There may be indirectly a small advantage in favor of the flat characteristic, as is well known.

The author also attempts to prove that the power loss due to leakage is constant for pumps of equal specific speed, irrespective of pump size and speed. This is of course ridiculous and entirely contrary to fact. He probably means that the power loss is to be expressed as a percentage of total power or something like that. Even then, the statement is open to question in view of the assumptions he has made; in particular, the statement between Equations [10] and [11]: "If similarity of the two pumps is extended to the wearing-ring diameters and clearances, then ......" Of course the similarity does not extend to the wearing-ring diameters and clearances in pumps found in practice, and variation can be large. A 6-in. pump would not ordinarily have twice the clearance of the 3-in. pump, etc.

In Fig. 9, it is shown that, at a specific speed of 2000, the maximum efficiency of a single-suction pump is 80 per cent, while the efficiency of a double-suction pump is 90 per cent; the latter also being single stage. This surely does not represent general pump practice. Also single-suction single-stage pumps of, say, specific speeds of 10,000 and above are really not centrifugal pumps at all. Hence it is misleading to compare them. Their basic laws of operation are different. In pumps with a specific speed of 2000, centrifugal force explains to a great extent the head developed, while in pumps with a specific speed of 18,000, centrifugal force has almost nothing to do with it.

The author very properly calls attention to the absurd value of the so-called manometric efficiency, or theoretical hydraulic efficiency, when computed from Equation [30]. From there on to the end of the paper, it seems clear that the attempts made to bring theory and practice into agreement are only guesses and not very shrewd ones at that. For example, might there not be something wrong with a theory which calls for a head of approximately U_2^2/g at small rates of flow, where actually only about $U_2^2/2g$ is realized? The writer tried to explain this in some detail in a paper[21] some years ago.

The paper is weakened by the statement just before the conclusion: "From the foregoing outline, it is seen that a study of losses for the entire head-capacity range of pumps of different specific speeds is really a difficult task, considering variation of head, capacity, efficiency, and brake-horsepower curve form with specific speed, illustrated in Figs. 2, 3, and 4." This statement is absolutely correct, and if put at the beginning might have changed the whole tone of the paper. The facts are simple, easily understood, and are about as follows: The difference between the water horsepower and the brake horsepower in a centrifugal pump constitutes the losses. In a single pump at constant revolution the curves are about as shown in Fig. 18 of this discussion. We now have all the losses in one curve but by careful work in the laboratory they can in part be separated and evaluated. When this has been carefully done the existence of another loss is revealed.

[21] "A New Theory for the Centrifugal Pump," by A. F. Sherzer, Trans. A.S.C.E., vol. 93, 1929, pp. 1–29.

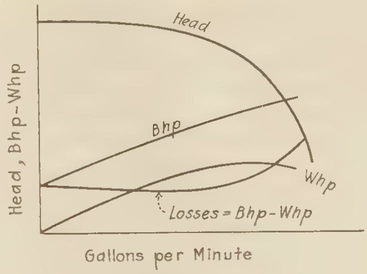

Fig. 18 Typical Curves Showing Losses in Centrifugal Pump

The power required to drive a pump at shutoff is composed of the following items:

1 Bearing friction
2 Stuffing-box friction
3 Disk friction
4 Pumping internal leakage
5 A loss which for want of a better name, the writer calls a "rotation loss."

The first four losses do not vary greatly with the output of a given pump; item 5 does. This explains the decrease in the total losses as the capacity is increased.

Assume a pump rated at 1300 gpm, 100 ft head, at 1750 rpm. The curves for such a pump built by a well-known manufacturer show a gross efficiency of 77 per cent. The specific speed of such a pump is 2000. According to data presented in this paper, the losses would be about as follows:

Bearing and stuffing-box friction, about 2 per cent
Power loss to pump leakage, about 1.5 per cent
Disk friction, about 2.5 per cent
Hydraulic losses, about 4 per cent.

This makes the total losses about 10 per cent. From the author's curves, the expected pump efficiency would be about 90 per cent. The actual efficiency by test was 77 per cent. This leaves a difference of unexplained losses of 13 per cent. The unexplained losses are greater than the sum of all the other losses put together. This should show the danger of any sweeping assumptions that operating performance of centrifugal pumps may be expressed as a function of specific speed alone. The effect of capacity as well should be considered at the same time as specific speed. Efficiency expressed as a function of either, exclusively, can be misleading.

Author's Closure

It is only natural that a divergence of views should exist on a broad subject dealing only with trends in the evaluation of losses in centrifugal pumps. To detect these trends, several simplifying assumptions and generalizations had to be made, and the discussion has to be confined to one definite class of pumps in sizes giving consistent performance irrespective of make of pumps. Deviations from the general trends were expected. Several discussions overlooked this fact.

The limited space does not permit individual reply to all discussions, therefore the author will comment only upon a few points brought up in the discussion which either were barely touched upon in the paper or entirely omitted.

The author gratefully acknowledges corrections to several errors in the paper brought to his attention by discussers.

In Fig. 6(a) and (b) of the paper, the clearance a was shown erroneously as a radial clearance; while it should have been a diametrical clearance, as given in the definitions, and used in Equations [3] and [4] and in Fig. 5. This error resulted in some

confusion, as evidenced in Col. Karl Eklund's and Professor Folsom's discussions.

No attempts were made by the author to correlate the values of the friction coefficient f and Reynolds number as defined in the paper with those used for pipe flow because of a great difference in the physical aspects of the flow in two cases:

1 There is no geometrical similarity between a round circular pipe section and a small annular clearance.

2 In all cases, except those of Schnekenberg and Becker, one circular wall forming the annular clearance was revolving.

3 There are numerous factors mentioned in the paper affecting the flow which do not exist in the case of pipe flow. Any generalization of the flow conditions under such circumstances would be more misleading then instructive.

Fig. 5 of the paper is primarily intended as an aid in the calculation of leakage for centrifugal pumps, and the units adopted were those used in commercial design work. Further refinements in presentation of the test results may be justified when more and better data are available. Use of a geometrical sum of the axial and peripheral velocities for calculation of the Reynolds number, as suggested by Professor Folsom, may bring more harmony in plotting the coefficient of friction f.

Replacing the speed factor ϕ in the author's Equation [26], as suggested by Mr. Karassik in his Equation [48], is a definite improvement, for the relationship between the factor ϕ and specific speed is a definite one.

Discussing the possible advantages of 1800-rpm pumps, as compared with 3600-rpm pumps for boiler-feed service, both Mr. Peterson and Mr. Karassik had tacitly assumed that 1800-rpm pumps are of lower specific speed than 3600-rpm pumps which is not necessarily true.

The author disagrees with Professor Knapp in that any of the power lost in disk friction in a centrifugal pump can be recovered, or that the latter may be responsible for an improved pump efficiency of pumps having liberal spacing between the casing walls and the impeller. Professor Knapp's reasoning disregards the following facts:

Each particle of water within the impeller carries more energy than that possessed by any particle at or near the shroud outside the impeller. All water particles in the space between the impeller and casing came from a zone of higher kinetic and pressure energy, this space being refilled by water from the volute many times a minute (10 to 30 times with a normal amount of leakage). Any particle outside the impeller which is able to augment its kinetic energy through direct contact with the shroud has to leave the shroud to effect any exchange in momentum. In this process it is immediately absorbed by water with a very low kinetic and pressure energy. Wide space between the impeller and the casing was never sought in modern designs but resulted from the use of more curved impeller profiles and mixed-flow vanes. A liberal clearance between the volute walls proper and the impeller periphery was introduced for mechanical (multistage pump impeller spacing) and hydraulic reasons.

Hydraulically, it has been found that the combined losses in the volute due to skin friction and diffusion are at a minimum if the average volute velocity is kept about one half that of the true absolute velocity leaving the impeller. Evidently, there is a velocity gradient established in the volute with a maximum at the impeller periphery and a minimum at the volute walls. The combined loss is a minimum when the impeller discharges into a body of revolving liquid rather than against the stationary casing wall.

It is an established fact accepted by the entire industry, here and abroad, that with pumps of medium and high specific speed in design where a close running clearance can be maintained (vertical mixed flow and propeller pumps), removal of the outer shroud improves pump efficiency. These latter facts are questioned by Professor Knapp. The author is unable to follow his reasoning to justify his claim. Without going into the details of the mechanism of the skin-friction loss, it is evident that with a shrouded impeller there is a hydraulic-friction loss inside the impeller, due to a relative velocity through the impeller with respect to the outer shroud, and a disk-friction loss outside the shroud. With the open impeller, the friction against the stationary wall due to the absolute velocity in the impeller channel is substituted instead of the friction, due to a relative velocity (relative and absolute velocities are approximately equal in high-specific-speed pumps), and the disk-friction loss is absent entirely. These facts became well established years ago and have found their way into a number of textbooks written within the last 12 years.[22]

Kaplan gives a gain in efficiency for straight propeller pumps with shrouds removed of 5 to 10 points. With mixed-flow impellers, this gain is less. For mechanical reasons, shrouded impellers are occasionally used on propeller pumps but always at the expense of efficiency.

Mr. Mockridge's discussion added some valuable information on the stuffing-box-friction loss. The method of testing and presentation of results are very interesting and instructive. A study of the variation of the brake horsepower at shutoff, such as described by Mr. Mockridge, indicates that the term "circulation loss," used in this paper, is most descriptive of the nature of the loss observed at partial capacities of centrifugal pumps. Anything which reduces chances for circulation within or beyond the impeller reduces the shutoff brake horsepower. As a result, propeller pumps had been designed with a flat brake horsepower curve with remarkably little sacrifice in efficiency. The ratio of b/D, used by Mockridge, is only one of the variables affecting the shutoff brake horsepower. It is no exaggeration to state that every element of the impeller design affects to a different degree the shutoff brake horsepower.

The fact that the shutoff brake horsepower is sometimes twice the normal brake horsepower indicates the magnitude of the "circulation" loss. Up to the present time, no way has been invented to express this loss in terms of known design elements. This is only one of the obstacles in the way of separating the hydraulic losses in centrifugal pumps. The author does not believe that such a state of affairs had ever retarded the progress in the development of centrifugal pumps or ever will. The remarkable degree of perfection of centrifugal pumps attained during the last 10 years serves as a proof of designers' knowledge of the nature and laws governing various losses without knowing their individual values.

The example, quoted by Professor Scherzer, i.e., a pump of 76 per cent 1300 gpm at 100 ft head and 1750 rpm, shows that this pump is of obsolete design as such pumps are rated at least 84 per cent by several pump manufacturers.

Professor Scherzer's statement that 90 per cent pump efficiency is "surely a maximum" shows that his information on the subject is not up to date. Accounts have been published showing pump efficiencies up to 92 per cent in sizes 8 in. and over.[23]

[22] (a) "Theorie und Bau von Turbinen-schnellaufern," by Victor Kaplan and Alfred Lechner, S. 142, R. Oldenbourg, Berlin,1931.

(b) "Centrifugal Pumps, Turbines, and Propellers," by Wilhelm Spannhake, Massachusetts Institute of Technology, Cambridge, Mass., 1934, p. 205.

(c) "Die Kreisel-Pumpen," by C. Pfleiderer, Julius Springer, Berlin, 1932, p. 298.

(d) "Turbinen und Pumpen," by F. Lawaczek, Julius Springer, Berlin, 1932, p. 179.

[23] "Centrifugal Pumps for the Colorado River Aqueduct," by R. L. Daugherty, *Mechanical Engineering*, vol. 60, 1938, pp. 295–299.

Professor Scherzer will find in the author's previously published paper (2) a proof that Equation [1] of this paper is correct and expresses the relationship between the input, output, and losses as defined by him. Professor Scherzer also raised an objection to including propeller and extreme mixed-flow pumps into this study of centrifugal pumps; as according to him different laws govern operation of these three types of pumps. The author wishes to point out that the term "centrifugal" pumps adopted in English is a rather unfortunate one. Germans abandoned this term, used in early literature (Newmann), in favor of "Kreiselpumpen" which suggests that the pumping is produced by the impeller rotation without any reference to centrifugal or propeller action. To divide all pumps into several groups like centrifugal, mixed flow, and propeller, besides being arbitrary, would not serve any purpose; as hydraulically all pumps form one perfectly continuous row, with laws governing their operation just as continuous as the design elements themselves. Difference in the theoretical treatment of centrifugal and propeller pumps only shows limitations of the theory rather than the difference in laws of nature governing two extreme representative groups of one type of machine.

The author had intentionally avoided discussion of the equation for the theoretical head developed by a centrifugal pump, as this has been done repeatedly and exhaustively in the past, and further arguments would be fruitless.

In connection with the article by Professor Scherzer, referred to in his discussion, the author can state that he is in complete agreement with Professor Scherzer's opponents. To defy Euler's equation is to question the validity of the laws of conservation of energy, matter, etc. The discrepancy between the actual pump performance and the Euler's theory has been well understood. Mr. Daily's discussion of this paper deals with one phase of this subject. On the other hand to accept a formula $U^2/2g$ for the theoretical head, as suggested by Professor Scherzer, leads to hydraulic efficiencies of over 120 per cent, for with medium- and low-specific-speed pumps the actual available heads at partial capacities are equal to or in excess of $U^2/2g$. One hardly can build up a "theory" around such a formula. It may qualify for a "rule of thumb," and even as such it is not very accurate.

The author wishes to thank all the discussers for their contributions, all of which marks another step toward a better understanding of the nature and trends of various losses in centrifugal pumps. This will lead to a further advancement of the art.

New Five-Bar and Six-Bar Linkages in Three Dimensions

By MICHAEL GOLDBERG,[1] WASHINGTON, D. C.

The author outlines the historical background for the study of linkages, both in a plane and of the three-dimensional type. The latter have not received very much attention, although some investigations of special types of space linkages have been reported. The present paper is confined to a discussion of two types of five-bar linkages and four types of six-bar linkages. While it is little known by the engineering field, the Bennett linkage is a useful and economic mechanism. An explanation of this linkage, its mathematical derivation, and combinations of its elements are discussed at some length. Among the possible applications of space linkages, the most obvious is the conversion of oscillating or rotational motion in one plane to motion in another plane. The Bennett linkage is the simplest for this purpose. Other uses of various types of linkages are mentioned.

Introduction

LINKAGES in the plane have been extensively investigated by engineers and mathematicians. An excellent bibliography of papers and books containing the principal original contributions has been compiled by R. Kanayama (1).[2] One of the most celebrated plane linkages is the linkage that can draw an exact straight line, disclosed by Peaucellier (2) in 1873, and for which he was awarded the "Prix Montyou," the great mechanical prize of the Institute of France. This linkage is described in many modern textbooks on kinematics (3). An independent straight-line mechanism had already been published in 1871, by Lipkin (4), a student of the great Russian mathematician Tchebycheff. But both of these were preceded by Sarrus (5), who exhibited, before the Paris Academy of Sciences, a linkage which was described in the transactions of that society in 1853. This linkage consists of an accordionlike arrangement of hinged plates joining two other plates. If one of the latter plates is held fixed, the other plate is constrained to move parallel to it. Not merely does a single point describe a straight line, but each point in the moving plate describes a straight line. From an engineering standpoint the Sarrus mechanism is much preferable since it is three-dimensional. The plane linkages, in practical applications, require additional constraints to confine them to their planes.

Three-dimensional linkages have not received much attention, except for those that are merely combinations of plane linkages. Studies of special types of space linkages (a shorter term for three-dimensional linkages) have been made; significant results have been few (6). This paper also is confined to a special type of space linkage. The new results consist of two types of five-bar linkages and four types of six-bar linkages.

Definitions

A "kinematic link" is a rigid body which can be connected to two or more other rigid bodies by means of kinematic pairs. A "ball-and-socket joint" is a "spherical pair." A "hinged joint" is a "turning pair."

The mathematical equivalent of a spherical pair is the restriction that a point which is fixed with respect to one body coincides with a point which is fixed with respect to the paired body.

The mathematical equivalent of a "hinge" is the restriction that a point and a line through that point, that are fixed with respect to one body of the hinged pair, coincide, respectively, with a point and a line through that point, that are fixed with respect to the other body of the hinged pair.

A "simple kinematic chain" is a set of rigid bodies or links which are joined in series, any two consecutive links being either spherically paired or hinged. (Sliding joints and screw pairs are omitted from this discussion.) In a "simple closed kinematic chain," the first link and the last link of a kinematic chain are joined.

More complicated linkage mechanisms may be considered as combinations of simple chain mechanisms. For that reason the study of the simple chains is of fundamental importance.

In plane hinged linkages, the hinge axes may be conceived as lines perpendicular to the plane of the linkage. Therefore, they are parallel and they remain parallel during the motion of the linkage. In space hinged linkages, however, the hinges in a link need not be parallel. The angle between the hinge axes in a link is called the "twist" of the link. In the linkage shown in Fig. 1 (K) the twist of each link is 90 deg. In Fig. 1(A), the twists are alternately 90 deg and 30 deg. Even though twists may be right hand or left hand, it is convenient to measure all twists in the same hand. For example, a left-hand twist of 60 deg is equivalent to a right-hand twist of 120 deg. Since a twist of 180 deg is equivalent to parallelism, the twist angles need not exceed 180 deg. In this paper, the symbols used for angles of twist will always be Greek letters.

The length of a link is the shortest distance between the centers of the kinematic pairs joining it to its neighbors. If one end is spherically paired and the other hinged, the length of the link is the perpendicular distance from the center of the spherical pair to the axis of the hinge at the other end. If both ends are hinged, the length of the link is the length of the common perpendicular between the hinge axes. In each case, this shortest line will be called the length line. In this paper, Roman letters will be used to indicate the lengths of the links.

Number of Degrees of Freedom

A body in space may be located and oriented with respect to a co-ordinate reference frame by six parameters. Three parameters may be the co-ordinates of a point in the body; two more co-ordinates may be the direction cosines of a line through that point, and the sixth co-ordinate may be the orientation of the body about that line. A random assembly of n bodies then requires $6n$ parameters to locate the bodies with respect to the reference frame. But since one of the bodies may itself be the reference frame, it requires only $6(n - 1)$ parameters to describe the configuration.

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[1] Ordnance Engineer, Bureau of Ordnance, Navy Department, Washington, D. C.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Production Engineering Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

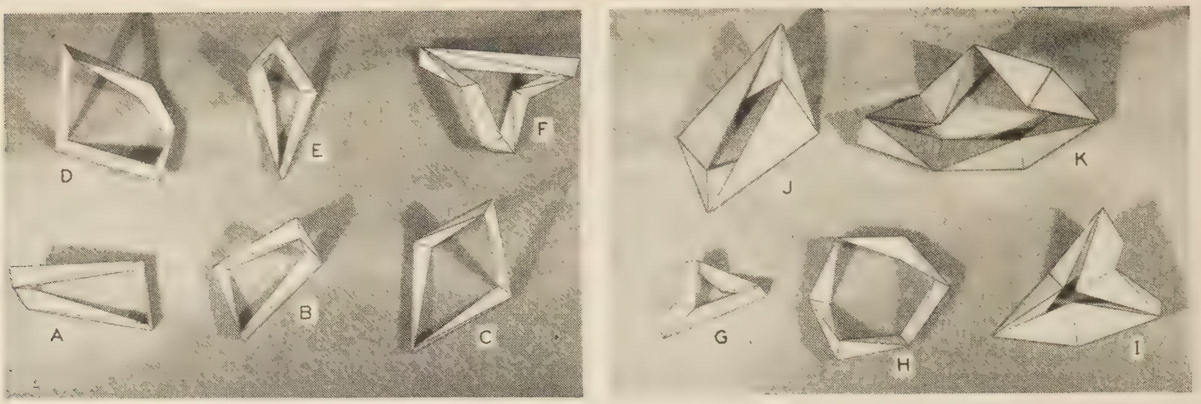

Fig. 1 Paper Models of Bennett and Other Typical Linkages
(The explanation of linkages A to K is given in Table 1.)

TABLE 1 DESCRIPTION OF LINKAGES SHOWN IN FIG. 1

	Type of linkage	Twist and length of links					
A	Bennett four-bar	90°	30°	90°	30°		
		$2a$	a	$2a$	a		
B	Bennett four-bar	90°	45°	90°	45°		
		$2a$	$\sqrt{2}a$	$2a$	$\sqrt{2}a$		
C	Bennett four-bar	60°	120°	60°	120°		
		a	a	a	a		
D	Goldberg five-bar	90°	60°	90°	30°	30°	
		$2a$	$2a$	$2a$	a	a	
E	Goldberg five-bar	75°	90°	45°	30°	90°	
		$(1+\sqrt{2})a$	$2a$	$\sqrt{2}a$	a	$2a$	
F	Goldberg six-bar	125°	90°	50°	30°	45°	90°
		$(1+\sqrt{2}+2\sin 50°)a$	$2a$	$2a \sin 50°$	a	$\sqrt{2}a$	$2a$
G	Rigid five-bar	90°	90°	90°	90°	90°	
		$4a$	$4a$	$4a$	$4a$	$\sqrt{97}a$	
H	Symmetric six-bar	90°	90°	90°	90°	90°	90°
		a	a	a	a	a	a
I	Bricard rectangular six-bar	90°	90°	90°	90°	90°	90°
	$A^2+B^2+C^2=a^2+b^2+c^2$	A	a	B	b	C	c
J	Bricard octahedral six-bar	From movable octahedron					
K	Seven-bar	90° twists, equal length links					

The constraint of a spherical pair imposes three conditions, namely, the equating of the three co-ordinates of a point in one link with the three co-ordinates of a point in the paired link. The constraint of a hinged joint imposes two more conditions, namely, the equating of two-direction cosines of the hinge axis in one link with the two-direction cosines of the hinge axis in the paired body.

Let λ be the number of free or undetermined parameters in a closed hinged kinematic chain linkage of n links. Since $6(n-1)$ parameters would describe the random configuration, and each of the n hinges fixes 5 conditions, the number of free parameters would seem to be given by the equality part of

$$\lambda \geq 6(n-1) - 5n \qquad [1]$$

or

$$\lambda \geq n - 6$$

However, the number of free parameters is not always exactly $(n-6)$. This is obviously a minimum, but there may be more free parameters because of some special conditions. One such special condition would be parallelism of the hinges which reduces a linkage to a plane linkage. For plane hinged linkages $\lambda = n - 3$.

The number of free parameters is sometimes called the number of degrees of freedom. When a linkage possesses only one degree of freedom, each point of the moving links will, in general, describe a curve if the linkage is moved while one of the links is fixed. If the linkage has two degrees of freedom, the moving points may, in general, describe surfaces. Of particular interest are those mechanisms which possess only one degree of freedom. Such linkages are useful because they can perform definite cyclic motions. Bricard (7) calls them "desmodromic" linkages from the Greek for "ribbon-pathed." American textbooks call them "constrained movable" or simply "kinematic" linkages but these terms are not sufficiently descriptive since they do not emphasize the single parameter.

Paradoxical Linkages

From the equality part of Equation [1], it is seen that a random closed hinged chain of six or fewer links in space would, in general, be a rigid structure or a locked linkage since λ would, in general, be zero or negative. Bricard undertook the study of those special cases in which the inequality of Equation [1] applied. Because of this special property, he called them "paradoxical" linkages. The following sections are devoted to a description and discussion of the known "paradoxical" hinged linkages of fewer than seven links.

The Bennett Linkage. Consider a four-bar linkage $ABCD$, shown in Fig. 2, in which the opposite links are equal in length and all the joints are ball-and-socket joints. This linkage can be made to assume the form of a skew quadrilateral in space. Let the length of one pair of opposite links be a, and the length of the other pair of opposite links be b. If the diagonal distance AC is held fixed, it is still possible to vary the other diagonal distance BD. Let the linkage $ABCD$ be so operated that the variable diagonal distances AC and BD are each maintained equal to a variable distance designated by x. Then, in the tetrahedron $ABCD$, the opposite edges are equal, the lengths being a, b, and x. The faces of the tetrahedron are congruent, each being a triangle

with sides a, b, and x. Let the area of this triangle be designated by k.

Drop the perpendicular from B upon the edge AD, whose length is a. Then the length d of this perpendicular is given by $d = 2k/a$. Designate by α the angle that this altitude line d makes with the plane ACD. The height H of the point B above the plane ACD is given by

$$H = d \sin \alpha = \frac{2k \sin \alpha}{a} \quad \text{[2]}$$

In a similar manner, let e be the length of the perpendicular from the point B to the line DC whose length is b. Then $e = 2k/b$. Designate by β the angle that this altitude line e makes with the plane ACD. Again, the height H of the point B above the plane ACD is given by

$$H = e \sin \beta = \frac{2k \sin \beta}{b} \quad \text{[3]}$$

Equating [3] and [2] gives

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} \quad \text{[4]}$$

Let us now replace the ball-and-socket joint at each vertex by a hinged joint connecting the two links. Let the hinge axis be perpendicular to the plane of the two links. The hinge axis at A is perpendicular to the plane DAB, and the hinge axis at D is perpendicular to the plane ADC. The angle between the planes DAB and ADC is equal to α. Therefore, in the link AD, the angle between the hinge axis at A and the hinge axis at D is equal to α. Similarly, in the link BC, the angle between the hinge axes is also α.

In a similar manner, the angle between the hinge axes in the link DC is shown to be equal to β. The angle between the hinge axes in link AB is also β.

Suppose, now, that the angle α is held constant. Then from Equation [4], the angle β will also be constant. This shows that it is possible to construct a movable hinged four-bar linkage in which the hinge axes are neither parallel nor concurrent. This linkage was described in 1903 by G. T. Bennett (8), a British mathematician.

In the Bennett linkage, note that the length of a link is the shortest distance between the hinge axes in that link, since it is their common perpendicular. The angles α and β are the twists of the links. The Bennett linkage may then be described as a space four-bar hinged linkage in which the opposite links have equal lengths and equal twists.

Figs. 1(A), (B), and (C) show easily constructed paper models of Bennett linkages. The links are tetrahedra which are hinged at opposite edges to the adjacent tetrahedra. The spatial character of the links and their twists is emphasized by the use of tetrahedra for the links.

Fig. 3 shows how a Bennett linkage may be used to convert rotation of a shaft S into rotation of a shaft T, when the shafts S and T are not parallel and not even in the same plane.

As a mechanism, the Bennett linkage has a fault in common with the plane four-bar linkage, namely, it has a dead center in which the connecting rod exerts no turning moment. For that reason it is not always desirable to use it for driving in continuous rotation. However, this difficulty does not arise when the Bennett linkage is used to transmit oscillating motion of a crank in one plane into oscillating motion of a crank in another plane. Note that only a connecting rod is used; no gears or universal joints are needed.

In spite of the simplicity of the Bennett mechanism, and its obvious utility and economy, it is rarely used. The explanation

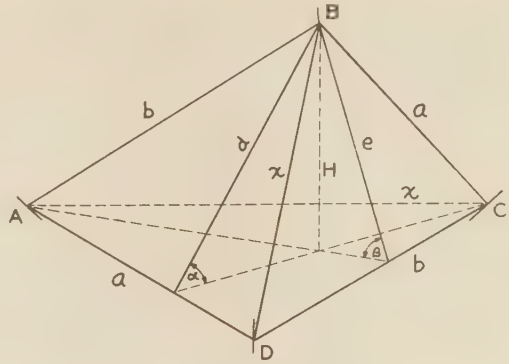

FIG. 2

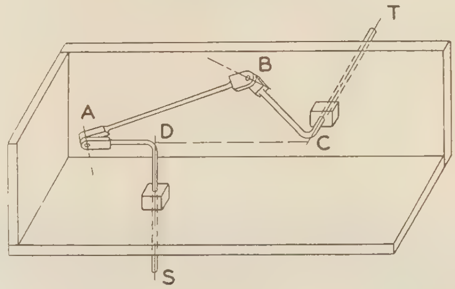

FIG. 3

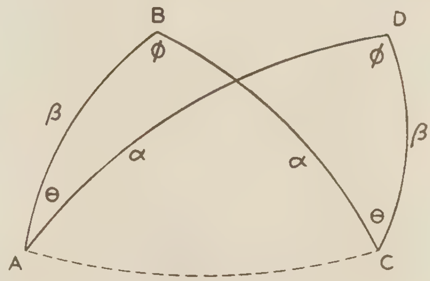

FIG. 4

for this failure seems to be merely that engineers are not aware of its existence. Except for Bricard's book, it does not seem to have been included in any textbooks or other published collections of useful mechanisms.

Motions of the Bennett Linkage. Form the spherical indicatrix of the Bennett linkage by constructing, in an arbitrary sphere, lines from the center of the sphere parallel to the hinge axes of the linkage. The lengths of the arcs of the great circles joining the ends of these radial lines are equal to the twists of the links as shown in Fig. 4. The angle between adjacent links is taken as the angle between the length lines. In Fig. 2, the angle between the links at A is angle DAB. Therefore, in Fig. 4, the corresponding angle is DAB. Call this angle θ. The opposite angle will also be θ. Call the other two included angles ϕ. Then, from the familiar Napier analogy for the spherical triangle ABC, we have the formula

$$\tan {}^1\!/_2(A - C) = \frac{\sin {}^1\!/_2(\alpha - \beta)}{\sin {}^1\!/_2(\alpha + \beta)} \cot {}^1\!/_2 B$$

But $A - C = \theta$ and $B = \phi$. Therefore

$$\tan \frac{\theta}{2} \tan \frac{\phi}{2} = \frac{\sin {}^1\!/_2(\alpha - \beta)}{\sin {}^1\!/_2(\alpha + \beta)} \quad \text{[5]}$$

For given values of the twists α and β, the right side of Equation [5] is a constant. Thus, if in a Bennett linkage, a link of length b is considered fixed, an adjacent link of length a is considered as the driving link, the other adjacent link is considered as the driven link, and the fourth link as the connecting link (or connecting rod), then the relation between the rotation θ of the driving link and the rotation ϕ of the driven link is given by Equation [5], where θ is the angle the driver makes with the fixed link, and ϕ is the angle between the fixed link and the driven link.

Five-Bar Linkages. Bricard does not list a movable five-bar linkage. However, a legitimate five-bar linkage can be made by the combination of two Bennett linkages (9), which have a link in common as shown schematically in Fig. 5. One Bennett linkage has a pair of links of length a and twist α, and another pair of links of length b and twist β, subject to the condition of Equation [4].

The other Bennett linkage has a pair of links of length a and twist α, and another pair of links of length c and twist γ, subject to a similar condition. Combined with Equation [4], it gives

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad [6]$$

If the common link, Fig. 5, which is shown in dotted lines, is held fixed, the two Bennett linkages are separately movable. Now, let us move the two Bennett linkages so that the length

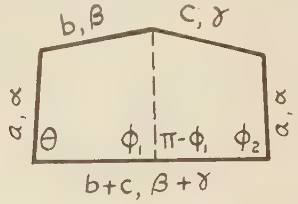

Fig. 5

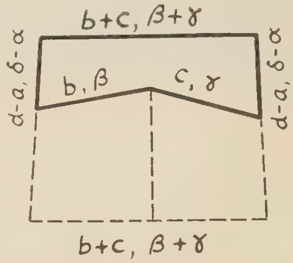

Fig. 6 Syncopated Five-Bar Linkage

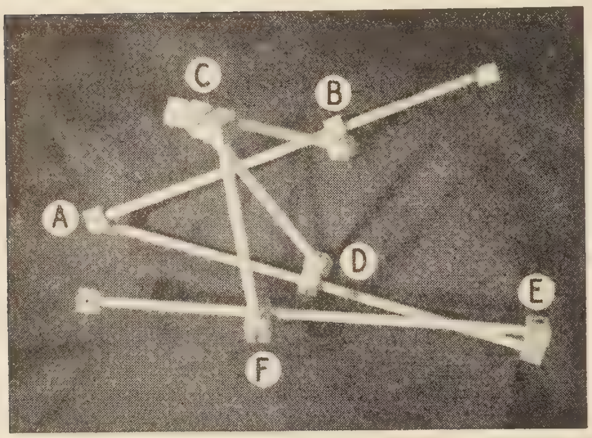

Fig. 5(*a*) Two Bennett Linkages *ABCD* and *CDEF* Which Have the Link *CD* in Common and in Which *AD* and *DE* Are Aligned (Compare with Fig. 5.)

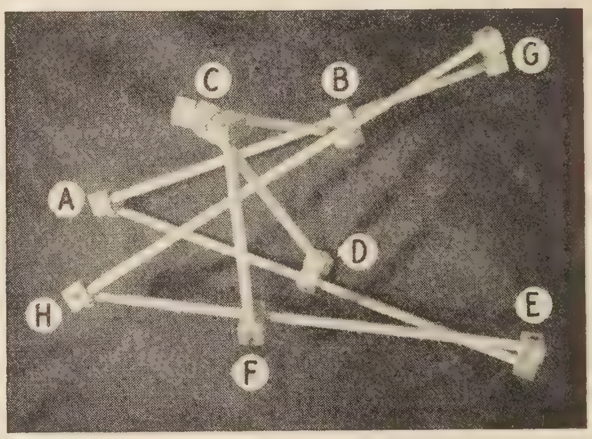

Fig. 6(*a*) *AB* Extended to *G*, and *EF* Extended to *H*; *G* Joined to *H* to Produce Bennett Linkage *AGHE* (Compare with Fig. 6.)

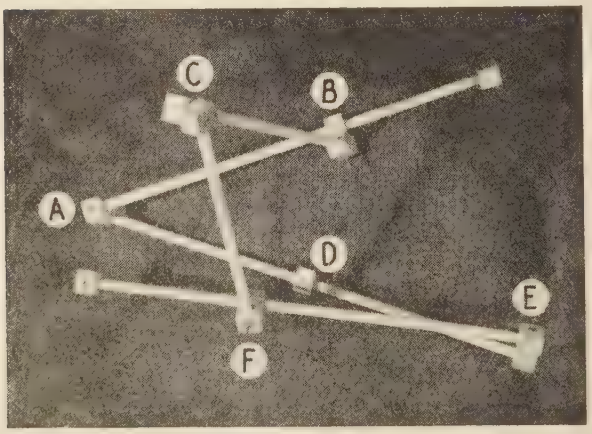

Fig. 5(*b*) Five-Bar Linkage *ABCFE*; Link *CD* Removed (Compare with Fig. 5.)

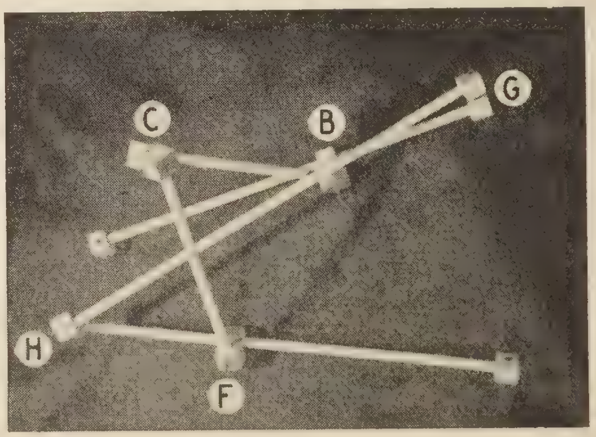

Fig. 6(*b*) Syncopated Five-Bar Linkage *BGHFC*; Links *AE* and *CD* Removed to Form a Five-Bar Linkage (Compare with Fig. 6.)

lines of the two links at one end of the common link are maintained along the same straight line. This is equivalent to locking these two links together so that it becomes a single link of length $b + c$, the algebraic sum of the two component lengths, and of twist $\beta + \gamma$, the algebraic sum of the component twists. The dotted link may now be removed, leaving a movable five-bar linkage.

From Equation [5], the following equations hold for the two Bennett linkages of which this five-bar linkage is composed

$$\tan {}^1/_2\theta \tan {}^1/_2\phi_1 = \frac{\sin {}^1/_2(\beta - \alpha)}{\sin {}^1/_2(\beta + \alpha)} = k_1 \text{ (a constant)} \ldots [7]$$

$$\tan {}^1/_2(\pi - \phi_1) \tan {}^1/_2\phi_2 = \frac{\sin {}^1/_2(\gamma - \alpha)}{\sin {}^1/_2(\gamma + \alpha)} = k_2 \text{ (a constant)} \ldots [8]$$

From Equations [7] and [8], we obtain

$$\tan {}^1/_2\theta \tan {}^1/_2\phi_2 = \frac{\sin {}^1/_2(\beta - \alpha) \sin {}^1/_2(\gamma - \alpha)}{\sin {}^1/_2(\beta + \alpha) \sin {}^1/_2(\gamma + \alpha)} = k_1k_2 = k_3 \text{ (a constant)} \ldots\ldots [9]$$

Equation [9] shows that the relation between the motions of the links adjacent to the $b + c$ link is the same as in a simple Bennett linkage. Therefore, the two links of length a can be extended and joined by another hinged link of length $b + c$ and of twist $\beta + \gamma$.

Let the length of the link joining the two $b + c$ links be d, and let its twist be δ; then δ and d can be determined from Equations [10] and [11]

$$\frac{\sin {}^1/_2(\beta + \gamma - \delta)}{\sin {}^1/_2(\beta + \gamma + \delta)} = \frac{\sin {}^1/_2(\beta - \alpha) \sin {}^1/_2(\gamma - \alpha)}{\sin {}^1/_2(\beta + \alpha) \sin {}^1/_2(\gamma + \alpha)} \ldots [10]$$

$$d = \frac{(b + c) \sin \delta}{\sin (\beta + \gamma)} \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

A second new five-bar linkage can be formed by the deletion of the first $b + c$ link and part of the d links for the length a, as shown schematically in Fig. 6. This shall here be called the syncopated five-bar linkage because it was formed by a contraction of the foregoing five-bar linkage.

Six-Bar Linkages. Bricard lists several movable six-bar hinged space linkages. They may be described by giving the lengths and corresponding twists of the links. They are as follows:

1 Six-bar, symmetric about plane
$(a, \alpha)\ (b, \beta)\ (c, \gamma)\ (c, -\gamma)\ (b, -\beta)\ (a, -\alpha)$

2 Six-bar, symmetric about line
$(a, \alpha)\ (b, \beta)\ (c, \gamma)\ (c, \gamma)\ (b, \beta)\ (a, \alpha)$

3 Six-bar, rectangular or trihedral type
$(A, 90°)\ (a, 90°)\ (B, 90°)\ (b, 90°)\ (C, 90°)\ (c, 90°)$
where $A^2 + B^2 + C^2 = a^2 + b^2 + c^2$

4 Six-bar, from articulated or movable octahedron (10).

However, Bricard does not list the following new six-bar linkages. The first new six-bar linkage is formed by arranging three Bennett linkages in series. The first two Bennett linkages have a link in common, and the opposite link of one of them is common with a link of a third Bennett linkage, as shown schematically in Fig. 7. The common links and the opposite links are each of length a and twist α, while the other links are pairs of length b, c, and d and twists β, γ, and δ, respectively, subject to the conditions

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{\sin \delta}{d} \ldots\ldots\ldots\ldots [12]$$

Three links of length b, c, and d in series have been arranged so that their length lines lie along the same straight line. Then these three links have been replaced by a single link of length $b + c + d$ and of twist $\beta + \gamma + \delta$. The common links, shown dotted, are removed to form a movable six-bar linkage. This shall here be called the six-bar linkage of the series type, since it was formed from a series array of three Bennett linkages.

The relation between the motions of the opposite pairs of movable links in the three component Bennett linkages are given by three equations similar to Equation [4]

$$\tan {}^1/_2\theta \tan {}^1/_2\phi_1 = \frac{\sin {}^1/_2(\beta - \alpha)}{\sin {}^1/_2(\beta + \alpha)} \ldots\ldots\ldots\ldots [13]$$

$$\tan {}^1/_2(\pi - \phi_1) \tan {}^1/_2\phi_2 = \frac{\sin {}^1/_2(\gamma - \alpha)}{\sin {}^1/_2(\gamma + \alpha)} \ldots\ldots\ldots [14]$$

$$\tan {}^1/_2(\pi - \phi_2) \tan {}^1/_2\phi_3 = \frac{\sin {}^1/_2(\delta - \alpha)}{\sin {}^1/_2(\delta + \alpha)} \ldots\ldots\ldots [15]$$

If Equations [13], [14], and [15] are multiplied together we obtain

$$\tan {}^1/_2\theta \tan {}^1/_2\phi_3 = \frac{\sin {}^1/_2(\beta - \alpha) \sin {}^1/_2(\gamma - \alpha) \sin {}^1/_2(\delta - \alpha)}{\sin {}^1/_2(\beta + \alpha) \sin {}^1/_2(\gamma + \alpha) \sin {}^1/_2(\delta + \alpha)} = k_4 \text{ (a constant)} \ldots\ldots [16]$$

As in the case of the five-bar linkage, described in the previous section, the relation between the motions of the links adjoining the $b + c + d$ link is the same as in a simple Bennett linkage. Therefore, the two links of length a can be extended and joined by another hinged link of length $b + c + d$ and of twist $\beta + \gamma + \delta$.

Let the length of the link joining the $b + c + d$ links be e, and let its twist be ϵ. Then ϵ and e can be determined from Equations [17] and [18] as follows

$$\frac{\sin {}^1/_2(\beta + \gamma + \delta - \epsilon)}{\sin {}^1/_2(\beta + \gamma + \delta + \epsilon)} = k_4 \ldots\ldots\ldots\ldots [17]$$

$$e = \frac{(b + c + d) \sin \epsilon}{\sin (\beta + \gamma + \delta)} \ldots\ldots\ldots\ldots\ldots [18]$$

A second new six-bar linkage can be formed by the deletion of the first $b + c + d$ link and part of the e link for the length a, as shown schematically in Fig. 8. This shall here be called the syncopated six-bar linkage of the series type, since it was formed by a contraction of a series array of three Bennett linkages.

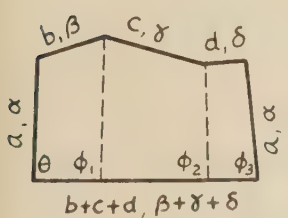

FIG. 7

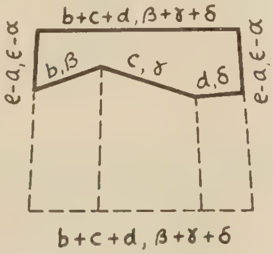

FIG. 8

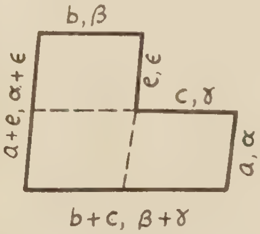

FIG. 9

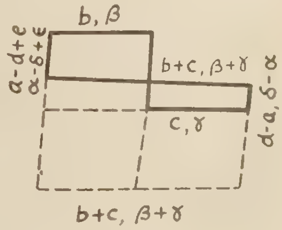

FIG. 10

Fig. 11

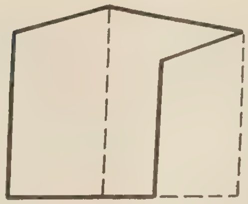

Fig. 12

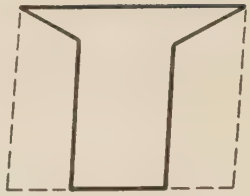

Fig. 13

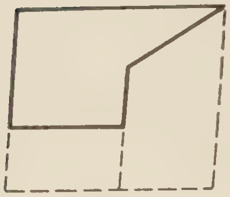

Fig. 14

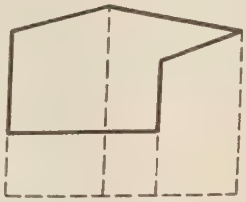

Fig. 15

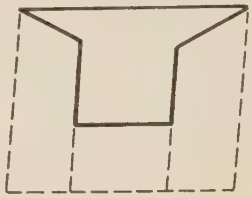

Fig. 16

Fig. 17

Fig. 18

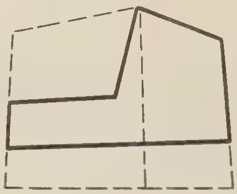

Fig. 19

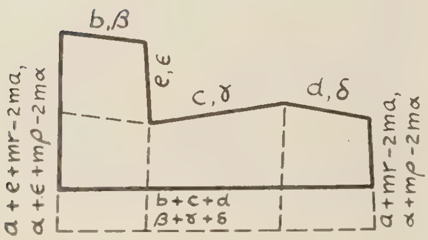

Fig. 20

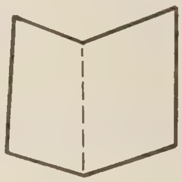

Fig. 21

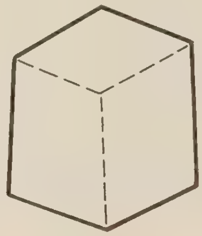

Fig. 22

A third new six-bar linkage can be formed by an L-shaped arrangement of three Bennett linkages, shown schematically, Fig. 9.

A fourth new six-bar linkage can be formed by replacing two of the Bennett linkages by the second five-bar linkage of Fig. 6, as shown schematically in Fig. 10.

Variations of New Linkages. The foregoing linkages can be varied by subtractions of Bennett linkages instead of additions. They are described best by the schematic illustrations. Fig. 11 shows the difference of two Bennetts. Fig. 12 is a Bennett subtracted from the sum of two Bennetts. Fig. 13 is two Bennetts subtracted from a Bennett. Figs. 14, 15, and 16 show variations formed by the syncopation of the linkages of Figs. 11, 12, and 13, in a manner similar to the formation of Fig. 6 from Fig. 5.

The L-shaped arrangement of three Bennet's can be varied also as shown in the illustrations. Fig. 17 shows a Bennett subtracted from the sum of two Bennetts. Fig. 18 and Fig. 19 are two different ways of syncopating Fig. 17.

Generalization of Bennett Linkage and Derived Linkages. A seven-bar linkage, from which the Bennett linkage and the derived linkages are obtainable as special or degenerate cases, can be constructed in the following manner: Choose arbitrary twists α, β, γ, δ, and ϵ. Compute the auxiliary angle ρ from α, β, γ, and δ by

$$\rho = 2 \tan^{-1}\left[\left(\frac{1-k_4}{1+k_4}\right)\tan {}^1/_2(\beta+\gamma+\delta)\right] \qquad [19]$$

k_4 given in Equation [16]. Choose an arbitrary length p and determine the lengths a, b, c, d, e, and r from

$$p = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = \frac{d}{\sin\delta} = \frac{e}{\sin\epsilon}$$

$$r = \frac{(b+c+d)\sin\rho}{\sin(\beta+\gamma+\delta)} \qquad [20]$$

Assemble the seven-bar linkage, shown in Fig. 20, with the following lengths, twists, and order of link

$$(a+e+mr-2ma,\ \alpha+\epsilon+m\rho-2m\alpha)(b,\beta)(e,\epsilon)(c,\gamma)(d,\delta)(a+mr-2ma,\ \alpha+m\rho-2m\alpha)(b+c+d,\ \beta+\gamma+\delta) \qquad [21]$$

where the m's must be all zero or all unity.

Then the special cases appear under the following conditions:

1 $\gamma = 0 \ldots\ldots m = 1$, syncopated six-bar linkage, L-shaped
2 $\epsilon = 0 \ldots\ldots m = 1$, syncopated six-bar linkage, series
3 $\gamma = \epsilon = 0 \ldots m = 1$, syncopated five-bar linkage
4 $\gamma = 0 \ldots\ldots m = 0$, six-bar linkage, L-shaped
5 $\epsilon = 0 \ldots\ldots m = 0$, six-bar linkage, series
6 $\gamma = \epsilon = 0 \ldots m = 0$, five-bar linkage
7 $\gamma = \delta = \epsilon = 0 \ldots m = 0$, Bennett linkage

From the section "Number of Degrees of Freedom," it is seen that any closed hinged chain of seven links has at least one degree of freedom, that is, it is movable unless the longest link is so long that the other links cannot be stretched out to join its ends. Since the twists α, β, γ, δ, and ϵ are purely arbitrary, the seven-bar linkage of Equation [21] may be varied continuously by varying these twists. Special cases, in which some of these twists and the corresponding lengths become zero, are identical with the cases of six or fewer links already discussed. The syncopated cases listed as numbers 1, 2, and 3 arise when m equals unity. The primary cases, listed as numbers 4, 5, 6, and 7 arise when m is equal to zero. Therefore, the linkage of Equation [21] may be considered as a generalized or parent linkage from which the four-, five-, and six-bar linkages of cases 1 to 7 are derived.

Linkages Possessing Two Degrees of Freedom. Each of the foregoing paradoxical linkages is desmodromic, that is, it possesses only one degree of freedom. However, paradoxical linkages possessing two degrees of freedom are easily formed by combinations of two or three Bennett linkages as illustrated in Figs. 21 and 22. These cases are now obvious, but they are mentioned here only to complete the list of known paradoxical linkages.

Linkages With Offset Links. The Bennett linkage was so constructed that the length lines met the hinge axis in the same point. In the five-bar linkage described in the section, "Five Bar Linkages," the length lines of two links were arranged along a single straight line. These links were then replaced by a link whose length line was the same straight line since this line is still the common perpendicular of the adjacent hinge axes. The same procedure was followed in the other five-bar and six-bar linkages described in the foregoing sections.

However, it is possible to construct movable linkages in which the length lines do not meet. If, in the section, "Five Bar Linkages," the two adjacent length lines are not arranged along the same straight line, but are locked together at some other position, as shown schematically in Fig. 21, the five-bar linkage so formed is still movable. If the common link of the two component Bennett linkages is held fixed, the two Bennett linkages can be moved while keeping the angle between the pair of adjacent links constant. The link which is equivalent to the pair of locked links has a new length line since the common perpendicular of its hinge axes has a new position and a new length. The ends of the new length line no longer meet the ends of the adjacent length lines. The ends of two length lines meet a hinge axis in two points which are separated by a distance which may be called an offset. The six-bar linkage, shown in Fig. 23, becomes an offset five-bar linkage if the links are clamped at the point A.

The other five-bar and six-bar linkages, formed by combinations of Bennett linkages, may be varied in form by the use of offset links.

Applications

Among the possible applications of space linkages, the most obvious one is the conversion of oscillating or rotational motion in one plane to motion in another plane as previously mentioned. The Bennett linkage is the simplest for this purpose. The five-bar and six-bar linkages may find application in clearing obstructions where the Bennett linkage would not clear (11). However, a uniform motion of the driving link does not produce a uniform motion of the driven link. This variability may be objectionable in some applications.

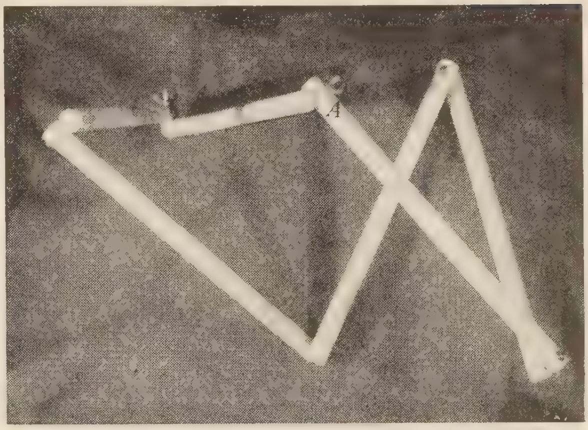

Fig. 23 Clamping of Two Links at A Converts This Six-Bar Linkage Into an Offset Five-Bar Linkage Possessing One Degree of Freedom

Another possible application is the use of the change in the rate of motion to produce a desired nonuniform motion. The special functional relationships that exist may sometimes take the place of cam-controlled motions. The linkages possessing two degrees of freedom may be used to give an output which is a function of two separate independent inputs.

In the applications of the mathematical linkages to real mechanisms, it should be borne in mind that it is not necessary that the material links lie along the length lines, nor that they even approximate them. The spatial relation between two hinges may be maintained by any solid body which joins them even though every part of the material link is considerably displaced from the length line. By taking advantage of this liberty, it becomes possible to construct linkages that permit continuous rotation of the driving link. Fig. 3 is an example of one such linkage which permits continuous rotation by permitting the links to pass one another in their motion.

In the design of rigid space structures it is necessary to avoid combinations which may inadvertently act as movable linkages. There may be special or novel applications in which a nonmovable hinged linkage, for example, the rigid five-bar of Fig. 1(G), may be used as a truss. Therefore, some knowledge of movable space linkages may help the structural designer to avoid blundering into the use of disastrous structures which may fail because they approximate movable linkages.

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3 "Kinematics of Machinery," by C. D. Albert and F. S. Rogers, John Wiley & Sons, Inc., New York, N. Y., 1931, p. 376.

4 "Uber eine Gelenk-Geradführung," by L. Lipkin, Bulletin Scientifique de l'Académie Impériale des Sciences de Saint-Pétersbourg, vol. XVI, 10, 1871, col. 57–60; *Der Naturforscher*, 1871, p. 179.

5 "Note sur la transformation des mouvements rectilignes alternatifs, en mouvements circulaires; et réciproquement," by P. T. Sarrus, Comptes rendus, vol. 36, 1853, pp. 1036–1038. (The spelling "Sarrut" is a misprint.)

6 Space linkages employing joints of one or more degrees of freedom have been discussed by the following:

"Kinematics of Machinery," by F. Reuleaux, Macmillan & Company, Ltd., London, 1876, pp. 549–555.

"Getriebe mit räumlicher Dreistabbewegung," by K. Stein, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 72, 1928, pp. 459–463.

"Die praktische Bedeutung der Raumgetriebe," by H. Alt, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 73, 1929, pp. 188–190.

"Beschleunigungsverhältnisse beim sphärischen Kurbeltrieb und verwandten Mechanismen," by F. O. Müller, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 73, 1929, pp. 117–125.

7 "Leçons de cinématique," by R. Bricard, Paris, vol. 2, 1927, pp. 7–12, 185–199, 311–332.

8 "A New Mechanism," by G. T. Bennett, *Engineering*, vol. 76, 1903, pp. 777–778; see also "The Skew Isogram Mechanism," Proceedings of the London Mathematical Society, vol. 13, 2nd series, 1913–1914, pp. 151–173; and "Zum Bennettschen Mechanismus," by J. Krames, *Sitzungberichte der Kaiserlichen Akademie der Wissenschaften in Wien. Mathematisch-Naturwissenschaftliche Klasse*, vol. 146, 1937, pp. 159–173.

9 Special cases of a five-bar linkage are given by F. E. Myard in "Chaines fermées a cinq couples rotoïdes, déformable au premier degré de liberté," Comptes Rendus, vol. 192, 1931, pp. 1352–1354, 1527–1528; and "Contribution a la géométrie des systèmes articulés," *Bulletin de la Société Mathématique de France*, vol. 59, 1931, pp. 183–210.

10 In addition to reference (7), refer to the following:

"Mémoire sur la théorie de l'octaèdre articulé," by R. Bricard, *Journal de mathématiques pures et appliquées, Journal de Liouville*, vol. 3, 1897, pp. 113–148; G. T. Bennett, Proceedings of the London Mathematical Society, 1911, pp. 309–343; J. Krames, *Monatshefte für Mathematik und Physik*, vol. 46, 1937, pp. 172–195.

11 "Converting Horizontal to Vertical Oscillation by Simple Mechanism," by M. Goldberg, *Machinery*, vol. 46, May, 1940, pp. 103–104; "Space Linkage Mechanism for Transmitting Oscillating Crank Motion," *Machinery*, vol. 47, August, 1941, p. 121.

Discussion

Frederick Franz.[3] The author's lucid explanation of the design of linkages will undoubtedly induce some designers to attempt employing some of the simpler forms when it is desired to transmit oscillating angular motion between two nonparallel and noncoplanar shafts. However, there are some valid reasons why even the four-bar Bennett linkages have not been, and probably will not be, generally employed when such motion is desired. Probably the most categorical one is that they possess no commercial advantage over the commonly designed combinations of plane linkages, constructed with ball-and-socket joints.

Other and more specific reasons are that their velocity-ratio computation is more involved; the graphical laying out on the drafting board of their space-time increments is more difficult; the machining of the pin holes at the ends of each member at the required angles is more difficult; it is impossible to design means of adjustment into the length of the link, as in plain linkages, to provide for ordinary errors of manufacture or for wear; and finally, for the same torque transmission, and for the same load in the connecting link, these Bennett linkages employ longer levers than corresponding ball-joint plane linkages, which means greater moments of inertia and greater shaft stresses from greater combined bending-and-torsion effects at the lever hubs.

When five-bar or six-bar Bennett linkages are proposed for the elements of a practical machine, these criticisms apply with even greater force. In addition, the removal of the redundant link or links (shown in dotted lines by the author) imposes a torsional load on the remaining links. Moreover, the excessive motion in space of the intermediate links, as related to the end motion actually obtained, interposes undesirable mass effects. These considerations greatly handicap such Bennett linkages in comparison with combinations of plane linkages when any appreciable load is to be transmitted or when high speeds and accelerations are involved.

These linkages seem to be more academic than practical, but it will be interesting to see whether or not the author's excellent paper on this subject will bring forth any number of practical and commercial applications.

[3] Consulting Engineer, New Haven, Conn. Mem. A.S.M.E.

A. E. R. de Jonge.[4] The author's paper is very interesting. Originally, the author presented this to the Society in 1941, but unfortunately in a form which was unsuitable for publication. The writer, then a member of the now defunct Subcommittee on Machine Design, did not want the Society to be robbed of the rare opportunity of having presented to it a paper describing a distinct advance in the theory of mechanisms, which, he believes, is one of the first occasions, if not actually the first, that an original contribution to this theory has been made in this country. Consequently, he made certain corrections and suggested numerous changes to the author, and he is very glad to see that the author has not only adopted most of them but has gone even further in extending his explanations and in describing more carefully the various steps in the derivation of the new mechanisms. The result has been the paper he has now presented. The writer is very happy to express to him his congratulations on this achievement.

The author has more or less adhered to the terminology indicated to him by the writer. This is of great importance, because the terminology used in kinematics and mechanisms in this country is very loose indeed, and any additional terms haphazardly chosen cannot but confuse the already confused situation still further. Undoubtedly, it will have cost the author a great deal of self-denial, but the result achieved is unquestionably gratifying.

However, there are still a few terms to which the author has adhered, which the writer considers out of place. The first is "twist" of a link. Twist is a concept strictly connected with a screw or with something that is "turned around and around." Nothing of the kind exists in the case of the hinge axes of a link. They are just two skew lines, and "skew" would have been a far more appropriate term and is going to be used by the writer throughout this discussion. It should be pointed out, however, that the term "twist" was not introduced by the author, but by Bennett, who used this term in his original description of his four-link skew mechanism. The writer agrees that it is difficult to tear oneself away from established precedent, but he feels that to use the more truly descriptive term "skew" is better than to perpetuate a term which has been carelessly chosen in the first instance.

There are other terms which the writer does not like, such as "space-hinged linkage" for "hinged space-linkage;" to measure all twists in the same "hand" for in the same "sense;" the shortest "line" for the shortest "distance;" hinged kinematic "chain linkage" in which "chain" is superfluous; and others. However, the writer must reject emphatically the author's use of the term "desmodromic" linkage. First of all, desmos means "bond, ligament, fastening, or fixing," while dromos means "path," thus "desmodromic linkage" means a linkage moving in fixed paths, and that is exactly what in modern kinematics is called "constrained movable," this term referring, as is shown in a paper by the writer,[5] to motions having a single parameter. To the author, this term, which he says is found in American textbooks, does not appear to be sufficiently descriptive. The writer does not know of any American textbook which deals with space mechanisms. In general, it is not advisable to adopt terms used by French mathematicians and kinematicians. The French delight in using high-sounding terms, while we are more modest and, generally, prefer the simplest and clearest terms.

[4] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[5] "A Brief Account of Modern Kinematics," by A. E. R. de Jonge, published on pages 663–683 of this issue of the Transactions.

In addition, the author uses another such term, which was previously used by the French kinematician Raoul Bricard, when he talks of "paradoxical" linkages. There is absolutely nothing paradoxical about them except that kinematicians have tried hitherto to make them conform to certain formulas or criteria which are said to describe movability fully. While this is true for plane mechanisms, in which case these formulas completely describe the degree of movability or freedom, because the axes of the hinged joints are parallel, this does not hold at all for space mechanisms. Of course, it is very intriguing to try to set up formulas for their constrained movability, in which only the number of links and the number of hinges enter. In space, this is not sufficient, however, and when the directions of the axes are taken into consideration, it will be found that a criterion can be set up which will cover all space mechanisms, including those which were called "paradoxical" by Bricard. The writer wishes to point this out here for the first time, because so far nobody seems to have considered such a possibility. Generally speaking, a nondescriptive term such as paradoxical signifies that its implications have not been fully understood, or that something is wrong somewhere.

The term "syncopated linkage" may be allowed to pass, although a more descriptive English term such as "contracted linkage" would be more appropriate.

Next, a few words on the historic notes are in order. The author makes it appear as if Lipkin were the first to originate the well-known straight-line motion usually called Peaucellier cell. This is not correct, for Peaucellier mentioned it and indicated how it works in a letter written in 1864, to the editor of the *Nouvelles Annales de Mathématiques*,[6] while Lipkin published his investigations only in 1871. In this letter Peaucellier makes much farther going claims than Lipkin ever did. It is true, however, that a clear description and drawing were not published by Peaucellier until 1873, but the aforementioned letter dispels all doubts as to the origin of this well-known mechanism.

On the other hand, we have to be thankful to the author for having drawn attention to the fact that the name of the originator of the first true space straight-line mechanism, Sarrus, is usually misspelled as Sarrut, due to its having appeared that way at the head of his paper (5). The author is correct, however, for the writer has confirmed from several directories of that period that "Sarrus" is the correct spelling. With regard to Sarrus-mechanism, the author states in the brief description given in the "Introduction:" "If one of the latter plates is held fixed, the other plate is constrained to move parallel to it." This, apparently, is an unintentional error, for the plate does not move parallel to the fixed plate, but perpendicular to it, so, however, that it always remains parallel to it.

Under "Definitions," the author states: "More complicated linkage mechanisms may be considered as combinations of simple chain mechanisms (the term 'chain' is superfluous)." This is not correct, for linkages are feasible and actually exist, which are not combinations of simple mechanisms, but, by their formation, are complex mechanisms not reducible to simpler units.

Passing now to the subject matter of the paper, the author describes first the Bennett linkage which, he states, has only been described in Raoul Bricard's book (7). Obviously, he has overlooked the book by Professor Dunkerley,[7] in which Bennett's linkage is included also.

The author has derived Equations [2], [3], and [4] for the Bennett mechanism correctly, and his Fig. 2 is likewise correct. However, when we come to Fig. 4 of the paper, and to his derivation of the formula for the motion of the links adjoining the fixed link, we encounter difficulties. The author has drawn his Fig. 4, in close agreement with that of Bennett,[8] who first used the "spherical indicatrix" for deriving the formula. Both show a crossed skew spherical quadrilateral, and Bennett makes the following statement: "The 'spherical indicatrix' of the skew isogram (the name by which he calls his skew mechanism) is formed by taking vertices H, K, H', K' representing the directions (of the hinge axes) h, k, h', k'. An arbitrary sense is given h, and those of k, h', k' are derived by successive twists, all measured in the same sense. The figure is a spherical crossed isogram with constant sides, and deforms simultaneously with the skew isogram." From this statement, it is not clear how the spherical indicatrix is actually drawn by him. The formula given by Bennett for the links adjoining the fixed link is identical with that given by the author as his Equation [5].

On the other hand, the author defines the "spherical indicatrix" as the geometrical form obtained on the surface of the sphere when parallels to the hinge axes are drawn from the center of the sphere. In that case, an ordinary spherical quadrilateral or a crossed spherical quadrilateral is obtained, according to whichever part of the axis is used to produce the intersection with the sphere. For the mechanism shown in the author's Fig. 2, an ordinary spherical quadrilateral (with no crossed sides) is obtained as follows:

Let Fig. 24 of this discussion represent a Bennett linkage, and let O, in Fig. 25, represent the center of a sphere of radius r, OD' a parallel to the vertical hinge axis at D in Fig. 24. Further, let OA' be the parallel to the hinge axis at A, which meets the sphere at A' in the upper forward left-hand octant, while OC', the parallel to the hinge at C, meets the sphere at C' in the upper forward right-hand octant. The parallel to the hinge axis at B is more inclined than either OA' or OC' and will meet the sphere in a point B', either in the upper forward right-hand or left-hand octant, as the case may be. If greatest circles are passed through each pair of the four successive points A', B', C', D', an ordinary (non-crossed) spherical quadrilateral is formed, in which the sides $D'A'$ and $B'C'$ are equal to α and the sides $D'C'$ and $B'A'$ equal to β, that is, to the skews of the respective hinge axes of the links shown in Fig. 24. If the internal angles of this spherical quadrilateral be designated by θ at A' and C', and by ϕ at B' and D', and if a further greatest circle be passed through A' and C', which divides the quadrilateral into two congruent spherical triangles, we get, by applying to any one of these triangles the proper Napier's analogy

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{\cos {}^1/_2\,(\alpha - \beta)}{\cos {}^1/_2\,(\alpha + \beta)}$$

In the original publication, in which Bennett described his mechanism (8), he gave the formula as

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{\cos {}^1/_2\,(\alpha + \beta)}{\cos {}^1/_2\,(\alpha - \beta)}$$

On the other hand, R. Bricard[9] as well as F. E. Myard[10] have derived Bennett's linkage from a torus (or ring) by using Villarceau's circles, in which a bitangent plane cuts the torus, see Figs.

[6] Lettre de M. Peaucellier, capitaine du Génie (à Nice), *Nouvelles Annales de Mathématiques*, second series, vol. 3, 1864, pp., 414–415.

[7] "Mechanisms," by Stanley Dunkerley, third edition, Longmans, Green & Co., London, 1920, pp. 406–414.

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[9] "Démonstrations élémentaires de propriétés fondamentales du tore," by Raoul Bricard, *Nouvelles Annales de Mathématiques*, fifth series, vol. 3, 1925, pp. 308–313.

[10] "Sur les chaines fermées à quatre couples rotoides non concourrants, déformable au premier degree de liberté—Isogramme torique," by F. E. Myard, Comptes Rendus de l'Académie des Sciences, Paris, vol. 192, 1931, pp. 1194–1196.

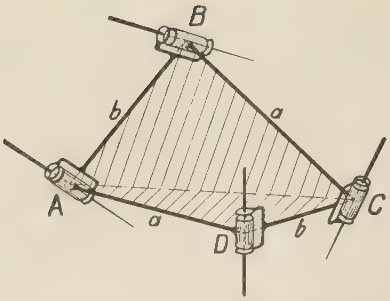

Fig. 24 Perspective View of a Bennett Linkage

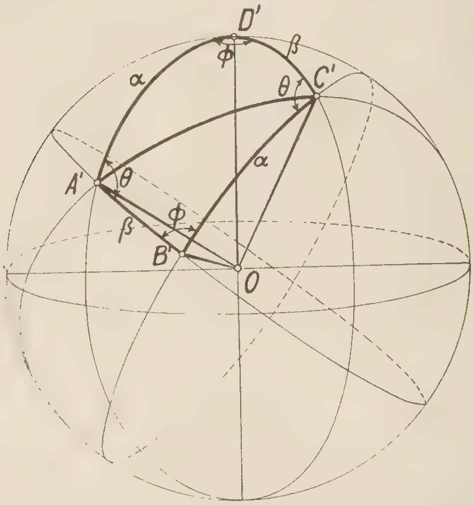

Fig. 25 Spherical Indicatrix for the Bennett Linkage Shown in Fig. 24

(The spherical indicatrix is a non-crossed spherical quadrilateral [spherical isogram]. Sides $A'D'$ and $B'C'$ are equal to α, the skew of the links AD and BC of Fig. 24, and the sides $A'B'$ and $C'D'$ are equal to β, the skew of the links AB and CD of Fig. 24. The angles θ are the angles between links a and b at A and C, while the angles ϕ are the angles between the links a and b at B and D.)

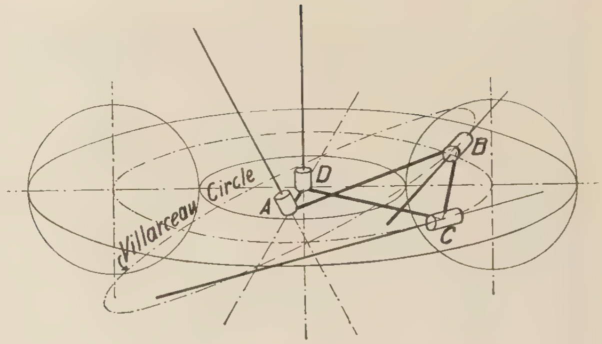

Fig. 26 Bennett Linkage Formed From a Torus by Using the Villarceau Circles–Perspective View

[The torus is indicated by two generating circles, the path of their center when moving about the vertical axis at D, and by the two limiting circles (inner and outer). The hinge axes and the Bennett linkage are shown for a random position.]

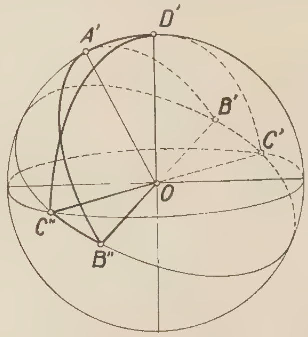

Fig. 27 Spherical Indicatrix for the Bennett Linkage Shown in Fig. 26

[Both the crossed (front half of sphere) and the non-crossed (rear half of sphere) spherical quadrilaterals are shown, depending on which parts of the axes at the B and C hinges are used.]

26 and 27 of this discussion. In that case, the hinge axes have inclinations which may result in a crossed spherical isogram and, hence, the formula given by Bennett[8] and by the author (his Equation [5]) may seem to hold, namely

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{\sin {}^1/_2\,(\alpha - \beta)}{\sin {}^1/_2\,(\alpha + \beta)}$$

Thus, apparently, three different formulas exist for the motion of the links adjoining the fixed link, and the question arises which of these is the correct one.

Since the meeting, at which the author presented his paper and the writer his discussion, the writer has investigated this question further and has arrived at the following conclusions.

It is evident that neither Bennett's definition of the spherical indicatrix nor that of the author is satisfactory and sufficient to prevent any ambiguity in obtaining the spherical quadrilateral. The ambiguity disappears when the directional sense of the hinge axes is properly specified, in which case only one type of spherical quadrilateral, namely, the non-crossed type, results. The proper specification of the directional sense of the hinge axes can be obtained as follows:

A plane surface passed through two adjoining links and that passed through the two opposite links of a Bennett mechanism intersect one another in a straight line, thus forming two congruent triangles having one side in common. The "broken surface" formed by these two triangles (shown hatched in Fig. 24 of this discussion) is to be considered as the "reference surface," and the parts of the hinge axes to one side of this reference surface have to be considered as the positive halves, and those to the other side of the reference surface as the negative halves of the hinge axes. If we define the "spherical indicatrix" so that it shall always be formed by the parallels drawn from 0 to the positive halves (or alternately by the negative halves) of the hinge axes, then the spherical indicatrix will under all circumstances be an ordinary (non-crossed) spherical quadrilateral in which the opposite sides and angles are equal. Any crossing over from one side of the reference surface to the other results in a crossed spherical quadrilateral and has to be avoided. This simple definition amounts to considering, as the skew of the hinge axes of each link, the acute angle of skew, the skew being alternately right-hand and left-hand skew when passing from link to link in cyclic sequence until the starting point is reached again. Provided the inner angles of the spherical quadrilateral (or of the Bennet linkage) are designated by θ and ϕ, respectively (θ at A' and C', ϕ at B' and D'), the correct formula to be used for the motion of the two links adjoining the fixed link is

$$\tan\frac{\theta}{2}\ \tan\frac{\phi}{2} = \frac{\cos {}^1/_2\,(\alpha - \beta)}{\cos {}^1/_2\,(\alpha + \beta)}$$

that is, the formula derived by the writer. Since no crossed spherical quadrilateral can occur under this definition, the formula, given by Bennett in his paper,[8] and by the author in his present paper, has to be abandoned.

There remains only the matter of clearing up the difficulty between Bennett's original formula and that of the writer. This is easily done. If the external angles instead of the internal angles of the Bennett isogram are designated by θ' and ϕ', respectively, then we obtain, by using the proper Napier's analogy

$$\tan \frac{180°-\theta'}{2} \tan \frac{180°-\phi'}{2} = \frac{\cos {}^1/_2 (\alpha - \beta)}{\cos {}^1/_2 (\alpha + \beta)}$$

or

$$\tan \frac{\theta'}{2} \tan \frac{\phi'}{2} = \frac{\cos {}^1/_2 (\alpha + \beta)}{\cos {}^1/_2 (\alpha - \beta)}$$

Thus, all difficulties have been cleared up satisfactorily. To the writer's knowledge, this has never been done before.

Next, the new linkages, derived by the author, have to be considered. It is not quite clear what ideas the author claims to be new. The idea of compounding two Bennett mechanisms to form a new one was hinted at already by Bennett himself, but was put into reality by the French kinematician F. E. Myard, although only in the form of two compounded symmetrical Bennett mechanisms. The author has to be credited, therefore, with having been the first to compound two unequal Bennett mechanisms, both of which, however, have one pair of opposite links of equal length, and with creating thereby a new five-bar linkage. Regarding his second or "syncopated" linkage, this, on closer inspection, turns out to be identical with the first, only that it is derived in a different way and is reversed. Hence, the two five-bar linkages of the author reduce to one only, a fact apparently not realized by the author, and, for the special case treated by Myard, this five-bar linkage was known already. What the author has discovered, however—and that is important enough to give his name a permanent place in the annals of kinematics—is: Two new and simple ways of deriving this linkage, namely, by a simple addition and/or subtraction of two ordinary Bennett linkages of any shape, having two pairs of corresponding links of the same size, and this he has achieved by his discovery that the product of the tangents of one half the angles between the fixed link on the one hand, and the driving and driven links on the other hand, is always a constant, or that

$$\tan \frac{\theta}{2} \tan \frac{\phi_n}{2} = k_n \text{ (a constant)}$$

this being the same as for an ordinary single Bennett linkage which, as an auxiliary linkage, he has derived also.

This discovery he has utilized, in addition, for deriving his six-bar linkages. The first one he describes, the series arrangement of three ordinary Bennett linkages, is undoubtedly new. The second one or his "syncopated" six-bar linkage suffers, however, from the same fault as his syncopated five-bar linkage, for on closer inspection it, too, is found to be the same as the first (his Fig. 7), that is, it also is simply a combination of three Bennett linkages in series. His *L*-shaped arrangement, on the other hand, leads to a new type of six-bar linkage (his Fig. 9). In fact, this is also a combination of three Bennett linkages of which two have one link a in common, while the third has one link b in common with the first. His Fig. 10 shows that, by subtraction of two Bennett linkages from the *L*-shaped linkage, he apparently obtains a new form. On closer inspection, this form again turns out to be identical with that of the L-shaped six-bar linkage, only that the lengths of the end links are different. The author's Figs. 11 to 19 show further forms which he calls "variations" of the new linkages described. They are obtained by subtraction of Bennett linkages from his five- and six-bar linkages. As stated already for the five-bar linkage, the important fact that the author has discovered is that Bennett linkages can be compounded by addition as well as by subtraction, or by a combination thereof, to form new constrained movable five- and six-bar linkages. In this respect, his Equations [9], [10], [11], [16], [17], and [18] assume importance, after they are suitably corrected, as explained previously for his Equation [5]. Equation [9], which should read

$$\tan \frac{\theta}{2} \tan \frac{\phi_2}{2} = \frac{\cos {}^1/_2 (\beta - \alpha) \cos {}^1/_2 (\gamma - \alpha)}{\cos {}^1/_2 (\beta + \alpha) \cos {}^1/_2 (\gamma + \alpha)} = k_1 \cdot k_2 = k_3 \text{ (a constant)}$$

shows that the product of the tangents of the internal angles made by the fixed links with the driving and driven links is always a constant, even after compounding. Equation [10], which should read

$$\frac{\cos {}^1/_2 (\beta + \gamma - \delta)}{\cos {}^1/_2 (\beta + \gamma + \delta)} = \frac{\cos {}^1/_2 (\beta - \alpha) \cos {}^1/_2 (\gamma - \alpha)}{\cos {}^1/_2 (\beta + \alpha) \cos {}^1/_2 (\gamma + \alpha)} = k_3,$$

can be written in the form

$$\tan \frac{\delta}{2} = -\frac{1 - k_3}{1 + k_3} \cdot \cot \frac{\beta + \gamma}{2}$$

from which δ can be calculated, that is, the skew of a link of an ordinary Bennett linkage which may replace the five-bar linkage as far as the motion of the two links adjoining the fixed link is concerned. The length of the link d itself is obtained from the author's Equation [11]. Similarly, for the six-bar linkage, the skew ϵ of a link e of an ordinary Bennett linkage, which may replace the six-bar linkage as far as the motion of the links adjoining the fixed link is concerned, can be obtained from the corrected Equation [17]

$$\frac{\cos {}^1/_2 (\beta + \gamma + \delta - \epsilon)}{\cos {}^1/_2 (\beta + \gamma + \delta + \epsilon)} = \frac{\cos {}^1/_2 (\beta - \alpha) \cos {}^1/_2 (\gamma - \alpha) \cos {}^1/_2 (\beta - \alpha)}{\cos {}^1/_2 (\beta + \alpha) \cos {}^1/_2 (\gamma - \alpha) \cos {}^1/_2 (\beta - \alpha)} = k_4$$

which can be written in the form

$$\tan \frac{\epsilon}{2} = -\frac{1 - k_4}{1 + k_4} \cdot \cot \frac{\beta + \gamma + \delta}{2}$$

The length of the link e of the equivalent Bennett linkage is obtained from the author's Equation [18].

Next, the author has presented a generalization of the Bennett and derived linkages, but he has failed to state the ideas which led him to this generalization, that is, to its derivation. Apparently, he has superposed the various linkages and has obtained thereby, as the common outline, a type of linkage shown in his Fig. 20. This linkage, obviously, is a seven-bar linkage, and since, as such, it is constrained movable no matter how the hinge axes are located, he has called it the "parent linkage." So far, no fault can be found with this procedure.

Then, he has, apparently, tried to obtain the various linkages from this "parent linkage" and, to that end, he has set up equations for the end links, as indicated in his Fig. 20. In these equations he has entered both, an auxiliary link r and an auxiliary skew ρ multiplied by a factor m, and to obtain all the mechanisms described by him, he had to deduct therefrom terms of $2\,ma$ and $2\,m\alpha$, respectively, from the length and the skew of the end links. By so doing, he actually obtains all the linkages described by him by putting either $m = 1$ or $m = 0$ and by suppressing other links.

Since it has been shown here that the syncopated linkages are no new forms, this procedure will have to be modified so that only the "actual" linkages are obtained, that is, only $m = 0$ need

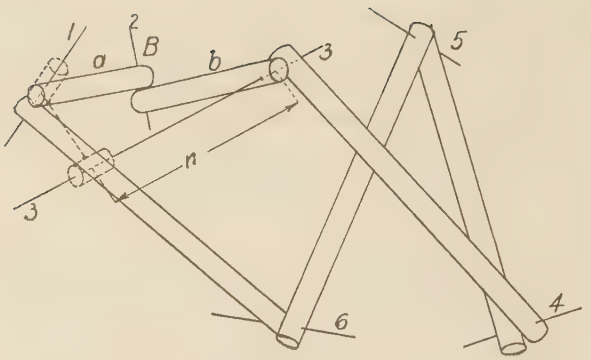

Fig. 28 Copy of Linkage Shown in Author's Fig. 23

(The hinge axes are indicated. The two links a and b are clamped at B thereby transforming the six-bar mechanism into a five-bar mechanism. The auxiliary link replacing a and b is shown between axes 1 and 3. This link rotates about axes 1 and 3 and the distance n is the constant projection of the links a and b onto axis 3.)

be used. This would mean that the terms containing m in the formulas for the end links are unnecessary and can be omitted thereby leading to considerable simplification. Perhaps the author can throw a little more light on this subject by stating the ideas which led him to the formulas for the end links and to his Equations [19], [20], and [21].

He, then, states that linkages with two degrees of freedom may be obtained by a combination of two or three ordinary Bennett linkages, as shown by his Figs. 21 and 22. To begin with, only six-bar linkages of two degrees of freedom can thus be obtained, and secondly, the author uses his Fig. 21 to illustrate two different cases, the one under consideration here and, a little later, that in which the offset length lines produce a constrained movable mechanism, that is, a mechanism with one degree of freedom only. Thus, his Fig. 21 illustrates both, the case of a mechanism with one degree of freedom and that of a mechanism with two degrees of freedom, and it is well to state this here clearly to avoid confusion.

Finally, the author discusses the possibility of constructing movable offset linkages. He states his case correctly and says finally: "The ends of two length lines meet a hinge axis in two points which are separated by a distance which may be called an offset." The important matter, however, is that the two points are separated by a constant distance and that, therefore, the mechanism remains still movable although it is reduced from a six-bar linkage to a five-bar linkage. Fig. 28 of this discussion, which is a copy of the author's Fig. 23, illustrates this clearly, but in Fig. 28, the two links which are locked are the links designated by a and b instead of the two shown locked at A by the author. This change was made, because the two links a and b show the condition of constant length between the foot points of the perpendiculars between hinge axes 1 and 3 and 3 and 4 more clearly, or that n = a constant. The length n is constant, because links a and b are locked at the hinge 2 (at B) and, therefore, represent a link of constant length the projection of which onto the hinge axis 3, about which it rotates, is constant.

Applications of these mechanisms have not been made so far. Once these mechanisms will be clearly understood, there is no reason why they should not find many applications, some of which the author has outlined. Since application of these linkages in practice requires that the designer should have a very clear conception of their properties, and since these properties were not all stated correctly in the paper presented by the author, the writer thought it necessary to clear up these properties as much as possible.

In spite of the shortcomings of the paper, and the many criticisms of the writer, the fact must not be lost sight of that the author has done an exceedingly useful piece of work by his discovery not only of several new space mechanisms, but also of simple methods for deriving them. For having undertaken this useful and arduous work on space linkages, which, in general, belong to one of the most difficult branches of the theory of mechanisms, particularly with respect to their kinematics, the author fully deserves the thanks not only of the Society, but also of that part of the engineering profession which is interested in mechanisms and their application to machinery and other apparatus.

Author's Closure

Mr. Franz's objections to the use of space linkages in power drives is quite valid. These mechanisms are not recommended where long links are subjected to heavy compression loads or severe torsional strains. However, they can be very useful in control mechanisms, in interlocks, and sometimes in computing devices. The author has seen a dozen instances where a Bennett linkage could have been used to replace a more complicated mechanism.

Ball-and-socket joints and Hooke's joints are more difficult to manufacture than the simple links of a hinged mechanism. The latter require only the drilling of holes at the proper angles. It is quite true that the drafting of twisted links is more difficult, but the engineer is letting "the tail wag the dog," if he allows the difficulties of drafting to influence the design.

The author is grateful to Professor de Jonge for his labors which have resulted in the publishing of this paper. His suggestions have contributed toward making the exposition more intelligible to the engineering profession. As an expert on the subject of kinematics, his opinions are not to be taken lightly, although his praise of the contributions of this paper seems excessive.

Professor de Jonge is well known to the Society for his papers on kinematics. The author agrees with him that many kinematic terms are ill-chosen, ambiguous, misleading, or too polysyllabic, and that better terms could be adopted. The author will leave that to the writers and compilers. The terms used in this paper, with few exceptions, had already been used by others. When new terms are necessary, the author prefers to follow the rule of choosing terms which are distinctive and descriptive, and to avoid common terms which have other connotations, even though these new terms may be exotic. However, some terms have become part of the language; for example, American engineers consider a left-hand screw to be "opposite hand" to a right-hand screw. Mathematicians prefer the expression "opposite sense."

"Shortest line" was used only when its position in space was intended. "Shortest distance" is a scalar quantity which is only the length of the shortest line, and does not imply a position. The hyphenated expression "space-hinged linkage" does not appear in the paper although the expression "space hinged linkage" was used to distinguish it from a hinged linkage in the plane.

The formulas for the number of degrees of freedom of a linkage, whether plane or three-dimensional, in terms of the number of links and the type and number of the couples, apply only to the general cases. There are exceptional cases, however, and these should be distinguished by a special name. The term "paradoxical" was well chosen by Bricard to describe these cases. It does not mean absurd or contradictory but, according to the dictionary, only "seemingly contradictory." The term is used in the same sense elsewhere in mechanics as, for example, in Ferguson's paradox on gears and the hydrostatic paradox.

Professor de Jonge is correct in stating that the syncopated

five-bar linkage is similar to the primary five-bar compounded of two Bennett linkages, although this result is not obvious. However, the syncopated six-bar, except for special cases, is distinct from the six-bar made of a series arrangement of three Bennett linkages. The following two paragraphs illustrate the difference:

The links of the syncopated linkage, which is associated with the five-bar linkage of Fig. 1(E) of the paper, are congruent to those of Fig. 1(E), except that the links of length $2a$ and twist 90 deg are replaced by links of length $0.44a$ and twist 12 deg 3 min, corresponding to d-a, and δ-α obtained by the use of the6 corrected forms of Equations [10] and [11]. The ratio of the length of the new link to the sine of its twist is $0.44a/0.22$ or $2a$. This ratio is the same as for the other two short links because $\sqrt{2}a/\sin 45$ deg $= 2a$ and $a/\sin 30$ deg $= 2a$. Since Equation [4] is satisfied for every pair of short links, this syncopated five-bar linkage may be resolved into two Bennett linkages.

The links of the syncopated linkage, which is associated with the six-bar linkage of Fig. 1(F), are congruent to those of Fig. 1(F), except that the links of length $2a$ and twist 90 deg are replaced by links of length $1.536a$ and twist 42 deg 46 min, corresponding to e-a and ϵ-α obtained by the use of the corrected forms of Equations [17] and [18]. In this case, however, the ratio of the length of the new link to the sine of its twist is $1.536a/$ sin 42 deg 46 min or $2.263a$. For the other short links in the syncopated linkage, the ratios are each $2a$. Since Equation [4] is not satisfied for every pair of short links, the syncopated six-bar linkage is not resolvable into three Bennett linkages.

The paper calls attention to the fact that the various methods of deriving the linkages did not always yield distinct linkages. An exhaustive investigation of the possibilities has not yet been made. The parent linkage of Fig. 20 of the paper is an attempt at summarizing the results by superimposing the new linkages. It does not, of itself, add any new results.

Professor de Jonge has performed a great service in pointing out the error in the evaluation of the constant k as it appears in Equation [7], and elsewhere in the paper. This evaluation has been beset by pitfalls and unfortunate mistakes since it was first conceived by Bennett. It was correct in Bennett's first paper although the associated figure was wrong. It was changed in his second paper to another formula which was incorrect. Since then, the literature has been confused by the repetition of both formulas. The formula derived by Professor de Jonge, to add to the confusion, is also incorrect.

The foregoing errors arise from two causes. One is the ambiguity in the formation of the spherical indicatrix, but this error can be avoided very easily by the use of other means as will be shown. The other difficulty is more fundamental, namely, the twist never exceeds 180 deg. A twist of 180 deg is the same as a twist of zero, while in the customary trigonometry an angle of 360 deg, but not 180 deg, corresponds to an angle of zero. These errors should serve as a warning to succeeding investigators.

Equation [4] was obtained by taking the Bennett linkage in mid-position. The movability of the Bennett linkage can be shown completely by beginning with the four-bar linkage $ABCD$, shown in Fig. 2 of the paper, in which all the joints are considered to be ball-and-socket joints. If AC and BD, not being kept equal, are allowed to vary so that the four-bar linkage remains symmetrical about a line, then

$$\text{Angle } BAD = \text{angle } BCD = \theta$$

and

$$\text{Angle } ADC = \text{angle } ABC = \phi$$

Then the volume V of the tetrahedron can be taken in two ways

$$6V = (ab \sin \phi)\, b \sin \theta \sin \alpha$$

or

$$6V = (ab \sin \theta)\, a \sin \phi \sin \beta$$

This shows that during the motion

$$b \sin \alpha = a \sin \beta$$

which is equivalent to Equation [4]. This relation is independent of angles θ and ϕ. Therefore, the angles α and β can be fixed twists while θ and ϕ vary.

The correct relation between θ and ϕ can be obtained from Fig. 2 as follows:

$$b \cos \phi = a - b \cos \theta = (b - a \cos \theta) \cos \phi + a \sin \theta \cos \beta \sin \phi$$

or

$$a(1 + \cos \theta \cos \phi) - b \cos \theta - b \cos \phi = a \sin \theta \cos \beta \sin \phi \ldots [22]$$

and

$$a \cos \phi = b - a \cos \theta = (a - b \cos \theta) \cos \phi + b \sin \theta \cos \alpha \sin \phi$$

or

$$b(1 + \cos \theta \cos \phi) - a \cos \theta - a \cos \phi = b \sin \theta \cos \alpha \sin \phi \ldots [23]$$

Adding Equations [22] and [23]

$$(a + b)(1 - \cos \theta)(1 - \cos \phi) = \sin \theta \sin \phi\, (a \cos \beta + b \cos \alpha)$$

Using Equation [4] of the paper, this becomes

$$\tan \frac{\theta}{2} \tan \frac{\phi}{2} = \frac{a \cos \beta + b \cos \alpha}{a + b} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha + \sin \beta}$$

$$= \frac{\sin (\alpha + \beta)}{\sin \alpha + \sin \beta}$$

$$= \frac{\cos {}^1/_2(\alpha + \beta)}{\cos {}^1/_2(\alpha - \beta)} = k$$

This equation should replace the incorrect formula of Equation [5] of the paper. The succeeding equations, involving the k's, have to be modified accordingly.

The sum of the interior angles of the Bennett linkage (or of any skew quadrilateral) is less than four right angles, since it can be formed from a plane quadrilateral by a fold about a diagonal. The process of folding reduces the interior angles at the ends of the diagonal. The sum of the interior angles of a spherical quadrilateral, on the other hand, is always greater than four right angles. Therefore, it is not the interior angles, but rather the exterior angles of the spherical indicatrix which correspond to the interior angles of the Bennett linkage. It is this difference which seems to have been the cause of Professor de Jonge s error. The correct spherical indicatrix diagram associated with the correct formula is shown in Fig. 29, which should replace Fig. 4.

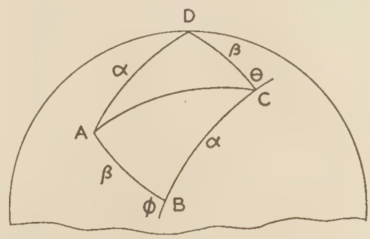

Fig. 29 Spherical Indicatrix Diagram

It is to be regretted that the error which appears in this paper was not discovered before it was presented to the Society. It is well, however, that this discussion has disclosed the error and similar ones which seem to have passed without comment, and frequently unnoticed, in the earlier literature. Fortunately for this paper, however, the error is only in the formula for the evaluation of a constant and it does not invalidate the new results which are essentially qualitative.

A Brief Account of Modern Kinematics

By A. E. Richard de Jonge[1]

The author has attempted to show what the various problems of plane kinematics are and how they have been approached and solved by simple means so as to make modern plane kinematics a usable tool, not for the mathematician or kinematician, but for the practical engineer. In this respect, the elements of a simple universally usable terminology are given, and the various branches of plane kinematics have been reviewed briefly to give the uninitiated an idea of the great simplicity of the modern methods. Quantitative kinematic synthesis has been presented briefly to the English-speaking engineer for the first time. A few remarks on the graphical methods used in space kinematics are added.

Previously, the author (1)[2] has outlined a "science of machinal devices" to which "kinematics," "kinetostatics," and "graphical dynamics" form subordinate sciences. The science of machinal devices will be dealt with in greater detail at a later date. At present, the challenge extended to the author by the various discussers of his previous paper (1) shall be met, and the author will show here the directions in which modern kinematics has been or is being developed.

Introduction

In order to present a clear picture, the entire structure of modern kinematics will have to be reviewed briefly. As was stated in the previous paper (1), the mistake made in the English-speaking countries is that mechanisms have been considered as primary units. These have then been treated by whatever methods were found most convenient. Thus, a multiplicity of methods has come into use, which confuse many an engineer. In Europe, on the other hand, the motion of a free system is considered first by a uniform method, and all mechanisms appear simply as special cases of this general motion.

Kinematics deals (*a*) with the motion of a material point, and (*b*) with the motion of a material system. The former need not be discussed here, as it is usually treated of extensively in most textbooks. The motion of a material system can take place in the plane or in space, and can be that of a free system, of a partly constrained or guided system, or of a fully constrained system. The latter system is equivalent with an elementary kinematic chain. We will here confine ourselves to considering the motion of plane systems, and will only very briefly, at the end, touch upon the methods used for investigating systems moving in space.

Before, however, taking up the subject of kinematics proper, it is necessary to say a few words about the terminology and expressions which are widely used at present, and the necessity for changing them.

Concise Terminology

To begin with, it is of importance to discuss the necessity for a clear and simple terminology in the English language. Numerous new concepts have been formed abroad and have been given simple names in foreign languages, while, up to the present, no equivalent terms exist for them in the English language. Moreover, other terms can be improved or simplified. Thus, while it is possible in foreign languages to use concise expressions for most concepts, cumbersome and lengthy descriptions have to be used for these concepts in the English language. Since economy in words is just as essential as economy in geometrical constructions, it becomes necessary to form new expressions or to modify existing ones so as to comply with this principle. This will be illustrated by a few examples.

The first term which immediately comes to mind is "instantaneous center (or axis) of rotation." This lengthy compound term of 11 syllables was found to be too cumbersome, and it was, therefore, shortened to "instantaneous center" (7 syllables), "virtual center" (5 syllables), "instant center" (4 syllables), and one author has even substituted for it "centro" (2 syllables). Obviously, the trend has been toward simplification, yet, as shown, there exists a multiplicity of terms for the same concept.

Furthermore, the path of the instantaneous center during the motion is a curve, or rather two curves, one in the fixed system and one in the moving system. The name for these curves, introduced for the first time by Professor Kennedy (2), was the incorrectly formed word "centroides," proposed by Professor Clifford (England), and this term was later rectified by being changed into "centrodes" (paths of the center). The two curves touch one another in the instantaneous center and have in it a common tangent and normal. This tangent would have to be designated as "tangent to the centrodes in the instantaneous center." How impossible such an expression is will be realized when it has to be referred to several times within one sentence or has to be used frequently. The need for simplification is obvious, and it can be obtained as follows:

Since, in projective geometry, a point about which lines rotate to create a geometrical form is frequently called a "pole," the use of this term suggests itself as a substitute for "instantaneous center." When a plane rotates about a fixed point, this point is customarily called a "center." When, however, a plane rotates about a point which continuously shifts its position, there is no reason why it could not be designated as a "pole" or, better still, as a "rotopole." That this is not just a fancy of the imagination is evidenced by the polar axis of the earth and the points at which it cuts the earth's surface, these points being commonly called the "poles." Neither the polar axis nor the poles are fixed, but change their position with time. Thus, the choice of the term "pole," or "rotopole," instead of instantaneous center seems appropriate, and the English language is brought thereby in line with German and other foreign languages.

Next, it becomes necessary to assign a name to the two curves described by the "rotopole" during the motion. Each can be called a "pole path" or, if one prefers the Greek equivalent, "polode" and, both together, the "polodes." Furthermore, the rotopole, the point of contact of the polodes, continuously changes its position on the polodes during the motion. It must, therefore, have a velocity of its own which, accordingly, is called the "velocity of change of the rotopole" or simply the "pole velocity." These terms would bring the English language again in line with numerous foreign languages. Having thus formed the simple expressions of "pole," "polode," and "pole velocity," it now is easy to choose a suitable term for the common tangent to the

[1] Adjunct Professor, Polytechnic Institute of Brooklyn. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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polodes. The "tangent to the polodes in the pole" can be designated yet more simply by "pole tangent." These terms establish a simple and coherent system of designations, the usefulness of which will soon become evident.

When, furthermore, in the parlance of projective geometry, a flat pencil (of rays) be laid through the rotopole, each of these "rays" would be normal to the elements of the paths described, in the fixed system, by all points of these rays located in the moving system. All these rays represent, therefore, "normal rays," and the one perpendicular to the pole tangent would, appropriately, be called the "principal normal ray."

A New Concept

It now becomes necessary to explain a new concept introduced into kinematic geometry by Prof. W. Hartmann (Germany). If A_1 and B_1 are two points of a normal ray in the moving system which, at a given instant, have the velocities v_a and v_b (Fig. 1),

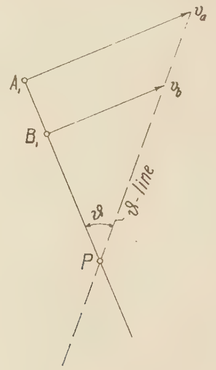

Fig. 1 Theta Angle and Theta Line

then, assuming rigid systems, the end points of the velocities of all other points of the normal ray lie on the line $\overline{v_a v_b}$ which meets $\overline{A_1B_1}$ in the rotopole P and makes with $\overline{A_1B_1}$ the angle ϑ. Hartmann has called this angle the "theta angle" and the line $\overline{v_a v_b}$ the "theta line." Because of the special nature of this angle, Hartmann selected for it the alternate symbol ϑ and not the usual symbol θ used for variable angles. The introduction of the ϑ-angle and ϑ-line simplifies descriptions of geometrical constructions considerably.

Thus, an essential terminology has been set up for the most fundamental elements of kinematic geometry. There is nothing unnatural or artificial about it since all terms fit in nicely with each other and with others as will be seen.

I—The Free Plane System

(*A*) *Polodes.* The coplanar motion of a free plane system relative to a fixed plane consists, as is well known, in the rolling motion of its polode on the fixed polode.

An important step in the development of kinematic geometry, as employed for the investigation of machinal devices, was the introduction, by Reuleaux, of a more complete and detailed investigation of the polodes and the application of the latter to the discussion and development of mechanisms. To be sure, polodes had been studied long before, yet always from the purely mathematical angle, and they seem to have scarcely been drawn for actual mechanisms. Reuleaux, however, was the first who applied them to the detailed study of crank mechanisms and thus laid the foundation for a new theory, namely, that of equivalent mechanisms.

One of his former students who afterwards, as Professor Hartmann, became his principal assistant and, later, successor at the Technical College of Charlottenburg, Germany, was the first who made extensive use of the polodes for the creation of mechanisms for a given purpose. In this respect, he may be considered as one of the first to have employed kinematic synthesis (although not quite in the sense this expression is used nowadays) for the geometrical design of mechanisms. Early in his career, when still a young "Regierungsbaumeister" (government constructor), Hartmann was given the task of investigating what was the trouble with a certain type of tender-locomotive which, in forward motion, would easily pass through a certain curve, but would, invariably, derail the tender when passing through it backwards. Hartmann used extremely ingenious reasoning and solved the problem by his masterly investigation of the locomotive-tender couplings (3). In this investigation, he determined a slider-crank mechanism which permitted correct motion between locomotive and tender. Since this mechanism proved unsuitable for other reasons, he had to replace it by another more suitable one. For the first one, he established the two polodes belonging to the two members involved and obtained the radii of curvature of the polodes for slight angles of deflection. This enabled him to determine the radii of curvature of the paths of points of the locomotive relative to the tender, and vice versa. These he used, for the first time, in a truly engineering way (not merely mathematically) as design elements (bars) for the desired coupling. In fact, for safety reasons, he used three bars, two to prevent sidesway and the third as the actual drawbar. In order to take up compressive forces, he added two rollers and proper tooth profiles on which they could roll. Thus, he obtained a theoretically accurate coupling. All who saw the locomotive and tender, however, with the three bars connecting them (without play), shook their heads in amazement and pointed out that, while two bars would produce a movable mechanism (four-link chain), the addition of a third bar would make the chain immovable, that is, rigid. Hartmann, however, had not chosen the lengths of the bars at random, but had made them equal to the radii of curvature of their respective point paths, and had thus obtained a redundantly closed chain, which is constrained movable within certain limits. Needless to say, the locomotive and tender completely fulfilled Hartmann's expectations and, at the same time, established his reputation. In the book cited, he has discussed exhaustively the entire subject of locomotive-tender couplings, using for them mechanisms "of the first kind" as well as those "of the second kind," as he called them.

By this investigation, Hartmann had shown that mechanisms, which have the same polodes, have the same motion and are, consequently, equivalent. If the polodes coincide within certain limits only, the motions are, also, identical within these limits only. It should be pointed out that, while Hartmann, thus, had established the theory of equivalent mechanism, he did not make extensive use of it and did not exploit it to the limits of its capabilities as has been done by this author. The importance of this theory and, consequently, of the study of the polodes hardly needs further elaboration.

(*B*) *The Quadratic Transformation.* Reference to the radii of curvature has already been made. In plane motion, a relationship, called the "quadratic transformation" assumes extraordinary importance. It may be defined briefly thus: "In the quadratic transformation, a point in the moving system and a point in the fixed system are interrelated so that each is the center of curvature of the path described by the other." The point in the moving system and its corresponding point in the fixed system are, therefore, doubly correlated and form, in the parlance of projective geometry, an "involution." In general, there corresponds to each curve of the n-th degree in one system a curve of the $2n$-th degree in the other system, hence the name "quadratic transformation."

This relationship, which is a purely geometrical one, can be derived most readily by means of projective geometry, that is, without considering time. Consequently, it is not a kinematic relationship, since velocities and accelerations do not play any part in it. Various authors have treated this subject either geometrically or analytically. A simple geometrical construction for the centers of curvature from a known pair was derived by E. Bobillier (4). An analytical and geometrical treatment of the quadratic transformation is contained in Signor Allievi's very original and masterly treatise (5), in which he derived, by the use of the analytical expression for the quadratic transformation, numerous important theorems which led him to the treatment of the higher properties of motion. Since American engineers are trained very one-sidedly—analytically—Allievi's treatment should appeal to them greatly, but it must be kept in mind that his analytical derivations are also based on projective methods and require, besides, a knowledge of the fundamental properties of higher algebraic curves.

On the other hand, it is by no means necessary to exclude "time" in deriving the quadratic transformation. In that case, a truly kinematic method results which involves velocities (or eventually accelerations). This method, which is due to W. Hartmann (6), is extremely simple mathematically. Since its derivation requires only the knowledge of elementary Euclidean geometry, it will here be given briefly for the specific case of a circle rolling upon a fixed circle.

In Fig. 2, let O, r_0, π be, respectively, the center, radius, and

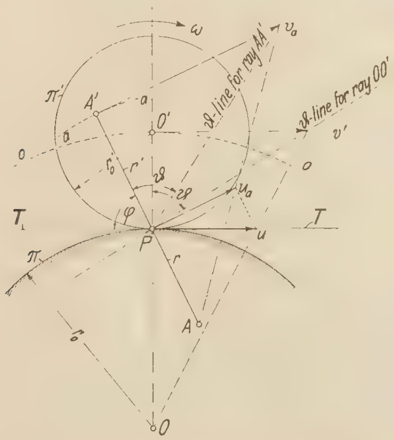

FIG. 2 CONSTRUCTION OF CENTER OF CURVATURE FOR THE PATH OF ANY POINT OF A MOVING SYSTEM

(According to Hartmann.)

circumference of the fixed circle, and O', r_0', π' those of the moving circle which rolls upon π. The point of contact P of the two circles is the rotopole for the motion of π' (and its entire plane) upon π and lies on $\overline{OO'}$, the principal normal ray which is normal to the pole tangent $\overline{PT}$. The point O' describes a circular path o-o about O. Let v' be the velocity which the center O' attains due to its instantaneous rotation about the rotopole P with the angular velocity ω. Hence

$$v' = r_o'\omega \qquad [1]$$

By joining the end point of v' with P, the ϑ-line $\overline{Pv'}$ of the principal normal ray of the moving circle π' is obtained and, at the same time, the ϑ-angle $O'Pv'$. Geometrically, we have, therefore, also the equation

$$v' = r_o' tg\vartheta \qquad [2]$$

Since the rotopole P always lies on the principal normal ray $\overline{OO'}$, and since the latter, due to the motion of O' on o-o, rotates about O, the rotopole changes its position continuously with a velocity which is obtained by joining the end point of v' with O. The intercept $\overline{Pu}$ on the pole tangent is the pole velocity u. From similar triangles, we get

$$\frac{r_o + r_o'}{r_o} = \frac{v'}{u}$$

and by substituting v' from Equation [1]

$$\frac{r_o + r_o'}{r_o r_o'} = \frac{1}{r_o'} + \frac{1}{r_o} = \frac{\omega}{u} \qquad [3]$$

If, furthermore, A' be any point in the plane of the moving system (circle π') and $\overline{A'P}$ its normal ray, its velocity v_a is, when $\overline{A'P} = r'$,

$$v_a = r'\omega \qquad [4a]$$

or geometrically

$$v_a = r' tg\vartheta \qquad [4b]$$

Let A be the center of curvature of the path a-a described, in the fixed system, by A' of the moving system. In analogy to the center of curvature O of the path o-o described by O', which can be found when v' and u are known, A can be obtained easily when the velocity u_a of the rotopole P perpendicular to the normal ray $\overline{A'P}$ is known. This velocity u_a, however, is the component of u perpendicular to $\overline{A'P}$. If the angle made by the normal ray $\overline{A'P}$ with the pole tangent $\overline{PT}$ be φ, then

$$u_a = u \sin \varphi \qquad [5]$$

The line joining the end point of v_a with that of u_a cuts $\overline{A'P}$ in A, the center of curvature of the path a-a of A'. This is the simple construction devised by Hartmann.

The analytical expression equivalent to this construction is easily obtained. When $\overline{PA} = r$, we get, from similar triangles

$$\frac{r + r'}{r} = \frac{v_a}{u_a}$$

and by substituting therein Equations [4a] and [5]

$$\frac{r + r'}{rr'} = \frac{\omega}{u \sin \varphi}$$

whence

$$\left(\frac{1}{r'} + \frac{1}{r}\right) \sin \phi = \frac{\omega}{u} \qquad [6]$$

Combining Equations [3] and [6], we obtain

$$\left(\frac{1}{r'} + \frac{1}{r}\right) \sin \varphi = \frac{1}{r_o'} + \frac{1}{r_o} = \frac{\omega}{u} \qquad [7]$$

the well-known Euler-Savary Equation (7), which represents the analytical expression for the quadratic transformation. This relation, which is not as readily obtainable analytically, is here derived with the greatest ease from Hartmann's construction.

Naturally, it is here not possible to give more than the derivation of this important law. Besides Hartmann's original paper cited (6), an extensive treatment of the quadratic transformation was published, in 1937, by W. Meyer zur Capellen (8).

It has been stated that the derivation was to be given for a special case, namely, for the rolling of one circle π' upon a fixed circle π. If these circles now be regarded as the osculatory cir-

cles of the respective polodes, the construction becomes perfectly general.

The simplicity of this method for finding the radius of curvature will be demonstrated by reference to an example. M. E. Martellotti (9) has determined the radius of curvature of the tooth path of a milling cutter by using the well-known formula[3]

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Anyone who wishes to try this method, starting from Mr. Martellotti's original equations, will be surprised at the great amount of calculation and transformation the differentiations entail. Only when all feasible simplifications are introduced at the earliest possible moment do the formulas become manageable. At any rate, the process is slow and cumbersome. By Hartmann's method, this problem is solved as shown by the

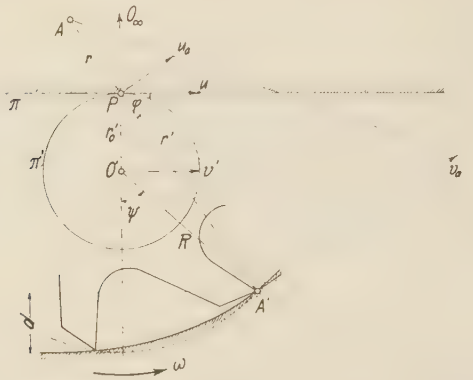

FIG. 3 DETERMINATION OF RADIUS OF CURVATURE FOR PATH OF A MILLING-CUTTER TOOTH
(According to Hartmann's method.)

geometrical construction in Fig. 3, and the analytical expressions are derived from it as follows:

In Fig. 3, let π' be the rolling circle (center O', radius r_o') which rolls upon the straight line π (circle with infinitely large radius, $r_o = \infty$). Point P is the rotopole, A' the cutting edge of the milling cutter tooth, $\overline{PA'}$ the normal ray, on which the center of curvature A must be located. Let the angle it makes with the pole tangent be φ. Let us assume further an angular velocity $\omega = 1$. Then, with the designations given in Fig. 3

$$u = v' = r_o' \qquad [a]$$

$$u_a = r_o' \sin \varphi \qquad [b]$$

$$v_a = r' \qquad [c]$$

and by Hartmann's construction, if $\overline{AA'} = \rho$

$$\frac{\rho}{r'} = \frac{r}{r' - r_o' \sin \varphi} \qquad [d]$$

Further

$$r' \cdot \sin \varphi = r_o' + R \cos \psi \quad \text{or} \quad \sin \varphi = \frac{r_o' + R \cos \psi}{r'} \qquad [e]$$

hence

$$r' - r_o' \sin \varphi = r' - \frac{r_o'}{r'} (r_o' + R \cos \psi) \qquad [f]$$

[3] Refer to Bibliography (9), Equation [6].

and the depth of cut

$$d = R - R \cos \psi \quad \text{or} \quad R \cos \psi = R - d \qquad [g]$$

and from Equations [f] and [g]

$$r' - r_o' \sin \varphi = r' - \frac{r_o'}{r'} (r_o' + R - d)$$

$$= \frac{r'^2 - r_o'^2 - r_o'(R - d)}{r'} \qquad [h]$$

Substituting Equation [h] into [d]

$$\rho = \frac{r'^3}{r'^2 - r_o'^2 - r_o'(R - d)} \qquad [i]$$

But

$$r'^2 = R^2 + r_o'^2 + 2Rr_o' \cos \psi$$

$$= R^2 + r_o'^2 + 2r_o'(R - d) \qquad [j]$$

whence

$$\rho = \frac{r'^3}{R^2 + r_o'^2 + 2r_o'(R - d) - r_o'^2 - r_o'(R - d)}$$

or

$$\rho = \frac{[R^2 + r_o'^2 + 2r_o'(R - d)]^{3/2}}{R^2 + r_o'(R - d)} \qquad [k]$$

Since, according to Martellotti's designations (9),

$$r_o' = \frac{F_tT}{2\pi}$$

we get finally

$$\rho = \frac{\left[R^2 + \left(\frac{F_tT}{2\pi}\right)^2 + \frac{F_tT}{\pi}(R - d)\right]^{3/2}}{R^2 + \frac{F_tT}{2\pi}(R - d)} \qquad [l]$$

This result, which has been derived here *without* differentiations, is the same as that found by Mr. Martellotti by the cumbersome process of differentiation and elimination.[3] There can be no doubt as to which method is the simpler one, the advantage lying entirely with the modern kinematic method as was stated by the author in his discussion of Mr. Martellotti's paper (10).

(*C*) *General Theory of Curvature.* As shown, the quadratic transformation teaches how to find the radii of curvature and the curves which correspond to given curves. Next, it becomes necessary to investigate what the state of curvature is of the point paths described by points in the various regions of the moving system. There exist points which, at the instant, describe the inflection points of their paths and others which describe cusps of their paths. These points lie on two circles which, respectively, are termed "inflection circle" and "cuspidal circle." Other points of the moving system describe portions of their paths, which have stationary curvature. All such points in the moving system lie on a curve of the third degree, which has been given various names, but which here may be called the "cubic of stationary curvature." To it corresponds a similar "cubic of stationary curvature" in the fixed system. These two cubics play an important part in modern kinematic geometry and also in the modern kinematic synthesis of mechanisms. They are very easily constructed as may be seen, for example, from Allievi's book (5). These cubics, which are focal curves, have been studied

extensively by Allievi and numerous German investigators. As Allievi has shown, these investigations lead to the establishment of several new concepts which, however, can here not be dealt with further, and to a systematic investigation of all elementary mechanisms, particularly also of those in special positions, as for example, in dead-point position, of symmetrical mechanisms, etc. Allievi has carried out this investigation in great detail and has classified the mechanisms into series according to their geometrical properties. On that account, his investigation is of greater usefulness to the practical engineer than are some of the other purely theoretical studies.

Furthermore, there exist regions in the plane, in which the curvature follows special laws, that is, the consecutive positions of points of the moving system fulfill certain conditions, namely, of lying on various curves, for example on straight lines, on circles, etc. Allievi and others also carried out investigations of these subjects. Other important studies include that of the evolutes of the point paths, Ball's point and Ball's curve, and the studies of the singularities of motion.

One problem deserves special mention, namely, the determination of the radii of curvature of the polodes. Time and again, mathematicians and engineers have tried to obtain them by means of the simple constructions that follow from the quadratic transformation. This is entirely erroneous, for the problem is far more complicated. It requires for its solution the investigation of four infinitely near positions or, what amounts to the same thing, the use of the evolutes of curve-envelopes or of the cubics of stationary curvature. A more direct method is due to Hartmann.

(*D*) *Velocities and Accelerations.* The problems considered so far are mostly problems of differential motion, that is to say, they deal with properties of the motion at a given instant. There are other problems, however, which investigate properties of the motion during a definite interval of time. The rolling of the polodes was such a problem. Velocities and accelerations of points of a moving system may be of either kind.

The state of the velocities and that of the accelerations of a moving free system at a given instant are briefly called "velocity state" and "acceleration state" respectively. They consider the velocities and accelerations of all the points of the system, that is, of the infinitely extended plane. Since any set of points composes a geometrical form, the investigation of these states has to concern itself with the determination of velocities and accelerations of such forms as straight lines, flat pencils, polygons, and curved figures. The sum total of the velocities and of the accelerations of any of these forms represents a "diagram of instantaneous velocities" or of "instantaneous accelerations" respectively. These diagrams may be of different kinds. Depending upon whether the velocities or accelerations are plotted in their proper positions, are turned through an angle, or are plotted from a certain point either parallel to their original positions or to their turned positions, local velocity (or acceleration) diagrams, turned velocity (or acceleration) diagrams, polar velocity (or acceleration) diagrams, and polar turned velocity (or acceleration) diagrams are obtained. The author calls the two latter more briefly "velocity (or acceleration) polygons" and "turned velocity (or acceleration) polygons" to differentiate them from the local velocity (or acceleration) diagrams which he calls simply "velocity (or acceleration) diagrams" and "turned velocity (or acceleration) diagrams. While the velocity diagrams described are well known in the United States, the respective "acceleration diagrams" are looked for in vain in American textbooks, except the polar-acceleration diagrams (or "acceleration polygons") which have been used in some of the more recent textbooks.

To elaborate this statement a little further, a few details of the theory of acceleration shall be presented. As will be clear from what has been said about the state of curvature of a moving rigid system, there exist points which move through the inflection points of their paths and, consequently, cannot have an acceleration component normal to this path. Other points exist which have no tangential component of the acceleration, that is, no component in the direction of the path. These two kinds of points lie on two circles respectively. From this fact alone, it will be clear that the entire state of acceleration of a moving system requires careful investigation. Laws very similar to those that hold for velocities are found to exist for the accelerations. As is well known, the rotopole is the only point of a moving system which, at the instant considered, has no velocity. The velocities of all other points are proportional to their distances from the rotopole. It might, now, be assumed that the rotopole has also no acceleration. This is erroneous, however. Yet there does exist a point which has no acceleration. As is clear, it must be the second point of intersection of the two circles mentioned before, points of which either have no normal or no tangential component of acceleration (the first point of intersection being the rotopole). This point was first discovered by A. Transon (11).

Much later, J. A. Chr. Bresse (12) carefully investigated the accelerations of a moving system. It was he who found the two circles mentioned, which today bear his name ("Bresse circles"), and who gave their second point of intersection the name "centre des accélérations." In conformity with our definitions of "center" and "pole" given at the beginning, it is better to call it the "acceleration pole" since it continuously changes its position like the rotopole. The accelerations of all other points, Bresse found to be proportional to their distances from the acceleration pole, and all accelerations make the same angle with their "acceleration rays," that is, with the straight lines joining them with the acceleration pole. Many other valuable facts about accelerations have been discovered by the numerous investigations made by French and German scientists. None of these developments can be found in American textbooks as was previously stated by the author and repeated hereinbefore. Thus he stands vindicated.

(*E*) *State of Continuous Motion.* Next the state of continuous motion of a rigid system has to be investigated. Of interest become now the questions how velocities and accelerations of given points change during a definite period of time. The analytical investigation of these changes is out of the question, because the formulas, even for very simple mechanisms, become so complicated as to be virtually unmanageable. The graphical investigation gives results in a relatively simple way. Since the velocities lie in the direction of the tangents to the path of the moving point, they can be plotted directly. Accelerations may also be plotted directly if their angular deviation from the respective normal is known. In general, it is better, however, to resolve them into normal and tangential components and to plot each of these separately. The idea of plotting velocities and accelerations in the respective positions of the moving point has to be credited to A. F. Moebius, but the name "hodograph" for the diagrams which thus result and their detailed investigation is due to Sir William R. Hamilton (13). Today, these diagrams are called "local hodographs" of the velocity or acceleration respectively. M. Grübler has shown that, by plotting the "turned velocities" ($\pm 90°$) to either side of the path, the total acceleration and its correct position can be obtained easily. The resulting velocity diagrams are called hodographs of turned velocities. If the velocities and accelerations are plotted, from a fixed point, parallel to their actual positions, polar hodographs of velocities and accelerations result. The laws of these hodographs have been investigated repeatedly.

The hodographs represent really velocity-displacement and acceleration-displacement diagrams over the curved path of the point. To get a better idea of the change of velocities and accelerations, it is customary to plot displacements, velocities, and accelerations against time in rectangular co-ordinates. Thus three types of well-known kinematic diagrams are obtained which are interrelated in the same way as are integral curve and differential curve. Other kinematic diagrams result when velocities and accelerations are plotted over the rectified path of the point, i.e., against displacements. Finally, an acceleration-velocity diagram is possible, and, from the foregoing, time diagrams plotted against displacement, velocity, or acceleration may be derived. All these diagrams are well known, yet some of them, particularly the hodographs, are generally not treated in American textbooks.

Several other interesting investigations have to be mentioned: The investigation of the possible states of acceleration due to L. Burmester, the investigation of the velocities and accelerations of similarly variable systems, which forms the bridge between the hodographs for various points, and between the hodographs and the motion of the plane system itself; furthermore, the investigation of the higher accelerations, and the investigation of angular velocities, angular accelerations, and the higher angular accelerations.

II—The Guided System

The free system has a threefold movability or, as it is generally expressed, three degrees of freedom. When the free system is subjected to restraint by guides, a "guided system" results. This may be of two types. If two degrees of freedom are left by the guide, the system is able to move so that each of its points can cover a certain area or range. Such a system is called a "singly guided" or "restrained movable system." If the system has only one degree of freedom left, all its points can move only on certain curves or paths. Such a system is called a "doubly guided" or "constrained movable system."

(*A*) *Restrained Movable System.* An example of a restrained movable system is the planimeter, or similar instruments as, for instance, the integrator, integraph, etc. One point in these instruments is either guided on a circle or on a straight line, while any other point, for example the tracer point, is free to assume any position within a limited range of the plane. The motion of the restrained movable system, which has two parameters, has been investigated by G. Koenigs, R. Bricard (France); A. J. E. Beth, W. Van der Wouden (Holland); F. Wittenbauer (Germany). The importance of the system is greater than may appear on first sight.

(*B*) *Constrained Movable System.* In this system, guiding can be effected by points, straight lines, or curves, and the guided elements may also be points, straight lines, or curves. Thus, a great variety of cases of possible constraint results, which have been investigated and classified, for instance, by F. Wittenbauer, and by R. Kreutziger. A special case, that of guiding two points of the movable system on two circles in the fixed system (crank-lever mechanism or four-link mechanism) has been treated of in almost every text book. The problems which this four-link mechanism creates are manifold. To begin with, the curves created by any point of the system (or coupler) having a general motion have to be investigated. These curves are now commonly called "coupler curves." In England they were called "three-bar curves." They have been extensively investigated, analytically, by S. Roberts (England), who has determined their order, class, and singularities and, in addition, those of the polodes and envelopes created by curves in the moving system. A. Cayley (England) has, likewise, extensively investigated the coupler curves. Recently, the coupler curves have been determined photographically or mechanically by K. Langen, K. Rauh, and others in Germany. One of the most important results found by S. Roberts is the threefold generation of the coupler curve, i.e., the generation of the same coupler curve by three different, but interrelated four-link mechanisms.

Special curves are produced when the four-link mechanism assumes special forms. If both guiding circles have infinitely large radii, the elliptic motion results and by its inversion the cardioidic motion (Pascal's curves). If one circle each, in the fixed and the moving systems, has an infinitely large radius, and the other two an infinitely small radius, that is, shrink together to points which travel on the respective straight lines of the other system, the conchoidic motion results. If both circles have the same radius, we get the parallel cranks or the antiparallel cranks, depending upon the way the coupler moves. Certain coupler curves have portions which approximate straight lines. These create the so-called approximate "straight-line motions" (not parallel motions as they are wrongly called in England). These have been investigated by very many scientists, and various forms were found by James Watt, O. Evans (England), P. L. Tchebycheff (Russia), S. Roberts (England). Systematic investigations not only of the straight-line motions but also of motions on other curves were carried out by R. Müller (Germany) and by L. Allievi (Italy). The latter has dealt also with approximate circular motions. Other problems are created by the determination of velocities and accelerations, for which latter, several methods exist, the simplest and most general ones being those by O. Mohr, R. Land, and W. Hartmann (all of Germany).

III—Several Systems

The relative motions of several coplanar systems form the basis for a general theory of plane kinematic chains. The fundamental relation for the motion of three systems S_1, S_2, S_3 is that the three rotopoles P_{12}, P_{23}, P_{13} for the relative motion of any two of these systems lie in a straight line, the "pole line," and that the algebraic sum of the angular velocities about these three rotopoles is zero. This relation follows from the composition of rotations, as discussed quite generally by Sir William R. Hamilton (14). The clear and concise formulation just given is due to S. Aronhold (15), and this theorem should, therefore, bear his name. However, it was later discovered independently by Prof. A. B. W. Kennedy and is now generally known by his name in the English speaking countries, although he has acknowledged the priority of S. Aronhold (16).

Interesting relations exist between the velocities of any three superposed points in the three systems. If the velocities of any two points in two systems, due to the rotation of either system about its common rotopole, be plotted in their true positions, or in the turned positions, two equal velocities of opposite senses are obtained. Thus, for all three rotopoles, six velocities result which determine a hexagon. This can be constructed if the three rotopoles and one velocity are known. Another interesting relation concerns the accelerations of three systems. This is the well-known law of Coriolis. A great number of proofs for this simple law have been given and are still advanced, many of them without the knowledge of those already published. Most of these proofs make this law appear far more complicated than it actually is. Proofs can be given in very simple form. It is here not possible to mention even the many authors who have published proofs. If the Coriolis accelerations are found for all three systems, they, too, like the velocities, follow certain simple relations involving linear constructions (17).

When more than three coplanar systems perform relative motions with respect to one another, the Aronhold-Kennedy relation

of the three poles holds for any three of these systems. Thus, several sets of three rotopoles on pole lines result. The entire set of rotopoles thus obtained and of the straight lines on which any three of them lie (pole lines) form, what is called, a "pole configuration," the general laws of which were first investigated systematically by L. Burmester, although for certain simple cases such investigations had been carried out previously. Other investigations of the pole configurations are due to E. Philipps and to G. Jung.

As a direct result of investigations of this nature, there follows the twofold generation of the cyclic curves which, in the English speaking countries, are usually called "trochoids," and on the continent "cycloids." The latter name, for which Reuleaux has put up a stiff fight, seems preferable as they originate from the rolling of circles. Because of the twofold generation of these curves, difficulties arise in regard to their terminology, no matter whether they are called "trochoids" or "cycloids" (in their respective form as "epi," "hypo," "ortho," etc.).

Whether a pole configuration is uniquely defined by a certain number of its poles has been investigated by L. Burmester, but his methods were not general enough to give a definite answer. By a different method, C. Rodenberg determined the most general conditions for the number of rotopoles and pole lines.

Many other interesting facts about relative motions of several systems have been studied, but cannot be mentioned here because of lack of space.

IV—Kinematic Chains and Mechanisms

(*A*) *Definitions and General Explanations.* The entire theory of the formation of kinematic chains, their types, and laws will be discussed, at a later date, in a separate paper which will deal with the theory of "Machinal Devices." Here, it may suffice to state that, regarded phoronomically, kinematic chains are combinations of systems which are in contact with one another with certain parts. These parts are called "elements" and the combination of any two in contact an "element pair." The systems themselves are called "links" and the combination of all links a "kinematic chain." Links are called singular, binary, ternary, quaternary, etc., according to whether they contain one, two, three, four, etc., elements. A chain is called "closed" when each link is connected to, at least, two other links, otherwise it is called "open." In an open chain, there exist singular links (which have but a single element). If a chain has n links, namely, n_1 singular links, n_2 binary links, n_3 ternary links, etc., then $n = \sum_1^i (n_i)$, and the sum of all elements of this chain is $e = \sum_1^i (in_i)$.

A chain in which all links move in parallel planes is called a "plane chain," otherwise it is a "space chain." A chain is "constrained movable" when the paths of the points of any of its links relative to any other link are definite curves. If one link of a kinematic chain is fixed relatively to the surrounding space, a "mechanism" results. In general, there can be formed n mechanisms from a kinematic chain of n links. A mechanism, one link of which sets all others in motion, is called a "drive." Since there are $(n - 1)$ free links in the mechanism, $(n - 1)$ drives may result from it and, consequently, $n(n - 1)$ drives from one kinematic chain of n links.

A machine, finally, is a drive, in which one of the $(n - 2)$ remaining free links performs useful work. Hence, $n(n - 1)(n - 2)$ types of machines result from one kinematic chain of n inks. This relationship is the reason for the many types of machines that exist or may exist. Mechanisms, drives, and machines are, therefore, special cases of kinematic chains, as was first shown by Reuleaux. Hence in general, it is only necessary to investigate the various types of kinematic chains in order to be able to deal with any kind of machine type.

The constrained movability of closed kinematic chains depends upon the types of their element pairs. Reuleaux distinguished "lower" and "higher pairs," according to whether the pairs have surface contact or only point or line contact. Lower pairs, or "wrapping pairs," can be subjected to "inversion" without change in their relative motions. Higher pairs, or "envelope pairs," when inverted, have different relative motions.

According to the element pairs they contain, kinematic chains may be subdivided into "lower element-pair chains" and "higher element-pair chains." The former contain only lower pairs, the latter, in addition, also higher pairs.

To find the conditions under which an element pair remains closed in its region of motion is the problem of the theory of restraint. This restraint has been investigated by Reuleaux for plane motion, by Reuleaux, by Beck, and by Grashof, although incompletely, for space motion. P. Somoff's studies of space restraint are quite general.

(*B*) *Criteria of Constrained Movability.* The problem of the constrained movability of kinematic chains belongs, to a great extent, already to the field of kinematic synthesis, particularly to that branch of it which is now called "number synthesis" (18). The latter teaches how a kinematic chain, having given elements, must be composed in order to be constrained movable. It also teaches how to find the total number of chains possible having a given number of links, and this is decidedly a problem of kinematic synthesis. Such problems have been investigated by Rittershaus (19) and Grübler (20). For plane chains, Grübler was the first to set up a criterion of constrained movability which is

$$2j - 3n + 4 = 0$$

where j is the number of hinged joints and n the number of links. The constrained movability, therefore, does not depend upon the size, nor upon the position of the links, but solely on their number and on the number and type of the joints. According to the type of the latter, auxiliary relations have to be observed, but it is not possible in this paper to go into details. When crossed links occur in a plane kinematic chain having both turning pairs and sliding pairs, conditions become more involved. A suitable criterion for the "degree of movability" has been derived by F. Wittenbauer and K. Kriso (21).

Kinematic chains in space were first dealt with very incompletely by Reuleaux. The treatment of simple space chains was attempted by T. Rittershaus with respect to their constrained movability. Grashof has investigated space chains further. A systematic treatment of space chains, comprising links of different degrees of freedom, by means of screw pairs as the most general lower pair is due to E. Delassus (22). Hochmann (23) gives as the criterion of constrained movable screw chains in space the relation

$$5s - 6n + 7 = 0$$

where s is the number of screw pairs and n the number of links. Later investigations by M. Grübler and R. Müller come to the same result. A general investigation of space chains and their movability was given by R. Kraus (24).

(*C*) *Compound Chains.* Simple elementary chains have been properly classified by Reuleaux and treated quite generally by him. The four-link chain has already been dealt with here. However, many other kinds of chains are possible. All of these are compound chains. They may be classified according to the number of links, type of links, type of joints, etc. Investigations of this nature are due to Grübler (25). A different way of creating chains has to be credited to W. Lynen (26). These chains are called "multicrank chains," because they have a number of cranks

through which motions can be introduced. According to the number of links through which such motions are introduced, they are called single, double, triple, . . . , n-fold crank chains. The laws of their formation and motion as well as of their reduction to single-drive crank chains cannot be discussed here.

A further type of chain exists, in which the number of links suddenly changes during the motion. These chains have links which come into coincident positions with other links during the motion so that they no longer determine the motions of the remainder of the links.

An interesting type of chain, based on considerations of "number synthesis," has been dealt with by K. Kutzbach (27) who has called them "Verzweigungsgetriebe" which may be translated best by "distribution chains," for they are really differential or equalizer chains which distribute an effort on to several branches. Their theory might be called the "theory of branching." As Kutzbach has shown, they can assume numerous forms. They, too, can only be touched upon here.

(*D*) *Excessively Closed Chains.* There exists another type of kinematic chain which, while constrained movable, does not satisfy the criteria of constraint, previously given, in that they have more links than are required for constrained movability. Since these chains have an excess of links, they are called "excessively (or redundantly) closed chains." An example of such chains has already been cited here in Hartmann's locomotive-tender couplings. These chains have been investigated by Peaucellier, L. Lipkin, J. J. Sylvester, H. Hart, S. Roberts, and many others, but their systematic treatment is due to G. Darboux (28), who was followed by M. Krause, K. Bleicher, and others.

If a kinematic chain has more links than are necessary for its constrained movability, it is, in general, immovable or rigid. When, however, the functional determinant of the equations for the rigidity of its links vanishes, it becomes movable, and this occurs when certain conditions for the dimensions of the links are fulfilled. Yet, the difficulties of arriving at the equations are, generally, so great that this criterion cannot be considered. Instead of starting from a rigid framework, it is preferable, therefore, to investigate the motions of points of the links to find out if the distances between some of them remain constant during the motion, in which case they may be connected by rigid links without affecting the movability. In general, this depends upon certain invariant relations which must exist to permit these points of being coupled by one or more redundant links. There are several such relationships.

To begin with, there is the relationship of "similarity" of motion which finds its expression in the well-known pantograph (O. Scheiner, 1631) and the plagiograph, or skew-pantograph (Sylvester). "Inversion" is another such relationship. It correlates two points A and B to a fixed point O so that $\overline{OA} \cdot \overline{OB} =$ a constant. The first mechanical generation of curves satisfying this relation is due to Peaucellier (1864) (29). It did not become generally known, however, and was later rediscovered (1871) by L. Lipkin, one of Tchebicheff's students. Other such devices are due to H. Hart and J. J. Sylvester. These devices can generate true circles, but may also be used to generate a circle of infinitely large radius, i.e., a straight line. Thus, Peaucellier was the first who, by mechanical means, could describe a true straight line by the use of his so-called "Peaucellier cell."

The question what other curves might be described by excessively closed kinematic chains was systematically investigated by J. J. Sylvester, A. B. Kempe, and H. Hart. Others have produced kinematic chains for special individual curves. Further excessively closed chains have been produced for the extraction of square roots (Sylvester, improved by W. Johnson), for the solution of cubic equations (St. Loup), for the evaluation of elliptic integrals (A. Emch), and for the representation of x^m for rational values of m (F. T. Freeland).

Several other types of plane compound chains have been investigated by Kempe.

(*E*) *Velocities and Accelerations of Kinematic Chains.* For the elementary kinematic chains, velocities and accelerations can readily be constructed by any one of several methods. As previously stated, such methods, particularly for the accelerations, are due to O. Mohr, R. Land, W. Hartmann, and a number of others. Since the practical application of compound chains does not depend only upon the type of the motion, that is, on the point paths produced, but also upon the velocities and accelerations of the points selected to bring about the desired effect, it is necessary to have simple methods for their determination. In compound chains, this is not as easy as it looks. Frequently, velocity and acceleration "polygons" will produce the desired result, but not always directly. In such cases, similar or projective point ranges have to be used together with a process of interpolation (30). Such methods have been developed by L. Burmester (1911), M. Grübler (1917), Th. Poeschl (1923), and F. Wittenbauer (1923). Recently, however, Prof. N. Rosenauer (31) of Riga has made known a method which is direct and obtains the velocities without interpolations. He also gave a method for obtaining the accelerations of complex chains. By his constructions, he outclassed completely those by Burmester, Grübler, Poeschl, and Wittenbauer.

(*F*) *Other Kinds of Kinematic Chains.* Reuleaux has classified the elementary mechanisms into screw trains, crank trains or linkages, wheel trains, cam trains, belt trains, and block trains (18). The corresponding chains bear similar names, i.e., screw chains, crank chains, etc. So far, only link chains have been considered, i.e., systems joined by turning pairs (or sliding pairs). Reference has been made also to the substitution, for turning pairs, of screw pairs as the most general type of lower pairs. When the screw pairs are coaxial, the elementary type of screw chains is obtained, when they are not coaxial, space chains may result. The coaxial screw trains have been dealt with in great detail by Reuleaux. These chains are lower pair chains. All the chains to be discussed next are higher pair chains.

Wheel trains consist of at least two curves which roll upon one another and enforce this rolling by supplementary higher element pairs. The plane systems of the rolling curves turn about fixed points, or axes, in the fixed system. Kinematically, their treatment is very simple, provided they are correctly formed geometrically. The greater part of the treatment usually found in textbooks on kinematics deals with the derivation of the correct geometrical form of toothed gears, as wheels with auxiliary higher elements, the so-called gear teeth, are usually called. The transmission ratios of gear trains are commonly derived analytically, but they can easily be obtained graphically by means of "angular-velocity diagrams," which are particularly useful for compound gear trains (epi- or hypocyclic gear trains). When the auxiliary higher elements of the gear wheels, the teeth, become infinitely small, friction wheels result. The laws of motion of friction-wheel trains are, consequently, identical with those of gear-wheel trains.

Cam trains consist usually of a rotating or oscillating system with higher element which is paired with the higher element of an oscillating or reciprocating body or system. Both systems rotate or reciprocate and rotate relatively to the fixed system. According to whether one element rotates continuously or whether both oscillate, one speaks of cam trains or of cam-lever trains. Their motion can always be reduced to that of an equivalent link chain by using the radii of curvature. Consequently, their laws of mo-

tion are obtainable from those of the respective equivalent link chains.

Belt trains are the simplest trains and scarcely require further explanation. Their laws of motion depend purely upon their geometrical layout and are thus easily derived.

In some of these types of chains, one system, or link, may be replaced by a fluid. This may occur especially in screw chains and wheel chains as Reuleaux has shown.

The largest class, and perhaps the most difficult one from the point of view of determining their precise motions, is the class of intermittent mechanisms or, as the author has called them, intermittors, block trains, or blockages (18). This is due to the fact that, for their functioning, they require the application of forces (springs, gases, gravity). Consequently, their motions depend not only upon the geometrical form of their elements, but also on these auxiliary elements. The kinds of blockages that Reuleaux has distinguished have been given suitable names by the author in his previous paper mentioned (18).

It may appear from the foregoing as if the author has put too great a stress on link chains and too little on the other five types of elementary chains. Since, however, with the exception of screw chains, all other types may be reduced, for the treatment of their motion, to linkages for the instant under consideration, it follows that their motions, that is, velocities and accelerations, can always be obtained by the methods outlined for link chains. This fact cannot be emphasized too strongly, because the error is made time and again to consider each of these four types of chains by separate methods while, in fact, this is unnecessary. Thus, a unified method of dealing with these different kinds of chains is obtained and thereby a great simplification in their treatment.

This concludes the brief review of plane kinematic chains.

V—Kinematic Synthesis

The most interesting, but least known branch of kinematics is kinematic synthesis. A brief account of its historical development has been given by the author in a previous paper (1).

The purpose of kinematic synthesis is to determine motions which fulfill given conditions and to obtain the machinal devices, or mechanisms, which realize these motions. Thus, the methods of kinematic synthesis are of particular value to the practical designer whose duty it is to solve just such types of problems.

As stated, there exist three different branches of kinematic synthesis. The first is called "type synthesis" and was developed by F. Reuleaux. It teaches how to obtain suitable machinal devices for a given purpose by purely mechanismal considerations. Few, however, have understood the real significance of this branch, and have put it to use intelligently. How such type synthesis can be carried out has been shown by the author for a particular case (18). In recent times (1930), R. Franke has tried, with some measure of success, to extend type synthesis also to electrical apparatus and electric circuits (32).

The second branch is the numerical kinematic synthesis, or briefly "number synthesis," discussed previously, which teaches how to obtain all kinematic chains of a given number of links, or the number of links in a chain, necessary for a given purpose. It also teaches how to find the numerical relations which exist between the number of links and the number of joints, sliders, etc. This branch was mainly developed by M. Grübler (25). That part of R. Franke's investigations, which makes use of the theory of combinations for obtaining all possible electrical circuits for a given purpose belongs also to this branch.

The third branch, finally, which teaches how to find the dimensions of the links of a linkage satisfying given conditions, that is, the branch which deals with quantitative kinematic synthesis, is called "size synthesis."

The early work on quantitative kinematic synthesis, published during the last quarter of the nineteenth century, dealt primarily with the generation of given curves by mechanisms, especially of approximate and accurate straight-line motions. A. B. Kempe proved that every algebraic curve can be generated by hinged linkages, and he showed how these linkages may be obtained. Into this category of investigations belong also those by L. Burmester on the relations between several positions of a system in coplanar motion, the aim of which was also the creation of approximate straight-line motions. These latter investigations have now become the basis of modern "size synthesis."

After this field had remained dormant for nearly two decades, as was shown by the author in a previous paper (1), it was brought to life again mainly by M. Grübler (25), who not only formulated a series of new problems, but also gave simple geometrical solutions for them. It was he who expressed the aims of kinematic synthesis with great clarity and who drew attention to the foregoing investigations by Burmester on the co-ordination of two or more different positions of two links which are to form a constrained movable kinematic chain, thus pointing the way for further development and indicating, at the same time, the method by which it could be accomplished. Investigations into this subject were taken up immediately by H. Alt who, followed later by R. Beyer, advanced this branch of kinematics considerably and coined for it the name "Masssynthese" which has here been rendered as "size synthesis."[4] The problems with which it deals are, in part, the following:

1 To generate given curves as accurately as possible by linkages, preferably with the least number of links;

2 To replace, by circles or conic sections, curves generated by linkages, the circles or conic sections hugging the curve in the region under consideration as closely as possible;

3 To select points on a link, for example on the coupler (connecting rod) of a linkage, so that the coupler curve described can be replaced, with sufficient accuracy, by straight lines, circular arcs, or arcs of conic sections;

4 To force, by a suitable linkage, a link to assume successively two, three, or more specified positions;

5 To force, by a linkage, two or more links to assume successively a number of given co-ordinated positions or, what amounts to the same thing, to select points on two or more links so that, when they are connected by rigid links, they will assume successively the given co-ordinated positions;

6 To design a four-link chain so that the velocity (or acceleration) of the end point of the lever bears a certain given ratio to the velocity (or acceleration) of the crank;

7 To arrange that one or more links come temporarily to rest at specified points of the motion cycle;

and various others.

Problems such as these require a kind of solution which differs from that used in ordinary kinematic problems. All problems discussed heretofore were concerned with continuous motion, no matter whether at a given instant or for a definite period of time. According to the problems outlined, kinematic synthesis deals mainly with certain finite positions of the moving system and does not take into consideration the intermediate course of the motion. The problems are solved when the systems, in their motions, take up the given positions, or when points of the system describe paths which pass through the given points, etc. The fundamental problems of this branch of kinematic synthesis, therefore, must deal with finite motions which will bring the system from one position to the next. Consequently, they are not concerned with infinitely small motions. It is necessary to state

[4] The literature on subjects of kinematic synthesis is widely scattered and, consequently, cannot be given here in detail.

this clearly, because some of the laws of finite motions are not the same in every respect as those of infinitely small motions.

To give a simple example, assume, in Fig. 4, a system to be given

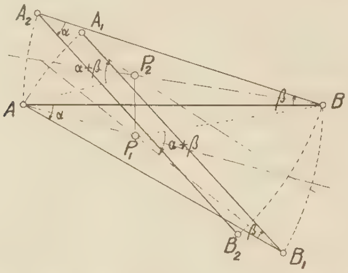

FIG. 4 IMPORTANCE OF SEQUENCE OF ROTATIONS IN CASE OF FINITE MOTIONS

by the position of two of its points A, B, that is, by the straight line $\overline{AB}$. Let this system be subjected to two rotations, one about A through an angle α, and one about B through an angle β, both rotations taking place in the same sense, say clockwise. If the rotations were infinitely small, the sequence of the rotations would make no difference to the resulting rotation. This is not the case, however, when the rotations are of finite magnitude. For if $\overline{AB}$ first turns about A through α, it comes into the position $\overline{AB_1}$, and if it then turns about B_1 through β, it comes into the position $\overline{A_1B_1}$. The resulting position $\overline{A_1B_1}$ could also be reached by a rotation through an angle $\alpha + \beta$ about P_1, this point being the intersection of the bisectrices of the two angles α and β. On the other hand, if the system $\overline{AB}$ first turns about B through β, it comes into the position $\overline{A_2B}$, and if it then turns about A_2 through α, it comes into the position $\overline{A_2B_2}$. The resulting position $\overline{A_2B_2}$ could also be reached by a rotation through an angle $\alpha + \beta$ about P_2, this point being the intersection of the bisectrices of the angles B_2A_2B and A_2BA. The two positions $\overline{A_1B_1}$ and $\overline{A_2B_2}$, while parallel, are not coincident, and the two rotopoles P_1 and P_2 lie symmetrical to the original position of $\overline{AB}$, as can be proved easily. The question of symmetrical positions of rotopoles plays an important part in all investigations of finite motions.

As has been stated, the methods which have to be used in quantitative kinematic synthesis (size synthesis) differ considerably from those of ordinary kinematic problems (kinematic analysis) in that, usually, a direct solution is not possible, or, involves, at least, constructions of an order higher than the second. Recourse must, therefore, be taken to geometrical loci which represent the geometrical conditions to which the system is to be subjected. Keeping this in mind, it will be easy to understand the nature of quantitative kinematic synthesis.

Let us assume, for example, that the plane of the coupler of a hinged four-bar linkage is to pass through a number of given positions. From Burmester's investigations, it follows immediately that no more than five positions can be specified beforehand (33). For these five positions, there exist four groups of five homologous points, each group being located on a circle. The centers of these four circles are called "Burmester points," and of these all four may be real, two real and two imaginary, or all four imaginary. In the first case, six different four-bar linkages result, according to the number of possible combinations; in the second case, but one four-bar linkage exists; and in the third, no solution is possible for the positions chosen.

1 *Two Positions.* To explain this a little more in detail, let first two positions S_1 and S_2 be given for the system S forming the coupler, which this link has to take up during the motion. There exist an infinite number of hinged four-bar linkages that will bring it, as their coupler, into the two given positions. In fact, this can even be done by a simple rotation of the link about a "rotopole" which is obtained by the two mid-perpendiculars to the chords defined by the original and final positions of the two endpoints of the link.

2 *Three Positions.* If three positions are given next, which the link $\overline{AB}$ is to assume during its motion, it is clear that rotations about three such rotopoles, which are defined, in the same manner as just given, by the three positions $\overline{A_1B_1}$, $\overline{A_2B_2}$, and $\overline{A_3B_3}$, will move the link into these three positions and return it finally into the starting position $\overline{A_1B_1}$. The three rotopoles P_{12}, P_{13}, P_{23} form a "pole triangle" which has some remarkable properties. In the first place, the angles of rotation of the system about these three rotopoles are twice as large as the corresponding angles of the pole triangle. In the second place, any rotopole, for example P_{23}, if referred to the original position S_1 of the moving system S, takes up a position P_{23}^1, which is symmetrical to P_{23} with respect to the line $\overline{P_{12}P_{13}}$. Such "symmetrical poles" are of importance if more than three positions have to be considered.

Next, any third point C of the moving system will take up three definite positions in the three given positions of the system, and through these three positions C_1, C_2, C_3, which are called "homologous points," a circle can always be passed, the center of which can be found easily. Furthermore, the mid-perpendiculars of the chords defined by the three positions of A, for example, meet in a point A_c, and those defined by the three positions of B meet in a similar point B_c. The circles about these centers A_c and B_c with $\overline{AA_c}$ and $\overline{BB_c}$ as radii pass through the three given homologous positions of A and B (A_1, A_2, A_3 and B_1, B_2, B_3 respectively). Consequently, the passage of the system through the three given positions can be enforced by a single hinged four-bar linkage $A_cA_1B_1B_c$. Through every other three homologous points, there pass also circles, and all the centers of these circles form what is called the "center system." The circles, by which A_1 comes into the positions A_2 and A_3 by rotations about P_{12} and P_{13} respectively, intersect in a further point $\bar{A}$ which can be obtained also as the symmetrical point to A_1 with respect to $\overline{P_{12}P_{13}}$. Thus if A_1 and the pole triangle $P_{12}P_{12}P_{23}$ are given, the position of A_2 can be obtained by first constructing the symmetrical point $\bar{A}$ to A_1 with respect to $\overline{P_{12}P_{13}}$ and then by constructing the symmetrical point A_2 to $\bar{A}$ with respect to $\overline{P_{12}P_{23}}$, and similarly the symmetrical point A_3 to $\bar{A}$ with respect to $\overline{P_{13}P_{23}}$.

The points P_{12}, P_{13}, P_{23} belong to the center system and are called its "principal points," and the three pole lines determined by them the "principal lines" of the center system. Likewise, the points P_{12}, P_{13}, P_{23}^1 which belong to the system S in its first position S_1 are called the "principal points" of this system S_1, and the three pole lines determined by them the "principal lines" of system S_1.

Taking the symmetrical poles into consideration, it can be proved easily that the pole lines (principal lines) of the center system touch a conic section, the foci of which are A_c and $\bar{A}$ for the circle passing through the homologous points A_1, A_2, A_3. For any other such circle the same condition prevails, but the conic section is different.

It can be proved further that the corresponding pairs of points of position S_1 of system S and of the center system appear under equal angles when viewed from P_{12}, P_{13}, and P_{23}^1 or P_{23}, or under angles which together are equal to two right angles. This relationship is of the greatest importance. It can also be shown that to the point P_{12} of the system S_1, there correspond all points of $\overline{P_{13}P_{23}}$ in the center system, similarly, to P_{13} of system S_1 all points of $\overline{P_{12}P_{23}}$ in the center system, and to P_{23}^1 of system S_1 all points of $P_{12}\bar{P}_{13}$ in the center system. Consequently, the two systems are in a mutual relationship to one another which, upon

investigation by means of projective geometry, is found to be a quadratic transformation in which P_{12}, P_{13}, and P^1_{23} of system S_1 and P_{12}, P_{13}, and P_{23} of the center system are, respectively, the co-ordinated principal points. If the three positions S_1, S_2, S_3 of system S become infinitely near, this quadratic transformation passes into the quadratic transformation between the centers of curvature in the fixed and moving systems, which was discussed in Section I (B) of this paper.

3 *Four Finite Positions.* The next step is the investigation of four finite positions S_1, S_2, S_3, S_4 of a system S or, as it may be expressed more simply, of four identical systems in different positions. It is obvious that, in general, four homologous points of these systems do not lie upon a circle. For, if a circle be passed through three of the homologous points, the fourth will lie more or less closely to the circle, depending upon the position of the point selected. It follows therefrom that there must exist points in system S_1 which, with their homologous points in the other three systems, lie on a circle. All such points of system S_1 lie on a certain curve which Burmester has called "Kreispunktkurve," that is, "circle-point curve," and it is convenient to call it by this name. It follows immediately that there must exist in each of the four identical systems S_1, S_2, S_3, S_4 such a circle-point curve, and that those for systems S_2, S_3, S_4 must be identical with that of system S_1, or as it is usually expressed, the four circle-point curves for systems S_1, S_2, S_3, S_4 are congruent. Since to each point in system S_1 there belongs one and only one center, these centers form a system of centers or, as it is called simply, a "center system." Consequently, there belongs to each point in system S_1 a certain point in the center system, and the same holds for all points of the other three systems. Conversely, to any point in the center system belongs a certain point in system S_1 and similarly in the other three systems. Between the center system and the other four systems there exist, therefore, quadratic relationships (or transformations). In particular, to the points of the circle-point curves in the four systems belong certain points in the center system, which lie on a curve that was called by Burmester "Mittelpunktkurve" or "center curve." To each point of the center curve there belong, therefore, four homologous points on the four circle-point curves of systems S_1, S_2, S_3, S_4, but so that these four homologous points lie on a circle.

These two curves, the circle-point curve and the center curve, play important roles in kinematic synthesis. Both curves are of the third order, that is, they are cubics, and since they pass through the imaginary circular points at infinity, they are "circular cubics," and in particular "focal curves," which can easily be constructed as such.

If the four positions of system S are not at a finite distance from each other, but are infinitely near to each other, these two curves pass into the cubics of stationary curvature discussed hereinbefore, and the quadratic relationship between the points of the moving system and those of the center system becomes the quadratic transformation of the points of curvature.

There exist further important relationships between the four systems S_1, S_2, S_3, S_4. Since four congruent plane systems, in general, have six rotopoles P_{12}, P_{13}, P_{23}, P_{14}, P_{24}, P_{34}, the respective rotations about which bring each system into the position of any of the others, the question arises how many of these rotopoles may be chosen at random so that by their choice the positions of the four systems S_1, S_2, S_3, S_4 are definitely fixed. It will be found that only four of these rotopoles can be chosen arbitrarily, that the fifth must lie on a certain straight line on which it may be chosen at random, and that the sixth rotopole is then definitely determined. If the rotopoles P_{13}, P_{14}, P_{24}, P_{23} are connected so that the rotopoles with no equal ciphers in their index form opposite angles of a quadrangle, and if such pairs P_{13}, P_{24} and P_{14}, P_{23} are called "contrapoles," it may be shown that the pairs of lines which pass through each of the other two contrapoles P_{12}, P_{34} and pass also through the end points of opposite sides of the pole quadrangle, make angles that are equal. Thus, if the pairs of contrapoles P_{13}, P_{24} and P_{23}, P_{14} are given, the other two contrapoles P_{12}, P_{34} must lie on a curve which is the geometrical locus of all points from which the opposite sides of the contrapole quadrangle appear under equal angles or under angles that are supplementary to each other, that is, form together two right angles. The curve on which the third pair of contrapoles lies has been called by Burmester "Pollagenkurve," that is, "pole-location curve." By using the foregoing property, the pole-location curve may be constructed easily. It is also a "circular cubic," in fact, it is identical with the center curve.[5]

Two further curves are of importance in quantitative kinematic synthesis, namely, the so-called R curves which have been introduced by H. Alt (34), and must be mentioned here. These are curves of those points of system S_1 which, together with their three homologous points in the other three systems S_2, S_3, S_4, lie on circles of a given radius. There exist again two curves, the R_1 curve in system S_1, which is identical with those in systems S_2, S_3, S_4, and the R_c curve which lies in the center system. Alt has shown also, that in the case of three systems, the R_c curve is a tricircular curve of the sixth order. There exist still further curves which are of importance as geometrical loci when a system, during the cycle of its motion, shall come to rest for a certain length of time, but these can only be mentioned here.

These are the elements of which quantitative kinematic synthesis, or size synthesis, makes extensive use. The idea in size synthesis, as will have become clear, then, is to find geometrical loci for points in the given system, which fulfill the given conditions. How this principle works will become clear from the following section.

4 *Five Finite Positions.* In the case of five given positions of system S, or of five identical systems S_1, S_2, S_3, S_4, S_5, the total number of rotopoles is ten. The relations of these ten poles to one another are already so complex that it is unlikely a direct solution ever will be found. An indirect solution of the problem is quite simple, however. For it is only necessary to consider first the four systems S_1, S_2, S_3, S_4 and to determine for them the center curve, and then to consider the four systems S_1, S_2, S_3, S_5 and to determine their center curve. Both center curves are cubics, and, therefore, they intersect one another in nine points. Since in the determination of both the same three systems S_1, S_2, S_3 were used, the rotopoles P_{12}, P_{13}, P_{23} must lie on both curves; i.e., the curves meet in these points. In addition, they have the two imaginary circular points at infinity in common, hence, the two center curves must intersect one another still in four points. These four points are the four "Burmester points" which were previously mentioned. They may either be all real, two real and two imaginary, or all four imaginary. Hence, in five given systems, there exist four groups of homologous points which lie on the four circles, the centers of which are the four Burmester points. Since five systems S_1, S_2, S_3, S_4, S_5 may be combined in five different ways to form groups of four, there exist five center curves, one of which belongs to each of the five systems, and these five center curves intersect one another in the four Burmester points. To each of these five center curves, there corresponds, in each system, a circle-point curve so that there exist also five such circle-point curves. Thus, numerous checks are possible.

While Burmester had utilized these curves only for the construction of approximate straight-line motions, Alt has used them for numerous other problems (34), some of which were indi-

[5] Most of the foregoing results are due to L. Burmester, see reference (33).

cated hereinbefore. The range of problems that can be solved by means of these various curves and others which could not be explained here is steadily extending.

5 *Other Advances in Size Synthesis.* In projective geometry, many new relations can be obtained from known results by the application of the "principle of duality," which co-ordinates to a given point a line, and to the line through two given points two lines which intersect in a point, etc. The well-known theorems of Pascal and Brianchon regarding conic sections are such that each can be obtained from the other by the principle of duality. Since the relations found by Burmester and Alt for the pole triangle and contrapole quadrangle, etc., for three and four positions, referred to points, to the lines which pass through them, and to points referred to these lines, it appeared that further results might be obtained by the use of the principle of duality. This idea has to be credited to R. Beyer (35), who, by replacing points (rotopoles) and principal lines by lines and points, obtained a considerable number of new theorems, thus advancing size synthesis considerably. By further investigating the properties of the pole triangle in the case of three given positions of a system, he found that this system could be made to pass through these positions by the use of Cardano's problem, that is, by the rolling of a circle inside a circle of twice the diameter. Thus numerous practical solutions become available to the designing engineer. As regards four given positions of the system, Beyer finds that the circles circumscribed to the four pole-triangles created by the six rotopoles, intersect in one and the same point, and that the straight lines corresponding to the "base line" pass through this point which lies on Burmester's center curve. This fact suggests at once a simple way of passing the system through the four given positions by means of a double-crossed slider chain, and leads to the simple construction of mechanisms in which one line comes to rest for a certain length of time during the cycle of the motion.

By considering three infinitely near positions, Beyer finds that the well-known properties of the inflection and cuspidal circles can be derived easily from the theory of three 'finite positions." The consideration of four infinitely near positions leads him to the discovery of the dual counterpart of "Ball's point," and this fact, again, can be utilized for the construction of a mechanism, in which one link remains at rest for some part of the motion cycle.

The question of five and more given positions is approached by Beyer in an entirely new way. Instead of using, as Burmester did, homologous points which lie on a circle, he considers homologous points which lie on an ellipse. Since an ellipse is determined by five given points, it follows that, through five given homologous points, an ellipse can always be passed. In general, a sixth homologous point will not lie on the ellipse, but, following Burmester's reasoning, the question may be asked if there do exist points in the system (position 1 of it, or system 1) which, together with their five homologous points (systems 2, 3, 4, 5, 6), lie on an ellipse, and, furthermore, on what kind of curve these points in system 1 lie. Beyer shows that there exist such points and that they lie on a C_1 curve which, in general, is of the eighth order. He shows further that this C_1 curve passes through the five rotopoles P_{12}, P_{13}, P_{14}, P_{15}, P_{16} and through the ten symmetrical poles P^1_{23}, P^1_{24}, P^1_{25}, P^1_{26}, P^1_{34}, P^1_{35}, P^1_{36}, P^1_{45}, P^1_{46}, P^1_{56}, so that the curve can be drawn fairly accurately. However, no general geometrical construction of this C_1 curve has been given so far. For several special cases, Beyer has derived simple constructions.

The value of these investigations lies in the fact that the ellipse can easily be produced by mechanical means, and Beyer shows by several examples how this can be accomplished. In some cases, the degree of movability is limited to the length of the elliptic arc as far as it follows the chosen crank circle closely. In such cases, oscillatory motions only can take place. However, he shows further how a curve can be passed through six given points by means of an ellipse generated by an isosceles slider crank.

Beyer gives also a brief indication of how seven positions may be handled. This can be done in a manner similar to that used by Burmester for five points, by determining two C_1 curves, one for the positions 1, 2, 3, 4, 5, 6 and the other for the positions 1, 2, 3, 4, 5, 7. The intersections of both C_1 curves, which do not coincide with the rotopoles common to both, are points of the system in position 1, which, together with their six homologous points, lie on a conic section. These points play, therefore, a role somewhat similar to that played by the "Burmester points."

Naturally, it has not been possible to give here more than just an indication of the problems with which size synthesis deals, and of the methods used in solving these problems. The fragmentary birds-eye view given will show, however, that this is virgin soil which, hitherto, has not been touched at all in the English-speaking countries, and to make this clear was the purpose of this brief exposé.

VI—Space Kinematics

The foregoing account of the geometrical methods used in plane kinematics, probably, will have shown that this field, in spite of its great simplicity, is a very extended one and is still growing. Vastly greater, however, is the field of kinematics in three-dimensional space or, as it is called simply, of "space kinematics." No attempt will here be made to give even an idea of what this branch of kinematics comprises. Only a few remarks regarding the type of the methods used in this field will be given to show the present status of approach.

Until about 15 years ago, the methods used for dealing with displacements, velocities, and accelerations in space were almost entirely analytical methods. While, in plane kinematics, the graphical methods had shown themselves to be greatly superior to analytical methods, because of their lucidity and general simplicity, only few attempts had been made up to that time to use graphical methods also in space kinematics. In this field, however, the great advantage of the graphical methods over the analytical methods, namely, their simplicity, disappears to a large extent because of the fact that, in general, two or three views (projections) have to be used to obtain the desired results. This requires a great deal of drawing work and careful planning of the investigations if they shall not become so complex as to be useless for practical purposes. For this reason, space kinematics has rather made use of analytical methods which are clearly outside the province of the practical engineer. Although much useful work has been done by means of these analytical methods, particularly by the French kinematicians G. Königs, R. Bricard, and others, their work has hardly become known in engineering circles due to the use of these very methods, because these do not lend themselves readily for applications to practical problems.

There exists however, a method, by which the geometrical methods of plane kinematic geometry can be made use of in a simple manner. W. Hartmann (36), at the suggestion of this author, has used, for this purpose, the isometric projection, and has obtained in that way, together with the author, very interesting results.

Essential progress was brought about by the introduction, into space kinematics, of new graphical methods which were first used in graphic statics in space, namely, the method by B. Mayor (37) and R. v. Mises (38), in which, to each vector in space, a vector in the plane is correlated so that the space vector may be said to be represented in all essential characteristics by a

plane vector. By using this method, K. Federhofer (Vienna) started a graphical kinematics and graphical kinetostatics of the rigid spatial system (39). The problems dealt with by him may also be solved in a simple manner by the construction of the moment vector and of the scalar product of two vectors as given by R. Beyer (40), who used for this purpose methods of descriptive geometry. Another method that may be used is that by R. Mehmke (41), who, starting with the plan view and side elevation, obtains the various vector values by further simple projections and the drawing of parallels.

As stated, all these methods have the common purpose of trying to obtain, with the minimum amount of drawing work, for a vector quantity in space, a representation in the plane, true as to magnitude and direction. In this, these methods have been successful but, nevertheless, they are, of necessity, much more complex than those of plane kinematic geometry. Although much has already been accomplished by these methods, still much more investigation work will have to be carried out before all the problems that, previously, have been solved by analytical methods can be solved with ease by these new graphical methods.

This is all that space permits the author to mention here about space kinematics. A review of the results accomplished so far in this field would necessitate a still more extended paper than the present one.

By the present paper, the author hopes to have fulfilled his promise of giving the average engineer an idea what modern kinematics consists of and how its problems can be approached.

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Discussion

A. H. Candee.[6] Comments of the writer are based upon more than 20 years of experience in applying mathematics and kinematics to the development of methods, machines, and tools for generating gear teeth. In this field, the geometrical relationships of moving surfaces and curves must be exactly known, and a high degree of dimensional accuracy is required.

The author has performed a valuable service in providing a review of what is contained in a numerous list of references

[6] Mechanical Engineer, Gleason Works, Rochester, N. Y. Mem A.S.M.E.

most of which are little known in this country. This paper will undoubtedly rekindle the discussion previously started, and should lead to beneficial results.

The principal points in the following comments are:

1 That the geometrical relationship of centers of curvature referred to in the paper as the quadratic transformation should be credited to Prof. Robert Willis.

2 That rolling curves, or polodes, should be considered in the same class with other curves which transmit motion by contact, and which generally both roll and slide.

3 That an important addition to the theory and practice of gearing has been made by the application of methods of investigation not described in the review of modern European kinematics given by the author.

The great importance previously ascribed to the relationship referred to as the "quadratic transformation" especially excited the writer's curiosity. Now that a description has been provided, this relationship is recognized as one long known but merely obscured by an unfamiliar name. The writer agrees wholly with the author that this relationship of centers of curvature should be taught in every course of instruction in kinematics.

The relationship of centers of curvature in gearing was first brought to the writer's attention by his associate Mr. Ernest Wildhaber, who had derived it by using a method of infinitesimal displacements. Afterward, the writer arrived at the same result by applying the theorem of three centers, and also by the equivalence of triangular areas. Still later, the complete geometrical anticipation was discovered in the old classic on kinematics by Willis.[7] Willis' Fig. 109 and test (reproduced herewith as Fig. 5) show the location of centers of curvature for gear profiles. He stated that the original suggestion came from Euler.

In the author's paper, a derivation is given with reference to his Fig. 2, which is based upon velocity vectors. No reason is indicated for introducing the concept of velocity into a relationship which can be shown most simply and directly by geometry.

In addition to the complete geometrical anticipation by Willis, there is a similar diagram in Reuleaux's work.[8] Centers of rotation of two gears and the construction to locate the centers of curvature of cycloidal gear teeth are shown.

Another reference showing the same basic diagram is contained in an article by H. E. Merritt.[9]

A fourth reference is to Stewart and Wildhaber.[10] Fig. 4 in this article shows the centers of curvature and the direction of motion of the point of contact in hypoid-gear teeth.

From the manner in which the author has presented what is called the quadratic transformation, a reader is likely to suppose that it is a rather recent discovery. The reference that has been given to Willis, however, shows that it is more than a hundred years old. The author has been very meticulous, and rightly so, in crediting Aronhold with the theorem of three centers, although in English texts it has usually been named after Kennedy. He will undoubtedly give Willis due credit for being first to present the geometrical relationship of centers of curvature, unless an earlier showing by someone else is found. The writer hopes, however, that instead of the mathematical term "quadratic transformation," one more natural to kinematics will ultimately be adopted; perhaps something like "curvature diagram."

A concept of curvature in the general theory of gearing, which has been very useful to the writer, is partially illustrated in Fig. 6 of this discussion. This is for plane motion only and may be described further as follows:

> Motions of rotation, at a varying ratio as well as at a constant ratio, may be transmitted by the contact of tooth profiles which, in general, roll and slide against each other.
>
> The sliding component between the profiles decreases to zero when the point of contact is on the line of centers.
>
> Pitch curves which transmit the same motion as the profiles, but which roll only and are called polodes in the paper, are merely a special case of the general group and are characterized by having the point of contact remain on the line of centers.
>
> The relationship of centers of curvature can be shown geometrically by applying the theorem of three centers, for all pairs of curves giving the same instantaneous motion.

The theorem of three centers is the basic tool, and the relationship of centers of curvature can be derived directly by it. The most general relationship is that of the relative curvatures, not only for the case of constant-velocity ratio, but also for a varying ratio. A problem which the writer has investigated is as follows:

Given any pair of gear profiles which produce rotations at a varying velocity ratio; required a pair of pitch curves giving the same instantaneous motion. By applying the relationships under discussion, the point of contact of the pitch curves, their common normal, and their relative curvature are readily obtained. There is then an infinite number of pairs of pitch curves which satisfy the requirements of the problem. A particular pair can be determined if additional information is stated in the original data, such as the length of the radius of curvature of one of the desired curves. The point of these remarks is that, in gearing, pitch curves and profile curves are treated by the same methods.

The reference in the paper to "cardioidic motion" was interesting, because the writer has used and patented devices which utilize such motion in guiding a diamond to form curved outlines on a grinding wheel. The theorem of three centers provided the basis for adjusting the moving parts to obtain curvatures of the wanted amounts.

The distinction between applied kinematics and kinematic synthesis is not very clear. The combining of known parts and mechanisms to obtain a required result appears to be described by both terms.

In regard to the emphasis usually placed upon the analytical method, the writer was once accused of being unscientific for recommending the geometrical and kinematic solution for radius of curvature. This was by an investigator who had received his basic education in Europe and is a thoroughly proficient mathematician. Perhaps it is more a matter of what subjects an individual happens to have selected for study than of where his education was received.

In generating bevel gears in which the teeth are oblique and curved, the problems encountered concern the relations of surfaces; that is, they are problems of space kinematics. Plane sections and the equations of curves are usually entirely inadequate for obtaining solutions. In some cases, spherical geometry has been used to advantage. The theorem of three centers and the "curvature diagram" have been applied on the spherical surface in the same way as in the plane.

The methods which are of greatest use, however, are one based on infinitesimal displacements in space, and another based upon the curvatures and tangency of surfaces. These are due to Ernest Wildhaber who has been previously mentioned, and were

[7] "Principles of Mechanism," Robert Willis, second edition, Longmans, Green & Company, London, 1870, pp. 129–132, and Fig. 109. The first edition was published in 1841.

[8] "The Constructor," Franz Reuleaux, translation from fourth enlarged German edition, by H. H. Suplee, Philadelphia, Pa., 1894, Fig. 577, p. 131.

[9] "The Art of Gear Design—Part VIII," by H. E. Merritt, *The Engineer*, vol. 162, Sept. 4, 1936, p. 222, Fig. 51.

[10] "Design, Production, and Application of the Hypoid Rear-Axle Gear," by A. L. Stewart and E. Wildhaber, Journal S.A.E., vol. 18, 1926, p. 577, also *American Machinist*, vol. 64, 1926, p. 857.

to be assigned to a toothed wheel of a given magnitude, and proportionably reduced the length of their acting sides, so that the circular approximation was rendered practically possible.

Perceiving this fact, I endeavoured in 1838 to follow out the views suggested by Euler's paper, and finally succeeded in discovering a practical method of finding a pair of centers with appropriate radii, for any given pair of wheels, by means of an instrument which I denominated an *Odontograph*. This instrument dispenses with all geometrical calculations and has been extensively employed in practice from the time of its publication in my paper 'On the Teeth of Wheels' in the *Transactions of the Institution of Civil Engineers*, vol. ii. 1838. The substance of that communication occupies the following pages.

Fig. 107.

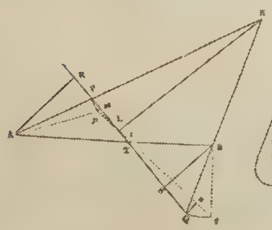
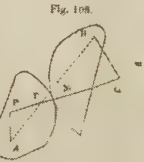


Fig. 108.



175. A simple construction is sufficient to give the centers and radii of the arcs in any required case. For it has been shown (Art. 30, Cor. 5) that the action of a pair of curves by contact is equivalent at every moment to that of a pair of radii AP, BQ (fig. 108) connected by a link PQ, P and Q being the respective centers of curvature of the curves at the point of contact. Now (fig. 107) the angular velocity ratio between the radii AP, BQ is that of the segments $BT : AT$, into which the link divides the line of centers (Art. 32); and if the rods be moved into a new position, this ratio becomes $Bt : At$, which is greater or less than the former, according as the point t moves to one side or other of the point T.

But if the point L, which is the intersection of two successive

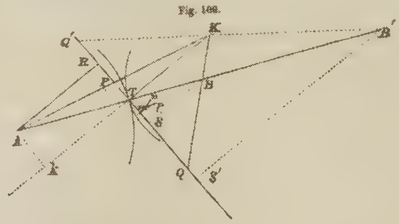
positions of the link, happen to coincide with T, the ratio of the segments will be the same in both positions, and the angular velocity ratio constant at that instant.*

If then the rods and links of fig. 107 be placed in such a relative position that L and T may unite, and the curves in contact be replaced by arcs of circles described from centers P and Q through *any point* M of the line PQ, the angular velocity ratio of these curved pieces will be perfectly constant at the moment of their reaching the position that makes M the point of contact, and the ratio will not vary essentially during a small angular motion on each side of this position.

176. As this constancy of the velocity ratio depends only upon the centers of the arcs, they may be struck through any common point of the line of action PQ, as at m, beyond both the centers. Only that if this point lie between the centers P, Q, as at M, the arcs and edges will be convex, but if the point lie beyond the centers, as at m, the edge corresponding to the most distant center P, will be concave.

177. It follows, that to find a pair of centers that possess the property of communicating motion in a constant velocity ratio, it is only necessary to construct the diagram (fig. 107) in such a manner, that the point L shall fall on the line of centers. But (by Art. 30, Cor. 5), L is that point of PQ which is met by a perpendicular from K, the intersection of the directions of the radius rods AP, BQ. Whence the following construction.

Fig. 109.



Let A, B be the centers of motion of the wheels, T the point of contingence of the pitch circles; through T draw PTQ

* *Vide* note, p. 18, above.

x 2

making any angle with the line of centers, and upon it assume P as a center, from which the circular side is to be described for a tooth of a wheel whose center of motion is A. To find the corresponding center for the wheel which turns upon B, draw TK perpendicular to PTQ, produce AP to meet it in K, join KB and produce it to meet PTQ in Q; then will Q be the required center.

And a small arc mn, struck from P as a tooth for the wheel whose center of motion is A, will work correctly with an arc mp, struck from Q through m, and employed as a tooth to the wheel whose center of motion is B.

If B be so placed that the angle KBT is acute, as for example at B', then will Q fall at Q' on the same side of T as P, but beyond it; the effect of this is to make the tooth mp concave instead of convex.

But if the angle $KBT = PTA$, KB will become parallel to PT, and the point Q being thus removed to an infinite distance, the arc mp or tooth of the wheel whose center of motion is B, will be a right line perpendicular to PT.

178. The distance of the centers from T may be calculated as follows.

Draw AR perpendicular to PT.

Let $KT = C$, $AT = R$, $PT = D$, $ATP = \theta$, then by similar triangles, ARP, PTK,

$$KT = \frac{PT \times AR}{PR} = \frac{PT \times AR}{TR - PT}$$

$$\text{or } C = \frac{DR \,.\, \sin\theta}{R \,.\, \cos\theta - D}; \ \therefore D = \frac{RC\cos\theta}{C + R\sin\theta}$$

and similarly, drawing BS perpendicular to TQ, and putting

$$BT = r,\ QT = d,$$

we have for the corresponding arc mp,

$$d = \frac{rC\cos\theta}{C + r\sin\theta}$$

But if a concave tooth be employed, draw $B'S'$ perpendicular to PTQ, then

$$KT = \frac{QT \times B'S'}{QT + TS'}, \text{ whence } d = \frac{Cr\cos\theta}{r\sin\theta - C}$$

179. If the side of the tooth be made to consist of a *single arc*, a very simple rule may be obtained; for suppose KT to be

infinite, then will AP and BQ become perpendicular to the line PTQ, and the points P, Q will come to R, S respectively. Let the arcs of the teeth be struck through T, let θ be the angle ATP, which the line PTQ makes with the line of centers, and let R be the radius AT of the wheel, and $D = TR$ be the required distance of the center of the tooth from the point T;

$$\therefore D = R\cos\theta$$

is independent of the wheel with which it is to work, as well as of the pitch and number of teeth of its own wheel.

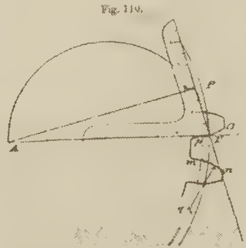
If therefore θ be made constant in a set of wheels, any two of them will work together, and their teeth are easily described as follows. Assume $\theta = 75° \, 30'$, which is a very convenient value;

$$\therefore D = \frac{R}{4};$$

for $\cos 75° \, 30' = \cdot 25038 = \frac{1}{4}$ very nearly.

180. Let A be the center, AT the radius of the pitch circle of a proposed wheel. Draw TP making an angle ATP of $75° \, 30'$

Fig. 110.



with the radius, and drop a perpendicular AP upon TP (or describe a semicircle upon AT and set off $TP = \frac{AT}{4}$), then will P be the center from which an arc sp, described through T, will be the side of the tooth required.

Or more conveniently, let a bevil of 75° 30′ be made of brass or card-paper, as in the figure, of which the side TP is graduated into a scale of quarter-inches and tenths. If this bevil be laid

FIG. 5 REPRODUCTION OF PAGES FROM WILLIS'[7] "PRINCIPLES OF MECHANISM," SHOWING FIG. 109, AND TEXT GIVING LOCATION OF CENTERS OF CURVATURE FOR GEAR PROFILES

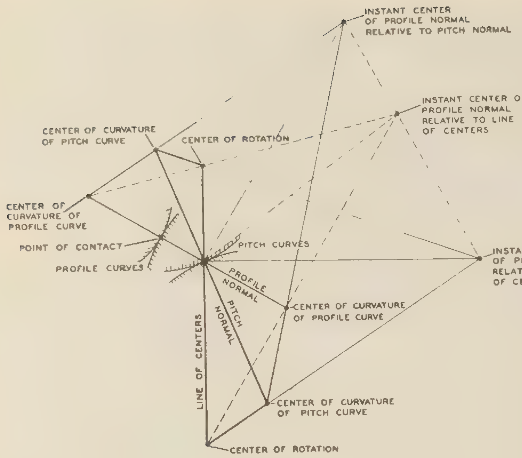


Fig. 6 Kinematic Elements in Toothed Gearing and Instantaneously Equivalent Linkages

(Profile curves, roll and slide, general case; pitch curves, roll only, special case.)

employed by him in solving the problem of gears with offset axes.

The attempts contained in textbooks and published articles known to the writer, to describe tooth surfaces for gears with axes not in one plane, other than worm gears, have all been incomplete and largely incorrect. They were limited to surfaces containing straight-line elements, and failed even to suggest that curved teeth are possible.

At the Gleason Works, beginning about 1925, Mr. Wildhaber has solved the whole problem, for teeth in any direction and of any curvature (straight, circular, or otherwise). Although the helicoidal basic member for hypoid gears, which corresponds to the basic rack for cylindrical gears and the basic crown gear for conical gears, had been partially described by others, he was the first to provide a complete and correct solution, not only for straight, inclined teeth, but also for curved teeth. This was accomplished by means of the methods which have been mentioned.

This work, in the opinion of the writer, represents the only basic advance in the theory of gearing in a great many years. It has been described only generally, and it is in a specialized field. Millions of hypoid gears are in use in automobiles, however, and many are installed in industrial equipment. In spite of this, very little notice of the successful generation of teeth in hypoid gears is taken in recent textbooks of kinematics, which at most merely mention that gears for offset axes are being made.

In addition to the Stewart and Wildhaber paper already mentioned, general descriptions and illustrations of hypoid gears are to be found in two papers by the present writer.[11]

M. E. Martellotti.[12] The author has presented an interesting review of the progress made in kinematics. He has indicated the trend in the treatment of kinematical problems and has pointed out the importance of kinematics in engineering.

He has also called attention to the fact that most of the terms now used, particularly in the English literature and textbooks, in defining kinematical elements are incorrect, and he has suggested that new terms should be devised to describe more appropriately and with a certain degree of brevity the intrinsic characteristic properties of these elements.

The author should be congratulated for bringing to the attention of his fellow engineers this rather neglected branch of mechanics. This paper will stimulate the interest of engineers on this subject, and this activity might well be co-ordinated by the Society in a division of applied and theoretical kinematics.

The author seems to prefer the geometrical or graphical method in studying problems in kinematics. The graphical method, however, is to a certain degree satisfactory in treating problems of plane kinematics, but fails to be equally valuable in the treatment of problems in space kinematics, as the author has stated in his paper.

Kinematics is but a branch of mechanics. This is the science of motion, and since motion involves space and time, it implies geometry as a necessary premise to which the concept of time must be added.

When analyzing variations in the geometric characteristics of a system of points, conceived either as rigid or deformable, in accordance with hypotheses suggested by the observation of motion of natural bodies and without regard to the forces and masses involved therewith, we refer this study to that branch of mechanics known as kinematics.

In a preliminary study of mechanical problems, we may be interested in the geometry of motion or change of position of a body without consideration of the temporal law under which these new positions are attained. In this case, the concept of time may or may not be used. This is a useful procedure in technical applications when it is desired to study the rigid motion in a plane, for instance, of a body relative to another, which is fixed.

In any event, if relations are established, they must of necessity be reduced to the analytical form, for this finally permits us to correlate in a formula or equation the various relations of the elements involved, and eventually to further the study of the dynamics of the bodies or of the mechanism under consideration.

Thus, the analytical and the geometric or graphical method of treatment of problems in kinematics are almost invariably jointly used in arriving at a solution of a given problem.

The author has taken particular exception to the analytical method used by the writer in the derivation of the radius of curvature of the milling-cutter-tooth path,[13] and he claims that the graphical derivation of the radius of curvature is simpler and more direct than the analytical one.

In this particular case, the writer does not fully agree with the author.

In deriving the radius of curvature of a milling-cutter-tooth path, the writer has used the following equation:[14]

$$\rho = \frac{\left[\left(\frac{dx}{d\alpha}\right)^2 + \left(\frac{dy}{d\alpha}\right)^2\right]^{3/2}}{\frac{dx}{d\alpha}\cdot\frac{d^2y}{d\alpha^2} - \frac{d^2x}{d\alpha^2}\cdot\frac{dy}{d\alpha}} \quad \ldots\ldots\ldots\ldots [8]$$

which is a more general equation than

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx}} \quad \ldots\ldots\ldots\ldots [9]$$

given by the author.

[11] "Large Spiral and Bevel Gears," by Allen H. Candee, Trans. A.S.M.E., vol. 51, 1929, MSP-51-9, pp. 59–68.

"Industrial Applications of Spiral Bevel Gears and Hypoid Gears," by Allen H. Candee, Trans. A.S.M.E., vol. 60, 1938, pp. 549–560.

[12] Research Engineer, The Cincinnati Milling Machine Co., Cincinnati, Ohio. Mem. A.S.M.E.

[13] "An Analysis of the Milling Process," by M. E. Martellotti, Trans. A.S.M.E., vol. 63, 1941, pp. 677–700.

[14] As mentioned in a private communication dated March 22, 1941, to Professor de Jonge.

Equation [9] is a special case of Equation [8] of this discussion and can be derived from the latter by making

$$\alpha = x$$

Equation [9] is used when the equation of the curve is of the type indicated by the following expression:

$$y = f(x)$$

which expresses a direct relation between the Cartesian co-ordinates; while the more general form Equation [8] of the radius of curvature applies to the case in which the equation of the curve is expressed as a function of the parameter, as is indicated in the equations

$$x = \varphi(\alpha)$$
$$y = \psi(\alpha)$$

expressing a relation between the co-ordinates of a point of the curve in terms of α, known as the parameter.

The writer has given the equation of a milling-cutter-tooth path in the following parametric form (Equation [2] of the paper[13] referred to)

$$\left.\begin{aligned} x &= r\,\alpha + R \sin \alpha \\ y &= R - R \cos \alpha \end{aligned}\right\} \qquad [10]$$

and he has used Equation [8] of this discussion for the derivation of the expression of the radius of a curvature. This was done in accordance with the following procedure:

$$\left.\begin{aligned} \frac{dx}{d\alpha} &= r + R \cos \alpha \\ \frac{dy}{d\alpha} &= R \sin \alpha \end{aligned}\right\} \qquad [m]$$

$$\left.\begin{aligned} \frac{d^2x}{d\alpha^2} &= -R \sin \alpha \\ \frac{d^2y}{d\alpha^2} &= R \cos \alpha \end{aligned}\right\} \qquad [n]$$

Squaring Equations [m]

$$\left.\begin{aligned} \left(\frac{dx}{d\alpha}\right)^2 &= r^2 + R^2 \cos^2 \alpha + 2r\,R \cos \alpha \\ \left(\frac{dy}{d\alpha}\right)^2 &= R^2 \sin^2 \alpha \end{aligned}\right\} \qquad [o]$$

and adding the results, it follows that

$$\left(\frac{dx}{d\alpha}\right)^2 + \left(\frac{dy}{d\alpha}\right)^2 = r^2 + R^2 + 2r\,R \cos \alpha \qquad [p]$$

which is the numerator of Equation [8] of this discussion.

By multiplying Equations [m] and [n], as indicated in the denominator of Equation [8], the following result is obtained:

$$\left.\begin{aligned} \frac{dx}{d\alpha} \cdot \frac{d^2y}{d\alpha^2} &= r\,R \cos \alpha + R^2 \cos^2 \alpha \\ \frac{dy}{d\alpha} \cdot \frac{dx^2}{d\alpha^2} &= -R^2 \sin^2 \alpha \end{aligned}\right\} \qquad [q]$$

The expression for the denominator of Equation [8] is now obtained by subtracting from the first, the second Equation [q]; hence

$$\left(\frac{dx}{d\alpha} \cdot \frac{d^2y}{d\alpha^2}\right) - \left(\frac{dy}{d\alpha} \cdot \frac{dx^2}{d\alpha^2}\right) = r\,R \cos \alpha + R^2 \qquad [r]$$

Substituting in Equation [8], the expressions obtained in Equations [p] and [r], it is found that

$$\rho = \frac{(r^2 + R^2 + 2r\,R \cos \alpha)^{3/2}}{r\,R \cos \alpha + R^2} \qquad [s]$$

But from Equations [10] of this discussion

$$\cos \alpha = \frac{R - d}{R} \qquad [t]$$

where d indicates the instantaneous depth of cut in place of y.

Therefore

$$\rho = \frac{[r^2 + R^2 + 2r(R - d)]^{3/2}}{r(R - d) + R^2} \qquad [11]$$

Thus the equation for the radius of curvature has been determined in a straightforward manner and without resorting to the additional steps which are required when following the graphical method.

Author's Closure

It is extremely discouraging that only two discussions of the author's paper were received by The American Society of Mechanical Engineers. These two discussions were presented by two practical engineers, Messrs. H. Candee and M. E. Martellotti, but none has been received from college professors or instructors. This fact seems to confirm the contention of the author, expressed in his previous paper (1)[15] and in his closing remarks thereto, that college professors either are ignorant of the modern development of kinematics and are unable, on that account, to discuss it, or that they do not care about the subject under discussion and hence their silence. This indifference is particularly unfortunate, because a valuable opportunity has been lost for eliciting further information from the author.

Mr. Candee's remarks require definite answers since he makes certain statements and levels some accusations against the author. The principal points of his comments, which shall here be taken up in detail, are as follows:

1 "That the geometrical relationship of centers of curvature referred to in the paper as the quadratic transformation should be credited to Robert Willis.

2 "That rolling curves, or polodes, should be considered in the same class with other curves which transmit motion by contact, and which generally both roll and slide.

3 "That an important addition to the theory and practice of gearing has been made by the application of methods of investigation not described in the review of modern European kinematics given by the author."

Mr. Candee continues: "The great importance previously ascribed to the relationship referred to as the 'quadratic transformation,' especially excited the writer's (Mr. Candee's) curiosity. Now that a description has been provided, this relation is recognized as one long known but merely obscured by an unfamiliar name. The writer (Mr. Candee) agrees wholly with the author that this relationship of centers of curvatures should be taught in every course of instruction in kinematics."

It is very unfortunate that, apparently, Mr. Candee did not study the author's paper very carefully, for had he done so, the author believes, he would scarcely have written as he did. Like

[15] Numbers in parentheses refer to the Bibliography at the end of the author's paper.

some of the discussers of the author's previous paper (1), he, too, is guilty of building up a straw man for the purpose of knocking him down.

To prove this, let us consider Mr. Candee's third point first.

As stated in the final sentence of the second paragraph of the paper, the author limited himself to reviewing only the motion of plane systems. That, at the end of the paper, he has briefly touched upon some new general and fundamental methods now used in Europe extensively for investigating space systems, but not known in this country, was done simply to show that the space branch also has been improved considerably. Thus, Mr. Candee's third point, which refers to a highly specialized branch of kinematics in space, namely, to hypoid gearing, is disposed of as being strictly outside the scope of the subject matter discussed, notwithstanding the fact that the methods used are methods of plane kinematics. There are many other similar applications of plane kinematics to space kinematics, none of which has been mentioned by the author on that account.

Let us now take up Mr. Candee's points 1 and 2. In point 1, he wishes the author to give due credit to Willis for having originated the "quadratic transformation."

Mr. Candee either does not seem to have read the author's definition in the third sentence of Part 1 (B), or he does not seem to have understood it properly. The definition of the quadratic transformation refers to a "reciprocal relationship" which not only correlates to any point in the moving plane system a definite point in the fixed plane system, but, at the same time, also correlates to this point in the fixed plane system the original point in the moving plane system. This is something quite different from finding the correlated centers of curvature of two tooth profiles in contact with each other, as described by Willis. It is true that, by a coincidence, these centers are also points of the quadratic transformation, but Willis neither referred to this fact in a general way, nor in any specific way other than having used the construction derived by him for obtaining the centers of curvature of two correlated gear-tooth profiles at the point in which they are in contact with each other. Still more important, however, is the fact that Willis was *not* the first to give a construction for these correlated centers of curvature of gear-tooth profiles in contact with each other. The first was Leonhard Euler who gave a general, although different, construction for this problem in his second paper on gearing.[16] Indeed, in the Transactions of the Institution of Civil Engineers,[17] in which Willis' original paper was published under the title "On the Teeth of Wheels," he himself says the following:

"Euler, in his second paper on the teeth of wheels (N. C. Pet. XI, 209),[18] has with his usual ability investigated the proper curves, by examining the relation between their radii of curvature at every point. This method has naturally conducted him to results of a similar nature to those which I have given in the following pages, and he suggests that a small arc of the circle of curvature would suffice in practice for the form of teeth. He has given some geometrical constructions for this purpose, and has then passed on finally to recommend the involute as the best curve, this paper being, in fact, the first in which that curve is pointed out as possessing the required properties. To Euler, then, belongs the merit of first suggesting the substitution of an arc of the circle of curvature for the real curve, a hint which has been, as far as I know, neglected by every succeeding writer. This may perhaps be attributed to the abstruse manner in which he has treated the subject."

Thus, Willis himself gives Euler credit for having been the first to derive a geometrical construction for obtaining the centers of curvature for the arcs of two correlated gear-tooth profiles in contact with each other. This is not all, however. As stated, nowhere in the paper cited, nor in his book, either in the first edition of 1841, or in the second edition of 1875, has Willis used his construction for the centers of curvature of two gear teeth at the point of contact, also for the determination of the center of curvature of a curve described by a point in the moving system. That he has not done so shows that he was unaware of the fact that it could be used for solving that problem. The same can be said of Euler, with regard to the paper cited.[16] However, while Willis gave no formulas whatever, Euler gave, in addition to his geometrical construction, also an analytical solution. Neither of these authors can, therefore, be credited with having been the first to derive a formula and construction for the radius of curvature of a curve described in the fixed system by a point of the moving system.

The first to have done this was Felix Savary (born October 4, 1797, at Paris, died at Estagel, Pyrenees, on July 15, 1841, that is, prior to the publication of the first edition of Willis' book), a distinguished student of the École polytechnique in Paris. After he had terminated his studies, he was made a professor in this famous institution, for courses in geodesy and in machinery, to the founding of which latter subject he had greatly contributed. Later he became a member of the Académie des sciences, and still later, of the Bureau des longitudes. His poor state of health neither allowed him to live a long scientific life nor to publish many papers. However, he will always be remembered by his famous theorem on the curvature of curves, described by the movement of a point connected with a curve rolling upon a fixed curve.[19] It is true that he himself never published this famous theorem, which he had taught his students for many years. However, a lithographic copy of this part of his lectures on gearing was made by his students and was issued with his approval after he had edited it, as will be shown hereinafter. His students used this theorem freely and gave it the name of "Savary equation." Since in Euler's treatise,[16] the equation is contained also, although in a somewhat different and more complicated form, the equation is, nowadays, called the "Euler-Savary equation." Since it relates further to the rolling of two curves, one upon the other, it holds true directly for the inverse rolling motion, and, hence, it is the complete analytical expression for the "quadratic transformation."

It is not quite clear when Savary first discovered this theorem, but it must have been in the late 1820's, or in the early 1830's during his professorship at the École polytechnique. M. Chasles[20] mentioned it in 1845, and used the equation to derive the radius of curvature of the envelope of a curve connected to the moving system, that is, to the solution of the problem previously solved by both Euler and Willis, showing thereby that the two centers of curvature of the curve and its envelope (which may be regarded as two tooth profiles in contact with each other) belong to the quadratic transformation as was stated previously. In fact, he says, in discussing the rolling of one polode upon the other, which he calls "roulette" and "base de la roulette" respectively:

"C'est la considération de ces deux courbes, la roulette et

[16] "Supplementum—De figura dentium rotarum," by Leonhard Euler, Novi commentarii Academiae scientiarum imperialis Petropolitanae (also known under the name of Akademiya Nauk), vol. 11, 1765, pp. 207–231; particularly fig. 6 and sec. 18.

[17] "On the Teeth of Wheels," by R. Willis, Trans. Institution of Civil Engineers (not the Proceedings), vol. 2, John Weale, London, 1838, pp. 89–112; particularly p. 95.

[18] Ref. 16, p. 209.

[19] Grand Dictionaire Universal par Pierre Larousse,Paris, 1865 edition, vol E, 1870, pp. 701–702, and vol. S, 1875, p. 286

[20] "Construction des rayons de courbure des courbes décrites dans le mouvement d'une figure plan qui glisse dans son plan," *Journal de mathématiques pures et appliquées*, vol. 10, 1845, pp. 204–208, particularly p. 205.

sa base, quí va nous servir pour résoudre le problème énouncé. Designons la base par B, et la roulette par A. Celle-ci importe, dans son mouvement, une courbe C; et c'est le rayon de courbure de la courbe enveloppe de l'espace parcouru par cette courbe C, que nous voulons construire à un instant quelconque du mouvement. Il est clair que pour cela nous pouvons substituer au deux courbes A et B leur cercles osculateurs en leur point de contact à cet instant. De sorte que la question se réduit à celle-ci: Un cercle A, roulant librement sur un cercle fixe B, on demande de déterminer le rayon de courbure de la courbe enveloppe de l'espace parcouru par la courbe C mobile avec le cercle A. Or cette question a été traitée par M. Savary dans ses Leçons sur les engrenages*."

*"Il a été fait, d'après la rédaction même de M. Savary, une lithographie de cette partie du Cours de Machines, professé à l'École Polytechnique."

Chasles then carries on, referring to the construction to be used, which he attributes to Savary. He says in fact:

"La construction de M. Savary est exprimée par la relation suivante, . . . " and in giving thereafter the Savary equation, he refers to it in a footnote marked***, which reads:

*** "Ou peut consulter aussi le Traité de Géométrie descriptive de M. Leroy; deuxième édition, 1842, page 384."

The construction used by Savary,[19] although derived in an entirely different way, proved to be the same that Willis found later for the two centers of curvature of correlated tooth profiles that is for a very different purpose. Thus, it is only accidental that Willis' construction is also a construction for the quadratic transformation, and, since the construction was found much earlier by Savary than by Willis, and since the latter did not notice its other possibilities, it is only natural to credit it to the former. Mathematically the construction by Savary or Willis is only a special case of the far more general construction for the quadratic transformation devised by Bobillier (4).

By the foregoing, it is shown that the "quadratic transformation" cannot be credited to Willis, as Mr. Candee has intimated, but must be credited to Savary, exactly as the author stated in his paper by referring to the equation as the "Euler-Savary equation."

Likewise, Mr. Candee's accusation, that the author merely tried to obscure a long known relationship by giving it a new and unfamiliar name, must be rejected emphatically, for, on the one hand, the quadratic transformation refers to a "reciprocal relationship" of a point in the moving system and a point in the fixed system, of which Willis was not aware, and, on the other hand, the term "quadratic transformation" is far over half a century old and occurs in many writings since that period, for example in Allievi's book (5) and many others. Thus, unfamiliarity cannot be blamed upon the term used by the author, but must be laid squarely upon Mr. Candee who, obviously, is unfamiliar with the extensive literature on that subject. Besides, the full meaning of the "quadratic transformation" might have become clear to Mr. Candee had he taken the trouble to study the references given by the author, particularly reference (8).

It is, therefore, not easy to understand how Mr. Candee can make the statement: "From the manner in which the author has presented what is called the quadratic transformation, a reader is likely to suppose that it is of rather recent discovery," for the author has given references (4), (5), and (7), which go back, respectively, to 1870, 1895, and 1765 as stated. That these dates cannot be called "recent" is obvious to every unbiased reader. In addition, there is no reason for changing the long existing term "quadratic transformation" into "curvature diagram," if the full meaning of the former is properly understood. In making the latter proposal, Mr. Candee, obviously, thinks only of the simple geometrical construction, but this is only a means to an end, namely, for finding the quadratic transformation. Apparently, he has not realized this.

Mr. Candee has drawn attention to another reference to lend weight to his argument. He cites Reuleaux's "Constructor" as translated by Suplee in 1894. This translation was made from the fourth German edition of this book, dated 1882–1889. Instead of giving this reference, Mr. Candee might better have referred to Reuleaux's first volume on "Theoretical Kinematics," translated by Prof. Kennedy, in 1876, under the title of "Kinematics of Machinery," in which Reuleaux describes Willis' method as one of seven for obtaining correlated profiles. It should not be lost sight of, however, that Reuleaux, too, used it only for obtaining the centers of curvature of two profiles continuously in contact with one another.

Regarding Mr. Candee's statement that no reason is given for the use of velocity vectors in the derivation of Hartmann's construction of the center of curvature, which the author explained in the text accompanying his Fig. 2, and regarding Mr. Candee's claim that this relationship (of a point describing a curve and the center of curvature of it in that point) can be shown most simply and directly by geometry, it must be said that this is not quite correct for the following reasons:

1 The geometrical construction (and we are here concerned only with a geometrical construction and not with a formula), which has been devised by geometry, is the Bobillier construction, or its special case the Savary-Willis construction, and this construction fails completely for the central position, that is, when the point describing the curve whose radius of curvature is to be found, lies on the line of centers. This is a very serious shortcoming, one which has been keenly felt ever since that construction was devised.

2 The author has stated clearly, at the top of the third page of the paper, that the construction for the quadratic transformation can be derived most readily by projective geometry and has cited sources.

3 In the following paragraph, he has stated that it is *not* necessary to exclude "time,"[21] and that, by introducing it, a truly kinematical solution, namely, the Hartmann construction, can be obtained.

4 The Hartmann construction, and this is of the greatest importance, is really a purely geometrical solution, for it pictures what happens during a differential of the motion that occurs. Since the construction would require the drawing of lines infinitely close to one another, which is impossible, he has simply used an enlargement by affine projection, by multiplying the displacements with the constant factor $\frac{1}{dt}$, to make visible the infinitely close array of lines. In doing this, he has not only derived a simple construction for the center of curvature, which does not fail for points on the center line, but, unconsciously and unknown to himself, has obtained thereby a projective construction which gives a complete representation of the "quadratic transformation." It is therein that the importance of Hartmann's construction lies. It is true that, by this process, the displacements appear in the form of velocity vectors, but what difference does that make? He has thus derived a truly kinematical solution in which the time is present as an element, but far from this being a drawback, it is a distinct advantage for, in all machine problems, time is present with respect to the motion that occurs. Assumptions simplifying

[21] See also the comment made by Mr. Martellotti who, like the author, feels that a kinematical solution which, in addition to geometry, includes time as a factor, is as justified as a purely geometrical solution.

the Hartmann construction still further can be introduced, as is shown in Fig. 3 of the paper and the accompanying text.

Let us now deal with point 2 of Mr. Candee's discussion:

Mr. Candee is correct in assuming that the polodes, or curves which roll upon one another, constitute a special case of all those profiles which both roll and slide upon one another if motion is transmitted by them. One of the important prerequisites, however, is that, for both profiles, the common normal must pass through the rotopole, that is, the point in which the two rolling curves are in contact with one another at the moment considered. By this requirement, the polodes are singled out as special curves. Furthermore, it follows that, for the profiles to act in a driving manner, the normal mentioned must have an inclination to the line of centers about which rotation, if any, of the rolling curves, or polodes, takes place. Consequently, the sliding component at the profiles transmitting motion can vanish only when the point of contact of the profiles, at the same time at which it is on the line of centers, lies also on the rolling curves. This is an important restriction, and Mr. Candee's statement that the sliding component vanishes when the point of contact of the profiles is on the line of centers, has to be modified to the effect that it also must lie in the rotopole of the instantaneous motion. His generalization is, therefore, not correct.

Although the polodes are a special case of the curves which both roll and slide, nevertheless, they form a group by themselves, for, as has been shown clearly in the paper, their centers of curvature at the point of contact cannot be determined uniquely by taking into consideration three infinitely near positions, as in the case of profiles for example, but can be found only by taking into consideration four infinitely near positions. Thus, it can be shown readily that a mechanism, which requires for its constitution the rotopole of two given polodes, or of their osculatory circles, can be obtained also from various other circles, which are the osculatory circles of other polodes than the given ones. Hence, again, Mr. Candee's generalized statement, that the polodes should be considered together with the profiles which both roll and slide, is incorrect and has to be modified to the effect, that although the polodes form a special case of the profiles, they must be treated in an entirely different manner. This is usually done in modern kinematics and was stated thus in the author's paper. In fact, Mr. Candee himself admits that polodes cannot be treated like profiles as will be shown a little later.

Mr. Candee's assertion that the theorem of the three centers is the basic tool and that the relationship of the centers of curvature can be derived directly by it, is fully true, but it has to be kept in mind that, historically, this was not the development which took place first, for the discovery of this theorem by Aronhold occurred only in 1872, and it did not become known in the English-speaking countries until 1886, after it had been rediscovered by Prof. A. B. W. Kennedy in England. That this basic theorem must be the appropriate tool for the solution of the problem of curvatures is evident from the fact that, in this problem, three systems are involved, namely, the moving system, the fixed system, and the system containing the radius of curvature, which connects the other two systems. Hence, the theorem that the three rotopoles of these three systems must be on a straight line applies, but this does not as yet lead to the Savary construction.

Mr. Candee asserts that he has investigated the most general relationship of relative curvatures not only for the case of constant velocity ratio, but also for varying ratios, and, in stating the problem, finds by its solution that an infinite number of polodes ("pitch curves," he calls them) satisfies the requirement of the problem. This is the reference to which attention was drawn a little earlier, and it is exactly what the author previously tried to make clear, but, while Mr. Candee wishes to use this as a proof that polodes, or rolling curves (or pitch curves), can be dealt with by the same methods as profile curves, he himself makes the admission that this is not possible and that "additional information" is required, to obtain the desired polodes, just as the author stated when he showed that polodes require separate and special treatment.

To Mr. Candee, the distinction between applied kinematics and kinematic synthesis is not very clear. No such distinction was made by the author who differentiates between "kinematic analysis," which takes a mechanism for granted and investigates its properties, and "kinematic synthesis," which considers the properties as given and determines the mechanism which will have these properties. This is the usual difference between analysis and synthesis.[22]

Regarding the use of analytical methods or of geometrical methods, it is naturally a question of personal preference, as Mr. Candee points out, and not so much of the country in which the student was educated. In Germany, however, the general preference has been to use geometrical rather than analytical methods, because the former generally give simple and valuable checks which guarantee the correctness of the solution, while this is not the case with the latter methods.

To sum up, it has been shown that:

1 Savary, and not Willis, was the first to give a general geometrical construction and formula for the quadratic transformation.

2 The polodes, although being a special case of the profile curves, must be treated separately because of their vastly different properties.

The answer to item 3 of Mr. Candee's discussion, which was strictly outside the scope of the paper, could not be given here. In any paper on "space kinematics," it would be proper to discuss, under the section of gears in space, Mr. Wildhaber's method dealing with hypoid gears by developing the gear as well as the pinion into a plane and treating it by plane methods of kinematical analysis.

To Mr. Martellotti, the author is grateful for the kind words he has said about the author's efforts to bring a very neglected field of engineering again to the attention of engineers in this country and to stimulate interest therein. The author is indeed gratified to find at least a few who appreciate his views and the value of his remarks, and who go even as far as to suggest that the Society might well co-ordinate this field of endeavor in a division of theoretical and applied kinematics. From his previous experience with the pronounced conservatism in this country, the author has not dared to suggest such a course, but he is endeavoring to create a division for "Design" (or "Design Division"), which would have mechanisms, kinematics, and dynamics as important branches of its activities. If farsighted engineers like Mr. Martellotti and many others who have approached the author with similar requests, would support such a venture, the Society would be enriched by a very important and worth-while activity to the advantage of both, itself and its members. The author takes this opportunity of issuing an appeal to all mechanical engineers interested in design or design problems, to support this new venture which, in due course, will be brought to their attention through the usual channels by the Society.

[22] It is unfortunate that Mr. Candee was not present at the meeting at which the author presented the paper. At this meeting, the author exhibited further slides which showed, in a number of cases, how such mechanisms can be determined from conditions which seem almost too complicated to be met with by any mechanism. Due to lack of space, these examples cannot be repeated here.

Mr. Martellotti seems to be in full agreement with the author on most points, such as the importance of kinematics to engineering, the publication of the new trends in kinematics, the creation of a simple, uniform, and expressive terminology instead of the cumbersome terminology still in use at the present time, and the fact that kinematic methods are just as useful as are purely geometrical methods (as advocated, for instance, by Mr. Candee).

On one point, however, Mr. Martellotti's ideas and those of the author seem to differ somewhat, namely, on the value of geometrical methods as compared with analytical methods. It is true that, historically, this science has been advanced frequently by analytical methods, but almost invariably much simpler geometrical methods were discovered thereafter. This has been due not so much to the inherently greater value of the analytical methods, as to the training which the particular investigator had received. There have been only comparatively few who were trained in geometrical methods, particularly in projective geometry, and the advances they have brought about, generally, have been much greater than those by kinematicians who used analytical methods. Mr. Martellotti says further that the author stated in his paper that geometrical methods fail in space kinematics to be equally valuable as in plane kinematics. This is not quite correct, for the author only said that the geometrical methods become more involved. However, the analytical methods are equally complicated in this case, or they require much more preparatory study to be absorbed so that a facility in their use will result.

One point, which has not been mentioned by either Mr. Candee or Mr. Martellotti, is the fact that geometrical methods usually permit of applying checks in a simple manner. Since numerous straight lines or circles have to be drawn, conditions arise frequently, in which certain points have to lie on straight lines or circles, or certain straight lines or circles must pass through certain points. Such checks form an extremely valuable criterion for the correctness of the derivations or constructions. No such checks exist in the analytical methods where the various analytical or numerical values have equal weight, and errors cannot be detected.

Perhaps Mr. Martellotti recalls that, when checking the original preprint of his paper[13] the author discovered, by his geometrical methods, several minor errors which had passed unnoticed and which were then eliminated in the final publication of the paper. This indicates clearly the great value of the geometrical methods. However, the author agrees with Mr. Martellotti that, in practice, geometrical constructions and numerical calculations have to be used side by side. Whether the numerical calculations are based on concise analytical formulas, or whether they involve no more than the use of similar triangles, etc., is immaterial. To the author's mind, the latter, as derived from the geometrical constructions, seem to offer the simpler way. However, in his own work, the author has frequently used both approaches, yet he feels from his experience that the geometrical methods and calculations based on them are, in general, preferable and simpler. An exception to this rule is formed by all problems or investigations of the higher forms of motion, which require higher differential quotients for their solution. In these cases, the analytical approach is either the only one possible, or it is the simpler one, as is shown, for example, by some of the investigations of Allievi, Mehmke, and others. Good judgment has, therefore, to be exercised as to which method is preferable. In most practical problems, however, the geometrical method is the simpler and more lucid one. This holds also in the case of the radius of curvature generally, although sometimes the purely mathematical solution also gives quick results.

The author, now, must revert to the problem of the path of the milling cutter and its radius of curvature. He owes Mr. Martellotti an unqualified apology, for it is perfectly true that Mr. Martellotti, in the private communication referred to,[14] pointed out the formula he had used for determining the radius of curvature for the parameter equations of the cutter path. Thus, in the paper, the author should not have stated that Mr. Martellotti used the common formula for the radius of curvature cited therein. The error is due to a contraction of the text which became necessary and which made it appear that Mr. Martellotti had used this formula. The facts are that, when the author put the problem to numerous other engineers and to some of his students, there was not a single one who had even a knowledge of the existence of the parameter form of the equation for the radius of curvature. Everyone used the common formula, as indicated in the paper, which leads to extremely complex expressions that must be handled very carefully if the final result is to be correct. It was due to this contraction of the text because of lack of space that the wrong sense was created. The author is glad that Mr. Martellotti has availed himself of the opportunity of giving his derivation, and takes pleasure in acknowledging that it is probably just as simple in this case as the kinematical solution according to Hartmann, although, in the latter solution, no differentiations at all are required, as was pointed out by the author in the paper.

In conclusion, the author desires to thank both Mr. Candee and Mr. Martellotti for their contributions and for the trouble they have taken to discuss the author's paper.

Rating Supercharged Engines on the Basis of the Mean Temperature of the Cycle

By RALPH MILLER,[1] BUFFALO, N. Y.

The author analyzes the temperature conditions of nonsupercharged and supercharged Diesel engines and bases the work on the assumption that the capacity of a prime mover which converts heat into work is limited by the temperatures to which its mechanism is subjected in the process. The exhaust-gas turbine used in the Büchi supercharging system may not be operated above 1020 F gas temperature. It is evident that the Diesel engine likewise reaches a limiting gas temperature as the load is increased. It is demonstrated that engines supercharged with blowers, having an adiabatic efficiency of about 75 per cent and operating without air cooling, cannot be loaded above 150 per cent of the brake horsepower of the naturally aspirated engine without exceeding its temperatures or maximum pressures. The paper further develops the importance of removing the hot clearance gases. With a clearance volume of 8 per cent, the indicated horsepower is increased 18 per cent with no supercharging, by removing the clearance gases only. The capacity of Otto-cycle engines with large clearance volumes and high residual-gas temperatures would be increased considerably more than 18 per cent by the scavenging method described.

SUPERCHARGING of four-cycle engines is being adopted on an ever-increasing scale as a means of obtaining more horsepower from an engine of given dimensions and weight. The development has been greatly accelerated by the war emergency.

It is the object of this paper to show how the thermodynamic conditions change with various degrees of supercharging and to establish a method by which the rating of supercharged engines may be calculated.

With few exceptions engines which are being supercharged at the present time have been designed to operate as naturally aspirated engines, and their horsepower ratings in most cases have been established by long field experience. This rating is limited by three factors, i.e., combustion efficiency, loads imposed by gas pressure, and the mean temperature of the cycle. If an engine has been so designed that all factors have the same margin for safe operation at full load it follows that when the unit is supercharged, the mean temperature and maximum combustion pressures of the standard engine must not be exceeded.

During the early period of development of the solid-injection system the load an engine could carry was limited to the point where exhaust smoke and incomplete combustion became troublesome. During this period research work was centered on combustion chambers and injection equipment. The discovery of the effect of turbulence and refinements in injection equipment has brought about such improvements in the combustion of the fuel that most modern engines have smoke limits far above the temperature limits. Just before this war, the author witnessed tests in England on an 8-in. × 12-in. 800-rpm engine which showed clear exhaust at 105 lb bmep. At this load the air-fuel ratio was about 20. However, the normal rating of the engine was 82 lb bmep with an air-fuel ratio of 27.

The simple, though admittedly highly controversial method of establishing the rated load by the appearance of the exhaust, as used in the days when the test engineer spent half of his time outside the building gazing at the exhaust pipe, is no longer being used.

When the smoke limit was lifted brake mean effective pressures were increased to values which have been established by tests and actual field experience for each particular type and size of engine in its particular service.

By this trial-and-error method the engine is rated at the maximum mean temperature it will stand with the chosen margin of safety required for the service. However, the actual value of the mean temperature is not found by this method.

While it may be of some value to know the mean temperature of the cycle of a given engine the information is of slight use as a means of rating projected designs because engines of different speeds, size, and design features, such as oil-cooled pistons, would not be rated at the same mean temperature.

However, when the output of a given engine is increased by supercharging, effective or useful increase in horsepower above the established rating as a nonsupercharged engine can be evaluated only by calculating the mean temperature of the cycle.

Büchi[2] shows by diagrams that the maximum and mean temperatures at all supercharging pressures remain the same as the temperatures in the nonsupercharged engine. To make that condition possible the initial temperature of compression must be kept constant which requires aftercooling of the supercharging air. Furthermore, the cutoff ratio must be kept constant and the maximum cylinder pressure increased with the supercharging pressure.

The analysis here presented is based on the assumption that the maximum combustion pressure remains constant at the value of the pressure used in the nonsupercharged engine.

Effect of Scavenging. In the nonsupercharged engine the residual gases in the clearance space remain in the cylinder and mix with the fresh charge. While it is true that neither the temperature of the residual gases nor the volume of the clearance space has any effect on volumetric efficiency, the initial temperature of compression is increased by retention of the residual gases. In a nonsupercharged engine this heating of the charge will amount to about 26 F at full load. With effective scavenging, this heating disappears and, as is seen by the accompanying graphs, a reduction of 26 F in T_1 permits an increase in load of about 10 per cent for the same mean temperature. The real benefit of scavenging is derived from this reduction of temperature only, and not from the addition of fresh air, because the excess air is already ample for good combustion.

The heat removed from the combustion-chamber walls and

[1] Chief Engineer, Worthington Pump & Machinery Corporation. Mem. A.S.M.E.

Contributed by the Oil and Gas Power Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[2] "Supercharging of Internal-Combustion Engines With Blowers Driven by Exhaust-Gas Turbines," by A. J. Büchi, Trans. A.S.M.E., vol. 59, 1937, pp. 85–96.

piston crown by the scavenging air as it sweeps through from the inlet to exhaust manifold during the valve-overlap period, is insignificant. Even if we assume that the temperature of the scavenging air is raised 100 F, only 75 Btu would be removed per hour per horsepower, while the heat liberated in the cylinder is about 7000 Btu per hr per hp.

It should also be realized that the greater the heating of the scavenging air the higher will be the initial compression temperature, because the air retained in the cylinder will also have been in contact with the hot walls.

The effect of the initial compression temperature is so great that a higher load would be carried at a given mean temperature if it could be assumed that the air did not receive any heat from the walls.

The low temperature read on thermocouples in the exhaust port with scavenging is no proof that the mean temperature of the cycle is lower than in a nonsupercharged engine at a load giving higher exhaust temperatures in the exhaust ports. The mass of the thermocouples is small and quickly responds to the relatively cold scavenging air.

In this analysis it has been assumed in all cases that the removal of the residual gases by scavenging is complete at all scavenging pressures.

Gas samples were taken at top dead center, 30 deg and 60 deg after top dead center in the scavenging period on an engine operating with 4 lb supercharging pressure. The result, plotted in Fig. 5 shows the scavenging to be complete at this pressure.

Determining Initial Compression Temperature T_1. The importance of determining T_1 accurately is evident when it is seen that the mean temperature bears an approximate ratio of 2 to T_1, and an increase of 48 deg in the mean temperature produces the same mean temperature as 10 per cent overload. To calculate the initial compression temperature T_1 in the nonsupercharged engine, the volumetric efficiency must first be determined.

From a light-spring-scale indicator card of the first half of the compression stroke, the compression exponent is found for the path between 45 and 90 deg after bottom dead center. Using the same exponent the pressure P_1 at V_1, the beginning of the compression stroke, is calculated. Cylinder heating is then

$$t_{ch} = T_{atm}\left(\frac{P_1}{P_{atm} \times E} - 1\right) \qquad [1]$$

The temperature of the charge is further increased by mixing with the retained residual gases. This heating may be calculated as follows:

$$t_{rh} = \frac{T_{atm} \times P_1}{P_{atm} \times E}\left(\frac{V_1}{V_1 - V_2 + \dfrac{V_2 \times T_{atm}}{E \times T_6}} - 1\right) \qquad [2]$$

The initial compression temperature T_1 is then

$$T_1 = T_{atm} + t_{ch} + t_{rh} \qquad [3]$$

where

t_{ch} = heating from hot walls, deg F
t_{rh} = heating of charge due to residual gases, deg F
P_1 = initial compression press, psia
T_1 = initial compression temperature absolute
T_{atm} = atmospheric temperature absolute
P_{atm} = atmospheric pressure psia
E = volumetric efficiency
V_1 = clearance + piston displacement
V_2 = clearance
T_6 = temperature abs, at point 6, Fig. 1
t_{rh} = is approximately 26 deg for a load of 94 lb mip with a clearance volume of 8 per cent. (The decrease in temperature due to the pressure drop from P_5 to P_6 is assumed to be that due to an expansion exponent of 1.23.) By using the known relations $\dfrac{P_1V_1}{T_1} = \dfrac{P_5V_1}{T_5}$ and $T_6 = T_5\left(\dfrac{P_6}{P_5}\right)^{(n-1)/n}$, T_6 can be calculated from the measured P_5 in the diagram, and the known exhaust back pressure P_6.

In the case of the supercharged engine, where residual gases are removed, determination of T_1 involves CO_2 analysis of the combustion product. The air delivered by the blower divides into

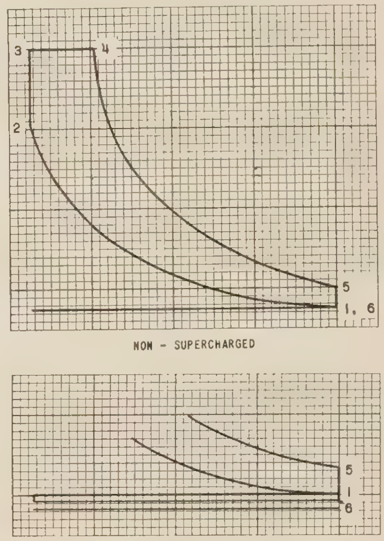


Fig. 1 Indicator Diagrams

two volumes, one passes through as scavenging air, the other volume remains in the cylinder for combustion. By weighing the fuel and determining the CO_2 of gas samples, taken with a timed sampling valve on the exhaust stroke, but before the inlet valve opens, the weight of air retained in the cylinder is calculated. Using the compression exponent, found by measuring the pressures at 45 and 90 deg from bottom dead center, P_1 at volume V_1 is determined by using the same exponent. The value of T_1 is then calculated.

When this method was tried it was found that gas samples taken through a timed valve placed in the combustion chamber did not show uniform results. Samples taken just before the opening of the exhaust valve would show as much as 2 per cent higher CO_2 than those taken half way up on the exhaust stroke. The sampling valve was, therefore, placed in the exhaust port and timed to take samples over the period between bottom dead center and 90 deg after on the exhaust stroke. This gave consistently good results.

Determining Mean Temperature of Cycle of Nonsupercharged Engine. Starting with T_1, the temperatures along the path of the compression and expansion stroke can be calculated by using the formula

$$T_2 = T_1 \times \frac{P_2V_2}{P_1V_1} \qquad [4]$$

On the expansion stroke a correction must be made for change in volume due to combustion. A four-cycle engine operating with a fuel consumption of 0.38 lb per bhp per hr with 86 per cent volumetric efficiency at a load of 75 lb bmep will have a correction factor of $C = 1.038$. Since this is an increase in volume the tem-

peratures on the expansion stroke must be divided by this value.

Mean Temperature of Exhaust Stroke. In the nonsupercharged engine with no scavenging effect, the temperature during the exhaust stroke is assumed to remain constant at T_6.

Mean Temperature of Suction Stroke. The temperature in the nonscavenging engine at the beginning of the suction stroke is that of the residual gases, or T_6.

The temperature at any crank position on the suction stroke is

$$T_s = \frac{T_{\text{atm}} \times P_1}{P_{\text{atm}} \times E} \times \frac{V_1}{V_1 - V_2 + \dfrac{V_2 \times T_{\text{atm}}}{E \times T_6}} \ldots\ldots [5]$$

where

T_s = absolute temperature at volume V_1
T_{atm} = atmospheric temperature absolute
V_1 = piston displacement at any given crank position plus clearance volume
V_2 = clearance volume
E = volumetric efficiency
T_6 = residual-gas temperature absolute

The temperatures for the four strokes are next plotted as shown in Fig. 2 and the mean temperature determined by using a planimeter.

Determining Mean Temperature of Cycle of Supercharged Engine. When a four-cycle engine is supercharged, the exhaust and inlet valves are given a large overlap for the purpose of scavenging the residual gases from the clearance space. This action starts when the inlet valve opens and the temperature begins to drop at that point until the exhaust valve closes.

Having determined the mean temperature of the standard engine or the temperature at which it is desired to rate the supercharged engine we have

$$T_{\text{mean}} \text{ st.} = T_{\text{mean}} \text{ sup.}$$

The supercharging pressure is selected and the atmospheric

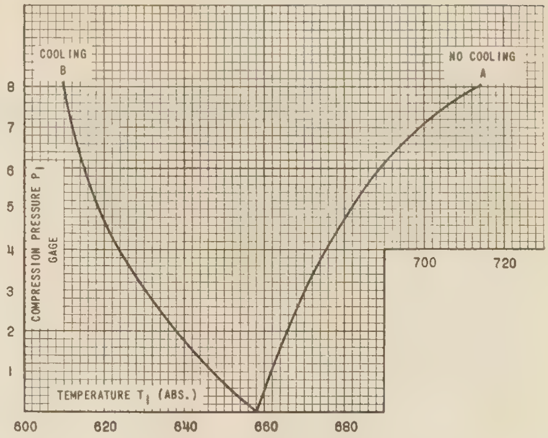


FIG. 4 INITIAL COMPRESSION TEMPERATURE T_1
(Intake temperature 90 F.)

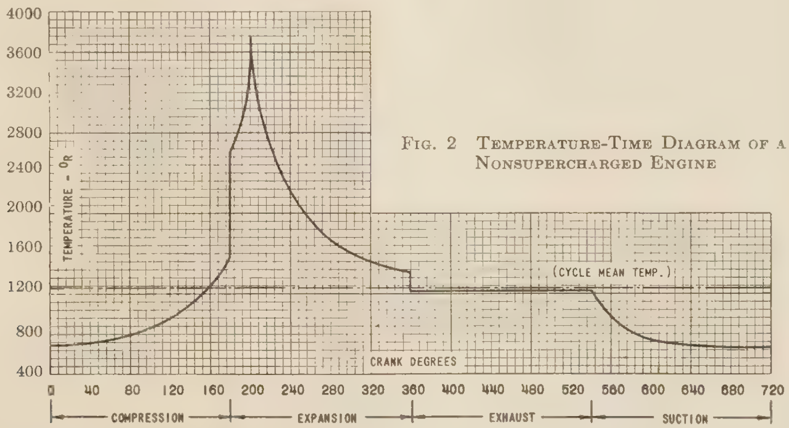


FIG. 2 TEMPERATURE-TIME DIAGRAM OF A NONSUPERCHARGED ENGINE

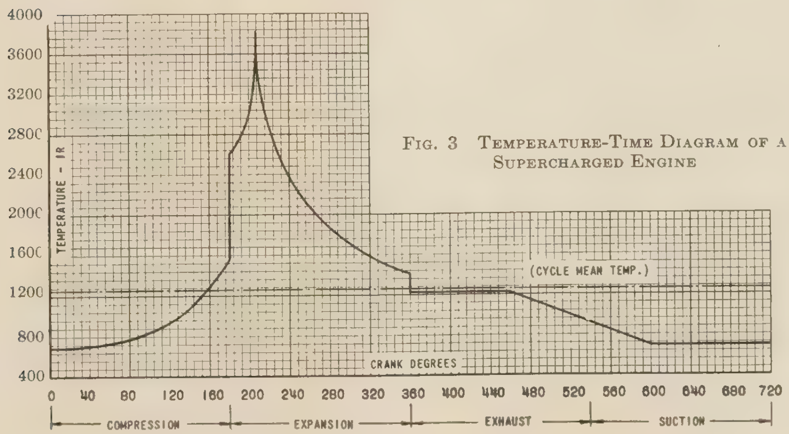


FIG. 3 TEMPERATURE-TIME DIAGRAM OF A SUPERCHARGED ENGINE

temperature taken equal to T_{atm} used in calculating the mean temperature of the nonsupercharged engine.

The intake-manifold temperature, T_{mf}, is calculated from the blower data or taken from actual tests.

With T_1 determined, the mean temperature of the supercharged engine can be calculated.

The compression and power strokes are calculated by the methods outlined for nonsupercharged engines.

The exhaust and suction strokes are calculated as follows:

Mean Temperature of Exhaust Stroke:

$$T_m = \frac{180-B}{180} \times T_6 + \frac{B}{360}\left[2T_6 - \frac{B}{A+B}(T_6 - T_1)\right] \quad [6]$$

Mean Temperature of Suction Stroke:

$$T_m = \frac{180-A}{180} \times T_1 + \frac{A}{360}\left[T_6 + T_1 - \frac{B}{A+B}(T_6 - T_1)\right] \quad [7]$$

where

A = overlap period after TDC, deg
B = overlap period before TDC, deg

It is assumed that the temperature drop from the exhaust to the initial temperature T_1 follows a straight line.

Fig. 3 is a typical temperature-time diagram for an engine with scavenging of the clearance volume. It will be noticed that mean temperatures on the exhaust and suction strokes are lower in the scavenged, supercharged cycle.

Example of Application of Method to Actual Test. To facilitate the work, theoretical *P-V* diagrams, as shown in Fig. 1 rather than the actual diagrams are used. Exponents 1.33 and 1.45 are assumed for compression and expansion, respectively. Combustion is assumed to progress at a rate that will produce vertical pressure rise and horizontal volume increase at constant pressure with sharp termination at point 4.

Calculating Mean Temperature of Nonsupercharged Engine. From the true diagram, P_1 was found to be

$$P_1 = 15 \text{ psia}$$

By measurement and checking by the CO_2 method the volumetric efficiency was determined as

$$E = 86 \text{ per cent}$$

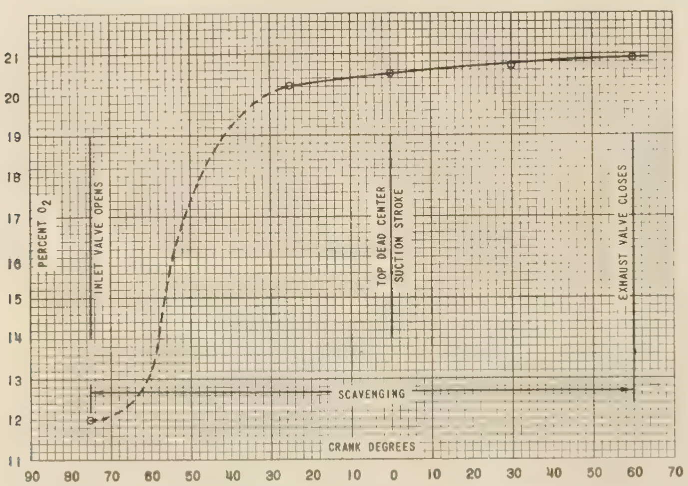


Fig. 5 Effect of Scavenging in Removing Exhaust Gases

also

Bmep	= 75.5 psi
Fmep	= 18 psi
Mip	= 93.5 psi
Ihp per cylinder	= 103.25
P_3	= 765 psia
Rpm	= 360

The cutoff ratio V_4 for 103.25 ihp is solved from the formula

$$1.45V_4 - \frac{V_4^{1.45}}{V_1^{0.45}} = \frac{0.45}{P_3} \times \left(\frac{P_2V_2 - P_1V_1}{0.33} + \frac{\text{Ihp} \times 33000}{N}\right) + V_2 \times 0.45 \quad [8]$$

where pressures are in pounds per square foot; volumes are in cubic feet; and taking

P_3 = (765 × 144) psia
P_1 = (15 × 144) psia
V_1 = 1.508 cu ft (piston displacement plus clearance)
V_2 = 0.112 cu ft (clearance)
N = power strokes per min

then

$$V_4 = 0.17 \text{ cu ft or } 11.3 \text{ per cent of } V_1$$

The *P-V* diagram can now be constructed. The initial compression temperature T_1 was calculated as explained and found to be

$$T_1 = T_{atm} + t_{ch} + t_{rh} = 550 + 110 + 26 = 686 \text{ R}$$

By calculating the pressures at increments of 7.5 deg along the compression and expansion paths, the corresponding temperatures were found from Equation [4] and plotted over crank degrees. The temperatures along the path of the suction stroke were calculated by using Equation [5] and plotted over crank degrees. The exhaust-stroke mean temperature was taken equal to T_6.

With the values given, the mean temperature of the cycle with the assumed theoretical *P-V* diagram was found to be

$$T_m = 1260 \text{ deg abs}$$

Finding Load at 1260 F When Supercharged. To facilitate the work of estimating the rating of the supercharged engine, a number of theoretical cards were constructed and formulas were developed. The data are plotted in Figs. 6 and 7.

Thus by finding T_1 (corrected to an atmospheric temperature of 90 deg), the cutoff ratio can be read in Fig. 6, for any initial compression pressure. Transferring this cutoff ratio to Fig. 7, the mean indicated pressure is read on the abscissa at the intersection of the cutoff ratio and the pressure (P_1). The temperature T_1 for 4 lb scavenging pressure was calculated from the following observed test data:

Pressure at 135 deg BTDC	= 22.6 lb abs
Pressure at 90 deg BTDC	= 37.8 lb abs
Compression exponent	= 1.3
Pressure at 180 deg BTDC	= 19 lb abs
Barometer	= 29.5 in. Hg
Blower-intake temperature	= 81 F
Intake-manifold temperature	= 133 F
Carbon dioxide	= 7.2 per cent by volume
Fuel burned per stroke	= 0.00383 lb

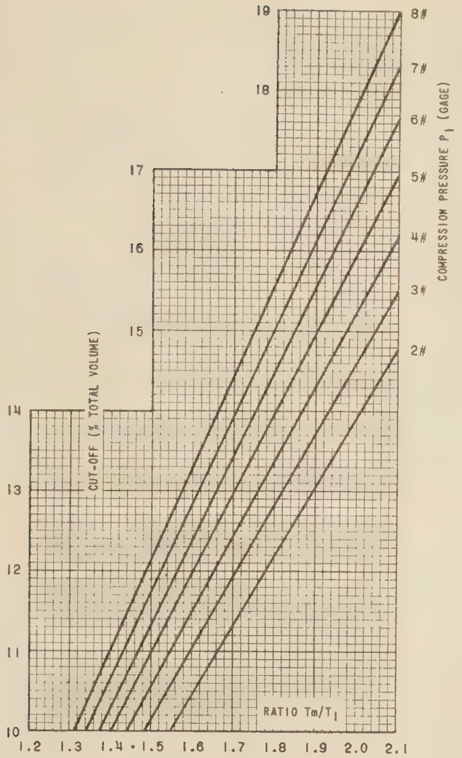


Fig. 6 Temperature Versus Cutoff

(T_m = mean temperature of cycle; T_1 = initial compression temperature.)

Fuel-air ratio = 0.033
Weight of air per stroke = 0.1161 lb
Equivalent volume at 14.5 lb and 80 F = 1.597 cu ft

From these data

$$T_{180} = 540 \times \frac{19 \times 1.508}{14.5 \times 1.597} = 669 \text{ R}$$

Cylinder heating is then

$$669 - (460 + 133) = 76 \text{ F}$$

Since the nonsupercharged-engine mean temperature was calculated for 90 deg atm temperature, and the supercharged engine operated with 81 deg atm temperature, the difference is added to T_{180} making

$$T_1 = 678 \text{ R}$$

We then have

$$T_m = 1260, \text{ and } T_1 = 678$$

$$\frac{T_m}{T_1} = 1.86$$

Interpolating between pressure lines for 4 and 5 lb, a cutoff ratio of 14.25 per cent is read at the intersection of temperature ratio 1.86 and 4.5 lb pressure in Fig. 6.

A cutoff ratio of 14.2 is read in Fig. 7 to give 124 lb mip.

This test shows that with a 4-lb supercharging pressure, the mean temperature of the nonsupercharged engine is reached at a load of 124 lb mip. Assuming that the fmp remains at a value of 18 lb, the bmep is 106 lb and the bhp 140.5 per cent of the nonsupercharged engine.

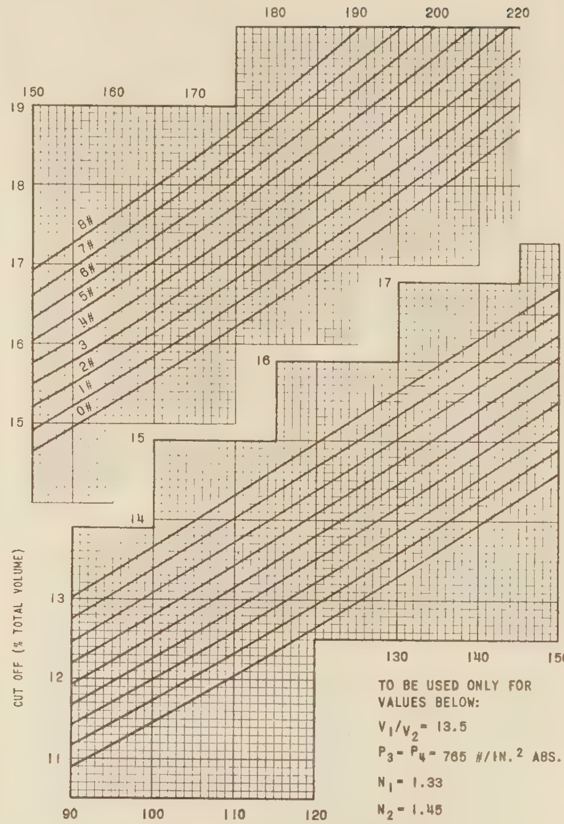


Fig. 7 Mean Indicated Pressure Versus Cutoff

Using the one established rating point at 4 lb supercharging, curve A in Fig. 8, has been plotted for pressures up to 8 lb by extrapolation. To do this, the initial temperatures at various supercharging pressures had to be calculated. It was assumed that the cylinder heating is a function of the total weight of air passing through the engine and that the ratio found at the test point would hold at all pressures. Then

Supercharged engines at 4 lb pressure:
Weight of air per cylinder per minute = 26.67 lb
Cylinder heating = 76 F
Standard nonsupercharged engine:
Weight of air per cylinder per minute = 15.8 lb
Cylinder heating = 110 F

From this it is seen that the cylinder heating at 4 lb supercharging pressure is

$$t_{ch} = 110 \times \left(\frac{15.8}{26.67}\right)^{0.71} = 76 \text{ F} \ldots\ldots \ldots\ldots [9]$$

Using Equation [9], and the known blower capacity at other selected pressures, the T_1 temperature curve A was plotted in Fig. 4. The horsepower at any pressure is then read in Fig. 7, at the intersection of the initial compression pressure P_1, and the cutoff ratio, after the latter has been found in Fig. 6 at the intersection of P_1 and the temperature ratio.

The point on curve A in Fig. 8, at zero supercharging pressure shows a nonsupercharged engine without valve overlap. Above the test point at 4 lb, the curve is extrapolated as explained.

Tests at higher pressures are needed to verify the extrapolation. The trend of the curve indicates that with normal heating of the air through the blower little is gained in horsepower above 4 lb.

The temperature ratio versus cutoff curves in Fig. 6 are based on $P_6 = 0.75 \times P_5$, that is, the exhaust back pressure is equal to $^3/_4$ of the scavenging pressure (gage). They therefore apply to the Büchi system where this relation is approximately held. An engine supercharged with a separately driven blower would expand to a lower P_6 and the mean temperature of the exhaust and intake strokes would be lower for the same T_1 and cutoff ratio. The negative work on the exhaust stroke would likewise be reduced by the reduction in P_6.

Cooling of Supercharging Air. The preceding analysis shows that a large increase in specific engine capacity may be obtained by reducing the initial temperature. This suggests the installation of an aftercooler in the blower discharge to reduce the air temperature in the engine inlet manifold.

Curve B in Fig. 8, shows the per cent mean indicated pressure of nonsupercharged engine rating, which is obtained when the air in the inlet manifold is cooled to blower-intake temperature, and T_1 follows curve B, Fig. 4.

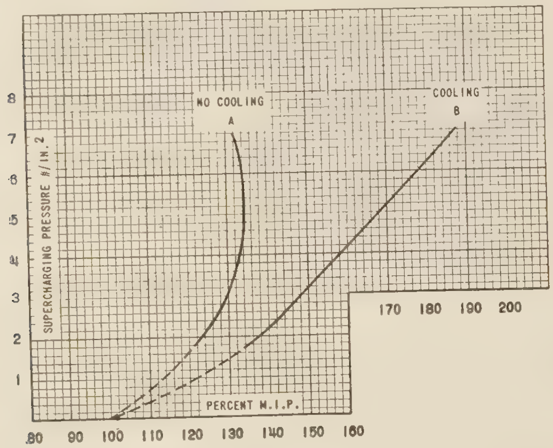


FIG. 8 RATING CURVE

The advantages gained by adding this simple equipment are so obvious that the introduction of intercooling of the scavenging air, especially on marine installations, would seem imperative.

Summary

Supercharged engines should be rated in accordance with pressures and mean temperatures as determined by design and established practice.

Increases in engine capacity obtained by oil-cooling the pistons, increasing the maximum combustion pressure or permitting the mean temperature to exceed established limits should not be credited to supercharging, because similar improvements or increases in temperatures and stresses may be applied to a nonsupercharged engine, thus increasing its normal rating the same amount.

When supercharged engines are rated at pressures and mean temperatures equal to the naturally aspirated engine, the orthodox system in which the manifold temperature is increased by the temperature rise through the blower shows a maximum possible increase over the standard engine mean indicated pressure of about 34.5 per cent at the optimum pressure of 5 lb. This will give an increase of about 42 per cent on the brake mean effective pressure.

Cooling of the air to reduce T_1 has a marked effect on engine capacity.

With the manifold air-cooled to atmospheric temperatures the indicated horsepower may be raised to 187 per cent with 7 lb pressure, and 70 per cent increase in brake horsepower is reached at 4 lb supercharging pressure. Without air cooling, 4 lb permits a 40 per cent increase, and thus for every horsepower gained by supercharging, 0.75 hp may be added by cooling the manifold air to 90 F.

In the land-transportation field, air-cooling to atmospheric temperature is impractical. However, if the air is cooled in this case to 20 deg above atmospheric temperature, the rating increases from 140 to 158 per cent of the nonsupercharged rating at 4 lb pressure. In this example, a locomotive engine rated 700 hp nonsupercharged would be rated 980 hp when supercharged without air-cooling, but 1110 bhp with air cooled to atmospheric temperature plus 20 deg.

Effective cooling of the manifold air opens the possibilities of increasing the brake horsepower 100 per cent without exceeding established practical limits of temperatures and pressures.

Discussion

MAX ESSL.[3] The author states that the rating of Diesel engines is established on the basis of heat limitation and not on the basis of smoke and combustion.

Gasoline engines, in spite of their poorer thermal efficiency, are rated much higher than Diesel engines, because they have no smoke or fuel-injection problems.

It is true that such gasoline engines must provide satisfactory valves, etc., to operate at these higher mean effective pressures. It is believed that Diesel-engine manufacturers would gladly provide sodium-cooled valves and other means necessary to operate at the higher loads (mean effective pressures) and mean temperatures if smoke and fuel consumption were not the limitation.

Regarding gas pressures as a limiting factor in output, the writer's understanding is that the rate of pressure rise but not the maximum gas pressure, as such, is a limitation. It is relatively easy to design bearings, and the like, for higher gas pressures, and the higher outputs obtainable with higher pressures would well justify the heavier design necessary for these pressures.

JOHN FULLEMANN.[4] This paper is particularly valuable, representing as it does an attempt to compute the effects not only of plain supercharging but also of scavenging the cylinder. It is generally known that scavenging has proved to be at least as important to the performance of modern turbocharged engines as the supercharging proper. Yet it is rather difficult to evaluate accurately the merits of scavenging by means of thermodynamic computations.

During the years the writer worked in Dr. Büchi's office in Switzerland, considerable theoretical work dealing with this type of supercharging was done. However, a great deal of additional test work and experience were required to enable us to predict the proper turbocharged ratings of a new engine with reasonable accuracy. Some engines can, with relatively minor adaptations, be rated 50 per cent or even higher turbocharged than with normal aspiration; in other cases, horsepower increases up to 40 per cent are feasible without excessive design changes of the engine. Just how much is allowable depends in most cases upon the scavenging capabilities of the engine. This indicates that scaven-

[3] Chief Consulting Engineer, The Baldwin Locomotive Works, Philadelphia, Pa.

[4] Engineering representative for Dr. Alfred J. Büchi, Winterthur, Switzerland (with Walter H. Young, Washington, D. C., U. S. Agent for Dr. Büchi).

ging is a rather involved matter. It cannot be evaluated accurately in a general thermodynamic study which deals with all types of engines alike, without regard to their special design features.

The author uses the mean temperature of the cycle as a basis for the allowable engine load. The writer is in agreement with him that this is a good measuring scale. However, it has one important disadvantage: The computations necessary for determining this mean temperature have to be based on a number of assumptions such as, for instance, the amount of heat picked up by the fresh charge before and during the suction stroke.

The author states that the benefit of scavenging is only due to the replacement of exhaust gas in the dead space by fresh air, and not to any measurable extent due to the cooling of the walls of the combustion space by the scavenging air. With this statement the writer cannot agree. The author attempts to prove this statement with a brief heat-transfer calculation which is probably correct except for the many initial assumptions upon which his computations have to be based. These assumptions, which of course influence the result of the calculations, are open to argument.

There are two reasons for stating that scavenging does more than merely replace the exhaust gas remaining in the dead space by fresh air:

1 We find that turbocharged engines with, say, 40 to 50 per cent scavenging-air excess can definitely be rated higher than those with only 20 to 25 per cent excess over the air used for combustion. But the smaller amount would be entirely sufficient to sweep out the exhaust gas in the dead space, which is only about 8 per cent of the swept volume.

2 The principal reason, however, why the writer disagrees with the author's conclusion will become evident when we examine some actual heat balances of nonsupercharged and Büchi-turbocharged engines (Table 1 of this discussion). The heat carried away in the cooling water (plus oil cooler) of a turbocharged and scavenged engine amounts to only from 15 to 20 per cent of the total fuel-heat content, the actual amount depending upon the scavenging capability of the engine. With nonsupercharged engines this figure is some 30 to 35 per cent. On the other hand, the scavenged engine carries off 35 to 38 per cent of the total heat in the exhaust gases and scavenging air, 6 to 8 per cent more than the corresponding nonscavenged engine. The author's theory of basing comparable ratings on the same mean temperature of the cycle does not explain this difference in the heat balance.

The improvement in mechanical efficiency of the supercharged engine may account for some 4 per cent drop in cooling-water heat. However, since a supercharged engine maintains higher pressures during most of the expansion stroke, the heat-transfer value, rising with the pressure, would tend to keep the cooling-water heat per horsepower at a constant figure. The fact that this is not so calls for an explanation, and the only reasonable explanation is the fact that the scavenging air must pick up heat from the hot cylinder walls before this heat can go through the walls into the cooling water.

Therefore it is felt that the author's theory of rating engines on the mean temperature of the cycle can be made to agree even better with actual experiences if greater emphasis is placed upon the effects of scavenging in cooling the cylinder walls.

The writer cannot quite agree with the statement in the paper that the low temperature readings on thermocouples in the exhaust port with scavenging are misleading. It is true that those thermocouples are exposed alternately to the hot gases as well as to the cool scavenging air, but, for example, so are the exhaust valves.

The author is correct in his conclusion that supercharging becomes more effective if we can cool the air after the blower. He refers to Dr. Büchi's first supercharging tests in the year 1910, in which the charging air was cooled before entering the engine. In these tests, charging pressures up to 28 psi were used, with mean effective pressures more than 200 psi. This was an experimental engine. Attempts were also made to cool the air on commercial applications, but it was then found impossible to produce an air cooler with sufficiently low pressure drop, sufficient effectiveness for the low temperature gradients available, and still of reasonable size.

Since then much progress has been made with air coolers, but even today a brief calculation will show that coolers for supercharging air will be impractically large, except in the case of rather high charging pressures, beyond, say, 7 psi. Thus far, charging pressures of 4 to 7 psi have proved to give best over-all performance in low-altitude supercharging. The indications are, however, that the practical range of air coolers will eventually be extended to lower pressures and the charging pressures may be increased, so that the resulting combination opens new possibilities of supercharging.

TABLE 1 HEAT BALANCES OF BÜCHI-TURBOCHARGED VERSUS NONSUPERCHARGED ENGINES

	MS *Polyphemus*,[a] Büchi-turbocharged		MS *Sycamore*,[b] nonsupercharged		MAN 6-cylinder,[c] 300/380 mm 700 rpm Büchi-turbocharged, mean effective pressure		
	100 per cent	75 per cent	100 per cent	75 per cent	80 psi	114 psi	148 psi
	Total heat, per cent						
Useful load	32.9	33.9	28.9	28.1	39	39.5	38
Engine cooling water	16.1	16.6	38.5	38.4	17	15.5	15
Turbine cooling water	3.3	2.3	....	....	3	2.5	3
Exhaust-gas heat	38.5	34.5	24.0	24.0	37	35.5	36
Air compressor	9.2	12.7	8.6	9.5	..	....	..

MAN 6-cylinder 300/380 mm 700 rpm[c]

	Mep, psi	Btu per hr	Btu per hp hr
Ne 700 hp (atm)	80	12100	1730
700 hp (Büchi)	80	8350	1190
1000 hp (Büchi)	114	10300	1030
1190 hp (Büchi)	136	12100	1020
1300 hp (Büchi)	148	13000	1000

[a] Marine Oil Engine Trials Committee, sixth report.
[b] Marine Oil Engine Trials Committee, first report.
[c] Dr. W. Pflaum, 1936.

W. M. Kauffmann.[5] The problem of supercharging a 4-cycle Diesel engine for maximum output is not a simple process of converting a standard engine by adding a blower at some convenient place, providing special piping, and replacing the camshaft with suitably timed cams having a specified overlap for scavenging. For a nominal supercharged rating between 35 and 40 per cent above the standard engine rating, such a conversion may be adequate and probably successful. However, where a 50 per cent increase in power for continuous full-load rating of the supercharged conversion is expected, we find that definite new design problems are experienced which did not appear at 40 per cent over standard rating.

For instance, let us consider a 4-cycle engine which had a commercial continuous full-load rating of 75 bmep and showed successful performance in numerous installations. When supercharged and operated at a sustained load rating of 112.5 bmep, this unit began to exhibit operating difficulties after a period of 30 hr running at this load. Piston rings began to fail, the rings wearing down to one half their original wall thickness in this short time. Subsequent tests with chrome plating, tinnizing, and changing ring width showed no improvement in ring wear. Finally, these standard uncooled cast-iron pistons were dis-

[5] Engineer, Engine Development Division, Worthington Pump & Machinery Corporation, Buffalo, N. Y. Mem. A.S.M.E.

mantled and a nozzle placed at the top of the connecting rod to admit a jet of lubricating oil to the underside of the piston.

This change required increasing the lubricating-oil rate of flow from the pump from 4 to approximately 7 gal per hp-hr for the oil-cooled pistons. In addition, a larger oil cooler and larger oil filter were installed, and oil grooves in the bearing shells were deepened. The ring wear after installation of these pistons was negligible and piston scuffing disappeared. This result pointed to a substantial increase in heat during the cycle when the 40 per cent rating was exceeded, as no difficulties were present at 105 bmep with an uncooled-piston design.

It was found also to be imperative that proper consideration be given to piston design, cooling, piston-top contour, and cylinder head and valve design for maximum output. Major changes such as these would indicate that for maximum output to be obtained from supercharging requires an engine specifically designed around the supercharger and its system.

Piston contours of standard engines must be altered to promote a smooth cross-flow of the scavenging air during the overlap period, and recesses must be provided to clear the valve heads. Cylinder-head-valve ports should be streamlined, particularly where valve cages are employed, as these are generally conducive to turbulent flow. Seating the valves directly in the head simplifies the problem and permits larger diameters of ports.

The paper is open to criticism in employing a theoretical card for the determination of mean temperature of the cycles. However, actual cards taken from a supercharged 4-cycle engine closely approximate the theoretical results.

The paper refers particularly to ratings for continuous full-load operation. The point may be made that a number of modern engines having oil-cooled pistons are rated at 80 psi with an additional overload of 10 per cent for short periods as guaranteed by the builders. It follows, therefore, that the mean temperature of the cycle is equal to that of the uncooled-piston engines at 75 bmep. On this basis, an increase of 40 per cent would give 112 bmep, which is equivalent to the 50 per cent rating over and above the standard uncooled-piston engine. Higher mean effective pressures are pointed to in recent developments abroad.

Possibilities in increasing engine ratings with 2-cycle engines were brought out during a discussion at the Oil and Gas Power Division Meeting of 1939. It was reported then by Mr. Schneitter that 250 imep were being developed on a combined 2-cycle engine exhaust turbine unit, where the exhaust turbine operated as the prime mover receiving its energy from the engine exhaust. This development by Sulzer Bros. has been given much publicity within the last year.

The suggested procedure of cooling the scavenge air has been applied successfuly to small automotive- and marine-type Diesels equipped with engine-driven blowers. A heavy-duty core-type air cooler, which uses raw water as coolant, is built into the supercharging layout. The application of intercooling of the scavenge air to larger engines of the stationary type presents a novel approach to higher output possibilities in engines for this service. The author has made a worthy contribution in calling this to the attention of the industry.

V. L. Maleev.[6] The elimination of certain discrepancies in the paper, would greatly enhance its value. Many data which the author used in his calculations, and which would permit a better check of various statements, are not included in the paper, while some are contradictory of one another.

The author's statement early in the paper, that neither the temperature of the residual gases nor the volume of the clearance space has any effect on volumetric efficiency, is contrary to what is generally understood. The temperature of the residual gases affects the temperature of the fresh charge and, hence, its specific volume, weight, and the volumetric efficiency. The volume of the clearance space affects the amount of residual gases left in the cylinder and, through it, the amount of fresh air which can be taken in. The numerical influence is rather involved, but is taken into account by introducing the compression ratio r in the expressions of volumetric efficiency.[7]

The author states that in order to find the initial compression temperature T_1, the volumetric efficiency, which later is designated by E, must be determined. Unfortunately, he does not say how he determined E, i.e., by what measurements and on what basis. Was use made of the A.S.M.E., N.T.P., meaning $p = 14.7$ psia and $t = 68$ F, or the more widely used S.A.E., N.T.P., meaning $p = 14.7$ psia and $t = 60$ F, or were the actual atmospheric conditions[8] used, in this case $p = 14.5$ psi and $t = 90$ F? He gives only its numerical value as 86 per cent. Later in the paper, he also gives the actual amount of air taken in as 15.8 lb per min, or 15.8/180 = 0.0878 lb per cycle. Using this value, the volumetric efficiency can be easily checked. On the basis of A.S.M.E., N.T.P. with the specific weight of air = 0.0752 lb per cu ft.

$$E = 0.0878/[(1.508 - 0.112) \times 0.0752] = 0.836$$

On the basis of the S.A.E., N.T.P., it would be $E = 0.814$. On the basis of outside conditions, $E = 0.883$. Thus, either the value $E = 0.86$ or the amount of fresh air, 15.8 lb per min, is in error.

Equation [1], which the author uses to find the temperature increase due to cylinder heating, actually gives the "total" temperature increase of the fresh charge, caused both by cylinder heating and mixing with the residual gases. This follows from the method of determining the theoretical pressure p_1 at the dead center. Since p_1 is found by means of p at 45 deg and p at 90 deg, where the charge is influenced both by cylinder heating and by the residual gases, therefore, p_1 is the pressure of the charge having the volume V_1 and the temperature $T_1 = T_{atm} + t_{ch} + t_{rh}$. There are several additional ways of proving it:

1 The temperature rise computed by Equation [1], with the data given at various places in the paper ($T_{atm} = 550$ R, $p_1 = 15$ psia, $p_{atm} = 14.5$ psia, $E = 0.86$), when accurately figured, is found to be 111.6 F, instead of 110 F as given, but this discrepancy is not important. A temperature increase of 112 F due to cylinder heating alone, when the exhaust temperature $T_6 = 1230$ R, Fig. 2, or 770 F, is much higher than ever encountered in a compression-ignition oil engine. When considered as the total temperature increase with 26 F of it due to residual gases, it gives a cylinder heating of (112–26) = 86 F, which is more likely, although still high. Thus, this method gives $T_1 = (550 + 112), = 662$ R.

2 With the numerical data given in the paper, T_1 can be computed directly. The weight of the fresh air per cycle, with 15.8 lb per min given is

$$W_a = 15.8/(360/2) = 0.0878 \text{ lb per cycle}$$

The weight of the residual gases, with $p_6 = p_1 = 15$ psia; the gas constant $R = 53.35 \times 1.038 = 55.38$; $V_2 = 0.112$ cu ft; and $T_6 = 1230$ R is

$W_r = 0.112 \times 15 \times 144/(55.38 \times 1230) = 0.00355$ lb per cycle and from the characteristic equation for point 1

[6] Senior Mechanical Engineer, U. S. N. Engineering Experiment Station, Annapolis, Md.

[7] "The Internal-Combustion Engine," by C. F. Taylor and E. S. Taylor, International Text Book Co., Scranton, Pa., 1938, p. 246; also "Internal Combustion Engines," by V. L. Maleev, McGraw-Hill Book Company, Inc., New York, N. Y., 1933, p. 82.

[8] Ibid., p. 227.

$$T_1 = (15 \times 144 \times 1.508)/(0.0878 \times 53.35 + 0.00355 \times 55.38) = 667.5 \text{ R}$$

3 It is regrettable that, in Fig. 1, the author gives a schematic diagram and not the theoretical diagram drawn to scale which he used in his calculations. Anyway, with the data given in the paper, $p_3 = 765$ psia, $V_4 = 0.17$ cu ft, $n = 1.45$, it is easy to find $p_5 = 32.2$ psia. Taking $T_5 = 1420$ R, Fig. 2, gives

$$T_1 = T_5(p_1/p_5) = 1420(15/32.2) = 661.5 \text{ R}$$

Thus for T_1 all three methods give values which are in satisfactory agreement, i.e., 661.6, 667.5, and 661.5 R, an average of about 22.5 F lower than computed in the paper.

In respect to cylinder heating of the supercharged engine, it is not clear why in Equation [9], the author introduced the exponent 0.71, when the temperature rise is inversely proportional to the first power of the weight heated.

Finally it would be interesting to obtain an explanation of how it did happen, that in a nonsupercharged engine the pressure at the beginning of the compression was higher than the atmospheric pressure, $p_1 = 15$ psia and $p_{atm} = 14.5$ psia.

R. B. Smith.[9] The problem of applying a supercharger to a Diesel engine is one that should be approached co-operatively between the engine designer and the supercharger builder. The allowable rating which an engine is capable of absorbing is within the control and knowledge of the designer only. Each manufacturer establishes his limits by a combination of analysis and practical experience. In the particular case under discussion, the author has proposed to limit his operation to an average static temperature, determined on the basis of an idealized indicator card with the engine working at a constant combustion pressure.

The determination of average operating temperature is not a function of temperature alone; it is more specifically one of heat transfer involving considerations of velocity, density, and time. Particularly is this true during the scavenging period, when a high-velocity stream scrubs the exposed surfaces of the piston head and cylinder walls. The influence of supercharging on the mechanism of combustion would also appear to be a consideration which cannot be neglected, inasmuch as the increased temperature of the charge alters the ignition time lag.

If the author's Fig. 2 represents a typical static temperature distribution of the cycle, it is interesting to note that approximately two thirds of the area beneath the curve is contributed by the compression, exhaust, and suction strokes. Over much of this time, the cylinder walls are probably being cooled rather than heated. A considerable volume of test data secured on Diesel engines points to the fact that the total heat absorbed by the water jacket and oil cooler is the same whether the engine be supercharged or nonsupercharged. While this does not establish the fact that the average temperature of the cylinder walls is identical under the two operating conditions, it would seem to lend weight to the interpretation that the average temperature varies by only a slight amount.

The author's Fig. 8 points to the rather interesting conclusion that with uncooled charging air, supercharging in excess of 4 psi is uneconomical. Experience with Büchi supercharging systems has shown that a charging pressure in excess of 4 to 5 psi is not a necessity. If a higher engine rating is desired, it has been possible to achieve it by an increase in the scavenging air and a decrease in the excess air for combustion. Roughly, this is represented by designing a supercharging system so that the product of the excess-air ratio and the scavenging-air ratio is a constant.

The author is quite free to admit that the trend of curve A, Fig. 8, is an extrapolation based on the analysis that he presents. If the trend of curve A had been such as to pass through 150 per cent increase in the mean indicated pressure for 6 psi of supercharging pressure, the conclusion reached regarding a maximum of 34.5 per cent increase in indicated pressure would have been altered. Working backward by means of the author's Figs. 6 and 7, it is apparent that this alteration in the trend of curve A can be achieved with only 40 to 50 deg change in the average temperature of the cycle.

Eichelberger has suggested on the basis of the analysis of numerous indicator cards that the combustion process is one which takes place partly at constant volume, partly at constant pressure, and partly at constant temperature. The nature of the combustion process depends upon a number of variables not yet clearly understood, but it is certainly influenced by conditions which may be altered when an engine is adapted to supercharging. It would appear, therefore, that any conclusion, based on a 50-deg temperature difference and established by an idealized set of circumstances, should be subject to close scrutiny before it is universally adopted by the profession.

There have been some installations on which piston trouble has developed after supercharging, even when the engine has not been abnormally rerated. The writer is of the opinion that these failures are in a large part attributed to a fatigue condition which starts at the material surfaces as a result of temperature variations. Turbine practice has established that the presence of a sharp corner in a stream of varying temperature will magnify the contact-temperature stresses which exist in the thinnest outer layers of material to such a degree that they may exceed the endurance limit of the material. The introduction of scavenging air may possibly attenuate the temperature variations to which the piston crown is subjected, and bring about a condition which on first glance may be attributed to an average temperature but is instead a localized surface phenomenon.

[9] Director of Research and Development, Elliott Company, Jeannette, Pa. Mem. A.S.M.E.

Author's Closure

In reply to Mr. Essl's comment we must first agree upon the reliability or endurance factor when speaking of standard ratings (mean effective pressure). The engine discussed in this paper is rated for continuous full-load operation at 76 lb bmep. Gasoline engines, or engines operating on natural gas, which have the same temperature characteristics, are rated below 76 lb bmep for continuous full-load service; usually between 65 and 70 lb bmep.

Analysis, completed by the author since the paper was presented, shows that a gas engine with 5 to 1 compression ratio reaches a cycle mean temperature of 1600 R abs at 65 bmep, which is 200 deg higher than the Diesel engine at 76 lb bmep. To carry this higher temperature, gas-engine pistons, and sometimes exhaust valves, are usually oil- or water-cooled. When it is seen that this derating and addition of cooling is carried out, even where Diesel engines are converted to operate on gas, it becomes obvious that load is limited by temperature not pressure.

To take advantage of the high smoke limit of well-developed combustion chambers, which may be as high as 100 lb bmep, such engines are being built with cooled pistons and other features permitting increases in mean effective pressures of 10 per cent or more. The author has long advocated the use of oil-cooled pistons and believes that this feature will be adopted generally in engine designs of the future.

The author does not deny Mr. Fullemann's statement that some cooling is effected by the scavenging air, but the benefit measured in horsepower is small. The mass of the thermocouple is but a fraction of the mass of the piston head, exhaust valve, etc. Therefore with the same quantity of air passing over both the temperature drop of the thermocouple will be much greater.

An engine which operated nonsupercharged with 725 F exhaust temperature was supercharged some 3 years ago by a Midwestern company. At 4 lb supercharging pressure, the load could be carried up to 132 lb bmep before an exhaust temperature of 725 F was reached. Obviously, this was on overload, and piston and valve trouble soon developed when it was attempted to operate at equal exhaust temperature. At 115 lb bmep, the temperature was 640 F, and this is approximately the load giving normal nonsupercharged cycle mean temperature.

The temperature of the charge during the compression stroke is readily determined with reasonable accuracy by the method outlined in the paper.

Mr. Fullemann states that 40 to 50 per cent scavenging definitely permits higher ratings than 20 to 25 per cent. However, the method by which this gain was recorded is not mentioned. If readings of thermocouple temperatures were used the gain was much smaller than indicated, as previously explained.

Mr. Fullemann seems to be confused in the use of quantities when expressed in per cent. When he states: "The author's theory of basing comparable rating on the same mean temperature of the cycle does not explain the difference in the heat balance," his Table 1 is given to prove this.

If a supercharged engine carries 150 per cent of the rating of the same engine nonsupercharged with the same cycle mean temperature, then the heat flow to the water jackets should be substantially equal in the two engines at these respective loads.

With equal fuel consumptions per indicated brake horsepower per hour, the supercharged engine will then have a total fuel heat 50 per cent greater than the total heat in the nonsupercharged engine. Therefore, if the heat balance of the latter shows 30 per cent of the total heat in the cooling water, this quantity of heat becomes 20 per cent of the total in the supercharged engine.

Measuring the heat flow to the cooling water provides a sound and practical method of determining the correct rating for a supercharged engine. Mr. Fullemann's Table 1 shows that at 136 lb mep, the heat flow to cooling water in the supercharged is the same as in the nonsupercharged engine at 80 lb mep; namely, 12,100 Btu per hr. Under these conditions the temperatures of the parts exposed to combustion must be equal.

On this basis of mean indicated pressure, the table shows an increase of 57 per cent for equal cycle mean temperature when supercharging the 300 × 380-mm M.A.N. engine.

The method outlined by the author shows an increase of 53 per cent with 5 lb manifold pressure without cooling the air, Fig. 9

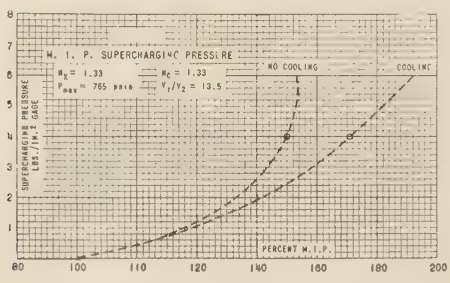


Fig. 9 Method of Determining Mean Indicated Supercharging Pressure for Equal Cycle Mean Temperature

of this closure. The difference of 4 per cent is probably due to increasing maximum cylinder pressures in the M.A.N. engine, whereas the author's calculations are based upon constant maximum pressure.

Thus when correctly interpreted, the data submitted by Mr. Fullemann actually verify and confirm the method developed by the author.

In reply to a point raised by Mr. Kauffmann, when a piston is cooled by circulation of oil against the underside of the crown, the heat flow through the piston crown is not necessarily increased above that of the uncooled piston, but the flow is diverted from the usual path to the piston rings, the cylinder liner, and cooling water into the piston-cooling oil. This is borne out by heat-balance tests which show that the total heat to jacket water plus the heat to the piston-cooling oil is approximately the same as the total heat to the jacket water in the engine with noncooled pistons. This is, of course, the reason for the much better performance of piston rings in oil-cooled than in uncooled pistons.

In answering Mr. Maleev's discussion, it is not the purpose of the paper to describe methods of determining the volumetric efficiency of a nonsupercharged four-cycle engine. Such methods are well known and tried. The volumetric efficiency of the engine discussed in the paper was found to be 86 per cent, that is, the weight of air taken in was equal to 86 per cent of the weight of a volume of air equal to the piston displacement at intake-manifold conditions. For determination of temperatures, manifold conditions were assumed to be 90 F and 14.5 psia, giving the lowest density at which sea-level performance is usually guaranteed.

In Equation [9] of the paper, the fraction in the bracket is the ratio of air taken in by the nonsupercharged and the supercharged engine. The values were taken from actual tests with inlet conditions of 76 F and 14.5 psia. This ratio is not affected by intake conditions so long as they are the same for both engines. The value 15.8 lb is therefore correct and shows a volumetric efficiency of 86 per cent.

The tests showed that cylinder heating is not inversely proportional to the first power of the weight of air. Evidently the heat-transfer rate from the cylinder walls to the air increases with its density. Equation [9] satisfied the conditions found in this particular engine at 4 lb supercharging pressure. The exponent 0.71 may be determined for engines having other characteristics.

Mr. Maleev outlines three methods by which he portends to prove that the author's formulas for determination of T_1 are incorrect.

The first is based on the opinion that a cylinder heating of 110 F is too high and that Equation [1] of the paper gives total heating. No proof is submitted.

The second method is correct, but Mr. Maleev uses a wrong value for the air weight; 15.8 must be changed to 15.4 as just explained for intake conditions of 14.5 lb psi. and 90 F. Then, Mr. Maleev's formula gives

$$T_1 = (15 \times 144 \times 1.508)(0.08558 \times 53.35 \times 0.00355 \times 55.38) = 683 \text{ R}$$

Now using Equation [2] for residual gas heating, t_{rh} is found to be 22 F (actually it is slightly greater because the formula purposely neglects the difference in specific heats, the error being small), and since

$$T_1 = T_{atm} + t_{ch} + t_{rh} \ldots\ldots\ldots\ldots\ldots [2]$$

$$t_{ch} = 683 - 550 - 22 = 111 \text{ F}$$

In the third method, it is assumed that the weight of the charge must remain the same during that part of the cycle between points 1 and 5.

Thus from

$$\frac{P_1V_1}{W_1R_1T_1} = \frac{P_5V_5}{W_5R_5T_5}$$

$$V_1 = V_5$$

$$W_1 = W_5$$

and therefore

$$T_1 = \frac{P_1 R_5 T_5}{P_5 R_1}$$

In this

$$R_1 = 53.35$$
$$R_5 = 55.38$$
$$P_1 = 15 \text{ psia}$$
$$P_5 = 32.4 \text{ psia}$$
$$T_5 = 1420 \text{ R}$$

so that

$$T_1 = \frac{15 \times 55.38 \times 1420}{32.4 \times 53.35} = 683 \text{ R}$$

The relation given by Mr. Maleev neglects the fact that the composition of the working substance changes during the cycle.

Mr. Maleev questions the correctness of the author's findings that when residual gas pressures are equal to P_1 the residual gases have no effect on volumetric efficiency in a four-cycle engine.

Since we are considering the effect of clearance gases only, let us assume that there is no heat flow between the fluid in the cylinder and the surrounding walls; also assume that P_1 equals P_{atm}. Let us say that the clearance volume is 8 unit volumes and the piston displacement 100 unit volumes. Let the temperature of the residual gases be 1000 R and the air 500 R.

At the end of the suction stroke the total cylinder volume will be 108 units. The temperature of the mixture of gas and air will be 519.23 R (Equation [2]).

The original volume of the air has then expanded to $\frac{100 \times 519.23}{500} = 103.845$ unit volumes. The gas has contracted to $\frac{8 \times 519.23}{1000} = 4.155$ unit volumes.

The sum of the volumes at 519.23 F is 108, that is, as the incoming air mixes with the residual gases, the latter are cooled and the air is heated. However, the contraction of the gases equals the expansion of the air. Therefore neither residual gas temperatures nor volume has any effect upon the volumetric efficiency.

Due to the difference in specific heats of the residual gases and air, there is actually a small loss in volumetric efficiency, but calculations show it to be less than 1 per cent.

If the exhaust back pressure is greater than the suction pressure P_1, the volumetric efficiency is reduced by the expansion of the gases from P_6 to P_1; however, by making proper substitutions in Equation [2], the t_{rh} value can be calculated.

If the manifold pressure is throttled below atmospheric pressure, then manifold pressure and temperature must be substituted for P_{atm} and T_{atm} in Equations [1] and [2].

Mr. Smith is correct in stating that the engine operating temperature is a function of velocity, density, and time, as well as temperature. However, when considering the same engine, supercharged and nonsupercharged, time and velocity factors remain constant. At low velocities, such as prevail in quiescent combustion chambers, the heat flow increases very little with density. During the suction stroke when heat flow reverses, the effect would be to increase the cooling of the walls, thus canceling the heating effect on the compression stroke. For lack of exact knowledge on the subject, it is assumed that any increase in heat flow, due to density during expansion and exhaust strokes, will be balanced by the heat removed from the walls by the scavenging air.

The compression exponent gives an indication of the rate of increase in heat transfer. When the suction pressure is increased from 14.5 to 18.5, the actual diagrams show that the compression exponent is greater in the supercharged engine, and it was found that the total heat lost at the end of the compression is only 5 per cent more than in the nonsupercharged engine. Had the exponents remained the same, the heat loss and therefore the heating of the walls would have increased by 28 per cent for the same mean temperature. Investigations of heating of the air during the suction stroke, when the heat flow is from the walls to the air charge, also indicate that the heat transfer is only slightly affected by change in density. In the nonsupercharged engine, this heat flow was found to be 2.33 Btu per stroke (110 deg heating). In the supercharged engine the heating was found to be 76 deg, indicating a heat flow of only 2 Btu or a reduction of heat flow with the higher density. This is probably explained by the fact that about 0.75 Btu is removed by the scavenging air prior to the suction stroke.

Engines with high turbulence, such as the dual-spray type SI or Ricardo Comet, probably will be affected more by the density increase caused by the high scouring velocity. The test engine with quiescent combustion chamber, as just explained, shows very little increase in heat flow with increased density.

To correct for density, when calculating a high-turbulence engine, the average gas-film heat-flow coefficient must be determined from the calculated heat loss on the compression stroke. This can then be converted into equivalent temperature difference between mean-stroke temperature and the cooling water. The cycle mean temperature, which will transmit the same heat to the

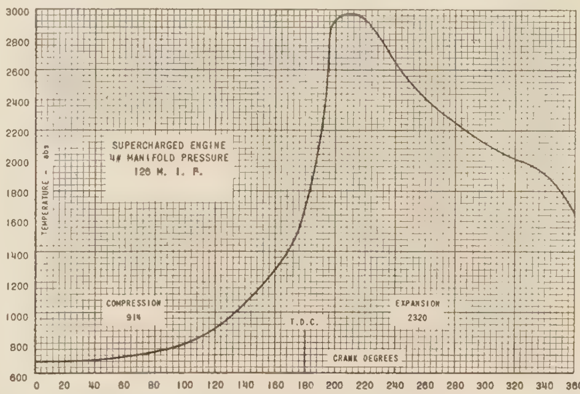


Fig. 10 Compression and Expansion Temperatures From Indicator Diagram of a Supercharged Engine

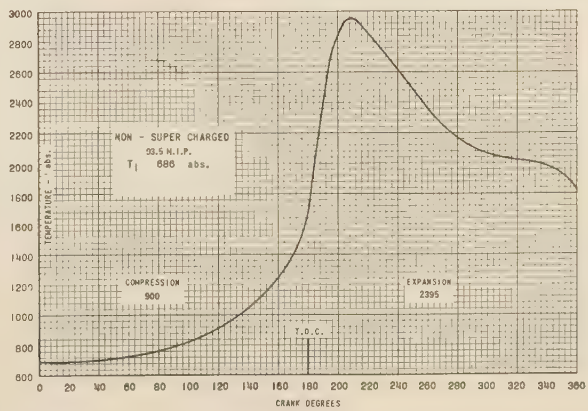


Fig. 11 Compression and Expansion Temperatures From Indicator Diagram of a Nonsupercharged Engine

walls as the nonsupercharged engine, can then be calculated. This is rather involved and the method has not been developed.

The simplest method of all is to measure the heat loss to the water jacket. Great accuracy is required, because a small difference in heat represents a large increment of power.

Mr. Smith correctly states that the proper rating of the supercharged engine is at the load where the heat loss to the water jackets is equal to that obtained from the nonsupercharged engine.

As shown in Fig. 4 of the paper, T_1 in the supercharged engine does not reach the value of T_1 which is calculated for the nonsupercharged engine until the pressure is 6 lb gage. The final compression temperature in the supercharged engine reaches the temperature in the nonsupercharged engine at 4 lb gage, however, due to the increase in compression exponent. The effect on ignition delay will therefore be negligible at 4 lb pressure. Actual diagrams also show that the combustion progresses along similar paths in both engines, Figs. 10 and 11 of this closure.

We know from a study of the diagrams that heat is transferred from the air to the cylinder walls during the compression stroke, and, since the mean temperature of that stroke is about 900 R, it follows that the walls must be cooler.

Heat flow from the walls to the air can only occur during that part of the suction stroke when the temperature of the air is relatively low, Fig. 12.

Fig. 8 of the paper is plotted for theoretical diagrams, as explained. When compared with actual diagrams, the cycle mean temperature at equal loads was found to be too low. Fig. 9 of this closure, is plotted from theoretical diagrams having compression exponents equal to the average exponent of the actual diagram and expansion exponents selected to give the same cycle mean temperature as the actual diagram at equal loads, supercharging pressure, and T_1.

Only one point, namely, 4 lb supercharging pressure is actually calculated. The loads for other pressures are interpolated and extrapolated, but only as to T_1 which is calculated for each pressure by using Equation [9] of the paper. When T_1 is determined the rating points are fixed. Moving the 6-lb rating to 150 per

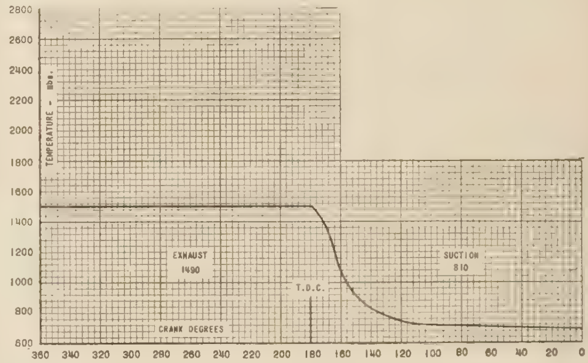


Fig. 12 Exhaust and Suction Stroke of the Nonsupercharged Engine

cent in Fig. 8, as suggested by Mr. Smith, can be done only by lowering T_1 from 689 to 660. The temperature rise through the blower is 74 F for 6 lb pressure, and, with atmospheric temperature 550 R, this would reduce the cylinder heating from 65 F as calculated by Equation [9] to 36 F. It is not probable that the cylinder heating will drop from 76 F as found at 4 lb pressure to 36 F at 6 lb pressure, when the blower capacity at 6 lb increases only about 16 per cent over that at 4 lb.

Inlet-Air-Temperature Correction in a Roots Supercharger

By F. A. Hiersch,[1] Ann Arbor, Mich.

This paper gives an Equation [1], in which volumetric efficiency may be substituted to obtain an inlet-air-temperature correction in a Roots supercharger. A typical application of Equation [1] is given at the close of the paper. Calculations from extensive test data[2] indicate that the use of this equation yields uniformly consistent values of *n* over the pressure range given. Comparison with the values of *n*, obtained by the use of a "speed coefficient,"[3] indicates a constant numerical difference in most cases; the correction by Equation [1] being characterized by lower numerical values of *n* than those obtained by means of the speed-coefficient correction.

Compression efficiency is usually determined on the basis of temperature and pressure measurements taken at the compressor inlet and outlet. The usual method of measuring the inlet-air temperature does not account for the effect of volumetric efficiency on the intake-air temperature.

When the volumetric efficiency is less than 100 per cent, the slippage of air from the outlet back into the next inlet charge will raise the temperature of the inlet air from T_1 to T_1', where T_1' is given by the equation

$$T_1' = T_1 + \Delta T(s + s^2) \qquad [1]$$

and where T_1 = intake-air temperature as usually measured

T_1' = corrected air temperature

$\Delta T = (T_2 - T_1)$

T_2 = outlet-air temperature

s = slippage per cent in decimal form

It follows from Equation [1] that

$$\frac{\Delta T'}{\Delta T} = (s + s^2) \qquad [2]$$

where

$$\Delta T' = (T_1' - T_1)$$

and

$$\frac{\Delta T_1'}{\Delta T} = [1 - (s + s^2)] \qquad [3]$$

where

$$\Delta T_1' = (T_2 - T_1')$$

These equations may be used to express the percentage of over-all temperature rise due, respectively, to slip and to compressor work in one cycle.

Likewise, $\frac{T_2}{T_1}$, $\frac{T_2}{T_1'}$, and $\frac{T_1'}{T_1}$ give the over-all temperature-rise ratio, the net compressor-temperature-rise ratio and the slip-temperature-rise ratio, respectively.

Having separated these compression components, we may allocate work input and efficiencies accordingly.

Derivation of Equation [1]

In a Roots supercharger, when the ratio of the discharge to inlet pressure is increased while all other variables remain constant, the air-temperature rise is greater than the pressure rise would lead one to expect. We may assume that this occurs primarily because air, heated by initial compression, slips from the discharge side of the impeller into the intake air, where it is assumed to mix at constant pressure, and increases the temperature of the intake charge. This mixture of slippage air and the new charge will be compressed and discharged in the second revolution, and a percentage of the mixture will slip back and be compressed in the third revolution, and so on.

The first slippage volume will contain particles of air from the first discharge volume only; the second slippage volume will be a mixture of particles from the first and second revolutions, etc. Each succeeding revolution will contain a smaller percentage of the first volume of air to slip back, but the final particle of the first slippage volume present in the *n*th revolution will have received *n* times the heat of compression received by a particle discharged in 1 revolution. The portion of the first volume of slippage of air present in the *n*th revolution is equal to the percentage of slippage air raised to the *n*th power. The total slippage volume in the *n*th revolution will be a mixture of particles from the first *n* slippage volumes, as given by Equation [4]. In the following development these arguments are applied in each revolution to every particle of air which slips back from discharge to intake.

If we neglect casing-temperature effects, the Joule-Thomson effect, frictional and eddying losses, and velocity, temperature, and pressure changes in the air during throttling back to intake, and maintain constant revolutions, we may then consider only the temperature change in the intake charge due to the addition of the percentage of the theoretical discharge air which slips back to the intake side of the impeller.

We might likewise determine the theoretical pressure rise in the air confined between the impeller and casing wall, assuming the addition of the slippage air to the geometric-displacement volume, and adiabatic-pressure equalization between the slippage air and the new charge. However, the physical situation at this point relative to pressure changes is not easily evident.

Assuming the conditions given, we may now determine the slippage-air weight and heat content in each revolution from 1 to *n*.

Let x per cent of the geometrical displacement equal slippage loss in the first revolution; then the volume of the displacement in the first revolution, which is returned to the intake charge in n revolutions, is

$$x_1 + x_1^2 + x_1^3 + x_1^4 + x_1^5 + \ldots\ldots + x = \frac{x_1(1 - x_1^n)}{1 - x_1}$$

[1] Department of Mechanical Engineering, University of Michigan.

[2] Data taken from "The Comparative Performance of Roots Type Aircraft-Engine Supercharger as Affected by Change in Impeller Speed and Displacement," by M. Ware and E. E. Wilson, U. S. National Advisory Committee for Aeronautics, Technical Report, No. 284, 1928.

[3] Ibid., p. 9.

Contributed by the Oil and Gas Power Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

Reading horizontally, Table 1 enables us to follow the history of an initial slippage volume of x per cent, as it is carried through n revolutions. Reading vertically gives the slippage content in each revolution.

Now, if the per cent slippage is small, powers of x, higher than, say, x_{ij}^5 may be neglected; however, whether the per cent slippage is large or small, we may solve for x_{ij} as follows:

TABLE 1

Rev. no. / Charge no.	1 2 3 4 5 6 7 8 $j \ldots\ldots n$
1	$x_{11} + x^2_{12} + x^3_{13} + x^4_{14} + \ldots\ldots + x^n_{1n}$
2	$x_{22} + x^2_{23} + x^3_{24} + \ldots\ldots + x^{n-1}_{2\,n-1}$
3	$x_{33} + x^2_{34} + x^3_{35} \ldots\ldots$
4	$x_{44} + x^2_{45} + x^3_{46} + x^4_{47} \ldots\ldots$
i	$\cdot \quad \cdot \quad \cdot \quad \cdot \quad x_{ij}[j-(i-1)]$
.	$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$
$n-4$	$x_{n-4,\,n-4} + x^2_{n-4,\,n-3} + x^3_{n-4,\,n-2} \ldots\ldots + x^5_{n-4,5}$
$n-3$	$x_{n-3,\,n-3} + x^2_{n-3,\,n-2} \ldots\ldots + x^4_{n-3,4}$
..	.. .. ..
..	.. ..
n	$x_{n,1}$

NOTE: The jth column gives the slippage content in the $(i + 1)$th intake charge.

TABLE 2 HEAT RETURNED PER REVOLUTION

Rev. no. / Charge no.	1 2 3 4 5 6 7 $\ldots\ldots$ i
1	$h_{11} + 2h_{12}x + 3h_{13}x^2 + \ldots\ldots + nh_{n}x^n$
2	$h_{22} + 2h_{23}x + 3h_{24}x^2 + \ldots\ldots + (n-1)h_{2,\,n-1}x^{n-}$
3	$h_{33} + 2h_{34}x + 3h_{35}x^2 + 4h_{36}x^3 + \ldots\ldots + (n-2)h_{3,\,n-1}x^{i-2}$
4	$h_{44} + 2h_{45}x + 3h_{46}x^2 + 4h_{47}x^3 + \ldots\ldots$
5	$h_{55} + 2h_{56}x + 3h_{57}x^2 + 4h_{58}x^3 + 5h_{59}x^4 + \ldots\ldots$
:	: : : : :
$n-1$	$+ 2h_{n-1},\, x_2$
n	$h_{n,1}$

$$(1 \text{ to } n):\ x_{11} = x_{22} + x^2_{12} = x_{33} + x^2_{23} + x^3_{13} = x_{44} + x^2_{34} + x^3_{24} + x^4_{14} = x_{55} + x^2_{45} + x^3_{35} + x^4_{25} + x^5_{15}, = \text{etc.}$$

where the successive expressions represent the slippage-air content in the corresponding intake charges after the first, second, third, fourth, and fifth revolutions

$$= \frac{x_i\,(1 - x_i^n)}{1 - x_i} = \text{slippage}$$

$$= \frac{x_i(1 - 0)}{1 - x_i}$$

for large n only.

The effect of the first several revolutions will ordinarily be small in comparison with the remaining revolutions in a test run, and, since the particles treated are identical in properties, we may drop the subscripts and write

$$\frac{x(1 - x^n)}{1 - x} = s = (\text{slippage}) = \frac{x}{1 - x} \ldots\ldots\ldots [4]$$

for large values of n only.

$$\therefore x = \frac{s}{1 + s} \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

(x may be described as that portion of the slippage air which has been returned only once. It is therefore the largest of the slippage portions making up the slippage sum s.)

Having solved for x in this expression, we now write an expression for the heat content of each particle as it is involved historically in the slippage. We note that if x per cent slips back in 1 revolution, then $(x)\cdot(x)$ per cent of that same particle will slip back in the second revolution, etc., as noted previously. We now note the additional fact that, in returning twice, the particle has been heated twice and, therefore, will have twice the heat content per unit weight added to it in the 2 revolutions. In 3 revolutions $(x)\cdot(x)\cdot(x)$ per cent will have three increments of heat added per unit weight, and so on. Table 2 traces the history of the heat value of each particle of air that is involved in the slippage volume, and an equation is given which sums up the enthalpy of the returned volume, where h_{ij} = enthalpy of the per cent slippage.

The "charge number," Table 2 reading horizontally, gives the history of the enthalpy returned in n successive revolutions, by a single particle starting with a given revolution. The "enthalpy returned," reading vertically, is the Btu returned to the intake in any given 1 revolution. For n revolutions the enthalpy returned in the jth revolution is equal to the enthalpy returned in the nth revolution, or

$$(h_{ij} - h_{1n}) = 0$$

Therefore, for the nth revolution, the heat returned in 1 revolution, is

$$H = (v)\,(\rho)\,(c_p)\,(x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \ldots\ldots + nx^n)$$

where v = displaced volume; ρ = air density; c_p = specific heat of air at constant pressure.

Let $(v)(\rho)(c_p) = k$ then

$$(H - xH) = k\left[\begin{array}{l} x + 2x^2 + 3x^3 + \ldots + nx^n \\ -\left(x^2 + 2x^3 + \ldots\ldots + [n-1]x^n + nx^{n+1}\right) \end{array}\right]$$

$$= k\,(x + x^2 + x^3 + \ldots\ldots + x^n - nx^{n+1})$$

$$(H - xH) = k\left[\frac{x(1 - x^n)}{1 - x}\right], \text{since } nx^{n+1} = 0,$$

therefore $$H = \frac{kx(1 - x^n)}{(1 - x)^2} \text{ Btu per deg F}$$

or $$H = \frac{kx}{(1 - x)^2} \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

for large values of n.

The results of Equation [6] could have been obtained by

arguing that the events experienced by any change in n revolutions are represented identically by the terms in the ith column; therefore we may sum a row instead of a column. However, this process also requires the assumption of equal heat increments per unit weight during each compression, i.e., $(h_{ij} - h_{1,n}) = 0$. Therefore the two processes are equivalent. For the total temperature difference between intake and outlet air

$$\Delta T = (T_2 - T_1)$$

$$\therefore H\,\Delta T \equiv H(\Delta T) = \frac{kx\,\Delta T}{(1-x)^2} \quad [7]$$

substituting Equation [5] in Equation [7]

$$H\,\Delta T = [k\,\Delta T(s + s^2)] \quad [8]$$

Since the temperature rise in the intake charge just before compression results from the enthalpy of the slippage air between the temperatures T_1 and T_2

$$T_1' = \frac{H\,\Delta T}{(\text{lb air disp/rev})\ (c_p)} + T_1 \quad [9]$$

$$= \frac{H\,\Delta T}{Dc_p} + T_1$$

$$= \frac{k\,\Delta T(s + s^2)}{Dc_p} + T_1 = \frac{Dc_p\,\Delta T(s+s^2)}{Dc_p} + T_1$$

$$T_1' = T_1 + \Delta T(s + s^2) \quad [1]$$

Equations [2] and [3] follow, as given previously.

Applications of Equations

Example: 8.25-in. supercharger; displacement = 0.382 cu ft per revolution; at 2935 rpm, measured delivery = 0.782 lb air per sec; T_1 = 519 F abs; T_2 = 624.36 F abs; p_2 = 14.7 psia; p_1 = 8.79 psia. Weight of air in intake at 519 F abs

$$w = \frac{(8.79)\ (144)\ (0.382)}{(53.35)\ (519)} = 0.01745 \text{ lb per revolution (displaced)}$$

$$w = \frac{(0.782)\ (60)}{2935} = 0.01600 \text{ lb per revolution (measured)}$$

volumetric efficiency = 91.65 per cent

$\therefore$ per cent slip = 8.35

$\Delta T = (T_2 - T_1) = 105.36$ F

Applying Equation [1]

$$T_1' = 519 + (106)\ (0.0835 + 0.0835^2)$$

$$= 519 + 9.54$$

$$= \underline{528.54\ °R} \quad [10]$$

$$\frac{\Delta T'}{\Delta T} = s + s^2 = \underline{0.0954} \quad [11]$$

$$\frac{\Delta T'}{\Delta T} = 1 - (s + s^2) = \underline{0.9046} \quad [12]$$

$$\frac{T_2}{T_1} = \frac{624.36}{519} = 1.203 \quad [13]$$

$$\frac{T_1'}{T_1} = \frac{528.54}{519} = 1.018 \quad [14]$$

$$\frac{T_2}{T_1'} = \frac{624.36}{528.54} = 1.180 \quad [15]$$

The rise in temperature between T_1', the corrected inlet-air temperature, and T_2, the discharge temperature, gives a temperature rise indicative of work done on the air in 1 delivery revolution, while the over-all change in temperature T_1 and T_2 must include work of compression done in 1 revolution plus work done on portions of slippage air during preceding revolutions.

For a polytropic

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} \quad [16]$$

Using the observed values of T and p

$$(1.671)^{\frac{n-1}{n}} = 1.203 \qquad n = 1.562$$

Using Equation [16] with the "speed coefficient," $C = 1.01$

$$C\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} \quad [17]$$

$$(1.01)\ (1.671)^{\frac{n-1}{n}} = 1.203$$

$$n = 1.514$$

Using T_1', the corrected value of T_1

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1'} \quad [18]$$

$$\frac{(14.7)}{(8.79)}^{\frac{n-1}{n}} = \frac{624.36}{528.54}$$

$$(1.671)^{\frac{n-1}{n}} = 1.180$$

$$n = 1.484$$

Using the data given by Ware and Wilson,[2] further calculations indicate that the values of n, obtained by substitution in Equations [1] and [18] in this paper, are usually lower than those obtained by use of Equation [17], in which a "speed coefficient" is applied.

It appears that Equation [1] may be useful in establishing performance standards for positive-displacement blowers including double-acting multistage reciprocating compressors.

Using an equation giving optimum rotor diameter[4] in terms of known factors, it should be possible to indicate the maximum "inlet-air-temperature correction" expected in a given design.

Discussion

W. L. H. Doyle.[5] Thinking particularly in terms of engine development, it is desirable to be able to compare performances of various makes and sizes of Roots type blowers on a common basis. It is significant that at this time there are decided differences in manner of measurement and interpretation of test results among the various blower manufacturers, and this obviously handicaps the engine builder where these blowers are parts of his supercharged-engine developments.

This condition to a large degree is due to the lack of a suitable standardized scheme for test-installation assemblies and to unsuitable instrumentation technique. In most of the test assem-

[4] "Das Rootsgebläse als Ladungsdichter an Mercedes-Benz Motern," by K. Schopper, *Automobiltechnische Zeitschrift*, vol. 10, 1935, p. 28.

[5] Research Engineer, Caterpillar Tractor Company, Peoria, Ill. Mem. A.S.M.E.

blies, it is the practice to apply some form of silencing and interconnecting piping for attaching to the actual blower intake and discharge. These test-assembly setups are never representative of the final blower installation on the engine to which they are ultimately applied, and in fact are frequently materially altered by the same builder from one test to another. Thus complex and unrepresentative surge- and flow-reaction conditions are frequently introduced at the blower intake and discharge, which are peculiar to the particular test assembly. The manner of determining pressures and temperatures is also not suitably representative of actual conditions. These various factors serve to explain some of the confusing discrepancies between test performances of the various sizes and makes of blowers and the performances in the actual engine assemblies.

This type of compressor is assuming a significant position as a supercharged-engine auxiliary. In the interest of its development, it becomes increasingly important, that results of testing of the device be obtained within given limits of accuracy and that they be expressed on a basis such that all performances may be made truly comparable. To this end the writer would recommend that steps be taken to set up a test procedure which will establish suitable standardized general details for test assemblies and for the important considerations involving instrumentation technique. It is further suggested that reported delivery rates be expressed in pounds and this, together with all other pertinent data, be plotted to the ratio of pressure rise through the blower as the common variable or abscissa, which brings out the significant factor of pressure depression at the blower intake.

Author's Closure

The author wishes to express his concurrence with Mr. Doyle in the need for "a standardized scheme for test-installation assemblies."

In the very near future, the author hopes to submit a paper to the A.S.M.E. entitled: "Proposed Expressions for Roots Supercharger Design Efficiencies," applied to blowers without and with built-in compression ratios. Should these expressions prove acceptable to manufacturers and users, they may aid in giving standard evaluations of various designs.

A Practical Way to Prevent Embrittlement Cracking[1]

By A. A. Berk[2] and W. C. Schroeder[3]

Suggestions are given for applying methods of preventing embrittlement cracking which are based upon the results of a great number of tests with the embrittlement detector on operating boilers. A summary is given in tabular form of more than 900 plant tests of this nature on variously treated boiler waters. The method of water treatment, the sources of supply of chemicals, and the most desirable conditions of application for sodium nitrate, quebracho extract, waste sulphite liquor, and zero-caustic alkalinity are discussed. These methods of chemical treatment for preventing embrittlement are compared.

The Embrittlement Symposium[4] at the 1941 Annual Meeting of The American Society of Mechanical Engineers indicated that several methods have been developed for preventing embrittlement cracking which gave satisfactory results in laboratory and plant testing. It is now possible to offer definite, practical suggestions for the application of these methods, based upon the results from a large number of tests with the embrittlement detector on operating boilers.

The embrittlement detector illustrated in Fig. 1 establishes under a test specimen the conditions that may cause embrittlement cracking in riveted seams, that is, boiler water concentrated in contact with highly stressed metal. Attached to a boiler, the apparatus exposes the specimen to the boiler water in the same manner as a riveted seam may be exposed under operating conditions. Embrittlement cracking of the test specimen indicates that the boiler water can cause failure in the boiler, provided similar conditions of concentration and metal stress exist in the seam. The detector test therefore permits differentiation between boiler waters that are capable of producing embrittlement cracking and those that are not, as well as testing of methods of chemical control to eliminate this tendency in the first class of boiler waters.

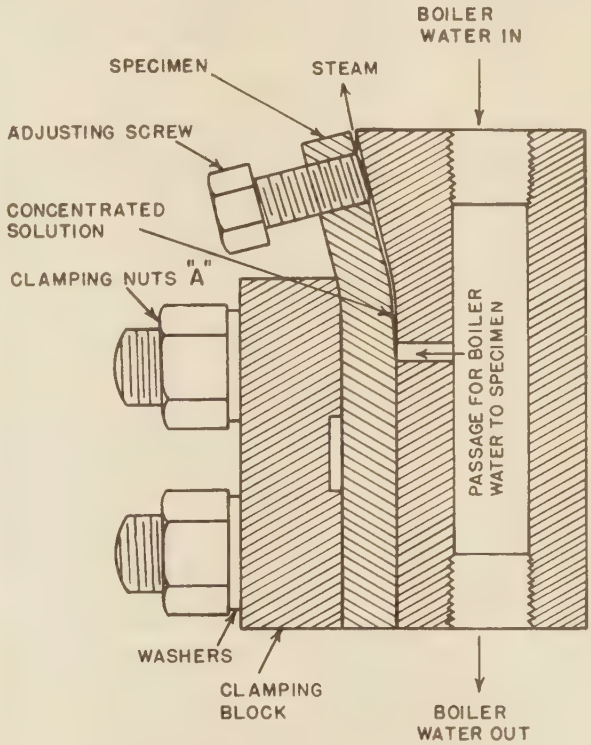


Fig. 1 Embrittlement Detector
(U. S. Patents 2,283,954 and 2,283,955.)

Table 1 summarizes the results of more than 900 plant tests on variously treated boiler waters. Thirty-five tests, in which none of the specimens was cracked, have been excluded from this table because the detector was known to have been improperly operated. Except for 187 specimens, exposed on 15 switch engines of the Chesapeake & Ohio Railway, the tests were all run by plant personnel on stationary boilers operated at pressures from 100 to 1450 psi.

Occasional difficulties were experienced in the operation of the detector, which was probably unavoidable in field testing by so many individuals in more than 200 plants; but less than 5 per cent of the tests had to be discarded. The co-operation has been wholehearted, and the results have been satisfactorily consistent. This paper presents the findings not of laboratory research but rather of a collective effort by engineers to solve a serious operating problem. The authors wish to thank the organizations and individuals co-operating in this work for their assistance in the accumulation of this information.

The data are grouped to show the results of the most satisfactory methods for controlling embrittlement cracking. Each method includes the concentrations or ratios that prevented cracking within the suitable pressure range. Sodium nitrate, quebracho extract, and lignin-containing paper-mill waste inhibited intercrystalline attack by caustic soda. The treatment designated as "zero-caustic alkalinity," on the other hand, substituted phosphate alkalinity for caustic alkalinity, and therefore eliminated the major chemical which causes cracking and made the use of an inhibitor unnecessary.

The chemicals used in this work to prevent cracking were present in the boiler water in a definite minimum ratio to the total alkalinity,[5] expressed as sodium hydroxide. The concentrations

[1] This paper is published by permission of the Director of the Bureau of Mines, U. S. Department of Interior.

[2] Associate Chemist, Eastern Experiment Station, Bureau of Mines, College Park, Md.

[3] Assistant Chief, Fuels and Explosives Service, Bureau of Mines, Washington, D. C. Mem. A.S.M.E.

[4] Trans. A.S.M.E., vol. 64, 1942, pp. 393–444.

Contributed by the Joint Research Committee on Boiler Feedwater Studies and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

[5] Since caustic alkalinity is the primary factor causing embrittlement cracking, the ratio should be expressed in terms of the concentration of this substance. There are several reasons, however, for including carbonate alkalinity in the ratio. These are (1) the possi-

were determined by chemical analysis rather than by the amount added, as losses from decomposition and coprecipitation could occur. References to analytical procedures that were used to determine inhibitor concentrations in this work are included in the table.

This paper briefly describes the method of water treatment, the source of supply for the chemicals, and the most desirable conditions of application for sodium nitrate, quebracho extract, waste sulphite liquor, and zero-caustic alkalinity. It also compares these methods of chemical treatment for preventing embrittlement.

Sodium Nitrate

The first line in Table 1 shows that 117 specimens were not cracked when the boiler water contained relatively high concentrations of sodium nitrate. In many of these tests the nitrate was added to waters that had previously caused cracking. In others it was naturally present in the feedwater. These data show that boiler waters containing a minimum ratio of sodium nitrate to total alkalinity of 0.3 did not cause embrittlement cracking of detector specimens.

The table also summarizes the experience of the Chesapeake & Ohio Railway where the nitrate treatment is in use on the entire railroad. Only one specimen was cracked with an approximate minimum concentration of sodium nitrate equal to 40 per cent of the total alkalinity, while 88 specimens were not cracked. A higher ratio is recommended for railroads than for stationary plants to offset the added uncertainties of road stress and water conditions inherent in the operation of locomotives. This railroad undertook the work, not from any academic interest in the embrittlement problem, but because up to 1939 an average of 30 locomotive boilers suffered embrittlement cracking and had to be repaired each year. Introduction of the nitrate treatment resulted first in the essential elimination of cracking in the embrittlement-detector specimens and, subsequently, a decrease in cracking in locomotive boilers, corresponding to a saving of more than 90 per cent of the engine time lost and the material and man-hours required for such repair work.

Sodium nitrate is an inexpensive inorganic chemical and is relatively easily handled and controlled. It is thermally stable and has not adversely affected boiler operation in the range 125 to 400 psi in which it has been used. In alkaline boiler water, it is apparently inert to sodium sulphite and the other salts usually present.

Quebracho Extract

The results obtained with quebracho extract are almost as satisfactory as those with nitrate. The 109 specimens not cracked were exposed on boilers operated at 130 to 700 psi. Only five specimens were cracked when a quebracho - total alkalinity ratio of 0.4 was reported to be maintained. Some of the difficulties with this material may have been due to improper analytical procedures or failure to analyze often enough to verify the maintenance of desired concentrations. As better control of the quebracho treatment is established upon the basis of proper analytical determinations, these exceptions should become less frequent. Concentrations present in boiler water can readily be determined by a rapid colorimetric procedure (reference *l* in Table 1).

bility that carbonate will lose carbon dioxide during evaporation of the boiler water in the seam and thus become hydroxide; (2) when a sample of boiler water is taken for analysis, some sodium hydroxide may be converted to carbonate by the carbon dioxide in the air. On the other hand, phosphate alkalinity should not form part of the ratio, and the total alkalinity, if it contains much phosphate, may be corrected for the amount of this substance present. Basing the ratios on the total alkalinity leans toward safety and an excess of the inhibiting chemical.

Quebracho extract is a tannin obtained from South American trees of the same name. The relatively crude extracts[6] have been found to be much superior to processed spray- or shelf-dried products. In addition to inhibiting cracking, quebracho frequently helps to retard corrosion and the formation of scale.

Waste Sulphite Liquor

There are no detector data for the use of waste sulphite liquor and other lignin-containing products in stationary boilers. The Chesapeake & Ohio Railway tested this method of embrittlement control on locomotive boilers, however, before turning to nitrate. The treatment was not entirely satisfactory in that protection could not be obtained at every location. Table 1 shows that while 60 specimens were not cracked 12 specimens were cracked when a ratio of inhibitor to alkalinity as sodium hydroxide of 0.4 was maintained.

Waste sulphite liquors are by-products of the wood-cellulose industries. The active ingredients are lignin-sulphonate compounds. Both concentrated liquors and derived solids are marketed.[7] While these have been used with apparent success in a very large number of locomotive boilers, they cannot be recommended at the present time for stationary boilers because of reported foaming difficulties.

Zero-Caustic Alkalinity

Although only eight specimens have thus far tested the treatment that eliminates caustic alkalinity, none has cracked. The operating range was 300 to 800 psi. All the tests were run on boilers using evaporated make-up, for which the method is especially suitable. The treatment is not, however, restricted to such plants, as caustic soda can be neutralized by mineral acids and so converted to mineral salts such as phosphates and sulphates.

Since caustic is the primary chemical responsible for embrittlement, its elimination from the boiler water would be the most obvious method of controlling cracking. A low concentration of alkalinity is considered desirable, however, to prevent corrosion and incrustation during boiler operation. The concentration of alkalinity is indicated by the difference between the pH of the boiler water and the neutral value of 7. The pH is raised above 7 by increasing the concentration of hydroxide ions. In so far as the environment is concerned, it makes no difference whether these hydroxide ions are furnished by caustic soda or some other chemical.

There are chemicals, such as trisodium phosphate, which react with water to release hydroxide ions. Concentration by evaporation of dilute boiler waters in which these chemicals are present reverses the reaction and the chemicals are reformed. The concentration of hydroxide ions present in water because of such chemicals, therefore, never increases very much as the result of evaporation. On the other hand when a boiler water containing caustic soda evaporates, the hydroxide ions may concentrate sufficiently to cause embrittlement cracking. Chemicals such as trisodium phosphate neither cause nor prevent cracking. The zero-caustic alkalinity treatment involves substituting such chemicals for caustic soda as the source of alkalinity in the boiler water.

[6] This investigation has shown that satisfactory quebracho extracts to prevent cracking are Argam (Crown) brand, distributed by The Tannin Corporation of America, Wilmington, Del., and Luna brand, distributed by International Products Corporation, 90 West Street, New York, N. Y.

[7] Spruce extract, distributed by Robeson Process Corporation, 500 Fifth Avenue, New York, N. Y. and MU, distributed by Marathon Chemical Company, Rothschild, Wisconsin, have proved to be satisfactory sources of lignin-sulphonate compounds. Waste sulphate liquors which result from alkali sulphide rather than neutral sulphite extraction of lignin from wood are not usually satisfactory.

TABLE 1 OVER-ALL SUMMARY OF PLANT TESTS WITH THE EMBRITTLEMENT DETECTOR[a]

Principal inhibitor present	Specimens not cracked	Ratio recommended,[b] inhibitor/total alkalinity	Specimens cracked, ratio maintained	Approximate useful[c] pressure range	Method of analysis
Sodium nitrate ($NaNO_3$)	117	0.3	0[d]	to 400 psi	(*k*)
Sodium nitrate ($NaNO_3$)[g]	88	0.4	1[e]		
Quebracho extract	109	0.4	5[f]	to 700 psi	(*l*)
Waste sulphite liquor[g]	60	0.4	12[h]	to 250 psi	(*l*)
Zero-caustic alkalinity	8	caustic 0–8 ppm	0	to 800 psi	(*m*)
Miscellaneous[i]	19	....	..	.....	..
No known inhibitor present in sufficient quantity[j]	184	....	286	.....	.

[a] There are 35 tests eliminated from this tabulation because the detector was known to be improperly operated or because the chemical conditions were entirely unknown.
[b] In the ratios "total alkalinity" represents the total alkalinity in terms of sodium hydroxide. Methods of analysis are given in reference (*m*), pp. 94–102.
[c] The detector has not been used at pressures below 100 psi.
[d] Thirteen specimens were cracked when the ratio was 0.2 or smaller.
[e] Three specimens were also lightly cracked when the ratio was not maintained.
[f] Thirty-seven specimens were cracked while lower ratios were maintained; no specimens cracked at pressures below 250 psi when a ratio of 0.3 was maintained.
[g] Data from tests on Chesapeake & Ohio Railway locomotive boilers.
[h] Twenty-three specimens were cracked when the ratio was low.
[i] Seventeen boiler waters contained high concentrations of organics derived from surface-drainage water and having a high tannin equivalent by the tyrosine determination (*l*); the other two contained high chromate concentrations.
[j] No nitrate analysis available for many of these tests.
[k] "Intercrystalline Cracking of Boiler Steel and Its Prevention," by W. C. Schroeder and A. A. Berk, Bulletin 443, Bureau of Mines, 1941, pp. 82–84.
[l] "Determination of Tannin Substances in Boiler Waters," by A. A. Berk and W. C. Schroeder, *Industrial and Engineering Chemistry*, Analytical Edition, vol. 14, June 15, 1942, p. 456.
[m] "Standard Methods of Chemical Analysis," eighth edition, American Public Health Association, 1936, pp. 95–98.

Although other chemicals could be used, trisodium-phosphate alkalinity is the logical substitute for caustic alkalinity, because it introduces no new chemicals. Phosphate has been in common use for many years to prevent scale in most boilers operated at higher pressures.

When phosphate alkalinity is used to replace caustic alkalinity, the method is best controlled upon the basis of the relationship of pH to PO_4 concentration.[8] For maximum economy the alkalinity should be maintained close to the theoretical pH corresponding to the phosphate concentration.[9] Occasional tests for caustic should also be made to insure its absence.[10]

The particular form of phosphate to be used will depend upon the feedwater. Trisodium phosphate, disodium phosphate, monosodium phosphate, and phosphoric acid are compounds that increase progressively in acidity. Provision for the use of two of these substances will increase the flexibility of operation.

In some cases, slightly acid or neutral condensate may be treated by recirculation of part of the boiler water to provide the alkalinity necessary to prevent corrosion of the economizer and feedwater system. In other instances, it may be more desirable to add caustic to the feedwater and an equivalent amount of monosodium or disodium phosphate to the boiler to reduce the caustic alkalinity to zero. No additional chemicals are required to control embrittlement. Total solids in the boiler water are generally decreased, and improvements in steam quality and turbine performance have been reported.

Tests Without Known Inhibitors

The last line in Table 1 shows that of 470 specimens tested under conditions where no known inhibitor was present in the boiler water in sufficient quantity to be included in the other groups, 184 (40 per cent) were not cracked. This figure could indicate that additional unknown inhibitors exist. However, 119 of the 184 uncracked specimens were tested at plants where cracking resulted from additional runs under better control (and in a few cases from longer test intervals). About one half of the remaining 65 specimens represent single runs that have not as yet been repeated, and almost all of the other tests were completed before nitrate was found to be a significant factor. No nitrate analyses are available for a majority of the tests completed before June, 1941.

Embrittlement cracking in a test specimen is invariably caused by an unsafe boiler water, but absence of cracking may be due to improper operation of the apparatus. It is therefore significant that more than one half of the specimens that were not cracked while the boiler water was properly conditioned by one of the methods in the table came from plants where cracking had resulted in companion tests of the uninhibited water.

Comparison of Methods and Conclusions

Sodium nitrate is relatively inexpensive compared with quebracho extract and phosphate. On the other hand, unlike these other substances it does not affect the character of precipitated solids[11] and therefore would not help to prevent the formation of scale. Furthermore, its usefulness has not as yet been determined at pressures above 400 psi, as compared to 700 psi for quebracho and at least 800 psi for the treatment that produces zero-caustic alkalinity. The lignin compounds are inexpensive but inferior to nitrate and quebracho by which they are largely being replaced.

Sodium nitrate and quebracho extract have stopped the embrittlement cracking of highly stressed steel specimens in embrittlement detectors on operating boilers. The maintenance of definite minimum ratios of the concentrations of these chemicals to the concentration of total alkalinity was required. By reducing the concentration of caustic to approximately zero, the need for an inhibitor was eliminated. Sodium-phosphate al-

[8] Reference 4, p. 401.

[9] Generally 60-100 ppm PO_4 will correspond to sufficient alkalinity for satisfactory boiler operation.

[10] Reference *m*, Table 1.

[11] Nitrate may be added to the water at any point in the system since it is not affected by chemical-softening processes. Quebracho on the contrary is destroyed by dissolved oxygen and adsorbed by sludge. The latter must, therefore, be added directly to the boiler or to the softened feedwater just before it enters the boiler. Concentrated quebracho-extract solutions (specific gravity of 1.20) are stable in the absence of proteins.

kalinity was successfully substituted for caustic alkalinity to achieve this purpose.

The methods of chemical treatment for preventing embrittlement that have been outlined in this paper are based upon testing with the embrittlement detector attached to operating boilers. Their indiscriminate use is not recommended. On the other hand, where cracking of detector specimens shows a particular supply of waters to be relatively unsafe, treatment is usually desirable. Detector tests should then be made to be sure that it is effective.

Discussion

E. W. Colbeck[12] and P. Hamer.[12] The authors have gathered together the results of a very large number of tests carried out with their embrittlement detector, and the data submitted provide clear evidence of the value of sodium nitrate and quebracho extract in inhibiting cracking under the conditions existing in the detector apparatus, providing this is adjusted according to their instructions. The authors state: "Embrittlement cracking of the test specimen indicates that the boiler water can cause failure in the boiler, provided similar conditions of concentration and metal stress exist in the seam." We would like to know whether the authors are satisfied that the conditions existing in the detector are always comparable with those present in the actual boiler. The writers believe that a condition of unequal distribution of stress[13] is an important factor in causing embrittlement, and that such conditions may not be present in the detector specimen. Again, in view of the authors' statement "absence of cracking may be due to improper operation of the apparatus," is it true that uncracked detector specimens of necessity indicate that the water is safe and will not under any circumstances cause intergranular cracking in the boiler itself?

It has been repeatedly stated that a characteristic feature of embrittlement cracking is the intergranular course pursued by the crack or cracks. We were therefore surprised at the complete lack of any evidence or comment in the paper regarding the nature of the cracking found in the 900 plant tests recorded.

What we have said will indicate that we are seriously disturbed with regard to the claims made for the embrittlement detector. In particular we feel that the following questions require an affirmative answer before this tool can be used with confidence:

1 Do boiler waters which have actually caused embrittlement cracks in the seams or rivets of a working boiler also crack the specimen in the detector, and has this experiment been carried out a sufficient number of times to place the issue beyond doubt or at least make it highly probable?

2 How often has the following sequence of events been observed:

(*a*) The detector fitted to a working boiler cracks.

(*b*) Cracks subsequently develop in the actual plates or rivets of the boiler.

3 Can a "safe" water be made "dangerous" so as to crack the detector specimen at will, and how can this be done?

In view of the human factor in the operation of the detector and the fact that it is impossible to know the mechanical and chemical conditions existing in a boiler seam, we do not think that absence of cracking in detector or boiler can be taken as evidence of the effectiveness of the detector as an indicator.

On the other hand, we must agree that, if, as a result of investigations with the detector, a substance such as sodium nitrate is shown to have marked inhibiting properties, and, subsequently, as a result of the addition of this substance to water in working boilers, the number of cracked boilers is substantially reduced, as is claimed on one of the American railroads, this is strong evidence of the usefulness of the detector.

Speaking frankly, we are uneasy about the position now reached with respect to sodium nitrate because it seems to us that it has a certain strong resemblance to a position reached with respect to sodium sulphate many years ago, and subsequently vacated. In this case another piece of apparatus was used as the guiding star. We still recommend that the ratio $Na_2SO_4/NaOH$ in the boiler water shall be maintained above 2.5 or at any rate in the nonembrittling area as defined by Straub's curves, and in the twenty or thirty cases of boiler embrittlement which we have investigated and confirmed in the last 4 or 5 years, in no case have these recommended conditions been consistently maintained. This is, of course, negative evidence.

The entire question of embrittlement of boilers is in our opinion still in a most unsatisfactory state from the point of view of the boiler engineer, and we therefore hope that the authors' recent discoveries will be amply confirmed in practice.

We have endeavored to make our contribution to the paper constructive, but we are fully conscious that we are 3000 miles from the scene of action, and that much may be said, done, or implied, which we do not hear about. We would like to congratulate the authors on the energy and persistence which have gone into the development of their research program and the collection of data.

H. E. Einert[14] and F. R. Owens.[14] The authors have summarized concisely, and we believe exactly, the present status of our knowledge of chemically inhibiting intercrystalline corrosion. Their title invokes an expectation of complete solution of the problem, which is not satisfied by their discussion. This leads us to express an opinion we have held for some time, namely, the published and expressed opinions available today very definitely reveal a recognition of the limitations of chemical inhibition.

The work of both Straub and Schroeder and their associates reveal uncertainties which may be due to stress or other unknown factors. Hence even though chemical passivity may be indicated with a given operation, through either or both types of detectors, there is not complete insurance against intercrystalline failure. We quote from the authors' summary[15] of the 1941 symposium on caustic embrittlement: "On the other hand, if the boiler water cracks the detector specimens, it will not necessarily crack the boiler, yet it is difficult to guarantee that conditions do not exist or will not arise in the boiler structure from which cracking may result."

The foregoing is not an attempt to detract from either the contributions of the authors or Straub. On the contrary, we endorse and encourage the use of the Schroeder and Straub detectors as engineering instruments for increasing the insurance in the safe and economical operation of steam boilers.

Aside from the relative merit of one of the present known chemical inhibitors over the others, let us consider the chemical limitations from an operating viewpoint. We have in mind a central station, generating steam at 1400 psi. Until a few months ago, the boiler concentrates would be of the following average composition in ppm: Phenolphthalein alkalinity, 202; total alkalinity, 239; sodium chloride, 10; sodium sulphate, 98; PO_4, 45; silica (SiO_2), 2; pH, 11.10.

[12] Research Department, I.C.I. (Alkali), Limited, Norwich, Cheshire, England.

[13] "Caustic Embrittlement," by E. W. Colbeck, S. H. Smith, and L. Powell, Proceedings of The Institution of Mechanical Engineers, Nov., 1942; also abridged. *Engineering*, Dec. 4, 11, and 18, 1942, pp. 455 and 478.

[14] Cyrus Wm. Rice & Co., Inc., Pittsburgh, Pa.

[15] "Symposium on Embrittlement; Summary," by W. C. Schroeder and A. A. Berk, Trans. A.S.M.E., vol. 64, 1942, pp. 427–430.

All Schroeder detectors revealed selective corrosion. If one were responsible for the feedwater treatment at this plant, and there was a desire on the part of the management to increase the insurance against intercrystalline failure, chemically, what method would he select?

Obviously, the choice lies between the use of nitrates or the phosphate-pH control proposed by Purcell and Whirl.[16] The former is doubtful at this pressure and also might affect turbine deposits adversely. Hence, the logical method would be phosphate-pH control, which is now under trial. The time interval to date does not permit a conclusive statement based on the detector tests.

Again, we cite an industrial plant using zeolite-treated make-up of a surface supply, to the extent of 80 per cent of the total evaporation. A supplementary treatment of monosodium phosphate and sodium sulphite is used. The boilers generate steam at 600 psi. The following is representative of the average composition of the boiler concentrates in ppm: phenolphthalein alkalinity, 245; total alkalinity, 310; sodium chloride, 75; sodium sulphate, 513; PO_4, 55; silica (SiO_2), 5; pH, 10.8.

Deposits on the turbine buckets, diaphragms, and nozzles necessitate periodic cleaning. Again, we must recognize the existing limitations with respect to chemical inhibition of embrittlement. Phosphate-pH control might be feasible, but it would be quite wasteful. The use of nitrates would further impair the turbine operation, which leaves no choice but the use of tannins. The use of tannins proved to be impractical, since we found the steam quality impaired to the extent that the specific conductance was increased on an average of $6^1/_2$ micromhos. This in turn decreased turbine capacity due to deposits twofold. Need we indicate the complete limitation of chemical inhibition in such instances?

We might continue citing similar experiences indefinitely without further serving our immediate purpose. We summarize our remarks by repetition; the published and expressed opinions recognize the limitations of chemical inhibitors, as at present known. Hence there is in the light of our present knowledge the need for "de-emphasis" of the chemical environment in our consideration of intercrystalline failure.

J. A. Holmes.[17] This paper is a very concise, complete, and easily understandable review of the work done and the results obtained at the Bureau of Mines. The authors are to be congratulated on their accomplishments.

There is one question we hear regularly and we hope the authors can give us some answer. Suppose one of these embrittlement detectors is put on a modern boiler of welded-type construction and the embrittlement detector shows that the water is embrittling. According to the latest information conveyed by the A.S.M.E. Boiler Code, it is suggested as unnecessary to observe any embrittlement protection in these types of boilers over 400 psi pressure. Should the operators go to all of the trouble and expense of correcting the boiler water because of the possibility of tube-end embrittlement, or is the possibility of any trouble so remote that embrittlement protection can be entirely disregarded?

In regard to the elimination of hydroxide ions by the use of phosphates to obtain the proper balance between pH, alkalinity, and phosphate to prevent embrittlement, the silica ions should also be taken into consideration. SiO_2 ties up caustic soda or Na_2O in the same manner as phosphates and also acts in the same manner to prevent embrittlement. If silica is disregarded, the amount of Na_2O in the boiler water may become so low that serious silica scale will result. For example, we know of a 400-psi-pressure plant which has a make-up water of 17 ppm total calcium and magnesium hardness as calcium carbonate, very little sulphate, and 100 ppm of silica. When this boiler water is treated with a phosphate-type treatment, the amount of alkalinity resulting from a correct balance of Na_2O and P_2O_5 is not near enough to prevent silica scale. It is necessary to increase the alkalinity considerably over the P_2O_5 demand. Theoretically, the boiler water should be embrittling with this higher alkalinity, but, according to all tests which can be made with either the Straub or the Schroeder testing unit, the water is not embrittling, evidently because of the soluble silica present. Therefore, where there are any appreciable amounts of soluble silica present, we believe it very desirable to have a ratio of alkalinity to phosphate and/or silica. Fortunately, the embrittlement-detector unit enables one to check under operating conditions any theoretical ratios or treatment calculated, and the authors of this paper are to be congratulated in providing an instrument for such testing and control.

J. J. Maguire,[18] H. L. Kahler,[18] and L. D. Betz.[19] The authors present in this paper, for the first time, a brief summary of the various methods which have been developed for combating embrittlement cracking. It is the first answer that has been given to the boiler operator in a concise form to his question of what he should do if and when he is troubled with this problem.

As the authors know, we have been carrying out numerous plant tests with the Schroeder embrittlement detector, and the following comments represent our opinion, together with the results achieved to date using the methods mentioned by the authors in their paper.

Sodium Nitrate. The use of sodium nitrate for combating caustic metal embrittlement appears to us to be the most practical method yet devised for the average industrial plant. It appears foolproof, inexpensive, and not too difficult to control. Definite proof should be established that it does not produce NH_3 in the steam in any appreciable quantity, and further results proving its effectiveness under all conditions should be provided.

Quebracho Extract. The results obtained through the use of quebracho extract look almost as good as with sodium nitrate, in pressures up to a higher level (700 psi). The test provided for tannin for control purposes is not too difficult, although this test does require delicate control to prevent misleading results. All such tests should be completed immediately upon sampling or some method of fixation provided for. One disadvantage which may be encountered in the use of this product for control of embrittlement is that the amount of quebracho used to maintain the tannin-NaOH ratio may be sufficient to produce a highly discolored boiler water. Such discoloration may produce difficulty in carrying out other water tests required for routine plant control for water conditioning.

Zero-Caustic Alkalinity. This method, as first proposed by Purcell and Whirl and reported[16] to this Society, we believe is misnamed. It should be noted that zero-caustic alkalinity refers to the absence of excess caustic in the solid material at the test

[16] "Embrittlement of Boiler Steel—Experiences With the Schroeder Detector," by T. E. Purcell and S. F. Whirl, Trans. A.S.M.E., vol, 64, 1942, pp. 397–402; also "Protection Against Caustic Embrittlement by Co-Ordinated Phosphate-pH Control," by T. E. Purcell and S. F. Whirl, Water Conference, Engineers Society of Western Pennsylvania, Pittsburgh, Pa., Nov., 1942.

[17] Director of Service, National Aluminate Corporation, Chicago, Ill. Mem. A.S.M.E.

[18] W. H. & L. D. Betz, Frankford, Philadelphia, Pa.

[19] General Manager, W. H. & L. D. Betz, Frankford, Philadelphia, Pa. Mem. A.S.M.E.

specimen and not to zero hydroxyl content in the boiler water. Waters with a pH between 10 and 11 as maintained in the boilers, reported upon by Purcell and Whirl, must have 1.7 to 17 ppm OH or 4 to 40 ppm NaOH formed from the hydrolysis of Na_3PO_4. Therefore, we believe it would be more correct to refer to this as the minimum-caustic-alkalinity method rather than the zero-caustic-alkalinity method, or that the name "zero-caustic alkalinity" should be explicitly defined as the condition which brings about zero-free caustic in the solid phase at the test specimen. True, the hydrolysis of Na_3PO_4 to form OH will be less than the ionization of NaOH which may account for the favorable conditions reported upon by Purcell and Whirl, but it must be remembered that, at the concentrations existing in contact with the stressed specimens in the embrittlement detectors, there will be OH present.

For example, it is well known that concentrations are extremely high at the test specimen. With a pH of 11, measured at room temperatures and not at boiler temperatures, (114 ppm PO_4) in the boiler water, we have calculated there is approximately 17 ppm hydroxyl ion present due to the hydrolysis of trisodium phosphate, and without the presence of any free caustic soda as measured by the strontium-chloride test used by Purcell and Whirl. If we assume 1000 cycles of concentration (purely arbitrary for illustrative purposes) at the test specimen, we will not have 17,000 ppm hydroxyl-ion concentration in the liquid phase due to repression of hydrolysis and ionization of these high concentrations. Although this has never been directly measured, it is reasonable to assume that it is relatively low in hydroxyl ion content and is not a dominant factor in caustic metal embrittlement. On the other hand, if we have 17 ppm excess hydroxyl over and above that produced by the hydrolysis of Na_3PO_4, we would have at the specimen almost 17,000 ppm hydroxyl. This clearly illustrates the necessity for close adherence to the curves of Purcell and Whirl in using this method and serves to explain why the presence of only a few ppm excess hydroxyl alkalinity, as measured by the strontium-chloride test, will mean the difference between an embrittling water and a nonembrittling water.

This method is primarily applicable to plants employing evaporated make-up water. For such plants, mainly central stations, this method seems to provide an advantageous solution to the embrittlement problem. We feel that it will be desirable to confirm the present results with a greater number of embrittlement-detector tests, but present data give every indication that this method will be successful and can be adopted as a standard means for embrittling prevention for plants employing evaporated make-up water.

For the average industrial plant, however, employing an appreciable percentage of raw water softened or unsoftened, this method is not a practical solution for the embrittlement problem. Sodium nitrate seems to provide the best solution. With feedwaters containing appreciable concentrations of hardness and alkalinity, the zero-caustic-alkalinity method may occasion a marked increase in the solids content of the boiler water and also represent a large cost increase compared with the use of sodium nitrate.

As an example, assume a zeolite softened make-up water and feedwater concentration as follows:

Hardness as $CaCO_3$: 0 ppm
P Alkalinity as $CaCO_3$: 0 ppm
M Alkalinity as $CaCO_3$: 20 ppm

This does not represent an unusual case. It is interesting to compare the conditions (Table 2 of this discussion), resulting from the use of phosphoric acid, monosodium phosphate, and disodium phosphate for the maintenance of zero-caustic alkalinity. Also shown are the same conditions employing sodium nitrate.

TABLE 2 COMPARISON OF TREATMENTS

	Use of H_3PO_4, ppm	NaH_2PO_4, ppm	Na_2HPO_4, ppm	$NaNO_3$, ppm
Trisodium-phosphate content of boiler water, as Na_3PO_4	440	656	1304	
Soluble-phosphate content of boiler water as PO_4	254	380	760	
Additional boiler solids[a] introduced by treatment, ppm	120	336	1184	96
Embrittlement treatment, lb per million lb feedwater	13.1	24.0	57	4.8
Cost of treatment, dollars per million lb feedwater[b]	1.13	1.85	3.65	0.10

[a] Based on considering untreated-boiler-water alkalinity as $20 \times 20 =$ 400 ppm total alkalinity as $CaCO_3$ = 320 ppm total alkalinity as NaOH.
[b] Based on costs of treatment as: H_3PO_4 (75 per cent), 6.5 cents per lb, equivalent to 8.65 cents per lb for H_3PO_4 (100 per cent); NaH_2PO_4, 7.7 cents per lb; Na_2HPO_4, 6.4 cents per lb; $NaNO_3$, 2 cents per lb.

As can be observed, the solids content of the boiler water introduced through the use of mono and disodium phosphate is quite appreciable. With the use of phosphoric acid, solids increase can be maintained at a lesser value, but the cost of treatment is still materially higher than with sodium nitrate. In addition, the question of handling and feeding phosphoric acid presents a problem.

For the average industrial plant, therefore, we feel that the so-called zero-caustic-alkalinity method does not provide as worth-while a solution as the use of sodium nitrate.

We believe that this method, while holding considerable promise, is still too new to conclude that it gives 100 per cent protection. A number of tests should be run and different types of water should yet be studied before adopting this method as the final word in caustic-metal-embrittlement control.

E. M. Partridge.[20] We note that one of the methods outlined which is spoken of as the "zero-caustic-alkalinity method" has been used to date only with boilers receiving evaporated water. We question that this method can be extended to water containing appreciable amounts of silica after softening by base-exchange materials or hot-lime-soda softening.

It has been our experience that waters containing silica need at least an equivalent amount of total alkalinity to hold the silica in solution in the boiler salines. A water containing 5 ppm of silica will at 20 concentrations have 100 ppm of silica in the boiler water, if the silica be held completely in solution. To hold this silica in solution will require a caustic-alkalinity content in the boiler water which will exceed the requirements of the zero-caustic-alkalinity method.

Where alkalinity reduction is desired, it can be obtained very effectively and completely by means of a combination of acid-and-salt-regenerated base-exchange material. We had an experience with equipment of this type which gives point to the need for the presence of alkalinity along with silica in the boiler water.

A West Coast plant uses a water for boiler feed which contains 25 ppm of silica. It was treated for some years with straight sodium zeolite with satisfactory results in the boilers. To reduce blowoff, by the removal from the water of a large portion of the dissolved solids present which cause a high alkalinity, part of the feedwater was passed through an acid-regenerated unit, and the alkalinity of the combined effluents of this unit and the salt-regenerated unit regulated to give a total alkalinity in the softened water of 15 ppm. With water of this alkalinity, a highly siliceous scale formed and caused the loss of boiler tubes. Upon increasing the proportion of sodium-zeolite water used so as to bring the

[20] Manager, Internal Treatment Division, the Permutit Company, New York, N. Y.

softened water up to an alkalinity of about 30 ppm, the scale formation stopped. It would not be practical to maintain this amount of alkalinity in the boiler water by the use of phosphate alone because of the expense involved.

Similar considerations apply to water softened by hot lime and soda. Also, water thus softened is ready for direct passage to the boiler. The addition of acid to bring the water to a point so near neutral that a slight excess will develop an actual acid condition is not good practice under these conditions.

The method is, however, very interesting, and its successful use, where distilled water is employed, indicates its importance. There is always the possibility that plant conditions will be found which will permit its application to other than evaporated make-up jobs.

T. E. Purcell[21] and S. F. Whirl.[22] One of the major outcomes of the many years of research in the field of caustic embrittlement was the development of the embrittlement detector. This device permits the operator to determine the embrittling characteristics of his boiler water under actual operating conditions and to test personally the effectiveness of the various methods of protective treatment. Indeed, its application is practical as attested by the varied experiences presented at the 1941 Embrittlement Symposium.[4]

Caustic embrittlement became a reality with the Duquesne Light Company in 1928.[23] It is only natural that since that date much time, study, and money have been devoted to this important problem. In 1938, an extensive program of embrittlement testing was begun. A total of 55 tests with the detector have been conducted on boilers operating at pressures from 175 to 440 psi.

The details of these tests and significant findings were reported in two papers.[16] The co-ordinated phosphate-pH control method of water treatment,[24] referred to by the authors as "zero-caustic alkalinity," was developed in our laboratory as a result of our studies.

While we are in general agreement with the material presented by the authors in this paper, we feel that several points require clarification.

Co-Ordinated Phosphate-pH Control Method. The fundamental principle of this method of protection against caustic embrittlement is to produce a boiler water which contains sufficient alkalinity for corrosion inhibition, and which upon evaporation does not yield caustic soda. This is accomplished by maintaining in the boiler water, phosphate alkalinity only, no caustic alkalinity being present. This treatment must not be confused with the unsuccessful use of trisodium phosphate as an embrittlement inhibitor in the presence of caustic. The graph, shown in Fig. 2 of this discussion, is the basis of control. By plotting the pH value and the PO_4 concentration of the boiler water on this graph, one can readily ascertain whether or not the boiler water contains caustic alkalinity. A point which falls on the curve indicates that all phosphate exists as trisodium phosphate (Na_3PO_4); a point which falls below the curve shows the presence of acid-phosphate salts; a point in the region above the curve is evidence that caustic is present. From another viewpoint, the PO_4-pH co-ordinates merely represent some stage in the neutralization between phosphoric acid and sodium hydroxide, as exemplified by the following equations:

$$H_3PO_4 + NaOH \rightarrow NaH_2PO_4 + H_2O \quad [1]$$
$$H_3PO_4 + 2\,NaOH \rightarrow Na_2HPO_4 + 2\,H_2O \quad [2]$$
$$H_3PO_4 + 3\,NaOH \rightarrow Na_3PO_4 + 3\,H_2O \quad [3]$$
$$H_3PO_4 + 3\,NaOH + X NaOH \rightarrow Na_3PO_4 + 3\,H_2O + X NaOH \text{ (excess)} \quad [4]$$

Equation [3] is associated with the curve of Fig. 2; both indicate that all the phosphate exists as trisodium phosphate; combinations of Equations [1], [2], and [3] yield points below the curve, while Equation [4], with excess sodium hydroxide, yields a point above the curve. The excess may be eliminated by Equations [1], [2], and [3], or if solid chemicals are preferred NaH_2PO_4 and Na_2HPO_4 may be used. If the PO_4-pH co-ordinates give a point below the curve, and the PO_4 concentration is at the desired maximum, the pH is increased by suitable additions of sodium hydroxide.

As we pointed out in our recent paper,[24] the methods of test for caustic alkalinity recommended by the authors are not satisfactory for low concentrations. This statement is based upon our experiences during fifteen tests with pH values below, on, and above the co-ordinated phosphate-pH-control curve. The method with which we have obtained the most consistent results is a modified Winkler test. Phenolphthalein is used as an indicator and strontium chloride is substituted for barium chloride and is added in the proportions recommended for its use with purple indicator.[25] The solution is then heated to boiling, cooled and titrated with N/50 acid. The fading end point, described by Schroeder and Fellows,[26] has not been experienced, except with synthetic solutions containing phosphates, carbonates, and silicates.

When using the modified procedure, we advise that no caustic alkalinity be tolerated in the boiler water instead of 0 to 8 ppm, recommended in this paper. However, it is our opinion that the curve, shown in Fig. 2 of this discussion, is a more practical and trustworthy guide than any of the methods of test for low concentrations of caustic.

That the amount of caustic alkalinity continuously permissible is extremely low is shown in Fig. 3 of this discussion. Here are plotted the actual pH values for a 95-day test, and the pH values from the curve of Fig. 2, corresponding to the PO_4 concentration. Both the cold- and the hot-rolled specimens cracked. The average caustic concentration by the modified Winkler method using strontium chloride and phenolphthalein indicator was only 3 ppm OH, with a maximum of 5 ppm OH. Because of these results, we strongly recommend that the pH be held below the curve value, not at zero-caustic alkalinity; i.e., that the boiler water have at all times some caustic-absorbing capacity. In this manner the danger of trespassing into the embrittling range is avoided.

Sodium Nitrate. While sodium nitrate appears to protect against embrittlement attack, it is not as easily controlled as implied in this paper. The analytical determination is both cumbersome and time-consuming. Approximately $1^1/_2$ hr of close attention are required for a single determination and, even under the most favorable conditions, the suggested method is prone to yield high results.

A word of warning should be sounded to operators regarding the use of sodium nitrate in evaporated make-up systems. Despite the authors' statement: "It is thermally stable and has not adversely affected boiler operation in the range 125 to 400 psi," there were indications of nitrate decomposition to yield am-

[21] General Superintendent, Power Stations Department of the Duquesne Light Company, Pittsburgh, Pa. Mem. A.S.M.E.

[22] Chief Chemist, Power Stations Department of the Duquesne Light Company, Pittsburgh, Pa.

[23] "Boiler Metal Cracking—A Case Study," by M. Hecht and D. S. McKinney, *Power*, vol. 70, 1929, pp. 633–636.

[24] Second paper of ref. 16.

[25] "Standard Methods of Water Analysis," eighth edition, American Public Health Association, 1936, pp. 97–98.

[26] "Determination of Carbonate, Hydroxide, and Phosphate in Boiler Waters" by W. C. Schroeder and C. H. Fellows, Trans. A.S.M.E., vol. 54, 1932, pp. 213–224.

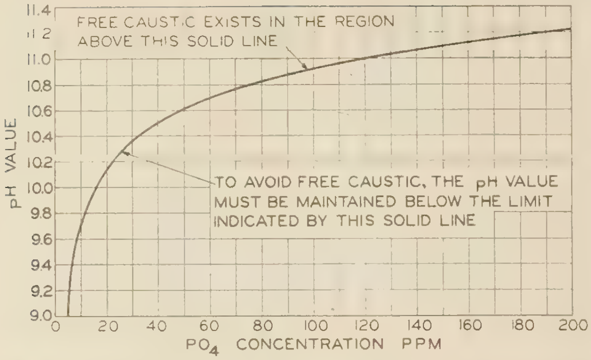


Fig. 2 Relation of pH Value to Trisodium-Phosphate Concentration

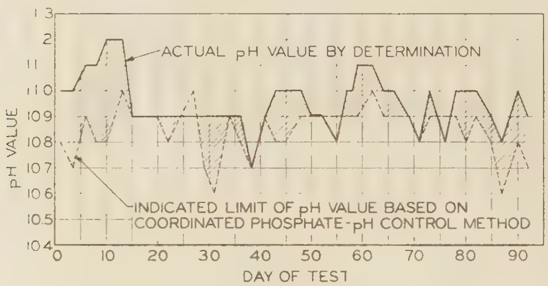


Fig. 3 pH Value During Test No. 108
(Both cold- and hot-rolled specimens cracked.)

monia in both our plant and laboratory tests. The sodium-nitrate requirement was found to be consistently higher initially than after the test was well under way, but even after 90 days, approximately $^1/_2$ lb was required per day for a 275-psi boiler, operating at approximately 200,000 lb of steam per hr.

Quebracho Extract. From our experiences with the use of quebracho, attention must be given to the choice of brands. Some brands appear to be good inhibitors; others do not prevent cracking. The quebracho itself is easy to use and control; the discoloration of the boiler water is objectionable, but does not unduly interfere with the determination of other constituents.

Duration of the Test. In the summary of their data in Table 1, the authors do not give the duration of the test periods. This, we believe, is an important omission from their paper. From our experience, it is quite probable that many of the specimens which did not crack in 30 days might have cracked had the test period been extended to 90 days or more. We found it necessary to extend the time, first from 30 to 60 days, and then to 90 days, in order to obtain reproducible results. Incidents of noncracking in less than 90 days might lead to false conclusions with respect to protective treatment.

In considering the cost of the various treatments, the authors write as follows: "Sodium nitrate is relatively inexpensive compared with quebracho extract and phosphate." This might, apply to some boiler waters treated for embrittlement protection alone; however, phosphate is almost universally used as internal treatment for the prevention of scale. Consequently, using it in the supplementary role of embrittlement protection increases the consumption only slightly in many boiler waters. In boilers operating with distilled water or with acid, neutral, or slightly alkaline feedwater, the cost of phosphate treatment is less than that of sodium nitrate.

Another statement made by the authors which is not entirely clear is, "A low concentration of alkalinity is considered desirable, however, to prevent corrosion and incrustation during boiler operation." We are accustomed to thinking of alkalinity in general as preventing corrosion, not causing it, and as having little, if any, effect on boiler incrustation. The authors may be referring to the action of caustic alkalinity in promoting embrittlement corrosion or to the type of corrosion identified as "steam-blanketing,"[27] in which cases, of course, their statement regarding corrosion would apply.

J. B. Romer.[28] Embrittlement cracking, like many other problems, divides itself into two phases, namely, cause and prevention or elimination. There is no question but that a full knowledge of the cause greatly simplifies the problem of prevention. However, it is not always necessary to understand fully the causes and their mode of operation in order to obtain adequate methods of prevention. That this is so is shown by the great strides made in preventing embrittlement cracking in low-pressure stationary boilers some twenty or more years ago.

Greater knowledge as to the part played by silica, and possibly other constituents present in the water, in the development of embrittlement cracks, might have been of considerable advantage to those carrying on the study of the causes and prevention of embrittlement cracking.

Prof. S. W. Parr early considered that he had considerable evidence to the effect that sodium sulphate, when present in adequate quantity, had a marked ability to reduce embrittlement cracking. Our operating experience yielded considerable evidence to the effect that the addition of sodium sulphate to waters that were deficient in this constituent would not only reduce embrittlement but, in many cases, stop it.

It is true that the possible effect of other salts which may have been present in waters that were classed as nonembrittling was not noted. Inasmuch as the addition of technically pure sodium sulphate secured the desired results in many instances, it is hard to believe that any other inorganic salt was involved in these particular cases.

During this period, and for a considerable period thereafter, the railroads felt that the use of sodium sulphate, in the quantities required in locomotive practice, led to undesirable effects and made it desirable to seek another embrittlement inhibitor in which these effects would be absent or reduced. This led to the railroads becoming vitally interested in the program being carried on by Dr. Schroeder, which work has been summed up by the authors in the present paper. During the progress of Dr. Schroeder's investigation, he developed the "embrittlement detector," and this particular instrument was of great aid in his subsequent laboratory studies, as well as in his survey of railroad locomotives. Normally the time required for embrittlement cracking to develop in an operating boiler, is such that a laboratory test had to be developed that would materially shorten the time. In this particular instance, this was accomplished by raising the stress. In so doing, Dr. Schroeder may have brought the embrittlement detector more in line with locomotive-boiler conditions and somewhat less in line with low-pressure stationary boilers. The data, submitted by the authors, would appear to warrant the belief that embrittlement cracking within the locomotive boiler can be controlled by the use of sodium nitrate to a much greater advantage than by any previously known treatment.

Irrespective of the ultimate merits of sodium sulphate, it does suffer from the fact that a very large quantity is required, as

[27] "Attack on Steel in High-Capacity Boilers as a Result of Overheating Due to Steam Blanketing," by E. P. Partridge and R. E. Hall, Trans. A.S.M.E., vol. 61, 1939, pp. 597–622.

[28] Chief Chemist, The Babcock & Wilcox Company, Barberton, Ohio.

compared with the quantity of sodium nitrate. This, in itself, is a definite advantage for sodium nitrate, and if the results of the authors' research have indicated some alleviation of caustic embrittlement through the use of sodium nitrate without foaming and carry-over from high concentration, then they have really accomplished something of benefit for the railroads.

There has been considerable discussion as to the thermal stability of sodium nitrate. Operating experience in stationary boilers has frequently indicated a definite relationship between corrosion and the presence of nitrates, this being particularly true where the nitrate appeared to be present in the form of calcium nitrate. However, an adequate control of the pH of the boiler water, plus the fact that the amount of nitrate required is relatively low, may control this characteristic of the nitrates to the point that the beneficial property of preventing embrittlement cracking will warrant its use in locomotive boilers.

G. C. Walker.[29] The practical method of preventing embrittlement cracking as outlined in the paper, is apparently to install embrittlement detectors on operating boilers, and when the detector specimen, through cracking, shows the particular water supply to be unsafe, to resort to treatment, with further detector tests to be made to determine the effectiveness of the treatment. The success of this method is predicated upon the proper operation of the detector apparatus, for, as the authors point out, the absence of cracking may be due to its improper operation. Thus, we might find cracking taking place in the boiler with the detector specimen giving no positive evidence.

We recognize fully the inestimable value of the detector apparatus, for without its use, it is extremely doubtful if correlation of the work of the many investigators in this field, and the approaching apparent agreement on the value of specific inhibitors could be attained. We do not, however, believe that its general use as a means for preventing embrittlement cracking is the practical method which boiler operators are seeking as a solution to this problem. As a tool of research or as an instrument of testing, in the hands of competent engineers, skilled in its use, or as a medium through which co-ordinating or supervising agencies may render competent advice to their clients on this subject, the detector apparatus is of utmost value in investigation and control.

Any method that is practical, in so far as the boiler operator is concerned, must be simple, positive, inexpensive, and its operation not too time-consuming. We must remember that the possibilities of embrittlement cracking are to be found in boilers having riveted seams, whereas most boilers of recent construction and those operating at the higher pressures have drums of welded construction, in which the problem can be ignored, with the exception of the remote possibility of tube-end cracking.

Thus, when dealing with embrittlement cracking, we are largely concerned with older type boilers operating mostly under 500 psi. A large proportion of these boilers probably do not have continuous blowdown or boiler-water recirculating lines, which are recommended points of detector locations.[30] The operator, in order to use this apparatus, would then have to install a take-off line from his boilers. When installed, the specimen must be carefully stressed, and continuous daily adjustments must be made. Then, if after a period of testing, the specimen exhibits cracking, the authors state "treatment is usually desirable." Then it is recommended that detector tests should continue to be made in order to be sure that it is effective.

Judging from experience in endeavoring to have boiler operators do necessary and advisable testing of boiler waters in a completely satisfactory manner, the prospect of converting them into research engineers on the subject of embrittlement in their respective plants as a practical means of preventing such cracking, conjures up difficulties of the first magnitude. Bird and Johnson,[31] have concluded that the embrittlement detector should be used only by skilled operators.

If treatment is to be required in the event that the specimen cracks, why not provide the treatment in the first place?

In Table 1, the authors present the results of more than 900 plant tests on variously treated waters. Their results, using sodium nitrate and quebracho extract as inhibitors, have been substantiated by other investigators. Thus, it would appear that we have two inhibitors available of known value. Either of these can be incorporated into the plant boiler-water-conditioning practice, and through maintenance of a proven ratio of inhibitor to total alkalinity, as determined by routine control analyses, prevent embrittlement cracking. Certainly this is a more advisable procedure for the powerhouse engineer to follow than the installation and operation of the detector apparatus, with its required qualifications, and the possibility that he might have to use inhibitor treatment anyway.

It is noted that all tests maintaining zero-caustic alkalinity were run on boilers using evaporated make-up, but the authors suggest that the use of this method is not restricted to such plants, since the caustic soda can be neutralized by mineral acids. While this is theoretically possible, as a practical method in an industrial plant operating with a substantial amount of make-up water of varying percentages and varying loads, the control necessary to maintain zero-caustic alkalinity under such changing conditions would present such difficulties as not to make this method feasible in such plants. We would like to see this system tried in such an industrial plant, with its final evaluation to include its effect on all other factors involved in correct boiler-water conditioning.

Tannins are quite universally used in boiler-water conditioning because of their known effect on retarding corrosion and for sludge control. If, through control tests, the ratio of tannin to total alkalinity in the boiler water can be held at a point where embrittlement cracking will not occur, the use of this material for this purpose would appear to be quite simple.

In the meantime, all of the data point to the inability of sodium sulphate to prevent embrittlement cracking, and particularly so in pressures exceeding 250 psi, and yet the apparently discredited sulphate-alkalinity ratios remain in the recommendations of the A.S.M.E. Boiler Code. This code is the operating authority for thousands of boiler plants and many insurance companies. There is slight wonder that confusion exists in the minds of operators.

It would appear to this writer that all independent investigators are approaching the point of agreement on an inhibitor. It is hoped that definite commitments without qualifications can soon be made, which will finally dispel the clouds of uncertainty, speculation, indecision, and confusion surrounding the subject of caustic metal embrittlement.

Authors' Closure

The authors were pleased with the interest which this short paper aroused, as evidenced by the extensive discussion. The presentation was purposely brief, emphasizing only the results of tests by operating engineers to show that there are practical methods for removing the tendency for boiler water to cause embrittlement cracking. A more elaborate publication is in preparation.

[29] The Bird Archer Co., New York, N. Y.

[30] "Embrittlement Detector Testing on Boilers," by W. C. Schroeder, A. A. Berk, and C. K. Stoddard, *Power Plant Engineering*, August, 1941, vol. 45, pp. 76–79.

[31] "Studies on the Cracking of Boiler Plates," by P. G. Bird and E. G. Johnson, Trans. A.S.M.E., vol. 64, 1942, pp. 409–416.

Fig. 4 Photomicrograph of Crack in Cold-Rolled Embrittlement-Detector Specimen; ×300

In this closure the remarks of the individual contributors are grouped under several general headings. Replies have been made to all the questions in so far as is possible at present.

Significance of Detector Tests. The embrittlement detector was designed to reproduce closely the conditions existing in an actual boiler seam. It provides a controlled rate of leakage of steam and contact of a stressed-steel specimen with the concentrated solution resulting from evaporation of the boiler water. It is questionable whether this is an accelerated test in the sense that one or more of the controlling factors was made deliberately to exceed the conditions that exist in a boiler seam. On the contrary, it is quite probable that the leakage, concentrations, and stresses in the boiler seam can equal those in the detector. The essential difference between the detector and the boiler is that the former is so constructed and operated that these three major factors act simultaneously, continuously, and under the most favorable circumstances to produce cracking, whereas in the boiler these factors are brought together only under unique circumstances. Furthermore, in the detector, cracking is produced in a small test specimen which can be periodically and thoroughly examined, while cracking in the boiler can be distributed through a considerable area which can be inspected only with difficulty.

The cracks produced in the test specimens are quite similar to those found in cracked seams. More than 100 failed specimens have been examined metallographically. Fig. 4 of this closure, shows a typical photomicrograph of an area that had been etched lightly to reveal the position of the fine cracks in relation to the grain boundaries. Cracks in a detector specimen are indicative of an embrittling condition in the boiler water.

In reply to questions 1 and 2 of the Colbeck and Hamer discussion, correlation of specimen and boiler cracking has been good. Representative samples of water from cracked stationary boilers have caused embrittlement failure in detector specimens in every case studied. The experience of the Chesapeake & Ohio Railway offers additional striking confirmation of the relation of the test to practical boiler experience. More recently, welded-drum boilers operated at pressures of 425 and 450 psi in two plants have lost tubes after detector tests had shown the waters to be embrittling. Tube-metal and detector specimens all showed the characteristic predominantly intercrystalline cracking.

Inasmuch as a personal factor is involved in the operation of the detector, questions relative to the meaning of a test that results in no cracking are especially pertinent. In general when the first specimen tested was not cracked at the end of 30 days, a new specimen was inserted in the detector and exposed for 60 days. If the second specimen also did not fail, a 90-day test was run, since in many cases it was found that cracking was delayed when the caustic alkalinity of the boiler water was low or a low concentration of preventive chemical was present. The probability that an embrittling water would not cause cracking in any of the three specimens can be calculated. For the calculation, an embrittling water was defined as one that had caused cracking at any time.

When tests of the proposed methods of treatment were eliminated, approximately 500 runs, made by the average plant personnel with detectors attached to stationary boilers, were available for consideration. For runs of 30, 60, and 90 days, the chances that an embrittling water would cause failure in the specimen were found to be respectively, 68, 84, and 95 in 100. Consequently, after successive 30-, 60-, and 90-day tests have been completed, the possible chance of error is considerably less than 5 per cent. In fact, if chance were the only factor that contributed to negative tests, the probability that the water was nonembrittling would be 99.7 per cent. In view of the limited number of data available, especially for 90-day tests, this figure represents an order of magnitude rather than a fixed probability.

On the other hand, if the test specimen is embrittled, the boiler water can be regarded as potentially dangerous. However, the hazard to welded-drum boilers would be restricted to rolled-in tube ends. For these and other modern boilers the question as to whether or not the water should be treated to remove its tendency to cause cracking is, to a large extent, one of economics. The expense of the treatment and its control must be weighed against the cost of possible trouble and perhaps peace of mind as well. No damage ascribed to embrittlement has yet been reported for boilers operated at pressures above 500 psi.

Treatment with inhibitors is *not* usually recommended unless there is definite evidence that the water is embrittling. There are several reasons for this. First, there may be substances naturally present in water that interfere with the inhibiting treatment. Until a sufficient background of experience has been accumulated, it is advisable that any treatment be checked with the embrittlement detector, at least during its initial period of use. Second, the boiler water may be naturally nonembrittling, and additional treatment would be poor economy. In addition to the cost of the chemicals and the expense of control, the increase in dissolved salts would tend to lower the steam quality. Third, nitrates and, to a lesser extent, quebracho extract have a current strategic importance and must be conserved. Indiscriminate use may lead to a state of affairs where treatment would not be available to boilers that require protection from embrittling waters. The zero-caustic treatment, on the other hand, can be applied to evaporated make-up systems with reasonable assurance of a safe and desirable boiler water, and without using material amounts of important chemicals.

Treatment With Nitrate. In the 6 months since the paper was written the proposed nitrate treatment has been used successfully at pressures from 100 to 600 psi. Application at one plant operated at 728 psi is being followed carefully. Here, too, the results have apparently been satisfactory and no difficulties have as yet been reported. No specimens have been cracked while the treatment was being maintained.

All the data obtained during this period confirm previous in-

dications that nitrates are stable salts in boiler water. Many of the waters for which this treatment has been used also contained sulphite, but no increase in sulphite requirements was reported. When nitrate decomposes, the disappearance of each pound of the chemical would be equivalent to the addition of 0.5 lb NaOH. No such phenomenon has been observed, although it was carefully sought by Purcell and Whirl during their test. On the other hand if the steam from their boiler carried over 0.1 ppm of the salt from waters having an average concentration of 70 ppm, it would account for the 0.5 lb of nitrate they could not find, since only one boiler was being treated with nitrate and the condensate was returned to the entire system. At present it seems desirable to withhold any decision as to the stability of nitrate as it is related to boiler-water temperatures until the test work is completed.

Einert and Owens call attention to the possibility that nitrates may increase the tendency of salts to deposit in turbines. While sodium nitrate melts at approximately the same temperature as sodium hydroxide, the concentration of the former is always relatively low, and thus far no trouble has been experienced. However, the possibility is being investigated, especially where the treatment is being used at the higher operating pressures.

Rapid colorimetric methods for the determination of nitrate[32] are quite suitable for control purposes. The more tedious procedure requiring careful manipulation and "$1^1/_2$ hr of close attention" is more accurate and was therefore prescribed during the initial research period.

Zero Caustic. The authors prefer the term "zero caustic" because it implies exactly what the treatment does. To an operator caustic means caustic soda rather than hydroxyl ion, and the treatment produces a water that contains no free caustic, that is, there would be no caustic soda in the residue if the water were evaporated to dryness. The initial laboratory experiments with a synthetic solution containing only trisodium phosphate gave unexpected results. Part of the phosphate formed an insoluble film with the iron of the walls of the autoclave, and sufficient free sodium hydroxide appeared in the solution to crack the specimen. For subsequent tests enough acid phosphate was added to the synthetic trisodium-phosphate solution to neutralize this free sodium hydroxide. None of these specimens was cracked, indicating that trisodium phosphate will not cause embrittlement in the absence of caustic.[33]

Purcell and Whirl at the Duquesne Light Company pioneered in the transition from laboratory tests to actual boiler operation, demonstrating that the treatment would work in a system using evaporated make-up. To this significant achievement they added an additional important contribution in the form of a curve for controlling the treatment on the basis of the relationship of phosphate concentration to pH.

Regardless of what this treatment is called, its application to high-pressure boilers using evaporated make-up has been entirely successful. Of the tests reported since the paper was written, several were run at 1400 psi in a plant where conventional treatment had resulted in eight severely cracked specimens. Zero caustic during its trial period proved so satisfactory that it has been made standard treatment for all high-pressure steam boilers belonging to the operators of this plant.

Several examples have been discussed of make-up waters for which the treatment would be unsuitable. The method was not intended for such waters but was developed primarily for use in boilers operated beyond what was thought to be the pressure range of the other proposed treatments and therefore for installations where distilled-water make-up is almost universal. However, its many advantages should encourage its use at lower pressures in those cases where a suitable water is available. No data have been obtained as yet for this method on boilers using other than evaporated make-up.

[32] One such method is given in reference *k*, Table 1 of the paper.

[33] "Embrittlement Detector Testing on Boilers," by W. C. Schroeder, A. A. Berk, and C. K. Stoddard, *Power Plant Engineering*, vol. 45, August, 1941, pp. 76–79.

The question whether boiler water containing only sodium-silicate alkalinity would cause cracking cannot as yet be answered. Laboratory tests were inconclusive, since silica tended to deposit on the walls of the autoclave and piping, the sodium content remaining in solution as caustic soda. There is also considerable uncertainty as to the solid phases formed during the evaporation of such water at boiler temperatures. Silica deposits from boiler water containing considerable caustic soda would tend to indicate that operating a high-pressure boiler with water such that the silicate alkalinity was a large proportion of the total alkalinity might be courting trouble from hard silica scale.

In the control of zero-caustic treatment, small quantities of sodium hydroxide are determined in the presence of large quantities of phosphate. This analysis has always been difficult Values reported by Schroeder and Fellows for the strontium-chloride procedure tended to be low with respect to known concentrations. By titrating to the lower-alkalinity phenolphthalein end-point, Purcell and Whirl obtain results that are more exact. There is some tendency, however, for strontium phosphate to form compounds analogous to hydroxyapatite and so cause low results. Barium phosphate, on the other hand, may contain some acid phosphate, a factor that, coupled with the tendency for the compound to supersaturate, causes high results. By allowing five minutes for equilibrium to be attained after the barium chloride is added, reasonably reproducible results can be obtained by the Winkler method.[34] The curve proposed by Purcell and Whirl is admirably suited to routine control purposes. Titration is definitely inferior to using the curve where there are no interfering substances.

Treatment With Quebracho Extract. Treatment with quebracho extract was mentioned only briefly in the discussion. Contradictory views were expressed as to its interference with routine analysis. Many laboratories have reported that dark water was troublesome but that standard procedures could readily be adapted to the special requirements of colored solutions. Other questions pertaining to the preferred brands of quebracho and to the preparation of a sample for future analysis have been adequately treated in the paper and the analytical reference *l* in Table 1.

Miscellaneous. Colbeck and Hamer ask in question 3 whether a safe boiler water could be made embrittling and how. In the simplest case, this is achieved in the laboratory by addition of caustic soda to distilled water. More generally an increase in the caustic alkalinity of a boiler water in an amount sufficient to overcome the action of any inhibitor present can make the water embrittling. For example, a plant in Illinois operates with a water naturally inhibited with nitrates. Yet several boiler drums were lost 16 years ago because sufficient caustic soda was generated in the water through overtreatment with soda ash so that the nitrate-alkalinity ratio was too low.

Attention has also been directed to the confusion between the methods for preventing embrittlement described in this paper and the recommendation set forth in the present A.S.M.E. Boiler Code for maintenance of ratios of sodium sulphate to sodium carbonate to achieve this same purpose. Code ratios will not prevent cracking of detector test specimens. The proposed methods of treatment would appear therefore to be more certain safeguards against embrittlement trouble.

[34] Private communication from C. E. Parker of the Public Service Electric & Gas Company.

Removal of Water-Insoluble Turbine Deposits by Caustic Washing

By W. L. WEBB[1] and R. G. CALL,[1] NEW YORK, N. Y.

In spite of the methods employed to reduce carry-over from boilers, both water-soluble and insoluble turbine deposits are prevalent. Where these deposits are water-insoluble, disassembly of the turbine with resultant capacity outage has been necessary. A caustic-soda-solution washing procedure, used by plants of the authors' companies for the removal of water-insoluble silica deposits, has resulted in regaining lost capacity and in restoring stage-pressure differentials to normal in a fraction of the time formerly required for removing deposits mechanically from buckets and diaphragm nozzles.

By far the majority of high-pressure plants are experiencing turbine deposits, the rate of deposition varying from that causing no measurable effect on turbine characteristics in a year of operation to rates so high that deposits must be removed many times a year to restore lost capacity or to hold stage-pressure differences within safe limits.

It is not within the scope of this paper to attempt to explain the mechanism of formation of turbine deposits. Actually, comparatively little is known about this controversial subject. As indicated by Schroeder,[2] some well-directed research on the cause and prevention of turbine deposits is highly desirable. Suffice it to say, deposits fall into two main classes, i.e., those which can be removed by water-washing and those which cannot.

Water-Soluble Deposits

Water-soluble deposits are commonly removed from turbines by reducing the inlet-steam temperature sufficiently to produce wet steam in the stages where the washing action is desired. Where a wide range of superheat control is available, this reduction may be made at the boiler. However, in the majority of installations, an auxiliary means for desuperheating the steam is necessary. Usually this is done by introduction of water through spray nozzles in the main steam line or preferably in an auxiliary line installed especially for washing the turbine. Where the steam is desuperheated in the main steam line under full temperature and pressure conditions, careful installation must be made to prevent damage to piping from thermal stress.

While washing, periodic analyses of hot-well condensate give an indication of the character of the deposit being removed from the turbine and show when further washing ceases to remove any deposit. It is good practice to isolate the hot-well condensate from the rest of the system and, where deposits are appreciable, it is advisable to dump it to waste. Provisions should be made so that vacuum can be maintained.

During the washing period the turbine is operated at subnormal speed.

In cases where it is permissible for the turbine washings to be returned to the boilers, turbines have been washed satisfactorily under load. This procedure involves a number of hazards even under very careful control.

The rotor of the high-pressure element of one of our compound units has been repeatedly washed by half-filling the turbine casing with condensate and turning the rotor by the turning gear until successive washings show no removal of solids. Care is taken that the temperature of the condensate used for washing is nearly equal to that shown by thermocouples peened into the turbine casing.

Water-Insoluble Deposits

The usual practice for removal of water-insoluble deposits consists of disassembling the unit and blasting the affected surfaces with fly ash or other suitable abrasive. In cases where deposits are extremely hard, the surfaces of buckets and diaphragm nozzles may be objectionably roughened by the abrasive. With this procedure, labor costs and capacity outages, particularly during the present war effort, dictate a search for speedier means of deposit removal.

At both high- and low-pressure plants of the companies with which the authors are associated, it has been common practice periodically to water-wash certain turbines to remove soluble deposits. Until recently there had been little difficulty with deposits not removable by water-washing. Where insoluble deposits did occur, they usually were in topping units and in such small amounts as to cause no appreciable change in turbine performance.

Windsor Plant Low-Pressure Turbines. The Beech Bottom Power Company's Windsor Plant capacity consists of four 1350-psi, 925 F boilers of approximately 750,000 lb per hr each, supplying two 60,000-kw topping units; these topping units exhausting to six 230-psi, 550 F, 30,000-kw condensing units.

The first major turbine-capacity losses occurred at Windsor following unusually heavy condenser leakage during a period when the Ohio River water used for condenser cooling contained in excess of 300 ppm of suspended matter, of which the analysis in Table 1 is typical. This high leakage made it impossible to blow down sufficiently to maintain the silica in the boiler water at its usual value of less than 10 ppm, the silica at times reaching 40 ppm. During this period, the steam quality, as indicated by conductivity measurements of gas-free samples, increased only slightly, whereas solids in the steam, determined gravimetrically, practically doubled. Rough checks on the residue from the gravimetric quality determinations showed it to contain almost 50 per cent silica.

TABLE 1 SUSPENDED MATTER

	Per cent
Silica (SiO_2)	22.58
Aluminum oxide (Al_2O_3)	42.06
Iron oxide (Fe_2O_3)	23.00
Calcium oxide (CaO)	0.24
Magnesium oxide (MgO)	0.18
Sulphuric anhydride (SO_3)	0.82
Organic matter	9.80
Total	98.68

Shortly after observing the higher solids in the steam, there was a slight loss in capacity both in the high- and low-pressure tur-

[1] Engineering Department, American Gas and Electric Service Corporation.

[2] W. C. Schroeder, Mem. A.S.M.E., discussion, Water Session, A.S.T.M. Meeting, Atlantic City, June, 1942.

Contributed by the Power Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1943, of The American Society of Mechanical Engineers.

bines. The nozzles on the steam-jet air pumps then became plugged and were mechanically cleaned.

Unit 5, one of the low-pressure units, was disassembled about this time for removal of the fifteenth-stage buckets. There was evidence of deposits throughout this machine but they were not extensive. A sample removed from the fifteenth-stage diaphragm nozzles was shown to be similar to the deposits on the steam-jet air-pump nozzles, the analyses of both being indicated in Table 2.

TABLE 2 ANALYSES OF DEPOSITS FROM EJECTOR AND TURBINE NOZZLES

	Deposit removed from No. 3 air-ejector nozzle, April 17, 1942, per cent	Deposit removed from fifteenth-stage diaphragm, No. 5 turbine, April 21, 1942, per cent
Silica	79.49	88.66
Ferrous oxide	1.07	0.86
Ferric oxide	2.10	1.56
Aluminum oxide	3.08	2.03
Calcium oxide	0.32	0.52
Magnesium oxide	0.01	0.03
Phosphorus pentoxide	0.26	0.38
Sulphuric anhydride	0.10	0.37
Sodium oxide	0.08	0.37
Organic and volatile	13.14	4.85
Total	99.65	99.63

Following the turbine inspection, the rate of accumulation of deposits proved to be greater than anticipated, and within a short time the capacity losses on four low-pressure units totaled approximately 17,000 kw. Although the analysis of deposits, indicated in Table 2, made it appear unlikely that capacity could be regained by water-washing, this method of removal was tried without success on Unit 4, which had lost 6000-kw capacity. During the water-washing period, conductivity readings of hot-well condensate indicated that no material affecting conductivity was being removed. Because plant load conditions would not permit extensive outages of units, attention was turned to methods other than water-washing for removal of deposits.

Reference had been made in Germany by both Geisler[3] and Goerke[4] to the use of caustic-soda solutions for the removal of silica deposits. Their practices included the spraying of a 10 per cent caustic-soda solution into the path of steam to the turbine and allowing this solution to remain in the turbine for 3 or 4 hours. Subsequent water-washing was said to have removed all deposits.

Laboratory work on the Windsor steam-jet air-pump nozzles indicated that if they were immersed in a boiling 10 per cent caustic solution for an extended period and then washed with water, the deposit was not removed. However, if the nozzles were maintained at a temperature considerably above the boiling point of the solution and then were repeatedly wetted with the 10 per cent caustic solution and allowed to dry for about 15 min after each application, the deposit was readily washed off with water.

The results of these experiments were put into practical use in caustic-washing Windsor low-pressure Unit 4 in May, 1942. As shown in Fig. 1, the below-seat drain of the turbine stop valve was replaced with a line for introduction of a 10 per cent caustic solution at a rate of about 2 gpm. While sufficient steam was admitted to hold the turbine speed between 100 and 200 rpm, the caustic solution was injected until the hot-well condensate, being dumped to the sewer, showed phenolphthalein alkalinity. This required about 45 min. The stop valve was then closed, and the caustic solution allowed to react with the deposits for a period of about 15 min after the unit came to rest. During the introduction of the caustic solution, the approximate steam conditions at the inlet to the stop valve were 230 psi, 550 F, and beyond the caustic-injection point were 140 F to 200 F at a pressure slightly higher than the condenser pressure of 5 psia.

[3] "The Desilification of Feedwater of the High-Pressure Plant in Höchst," by W. Geisler, *Vom Wasser*, vol. 12, 1937, pp. 381–386.

[4] "Salt and Silica Deposits in Steam Turbines," by H. Goerke, *Elektrizitätswirtschaft*, vol. 38, 1939, pp. 614–616.

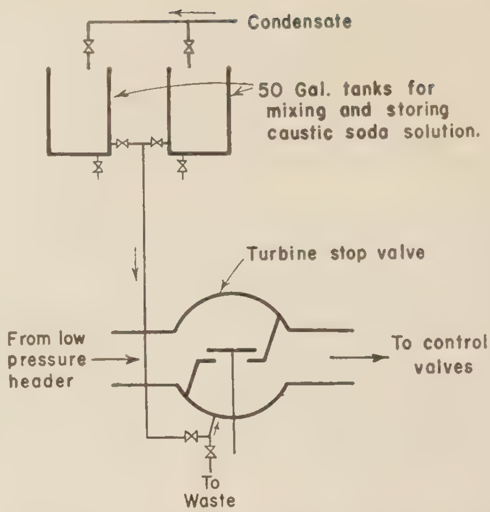


Fig. 1 Temporary Piping for Introduction of Caustic-Soda Solution and Condensate to Windsor Low-Pressure Turbines

Eight additional 5- to 10-min shots of caustic solution were introduced while sufficient steam was being admitted to hold 100 to 200 rpm, each followed by the 10- to 15-min reaction period. The entire caustic-washing period required about 3 hr and consumed 350 lb of flake caustic soda.

Steam was then admitted to the unit to maintain 100 to 200 rpm, and condensate was injected at the caustic-solution injection point at a rate of about 2 gpm. After a 3-hr washing period, the hot-well condensate showed approximately normal conductivity and the unit was then returned to service.

Prior to caustic washing, the maximum load obtainable from the unit was 27,000 kw with both control valves wide open. After washing, the unit was brought to a load of 33,000 kw, which is the limiting load, although the secondary valve was not in its full-open position. The stage-pressure data, shown in Table 3, indicate general distribution of deposits throughout the unit prior to the washing operation. In interpreting these data, it should be observed that, at a load of about 27,500 kw, the secondary control valve admitting main steam to the fifth stage was wide open before washing, whereas after washing it was only partly open.

TABLE 3 WINDSOR LOW-PRESSURE UNIT 4: CONDITIONS BEFORE AND AFTER WASHING

	Before washing				After washing
	4/30/42	5/1/42	5/4/42	5/5/42	5/12/42
Load, kw	27830	27560	27440	27410	27400
Condensate flow, M lb per hr	345.67	330.8	334.8	332.9	313.5
Throttle steam temp, F	571	608	566	577	569
Throttle press., psia	253.7	251.2	253.1	251.7	257.2
Turbine head press., psia	237.8	236.1	236.1	235.1	240.0
Fifth-stage press., psia	235.3	233.6	234.4	232.8	184.1
Eleventh-stage press., psia	57.0	56.6	56.1	56.8	44.7
Thirteenth-stage press., psia	18.75	18.50	18.80	18.70	16.30
Condenser press., psia	0.57	0.66	0.80	0.82	0.66
Stage press. differences, psi:					
Turbine head to 5th stage	2.5	2.5	1.7	2.3	55.9
5th to 11th stage	178.3	177.0	178.3	176.0	139.4
11th to 13th stage	38.3	38.1	37.3	38.1	28.4
13th stage to condenser	18.18	17.84	18.0	17.88	15.64

During the washing operation, samples of hot-well water were taken at intervals for analysis, as given in Table 4. These analyses include material removed both from the turbine and the

TABLE 4 DEPOSITS REMOVED FROM WINDSOR UNIT 4 AS INDICATED BY ANALYSIS OF HOT-WELL WATER

				Residue on evaporation,[a] per cent					
Sample no.	Time	Sodium hydroxide, ppm	Silica, ppm	Silica	Iron and aluminum oxides	Calcium oxide	Magnesium oxide	Phosphorus pentoxide
1	After second shot of caustic	3700	63	32.4	38.5	4.1	7.3	17.6
2	After third shot of caustic	..	495	59.7	26.5	2.4	4.7	6.6
3	Beginning of last shot of caustic	..	382	70.0	16.5	1.8	2.8	8.9
4	15 min after last caustic shot	..	180	50.1	18.7	9.5	9.5	12.2
5	Beginning of water-washing	..	617	73.0	23.3	1.9	1.3	1.6
6	After 20 min of water-washing	..	431	77.1	13.8	1.9	1.4	5.7

[a] After subtracting sodium hydroxide, sodium chloride, and sodium sulphate.

condenser and indicate the silica concentrations, as well as the compositions, expressed as per cent, upon evaporation to dryness, after subtracting sodium hydroxide, sodium chloride, and sodium sulphate. Most of the chloride and sulphate found apparently came from the caustic soda used, inasmuch as it contained 0.2 per cent of sodium sulphate and 0.54 per cent of sodium chloride.

To date all six of the Windsor low-pressure units have been caustic-washed and the lost capacity restored. Two of these units were washed just prior to the annual overhaul, which permitted inspection of rotors and diaphragms. The buckets of one rotor still showed thin friable deposits near the roots and, in some stages, the buckets were covered with a thin powdery material which could be removed by wiping. Thin adherent deposits were still evident on some diaphragm nozzles. The other unit inspected showed the diaphragms and the buckets to be practically free of deposits.

It should be recognized that turbine deposits may unbalance the thrust forces sufficiently to throw an excessive load on the thrust bearing. Thus, in several instances, Windsor low-pressure-turbine deposits have made necessary the reduction of turbine output to limit thrust-bearing oil temperatures.

Philo Low-Pressure Turbine. The new extension of the Philo Plant of The Ohio Power Company consists of two cross-compound units, each totaling 90,000 kw capability, the load being approximately equally divided between the high- and low-pressure elements of each unit. An arrangement is provided whereby the low-pressure turbine may be operated independently of the high-pressure turbine by high-pressure desuperheated steam through the auxiliary control valve and pressure-reducing arrangement incorporated in its governing mechanism. The high-pressure-turbine exhaust conditions vary from about 120 psi, 450 F at full load to about 36 psi, 300 F at one-third load. Each unit is served by two 450,000-lb per hr boilers delivering steam at 1375 psi and 950 F total temperature. Boiler superheat is controlled by means of spray-type attemperators located between the primary and secondary superheaters.

Shortly after compound Unit 4 was put into service, the pressure drop across the twin steam strainers at the inlet of the low-pressure turbine was observed to be increasing. Likewise, the difference between first- and third-stage pressures of the low-pressure turbine showed an increase but not at as high a rate as the strainer pressure increase. The strainer pressure drop finally rose to a value which made necessary the shutdown of the unit to clean the strainers. The deposit was found to be very thin, hard, and tightly adherent. Water-washing was employed without effect. An analysis of the deposit showed it to be high in silica with appreciable quantities of iron and calcium and small amounts of magnesium and phosphate. The screens were cleaned with wire brushes and scrapers.

Over the period during which deposits were accumulating, both on the strainers and in Unit 4 low-pressure turbine, silica in the boiler waters was abnormally high due to unusual operating conditions.

Following the cleaning of the strainers, the first-to-third-stage pressure difference in Unit 4 low-pressure turbine continued to increase until the limit set by the turbine manufacturer was reached and slightly exceeded at high load. Because of the belief that the deposit could not be removed by water-washing and, in view of limited permissible outage, it was decided to proceed with caustic washing, following in general the Windsor procedure. This was done on June 7, 1942. As indicated in Fig. 2, the caustic solution was introduced directly below the two steam strainers in the low-pressure steam leads to the turbine and was atomized by means of nozzles which extended up through the drain-connection openings. These nozzles were designed to give a flow of approximately 1 gpm each under a total head of 65 ft. Prior to the installation of the nozzles, a check test was made of the flow rate and effectiveness of atomization.

Caustic-solution injection below the strainers was preferable to injection into the below-seat drain of the control valve, thereby avoiding caustic contact with the valve-guide bushings and possible interference with free valve movement.

The turbine was operated independently of the high-pressure element by means of the auxiliary steam admission valves using high-pressure steam at 1100 psi, desuperheated to 600 F, and controlled to give 100 to 200 rpm. The steam conditions beyond the caustic-solution injection point were 250 to 350 F and 3 to 6 psia.

Five injections of 10 per cent caustic-soda solution were made,

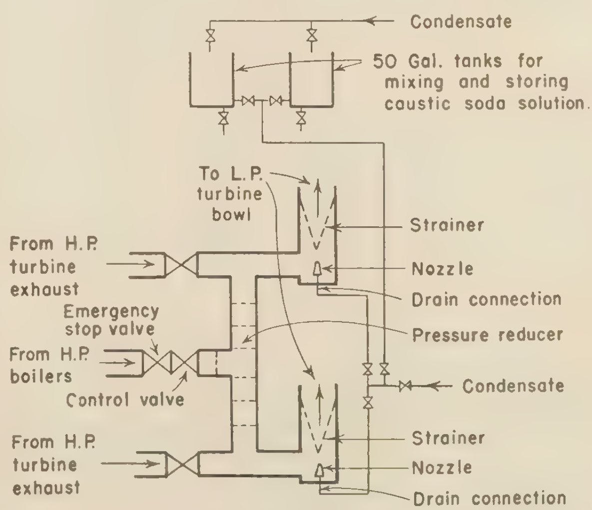


Fig. 2 Temporary Piping for Introduction of Caustic-Soda Solution and Condensate to Philo Low-Pressure Turbine

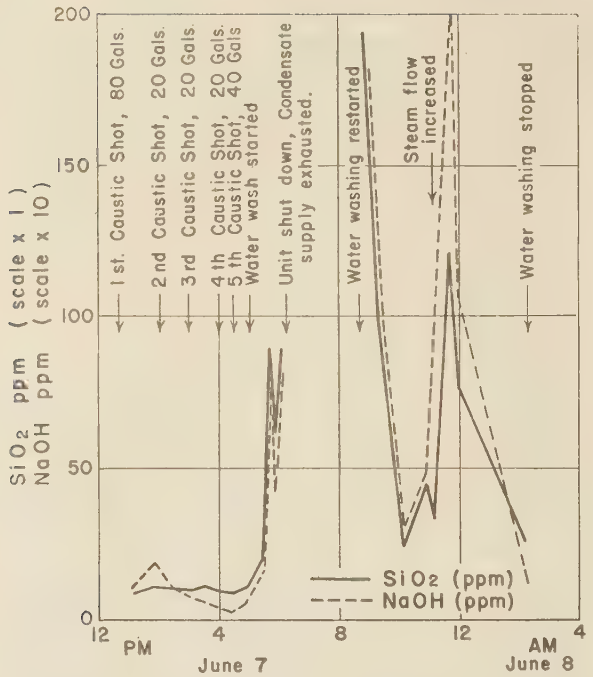


FIG. 3 SILICA AND CAUSTIC-SODA CONCENTRATIONS IN HOT-WELL WATER DURING WASHING OF PHILO UNIT 4 LOW-PRESSURE TURBINE

requiring approximately 150 lb of flake caustic. The unit was then water-washed with condensate through the atomizing nozzles for a period of 10 hr. For reasons not yet determined, the water-washing operation required a much longer period than anticipated. A log of operations and concentrations of NaOH and SiO_2 in the hot-well water is given in Fig. 3.

Table 5 indicates the stage-pressure differences before and after washing. According to these data, the deposits were confined mainly to the first to third stages. Caustic washing restored stage pressures substantially to design conditions.

TABLE 5 PHILO UNIT 4 LOW-PRESSURE STAGE-PRESSURE DIFFERENCES WITH 840,000-LB PER HR STEAM FLOW TO UNIT 4 HIGH-PRESSURE, AND WITH ALL BLEED HEATERS IN SERVICE

Date	Stage-pressure differences, psi			
	Bowl to 3	3 to 6	6 to 9	9 to condenser
Before washing				
March 26	60	34.4	17.0	2.6
April 28	65	36	17.8	2.0
May 7	67	36	17.2	2.2
Steam strainers hand-cleaned, on May 10				
May 19	68.8	37	17.8	1.8
May 27	67	37	17.0	2.2
May 31	69	36.5	17.4	1.9
June 1	69.5	36.5	17.4	1.8
June 3	70	36.5	17.6	2.0
June 4	69	36.5	17.6	1.8
After washing				
June 9	60	34.2	16.5	1.6

Windsor Boiler-Feed-Pump Turbines. The three Windsor feed pumps of Unit 7 boilers are driven by 1250-hp 3500-rpm turbines, receiving steam at 230 psi 550 F and exhausting at 8 to 16 psia. These turbines were washed with caustic-soda solutions, using the established procedure, except that during washing the turbines exhausted to the atmosphere. It was therefore necessary to pump in the caustic solution and condensate.

The caustic solution was injected in two shots through a pressure-gage connection in the pipe between the throttle valve and the steam chest. The steam-chest control valves were blocked wide open, and the steam flow controlled by means of the throttle valve to hold 100 to 300 rpm. Condensate was then injected at the same point until the exhaust drips showed substantially normal conductivity. The entire operation required 6 to 7 hr per unit. Typical results are as follows:

	First-stage pressure, psia	Pump output, lb per hr
Before washing	156	528,000
After washing	90	630,000

GENERAL RECOMMENDATIONS

Inasmuch as there has been comparatively little experience with the caustic washing of turbines, no specific recommendations can be made for a given application. Consideration of the turbine design and arrangement, together with further study of the washing procedure itself, must be the guide in its use. However, based upon the authors' experience to date, the following suggestions are offered for those who wish to attempt the removal of water-insoluble deposits from condensing units:

1 Since it is necessary, among other things, for the successful removal of deposits, to coat the affected surfaces with caustic soda, the method of introducing the solution should assure its thorough distribution over these surfaces. This is believed to be effectively accomplished by atomizing the solution and introducing it into the main steam flow.

2 The design of packing glands on turbine stop and control valves should be carefully reviewed when the caustic-washing procedure is contemplated. Where the point of caustic-solution introduction is such as to permit the stem packing to be flooded with solution, some difficulty has been experienced with the sticking of stems and in removing the packing. This has occurred only on valves having stems which extend downward.

Consideration is being given to the use of a temporary soft packing during the washing operation, this packing to be replaced before the turbine is returned to service.

3 The foregoing precaution is also applicable to the more recent designs of packing glands which depend upon steam leakoff through close-clearance bushings. In one instance, some of the control valve stems froze in the bushings and were extremely difficult to remove. Water-washing was ineffective in removing the solidified caustic. Depending upon turbine design, the proper selection of the caustic-solution injection point may forestall this difficulty.

4 For low-pressure units, the caustic solution usually can be most conveniently introduced by gravity flow. However, if advisable, it may be pumped.

5 During the caustic injections, the shell drains should be closed, and during the subsequent washing with condensate, these drains should be open to the condenser.

6 All turbine instrument and bleed connections should be closed to prevent access of caustic to external equipment.

7 Excessive rates of temperature change of turbine parts should be avoided with the same care as used in normal operation.

8 The authors' experience with turbine deposits has been limited to those which are comparatively thin. To assure the removal of heavy deposits, it may be advantageous to repeat the entire washing cycle. As both physical and chemical characteristics of deposits are changed by caustic treatment, a unit so treated should not be returned to service before receiving a thorough water-wash. If a turbine is returned to service with heavy coatings treated by caustic, these coatings may become detached causing damage to the unit.

9 Although the authors have not used agents to improve the

wetting properties of the caustic-soda solution, their use is contemplated in the near future. Such agents should be particularly effective where oily deposits exist on the latter stages.

Conclusions

Experience to date with seven low-pressure turbines and with three feed-pump turbines of the American Gas and Electric Company system indicates that the variety of water-insoluble turbine deposits encountered can be removed from turbine buckets and diaphragm nozzles by washing with caustic-soda solutions to regain lost capacity and to restore stage pressures to normal. This has been accomplished by using available connections as injection points and without definite knowledge of conditions for speediest and most complete removal of deposits. Studies are under way to determine, for specific types of deposits, the optimum conditions for their removal.

Although so far there have been only slight indications of water-insoluble deposits in our topping units, there is every reason to believe that these units or any high-pressure condensing units can be successfully washed, provided (1) facilities can be made available for introduction of the caustic solution at suitable points; (2) temperatures can be properly controlled; (3) wash water can be injected where it will remove soluble deposits and residual caustic effectively; and (4) the washings can be run to waste.

Fear has been expressed that the caustic soda may damage turbine parts, either from caustic embrittlement or corrosion. The authors grant that such conditions may occur but feel that the short period of caustic contact, the probable absence of high stresses if temperature conditions are properly controlled, and the thorough subsequent water-washing would make the likelihood of damage improbable. At the same time, they recognize that a thorough investigation to prove or disprove the possibility of such attack is highly desirable.

From a turbine-outage standpoint especially, the procedure outlined in this paper for removal of insoluble deposits from turbines without disassembly should be of primary interest to all operators of high-pressure steam plants. Many have shown great interest in it and some have used it. It is urged that those who use chemical means for the removal of insoluble turbine deposits make their experiences and improvements in technique available to the industry as a whole.

Chemical Removal of Scale From Heat-Exchange Equipment

By F. N. ALQUIST,[1] C. H. GROOM,[2] and G. F. WILLIAMS[3]

The use of inhibited hydrochloric-acid-base solvents to remove scale from various commercial units, chiefly boilers and other heat-exchange equipment, is comparatively new. A sample of the scale to be removed is subjected to X-ray and spectroscopic analysis and solubility tests as an aid in determining the conditions of treatment. Four treatments of commercial equipment and results obtained are described, the mobile equipment used in chemical cleaning is illustrated, and tables of scales and their occurrence are presented.

Commercial Uses of Inhibited Hydrochloric Acid

THE removal of surface scale from iron and steel by immersion in acid is an old art which owes its growth to the discovery that the addition of certain materials called inhibitors practically prevents attack on the base metal but permits fairly rapid solution of the scale. Study of the literature shows innumerable references to substances possessing inhibiting qualities in varying degrees including aldehydes, glue, dextrin, glucose, crude anthracene, anthracene residues, waste sulphite lyes, coal-tar products, thiourea, quinoline, pyridine, and many others too numerous to mention. Recent discoveries have produced several exceedingly potent inhibitors, each appropriate for the particular type of work for which it was devised.

However, inhibitors alone do not make a good scale solvent. Other agents must be added to the inhibited acid to adapt it to the particular type of scale to be removed.

In the latter part of the nineteenth century, a United States patent was granted on a method for increasing the flow of oil wells in limestone formations by treating the formation with acid, particularly hydrochloric acid. The method, although effective, was not widely used because of the corrosive action of the acid on the metal parts of the oil well. Later this objection was eliminated by the use of inhibited acid.

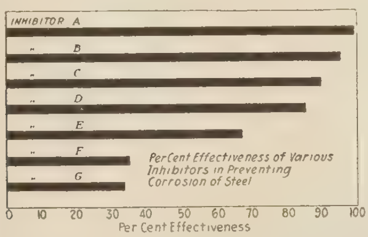


Fig. 1 Comparative Value of a Few Inhibitors

Research on the use of inhibited acid has continued, with the result that in the last few years inhibitors have been developed for almost every type of metal normally utilized in the construction of heat-exchange equipment at various acid concentrations, temperatures, and time intervals. This research program has developed a knowledge of what types of metals can be safely contacted with inhibited acid, which makes it possible to remove scale deposits from heat-exchange equipment with chemical solvents. The comparative value of a few inhibitors is shown in Fig. 1.

Industry itself has made many attempts to supplement mechanical cleaning with chemical cleaning, the results being varied and occasionally unsatisfactory, due mostly to incomplete knowledge of the materials concerned and lack of adequate equipment. The introduction of a scale-solvent-removal service was welcomed by industry, and it is due to the willing co-operation of oil refineries, power companies, and other industries possessing heat-exchange equipment that the knowledge of chemical scale solvents has reached its present state of advancement.

Sampling of Scales

A solvent treatment for the removal of scale is based upon the particular scale to be removed. Naturally, the scale sample is very important and should be representative of the scale in the equipment. It should be taken from surfaces where it is the most difficult to remove. A scale sample from a boiler should represent the hardest, most dense scale found in the boiler and probably should come from the hottest tubes. Scale samples from condensers and similar equipment should come from near the inlet where the heaviest deposits are formed. The sample should represent the deposit down to the bare metal. The preferred type of sample is a small section of tube or pipe containing the scale; however, as in most cases this is neither necessary nor practical, chunks of scale are most frequently obtained and are adequate. Powdered scale in most instances is sufficiently satisfactory, but chunks of scale make the design of a treatment much more certain.

X-Ray Analysis of Scale Samples

The scale sample is first examined for its physical characteristics, such as density, porosity, thickness, manner of formation, etc., and whether it appears to consist of more than one type, as is usually the case with scales composed of different layers. After this general examination, the chemical compounds composing the scale are identified by means of X-ray diffraction patterns. This method of analysis results in the determination of actual compounds as they exist in the scale, which is not possible by other analyses.

The X-ray equipment used is a multiple diffraction unit consisting of a transformer, X-ray tube with molybdenum target, and cameras. Twelve slits around the circumference of the cylinder containing the X-ray tube define the X-ray beams. There is also a switchboard on which are mounted the operating switches, meters, filament-current stabilizer, a water-pressure switch, and an overload circuit breaker. In addition to this standard equipment, there is a recording milliammeter, which

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Contributed by the Power Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1942, of The American Society of Mechanical Engineers.

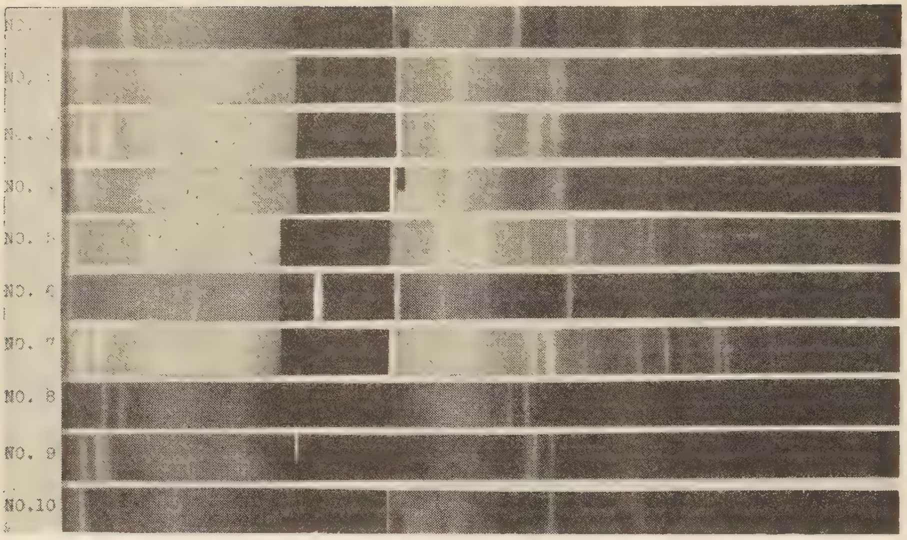

Fig. 2 X-Ray Diffraction Patterns of Ten Typical Scales

6 The analysis is quickly and cheaply made. This is of importance when time and expense are controlling factors.

At the present time, X-ray diffraction gives a semiquantitative analysis. The accuracy varies widely but usually allows a determination to within 5 per cent of the amount present.

Sometimes a scale is encountered which gives a diffraction pattern which does not match any of the reference patterns. In these instances the X-ray analysis is supplemented by a spectroscopic analysis which shows semiquantitatively the metallic elements present. Whether the metallic elements are combined or uncombined makes no difference as the total amount is obtained. As an example, scale found on the Admiralty metal tubes of several large condensers gave an X-ray diffraction pattern which did not match any of the reference patterns but had a distinct relation to spinel, a mineral which is the double oxide of magnesium and aluminum, $MgO \cdot Al_2O_3$. A spectroscopic analysis indicated that the scale had a high iron and copper content, 1 to 5 per cent zinc, 0.1 to 1 per cent aluminum, 0.05 to 0.5 per cent lithium, calcium, and magnesium. Thus the metals present in the scale in significant amounts were iron, copper, and zinc, while aluminum and magnesium were present in insignificant amounts. If the scale were a spinel-like material as the X-ray diffraction patterns indicated, it is most likely that it is a double oxide of zinc and iron with the copper present in a separate compound, or more likely the scale is a combination of the oxides of zinc, copper, and iron.

Table 2 shows a few of the scales which have been encountered.

gives a record of the tube current during exposure; a time switch, by which an exposure can be stopped at a predetermined time; and a temperature switch which cuts off the voltage when the cooling-water temperature reaches 100 F.

The specimens are powdered and loaded into capillary tubes, inserted in the camera, placed on the diffraction unit, and exposed for 6 hr at 30,000 v and 20 ma. Development of the films requires about $1^1/_2$ hr. The diffraction pattern on the developed film is then compared with patterns of known crystalline substances, thousands of which are available in a comprehensive filing system much like a fingerprint file.

TABLE 1 ANALYSES RESOLVED FROM PATTERNS SHOWN IN FIG. 2; BY PERCENTAGES

Sample no.	Calcium carbonate, $CaCO_3$	Calcium hydroxy phosphate $Ca_{10}(OH)_2(PO_4)_6$	Iron oxide, Fe_3O_4	Iron oxide, Fe_2O_3	Basic iron oxide, FeOOH	Magnesium hydroxide, $Mg(OH)_2$	Hydrated magnesium silicate, $Mg_3Si_2O_7 \cdot 2H_2O$	Calcium sulphate, $CaSO_4$	Miscellaneous
1	100[a]								
2			30		50				20 $FeCO_3$
3		40		40			15		5 SiO_2
4					80				20 FeO
5						60	10	30	
6	25					50		25	
7		20	10	60			10		
8	40	40		10			10		
9		30	5	60			5		
10			60		40				

[a] Calcite, 80 per cent; aragonite, 20 per cent.

Fig. 2 shows the X-ray diffraction patterns of ten typical scales, while Table 1 contains the analyses resolved from the X-ray diffraction patterns shown in Fig. 2.

Some of the advantages of the X-ray method of analysis are as follows:

1 Substances are shown in their true state of chemical combination.

2 The analysis is conclusive, even though only small amounts of material are available. This is valuable for scale studies, where it is sometimes quite difficult to obtain a large sample without dismantling the equipment.

3 Samples are analyzed as received and are not destroyed.

4 Preparation of sample and procuring of the pattern are simple and economical.

5 A permanent record of data is always on file in the form of the pattern.

TABLE 2 TYPES OF HEAT-EXCHANGER SCALE DEPOSIT

Name of scale	X-ray analysis	Where found
Calcium hydroxy phosphate....	$Ca_{10}(OH)_2(PO_4)_6$	Boiler
Pectolite....................	$4CaO \cdot Na_2O \cdot 6SiO_2H_2O$	Condenser
Magnesium hydroxide.........	$Mg(OH)_2$	Brine evaporator
Iron oxide..................	Alpha FeOOH	Bubble tower
Iron oxide..................	Fe_3O_4	Oil cooler
Iron oxide..................	$Fe_2O_3 \cdot H_2O$	Polymerization cooler
Iron sulphide................	FeS_2	Gas pipe line
Magnesium oxide..............	MgO	Boiler
Noselite.....................	$Na_5Al_6Si_6O_{24}.SO_4$	Heat exchanger
Hydrated magnesium silicate...	$Mg_3Si_2O_7 \cdot 2H_2O$	Firetube boiler
Analcite.....................	$NaAlSi_2O_6 \cdot H_2O$	Watertube boiler
Aragonite....................	$CaCO_3$	Evaporator
Calcite......................	$CaCO_3$	Miscellaneous
Anhydrite....................	$CaSO_4$	Open-box heat exchanger

Field Data on Boiler and Heat-Exchanger Scales

Tables 3 and 4 show the compositions of scales found in various heat-exchange equipment. The data are selected from a large number of scale samples investigated over a period of years.

TABLE 3 SCALES FROM BOILERS[a]

Compound	Percentage of scales containing compound	Percentage of scales containing compound in concentrations indicated 1–25 per cent	26–50 per cent	51–75 per cent	76–100 per cent
$CaCO_3$	54	31	15	23	31
Silicates	41	67	15	0	18
$Ca_{10}(OH)_2(PO_4)_6$	33	20	30	8	42
Iron oxides	32	40	29	14	17
$CaSO_4$	28	20	16	17	47
$Mg(OH)_2$	14	33	47	20	0
$Ca(OH)_2$	1	100	0	0	0

[a] Scales from boilers have been analyzed and tabulated to show the types of scales predominant in boilers.

TABLE 4 SCALES FROM CONDENSERS, EVAPORATORS, AND HEAT-EXCHANGE EQUIPMENT[a]

Compound	Percentage of scales containing compound	Percentage of scales containing compound in concentrations indicated 1–25 per cent	26–50 per cent	51–75 per cent	76–100 per cent
$CaCO_3$	54	33	27	3	37
Iron oxides	54	17	13	17	53
$CaSO_4$	18	20	30	30	20
Silicates	18	40	40	0	20
NaCl	7	75	25	0	0
$Mg(OH)_2$	7	50	25	0	25
Sulphides	5	67	0	0	33
$Ca_{10}(OH)_2(PO_4)_6$	4	50	50	0	0

[a] Scales from condensers, evaporators, and heat exchangers have been analyzed and tabulated to show types of scales predominant in this type of equipment.

It is interesting to note that calcium-hydroxy-phosphate is characteristic of boilers as a result of feedwater treatment, while it rarely occurs in other heat-exchange equipment. Iron oxides, calcium sulphate, calcium carbonate, and silicates are common to all equipment.

Selection of Solvent

From the physical and chemical characteristics of the scale, the various solvents suitable for dissolving and disintegrating that type of scale are readily selected.

The usual solvent base is muriatic acid, which is rendered noncorrosive by the addition of special inhibitors applicable for the particular metals and scales to be encountered. To this acid are then added catalysts, wetting agents, intensifiers, reaction-control agents, etc., to adapt the solvent to the scale to be treated. Different scales require different auxiliary chemicals, which fact accounts for the importance placed upon the identification of the scale. Fig. 3 shows the effect of a wetting agent.

Solubility determinations are then made under varying conditions similar to those to be encountered in the treatment. Time, temperature, and manner of application of the solvent are all considered. These tests not only verify the choice of solvent make-up but they also indicate the best treating technique.

Treating Technique and Equipment

The equipment available for the chemical cleaning of heat-exchange equipment is the result of years of experience in the chemical treatment of oil wells. Motorized units are equipped with large metal tanks for transporting the necessary chemicals and pumps for pumping the prepared solvent into the industrial equipment to be cleaned. Each treating unit has a proportioning apparatus for diluting the solvent with water which is obtained on the premises and also apparatus for adding auxiliary chemicals. In this manner, the awkwardness of handling acid in glass carboys and the need for mixing tanks are eliminated. A heater which raises the temperature of the prepared solvent is also part of the equipment. Fig. 4 shows a treating unit, while Fig. 5 shows a typical treating hookup.

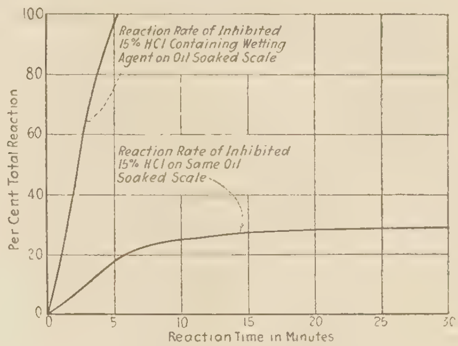

Fig. 3 Effect of Wetting Agent

In removing scale from a heat-exchange unit, the unit is taken out of service, drained, and isolated from all portions not to be cleaned, by closing all connecting valves or blank-flanging piping. Prepared solvent at the prescribed temperature is pumped into the unit through one of the lower connections until the unit is full. The solvent may then be circulated continuously, intermittently, or allowed to stand according to the treatment prescribed.

Samples of solvent for strength and temperature determinations are taken at regular intervals as the treatment proceeds. With scales of calcium carbonate or similar types, the solvent strength indicates when the scale is all removed, Fig. 6.

When the solvent treatment is finished, the solvent is drained to the sewer and all traces of solvent flushed out of the treated equipment with water. The water flush is often followed by an additional chemical neutralizing solution. After inspection, the unit is ready to be put back into service.

Chemical-solvent treatments made to date include nearly 100 different types of industrial equipment. Among these have been different types of boilers—stationary, marine, and locomotive, also heat exchangers, economizers, superheaters, water softeners, filters, evaporators, heaters, water lines, sewers, condensers, and cooling systems on compressors and internal-combustion engines.

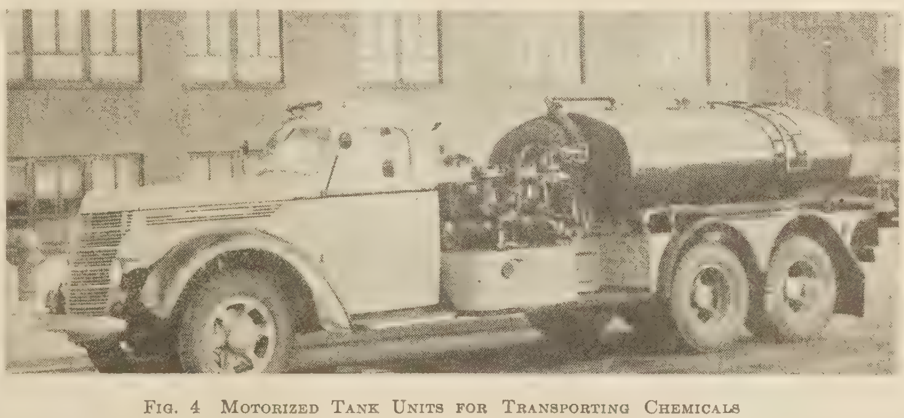

Fig. 4 Motorized Tank Units for Transporting Chemicals
(Motorized units bring chemicals to the equipment to be descaled in sufficient quantity to fill fouled space completely with solutions of necessary strength.)

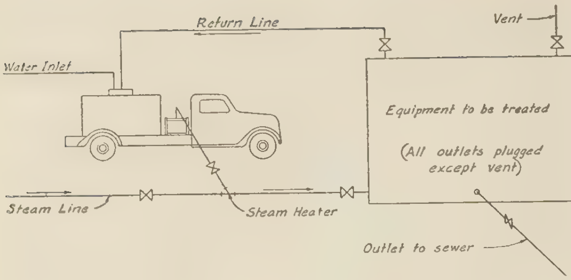

FIG. 5 TYPICAL TREATING HOOKUP

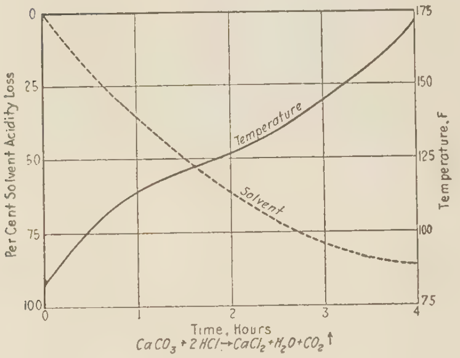

FIG. 6 SOLVENT STRENGTH DURING TREATMENT

Limitations of Chemical Cleaning Methods

Naturally, most mechanical engineers have had little experience with chemical cleaning methods. As the materials used are reactive and in concentrated form, they must be properly handled. Experience in this field is advantageous. Power-plant engineers depend upon their chemists for guidance in handling chemicals.

All types of scales cannot be readily removed, and it is therefore necessary to determine the composition of the scale before treatment. The types of scales difficult to remove are calcium sulphate, certain silicates, and some refinery scales. However, scales containing certain percentages of calcium sulphate, and about 80 per cent of the silicate scales encountered may be satisfactorily removed.

Evaluation of Chemical Methods for Removal of Scale by Examples

Illustrative of the wide application of the chemical cleaning of heat-exchange equipment are the following examples:

1 A Babcock & Wilcox integral-furnace boiler, supplying process steam at 225 psi pressure, was scaled over with a $^3/_{16}$-in. deposit which X-ray analysis showed to be mostly calcium-hydroxy-phosphate, $Ca_{10}(OH)_2(PO_4)_6$, with a small amount of a similar compound containing magnesium in place of the calcium, and fluorine in place of the hydroxyl. Treatment of this boiler with the prepared solvent removed approximately 1500 lb of scale which was determined by analysis of the used solvent. Visual inspection revealed that all scale was removed. Less than 24 hours after the start of treatment, the boiler was ready to go on the line.

2 A utility boiler, generating 350,000 lb of steam per hr at 1250 psi pressure operates on condensed steam and evaporated make-up with internal phosphate treatment. Twice a year the boiler is taken out of service and the tubes turbined. Because of the length and construction of some of the tubes, turbining is not satisfactory. X-ray analyses of scale samples taken from various parts of the boiler showed the following:

Calcium-hydroxy-phosphate, $Ca_{10}(OH)_2(PO_4)_6$, 50 per cent
Iron oxide, Fe_3O_4, 40 per cent
Iron oxide, Fe_2O_3, 10 per cent

Prepared solvent, totaling 12,500 gal, dissolved approximately $^1/_2$ ton of scale (by analysis of used solvent). Turbinings from several of the tubes after the treatment showed only a trace of scale.

3 A Babcock & Wilcox return-bend integral economizer on a medium-pressure boiler, supplying process steam, requires cleaning every 6 months. When cleaned mechanically, the boiler has to be taken out of service for a considerable time, and 440 U-bends have to be removed, the turbines run, and 880 copper gaskets replaced. Mechanical cleaning of this economizer was definitely not satisfactory, so chemical cleaning was tried, which worked out very successfully. The scale deposited in the economizer averages approximately:

Calcium-hydroxy-phosphate, $Ca_{10}(OH)_2(PO_4)_6$, 75 per cent
Iron oxide, Fe_2O_3, 25 per cent

The total solvent cleaning time is less than 8 hr. Repeated treatments (ten to date) of this economizer show no ill effects.

4 A novel treatment which illustrates the varied applications of chemical-solvent cleaning was that of the underground piping of a waste-disposal system made up of 1714 ft of 16 to 30-in. pipe. X-ray analysis showed the scale in the system to be:

Iron carbonate, $FeCO_3$, 75 per cent
An amorphous iron compound, 15 per cent
Chemical analysis showed 10 per cent organic material

Prepared solvent totaling 42,000 gal was used to clean the system which was back in service within 24 hr. The pumping rate was increased from 11.6 to 14.4 million gal per day.

Advantages of Chemical Cleaning

The advantages of the chemical cleaning method include:

1 Outage time is materially reduced.

2 Dismantling operations are eliminated, since solvents can circulate through existing connections and penetrate wherever steam and water flow, cleaning all inaccessible locations. This is of considerable importance as much equipment is being designed for maximum economy and efficiency rather than accessibility.

3 No extra labor or equipment is necessary.

4 Equipment replacements, such as gaskets, turbines, etc., are greatly lessened.

Chemical cleaning, with its accompanying saving of downtime, man-hours, and metals, fits in with the nation's war effort.

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Delamination Tests of Plywood and a Proposed Specification

By C. W. MUHLENBRUCH,[1] PITTSBURGH, PA.

In order to overcome drawbacks of the standard test to determine delamination of plywood, which is dependent upon visual examination of a sample which has been subjected to a standard manner of soaking, the author has conducted tests designed to show that an actual physical test for modulus of rupture or modulus of elasticity will provide a quantitative measure of loss in strength due to wetting, and consequent delamination. Such a test is demonstrably far superior to the qualitative visual test now in use to forecast the ability of a plywood to withstand atmospheric exposure. As a result of the tests, the author proposes that a specification which is outlined in the paper be made a part of the delamination test for plywood.

THE standard test for delamination of plywood depends for its consummation upon the visual examination of a sample which has been subjected to a standard manner of soaking. Swelling of the plys often removes the external evidence of delamination. Core voids permit the entrance of water to the interior of the sample, where serious delamination may occur and yet be unobserved from the outside. The tests made in connection with this report were intended to show that an actual physical test for modulus of rupture or modulus of elasticity will provide a quantitative measure of the loss in strength due to wetting, and any consequent delamination, and that such a quantitative test is far superior to the qualitative visual type of test now in use to forecast the ability of a plywood to withstand atmospheric exposure.

Treatment of the Douglas-fir samples in various ways intended to repel the water give interesting results as to what may be expected from such treatment.

Materials Used and Treating Preparations

A moisture-resistant grade of Douglas-fir plywood, glued with a vegetable glue into a 5-ply panel, was cut to make 42 specimens 12 in. long, 4 in. wide, and $^1/_2$ in. thick. The panel was sound both sides with both sides sanded. All samples were cut from the original panel in such a way that no core voids were present or were so close to the ends of the specimen that they could be ignored. No core voids larger than $^1/_{16}$ in. were permitted. Six specimens were prepared for each condition of testing.

One set of samples was soaked without treating. Three other sets were treated with commercially available preparations intended to repel the water. These preparations will be identified as follows:

Preparation A. A toxic preservative which may be used as a vehicle for prime painting.

Preparation B. A synthetic-resin type of preservative which is a better thinner and vehicle than type *A*.

Preparation C. A mixture of 3 parts by volume of boiled linseed oil and 1 part by volume of synthetic turpentine.

The samples were dipped in preservative for 10 sec and then allowed to dry for 24 hr. Both the samples and the preservatives were kept at room temperature prior to and after dipping.

Method of Testing

Untreated as well as treated specimens were submerged in water at room temperature for a period of 4 hr. They were then dried at room temperature for a period of 20 hr. This cycle of wetting and drying was then repeated giving a total of 2 cycles for each specimen. Following the last period of drying, the specimens were tested after a final 24-hr additional drying period.

The panels were tested as simple beams on a span of 10 in. with a concentrated load at mid-span. Load was applied through a cast-iron block tapered to a $^3/_8$-in. bearing surface at the point of contact with the beam. The specimens were supported on similar blocks. Deflections were obtained with a dial gage reading directly to 0.001 in., and by estimation to 0.0001 in. Load was applied with a mechanical type of testing machine reading to 1 lb. For each 25-lb increment of load, the corresponding deflection was read to furnish the data for a load-deflection curve.

Theory and Interpretation of Test Results

If a plywood panel delaminates upon soaking, this delamination will be best observed in either a shear or a bending test. The shear test, because of the size of test panel necessary to secure dependable results, is not practical. Since horizontal shear is developed in a beam supporting bending loads, the beam test will subject the specimen to the shearing type of stress necessary to detect glue failures. The evidence of delamination may be observed both in the modulus of elasticity and the modulus of rupture. That this is true may be seen from the analysis presented herewith.

The deflection at mid-span of a simply supported beam loaded at the center may be determined from the following relationship

$$\Delta = \frac{PL^3}{48\,EI}$$

in which Δ = deflection at mid-span, in.
P = load at mid-span, lb
L = span of beam, in.
E = modulus of elasticity in bending, psi
I = moment of inertia of cross section, in.^{4}
$= \frac{bd^3}{12}$

This relationship holds, within the proportional limit, for a solid beam, or for a beam composed of two separate parts but having these parts keyed together in such a fashion that sufficient shearing resistance is offered by the keys to prevent separation along the joint, as shown in Fig. 1(*a*) (*b*). In a solid plywood panel, the glued joint is the counterpart of these keys. When the keys, or glue in the case of delamination, are not present, the resulting loss in stiffness (modulus of elasticity) and flexural strength (maximum load) occurs at a relatively fast rate, as will be shown.

The deflection of a two-section beam without keys, as shown in

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Contributed by the Wood Industries Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1943, of The American Society of Mechanical Engineers.

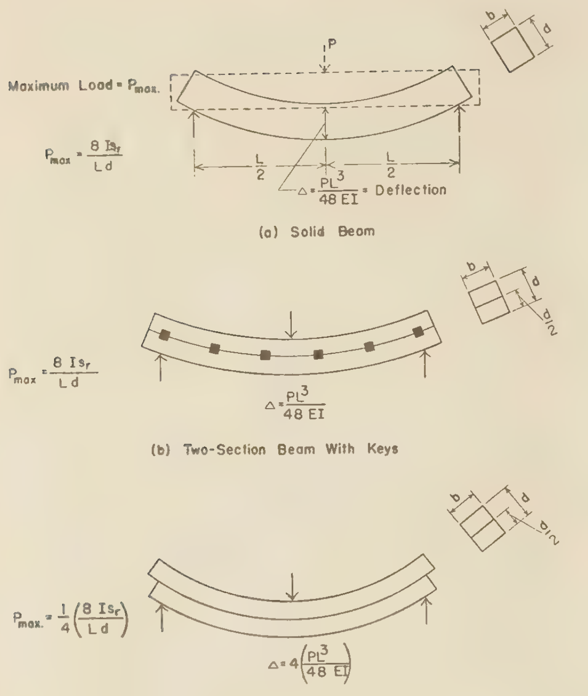

Fig. 1 Comparison of Deflections and Ultimate Loads for a Solid Plywood Panel Beam and a Delaminated Two-Section Beam

(This comparison also applies to a two-section beam with keys. The properties shown vary as the square of the number of plys.)

Fig. 1(*c*), is equal to one half the deflection of either equal section (neglecting friction). The deflection of one half the beam would be

$$\Delta'' = \frac{PL^3}{48EI'}$$

in which P, L, and E = as before
Δ'' = deflection at mid-span of one-half beam, in.

$$I' = \frac{b(d/2)^3}{12} = \frac{bd^3}{96}$$

and the deflection of the entire two-section beam would be

$$\Delta' = \frac{1}{2}\left(\frac{PL^3}{48EI'}\right)$$

Substituting the value for I'

$$\Delta' = \frac{PL^3}{Ebd^3}$$

Replacing these values for cross-sectional properties with the solid section value of $I = bd^3/12$ gives

$$\Delta' = \frac{PL^3}{12EI} = 4\left(\frac{PL^3}{48EI}\right)$$

or the deflection of the two-section beam without keys is 4 times that of the solid or properly keyed (glued) beam.

A similar analysis for the maximum load, which the various types of beams will hold, may be made with the ordinary flexure formula. The relationship for stress in a solid or properly keyed beam is

$$s = \frac{Md}{2I}$$

in which s = stress at outer fiber of beam, psi
M = bending moment, in-lb
d and I = as before

By substituting the bending moment at mid-span, $M = PL/4$, this may also be written

$$s = \frac{PLd}{8I}$$

Solving for the maximum load which the beam will support

$$P_{max} = \frac{8Is_r}{Ld}$$

in which

s_r = modulus of rupture, psi, the fictitious maximum fiber stress which exists at the ultimate load. Since the flexure formula may be applied only to stresses within the proportional limit, the actual maximum load is much higher than that obtained using the ultimate strength of the material for s_r.
I, L, and d = as before

Replacing the moment of inertia in this relationship with that for two half sections of a two-section beam without keys, as was done for deflection, and solving for the maximum load gives

$$P'_{max} = \frac{1}{4}\left(\frac{8Is_r}{Ld}\right)$$

or the maximum load which the two-section beam without keys will support is one fourth that which the solid or properly keyed beam will support.

From a similar analysis, it may be shown that, for a beam made up of four unkeyed sections, the deflection is 16 times that of a solid beam, and the maximum load which the beam will support will be $^1/_{16}$ that which a solid beam will support. It may be stated therefore that these properties vary as the square of the number of unkeyed sections or laminations. This leads to the general formula for deflection

$$\Delta = n^2\left(\frac{PL^3}{48EI}\right)$$

in which n = number of unkeyed or unglued laminations
P, L, E, and I = as before, with I based on depth of all plys included in term n.

and for maximum load

$$P_{max} = \frac{1}{n^2}\left(\frac{8Is_r}{Ld}\right)$$

These relationships show that both the deflection and maximum load are very sensitive to change caused by delamination and would therefore make a desirable quantitative test to present the results of exposure.

Results of Tests

The first tests were made to determine the properties of the untreated and unsoaked panels. The results of these tests are summarized in Table 1 for those specimens designated by the symbol P. As noted in this table excessively high or low values

TABLE 1 MODULUS OF ELASTICITY AND MODULUS OF RUPTURE OF 1/2-IN. DOUGLAS-FIR PLYWOOD-PANEL SPECIMENS, TREATED WITH VARIOUS WATER-REPELLENT PREPARATIONS

Preservative	Specimen	Modulus of elasticity, psi	Average modulus of elasticity, psi	Modulus of rupture, psi	Average modulus of rupture, psi
Untreated	P-1	997500[b]		7470[b]	
	P-2	1168000		9300	
	P-3	1178000	1198000	9300	8970
	P-4	1308000	or	9300	or
	P-5	1240000	1238000[a]	9220	9270[a]
	P-6	1297000		9220	
Preparation *A*	A-1	1120000[b]		9150	
	A-2	1308000[b]		7320	
	A-3	1201000	1210000	8380	8230
	A-4	1235000	or	8840	or
	A-5	1226000	1209000[a]	9300	8600[a]
	A-6	1172000		6400[b]	
Preparation *B*	B-1	1133000		8840	
	B-2	1472000[b]		8380	
	B-3	1183000	1226000	9000[b]	8590
	B-4	1030000[b]	or	8230	or
	B-5	1264000	1214000[a]	7920[b]	8580[a]
	B-6	1274000		8850	
Preparation *C*	C-1	1409000[b]		9000	
	C-2	1175000		8180	
	C-3	1370000	1262000	9000	8660
	C-4	1266000	or	9600[b]	or
	C-5	1190000	1232000[a]	8380	8640[a]
	C-6	1160000		7770[b]	

[a] Excessively high or low values not included in average.
[b] Not included in average indicated by (*a*).

TABLE 2 MODULUS OF ELASTICITY AND MODULUS OF RUPTURE OF 1/2-IN. DOUGLAS-FIR PLYWOOD-PANEL SPECIMENS, TREATED WITH VARIOUS WATER-REPELLANT PREPARATIONS AND THEN SUBJECTED TO 2 CYCLES OF 4 HR SOAKING, FOLLOWED BY 20 HR OF DRYING

Preservative	Specimen	Modulus of elasticity, psi	Average modulus of elasticity, psi	Loss in E, per cent	Modulus of rupture, psi	Average modulus of rupture, psi	Loss in s_r per cent
Untreated and unsoaked	P-1	997500[b]			7470[b]		
	P-2	1168000			9300		
	P-3	1178000	1198000		9300	8970	
	P-4	1308000	or	0.0	9300	or	0.0
	P-5	1240000	1238000[a]		9220	9270[a]	
	P-6	1297000			9220		
Untreated and soaked	P-1-s	800000			4120[b]		
	P-2-s	770000			4580[b]		
	P-3-s	857000	780000	34.9	4350	4370	51.3
	P-4-s	844000	or	or	4350	or	or
	P-5-s	755000	805000[a]	35.0[a]	4420	4390[a]	52.6[a]
	P-6-s	655000[b]			4420		
Preparation *A*	A-1-s	912000[b]			7550		
	A-2-s	1205000[b]			7010		
	A-3-s	1125000	1065000	11.1	8310[b]	7460	16.8
	A-4-s	996000	or	or	7310	or	or
	A-5-s	972000	1068000[a]	13.7[a]	6700[b]	7430[a]	19.8[a]
	A-6-s	1180000			7850		
Preparation *B*	B-1-s	841000			5340		
	B-2-s	871000			5110		
	B-3-s	811000	833000	30.4	5420	5470	39.1
	B-4-s	749000	or	or	5490	or	or
	B-5-s	989000[b]	801000[a]	35.3[a]	6480[b]	5340[a]	42.4[a]
	B-6-s	735000			4960[b]		
Preparation *C*	C-1-s	726000[b]			5190		
	C-2-s	760000			6480[b]		
	C-3-s	738000	785000	34.5	5340	5370	40.1
	C-4-s	844000	or	or	4960	or	or
	C-5-s	870000[b]	778000[a]	37.1[a]	4880	5140[a]	44.5[a]
	C-6-s	770000			5340		

[a] Excessively high or low values not included in average.
[b] Not included in average indicated by (*a*).

may be ignored in arriving at satisfactory averages for modulus of rupture and modulus of elasticity.

The modulus of elasticity in bending was determined from the load-deflection diagram drawn from the test data. Enough loads and corresponding data points were obtained to plot the straight-line portion of the diagram. Although 12 to 15 points were thus obtained, as few as 6 readings would have been sufficient to establish the line properly. In order to calculate the modulus of elasticity, the deflection for a relatively high load was taken from the curve (not necessarily an actual data point) and substituted in the formula for the deflection of a solid beam. Since the span and properties of the cross section were actually measured, the modulus of elasticity could readily be obtained by calculation.

Before the effect of soaking in water could properly be determined it was necessary to find the effect of the preservatives upon the physical properties. A set of six specimens was therefore selected for treatment with each of the three preparations used. The results of these tests, which are also summarized in Table 1, show that both the modulus of rupture and the modulus of elasticity remain about the same when treated, although the former is lowered a small amount. This small reduction may be due to the wetting action of the preparations which would naturally tend to reduce the properties. It follows, therefore, that any loss which might be found after soaking could be attributed solely to delamination.

The results obtained for the specimens subjected to soaking are summarized in Table 2. As may be seen from Table 2, the reduction in modulus of elasticity due to soaking was 35 per cent for the untreated panels, and the reduction in modulus of rupture was 52.6 per cent. Most of this reduction was due to delamination which was not visible before the test. A small amount of the reduction was undoubtedly caused by the increased moisture content, although the total drying time of 44 hr after the last period of soaking had reduced the moisture content until it was approximately 5 per cent more than that of the treated but unsoaked specimens.

The ability of the various preparations to repel the water is apparent from Table 2. Preparation *A* appears to be the most

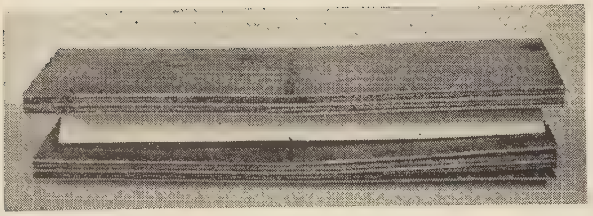

Fig. 2 Characteristic Fractures for Sound and Partially Delaminated Plywood Panels When Subjected to Bending Loads

(Sound panel is shown at top. Delamination shown was not visible before test.)

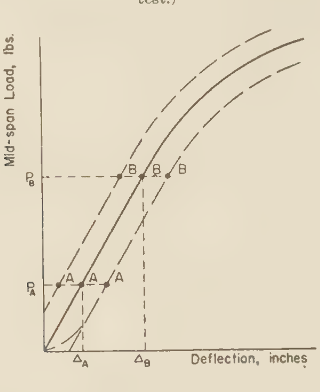

Fig. 3 Characteristic Load-Deflection Curves for Beams Showing Points Which May Be Used to Determine Modulus of Elasticity

effective. For this treatment, the modulus of elasticity was reduced only 13.7 per cent, and the modulus of rupture 19.8 per cent. The linseed-oil-turpentine combination used for preparation C was the least effective. This treatment did not increase the modulus of elasticity over that obtained for the untreated and soaked panels, but it did permit a slightly higher breaking load, as evidenced by the higher modulus of rupture. Preparation B was slightly better than C but not as satisfactory as A. More serious delamination, that is, deeper and longer separation, was observed for untreated specimens and for those treated with preparations B and C than was found for those treated with preparation A. In many cases, the delamination was entirely internal and was caused by the entrance of water through a core void. It should be pointed out that a longer time of dip for the linseed-oil-turpentine combination would undoubtedly have permitted better penetration of this higher-viscosity preparation. For comparative purposes, however, the time of dip was held constant for all preparations.

These test results seem to indicate that a worth-while and properly applied preservative might be expected to protect plywood panels (and undoubtedly ordinary cut lumber) against loss in strength caused by exposure to severe moisture conditions. That the physical properties of untreated plywood are severely affected by such exposure is also evident.

Characteristic fractures obtained for a sound and a partially delaminated panel are shown in Fig. 2. As was true in all other cases, the delamination, so evident in Fig. 2, could not be detected by surface inspection prior to the completion of the test. These specimens were of the same set and were numbered P-1-s and P-6-s, the latter being the delaminated panel. It may be seen from Table 2 that these specimens differ considerably, yet both would have passed the visual-inspection type of test. Whether the amount of delamination was sufficient to warrant rejection on the basis of loss in physical properties may be a matter of opinion. The point has been established that the quantitative type of test employed here is far superior to the visual qualitative test now used.

Test to Determine Loss in Modulus of Elasticity

In order to determine the loss in modulus of elasticity or modulus of rupture by means of a rapid laboratory test, adaptable to routine analysis, a much simpler but just as dependable type of test could be used. Since the modulus of elasticity is merely a numerical statement of the stiffness or deflection observed during the test, the actual deflection may be more conveniently reported. Thus, the data for this property in Tables 1 and 2 might have been presented as some observed deflection at some specified load. This method would be satisfactory for all specimens having load-deflection curves which passed through the origin, as shown by the solid curve in Fig. 3. Very often, however, the load-deflection curves will be similar to those shown as dotted in Fig. 3. This type of curve is usually due to warping of the test panel so that it cannot be properly seated on the supports nor can load be applied across the entire specimen at mid-span. This condition will result in a diagram which "brooms-out" near the origin but quickly establishes a straight-line relationship. If the dial with which the deflection is obtained is not properly placed at the zero load position, the load-deflection curve will not pass through the origin as shown in Fig. 3. Regardless of the type of curve then, the modulus of elasticity is the slope of the straight-line portion of the load-deflection curve. In Fig. 3, the slope of any curve from A to B would be used to obtain the modulus of elasticity. The test procedure would be as follows:

1 Place sample panel in test position and apply load P_A. Read deflection dial.

2 Increase load to P_B and read deflection dial.

3 Substitute difference in loads and difference in deflections in the formula

$$E = \frac{PL^3}{48\Delta I} = \frac{(P_B - P_A)L^3}{48(\Delta_B - \Delta_A)I}$$

and solve for E. By this method, the modulus could be determined quickly and with a minimum of observation. Thus, the test would be readily adaptable to routine laboratory analysis.

The loads P_A and P_B to be used in the previous formula will vary for different thicknesses of plywood, width of panel, and length of span. It would therefore be desirable to standardize the last two and specify the two loads required for various thicknesses of plywood.

It may be seen from Table 2 that the modulus of rupture is affected to a greater degree by soaking than is the modulus of elasticity. For some uses then, it may be preferred over the modulus of elasticity. A test for breaking load, from which the modulus of rupture is obtained, is quite easy to make and is less costly than the deflection test. The results of the test need not be reported as modulus of rupture, but merely as the breaking load obtained for a stated thickness and width of panel and length of span.

Conclusions and Recommendations

As a result of the data and theoretical considerations presented in this report, the following conclusions seem justifiable:

1 A quantitative type of test for modulus of elasticity (deflection) or modulus of rupture (breaking load) is far superior to the visual qualitative type of test now used to determine the amount of delamination in a plywood panel subjected to water soaking.

2 The physical properties of untreated plywood panels are lowered when subjected to water soaking.

3 The presence of core voids will increase the delamination and consequent loss in physical properties.

4 Worth-while and properly applied preservatives may be expected to reduce this loss.

It is further proposed that a specification of the following general type be made a part of the delamination test for plywood:

Six plywood panels 4 in. wide and 12 in. long (for example), having no serious core voids within the middle third of their length, shall be maintained at a constant room temperature and humidity for 24 hr after which they shall be submerged in tap water at room temperature for 4 hr. The panels shall then be removed from the water and dried at the constant room temperature and humidity for 20 hr. This cycle of 4 hr soaking and 20 hr drying shall then be repeated once (a total of 2 cycles). Following an additional 24 hr drying at the constant room temperature and humidity, the six panels shall be tested as beams with the thickness vertical and on a 10-in. span (for example). A concentrated load shall be applied at mid-span. The average decrease in modulus of elasticity and breaking load, compared to the average of six unsoaked specimens, shall not be more than that given in Table 0 (to be provided by the writer of the specification) for the thicknesses of panel indicated.

A sample form for Table 0 would then be as follows:

TABLE 0 MAXIMUM AVERAGE LOSS IN MODULUS OF ELASTICITY[a] AND ULTIMATE LOAD FOR PLYWOOD PANELS SUBJECTED TO WATER SOAKING AND TESTED AS SIMPLE BEAMS

Thickness of panel and number of plys	Average loss, per cent	
	Modulus of elasticity	Ultimate load

[a] To be obtained in a manner similar to that outlined in this paper.

Acknowledgments

The author wishes to acknowledge the valuable assistance and suggestions made in connection with these tests by Mr. Hal Keely of the Hal Keely Plywood Company, Pittsburgh, Pa., who also furnished the plywood test panels and two of the treating preparations.

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Some Observations on Density and Shrinkage of Ponderosa Pine Wood

By R. A. COCKRELL,[1] BERKELEY, CALIF.

California timber has largely been obtained from original old-growth forests of redwood, ponderosa pine, and sugar pine, to the point where these forests are rapidly becoming depleted. Future supplies of these species must come from second-growth forests established on cutover land. However, prejudices have developed against the use of second-growth timber, especially redwood and ponderosa pine. Some of these may be justified, but others are based solely upon false assumptions and misinformation. To clarify the situation, this paper presents the results of a preliminary survey of density and shrinkage percentages of California ponderosa pine, as related to growth rate, position in tree, and age. These properties considered together constitute a fairly reliable index of the mechanical and physical characteristics of wood. Strength, hardness, and ease of working with tools are related directly to wood density, and the seasoning behavior of the wood is associated with its differential shrinkage.

PRACTICALLY all of the timber produced in California has been obtained from the original old-growth forests and largely from the three species, redwood, ponderosa pine, and sugar pine. These original forests are rapidly being depleted and, in the not too distant future, wood supplies of these three species must be obtained from second-growth forests established on cutover lands. Such timber will be utilized when younger and, consequently, smaller than the trees now available in the original forests.

Already a prejudice has grown up against the second-growth wood, particularly redwood and ponderosa pine, as compared with the old-growth material. Undoubtedly, there is justification for some of the views held, but others appear to be based entirely upon notion or false assumptions from observation of timber products improperly manufactured. Since, in some areas, the trees from these younger stands are already being utilized, it behooves us to learn more about the variation of the technical properties of the wood of these species resulting from differences of environment, position in tree, and age and vigor of the tree, so that we may be in a better position to manage the stands properly and to make most effective use of the wood.

This paper reports the results of a preliminary survey of density and shrinkage percentage of California ponderosa pine, particularly as related to growth rate, position in tree, and age. These properties were selected because together they constitute a fairly reliable index of the mechanical and physical characteristics of wood. Strength, hardness, and ease of working with tools are all directly related to wood density, and the seasoning behavior of the wood is associated with its differential shrinkage.

The relationship of density and shrinkage to ring width (growth-rate) has been the subject of studies on many different species. In his study of the relation of growing conditions to specific gravity, Paul (1)[2] states: "Both very wide and very narrow rings in conifers usually contain a larger proportion of the springwood layer, so that in these species wood representing either extreme of growth may be low in specific gravity. The wood of intermediate growth rate is usually the heavier." On the other hand, Lodewick (2), in a study based upon southern pine, presents data that show the summerwood percentage of the ring increasing with decrease in ring width, while Turnbull (3), working with exotic pines grown in South Africa, states that density, which is a function of the percentage of summerwood, is not determined by rate of growth but follows a regular pattern which is specific for each individual tree. He points out that since summerwood in a particular ring narrows in ascent while springwood widens in ascent the summerwood percentage would decrease with ascent and, accordingly, for any given cross section the summerwood percentage would increase from the pith to the periphery. Successive sections taken upward in the stem would have progressively less summerwood, and, since density is a function of summerwood ratio, the specific gravity would decrease progressively up the stem.

This is further corroborated by Siimes (4) working with *Pinus silvestris*, who states, ". . . the weight of pine decreases from the base to the top, as is also true of the summerwood percentage and accordingly the tangential and radial shrinking percentages." Markwardt and Wilson (5), in discussing the effect of various factors on the strength of wood, state that the lighter-weight, slower-growth coniferous wood shrinks and swells less (laterally) than the heavier material. With regard to longitudinal shrinkage, Koehler (6), investigating southern pine, concluded that extremely light, rapidly grown wood (less than

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

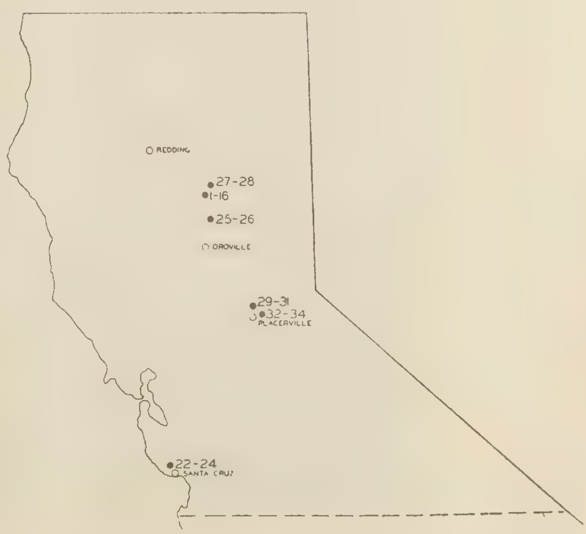

Fig. 1 Location of Trees

[1] University of California; temporarily at Forest Products Laboratory, Madison, Wis.

Contributed by the Wood Industries Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1943, of The American Society of Mechanical Engineers.

TABLE 1 GENERAL DESCRIPTIVE DATA

Tree no.	D.B.H. (ob)	Stump[a] diam, (ib)	Height, ft	Rings at stump	Dunning tree class	Elevation	Stand condition
1[b]	33.0	28.2	127	133	1	4200	
3	17.7	17.0	81	56	2	4200	
6	31.8	27.1	126	114	3	4200	
7	19.9	16.0	92	105	2	4200	
9	20.9	17.9	114	100	2	4200	All-aged mixed conifer, large pines cut out about 60 years ago
10	24.3	22.3	109	87	1	4200	
11	26.7	25.7	124	119	2	4200	
12	30.2	27.7	127	119	2	4200	
15	20.6	19.5	107	82	1	4200	
16	21.4	18.9	105	85	1	4200	
22[c]	21.6	20.4	93	79	2	1700	On edge fairly open stand, ponderosa and knobcone
23	21.8	21.0	114	117	4	450	Open stand on upper slope, ponderosa and knobcone
24	18.1	17.2	97	94	1	450	Edge of a group of 12 trees; stand rather dense
25[d]	36.2	34.0	148	193	4	3750	Fairly open mixed conifer, old growth
26	38.8	32.4	163	203	3	3750	Fairly open mixed conifer, old growth
27	59.5	54.8	173	365	5	4880	Mixed conifer, predominantly fir, well stocked
28	52.0	47.0	167	382	5	4880	Mixed conifer, predominantly fir, well stocked
29[e]	23.4	19.6	112	73	2	2600	Interior, fairly dense pure pine second growth
30	24.2	22.3	120	78	1	2600	Interior, fairly dense pure pine second growth
31	27.1	24.0	128	59	1	2600	Margin, open stand pure pine second growth
32	38.7	36.0	160	144	3	3300	Open stand, low brush understory
33	36.5	33.8	149	143	4	3300	Open stand, low brush understory
34	30.5	27.8	140	148	4	3300	Stand of fair density, low brush

[a] Diameter measured along strip from which samples were prepared.
[b] First 10 trees obtained from second-growth pine mill study through the California Forest and Range Experiment Station.
[c] Trees 22 to 24 obtained through co-operation of State District forest ranger at Felton, Calif.
[d] Trees 25 to 28 obtained through co-operation of Diamond Match Company.
[e] Trees 29 to 34 obtained through co-operation of U. S. Soil Conservation Service.

$3^1/_2$ rings per in.) usually shrank along the grain in excess of 0.25 per cent.

In this preliminary study, wood was obtained from several trees, representing different age classes and from different localities, on which to determine the variation in density and shrinkage from green to air-dry and oven-dry condition with respect to rate of growth and position in the tree.

Material and Methods

The location of the twenty-three trees studied is shown on the outline map, Fig. 1. The general descriptive information is given in Table 1. Disks 8 in. thick were cut at the stump level and at convenient intervals along the trunk and were carefully kept in the green condition until they were worked up into samples. Test pieces to determine longitudinal, radial, and tangential shrinkage were prepared as shown in Figs. 2 and 3. For the longitudinal shrinkage, pieces 1 in. wide × 4 to 6 in. long were cut along the diameter of each disk to simulate the cutting of a log into 1 in. boards. Every effort was made to select clear wood free from the influence of knots or other defects, and values from

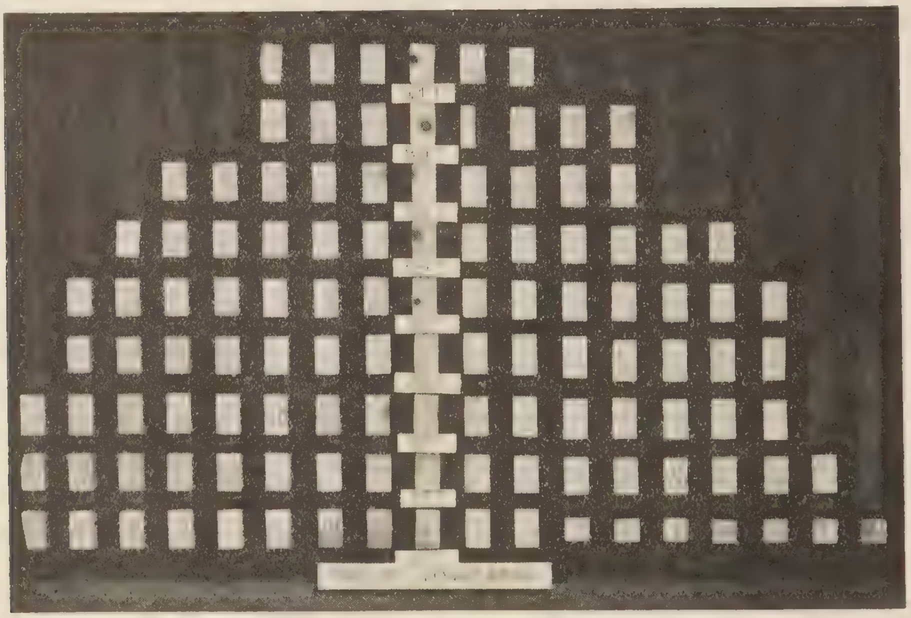

Fig. 2 Longitudinal-Shrinkage Samples From Tree 29
(Samples separated laterally by $^3/_{16}$ in. saw kerf.)

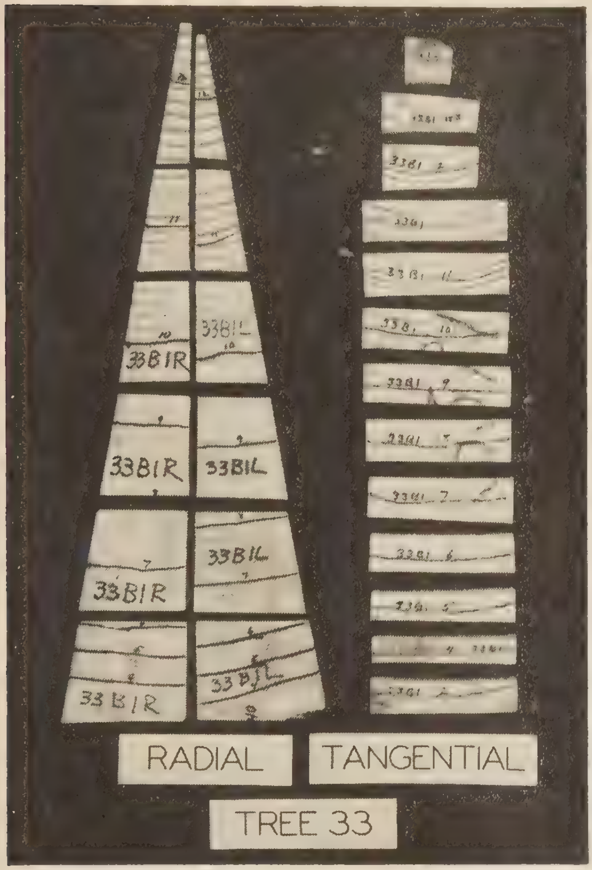

Fig. 3 Radial and Tangential Samples From Tree 33

test pieces with any irregularities of grain were not included in the final computations. Where the disks were cut from the ends of logs on a commercial operation it was difficult to avoid cutting at the nodes in many cases, and often near the center of the disk cut at the stump level there were knots which were invisible from the outside.

Prior to cutting, the sequence number of a growth ring was marked on every test piece to locate the medial ring for use later in establishing the trend of values with relation to ring position in the cross section. The weight, volume, and length of the test pieces were taken for the green, air-dry (exposed in room until constant weight was reached), and oven-dry conditions. All weights were taken to 0.1 g on a standard gravimetric balance. Volumes were obtained by determining the weight of displaced mercury, being careful to make corrections for density with change in temperature. Lengths were recorded to the nearest 0.001 in. All test blocks were measured with a Starrett micrometer caliper, except those of trees 29 through 34 which were measured with a bench gage, using a dial micrometer. Specific-gravity determinations were based upon green volume and oven-dry weight, and shrinkage was expressed as a percentage of the green dimension.

Results of Tests

Longitudinal Shrinkage. On the basis of measurements made on approximately 1800 test blocks, it was found that the longitudinal shrinkage from the green to the oven-dry condition was greatest for the blocks taken from near the pith of the stump disks. In Fig. 4, the values for longitudinal shrinkage green to oven-dry of basal and second disks from several trees are plotted with respect to the number of rings from the pith out to 100 rings. From this it may be observed that the blocks taken from the basal disk exhibited the greatest range in values, and the high values extended farthest from the pith. A few of the blocks from the disks above the stump level had longitudinal shrinkage above 0.3 per cent up to twenty rings from the pith, but beyond this point they remained quite consistently within a very narrow range below this value. The shrinkage percentage for the rings beyond 100 from the pith followed the same general pattern established in the 50 to 100 interval.

Using the 1-in. test blocks to define the limits, the dimensions of the central core of wood, having longitudinal shrinkage in excess of 0.4 per cent for comparable height disks of all the trees, are given in Table 2. It is evident from these figures that the central zone of wood, as well as the number of rings from the pith having excessive axial shrinkage, is conspicuously greater in

TABLE 2 CENTRAL CORE OF WOOD ABOUT PITH WHERE LONGITUDINAL SHRINKAGE FROM GREEN TO OVEN-DRY EXCEEDS 0.4 PER CENT

	Stump Height			18 feet			34 feet			64 feet		
Tree no.	Central core diam (or radii), in.	Rings from pith	Rings per inch in central core	Central core diam (or radii), in.	Rings from pith	Rings per inch in central core	Central core diam (or radii), in.	Rings from pith	Rings per inch in central core	Central core diam (or radii), in.	Rings from pith	Rings per inch in central core
1	3	31	16	..	...	...	1/2:7 1/2 [a]	2:67	7	1	2	4
3	4 [b]:7 [b]	22:55	6	..	...	...	2	3	3	1/2:1 1/2 [d]	3:5	3
6	4	18	8	..	...	...	0:2	0:7	3	1 1/2 [b]:4 1/2 [b]	8:39	4
7	2 1/2:7 [b]	29:105	11	..	...	...	2:6 [c]	14:67	8	1:3 [c]	3:22	6
9	1 1/2 [b]:2 1/2 [b]	18:25	8	..	...	...	1/2:1 1/2	3:6	4	1 1/2 [b]:2 1/2 [b]	10:16	4
10	1:2	6:14	7	..	...	...	1	1	2	1 [e]	3	3
11	2 [b]	13	8	..	...	...	1	3	4	2	8	4
12	4	15	7	..	...	...	3 [b]	5	3	1	3	3
15	6 [b]	35	8	..	...	...	0	0	...	1/2:2 1/2	3:13	4
16	6 [b]	29	8	..	...	...	2	3	4	1	3	7
22	8 [b]	30	8	1	2	4	1	3	4	1	4	7
23	6	27	7	0	0	...	0	0	...	0	0	
24	4 [b]	21	12	1	3	6	0:2	0:10	6	1	5	8
25	2 [b]	21	20	..	...	...	1	3	6	2	7	5
26	4	16	7	..	...	...	1	3	5	3	6	5
27	4	31	17	..	...	...	1	5	7	0	0	..
28	4	22	11	..	...	...	1	3	7	1	7	6
29	5	12	4	1 [f]	2	2.5	1	3	3	1	3	4
30	3:4 1/2 [b]	17:24	5	1 [g]	2	3	1	2	3	1	3	5
31	2:3 [c]	6:9	3	1	2	2.5	1	2	2.5	1	3	2.5
32	1/2:1 1/2 [b]	3:8	5	1	2	4	...	...	...	...	..	..
33	3	12	8	1	2	3	1	2	3	...	...	
34	3	12	8	1	2	4	1	2	4	...	...	

[a] When zone of excess longitudinal shrinkage does not center at pith, separate figures for rings and radius given for each side along test strip.
[b] Irregularity of grain from defects affects part or all of these blocks.
[c] Location of pith within disk eccentric, values from short and long radii.
[d] Disk cut at 52 ft above stump.
[e] Disk cut at 58 ft above stump.
[f] Disk cut at 9 ft above stump.
[g] Disk cut at 14 ft above stump.

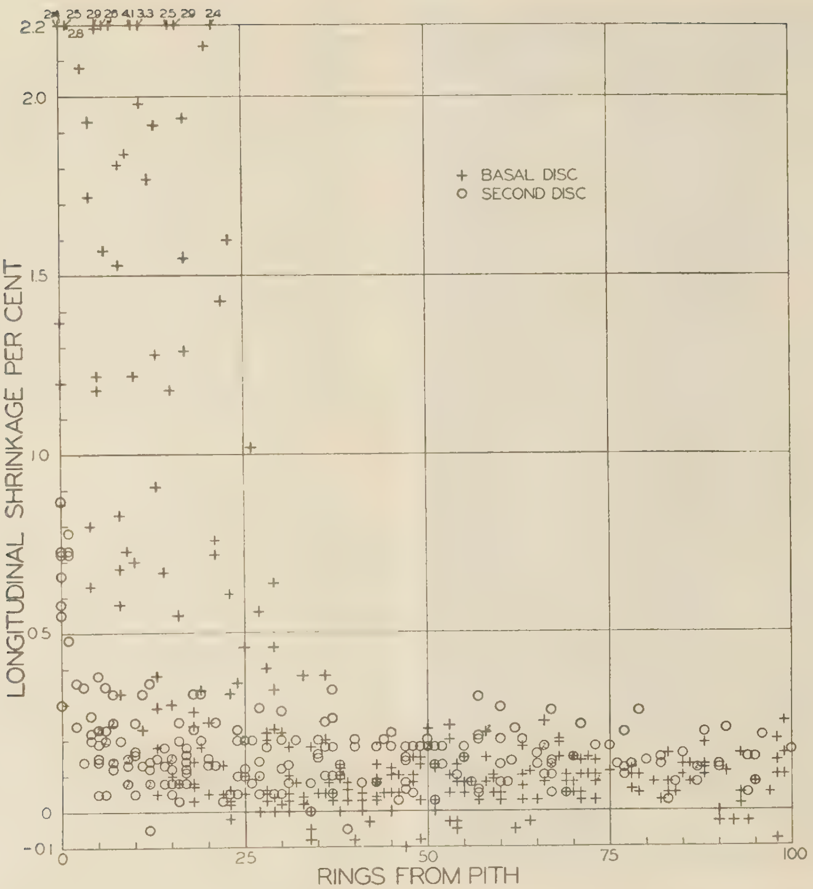

Fig. 4 Relationship Between Longitudinal Shrinkage Green to Oven-Dry, and Rings From Pith for Basal and Second Disks of Trees 22–31, 33, and 34

the base than higher in the tree. Unfortunately, many of the basal disks had knots, wound scars, or other defects in the wood near the pith or were slightly asymmetrical, which undoubtedly accentuated some of the shrinkage values or extended the excess-shrinkage zone. Nevertheless, trees such as 1, 6, 10, 12, 23, 27, 28, 29, 33, and 34 with perfectly clear sound wood in this region would indicate that ordinarily a zone of from 3 to 6 in. of wood with excess longitudinal shrinkage can be expected in the center of the stump or large end of the butt log. Depending upon the rate of growth, somewhere from 12 to 30 growth rings about the pith were included in this zone. In the disks higher in the tree, this zone seldom exceeded 1 in. and included only a few rings from the pith.

It is not possible from the data assembled in this work thus far to state precisely within what limits the zone of wood with excess shrinkage usually narrows from the condition prevailing in the stump to that characteristic of the disks higher in the tree. It is reasonably certain, however, that the condition does not extend very far up the trunk in the normal forest-grown tree, since the second disks, taken in trees 29 and 30, cut at approximately 5 ft above the ground, showed a longitudinal-shrinkage pattern near the pith typical for the upper disks of the trees studied rather than the basal disk (see Table 3).

Beyond the central zone of wood which shrank an excessive amount longitudinally, most of the blocks shrank less than 0.3 per cent, while for a few trees the values were mostly below 0.2 per cent. Some of the blocks showed no change of length when dried from the green to the oven-dry condition, and 39 of the blocks actually increased slightly in length (less than 0.1 per cent). Of the blocks which failed to shrink, 31 were cut from the basal disks.

Measurements of longitudinal shrinkage from the green to the air-dry (9 to 12 per cent moisture content) condition follow the same pattern as the green to oven-dry figures, the highest shrinkage taking place in the central blocks, with relatively uniform values prevailing toward the periphery. The relationship between shrinkage and number of rings from the pith for seasoning to the air-dry state out to 100 rings is shown in Fig. 5. Excepting the blocks with high shrinkage near the pith, practically all of the values are below 0.2 per cent, with a considerable number remaining constant in length, or showing elongation. As in shrinkage to the oven-dry condition, the majority of those remaining constant or lengthening were cut from the basal disks, but for this stage in the drying this failure to shrink was somewhat more general, and tree 26 had one or more blocks that increased in length in each disk.

Between thirty and fifty rings from the pith in Fig. 5, there are plotted points for twice as many blocks from the basal disks that lengthened in drying as blocks that shortened. The concentration of blocks from the disks above stump height, showing elongation between five and twenty rings from the pith in Fig. 5, were mostly from trees 29 and 30. The second disks of these trees were taken about 5 ft above the ground, and even though the blocks near the pith did not show excessive shrinkage,

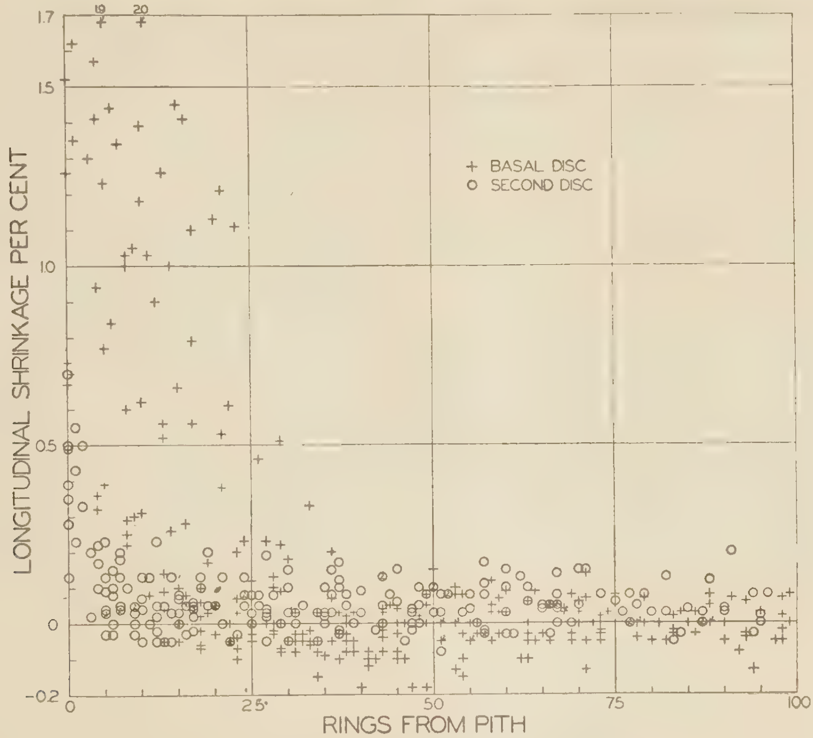

FIG. 5 RELATIONSHIP BETWEEN LONGITUDINAL SHRINKAGE GREEN TO AIR-DRY, I.E., 9 TO 12 PER CENT MOISTURE CONTENT, AND RINGS FROM PITH FOR BASAL AND SECOND DISKS OF TREES 22–31, 33, AND 34

the other blocks did tend to remain stationary or elongate in a similar manner to the test pieces from the basal disk. It is rather significant to note in respect to this disinclination to shrink axially in drying that all of the trees except one (tree 7; no lean evident when standing but wound scars present and bole asymmetrical; see Table 2) had some blocks that remained constant in length while nineteen of the twenty-three trees had blocks which elongated (mostly 0.05 to 0.1 per cent, maximum 0.18 per cent).

In Table 3 are listed the green to air-dry shrinkage percentages for the blocks including the pith for comparable disks for several trees representing different localities and age classes. The values for the pith blocks in the basal disks are conspicuously higher than for the others, while in many instances the values for pith blocks in the upper disks are within the limits of the majority of the blocks farther out from the pith. Some of these blocks of intermediate shrinkage (0.36 to 0.50 per cent) contained only the pith ring and the adjoining ring, while several of the others with little shrinkage (about 0.10 per cent) contained only three to four rings from the pith. In most cases above the stump level only the pith ring and perhaps the adjoining one or two rings had abnormally high axial shrinkage.

TABLE 3 LONGITUDINAL SHRINKAGE PERCENTAGE FROM GREEN TO AIR-DRY STATE FOR BLOCKS INCLUDING PITH AT DIFFERENT HEIGHTS ABOVE GROUND

Tree no.	2 feet	5 feet	34 feet	66 feet	80 + feet
1	2.23	..	0.36	0.98	0.56
24	2.04	..	0.28	0.25	0.10
25	0.90	..	0.50	0.75	0.60
26	1.62	..	0.55	0.84	0.70
27	1.00	..	0.46	0.10	0.08
28	1.18	..	0.23	0.36	0.56
29	0.67	0.08	0.46	0.36	0.30
30	1.23	0.50	0.41	0.43	0.28
31	1.35	..	0.36	0.36	0.13
33	1.39	..	0.36	..	..
34	0.61	..	0.47	..	..

The percentages for green to air-dry and green to oven-dry shrinkage for identical blocks did not indicate any uniform relationship between moisture content and longitudinal shrinkage. In the case of those blocks that remained constant or increased in length to the air-dry condition, and then shrank with further drying to the oven-dry condition, this disparity was quite marked.

In order to test for a relationship between longitudinal shrinkage and specific gravity, the shrinkage values of the sapwood blocks from all the disks of trees 22 to 31, inclusive, were segregated by specific-gravity classes and averaged. The resulting figures listed in Table 4 show, though somewhat obscurely, a

TABLE 4 AVERAGE LONGITUDINAL SHRINKAGE PER CENT OF SAPWOOD, GREEN TO OVEN-DRY, BY SPECIFIC GRAVITY CLASSES EXCLUSIVE OF PITH REGION

Tree no.	Specific gravity 0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58
22	..	..	..	..	..	..	0.16	0.18	0.18	0.23	..	0.03	0.08	0.12	0.07	0.03	....
23	..	..	..	..	..	..	..	0.18	0.25	0.25	0.20	0.07	0.03	0.19	0.02	0.11	—0.04
24	..	..	..	..	0.18	0.18	0.14	0.14	0.09	0.05	0.08	0.05	0.05	..	..	..	....
25	..	..	0.15	0.17	0.13	0.10	0.10	0.04	0.13	0.12	0.03	0.08	..	..	..	..	....
26	..	..	..	..	0.10	0.08	0.13	0.12	0.08	0.07	0.09	0.05	0.08	..	..	..	....
27	0.20	0.23	0.21	0.25	0.16	..	0.17	0.15	0.12	0.10	0.05	..	..	..	..	..	....
28	..	0.17	0.18	0.26	0.16	0.14	0.15	0.10	..	0.17	0.17	..	..	..	..	..	....
29	..	..	..	0.16	0.15	0.15	0.15	0.11	0.13	0.06	0.08	0.01	0.05	0.10	..	..	....
30	..	..	..	0.23	0.15	0.15	0.15	0.11	0.08	0.12	0.07	0.09	0.02	..	..	..	....
31	..	..	..	0.14	0.14	0.17	0.14	0.13	0.07	0.09	0.04	0.09	..	..	..	..	....

TABLE 5 RELATION BETWEEN AXIAL AND TRANSVERSE SHRINKAGE[a]

Tree no.	Height above ground, ft	Rings from periphery	Longitudinal shrinkage, per cent	Tangential shrinkage, per cent	Radial shrinkage, per cent
33	18	60	0.14	5.8	Samples not matched
		70	0.32	5.3	
		80	0.14	6.2	
		90	0.34	5.2	
		100	0.10	5.9	
33	34	60	0.15	5.6	5.3
		70	0.29	4.7	2.8
		80	0.12	5.3	3.6
		90	0.31	5.3	2.1
		100	0.14	4.0	3.4
34	2	110	0.14	5.7	4.6
		120	0.22	5.4	3.7
		130	0.07	7.1	4.5

[a] Green to oven-dry.

general trend toward increasing axial shrinkage with decreasing density. This same trend is evident when the individual block values are plotted, with the scatter of the points quite uniform rather than showing a concentration toward the center. When the specific gravity and axial shrinkage of a particular set of rings are compared for successive disks upward in the tree, the specific-gravity values almost always decrease, but the shrinkage values seldom follow any regular trend. Likewise, in progressing from the pith to the periphery of a cross section, there is often an observable trend in specific-gravity values but no corresponding trend in axial shrinkage.

Tangential and Radial Shrinkage. Within any one cross section, the percentage values of tangential and radial shrinkage from green to oven-dry condition were at a minimum in the rings closest to the pith. They increased rather abruptly in the rings farther from the pith, and then with minor fluctuations showed a slight decrease or remained relatively uniform in progressing to the periphery. This trend is shown for the basal and 32-ft disks of tree 26 in Fig. 6. The tangential values for the 32-ft disk increased much more abruptly outwardly from the pith than those of the basal disk, in this respect behaving similarly but in inverse fashion to the longitudinal shrinkage.

This inverse relationship between longitudinal and lateral shrinkage is evident in many instances where the values fluctuate widely in adjoining blocks and where the longitudinal, radial, and tangential test specimens are matched (contain the same rings and come from the same position in the disk). Three illustrative examples of this relationship are given in Table 5. The two sets of figures given for tree 33 come from blocks that were aligned vertically in the trunk but 16 ft apart, and, interestingly enough, the corresponding blocks in the basal disk showed the same variation in axial shrinkage.

Incidentally, it might be mentioned that the only points from the second disks beyond 20 rings from the pith in Fig. 4 that are above 0.3 per cent are those representing the blocks 70 and 90

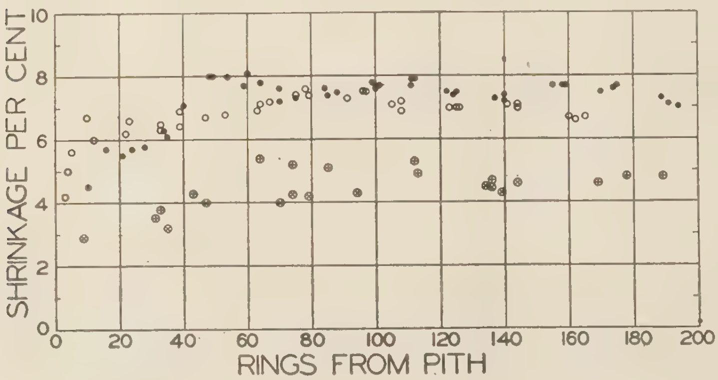

Fig. 6 Relationship Between Tangential and Radial Shrinkage and Rings From Pith for Basal and 32-Ft Disks of Tree 26

(Solid and hollow dots represent tangential shrinkage, while encircled + and × represent radial shrinkage for basal and 32-ft disks, respectively.)

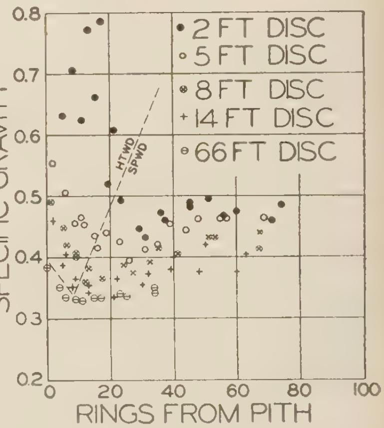

Fig. (7b)

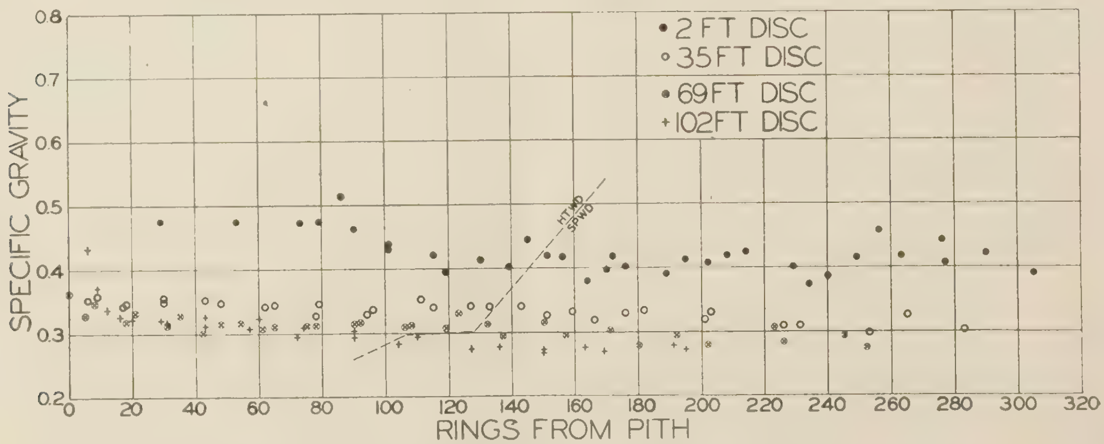

Fig. (7a)

Fig. 7 Relationship Between Specific Gravity and Rings From Pith for Selected Disks of Trees 27 and 30

rings from the periphery of the 16-ft disk of tree 33. The blocks containing the same rings on the opposite side of the pith did not show a similar fluctuation. Because of the relatively large number of rings that were included in each radial-shrinkage block (Fig. 3), the plotted points for this shrinkage category really represent average effects, and the lowest values for the rings adjacent to the pith were increased somewhat by the effect of the other rings included in the test block.

The average values for tangential and radial shrinkage are recorded for comparable heights for several trees in Table 6. From these figures it is quite evident that lateral shrinkage decreases in the wood above the butt section, the trend of shrinkage values in this case paralleling somewhat that of specific gravity, although showing a more consistent decrease with increasing height (see Table 8). This trend is illustrated more completely for one tree in Table 7.

Specific Gravity. Specific gravity varied considerably both with respect to position in the cross section and height in the tree. Values for selected disks for two trees are plotted over rings from pith in Fig. 7. There was a rather definite decreasing trend in specific gravity in most disks from the pith to the boundary between heartwood and sapwood, but, from this point through the sapwood to the periphery, the values were relatively constant, with minor fluctuations or slight upward or downward trends for certain disks. In practically every tree, the values for the heartwood blocks of the basal disks were conspicuously higher than the similarly located blocks of the other disks.

The average specific-gravity values for comparable heights in the trees are listed in Table 8. These figures show that, as a general rule, specific gravity is relatively uniform in the upper portion of the tree, with a rather pronounced difference between the basal disk and the disks at the higher levels. This can be accounted for in part by the high specific gravity of the stump heartwood, but, irrespective of this, there is usually an appreciable difference between the density of blocks cut from comparable rings in the sapwood, as, for example, in trees 27 and 30 (see Fig. 7).

In order to ascertain if any relationship existed between rate of growth and density, the specific-gravity values of the sapwood blocks from the basal and 32-ft disks of several trees were plotted over rings per inch in Fig. 8. Within this sample, no relationship between these two features was evident. As indicated also in Table 8 and Fig. 7, however, the values plotted for the blocks from the basal disk were distinctly higher than those from the 32-ft level. The only blocks from the 32-ft disks having specific gravity above 0.45 were those from tree 23; this would be expected, of course, since in Table 8 the average values for this tree are quite distinctly higher than those for corresponding heights in the other trees.

TABLE 6 AVERAGE TANGENTIAL- AND RADIAL-SHRINKAGE PERCENTAGE AT SPECIFIED HEIGHTS ABOVE GROUND

Tree no.	Tangential 2 ft	34 ft	66 ft	80+ ft	100+ ft	Radial 2 ft	34 ft	66 ft	80+ ft	100+ ft
22	6.8	6.1	5.7	...	...	5.4	4.3	4 3	...	...
23	7.4	6.6	6.1	6.0	...	6.0	4.5	4.6	4.5	...
24	7.4	6.7	6.4	...	...	4.4	3.9	2.9	...	...
25	6.6	5.6	5.4	4.4	4.9	4.4	2.3[a]	3.1	3.0	2.9
26	7.1	6.7	6.1	...	4.9	4.8	4.0	3.6	...	3.3
27	7.0	6.0	5.3	...	5.4	4.6	3.9	3.4	...	3.2
28	7.2	6.2	5.6	...	5.0	4.0	3.8	3.4	...	3.2
29	6.7	6.1	5.8	5.8	...	4.8	3.9	3.6	3.8	...
30	7.4	6.4	6.1	5.3	...	4.1	3.2	3.2	3.0	...
31	6.9	6.2	5.9	5.8	...	4.7	3.5	3.5	3.9	...
32	7.4	6.8[b]	...	...	...	4.3	3.5[b]	...	...	...
33	6.5	5.7	...	...	...	4.9	3.8	...	..	...
34	6.5	6.3	...	...	...	5.0	4.1	...	...	...

[a] These blocks immersed and redried to check measurements.
[b] Disk from 18 ft above ground.

TABLE 7 RELATIONSHIP OF SPECIFIC GRAVITY AND SHRINKAGE[a] TO HEIGHT IN TREE 30

Tree height, ft	2.5	5.5	8.5	14.0	21.0	32.5	50.0	66.0	77.0	88.0
Specific gravity	0.50	0.45	0.42	0.38	0.38	0.36	0.35	0.34	0.35	0.35
Tangential shrinkage	7.4	7.5	7.2	6.9	6.4	6.4	6.1	6.1	5.6	5.3
Radial shrinkage	4.1	3.9	3.6	3.3	3.2	3.2	3.3	3.2	3.0	3.0
Longitudinal shrinkage, entire disk	0.58	0.13	0.11	0.21	0.25	0.25	0.21	0.21	0.20	0.25
Longitudinal shrinkage, outer 40 rings	0.08	0.08	0.06	0.16	0.15	0.14	0.16	..	..	..

[a] Per cent green to oven-dry based on green values.

TABLE 8 AVERAGE SPECIFIC GRAVITY[a] AT SPECIFIED HEIGHTS ABOVE GROUND

Tree no.	2 ft	18 ft	34 ft	66 ft	80+ ft	100+ ft
1	0.46	..	0.37	0.36	0.37	..
3	0.43	..	0.35	0.40	..	..
6	0.46	..	0.35	0.38	..	..
7	0.42	..	0.37	0.40	..	..
9	0.43	..	0.38	0.36	..	..
10	0.42	..	0.34	0.35	..	..
11	0.46	..	0.34	0.34	..	..
12	0.40	..	0.35	0.33	0.33	..
15	0.42	..	0.33	0.34	..	..
16	0.44	..	0.38	0.33	..	..
22	0.52	0.42	0.41	0.42	0.40	..
23	0.56	0.50	0.48	0.44	0.45	..
24	0.45	0.39	0.37	0.35	0.36	..
25	0.42	..	0.35	0.34	0.32	0.32
26	0.46	..	0.43	0.38	..	0.38
27	0.46	..	0.33	0.30	..	0.29
28	0.43	..	0.35	0.30	..	0.32
29	0.47	0.40[b]	0.35	0.37	0.37	..
30	0.50	0.38[c]	0 36	0.34	0.35	..
31	0.44	0.37	0.37	0.37	0.37	..
32	0.48	0.38	..	..	..	..
33	0 49	0.40	0.40	..	..	..
34	0.51	0.42	0.41	..	..	..

[a] Specific gravity based on green volume and oven-dry weight. Average specific gravity determined by weighting each block in proportion to area of cross section as its position on radius indicated. No correction made for differences in resin content. Blocks with defects omitted from calculations.
[b] Disk cut at 9 ft.
[c] Disk cut at 14 ft.

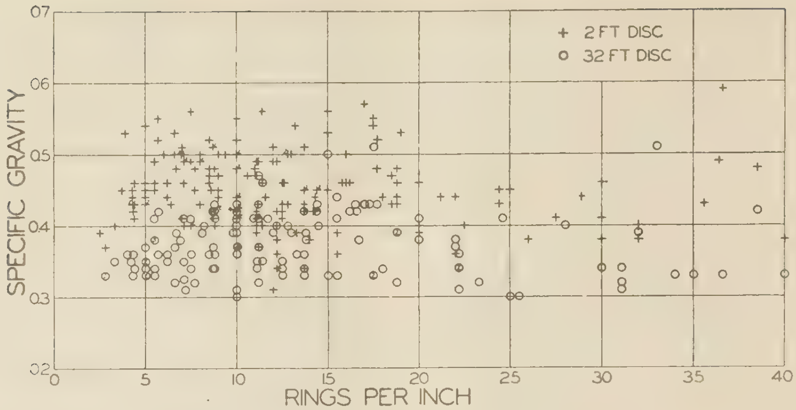

Fig. 8 Relationship Between Specific Gravity and Rate of Growth for Sapwood From Basal and 32-Ft Sections of Trees 22–31, 33, and 34

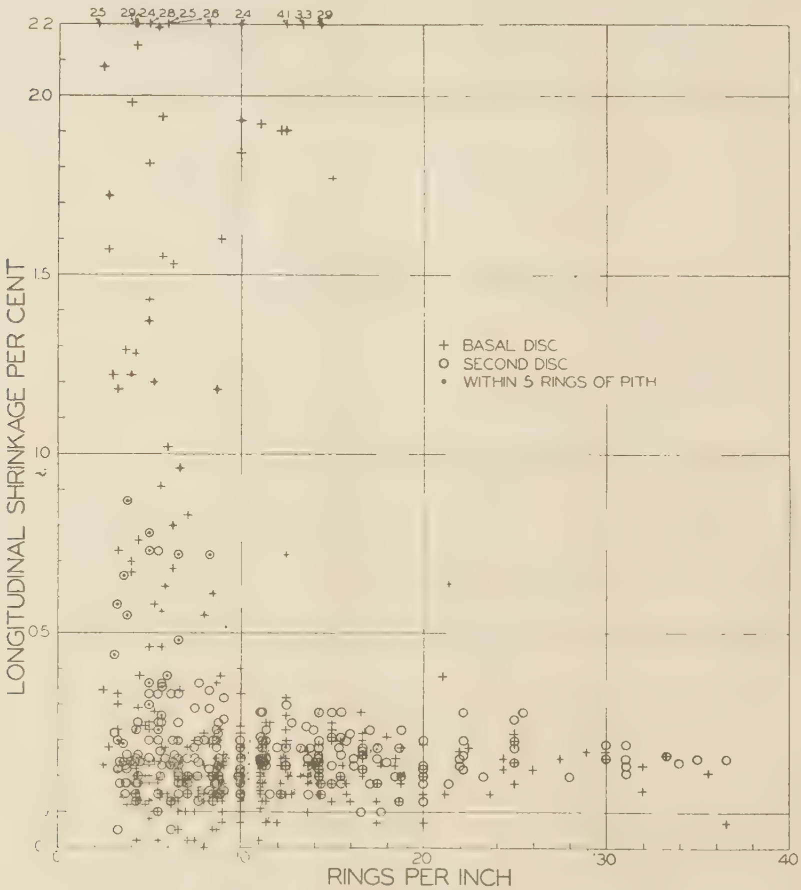

Fig. 9 Relationship Between Longitudinal Shrinkage and Rate of Growth for Basal and Second Disks Out to 100 Rings From Pith for Trees 22–31, 33, and 34

Within the limited sample studied, the density of trees from the same stands showed almost as much variation as existed between trees in different stands. It might be of interest to note, however, that the lightest disks were obtained from the upper levels of trees 27 and 28 which grew at comparatively high elevation, while the heaviest disks were obtained from the base of trees 22, 23, 30, and 34, all of which came from appreciably lower elevations.

Rate of Growth. In most of the disks, the growth rate was highest at the pith and decreased gradually toward the periphery. In the basal disk, wide rings were not always present near the pith, but the disks above this in the trees examined in this study usually had the widest rings near the pith. Tree 1 illustrated this variation in its basal disk, with the lowest growth rate (20 rings per inch) occurring at the pith and the highest growth rate ($4^1/_2$ rings per inch) occurring over 100 rings from the pith. The disk taken at the top of the second log in this tree, however, showed the usual pattern with growth rate decreasing away from

the pith. Undoubtedly, this tree was suppressed in its early life, and release from suppression at about 70 years of age accounts for the changing growth condition in the basal disk. The widest growth rings were found in trees 29 through 33, with tree 31, because of its location on the edge of an open stand, having the wide rings most sustained both in height and diameter.

There was no relationship apparent between longitudinal shrinkage and rate of growth. When longitudinal shrinkage is plotted over rings per inch, as in Fig. 9, it can be seen that there is an appreciable spread in shrinkage values for the wide-ringed blocks, and most of those with the greatest shrinkage can be identified as being from the basal disk and from near the pith. In the case of trees 1, 25, and 27, the greatest shrinkage occurs in conjunction with the slowest growth rate. In this respect, it is interesting to refer to Tables 2 and 3, where the growth rate recorded for the centers of most of the upper disks is greater than the growth rate for the center of the basal disk of the corresponding trees, although the shrinkage values are higher and the zone of wood with high shrinkage more extensive in the basal disk.

Discussion of Results

Longitudinal Shrinkage. The fact that wood near the pith of some conifers is characterized by abnormally high shrinkage along the grain is reported by Koehler (7), who states: "Boards and planks of the southern and western yellow pines, and occasionally other species, which happen to be sawed lengthwise through the pith, frequently show cross-breaks extending for a short distance at right angles to the pith. Such breaks are due to the greater longitudinal shrinkage of the wood near the pith. For the same reason, narrow strips cut out so that the pith runs along one edge will bow as they dry." Similarly, high longitudinal shrinkage is also a characteristic of "compression wood," hence it is of interest to make a comparison of the two. The growth rings near the pith lacked well-defined summerwood and were much easier cut and worked (less dense and hornlike) than the wood produced further from the pith. This homogeneity and ease of cutting was especially conspicuous when the wood was sectioned on the microtome or when the ends of blocks were cut across the grain. Microscopic examination disclosed large fibril angles, those of the summerwood tracheids in rings near the pith measuring 40 to 55 deg, in contrast to angles of 10 to 25 deg in normal summerwood tracheids. All of these features are pointed out by Pillow and Luxford (8) to be typical of "compression wood," hence it is logical to consider this wood near the pith as a form of this abnormal tissue.

The recently reported observations of Wershing and Bailey (9) on the formation of "compression wood" in white-pine seedlings, following treatment of the cambial region with auxin, suggests that the cambium in the vicinity of the apical meristem may be supplied with higher concentrations of growth hormone than that more distant down the stem, with consequent formation of varying degrees of "compression wood." Usually, this wood near the pith has the character of mild "compression wood," but in several disks cut at the stump, sectors of a few rings formed pronounced "compression wood." Apparently this is not unusual, for Pillow and Luxford (8) state that the most common occurrence of "compression wood" in cross section is within a few annual rings of the pith.

Increase in length along the grain, following drying from the green to the air-dry and even oven-dry condition, is not commonly expected of wood. In a study of longitudinal shrinkage, however, Koehler (7) reported that "some of the specimens showed no shrinkage or even elongated in the first and second stages of drying, but when all moisture was removed they were shorter than when wet."

In a study on the longitudinal variation of Australian timbers in seasoning, Welch (10) became interested in the fact that several woods elongated axially in drying to the air-dry condition and was stimulated to conduct a further study (11). His results here led him to conclude that below the fiber-saturation point, light timbers showed greatest liability to shrinkage, and heavy timbers were inclined to remain stationary or to swell. The same effect was observed by Hartig, quoted by Kollmann (12), in axial shortening of tension wood from knots with water absorption.

Most of the shrinkage specimens of ponderosa pine that performed in this fashion first increased in length to the air-dry condition and then shrank as the wood approached the oven-dry condition. Also, this elongation was predominantly a characteristic of the denser wood formed in the butt of the tree and could be repeated by successive wetting and drying. In a sense, this confirms both Koehler's (7) and Welch's (11) observations of this shrinkage anomaly. Perhaps the best attempt at explaining this phenomenon would be to ascribe it to the physical cell-wall structure as envisaged by Frey-Wyssling and reported by Bailey (13). According to this theory, cellulose is a "continuous matrix of overlapping chain molecules which is perforated by a continuous system of intermicellar capillaries." If these chain molecules are inflexible, as seems probable, in view of the limited and essentially reversible swelling and shrinking of wood, their lateral distension by adsorbed water molecules must be accompanied by a pulling together of the affixed ends. Such a movement would act counter to any change of dimensions along the grain resulting from the axial linear vector of the lateral distension of the fibrils (chain molecules). It would exert its maximum shortening effect when the fibrils were vertically aligned, in which position the vertical component of movement due to lateral distension of the fibrils would be zero. The net axial movement in any situation would be equal to the algebraic sum of the axial components of these two factors.

The inverse relationship reported as existing between longitudinal shrinkage and density by Koehler (7) and Siimes (4) was borne out in a general way by the results of this study. Even when the samples were confined to a certain series of growth rings within a single tree, however, there was considerable variation in longitudinal shrinkage percentage among samples of identical specific gravity. It is possible, however, that further studies can explain such inconsistencies on the basis of varying fibril orientation or different summerwood-springwood ratio.

Tangential and Radial Shrinkage. The tangential-and radial-shrinkage measurements conformed generally to the observations of Koehler (14), decreasing in magnitude with decrease in density, and, within a cross section, being lowest where the axial shrinkage was highest. The ratio of tangential to radial shrinkage varied inversely with the density, in this respect agreeing with the work of Siimes (4) on *Pinus sylvestris.*

Specific Gravity. The specific gravity of all the trees investigated was perceptibly greater in the butt region than higher in the tree. From a point about 16 ft above the ground (based on trees 22–24, 29–31, 33, and 34) and upward, however, density was relatively constant. In many instances, especially in the butt, the heartwood was appreciably denser than the sapwood. This would have to be taken into consideration in any use of density values as an index of strength, for the wider rings with larger amounts of springwood in the center of the log were often, because of this, denser than the structurally superior wood farther out in the stem. For comparable heights in the tree and considering only sapwood to eliminate the variable effect of heartwood on density, specific gravity appeared to be relatively independent of growth rate. The number of trees studied was not large enough to give any significant indication of the effect of locality on density.

Effect of Age and Growth Rate. No significant differences in shrinkage or density were observed that could be attributed to age. Trees suppressed in youth showed narrower rings at the center near the ground, with a greater tendency for pronounced "compression wood" to be present near the pith. In trees of rapid early growth, there was no evident pronounced "compression wood," but the rings near the pith did have large fibril angles in both summerwood and springwood that caused rather high longitudinal shrinkage to take place. It should be noted, however, that this wood did not shrink as much as the pronounced "compression wood" occurring near the center of the butt of the initially slower growing trees. Within the limits of growth rate observed in the sample studied (from $2^1/_2$ to over 50 rings per in.), there was no perceptible influence of rate of growth on longitudinal shrinkage.

In older trees, the central wood in the lower sections, which in some cases were as large as the entire cross section of the younger trees, was all changed over to heartwood. Consequently, it was usually heavier and more resinous than wood of the same growth period of the younger trees.

Application

In butt logs, the combination of high longitudinal shrinkage near the pith and slight shrinkage or elongation of the wood further out can cause extreme counter tension in seasoning which would result in pronounced warping of boards cut with one edge along the pith. Although the cross grain surrounding knots would, to a greater or lesser degree, offset the slight shrinkage or elongation of the outer wood, no lumber can safely be cut with the pith along one edge without incurring the risk of degrade in seasoning. It would therefore be good practice to saw logs, and especially butt logs, so that the pith is approximately in the center of a board or timber.

Conclusions

1 Excepting irregularities of grain, caused by knots and other defects, and "compression wood," caused by lean, excessive longitudinal shrinkage is usually restricted at the base of the tree to a 3 to 6-in. zone containing from 12 to 30 rings, while, at points higher in the tree, it is restricted to the first-formed two or three rings in the vicinity of the pith.

2 An appreciable portion of the denser ponderosa pine wood, particularly that in the basal portion of the tree, instead of shrinking, remains stationary or elongates axially in losing moisture from the green to the air-dry condition. In some instances this elongation is maintained even when drying is continued to the oven-dry condition.

3 Specific gravity, along with tangential and radial shrinkage, is at a maximum in the basal portion of the tree; it decreases rather abruptly above the base and then decreases very slightly or remains relatively uniform in the upper part of the tree. Specific gravity is apparently independent of rate of growth.

4 Excessive longitudinal shrinkage appears to be more directly related to position in the cross section with respect to proximity to pith and to height in stem rather than to rate of growth or age.

5 Although the results of this study point more to such factors as site variability, especially as affected by elevation and exposure, and stand density having a controlling influence on wood quality rather than the broad and loosely defined categories of second growth and old growth, it is recognized that wood from a much larger number of trees representing a greater diversity of age classes and growing conditions must be examined to establish this contention indisputably.

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Discussion

C. C. Forsaith.[3] This paper adds to the evidence in support of the contention that the determination of quality in wood presents a more complicated problem than that involved in a selection from old-growth or second-growth stock. Variations in the structure and properties of wood, as he points out, are brought about by many factors including age, rate of growth, site, position in a stand, elevation above sea level (exposure to sun and wind), location in the tree, and many others. Variation between trees suggests a possibility that heredity may also play a part.

The determination of the location and magnitude of longitudinal shrinkage is of special interest not only because it confirms similar statements made by other investigators working with different species but also because it is an important defect which should receive greater attention in the production and utilization of lumber. Longitudinal shrinkage as such may not be highly significant but even a small difference in degree on opposite edges or faces of a piece may cause pronounced bowing or checking.

In the writer's opinion, the author has used the proper technique for the solution of his problem, viz., a formal or informal statistical study of samples, in order to establish trends upon which his conclusions are based. Furthermore, the number of specimens was sufficiently large, since a reference to his figures indicates curves which are decisive. It is doubtful if any appreciable change would have occurred even though a larger number of samples had been measured.

Arthur Koehler.[4] In this paper more space is devoted to

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discussing longitudinal shrinkage than to transverse shrinkage or density. To the so-called practical man this may seem like "straining at a gnat and swallowing a camel," because longitudinal shrinkage of wood is commonly considered negligible and any woodworker knows that density and transverse shrinkage are important factors.

It is not the shortening, however, that is the important matter but, as the author points out, the crooking that results therefrom when a board or strip has wood with high longitudinal shrinkage at one edge and normal shrinkage at the opposite edge. To the mill man that means degrade or waste of lumber; to the manufacturer and user it means trouble and disgust with the perversity of wood. Even the author's suggestion to saw lumber and dimension stock so that the wide rings around the pith are at the center of a board does not completely solve the problem for the mill man. The longitudinal stresses set up in drying such lumber may cause warping or the development of long diagonal cracks in the center portion, especially when it is being planed, as the writer has observed to be the case in southern pine.

Excessive longitudinal shrinkage also denotes some other eccentricities of wood as a rule, namely, wood with high shrinkage along the grain often will bend a great deal more before breaking than normal wood, but when it breaks it usually fails suddenly and completely, flying into a number of pieces. Its shock resistance may be high but its stiffness and load-carrying capacity as a beam are low. More information along this line is needed, however, and if the author intends to do any further work along the line of his paper, some strength tests might well be included.

Very little can be done even under intensive forest management to prevent the formation of wide rings at the center of trees. Under selective logging the young vigorous trees are left standing as a rule, and they are the ones which usually have wide rings at the center. When planting is resorted to, the initial spacing of necessity must be large to reduce costs, which makes for vigorous early growth.

Nevertheless, to know the cause of the trouble and that it is not due to poor dry kilns, faulty operation of kilns, or improper planing practices is worth a good deal. Before the depression of the 1930's someone said: "The man who knows how will always have a job, but the man who knows why will always be boss."

Overfire Air Jets

By R. B. ENGDAHL[1] and W. C. HOLTON,[2] COLUMBUS, OHIO

For many years overfire air jets have demonstrated their effectiveness for smoke abatement as well as for increasing the ratings of existing furnaces. Except for the chain-grate stoker on which fan-driven overfire air jets have become standard equipment, jets in this country have been actuated mainly by high-velocity steam from nozzles. Their wide use has been due largely to a minimum of engineering and auxiliaries, and in spite of demonstrably poor efficiency and high steam consumption. Because of uncertainty as to the effects of variables involved in a steam jet, there has been reason to believe that among the available types, there are some which might compare favorably in economy with fan-driven jets. To determine the relative performances of steam jets and fan-driven overfire jets, the authors undertook a laboratory study of the problem at Battelle for Bituminous Coal Research, Inc. The results are given in this paper, as well as a comparison with the meager information which had been made available from earlier investigations.

Many years of successful experience with jets of air directed over beds of burning solid fuel have justified the common acceptance of their effectiveness for smoke abatement. This has been so despite the very meager and even negative (1)[3] data which have been available concerning them.

Except for the chain-grate stoker on which fan-driven overfire air jets have become standard equipment (2), overfire jets in this country have usually been actuated by high-velocity steam from nozzles. For existing and small new boiler furnaces, involving a minimum of engineering and auxiliaries, the choice between steam- and fan-driven jets has usually been in favor of the simpler steam jet. This has been so despite repeated assertions of their low efficiency and published calculations showing a fairly high rate of steam consumption. But uncertainty over the effects of the many variables involved in a steam jet has left ample reason for the widespread feeling that there are types of steam jets of optimum performance which compare favorably in economy with fan-driven jets. However, no data have been available to show what those optimum types are.

At present the smoke-abatement feature of overfire jets is of less interest than their widely recognized effect of increasing ratings of existing furnaces. It seems reasonable that, with a given combustion space, heat release in a furnace can be increased by turbulent mixing of stratified gas streams with sufficient air. But the current unavailability of fans for such purposes leaves steam jets as the only means of doing this. They should be so installed as to use the least steam, but how to do this has not been clear. The results of the following laboratory study conducted by Battelle for Bituminous Coal Research, Inc., should answer many questions on steam and fan jets.

The first published attempt to answer some of the many questions of detail on the construction and performance of steam jets was by Kugel (3), who reported a thesis study of DuPerow and Bossart (4), in 1928. Since then Stern (5) has briefly reported some actual tests of a practical application of steam jets.

The present work has been an attempt to expand further the small amount of knowledge of jets by a laboratory study of the performance and action of jets typical of those used in practice. Comparison is made with the results of earlier studies.

Experimental Apparatus

Fig. 1 shows the laboratory apparatus for measuring the performance of steam jets. The jet from the steam nozzle aspirated air from the plenum chamber through the air tube. To maintain the chamber at atmospheric pressure, a fan supplied air to it at a rate which just balanced the rate of removal. The amount of air supplied was measured at an orifice which thus gave the performance of the steam jet acting as a blower.

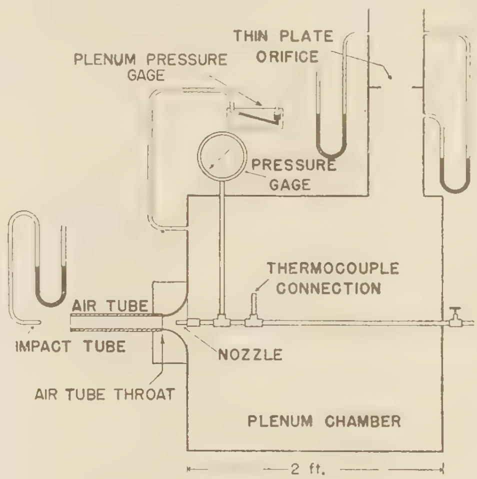

Fig. 1 Experimental Apparatus

Steam pressure was measured near the nozzle by means of a Bourdon gage. Steam temperature was obtained from a thermocouple immersed in a well in the steam line near the gage. A small heater was used to re-evaporate the moisture in the wet steam as it came from the building heating boilers in order to obtain dry or superheated steam at the nozzle.

The rate of steam flow was computed from Grashof's empirical formula

$$W_s = \frac{ap^{0.97}}{60.5}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{1 + 0.00065T_s}\right) \quad \text{[1]}$$

where

W_s = weight of steam flowing, lb per sec
a = area of throat of nozzle, sq in.
p = steam pressure, psia
x = quality of steam if wet, as decimal
T_s = amount of superheat, deg F

This simple equation was carefully checked against a more elaborate and accurate method (6) and was found to give values

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[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

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within 1.5 per cent of the latter over the range of conditions covered in these tests.

Steam pressure was held constant at 48 psi gage for most of the tests. Following determination of the optimum proportions of jet the pressure and superheat were varied over a wide range. Through the courtesy of R. N. Tucker, superintendent of the Division of Electricity of the City of Columbus, the high-pressure steam at the Municipal Light Plant was made available for the high-pressure tests.

For the study of fan-driven jets, a bellmouth orifice was installed at the end of a large duct which supplied air at pressures up to 14 in. water.

The velocity distribution in the streams produced by the various jets was measured by means of a simple impact tube and inclined or vertical manometers.

Because of their dual role of providing additional air and inducing turbulence, the laboratory jets were rated both on their performance as air blowers and their ability to penetrate a region with a stream of high velocity. The first is expressed as entrainment ratio, pounds of air per pound of steam, and the latter as penetration or throw, the distance over which all velocities are above some arbitrary minimum.

Entrainment of Steam Jets

Air Tube. When steam emerges into a space from a nozzle, the jet spreads and mixes with the surrounding fluid which is thus set in motion. Measurements (7, 8) have shown that the momentum of the mixing stream remains constant as more surrounding fluid is drawn into the mixture. Thus as the mass of the jet increases, the velocity decreases. When a high velocity is desired, the mixing action must be stopped as soon as enough surrounding fluid has been entrained to serve the purpose intended. This is done by enclosing the expanding stream in a tube. Then as the primary jet meets the tube wall no further secondary fluid is brought in and the mixture flows on through the tube at constant velocity. Because of the predominance of the proportions of the air tube over the other variables, it will be discussed before the steam nozzle. While studying the air tube, the nozzle was located 1 air-tube diam back of the throat of the air tube, the position recommended by DuPerow and Bossart.

Air-Tube Entry. Because both flow and mixing of fluids occur in the air tube, it is conceivable that what hinders turbulence might improve flow or vice versa. Although Young (9) found that in the locomotive front end the use of nozzles which increased turbulent mixing in the steam stream were of no benefit, it was felt that the special conditions in that type of jet might not be applicable here.

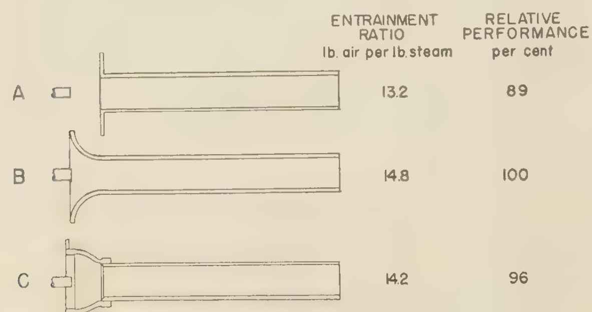

Fig. 2 Effect of Air-Tube Entry
(Nozzle diam = $^9/_{64}$ in.; steam pressure = 48 psi; air-tube diam = 2 in.; length = 10 in.)

The shape of the mouth of the air tube doubtless affects both mixing and flow, and to check on their effects a number of entries were tested. Fig. 2 shows the shapes tried and the results obtained for a $^9/_{64}$-in. steam nozzle discharging from 48 psi gage into a 2-in-diam air tube 5 diam long. The bellmouth gave the best performance, the sharp entry the poorest. It is interesting to note that the relative performances of these two shapes are in approximately the same ratio as their respective discharge coefficients when serving as simple orifices (10). The good performance of shape C which was a common 3-in. × 2-in. reducing coupling indicates that this is an effective as well as a readily available type of entry.

Stern (5) showed a conical entry, shape D, which was first recommended about 1928, by the Cleveland Department of Smoke Abatement, then headed by Col. E. H. Whitlock, following the work of DuPerow and Bossart which was begun at his suggestion. This form was not tested in the present study, but on the basis of the other results its performance is estimated as 98 per cent, or practically as good as the bellmouth. This conical entry has another feature to recommend it in that a strut cast integrally with the body serves to locate the steam nozzle rigidly.

Air-Tube Length. Measurements have been made previously (4) of the effect of varying the length of air tube which indicated the optimum length to be approximately 7 diam. However, recognition by many of the function of the air tube as a mixing chamber had produced a number of suggestions for nozzle and air-tube modifications which might so facilitate mixing that the optimum length of tube might be reduced. Hence the optimum length of air tube was first verified with the present apparatus before these modifications were tried. DuPerow and Bossart recommended that for best performance a 2-in. air tube should be between 15 and 30 in. or 7.5 to 15 diam long. For the air ejector, Keenan and Neumann (11) found that 7 diam was the optimum but that anywhere between 5 and 10 diam gave within 3 per cent of the optimum performance. They found that, although the optimum length increased slightly with pressure and air-tube diameter, the increase was less than 1 diam even when these factors were doubled. Fig. 3 shows the relation of the entrainment

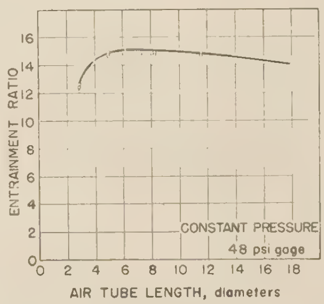

Fig. 3 Relation of Entrainment Ratio to Length of Air Tube for 2-In. Standard-Pipe Air Tube

ratio, pounds of air per pound of steam, to the length of the air tube for a 2-in. standard-pipe air tube at 48 psi gage with a $^9/_{64}$-in-diam (0.141) steam nozzle located 2 in. from the throat of the air tube. It is evident that performance decreases much more rapidly with tubes which are too short than with those which are too long. The optimum length is about 7.5 diam and any length between 4 and 14 diam will give within 4 per cent of the optimum performance.

Air Tube With Diffuser. A frequently suggested modification of the air tube to improve performance is to make it Venturi-shaped. The well-known excellent flow characteristics of the Ven-

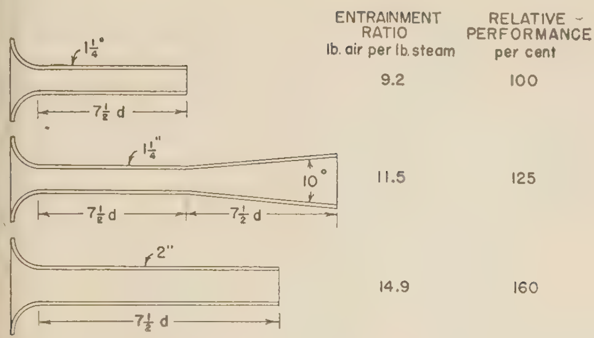

Fig. 4 Effect of Diffuser on Entrainment
(Dimensions are nominal pipe sizes; nozzle diam = $^9/_{64}$ in.; pressure = 48 psi.)

turi-shaped duct were expected to enhance the entrainment and possibly reduce the length of air tube needed for good mixing.

In order to test this idea, a diffuser with an included angle of 10 deg was added to an air tube made from $1^1/_4$-in. pipe, $7^1/_2$ diam long with bellmouth entry. Fig. 4 shows the results in comparison with straight tubes of $1^1/_4$ and 2-in. pipe. Although the diffuser improved the performance of the $1^1/_4$-in. jet by 25 per cent, a larger air tube can give even greater improvement, 60 per cent, if a 2-in. tube is used. It was found that a decrease in length of either the straight tube or the diffuser reduced the performance.

The reason for this apparent anomaly is clear if the differing purposes of the overfire jet and Venturi diffuser are examined. The purpose of the Venturi diffuser is so to decelerate gently the fluid at the throat that a maximum of kinetic energy at the throat is converted to potential energy in the form of static head at the outlet. Thus with the outlet usually at atmospheric pressure, the prime mover can operate against a reduced pressure in the system. But when the diffuser is applied to an air tube of a steam jet, the reduction in pressure against which the steam from the nozzle has to operate is not enough to compensate for the restricted mixing area at the throat. Thus, compared on the basis of outlet diameters the straight tube gives greater entrainment.

Even if the air tube with diffuser is made large enough to supply the desired quantity of air, the decelerating feature of the diffuser is usually undesirable because penetration and turbulence are both reduced by it. This disadvantage of the Venturi shape applies both to fan and steam jets.

Other Air-Tube Modifications. In practice it is sometimes desirable to bend the air tube, either to avoid structural difficulties or to reduce the amount of projection of the air tube where the walls, such as in a locomotive firebox, are too thin to contain an air tube of optimum length. To find the effect of a partial bend, a common 2-in. 45-deg pipe elbow was added to two 2-in. air tubes both 2.9 and 5 diam long, both less than the optimum length. This did not impair entrainment as was expected but instead raised the entrainment by approximately 3 to 5 per cent. On a tube 11.6 diam long, or about 4 diam in excess of the optimum length, entrainment was reduced slightly by the addition of the 45-deg elbow. A 90-deg elbow decreased entrainment in all cases. When a length of pipe 1 diam long was added, following the 45-deg elbow on the short tubes, entrainment dropped 8 per cent. Observation indicated that the stream was deflected only 20 or 30 deg by the 45-deg elbow alone. Apparently, for short tubes the added mixing in the elbow improved performance more than the added resistance impaired it. Addition of pipe following the elbow, however, caused the change in the direction of the stream to approach 45 deg. The added resistance of this turn offset improved mixing in the elbow.

These tests indicate that where the direction of the stream must be altered, a change up to 30 deg can be made without sacrificing entrainment if the air tube is about 5 diam long before the elbow. Greater angle or added length of tube either before or after the bend reduces entrainment.

Steam Nozzle. Data are available (12) for proportioning the steam nozzle to convert a maximum of the enthalpy of steam into kinetic energy acting in one direction. For steam turbines this has been shown to be important. Shock losses and useless lateral expansion result from a poorly shaped nozzle. However, for the jet, mixing appears to be impaired very little by the use of a simple nozzle which neglects this refinement. DuPerow and Bossart tested both a simple sharp-edged orifice formed by drilling a $^1/_8$-in. hole in a $^1/_4$-in-pipe plug and a "theoretical" convergent-divergent nozzle of the same throat diameter. They found no difference in the performance of the two. This might be attributed to the fact that their "theoretical" nozzle was designed for pressures much higher than those actually used. In the present study, however, nozzles designed according to Marks (12) for specific pressures were found to give performance only slightly better than that of simple straight-hole nozzles at pressures up to 170 psi gage. Possibly at higher pressures the differences might increase. Fig. 5 shows the nozzles tested.

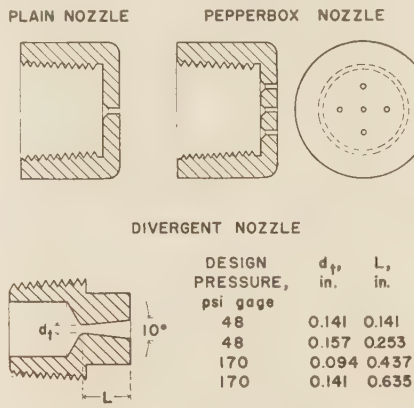

Fig. 5 Steam Nozzles

Several times during the tests, the performance of the jet decreased abruptly without any change in conditions. Inspection usually revealed that the flow of steam was being reduced by small bits of pipe scale or other impurity lodged in the throat of the steam nozzle. In practice, foreign materials may be built up at the corner between the nozzle and the steam-pipe wall and eventually clog the throat. A machined nozzle with a convergent approach so that this corner is eliminated will reduce clogging, but it is probably well to clean the nozzles regularly for best performance.

Nozzle Location. For all of these tests of air tubes, the steam nozzle was located 1 air-tube diam from the throat of the tube, the point where the entry merges with the cylindrical air tube. This location was recommended as about the best by DuPerow and Bossart. However, for very short tubes, they recommended a slightly greater distance. The effect of the greater distance was found in these tests to be measurable but small. Fig. 6 shows curves of the effect of nozzle position for constant air-tube length. It is evident that the optimum distance from nozzle to air-tube throat increases slightly as length of air tube decreases.

DuPerow and Bossart found that it was important to have the nozzle centered with the air tube. This was verified in the present study. Also, it was observed that the steam should not be

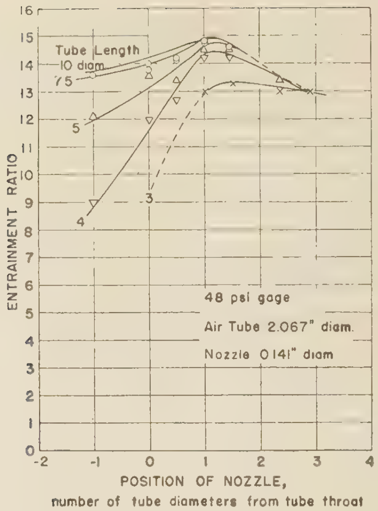

Fig. 6 Effect of Nozzle Position on Entrainment

directed at an angle with the axis of the air tube. For a 2-in. air tube 7.5 diam long, an eccentricity of $^1/_{16}$ in. or an angularity of 2 deg each decreased entrainment by 2 per cent. The effect of these small deviations was not measurable on an air tube 11.6 diam long, or 4 diam greater than the optimum length. Apparently, then, the orientation of the nozzle is less critical with long air tubes than with short ones.

Nozzle Body. Flow of air into the air tube and mixing with the jets of steam will occur with least disturbance when the outside diameter of the nozzle is as small as possible. On the other hand, it is simplest to use standard pipe plugs or fittings for the nozzle body. No small, thin-walled nozzles were used in these tests, but Fig. 7 compares the curves for thick- and thin-walled nozzles from Keenan and Neumann with a comparable curve from Fig. 6. The thin-walled nozzle was tapered on the outside so that the tip was little larger than the nozzle outlet. The thick-walled ones were all approximately of an outside diameter $5^1/_2$ times the nozzle opening.

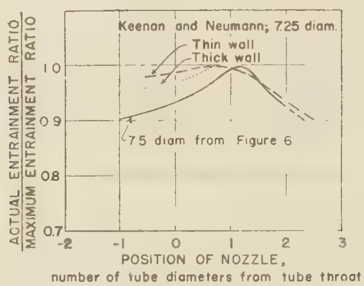

Fig. 7 Effect of Nozzle Position for Thick- and Thin-Wall Nozzles

The curves show that the thin-walled nozzle is of advantage only where the nozzle is less than 1 air-tube diam from the throat.

Pepperbox Nozzle. A suggestion which is often made is the use of a pepperbox nozzle. This idea stems logically from the accepted understanding of the action of the steam jet. The high-velocity steam jet is visualized to spread as it moves away from the nozzle, and as it spreads it mixes with surrounding air. If the jet emerges from the air tube before it has spread enough to meet the tube wall, mixing is not complete and entrainment is hindered. Hence, this explanation indicates that if the steam emerges from the nozzle over a wide area, as from the multiple jets, it will more quickly fill the air tube and mixing will be completed in a short length. Young (9) found that, in the special case of the locomotive front end, where the length of air tube or stack is limited, the pepperbox gave distinctly better entrainment than any other nozzle. The present tests showed that the optimum length of air tube is appreciably less with the pepperbox nozzle. Fig. 8 shows

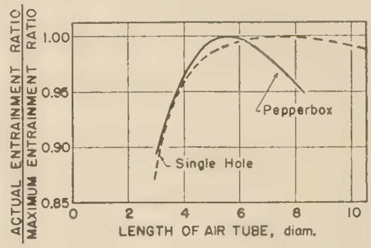

Fig. 8 Effect of Length of Air Tube for Plain and Pepperbox Nozzles
(Air tube made of 2-in. standard pipe.)

that the optimum length for the 5-hole nozzle shown in Fig. 5, having a total area equivalent to a plain nozzle 0.135 in. diam, was $5^1/_2$ diam. This is compared to 7.5 diam for the plain nozzle. However, with the pepperbox, a decrease of air-tube length below the optimum causes a rapid decrease in entrainment. Thus the entrainment of the two nozzles is practically the same for air tubes 4 diam long. Noise measurements were not made with the pepperbox, but it was observed that noise was definitely less than that from a single-hole nozzle.

There is an important practical disadvantage to the small pepperbox, which Young did not encounter in his large nozzle. Impurities in the steam and piping will occasionally clog any nozzle. In a 5-hole pepperbox equivalent to a simple $^1/_8$-in. nozzle, the small holes are only 0.056 in. These holes will clog much more readily than a plain $^1/_8$-in. nozzle.

Effect of Nozzle and Air-Tube Areas. The work of Keenan and Neumann showed that, for the air ejector, the entrainment ratio at a constant pressure increased as some power of the ratio of air-

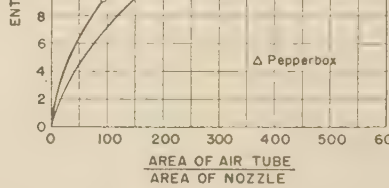

Fig. 9 Relation Between Entrainment and Ratio of Air-Tube Area to Nozzle Area

tube-throat area to nozzle-throat area. The present study shows that a similar relationship holds for the steam jet. Fig. 9 shows points obtained at a constant steam pressure of 48 psi gage, with the nozzle always 1 diam back of the throat of a 7.5-diam air tube having a rounded approach. Nozzle diameter was varied from

0.094 ($^3/_{32}$) to 0.157 in. ($^5/_{32}$) and air-tube diameters were varied from $1^1/_4$ in. to $2^1/_2$ in. standard pipe. The resulting combinations gave the points plotted. The slope of this curve plotted logarithmically is identical with the curve for the air ejector which was interpolated from the data of Keenan and Neumann for a pressure of 48 psi gage. Thus in this respect the air and steam jets are similar. However, for the same pressure, the entrainment of the steam jet is approximately 30 per cent higher than that of the air ejector, presumably because of the higher available energy with steam.

The equation of the curve for the steam jet in Fig. 9 is

$$\text{Entrainment ratio} = 0.74\ (\text{area ratio})^{0.56} \qquad [2]$$

This applies only to steam at 48 psi gage. Although these tests did not include area ratios above 500, the air-ejector curves were carried up to 1024. It is assumed that the steam and air curves would parallel one another at least up to this point and that the equation would apply for steam throughout.

It is evident from Fig. 9 and Equation [2] that there is no optimum ratio of air-tube to nozzle size. The larger the air tube for a given nozzle, the better the entrainment; a larger nozzle will give more air but at a poorer entrainment ratio.

Steam Pressure. As stated previously, all of the conclusions on the construction of the jet are based on tests run at a constant pressure of 48 psi gage steam, dry, saturated, or slightly superheated.

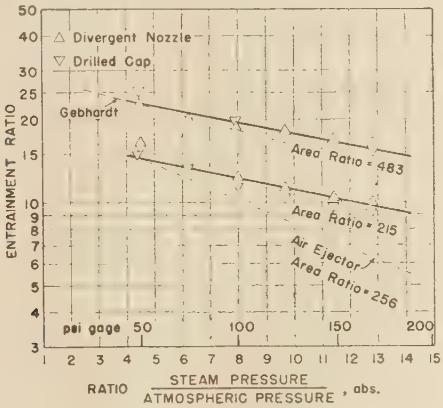

FIG. 10 RELATION OF ENTRAINMENT TO STEAM PRESSURE FOR TWO AREA RATIOS

The study of the air ejector (11) showed that increasing the pressure reduced the entrainment. In the present study, a similar, though not identical, effect was found for the steam jet.

In order to operate at higher pressures, the apparatus was moved to the Municipal Light Plant of the City of Columbus where steam was available at 170 psi gage and 500 F. Arrangements were made to supply dry, saturated, or slightly superheated steam to the nozzle at pressures up to 170 psi gage. The results of the measurements of entrainment ratio are shown in Fig. 10, for two area ratios. The rate of decrease in entrainment with increasing pressure is much less with steam than with air. The slope of the curves for steam is partially verified by the curve shown from Gebhardt (13) which was for a multiple-inlet steam jet of unstated area ratio.

Both simple drilled pipe caps and convergent-divergent nozzles were used. The latter were advantageous only above 100 psi gage. In that region, they were 3 to 6 per cent better than the simple drilled caps. Possibly the advantage would be greater at pressures above 170 psi gage. The statement is sometimes made that steam jets undergo a sharp decrease in performance below 100 psi gage. These tests do not bear this out.

The lower experimental curve for area ratio of 215 ($^9/_{64}$-in. nozzle in 2-in. pipe) of Fig. 10 is represented for the range of pressures studied by the equation

$$E = 17.9\ (0.953)^P \qquad [3]$$

in which P = ratio of initial steam pressure, psia, to atmospheric pressure, psia.

Effect of Superheat. The mass flow through a nozzle decreases with increasing superheat, but because the enthalpy increases, the energy available in the jet remains about the same. At 40 psi gage, the steam temperature was raised to 420 deg above the saturation temperature. The quantity of air entrained remained constant. The result was an increase in entrainment ratio inversely proportional to the decrease in rate of steam flow. Similarly, there was no effect on air flow when the nozzle was supplied with wet steam.

Under the usual test condition with dry saturated steam at 48 psi gage, the first few feet of the mixture emerging from the air tube appeared quite foggy and did not become completely clear until about 8 ft from the tip. Room temperatures ranged from 60 to 80 F. However, when the steam was superheated about 75 F, the jet usually became completely clear. A clear jet indicated that all of the droplets produced by the steam nozzle were re-evaporated because of the high temperature of the mixture. Whether this occurred depended upon the enthalpy of the entering air and steam and the entrainment ratio.

In a practical jet, if these factors do not combine to introduce a clear jet to the furnace, some heat is absorbed from the fuel bed to evaporate the drops of water. Thus in addition to the obvious advantages of preheating the air supplied for combustion, the higher-temperature air insures a clear jet so that the fuel bed is not called upon to supply heat for evaporating the drops of water.

Calculated Entrainment of Typical Jets. Equations [2] and [3] may be combined into the general equation

$$E = (0.89A^{0.56})\ 0.953^P \qquad [4]$$

This empirical equation then gives, for the range most common, the entrainment ratio for any steam jet of optimum proportions: Bellmouth entry or equivalent, air tube about 7.5 diam long, nozzle approximately 1 tube diam back of the air-tube throat. Table 1 gives the weight of air delivered by some typical jets computed from Equation [4] for saturated steam.

TABLE 1 AIR DELIVERY FOR TYPICAL STEAM JETS

Nozzle diam, in.	Pressure, psi gage	Steam flow lb per hr	Air Quantity, lb per hr — Nominal air tube diam, in. $1^1/_2$	2	3	4
3/32 (0.094)	50	23.4	404	537	832	1139
	75	32.4	516	686	1067	1448
	100	41.1	602	801	1239	1699
	150	58.4	724	961	1491	2036
	200	72.8	763	1013	1568	2139
	300	106.7	800	1064	1654	2254
1/8 (0.125)	50	41.6	525	692	1080	1472
	75	57.2	667	878	1368	1868
	100	72.7	799	1026	1600	2180
	150	103.3	936	1232	1923	2620
	200	134.5	1030	1359	2115	2885
	300	194.0	1063	1402	2185	2983
5/32 (0.156)	50	64.7	636	845	1350	1782
	75	88.1	797	1060	1694	2235
	100	113.2	943	1253	2000	2643
	150	160.9	1133	1504	2405	3175
	200	208.1	1241	1648	2545	3475
	300	302.5	1292	1718	2741	3625
3/16 (0.188)	50	94.0	748	986	1522	2105
	75	129.4	949	1250	1932	2675
	100	164.3	1108	1461	2255	3125
	150	233.8	1332	1792	2710	3755
	200	302.1	1458	1918	2965	4100
	300	439.0	1520	2000	3090	4280

Penetration by Fan and Steam Jets

Overfire jets help to mix strata of air and burning gases above the fuel bed in addition to providing secondary air. The extent to which they mix depends upon the velocity pattern of the emerging jets. While the actual boundary of zero velocity is not usually of interest, the boundary beyond which the velocity is too low to induce appreciable mixing is of practical importance. Unfortunately, the velocity at which this occurs is not known. In this study, this limit was arbitrarily chosen as 1000 fpm (16.7 fps) and the point on the axis of the symmetrical jet at which the velocity reached this value was taken as the point of maximum penetration. The distance from this point to the tip of the jet was called the penetration or throw. If later studies show some other velocity to be the limit, enough data are available to determine the corresponding penetration.

From theoretical considerations, Tollmien (14) developed an expression for the velocity V_x at a point along the axis of a constant-temperature stream as

$$V_x = \frac{kdV_0}{x + 1.9\,d} \qquad [5]$$

where

k = empirical constant
d = diameter of outlet
V_0 = velocity at outlet
x = distance, outlet to point x

Kuethe (16) showed this to hold for all points greater than 8 diam from the outlet. If V_x is assumed at 1000 fpm then L, the throw, becomes

$$L = \frac{k}{1000}\,dV_0 - 1.9\,d \qquad [6]$$

With overfire jets, L is large compared to $1.9\,d$, hence this term can be safely ignored, or

$$L = k_1 dV_0 \qquad [7]$$

Mackey (7) and others have shown that for air-conditioning outlets, this equation applies very well. All of these equations apply for constant-density fluid discharging into a fluid of the same density, that is, at the same temperature and pressure. In terms of the weight of air flowing

$$L = k_1 d\,\frac{W_0}{\delta_0 A_0} \qquad [8]$$

where

W_0 = weight of air
δ_0 = density of air
A_0 = area of outlet

and with the pressure practically constant and atmospheric

$$L = Kd\,\frac{W_0}{1/T_0\,d^2} \qquad [9]$$

$$L = K\,\frac{W_0 T_0}{d} \qquad [10]$$

Thus the temperature of the overfire air affects throw, and the obvious economy of preheated secondary air is supplemented by the greater penetration possible.

Davis (15) has pointed out that the furnace temperature influences the throw, for the secondary-air jets are heated soon after emerging from the air tube. Hence he recommended multiplication by the factor $(T_f/T_0)^{1/3}$ where T_f and T_0 are the absolute temperatures of furnace and overfire air, respectively. So Equation [7] becomes

$$L = k_1 dV_0 (T_f/T_0)^{1/3} \qquad [11]$$

and Equation [10] becomes

$$L = k\,\frac{W_0}{d}\,T_f^{1/3}\,T_0^{2/3} \qquad [12]$$

It is evident that this temperature term is of appreciable influence, for at a furnace temperature of 2000 F, the throw is increased 71 per cent over that at 70 F. Davis made calculations for an actual furnace which showed that the flame path could be predicted from these equations when this factor was included.

Measurement of Penetration. The constant K must be determined empirically. Davis computed a value from some work of Keuthe (16) which was limited to very short jets and he introduced an approximate shape factor to account for the rectangular shape of outlet. The present study was confined to circular outlets and throw was measured for both steam and air jets. The effect of moisture in the steam-air jet is ignored.

Velocity measurements were made by means of a simple impact tube patterned after the standard pitot tube (17). Preliminary static pressure traverses by means of a standard static tube revealed that the static pressure in the jet was definitely below atmospheric pressure. It was observed that, at all points in the free jet, the static pressure was approximately 10 per cent of the impact pressure at the same point but opposite in sign. In order to avoid the labor of making separate static and impact traverses, this simple relationship was used in the computations for velocities from the impact traverse alone. Another approximation which it is felt was justified was the use of room-air density in computing these velocities, thus neglecting both the moisture and temperature of the jet. For example, at 12 in. from the tip of the air tube, the temperature was 108 F when room temperature was 60 F and when the initial steam was 309 F at 48 psi gage. At that point, the air-steam mixture was saturated, and there was much free moisture in the form of a fine spray. Measurement of the true velocity at that point would have been difficult. Fortunately it is the velocity farther out in the stream which is of much more importance. This is less subject to error because the mixture at a distance of 3 ft was cooled to 73 F and most of the free moisture was already re-evaporated into the room air which had been drawn into the mixture up to that point. Hence it is felt that these simplifications in procedure did not seriously affect the accuracy in the region of most interest.

Relation of Air Quantity to Penetration. Fig. 11 shows the variation of velocity at the center of the jet with distance from the outlet for various nozzles, pressures, and air tubes. Curves 1 and 2 are for a $^9/_{64}$-in. nozzle without air tube at 48 and 170 psi gage, respectively. They show that where supplementary air is not needed, open steam jets will give ample mixing. The increase in pressure by a factor of 3.5 did not quite double the penetration. Curves 3 and 4 show, respectively, the velocity curve for this same nozzle at 48 psi gage, a 2-in. air tube and a $1^1/_4$-in. air tube with diffuser. The air tube, curve 3, increased the throw over the open jet, curve 2, and the diffuser, curve 4, reduced the throw. Curves 5 and 6 duplicate 1 and 3, respectively, except that a $^3/_{32}$-in. nozzle was used; they show that a 55 per cent decrease in nozzle area, and hence in steam consumption, decreased the throw by only 35 per cent. Curve 7 is for a 2-in. bellmouth passing air from a fan at a static pressure of 10.3 in. water.

These comparisons show the effects of a few specific changes, but it would be difficult to compare a large number of variables in this way. Equations [9] to [12] indicate that all but the open steam jets can be compared on the basis of air quantity and outlet diameter.

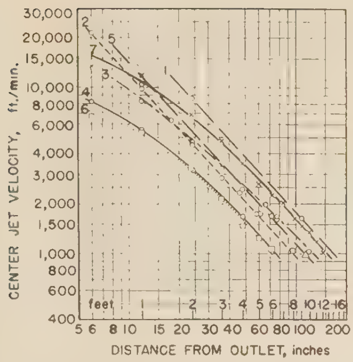

FIG. 11 RELATION OF VELOCITY ALONG AXIS OF JET TO DISTANCE FROM OUTLET FOR VARIOUS JETS

1—0.141-in. nozzle, 170 psi gage, no air tube
5—0.094-in. nozzle, 170 psi gage, no air tube
2—0.141-in. nozzle, 48 psi gage, no air tube
7—Fan jet, 2-in. air tube, 10.3 in. water static pressure
3—0.141-in. nozzle, 48 psi gage, 2-in. air tube
4—0.141-in. nozzle, 48 psi gage, $1^1/_4$-in. tube (7.5 diam) diffuser
6—0.094-in. nozzle, 48 psi gage, 2-in. air tube

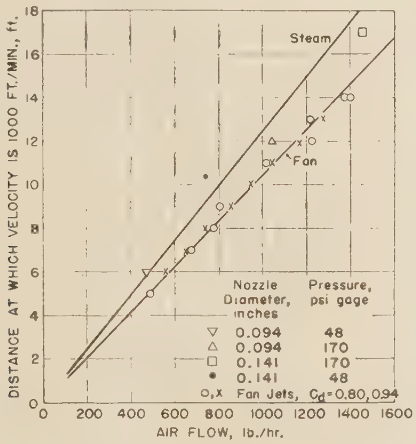

FIG. 12 EFFECT OF AIR RATE ON THROW ALONG CENTER LINE OF FAN AND STEAM JETS
(All for 2-in. standard pipe.)

Fig. 12 shows the throw for a number of steam jets and two fan jets plotted against air flow. All are for an outlet made of 2-in. pipe. The penetration with the steam jet is about 10 per cent greater than for a fan jet passing the same air quantity. From Equation [10], it is evident that the higher outlet temperatures with the steam jet are mainly responsible for the difference. Temperature measurements at the outlet were made in only a few tests. For the $^9/_{64}$-in. nozzle at 48 psi gage and a 2-in. air tube, the outlet temperature was approximately 130 F. Compared to the fan jet with an outlet temperature of 70 F, the higher-temperature steam jet should have $\left(\frac{460 + 130}{460 + 70} - 1\right)$ $\times 100 = 11$ per cent greater throw, which agrees well with Fig. 12.

The temperature would vary with different steam temperatures and entrainment ratios. Also the presence of water vapor and the high center velocity at the outlet of the steam jet may contribute to the increased penetration.

The two fan jets in Fig. 12 were both bellmouth outlets, with straight throats 7.5 and 1 diam long and discharge coefficients of 0.80 and 0.94, respectively. Fig. 12 shows that this difference in the type of nozzle did not influence throw.

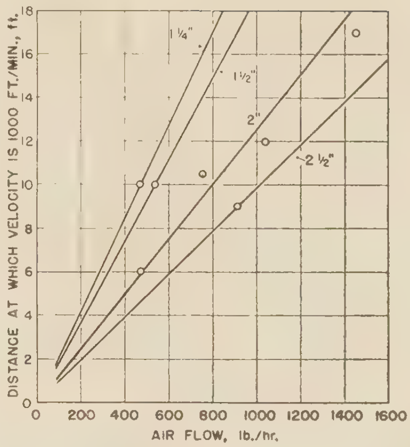

FIG. 13 EFFECT OF AIR FLOW ON PENETRATION FOR VARIOUS AIR TUBES

Influence of Diameter. Fig. 13 shows the influence of outlet diameter for steam jets. For a given air quantity, the smaller the outlet, the greater the throw. This is true only if steam pressure or nozzle size is varied to keep the air quantity constant. In the practical condition, when steam pressure and nozzle are fixed, throw does not increase as the size of air tube is decreased because the air quantity is decreased at the same time; the two opposing effects almost balance so that penetration is nearly independent of tube diameter.

Tests of fan-driven jets with varying diameters were not made. However, it can be shown that the effect of decreasing the outlet diameter is not the same as with steam jets. The outlet acts as a simple orifice with a discharge coefficient between 0.6 and 0.95 depending upon how well it is made (10). For a given air pressure

$$w = ad^2 \quad [13]$$

where

w = air flow
a = constant
d = diameter

but from Equation [10], for constant temperature

$$L = b\frac{W_0}{d} \quad [14]$$

thus

$$L = (ab)d \quad [15]$$

Hence if the fan is capable of maintaining a constant pressure at the outlet as diameter is increased, throw will increase directly as the diameter. Both curves in Fig. 12, may be expressed within 3 per cent by the equation

$$L = 0.51\frac{W_0T_0}{1000d} \quad [16]$$

where

L = penetration, in.
W_0 = air flow, lb per hr
T_0 = air temperature, deg F abs
d = outlet diameter, in.

The use of the Davis factor for furnace temperature effect together with this equation will give fairly accurately the penetration for an actual furnace. Equation [16] gives values of penetration about 10 per cent higher than values computed from the data of Cleve for fan-driven jets. This disagreement is not

large considering the uncertainty of velocity measurements under the conditions of irregular, fluctuating flow obtaining in the stream.

Fan Pressure. The use of comparatively high air pressures for fan-driven jets is becoming more widespread. Flow from an outlet of given diameter is proportional to the square root of the pressure in a large duct supplying the outlet, hence doubling the pressure will give a 41 per cent increase in air flow and penetration. It should be emphasized, however, that the ducts leading from the fan to the port should be of ample size. The authors know from laboratory experiences and field experiences of others that these ducts are likely to be undersized if they are chosen by guesswork rather than by calculation. Thus much of the pressure available at the fan can be easily dissipated in high friction losses occasioned by excessive velocities. A safe velocity to use for the design of the ducts is 2000 fpm.

Width of Penetration. The distance to which the stream spreads from the center line of the jet outlet is important in that it indicates the optimum spacing of jets if the width of the firebox is to be covered adequately. Here again a knowledge of the minimum velocity necessary for satisfactory mixing would be helpful. Lacking this information, the same arbitrary limit of 1000 fpm was used to determine the spread.

Fig. 14 shows three typical velocity patterns, i.e., that of a

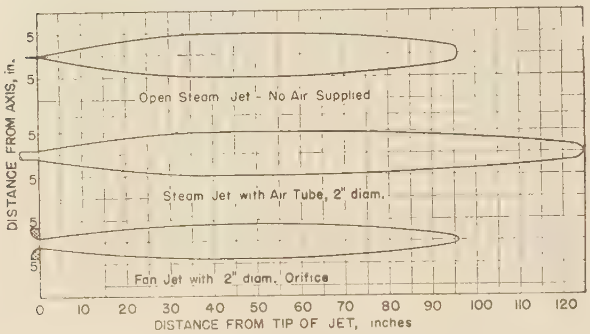

Fig. 14 Typical Velocity Patterns for Typical Jets
(Steam jets at 48 psi gage, 0.141-in. nozzle. No air for top jet. Air rate for others 740 lb per hr.)

Fig. 15 View of Typical Jet

free steam jet without air tube, a jet from the same nozzle with optimum size of air tube, and a jet from the same air tube supplied by a fan delivering approximately the same air quantity as the steam jet delivered. The curves are the envelopes or lines connecting all points at which the velocity was 1000 fpm. As for curves 1 and 2 in Fig. 11, the open steam jet is seen to produce a considerable region of high velocity. Although the penetration of the various open jets observed averaged 10 per cent less than that of the corresponding jets with air tubes, these results are in distinct contrast to the conclusions of DuPerow and Bossart that "air is required to give body to the jet." The tests from which this conclusion was drawn were made with the entrance to an air tube sealed off so that the jet was serving as a vacuum pump for the mixing region rather than as a blower.

The inferior penetration of the fan-driven jet shown in Fig. 14 has already been shown, in the case of Fig. 12, to be caused mainly by the lower temperature of the fan-driven jet.

Fig. 15 illustrates a fan-driven jet colored with ammonium-chloride smoke. The visible envelope is greatly different from those measured and shown in Fig. 14, because the slow mixing which occurs between the 1000-fpm envelope and zero velocity or still air is indicated. The total angle of spread measured on the figure is 20 deg. The angle of spread measured for the emergent jets in Fig. 14 varied from 12 deg for the fan-driven jet to about 15 deg for the steam jets with air tubes, 18 deg for the unenclosed steam jet. It will be noticed that the maximum width of the 1000-fpm envelope is about 12 in. for these jets operated at 48 psi gage. It is not expected that higher pressure and the accompanying higher air quantity would increase the width in any greater proportion than penetration is increased. In other words, the maximum useful width of any jet is approximately one tenth of the throw if the minimum practicable velocity is 1000 fpm.

Noise of Steam Jets

The noise produced by steam jets is frequently cited as discouraging their use. A simple silencer has been found to reduce

TABLE 2 COMMON INDOOR NOISES (19)

Decibels	
130	Painful sound
97	Boiler works
85	Very noisy factories
80	Very loud radio music
70	Stenographic room
68	Average of six factory locations
57	Noisy office
50	Few places where people work are below this
40	Soft radio music
0	Threshold of audibility

jet noise greatly without hindering the performance of the jet in delivering air.

With dry steam flowing through a $^9/_{64}$-in. nozzle under a pressure of 45 psi gage, the noise level measured at a point 34 in. from the outlet of the nozzle was 83 decibels. Fig. 16 shows the

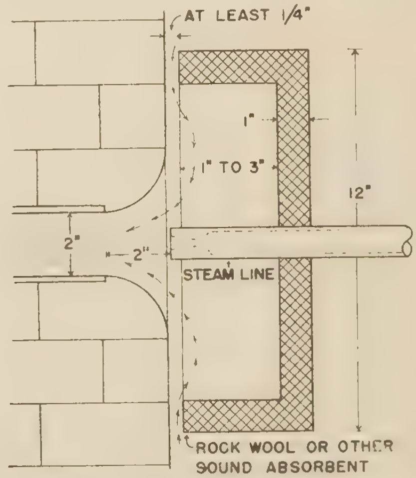

Fig. 16 Simple Silencer Tested in Laboratory

silencer which was located concentrically about the jet. It consisted of a sheet-metal cylinder lined with hair felt. The noise level was reduced to 70 decibels. This corresponds to a reduction

in noise intensity of 95 per cent. Throughout these measurements, the laboratory noise level was 65 decibels. The level of 70 decibels is less than that prevailing in many boiler rooms and is about the noise level of a busy city street. Table 2 gives the noise level of some common sounds.

It was found that moving the lip of the silencer too close to the front wall so throttled the flow of room air into the jet that the performance of the jet was impaired, but this did not occur until the lip was $^1/_4$ in. from the front wall. The velocity past the lip at this point was calculated to be 2060 fpm.

The rate of air flow would be increased by higher steam pressure, a large steam nozzle, or a larger air tube. Hence in such cases the area allowed between the lip of the silencer and the front wall should be greater.

In many cases it would be desirable to make the diameter of the silencer less than 12 in. This would be practicable without sacrificing much in noise reduction, but the distance of the lip from the front wall should be increased in order to keep the air velocity low.

The inside depth of the silencer tested was 3 in. This distance could be decreased considerably without affecting performance, so long as the air flow was not throttled.

Additional inverted buckets lined with sound-absorbent material could be arranged concentrically with the jet to provide a zigzag passage with additional absorbing surface, but indications were that the noise reduction by the first silencer was so large that little room was left for improvement.

The sound-absorbent material used in these tests was hair felt. To avoid fires, rock wool should be used which has the same sound-absorption coefficient as hair felt, i.e., 0.50. Some commercial fireproof sound absorbents give better absorption; a very few have almost double the sound absorptivity of rock wool.

Another effective way to reduce the noise of a number of jets close together is to install a trunk air duct leading to the jet inlets. The duct inlet can be located outdoors or above the floor so that the noise is not so apparent on the floor, or it can be connected to a preheater to permit supplying preheated air over the fire. In any such duct system, the design should permit low air velocities so that the steam jet does not have to work against a partial vacuum at its inlet.

Laboratory tests showed that a reduction in pressure of only 1 in. of water below atmospheric reduced the entrainment of the jet approximately 5 per cent. A safe velocity to use in any extended duct system is 2000 fpm.

Efficiency of Steam Jets

Based upon the kinetic energy alone of the air entering the furnace, the efficiency of even the optimum steam jet is low. The remainder of the total energy supplied goes to preheat the air. However, this preheating cannot be considered of much advantage; it occurs largely at the expense of the condensing vapor which immediately absorbs energy from the fuel bed upon reevaporation.

Table 3 gives the efficiency of typical jets calculated according to Equation [17]

TABLE 3 EFFICIENCY OF TYPICAL JETS, SATURATED STEAM $^1/_8$-IN. NOZZLE

Steam pressure, psi gage	Air-tube diam, nominal diam	Efficiency, per cent
100	1	1.2
	2	0.8
	3	0.6
	4	0.5
300	1	2.6
	2	0.7
	3	0.6
	4	0.5

Assume outlet-air temperature, 100 F
Feedwater temperature, 60 F

$$\text{Efficiency} = \frac{\text{Kinetic energy of air}}{\text{Energy supplied}} = \frac{\dfrac{WE}{2g}\left(\dfrac{WE}{\delta A}\right)^2 \times 100}{778W(h_s - h_{fw})} \quad \text{[17]}$$

where

W = steam flow, lb per sec
E = entrainment ratio
g = acceleration of gravity
δ = air density in air tube, lb per cu ft
A = area of air tube, sq ft
h_s = enthalpy of initial steam, Btu per lb
h_{fw} = enthalpy of feedwater, Btu per lb

It is evident that the efficiency is nearly independent of the pressure but decreases with increasing air-tube diameter.

Cost of Operation

Although a fan is many times as efficient an air mover as the steam jet, it is well to examine the other efficiencies which enter into the use of a fan. If a plant generates its own power, the steam costs the same whether supplied to the jet or to a turbine. This cost will range from 23 to 37 cents per 1000 lb for the usual range of coals, 10,000 to 14,000 Btu per lb, \$3.50 to \$4.50 per ton, and boiler efficiencies from 50 to 85 per cent. If the efficiencies of the turbine, motor, and fan are 20, 75, and 60 per cent, respectively, the over-all efficiency of this combination, neglecting slight electrical and air-pressure losses, is 9 per cent. Thus, the fan-motor-turbine combination requires only about one ninth the energy required by a steam jet to supply the same air.

For a $^1/_8$-in. steam nozzle at 100 psi gage using 72.7 lb of steam per hr, the yearly cost would be \$191 if operated continuously with steam at 30 cents per 1000 lb. If operated on plant-produced electrical energy, a fan would cost \$21 per year to operate. Thus based upon continuous operation, \$170 per year would be saved by the fan. If operated only part of the time, savings would be proportionately smaller.

If electrical energy is purchased from outside, the saving is not so great. These energy costs can be compared on the basis of 1000 Btu.

If power costs 2 cents per kwhr, the corresponding cost per 1000 Btu is 0.58 cents. The motor-fan combination will use this energy with an over-all efficiency of $60 \times 0.75 = 45$ per cent. Thus, per useful 1000 Btu, the cost would be 1.29 cents.

At 30 cents per 1000 lb, the energy cost of steam, at say, 1150 Btu per lb is 0.026 cents per 1000 Btu. The steam jet will use this energy with an approximate efficiency of 1 per cent; or the cost per 1000 useful Btu will be 2.6 cents or twice the fan power costs. For an output equivalent to that of the typical $^1/_8$-in. steam jet costing \$190 per year, the fan would thus cost \$93 per year, saving \$97 per year. Table 4 gives similar figures for various power costs.

TABLE 4 COMPARATIVE OPERATING COST, STEAM JET[a] VERSUS FAN

Power cost, cents per kwhr	Fan operating cost: Cost of energy[b] consumed, cents per 1000 Btu	Fan operating cost: Cost of useful[c] energy, cents per 1000 Btu	Steam Jet Operating cost: Cost of energy consumed, cents per 1000 Btu	Steam Jet Operating cost: Cost of useful[d] energy, cents per 1000 Btu	Cost ratio, steam jet to fan	Yearly saving by use of fan, dollars
1	0.29	0.65	0.026	2.6	4.0	143
2	0.58	1.29	0.026	2.6	2.0	97
3	0.88	1.95	0.026	2.6	1.3	44

[a] Based on air delivery equivalent to that of $^1/_8$-in. steam nozzle at 100 psi gage, 2-in. air tube; steam rate 72.7 lb per hr.
[b] 1 kwhr = 3415 Btu.
[c] For fan efficiency = 60 per cent; motor efficiency = 75 per cent; assumed ducts large enough to make pressure losses negligible.
[d] Steam cost: 30 cents per 1000 lb, corresponding to coal at \$4 per ton, 12,000 Btu per lb; boiler-furnace efficiency = 65 per cent; enthalpy of steam = 1150 Btu per lb.

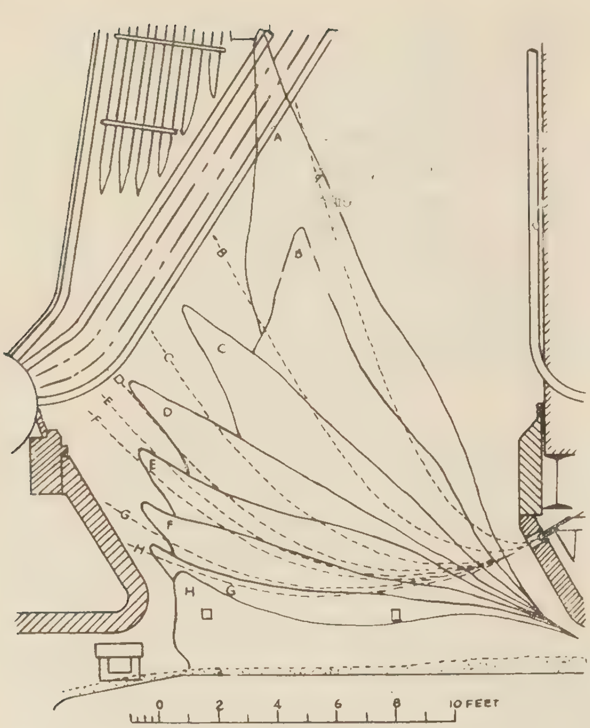

Fig. 17 Effect of Varying Percentages of Overfire Air for Fixed Nozzles Supplied by Fans

(Dotted curves are calculated trajectories of air from Davis. Table 5, from Davis, gives percentages corresponding to curves.)

TABLE 5 DATA USED BY DAVIS IN CALCULATION OF AIR TRAJECTORIES IN FIG. 17

	Overfire air as percentage of total air	Percentage of carbon dioxide at boiler exit	Air temperature, deg F	Total overfire, air, cfm	Initial velocity, fps	Average upward velocity of furnace gases, fps
A	5.65	10.1	222	1990	33.7	9.4
B	10.5	9.3	234	3780	64.0	8.9
C	16.6	9.0	242	6050	102.5	8.3
D	20.0	8.8	244	7300	123.5	8.0
E	21.4	9.0	244	7780	132	7.8
F	22.8	8.9	242	8300	140.5	7.7
G	26.8	8.5	244	9750	165.0	7.3
H	28.8	9.5	246	10500	178	7.1

Note: Number of nozzles, 18; size $3^1/_2$ in. × $2^1/_4$ in.; total area, 0.985 sq ft; jet inclination, 30 deg downward; furnace 15 ft × 16 ft; average furnace temperature, 2100 F.

Application

This study has not included tests of the effectiveness of overfire air jets in reducing smoke or combustible losses. Tests have been proposed to investigate the effect of location, number, angle, air quantity, and velocity on rating, smoke, and efficiency. A few published data giving results from specific applications are available.

Fig. 17 shows the effect of location, angle, and air quantity on an English installation, described by Robey and Harlow (2). Davis computed the expected trajectories of the air, shown dotted. Table 5 from Davis indicates the furnace conditions which existed. A maximum of 28.8 per cent of the total air supply as secondary air was used. In addition, successful use of an amount of secondary air in excess of 40 per cent of the total was reported. It was contended that the thicker fuel beds and lower grate speeds possible with high secondary-air quantity offset the reduced cooling effect of decreased wind box air so that no trouble was encountered from overheating the grate.

The effect of increasing the amount of secondary air in the case shown was to reduce the length of flame and force it toward the rear wall. This, however, is not necessarily a function of percentage of overfire air alone. Fans were used, and the same effect might have been obtained by decreasing both the air quantity and size of jet opening according to Equation [10]. This is nearly true also for steam jets, except for the slight reduction in throw which occurs with a decrease in the size of air tube.

Fan-jet pressures ranging from wind box up to 30 in. of water (20) have been advocated. Provided the air ducts do not throttle flow, the higher pressures permit use of less air to produce the same velocity distribution or penetration. On the other hand, on narrow fireboxes, especially those on which jets can be installed in both sides or front and rear, even the pressure difference afforded by induced draft may give sufficient penetration without introducing excessive amounts of unnecessary air.

Effects of Overfire Air

Carroll (21) has emphasized the accuracy required in gas analyses in order to measure the small gains in over-all boiler-furnace efficiency effected by overfire air. Christensen and Strahl (22) obtained a variation in improvement of 1 to 10 per cent from identical tests on the same furnace; these variations were random and not because of some specified change. Gas analyses made without the jets revealed very high excess air, hence overfire air could not have improved burning very much. On very poor installations where there is a definite deficiency of overfire air, large increases in efficiency might be obtained because of the low efficiency; but more fundamental changes than overfire air might do as well at lower cost in the long run.

Determination of the increase in rating and decrease in smoke is less difficult. Carroll reported on one installation an increase of 25 per cent in steam production. Gilg (23) measured an increase in burning rate without smoking from 40 to 48 lb per sq ft per hr on a chain-grate stoker; also an increase in the "no-smoke" limit of steam production of 27 per cent on a large cross-drum boiler fired by an underfeed stoker.

Thus the output of even well-designed and operated existing installations can be increased substantially without any increase in smoke and with equal or slightly better efficiency.

Conclusions

The experimental data on the performance of steam and fan air jets designed to provide mixing or secondary air, or both, show the following:

1 The optimum length of air tube on a steam-air jet is about 7.5 diam, and the optimum nozzle location is about 1 air-tube diam back of the throat of the air tube. The tube entry should be rounded or funnel-shaped.

2 For a given steam pressure and a given nozzle, the larger the air tube, the greater is the weight of air delivered. Also, for a given steam pressure and a given air tube, the larger the nozzle, the greater is the steam flow and weight of air delivered.

3 A Venturi-shaped air tube is less effective than a straight tube of the same outlet diameter, both from the standpoint of air flow and penetration.

4 The pepperbox nozzle is quieter than a single-hole nozzle and permits shorter air tubes, but the small holes are likely to become obstructed by foreign materials.

5 The ratio of weight of air entrained to weight of steam used decreases with increasing steam pressure.

6 For a given mass rate of flow of air, the penetration is directly proportional to the temperature at the outlet and inversely proportional to the diameter, for both fan and steam jets. However, for steam jets at constant steam pressure, the penetration is practically independent of air-tube diameter.

7 With fan jets, penetration is directly proportional to outlet diameter at constant pressure.

8 A steam jet which entrains no air gives nearly as much penetration as a jet with air tube.

9 The width of penetration is only about $^1/_{10}$ the depth of penetration.

10 The noise from the steam jet can be effectively reduced by a very simple silencer.

11 For most installations, the low efficiency of even the best steam jet makes its operating cost higher than fan jets.

12 Further tests on specific installations are proposed to determine the optimum number, size, and orientation of jets.

ACKNOWLEDGMENTS

The authors wish to acknowledge the assistance particularly of T. E. Pochapsky for a fundamental analysis of jet performance; also the co-operation of others of the Battelle staff and many interested fuel engineers.

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Discussion

William G. Christy.[4] The authors are to be congratulated on a very interesting paper covering quite a complete investigation of overfire air jets. While jets have been used for many years, there has not been too much authoritative information available about them.

It is a little disappointing to find that most of the tests conducted were with a steam pressure of 48 psi. In practice, we have found that steam-air jets are not effective in eliminating smoke with less than 75 psi steam pressure. A minimum of 100 psi pressure is better. On boilers operating with 45 to 50 psi pressure, the jets do not eliminate the smoke.

During the last 12 years, jets similar to Type D, Fig. 2 of the paper, have been used in Hudson County. In the great majority of cases, they have been successful, provided they are used.

It is good to know that these tests confirm the fact that it is important to have the steam jets in the exact center of the air induction tube and 1 diam away. Also, that the optimum length is 7.5 diam.

Attention should be called to the fact that the cost figures mentioned are based on continuous use of jets. In practice on most jobs they are used only 10 to 20 per cent of the time. This would reduce the estimated cost of operation.

The writer was disappointed that the Bibliography does not include reports of tests made by the Pennsylvania Railroad at the locomotive test plant at Altoona, some 30 years ago. These tests made on locomotives were quite extensive.

Those of us who are interested in the application of overfire air jets hope this work will be carried on and that tests will be run on boiler installations to show the effects on smoke, fly ash, and boiler efficiency.

In practical application of such jets, the great objection is that the human element is too important a factor. In other words, many jets are installed and not used regularly. The operators forget, do not bother, or do not like the noise. The human element could be overcome by use of a control which would turn on the jets whenever the firedoor is opened. Many such installations have been made, particularly in Europe. A considerable increase in the cost of installation is involved which makes it difficult, or impossible, to show any return on the investment.

F. X. Gilg.[5] As stated by the authors, steam jets and steam-jet air aspirators have been used for many years and it is surprising how little information has been available on their design and effectiveness. Therefore the authors are to be commended for their work in presenting the data obtained from their tests at Battelle Memorial Institute. They have done considerable debunking of steam-air-jet design and reduced it to practical terms which can be used by any one familiar with boiler-room requirements. When it is realized that an effective, economical steam air jet can be made from a piece of 2-in. pipe with a 3-in. × 2-in. reducer on the inlet end containing a small capped end of a steam pipe drilled with a $^1/_8$-in. hole as the steam nozzle, and obtain a 12,000-ft-velocity air jet with a steam consumption of just slightly more than 1 lb per min at 100 psi pressure, we have a rough idea of the fine work these authors have done in supplying the information.

[4] Smoke Abatement Engineer, Department of Smoke Regulation, Hudson County, Jersey City, N. J. Mem. A.S.M.E.

[5] Mechanical Engineer, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

Their work on penetration of fan and steam jets is particularly enlightening and will explain why so many early overfire air systems, using stoker wind-box pressure, proved inadequate. Effective penetration requires velocities higher than the velocity of the gases in the furnace. While the average velocity in a furnace might very well be 1000 fpm, there are zones at the front over the active burning zone where the actual velocity is many times more than the average. Between these high-velocity high-excess-air strata and the arch, we usually find the unburned volatile gases which are deficient in air. To burn these gases it is only necessary to mix these two strata but to do this, high-velocity jets are necessary to penetrate into the high-velocity gas and air streams. Thus the quantity of overfire air is not as important as the proper turbulence.

The quantity of overfire air adds to the total air for combustion and, in some installations with limited fan capacity, increased steaming capacity can be obtained with steam-air jets. During this emergency period when fans are scarce, this feature of the steam-air jet cannot be ignored. To obtain the added air for combustion the tendency will be to use the larger air tubes which, from Table 1 of the paper, show a greater entrainment of air for the same steam consumption. However, the velocity leaving the tube is considerably less than from the smaller air tubes and therefore they might not be as effective as overfire air nozzles. For instance, a $^1/_8$-in. steam nozzle at 75 psi pressure will aspirate 878 lb of air per hr through a 2-in. pipe at a velocity of about 11,000 fpm. The same steam nozzle and steam conditions will aspirate 1868 lb per hr through a 4-in. pipe, but the velocity leaving the nozzle will only be about 5100 fpm. The ratio of quantity increase is about 2 to 1, whereas the area increase is 3.8 to 1, hence the lower velocity. Since penetration is primarily a function of velocity leaving the air nozzle, the writer cannot agree with the authors' statement: "Penetration is nearly independent of tube diameter." It does not check with their data. However, the only reason for mentioning it is to emphasize the necessity of choosing high-velocity air jets if they are to function as overfire air nozzles rather than simply as air movers.

The simple silencer design is a welcome device and will remove one of the objectionable features of steam-air jets in boiler rooms. The writer has seen more than one job with steam-air jets turned off by the fireman because he could not stand the noise.

The fact that steam-air jets are not as efficient as overfire-air fans is not too disturbing during these abnormal times when it is considered that many plants can obtain the well-known benefits of overfire air by the addition of these simple devices which they can design from the data presented by the authors and fabricate in their own plants.

Proper location of overfire air jets is important, but, unfortunately, there is not yet a complete answer which may be used as a guide in all cases. Experience is the best teacher and the results will show in the performance of the units, perhaps as (*a*) reduced smoke; (*b*) higher efficiency; (*c*) higher capacity; (*d*) shorter flame travel and attendant benefits; (*e*) better ignition at the stoker.

M. A. Goetz.[6] This paper comes at a time when most steam plants are being pushed to the limit of their capacity. If a way to increase rating by the introduction of overfire air jets can be worked out, it should find wide application throughout the entire country. Not only is this information urgently needed now, but after the present emergency passes it will be equally valuable, as civic authorities are bound to renew their interest in the problem of smoke abatement. The authors are to be congratulated on the clear and understandable presentation of their studies; and it is hoped they will be able to proceed immediately to the application of their jets on actual boilers. Under the present manpower stringency, small boiler plants simply do not have the necessary personnel to carry on experiments, and the overfire-air installation must be handed to them as a completed job.

[6] Consolidated Edison Company of New York, Inc., New York, N. Y.

In connection with this study, the experiments of the writer's organization, commencing back in 1936, and extending over a period of a year and a half, may be of interest. We were then interested in developing a means of increasing the smokeless capacity of stoker-fired boilers in which bituminous coal is burned.

Preliminary Laboratory Experiments. Experiments were commenced in the laboratory and they indicated that a fairly large quantity of air was required to eliminate smoke completely, and that cold air was just as satisfactory as hot air. It also appeared that a large nozzle was more effective than a number of smaller ones of equivalent area.

Experimental Boiler Installation. The experimental installation was made on a horizontal cross-drum boiler having a total evaporating surface of 24,700 sq ft. The furnace has a volume of 8500 cu ft and is enclosed by waterwall tubes covered with refractory-type blocks on the rear and side walls. It has a depth of approximately 20 ft from the rear to front wall and a width of 26 ft. The unit has an interdeck superheater, economizer, forced- and induced-draft fans, and a wet-type cinder catcher. It is provided with a 15-retort 33-tuyère underfeed-type stoker with an overfeed undulating-grate section at the end. It has a nominal generating capacity of 320,000 lb of steam per hr.

Six overfire air nozzles $2^3/_4$ in. wide × 18 in. high were equally spaced across the rear wall of the boiler at an elevation about equal to the head end of the downward sloping fuel bed. They were so designed that their vertical height could be varied from 18 in. down to zero, and so that the air could be directed in an upward or downward direction. Each nozzle was supplied from a main header by an individual branch duct containing a damper, and a turbine-driven blower of sufficient capacity was temporarily set up to furnish the overfire air at pressures up to 26 in. of water.

Tests to Determine Optimum Opening of Rear-Wall Nozzles. Observations indicated that nothing was to be gained by directing the air upward or downward, hence the effect of various nozzle sizes was determined with the air directed horizontally across the furnace toward the head of the fuel bed.

Nozzle heights of 18, 14, 12, 10, 8, and 6 in. were tried. Table 6 of this discussion shows the amount of overfire air required when using all six nozzles to eliminate completely very dense smoke created by purposely disrupting the fuel bed.

TABLE 6 AMOUNT OF OVERFIRE AIR REQUIRED TO ELIMINATE SMOKE

Vertical opening or height of six rear overfire-air nozzles, in.	Area of each nozzle, sq in.	Header air pressure, in. of water	Total overfire-air flow, cfm of free air	Ratio of overfire air to total air supplied to boiler, per cent
6	16.5	21.0	21900	26.6
8	22.0	17.8	23200	28.3
10	27.5	15.1	24700	30.0
12	33.0	12.7	25000	30.5
14	38.5	10.8	27000	32.9
18	49.5	9.3	27200	33.2

Tests were also run with some of the nozzles cut out, but the results were not as satisfactory as with all six in use.

Tests With Side-Wall Nozzles. A pair of side-wall nozzles, one on each side, was installed about $4^1/_2$ ft from the rear wall so as to direct air over the tail end of the fuel bed. They were placed at a slightly higher elevation than the rear nozzles so that the jets of air therefrom would not collide with those coming in at right angles from the rear wall.

No beneficial results could be observed by the use of these two side-wall nozzles. They were subsequently moved up to within

about $2^1/_2$ ft from the front wall, but here again the six rear nozzles appeared to do just as good a job without them.

Permanent Installation. For the permanent installation on eight boilers, two variable-speed fans, each capable of delivering 135 000 cfm of air at a static pressure of 24 in. of water were selected for the air supply. Six nozzles $2^3/_4$ in. × 12 in. were uniformly spaced across the rear wall of each boiler.

In actual operation, it has usually been possible, even at maximum ratings on the boilers, to eliminate smoke entirely with very much less volume and pressure of overfire air than used in the experiments with artificially created smoke. In most cases, a volume representing 14 per cent of the total air supplied and a pressure of from 3 to 6 in. of water are sufficient.

In addition to the elimination of smoke, there is also a reduction in the cinder loss at the higher ratings on the boiler at which the overfire air is used.

From a maintenance standpoint, there is an advantage in not using any more overfire air than necessary. The reason for this is that on an economy basis the total air-fuel ratio should not be increased, or in other words the air flow through the fuel bed must be reduced by the amount that goes over the fire. This means that with overfire air the fuel bed must be thickened in order to carry the rating, resulting in higher temperatures in the fuel bed and furnace, which in turn cause increased maintenance on the refractory front wall, stoker, and pit.

In conclusion, our experience has shown that the use of overfire air is effective in increasing materially the smokeless rating on a large industrial boiler equipped with an underfeed stoker and burning bituminous coal. The cinder loss at the higher boiler ratings is reduced. Care must be taken that refractory and stoker-iron maintenance is not increased, as this will take place when the volume of overfire air is large over an extended period of time.

H. K. Kugel.[7] It has been said many times that steam jets are as "old as the hills," but it is certain that very little has been known about the scientific principles surrounding their design and use. The present paper therefore is more than appropriate in throwing further light on the subject with the addition of information on fan jets. The investigation of DuPerow and Bossart at Case School of Applied Science in 1927 was the first work ever done on this subject, but except for the standard Cleveland Smoke Department design shown in Fig. 2, (C) and (D) of this paper, there has been little or no use made of this information. Design (C) was developed in order to enable users to make the heads in their own shop and thus avoid paying the excessive cost of some of the commercial steam jets in use in Cleveland at that time. Installations were being made under horizontal return-tubular boilers for as much as \$500 to \$600 per boiler. Design (D) was developed as a casting which was sold by a local shop and extensively used by the steamship companies. Some of the jets now used by the stoker companies leave much to be desired, and it is believed that they should avail themselves of the information contained in this paper.

In using the Cleveland type jets in hand-fired boilers in various schools in Cleveland, it was found that they were ineffective in reducing smoke at less than 40 psi. Various devices were used to muffle the noise of the jets which is often objectionable to firemen, particularly in the closed fire rooms of Lakes vessels. A sheet-metal duct open at both ends was one of the cheapest and most effective mufflers which apparently satisfied the firemen. A duct leading up to and surrounding a section of the breeching also was very effective and gave a certain amount of preheat to the air. Another type pulled the air through air spaces in the side walls.

[7] Chief Engineer, Smoke Regulation and Boiler Inspection, District of Columbia. Mem. A.S.M.E.

One of the most striking cases known to the writer was that of a Coxe chain-grate stoker installed under a 650-hp water-tube boiler at the central furnaces of the American Steel & Wire Company in Cleveland to burn coke breeze. Due to a change in operations, it became necessary to use high-volatile coal on this stoker. It then produced a No. 4 smoke continuously. An installation of steam jets through the arch was made which resulted in a clear stack and an increase in boiler output and efficiency. The improvement seemed almost unbelievable to the company.

It is gratifying to note that many of the conclusions established in the earlier study have been verified by this later and more extensive research, and it is to be hoped that further work will be done along the lines most interesting to smoke inspectors, particularly on underfeed stokers.

E. D. Benton.[8] The authors are to be congratulated upon rationalizing design data of the important but little understood steam-air jet. Air jets play such an important part in smoke abatement, it is indeed strange that until now practically no data have been available from which competent design could be calculated. The exploding of the theories regarding the Venturi-shaped induction tube and the fact that a steam jet, without benefit of air tube, is able to penetrate practically as far as one with an air tube, should promote better application than in the past. It has been the writer's experience, in certain instances where turbulence only was necessary to abate smoke, that additional overfire air was detrimental to efficiency. This is particularly true with natural-draft chain grates operating with short fires.

The steam-air jet, because it is cheap and simple to apply, will be used in many installations in spite of its cost of operation, as compared to high-pressure fans. This is particularly true on locomotives. In the case of the locomotive, a table giving air capacities of short induction tubes for various pressures would be useful in spite of the low ratios of entrainment.

The question of quantity of overfire air will vary somewhat, depending upon coal characteristics, boiler, furnace, and stoker design. The relationships of these factors do not lend themselves to mathematical analysis, so that practical experience, together with trial-and-error methods, will still play an important part in abating smoke, increasing capacity or efficiency. It is hoped, however, that further research at Battelle will suggest certain fundamental installation features which will clarify some of the uncertainty of their application.

To anyone who has worked around open steam jets, noise is their principal objection. The writer has built a number of silencers and it has been his experience that ruggedness of construction is of prime importance. The design used is patterned after the automobile muffler and made of pipe. Where efficiency of the jet (entrainment ratio) is important, findings show the necessity of generous proportioning of the air passages. Extending the intake pipe 18 to 20 ft above the boiler-room floor is now definitely obsolete practice.

Authors' Closure

The supplementary information given by the discussers forms a valuable addition to the laboratory results and the authors appreciate the interest shown.

Although the highest steam pressure available in the laboratory was 48 psi, the use of this pressure for the jet-dimension tests should not be taken as an endorsement of it for general practical use. The penetration tests showed that air quantity and tube diameter are the fundamental factors, not steam pressure. Mr. Christy's recommendation of a 75 psi minimum probably means that in larger furnaces the quantity of steam required at less than

[8] Fuel Engineer, Louisville and Nashville Railroad, Louisville, Ky.

75 psi in order to give adequate penetration is excessive. Mr. Kugel's recommendation of 40 psi minimum, on the other hand, indicates that on smaller furnaces, adequate mixing with reasonable steam consumption is obtainable as low as 40 psi.

Mr. Gilg has rightly emphasized the great variation of upward velocity in boiler furnaces. This condition may defy all simple rules for jet spacing and locating. Perhaps it is responsible for the seeming anomalous conclusion drawn by Mr. Goetz from actual tests that a large nozzle was more effective than a number of small ones of equivalent area. Optimum distribution and mixing would be expected from the latter if the furnace gases were uniform. However, if smoke is occurring in long lanes, judicious location of a few large jets could be superior to many small ones.

Perhaps the paper seems obscure regarding constancy of penetration with varying diameters. This was found to apply to steam jets at constant pressure and nozzle diameter only. Fig. 13 of the paper does not contradict this, because there both pressure and nozzle were varied.

Since presentation of the paper the Altoona test data have been located through the assistance of W. G. Christy. They are incorporated in a report of the Committee on Smoke Abatement in the Proceedings of the American Railway Master Mechanics Association for 1913, pages 309–376. The tests were made on a switch engine in connection with a study of the use of an arch in the firebox. Only one length and diameter of air tube was tried in several locations and at pressures of 15 to 125 psi gage. Location above the firing door was recommended as best but no conclusions were drawn regarding pressure or penetration. Based on necessarily rough measurements of the air flow to the furnace it was concluded that for this service 30 to 45 per cent of the total air supplied should be overfire air.

Heat Transfer and Pressure Drop in Annuli

By ELMER S. DAVIS,[1] NEW YORK, N. Y.

This paper presents new equations for heat transfer and pressure drop in annular spaces for turbulent and laminar flow. The equations have been checked against most of the data available in the literature and are found to give excellent agreement. For heat transfer, the D_2/D_1 range tested is from 1.18 to 6800, and for pressure drop from 1.0044 to infinity. The heat-transfer and friction factors vary with the same powers as the corresponding cases for flow inside tubes. The friction equation reduces exactly to the conventional equation for flow inside tubes when D_1 approaches zero. For most commercial cases, the new equations give higher rates than the equation for flow inside tubes of the Tubular Exchanger Manufacturers Association, using hydraulic diameter based on either heated or wetted perimeter.

Nomenclature

THE following nomenclature is used in this paper:

c = specific heat, Btu per lb per deg F
D_1 = inside annular diameter, ft
D_2 = outside annular diameter, ft
D_a, D_b = annular characteristic dimensions, ft
f = friction factor, dimensionless
F = friction, ft-lb per lb $= \dfrac{\Delta P}{\rho}$
G = mass velocity lb per hr per sq ft
g_c = acceleration due to gravity = 32.2 fpsps
h = heat-transfer coefficient, Btu per sq ft per deg F per hr
i = exponent, dimensionless
j = dimensionless function $(hD/k)\,(c\mu/k)^{-1/3}\,(\mu/\mu_w)^{-0.14}$
k = thermal conductivity, Btu per hr per sq ft (deg F per ft)
L = length, ft
N = exponent in friction equation, dimensionless
ΔP = pressure drop, psf
S = spacing, $D_2 - D_1$, ft
V = linear velocity, fps
α = proportionality constant, dimensionless
θ = time, hr
μ = viscosity, lb per ft per hr
μ_w = viscosity at wall, lb per ft per hr
ρ = density, lb per cu ft
ϕ_1 = $\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$, viscosity ratio, dimensionless
ϕ_2 = $\left(\dfrac{\mu}{\mu_w}\right)^{0.25}$, viscosity ratio, dimensionless

Introduction

The problem of determining the proper equations to be used for heat transfer and for pressure drop in annuli has presented an obstacle which has not as yet been adequately solved. Various authors have made correlations which are satisfactory over a limited range of D_2/D_1 for either heat transfer or pressure drop. However, all of these equations give rather erroneous results when applied to cases beyond the limited range of tests on which they were based.

With one exception, most authors have used some type of hydraulic diameter as the shape factor in the equations. Two main types of hydraulic diameter have been used. More commonly, the type which defines the hydraulic diameter as 4 times the net free area over the wetted perimeter has been used. Others have used 4 times the net free area over the heated perimeter. One exception to this general trend may be noted in the literature, namely, the correlation of Mueller (1)[2] who has used the inside diameter as the shape factor in both the Nusselt and Reynolds numbers. Many of the correlators have attempted to correct for the variation in j with D_2/D_1 by introducing a constant in the Dittus-Boelter equation, which took the form of $(D_1/D_2)^i$. These have been satisfactory in explaining a limited set of data but have been of no use so far in covering a very extended range.

Wiegand and Baker (2) have given extensive plots of the data in the literature as correlated by the original investigators. The reader may examine this excellent paper to see how divergent are the individual data.

Recently, Mueller (1) published data on turbulent flow in annuli where the inside pipe was a wire of small diameter. His suggested correlation involved a dimensional group and was a curved line which, upon extension to higher Reynolds numbers, could be made to pass through data obtained by Monrad and Pelton (3) on much larger wires by bending the curve still further. However, an extension beyond wires led to exceedingly high j values in the range of commercial sizes. Moreover, Mueller's (1) suggested correlation is unsatisfactory because of the dimensional group mentioned; however, his alternative equation is of highly satisfactory form, which is in fact similar to the present proposed equation with the powers modified. In this paper, a totally dimensionless equation is applied to a very large range of D_2/D_1 and is substantiated by the correlation of available published data for transfer at the inner wall of annuli.

Attempts have been made to correlate pressure drop in annuli. However, no satisfactory correlation has covered the entire D^2/D_1 range in both the turbulent and viscous regions. In this paper an equation is proposed for this also and is found to compare favorably with the data available in the literature.

Heat Transfer at Inner Wall of Annuli in Turbulent Flow

The equation derived for heat transfer in annuli for turbulent flow is

$$\frac{hD_1}{k} = 0.038\left(\frac{D_1G}{u}\right)^{0.8}\left(\frac{c\mu}{k}\right)^{+1/3}\left(\frac{\mu}{\mu_w}\right)^{+0.14}\left(\frac{D_2}{D_1}\right)^{+0.15} \ldots [1]$$

This general form was derived by dimensional analysis and the powers determined by comparison with the experimental data. If the general equation involves $D_2 - D_1$, D_2, and D_1, as in the correlations of Monrad and Pelton (3), and Foust and Christian (4), the value of h will be determined by dependent and independent variables since only two of these quantities are needed to completely describe the geometry of the system. Mueller's equation

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[1] Development Engineer, American Locomotive Company, Alco Products Division. Deceased.

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

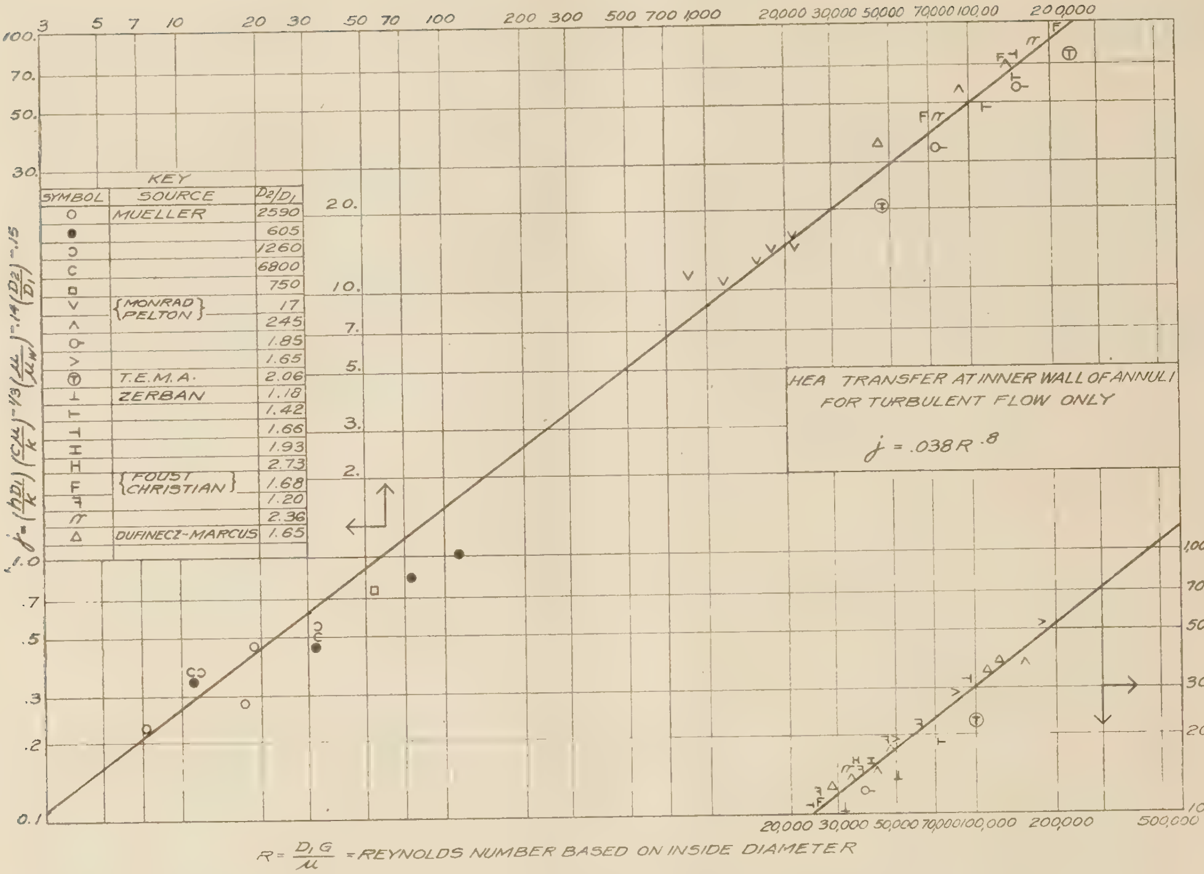

Fig. 1 Heat Transfer at Inner Wall of Annuli in Turbulent Flow

has only D_2 and D_1 for shape factors and is a great step forward. It is of the same general form as Equation [1] with the powers modified and the range of applicability greatly extended. A better correlation of Mueller's (1) data would be obtained if the curve were bent below $Re = 100$. However, this would be only a fictitious improvement since the data are a combination of crossflow and annular flow and should not fall directly upon the annular line. Fig. 1 shows the agreement between the literature cited and the proposed equation.

It should be mentioned that powers of 0.1 and 0.2 were also tried for D_2/D_1 but 0.15 was found to give the best correlation. Obviously, it may be that the power is 0.14 or 0.16, but this is impossible to say definitely. The use of D_1 in the Nusselt and Reynolds numbers was decided upon by trying both D_1, D_2, and $D_2 - D_1$ in these groups and finding which gave the most satisfactory correlation. Since transfer occurs only at the inner surface, the use of D_1 in the Reynolds number appears reasonable, hence leaving the effect of D_2 solely in the correction factor. This will not be true if $D_2 - D_1$ is used in Re, and when D_1 becomes small, the use of $D_2 - D_1$ or even D_2 will give an impossible correlation. The j value varies as the 0.8 power of the Reynolds number, as expected, even though the Reynolds number is based on D_1. The powers of $c\mu/k$ and of μ/μ_w are assumed to be the same as those in previous correlations.

The use of $(\mu/\mu_w)^{0.14}$ in the heat-transfer correlation as introduced originally by Sieder and Tate (5) is strictly valid only for flow inside tubes; however, this group accounts for increased or decreased velocity gradients at a heated or cooled wall and should be satisfactory even if another wall is placed opposite it as in an annulus. The 0.14 power is valid in crossflow, according to Sieder (6). It does not seem probable that the presence of another wall should affect appreciably the velocity gradient at the heated or cooled wall. In the case of pressure drop where transfer occurs at two walls, μ_w should be calculated based on an areal weighted μ_w for the two walls. The magnitude of $(\mu/\mu_w)^{0.14}$ used in the data correlated by this paper is 1.1 maximum and has only a minor effect on the correlations.

The data of Mueller (1) for thin wires in annuli do not correlate quite as well on this plot as they did on his plot. However, Mueller has derived only a special equation for heat transfer, due to crossflow and parallel flow over vibrating wires, applicable over a limited range, which is of no use over the entire possible D_2/D_1 range. Moreover, his alternative curve does not vary with the 0.8 power of the Reynolds number which is a fairly well established fact in turbulent flow. On the present correlation, Mueller's data fall on both sides of the suggested line. Deviations from this line may easily be accounted for by the fact that wires down to the size of 0.000058 ft diam are used in ducts up to 1.717 ft. Under such conditions with gas flow up to 77 fps, the magnitude of a turbulent vortex and wire vibration will easily be greater than 1 wire diam, which Mueller has confirmed; hence producing some crossflow. The fact that these deviations are largest for the smallest of Mueller's wires lends credence to this supposition. Although the Mueller equation has been appreciably altered, it was of great help in determining the final Equation [1] of this paper.

Monrad and Pelton data (3) on a much larger wire correlate very well on the proposed line. Monrad and Pelton's data for

commercial sized annuli with water flowing also correlate satisfactorily on this line.

Zerban (7) has presented extensive data on five different sized annuli with air flowing. He obtained five different lines for these data and made no attempt to correlate on one line. When the data are correlated on the Reynolds number of Equation [1], they also fall quite well on the proposed line.

Extensive data have been published by Foust and Christian (4) on various sized annuli covering a small D_2/D_1 range. They propose modifying the Dittus-Boelter equation by a constant equal to 0.0322 D_2/D_1. They have obtained their water-side coefficient from the over-all coefficient by means of a semigraphical method. This semigraphical method results in a steam coefficient varying from 900 to 2600. The true steam coefficient has rarely been found to vary in the 900 to 1500 range, except in extreme cases, and in practice it is usually taken around 2500. For this reason, an average value of the steam coefficient of 3000 has been used to recalculate the water-side coefficient in the annuli. When this was done, the Foust and Christian (4) data were found to correlate exceedingly well on the proposed line. Use of a steam coefficient from 2000 to 4000 will not produce an error greater than 10 per cent and usually of 5 per cent in the recalculated rates. Concerning the proposed equation of Foust and Christian, it may be said that rates calculated from this equation

TABLE 1 HEAT-TRANSFER-DATA CHART

	Source and material	D_1	D_2	$\frac{D_1G}{\mu}$	V	$\left(\frac{D_2}{D_1}\right)^{0.15}$	j
1	Mueller	0.000058	0.151	2.6	7.4	3.34	0.163
2	Air	......	....	7.5	20.9	...	0.214
3		......	....	17.4	49.9	...	0.276
4		0.00025	....	11.0	7.4	2.62	0.344
5		......	....	31.3	20.8	...	0.474
6		......	....	76.0	49.9	...	0.825
7		......	....	116.5	77.5	...	1.00
8		......	0.315	11.3	7.5	2.92	0.365
9		......	....	31.5	20.8	...	0.548
10		......	1.717	11.2	7.7	3.77	0.376
11		......	....	30.7	21.2	...	0.520
12		0.00042	0.315	19.2	7.5	2.71	0.431
13		......	....	51.4	20.3	...	0.735
14	Monrad-	0.00521	0.0885	2110	72	1.53	15.1
15	Pelton	......	....	1790	61	...	13.7
16	Air	......	....	850	29	...	11.0
17		......	....	2100	72	...	14.0
18		......	....	1580	54	...	12.6
19		......	....	1160	39.6	...	10.5
20	Water	0.104	0.256	49000	1.81	1.144	180
21		......	....	13900	0.528	...	70
22		......	....	41500	1.58	...	150
23		......	....	9150	0.35	...	56.5
24		......	....	155000	4.31	...	370
25		......	....	33600	1.66	...	142
26	Outer wall	0.045	0.0843	7500	1.95	1.098	33.3
27		......	....	38400	10.2	...	120
28		......	....	15000	3.96	...	57.5
29	Inner wall	0.104	0.172	50000	...	1.078	183
30		......	....	86500	...	...	289
31		......	....	180000	...	...	525
32		......	....	29800	...	...	122
33	T.E.M.A.	0.0833	0.172	24200	50	1.115	74.0
34	Air	......	....	4850	10	...	18.0
35		......	....	48400	100	...	129
36	Water	......	....	100000	5.0	...	229
37		......	....	20000	1.0	...	62.5
38		......	....	10000	0.50	...	34.3
39	Zerban	0.0875	0.1035	32800	...	1.025	100
40	Air	......	....	54500	...	...	133
41		0.0875	0.1243	15500	...	1.054	56
42		......	....	11900	...	...	45.5
43		......	....	71400	...	...	180
44		0.0875	0.1453	23200	...	1.07	105
45		......	....	92000	...	...	334
46		......	....	15200	...	...	73.5
47		0.0875	0.1690	42800	...	1.104	151
48		0.0875	0.2390	35800	...	1.162	154
49	Foust-	0.052	0.0875	6700	0.758	1.08	46
50	Christian	......	....	13600	1.738	...	73.5
51	Water	......	....	21000	2.92	...	93.5
52		......	....	24900	3.52	...	106
53		0.0729	0.0875	25600	2.00	1.037	122
54		......	....	37300	3.02	...	145
55		......	....	47200	3.82	...	194
56		......	....	62500	5.23	...	210
57		0.0729	0.172	7550	0.761	1.138	55.0
58		......	....	17500	2.04	...	87.0
59		......	....	33500	4.65	...	152.0
60	Dufinecz	0.104	0.172	111000	...	1.078	353
61	Marcus	......	....	120000	...	...	385
62	Water	......	....	27000	...	...	131
63		......	....	4250	...	...	37.3

are exceedingly high even in the commercial range and would be totally absurd when extended down to wire sizes.

Data presented by Dufinecz and Marcus (8) also correlate well. All heat-transfer data from the foregoing references are given in Table 1.

If the customary hydraulic diameter is used in the Tubular Exchanger Manufacturers Association (T.E.M.A.) (9) chart 31 for flow inside tubes, a different rate will be predicted from that predicted by this equation for annuli. This is done by many persons without justification since T.E.M.A. does not claim applicability to annuli for its chart 31. Moreover, the deviation may be either positive or negative, depending upon the case selected. Actually, T.E.M.A. predicts higher rates when the inside of the annulus becomes small or when the spacing becomes comparatively small. As the size becomes larger or the spacing wider, the T.E.M.A. values become lower than those given by the new equation.

It must be noted that this correlation is for turbulent flow only, and the Reynolds number used is in no way intended to be a measure of the turbulent or laminar state of flow. This fact must be determined by the use of the conventional hydraulic diameter before applying the equation just referred to.

The equations presented here have been tested against most of the available data in the literature on heat transfer in annuli and found to give a satisfactory correlation over the entire range of D_2/D_1 tested from 1.18 to 6800. It is recommended that this equation be used hereafter for all annular calculations except those where extended surface exists. In the case of extended surface, where only a limited range of D_2/D_1 is used commercially, it still will be better to correlate experimental data on the hydraulic diameter and use the equation thus obtained, over the range only for which original data were obtained. The difference in hydraulic diameter, calculated on heated and wetted perimeter, is small in these cases, so it is immaterial on which basis D_e is computed as long as consistency is observed.

Heat Transfer at Inner Wall in Laminar Flow

Concerning heat transfer in the laminar-flow region, sufficient data are not at present available to afford any satisfactory correlation. However, the same general form of the equation will probably be

$$\frac{hD_a}{k} = \left(\frac{D_aG}{\mu}\right)^a \left(\frac{c\mu}{k}\right)^{+1/3} \left(\frac{\mu}{\mu_w}\right)^{+0.14} \left(\frac{D_b}{D_a}\right)^e \left(\frac{L}{D_e}\right)^f \ldots [2]$$

These powers will have to be determined through experiment, and at present experiments are being conducted to determine them. Conclusive and sufficient data are not available in the literature at present to determine the powers. Jakob and Rees (10) have developed this phase of heat transfer by theoretical means. Unfortunately, the functions involved are not very simple and but few data are presented which are not in good agreement with the derived equation. It is hoped that Equation [2] may be usable as a method with commercial applicability. Only test data will settle this point.

Heat Transfer at Outer Wall in Turbulent Flow

All the previous equations apply to heat transfer at the inner wall of the annuli, which is by far the most important commercial case. When heat transfer takes place at the outer surface, some modified equation will be necessary.

Actually, from the boundary condition that the heat-transfer equation must be that for bare tubes when D_1 approaches zero, and from the one set of data of Monrad and Pelton (3) on heating at the outer wall of annuli with D_2/D_1 = 1.85, it may be predicted that the equation will be

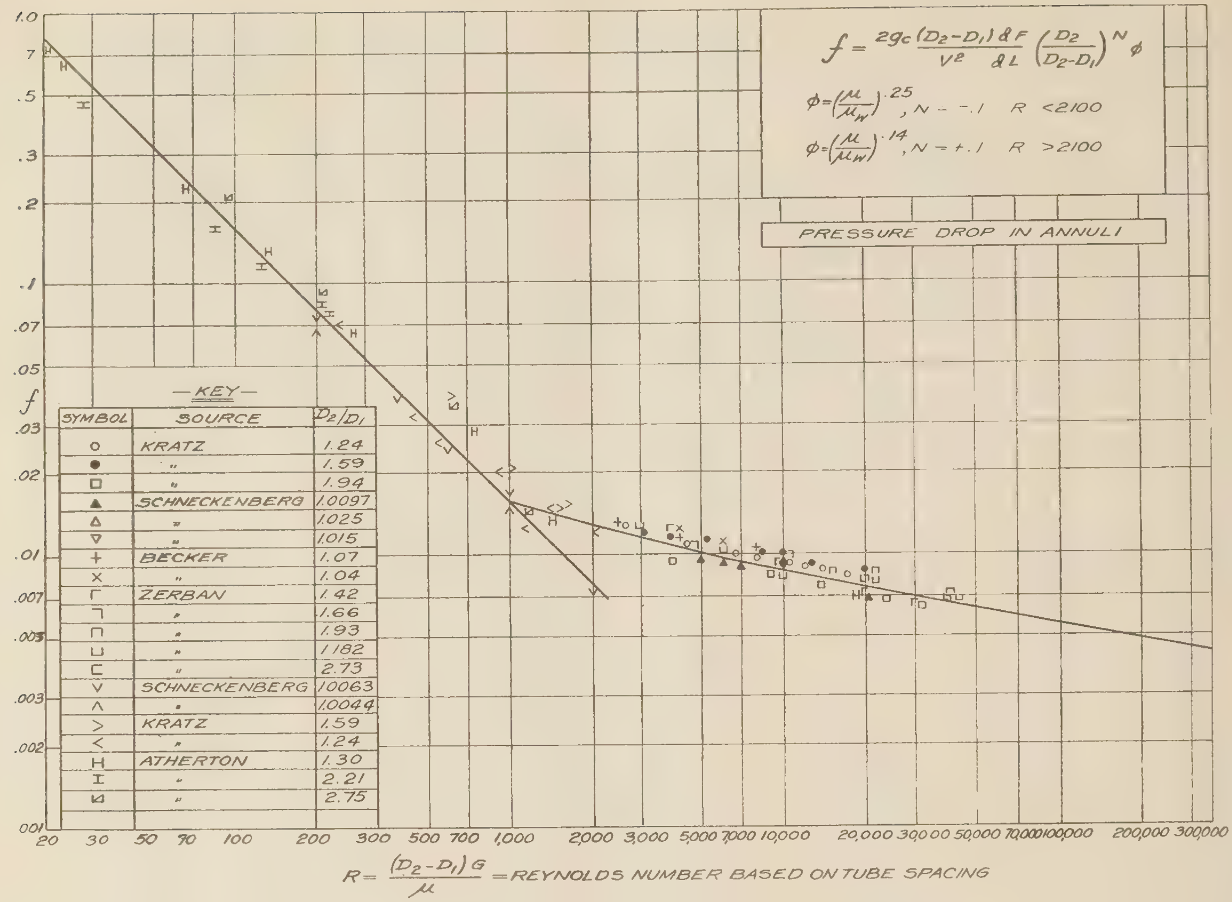

Fig. 2 Pressure Drop in Annuli

$$\frac{hS}{k} = 0.0225\left(\frac{SG}{\mu}\right)^{0.8}\left(\frac{c\mu}{k}\right)^{+1/3}\left(\frac{\mu}{\mu_w}\right)^{+0.14}\left(\frac{D_2}{D_2 - D_1}\right)^{-n} \ldots [3]$$

where n will lie between $+0.05$ and $+0.15$ approximately.

Experimental data for heat transfer in annuli at the outer surface are necessary to establish this form and the powers.

If heat transfer occurs at both walls, the rate at each wall may be calculated and the areal weighted-average rate used.

Pressure Drop

The general equation presented for pressure drop is

$$\frac{g_c S}{2\rho V^2}\cdot\phi\cdot\frac{dp}{dL} = \alpha\left(\frac{\mu}{SV\rho}\right)^f\left(\frac{D_2}{S}\right)^b \ldots\ldots\ldots\ldots [4]$$

where $f = +0.2$ and $b = -0.1$ in the turbulent region, and $f = +1$ and $b = +0.1$ in the viscous region. Use $\phi_1 = \left(\frac{\mu}{\mu_w}\right)^{0.14}$ in turbulent flow, and $\phi_2 = \left(\frac{\mu}{\mu_w}\right)^{0.25}$ in laminar flow.

This correlation, which is shown in Fig. 2, has been tested over the range of D_2/D_1 from 1.0005 to infinity where the possible range is from 1 to infinity. One of the limiting cases is the case where D_1 approaches D_2. This has been found to correlate satisfactorily using the data of Schneckenberg (11) on water. The data up to D_2/D_1 of 2.75 correlate well. Beyond this the correction factor becomes negligible and the equation becomes that of flow inside tubes at D_2/D_1 of infinity.

The data of Kratz, MacIntire, and Gould (12) for three different annuli have been correlated on this plot. Slight deviations from the mean line may be accounted for by the varying roughness of the different outside pipes used. The data of Zerban (7) for friction in smooth annuli have been correlated by taking into account the variance in smoothness as shown by the data he gives for the round pipes alone. By this means, his data are found to correlate well by the proposed equation.

The data of Atherton (13) on three different sized annuli do not appear to be reliable, inasmuch as for the same Reynolds number and the same pipe combinations, entirely different friction factors are found by the report for air, water, and oil. The data on oil in the viscous range seem to be the most reliable and correlate fairly well for all annular sizes on the proposed line. It is to be noted that these are isothermal data. The data of Becker (14) on friction of water in thin annuli also have been used in establishing this proposed equation. All friction data from the foregoing papers are given in Table 2.

As D_1 approaches zero, the proposed equation reduces to the equation for friction in round pipes and the proposed curve is the same as that generally accepted for flow inside round pipes. Hence the equation is valid whether the annulus has an inner core or not. This means that the slope of the proposed line is approximately -0.2 in the turbulent region and -1 in the viscous region when plotted against Reynolds number, which are the slopes of the curves for isothermal flow inside pipes.

TABLE 2 FRICTION-DATA CHART

	Source and material	$\frac{SG}{\mu}$	$\frac{D_2}{D_2-D_1}$	$\frac{D_2}{D_1}$	$\left(\frac{D_2}{D_2-D_1}\right)^{0.1}$	f
1	Schneckenberg	10000	40	1.025	1.43	0.0097
2	Water	....	66	1.015	1.52	0.00915
3	Isotherma	4000–6000	103	1.0097	1.59	0.00955
4	Kratz, et al	2692	5.13	124	1.176	0.0128
5	Isothermal	4520	..	....	...	0.0109
6		6848	..	....	...	0.0099
7		8240	..	....	...	0.00965
8		10640	..	....	...	0.00907
9		12400	..	....	...	0.00895
10		14000	..	....	...	0.00876
11		17520	..	....	...	0.00805
12		3890	2.69	1.59	1.103	0.01165
13		5350	..	....	...	0.0113
14		8450	..	....	...	0.00995
15		10200	..	....	...	0.00982
16		12600	..	....	...	0.00915
17		20500	..	....	...	0.00850
18		3100	..	....	...	0.0119
19		9160	2.05	1.94	1.074	0.00895
20		13360	..	....	...	0.00773
21		20920	..	....	...	0.00705
22		23360	..	....	...	0.00688
23		31300	..	....	...	0.00650
24		3830	..	....	...	0.00955
25	Becker	4000	14.8	1.073	1.309	0.0115
26	Water	8000	..	....	...	0.01085
27	Isothermal	2500	..	....	...	0.01309
28		4000	25.8	1.04	1.384	0.0116
29		6000	..	....	...	0.0112
30	Zerban	10000	3.38	1.42	1.129	0.00915
31	Air	20000	..	....	...	0.00740
32	Heating	3800	..	....	...	0.0124
33		30000	..	....	...	0.00692
34		10000	2.51	1.660	1.096	0.00965
35		40000	..	....	...	0.00700
36		4800	..	....	...	0.01096
37		20000	..	....	...	0.00830
38		15000	2.08	1.93	1.075	0.008[illegible]5
39		20000	..	....	...	0.00862
40		40000	..	....	...	0.00728
41		10000	6.50	1.182	1.206	0.00885
42		20000	..	....	...	0.00710
43		3000	..	....	...	0.01235
44		6000	..	....	...	0.0103
45		20000	1.48	2.73	1.04	0.00790
46		40000	..	....	...	0.0068
47	Schneckenberg	2000	160	1.0063	1.66	0.00723
48	Water	1000	..	....	...	0.01660
49	Isothermal	600	..	....	...	0.0241
50		200	..	....	...	0.0725
51	Schneckenberg	400	160	1.0063	1.66	0.0386
52	Water	200	228	1.0044	1.71	0.0648
53	Isothermal	1000	..	....	...	0.0141
54	Kratz, et al	644	2.69	1.59	1.103	0.0392
55	Water	1520	..	....	...	0.0147
56	Isothermal	1033	..	....	...	0.0217
57		1680	..	....	...	0.0155
58		243	5.13	1.24	1.175	0.0693
59		464	..	....	...	0.0325
60		588	..	....	...	0.0254
61		915	..	....	...	0.0159
62		2000	..	....	...	0.0120
63		1460	..	....	...	0.0150
64		1160	..	....	...	0.01280
65	Atherton	24.3	5.22	1.30	1.175	0.688
66	Oil	20.2	..	....	...	0.770
67	Isothermal	69.5	..	....	...	0.235
68		132	..	....	...	0.130
69		274	..	....	...	0.066
70		764	..	....	...	0.0296
71		1415	..	....	...	0.0138
72		218	1.83	2.21	1.062	0.0765
73		200	..	....	...	0.0807
74		123	..	....	...	0.116
75		27	..	....	...	0.45
76		82	..	....	...	0.155
77		211	1.57	2.75	1.046	0.091
78		96	..	....	...	0.210
79		1172	..	....	...	0.0145
80		607	..	....	...	0.0389

Wiegand and Baker give a plot[3] of a theoretical diameter-ratio function to apply to problems of transfer in annuli. The proposed function drops from 1.5 to 1, over the D_1/D_2 range from 0.85 to 1, which is in contradiction to much of the data of Becker (14) and Schneckenberg (11). Inasmuch as Equation [4] satisfactorily correlates their data, it appears desirable to use it rather than the theoretical equation.

It appears that the friction equations obtained for both turbulent and viscous flow are highly satisfactory for annuli from D_2/D_1 of 1.0044 to infinity and afford much better correlations than can be obtained by any other equation available.

[3] Bibliography (2), Fig. 14.

General Conclusions

1 Heat transfer at the inner wall of annuli is correlated by a new equation in the turbulent region for D_2/D_1 from 1.18 to 6800, the use of which is recommended for the solution of all problems in this range.

2 A tentative equation for heat transfer at the outer wall of annuli in the turbulent region is proposed. However, to establish it, further tests are required.

3 Heat transfer in the viscous region is discussed, and the type of equation to use in developing a correlation from future data is outlined. Tests are now being conducted with this in view.

4 Pressure drop in annuli is correlated by a new equation for both turbulent and viscous flow for the D_2/D_1 range of 1.0044 to infinity, and it is recommended that it be used for all cases within the range.

Acknowledgment

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Discussion

MAX JAKOB.[4] The author has correlated the available experimental data under turbulence conditions by an empirical formula of the type created by Sieder and Tate for tubes. He recommends using the same pattern for laminar flow in an annulus and this may work as satisfactorily as it does for tubes. With five dimensionless groups as dependent variables and one empirical constant factor good results are to be expected, and should the free convection play a part then the Grashof number will have to be used as a sixth dependent variable.

Referring to the paper of Jakob and Rees,[5] the author might have mentioned that this does not deal with heat transfer at uniform wall temperature over the entire length, but with uniform heating or cooling. Theoretically, this is a much simpler case and so is the solution. We succeeded in representing the Nusselt number as a unique function of the ratio of the radii r_1/r_2; in other words, by one single independent variable compared with four suggested by the author, omitting the viscosity ratio which we did not need for gases and small temperature differences, and which the author would not need either in this case. It must be further mentioned that our equations do not contain any empirical factors.

In this way we are at the mercy of what the outcome of the theory may be, whereas the author's method allows an adjustment by the choice of one empirical factor and four exponents of his dependables. So it is not surprising that our experimental results of the Nusselt number, obtained under unfavorable conditions, as explained in Part II of the writer's paper,[5] deviate considerably (as an average +13 per cent) from the unique value required by the theory. Until more experimental data covering the cases of uniform heating and cooling become available the writer recommends the use of the theoretical formulas and their simple graphical representation as given in the Jakob and Rees paper.[5]

[Mr. Davis, the author of the foregoing paper, died on June 30, 1943.—EDITOR.]

[4] Research Professor of Mechanical Engineering, Illinois Institute of Technology and Armour Research Foundation, Chicago, Ill., Mem. A.S.M.E.

[5] Author's Bibliography (10).

Creep of Metals at Elevated Temperatures—the Hyperbolic-Sine Relation Between Stress and Creep Rate

By P. G. McVetty,[1] East Pittsburgh, Pa.

This paper recommends further use of the hyperbolic sine to express the relation between stress and creep rate at constant temperature. Evidence is submitted to show that the commonly used straight-line relation in the log-log plot becomes unreliable when extrapolated over a considerable range into the field of low stresses. Reference is made to theoretical considerations in favor of the hyperbolic sine and to tests on a wide variety of materials which appear to support its use. For specific examples, comparative curves are shown to illustrate application to creep data representing a wrought 18 Cr-8 Ni alloy tested at 1500 F. The two methods of extrapolation are compared with special reference to their reliability for selection of safe working stresses. To stimulate application of the hyperbolic-sine relation to practical problems, a graphical solution is proposed, and a chart is given to facilitate determination of the required constants from creep tests at two or more stresses at the same temperature.

Introduction

While it may appear at first sight to have a very limited significance, the most effective use of available creep-testing equipment depends largely upon our knowledge of the relation between stress and creep rate at a constant elevated temperature. Accelerated tests of various kinds have their place in the preliminary sorting of heat-resisting alloys, but the final appraisal is usually based on the long-time creep test. Any form of analysis, correlation, and interpretation which reduces the duration and requisite number of such tests, releases equipment for study of new alloys and contributes thereby to the war effort. The fundamental basis for such analyses lies in close study of known theories and their application to existing test data. The practical significance of these preliminary studies appears in the formulation of effective testing programs and in reliable appraisal of materials and choice of safe working stresses.

Factors Affecting Stress - Creep Rate Relations

The use of the creep rate as a measure of creep resistance is quite general in applications in which maximum allowable deformation is the criterion of failure. It is obvious that a rapid change of length with respect to time under the combined effects of stress and temperature is undesirable, and poor heat resistance is indicated. Similarly, a slow length change or low creep rate is indicative of good heat resistance. On this basis, there is no difficulty in establishing the superiority of certain alloy steels over plain carbon steel at high temperatures. When, however, the difference is less marked, a fair comparison requires detailed consideration of the past history of the materials, the conditions surrounding the tests, and all that is known about the action of similar materials, and laws and hypotheses derived therefrom. With this background, appraisals, comparisons, and choice of working stresses may be based upon logical analysis; without it, conclusions based on creep tests may be misleading and often dangerous.

A detailed discussion of the variables affecting creep tests is beyond the scope of this paper; only a few of the contributing factors may be mentioned here. One of the earliest concepts of creep as a balance between the opposing forces of strain hardening and annealing is indicated in Bailey's definition (1)[2] of creep as "the outward manifestation of the balance in the destruction of strain hardening by thermal influence and its recreation by further slip." The use of the creep rate as a measure of the integrated effects of these two opposing forces is applicable to an annealed plain carbon steel. It is less reliable as a measure of heat resistance in the case of other heat-treatments, prior cold work, or when other alloying elements are present in appreciable amounts.

Another concept due to Jeffries and Archer (2) is that of the "equicohesive temperature." This would establish a temperature boundary above which flow is confined to material between the crystals; below this temperature, flow is assumed to occur within the crystals by slipping along crystallographic planes. It is obvious that the possibility of such a boundary condition may introduce a transition from one law of flow to another with changes in temperature. Subsequent studies indicate that the transition is gradual over a considerable range of temperature. Care must be used in estimating temperature effects because tests have shown 100 per cent increase in creep rate for a temperature rise of as little as 6 deg F. This paper is confined to a discussion of the relation between stress and creep rate at a constant elevated temperature.

One of the earliest presentations of stress - creep rate relations is that of Norton (3). Test data for a number of different alloys at several temperatures indicated a relation of the form

$$v = v_1 \left(\frac{\sigma}{\sigma_1}\right)^n \qquad [1]$$

in which v = creep rate

σ = applied stress

v_1 and n are constants dependent upon the material and the temperature, and σ_1 is an arbitrary constant with the dimensions of a stress.

The value of this relation is proved by its wide use for many years. It was considered the best method of plotting creep data at the time of publication of the 1938 Creep Data Book (4). At that time, considerable evidence had been collected which indicated that the power-function relation might not be satisfactory

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Contributed by the Research Committee and Power Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1943, of The American Society of Mechanical Engineers.

[2] Numbers in parentheses refer to the List of Selected References at the end of the paper.

at lower stresses and temperatures. The problem of distinguishing between two functions by means of test data was complicated by the large effect upon creep rate of small temperature variations. Even in tests at the same nominal or average temperature, slight differences in cyclic temperature variations have a marked effect upon creep when such effects are integrated over long periods of time (5, 6). Tests at low stresses and temperatures and correspondingly low creep rates are slow in producing the data necessary to establish functional relations over a wide range of creep rates.

It is fortunate that the committee, which compiled the 1938 Creep Data Book (4), recognized the possibility of other methods of analysis. In addition to the log-log plot of stress - creep rate relations, all essential fundamental data are recorded also. This makes it possible to try other functional relations and to study the initial stage of creep. (The latter problem is also of great importance but it must be left for discussion at a later time.) This book is an ideal source of data for study of functional relations, since it represents co-operative investigations in which sixteen different laboratories participated. This permits conclusions wider in scope than would be possible from studies of data from a single laboratory.

A rather hurried survey of the Creep Data Book (4) indicates a total of 364 curves plotted, of which 201 are straight lines in the log-log plot, while 163 show a downward trend at low stresses. Further examination shows that only 78 of these straight lines pass through three or more points, while 123, which pass through one or two points, may be considered indeterminate in any attempt to establish functional relations. This survey indicates then that 34 per cent of the 364 curves cannot be used for this purpose; 78 or 21 per cent favor the power function; while 163 or 45 per cent favor some function which leads to more conservative stress estimates in the low-stress field than those obtained by extrapolation of a power function.

The Hyperbolic-Sine Relation Between Stress and Creep Rate

For many years, Dr. Nadai has objected to careless use of the power function to express the relation between stress and creep rate. This objection is based upon theoretical grounds in addition to test data. A summary of his views on this subject was published (7) soon after the Creep Data Book (4) and discussed further in a subsequent paper (8). The hyperbolic-sine relation, suggested by Dr. Nadai, receives strong support from the logarithmic speed law of Ludwik (9), based upon tests of tin wires, the lattice-structure studies of Prandtl (10), and the chemical-rate theory of Eyring (11). Further supporting evidence based upon tests of single crystals of tin is found in the work of Chalmers (12), and upon nonmetallic materials, such as celluloid, in the work of Mussmann (13). Investigations at the University of Illinois (14) on lead and lead alloys indicate that the hyperbolic-sine relation is applicable.

It has been found, in many cases, that theories derived from studies of tin, lead, and similar materials can be applied to heat-resisting alloys at high temperatures. This concept of "homologous temperatures" proposed by Ludwik has been discussed by Nadai (15). The hyperbolic-sine relation has been applied by the author to a wide variety of data with good evidence of agreement. It avoids the objectionable characteristics of exponential and power functions at low stresses while retaining desirable exponential characteristics at intermediate stresses. It is not recommended for the very high strain rates associated with impact tests, but this is not considered a serious practical objection to its use.

It is not the intention of this paper to discard the power function as a useful relation between stress and creep rate. Reference has been made to the fact that 21 per cent of the 364 curves in the Creep Data Book (4) follow such a relation. It has been shown by Dr. Nadai (7) that the power function defines the tangent to the hyperbolic-sine curve and that, under certain conditions, the tangent is a close approximation to the curve itself. One would expect therefore that either function would fit experimental data over a limited range of stress and creep rate. Under these conditions, the power function is extremely useful in the mathematical treatment of combined stresses and other problems. It is its use for extrapolation to low stresses which leads to erroneous and possibly dangerous conclusions.

Determination of Constants

The hyperbolic-sine relation was expressed by Dr. Nadai as follows:

$$v = v_0 \sinh\left(\frac{\sigma}{\sigma_0}\right) \qquad [2]$$

in which

v = creep rate

σ = applied stress

v_0 and σ_0 = constants dependent upon the material and the temperature.

The problem involved in the application of this relation is one of determining the constants v_0 and σ_0 having given two points, v_1, σ_1 and v_2, σ_2 obtained from creep tests at two different stresses.

To those who had used the straight-line log-log plot for many years, a change to the hyperbolic-sine relation presented considerable difficulty. Any interpolation between stresses or extrapolation to lower stresses required determination of two constants. An approximate method proposed by Dr. Nadai (7) worked very well if the known test points fell upon that part of the curve which was essentially the same as an exponential. If part or all of the test data were in the low-stress region, the assumed exponential relation did not fit the experimental data. Attempts to solve the transcendental equations by trial-and-error methods were quite laborious.

A considerable simplification of the problem is due to R. A. Bice who, in 1941, proposed a graphical solution for the hyperbolic-sine constants when two sets of observations of stress and creep rate are known. This simplification is developed easily from Equation [2] by substitution of the known values of v_1, σ_1 and v_2, σ_2.

$$v_2 = v_0 \sinh\left(\frac{\sigma_2}{\sigma_0}\right) \qquad [3]$$

$$v_1 = v_0 \sinh\left(\frac{\sigma_1}{\sigma_0}\right) \qquad [4]$$

Dividing Equation [3] by [4], we have

$$\frac{v_2}{v_1} = \frac{\sinh\left(\dfrac{\sigma_2}{\sigma_0}\right)}{\sinh\left(\dfrac{\sigma_1}{\sigma_0}\right)} = \frac{\sinh\left(\dfrac{\sigma_1}{\sigma_0} \times \dfrac{\sigma_2}{\sigma_1}\right)}{\sinh\left(\dfrac{\sigma_1}{\sigma_0}\right)} \qquad [5]$$

For convenience, we may replace the known ratios $\frac{\sigma_2}{\sigma_1}$ by m and $\frac{v_2}{v_1}$ by n and the unknown ratio $\frac{\sigma_1}{\sigma_0}$ by x.

We have then

$$n = \frac{\sinh mx}{\sinh x} \qquad [6]$$

in which the only unknown to be found is x.

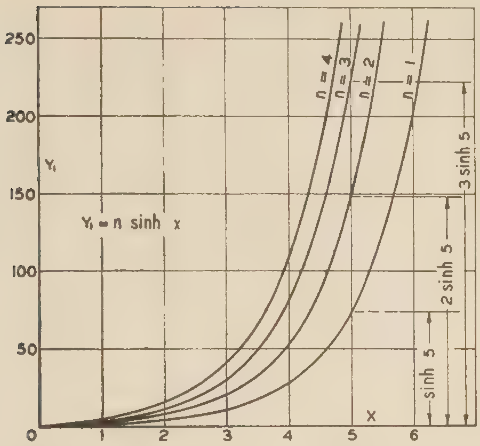

FIG. 1 EFFECT OF CHANGE OF n ON HYPERBOLIC-SINE CURVES

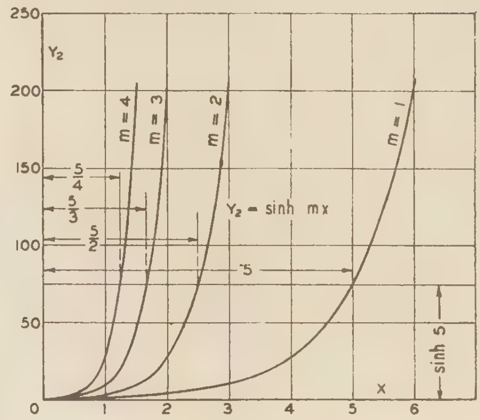

FIG. 2 EFFECT OF CHANGE OF m ON HYPERBOLIC-SINE CURVES

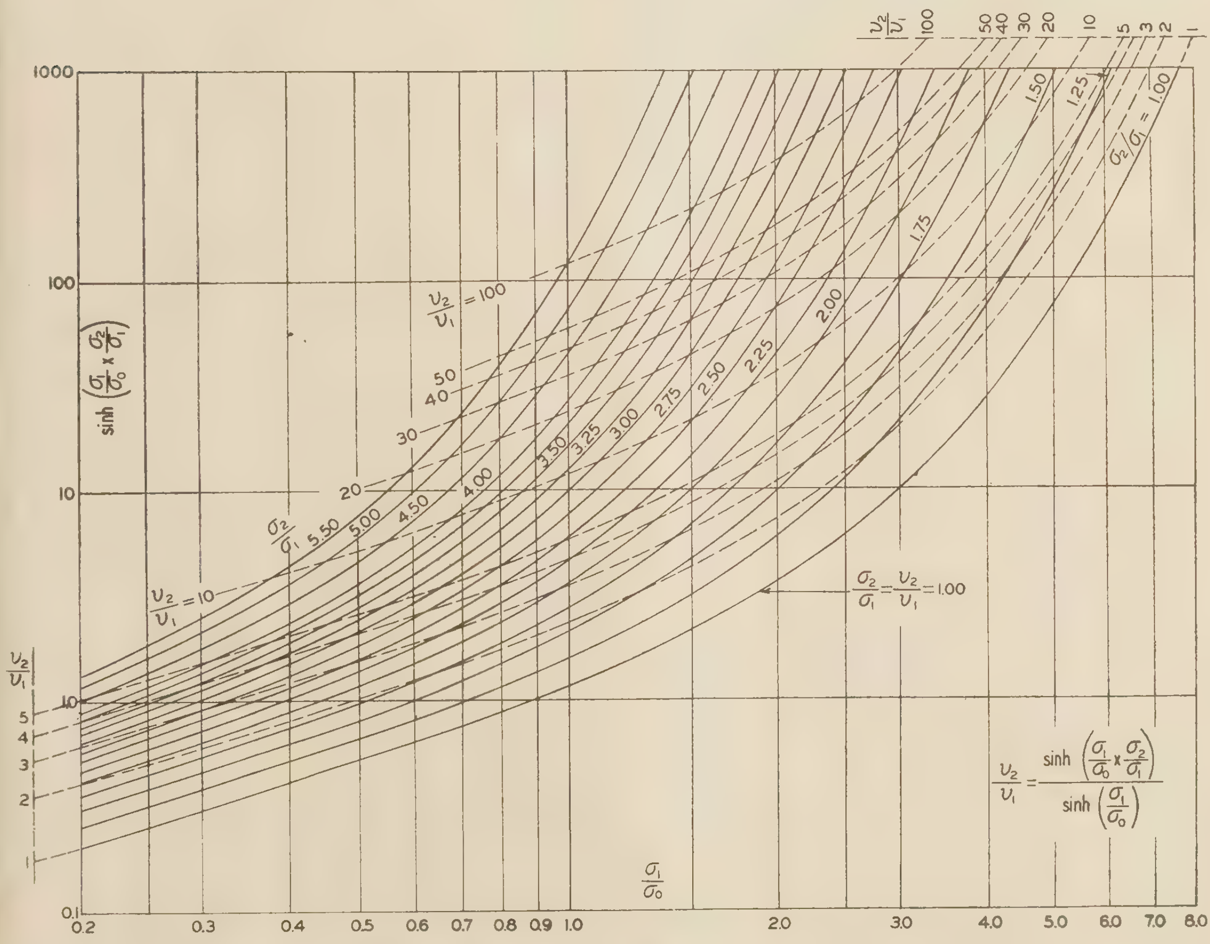

FIG. 3 CHART FOR DETERMINATION OF CONSTANT σ_0 IN HYPERBOLIC-SINE EQUATION

(Hyperbolic-sine equation, $v = v_0 \sinh\left(\frac{\sigma}{\sigma_0}\right)$; when v_1 σ_1 and v_2 σ_2 are known from creep at two stresses at the same temperature.)

To explain the graphical solution, we may set up two sets of functions

$$y_1 = n \sinh x \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

and

$$y_2 = \sinh mx \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

In Equation [7], $n = \frac{v_2}{v_1}$ is a parameter representing only a change in the vertical scale. This is represented in Fig. 1. Similarly in Equation [8], $m = \frac{\sigma_2}{\sigma_1}$ is a parameter representing only a change in the horizontal scale. This is represented in Fig. 2.

The desired value of x is found from the intersection of the particular curve in Fig. 1, in which $n =$ the known $\frac{v_2}{v_1}$ ratio with the curve in Fig. 2, in which $m =$ the corresponding known $\frac{\sigma_2}{\sigma_1}$ ratio.

Fig. 3 represents a combined plot of these two sets of functions so that the value of $x = \frac{\sigma_1}{\sigma_0}$ corresponding to the intersection of the

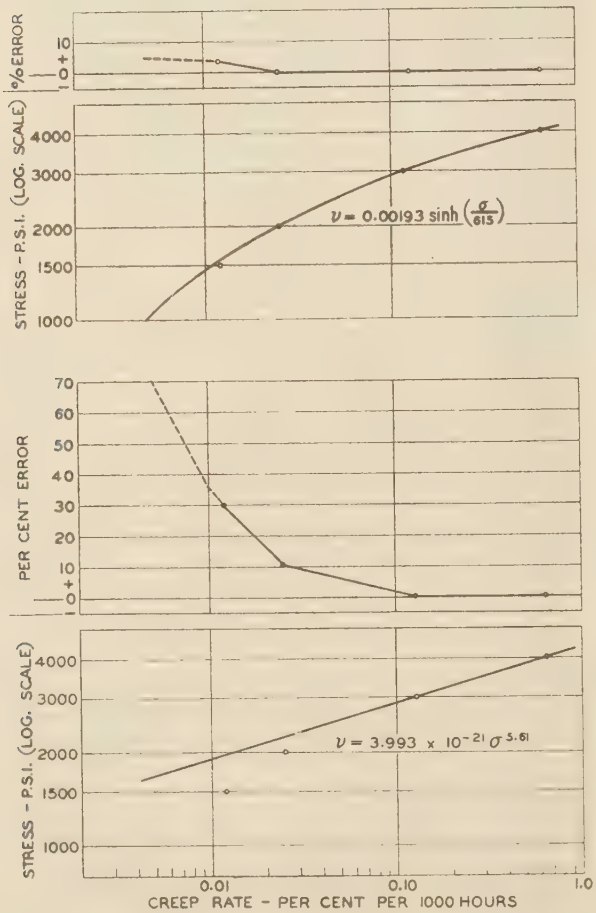

Fig. 4 Computed Errors for Hyperbolic-Sine and Power Functions, Assuming Test Data at 3000 and 4000 Psi Only Are Known

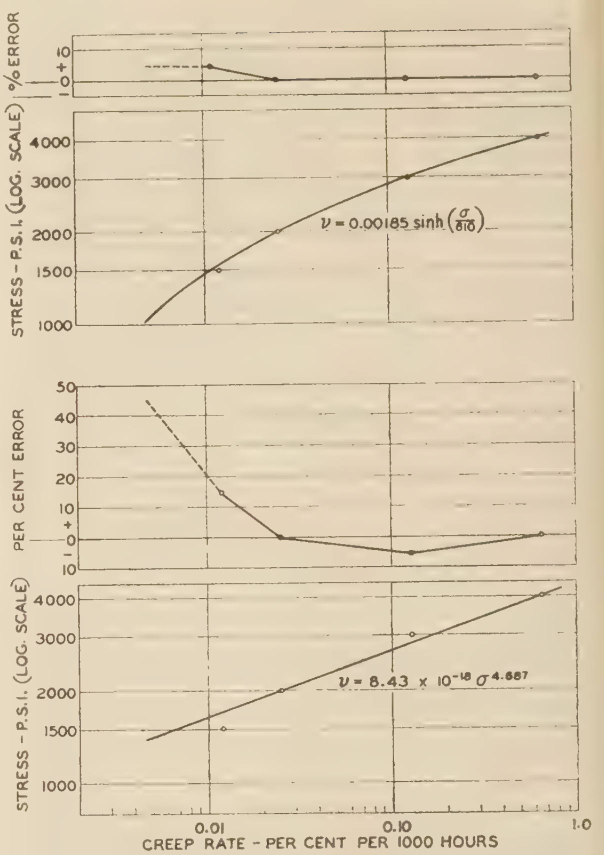

Fig. 5 Computed Errors for Hyperbolic-Sine and Power Functions, Assuming Test Data at 2000 and 4000 Psi Only Are Known

$n = \frac{v_2}{v_1}$ curve with the $m = \frac{\sigma_2}{\sigma_1}$ curve may be read conveniently from the scale of abscissas. As soon as $x = \frac{\sigma_1}{\sigma_0}$ is known, the value of σ_0 follows since σ_1 is known. Substitution of σ_0 in Equations [3] and [4] gives two values of v_0 which may be averaged. If several sets of observations are available, stress and creep-rate ratios may be taken in pairs and the resulting constants averaged to give the best values. Other methods are possible if time permits their use.

The charts proposed by Mr. Bice used $\frac{v_1}{v_2}$ as ordinates and $\frac{\sigma_1}{\sigma_0}$ as abscissas, both plotted to Cartesian co-ordinates. The stress ratio $\frac{\sigma_1}{\sigma_2}$ was used as a parameter. The shape of the curves and their relative positions led to considerable difficulty in interpolating actual stress ratios, and it is proposed to rearrange the chart as shown in Fig. 3. It will be noted that the quantity $\frac{\sigma_1}{\sigma_0}$ is plotted as abscissas to a logarithmic scale. When $\frac{\sigma_2}{\sigma_1} = \frac{v_2}{v_1} = 1$, the lowest curve represents as ordinates the hyperbolic sine of the quantity $\frac{\sigma_1}{\sigma_0}$ also plotted to a logarithmic scale. Using $\frac{\sigma_2}{\sigma_1}$ as a parameter, any desired number of stress ratios may be plotted. Reference to Equation [5] shows that any known $\frac{v_2}{v_1}$ ratio is

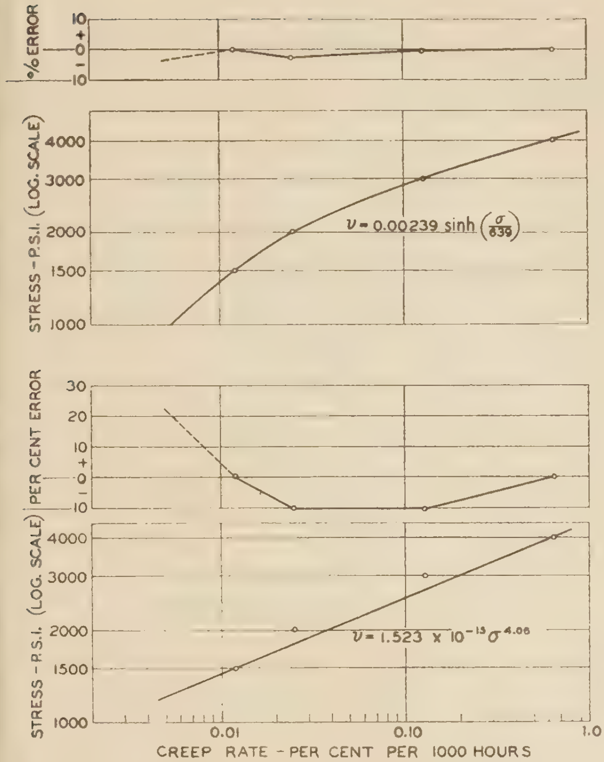

FIG. 6 COMPUTED ERRORS FOR HYPERBOLIC-SINE AND POWER FUNCTIONS, ASSUMING TEST DATA AT 1500 AND 4000 PSI ONLY ARE KNOWN

represented as a vertical distance from the curve for which $\frac{\sigma_2}{\sigma_1}$ = the corresponding stress ratio to the curve for which $\frac{\sigma_2}{\sigma_1} = 1.00$. Several values of $\frac{v_2}{v_1}$ have been plotted as another parameter. Reading vertically down from the intersection of corresponding $\frac{v_2}{v_1}$ and $\frac{\sigma_2}{\sigma_1}$ curves gives the $\frac{\sigma_1}{\sigma_0}$ ratio from which σ_0 is obtained easily. Since it is difficult to interpolate accurately between $\frac{\sigma_2}{\sigma_1}$ curves, it is desirable to use as many pairs of points as possible and substitute back in Equations [3] and [4] for the best value of v_0.

The usual method of selecting test stresses leads to a wide variety of stress ratios and complicates either the chart or the problem of interpolation. A valuable suggestion has been made by F. T. Hague. If the A.S.A. Z17.1-1936 American Standard Preferred Numbers system is used in assigning test stresses, the number of possible stress ratios can be reduced materially. For best results, it would be desirable to use exact computed values of stress instead of the rounded values normally listed as preferred numbers.

APPLICATION TO CREEP-DATA BOOK

It is of interest to illustrate the application of the hyperbolic-sine relation to actual creep data and compare it with the commonly used power function. For this purpose, it is convenient to use data for a wrought 18 per cent chromium 8 per cent nickel steel tested at 1500 F. The log-log plot[3] of stress versus creep rate and the basic creep data[4] are given in the Creep Data Book. From this source, we obtain the following experimental data:

Stress, psi	Creep rate, per cent per 1000 hr
1500	0.012
2000	0.025
3000	0.128
4000	0.644

Since creep tests are slow and expensive, it is always desirable to obtain as much information as possible from a limited number of tests. Under present war conditions, the possibility of check tests is curtailed and analyses are based frequently upon tests at only two stresses at each temperature. If we consider any two of the four available points as known and apply both the power and hyperbolic-sine functions to them, it is possible to measure the errors at each of the other two points. From four tests, six pairs of points may be used. Figs. 4, 5, 6, 7, 8, and 9 illustrate this application. In each case, two of the known points are used to determine the hyperbolic sine and the power function passing through them. The other two stresses are then computed and compared with the known values. The error curve above each of the assumed functions shows the percentage error involved in estimating the stress to produce the two other known creep rates. In all cases shown here, the error in the hyperbolic-sine assumption is less than 5 per cent, while that of the power function reaches 78 per cent.

It is recognized that the measured creep rates may be incorrect for various reasons, chief among which are the effect of small temperature variations and lack of perfect uniformity among test specimens. The data here used have been chosen as repre-

[3] List of Selected References (4), p. 207.

[4] Bibliography (4), p. 217.

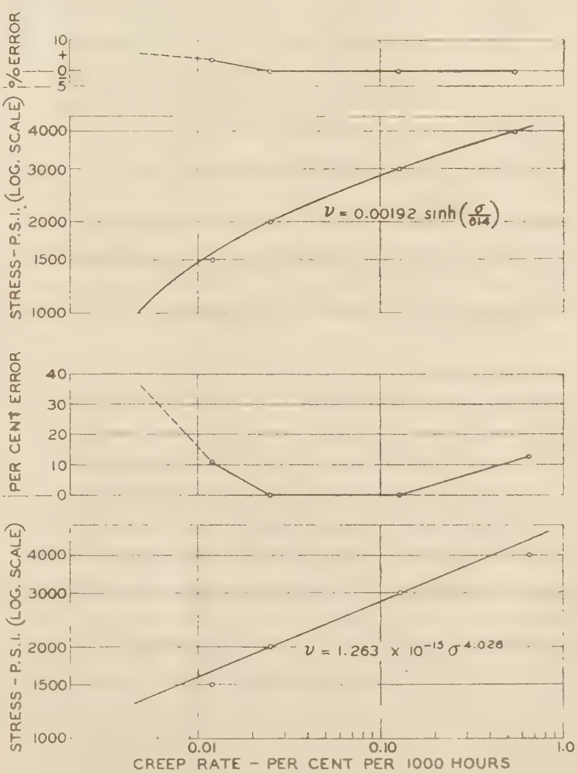

FIG. 7 COMPUTED ERRORS FOR HYPERBOLIC-SINE AND POWER FUNCTIONS, ASSUMING TEST DATA AT 2000 AND 3000 PSI ONLY ARE KNOWN

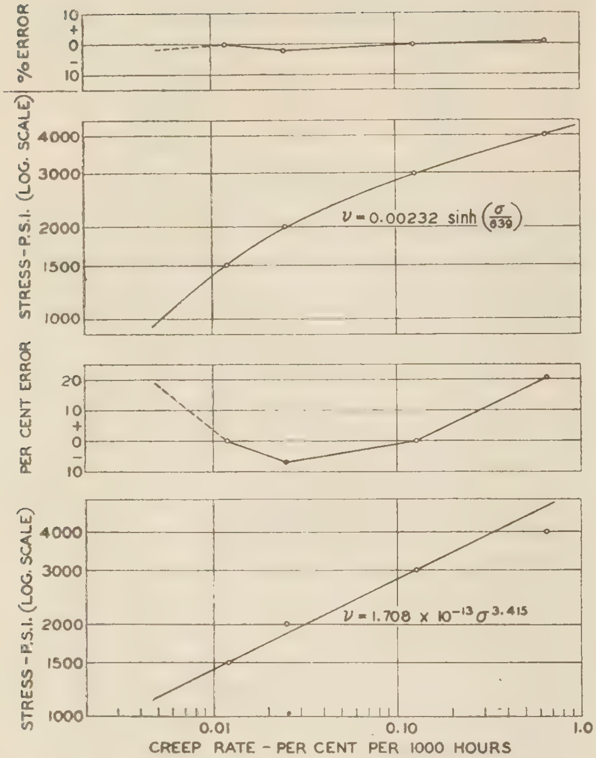

FIG. 8 COMPUTED ERRORS FOR HYPERBOLIC-SINE AND POWER FUNCTIONS, ASSUMING TEST DATA AT 1500 AND 3000 PSI ONLY ARE KNOWN

sentative of a large number of cases to which this comparison has been applied.

To illustrate the application to more extensive test data, Fig. 10 shows the point-to-point straight-line plot of the Creep Data Book,[8] compared with the hyperbolic-sine curve in which four tests are used to determine the constants. The best extrapolation of the power function to lower stresses would use the 1500- and 2000-psi points without any help from the tests at the two higher stresses. Extrapolating to 0.005 per cent per 1000 hr and assuming that the hyperbolic-sine assumption is correct, the power function leads to an estimate 8 per cent high. Using other pairs of points, the errors of the power-function assumption at 0.005 per cent per 1000-hr creep rate reach 71 per cent, as shown in the dotted error curves of Figs. 4, 5, 6, 7, 8, and 9. The corresponding errors in the hyperbolic-sine assumption do not exceed 5 per cent.

Another example may be taken from Fig. 4, which might represent a steam-turbine application. In this case, the maximum allowable creep rate is often taken as 0.1 per cent in 100,000 hr. For the purpose of this example, the error due to neglect of the initial stage of creep may be disregarded, and we may assume that 0.001 per cent per 1000 hr is equivalent to 0.1 per cent per 100,000 hr. Extrapolation by means of the power function leads to an estimate of 1260 psi as the stress which would produce this creep rate. Application of the hyperbolic-sine assumption to the same data leads to an estimate of 306 psi. If we assume that the hyperbolic-sine curve of Fig. 10 is the best basis for extrapolation, the estimated value of maximum safe stress is 278 psi. The error introduced by basing estimates for a steam-turbine application upon only two relatively high-stress tests would be 10 per cent for the hyperbolic-sine assumption and 340 per cent for the commonly used straight-line extrapolation in the log-log plot. An error of 10 per cent under these unfavorable conditions would not be considered excessive, but an estimate 340 per cent high would undoubtedly lead to failure in service.

CONCLUSIONS

While space limitations permit the inclusion of only a few examples, it is felt that these data are representative of an important type of practical applications. Further study may show that the hyperbolic-sine relation is limited to certain materials or to definite stress and temperature ranges. It is obvious that some care must be used in its application when test data at only two stresses are available. It is not intended as a replacement for the power function in all cases. The author considers it the best means of interpolation and extrapolation known today for stress variation at constant temperature in the field of low stresses and creep rates. The chart shown in Fig. 3 is offered primarily to simplify its application to the large mass of creep data now being collected, as well as the correlation of existing data. If this simplification stimulates study of relations between stress and creep rate other than the commonly used power function, this paper will have served its purpose.

ACKNOWLEDGMENT

The author wishes to acknowledge his indebtedness to Dr. A.

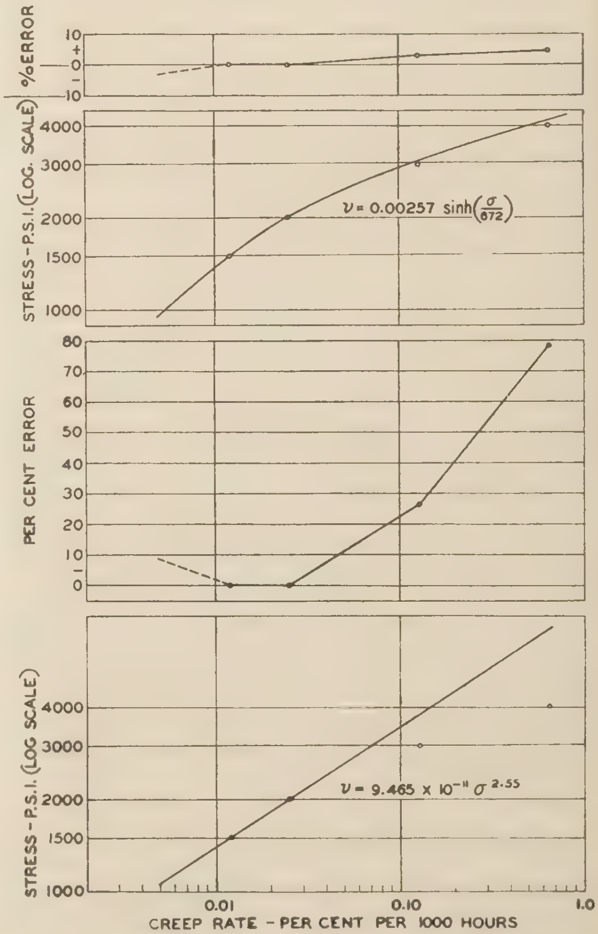

FIG. 9 COMPUTED ERRORS FOR HYPERBOLIC-SINE AND POWER FUNCTIONS, ASSUMING TEST DATA AT 1500 AND 2000 PSI ONLY ARE KNOWN

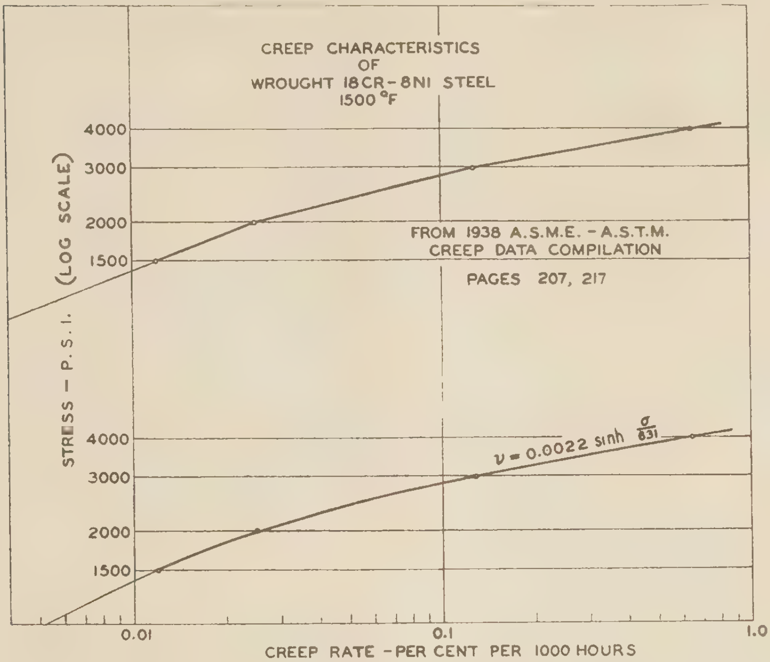

Fig. 10 Comparison of Point-to-Point Plot of Creep Data Book With Hyperbolic-Sine Curve in Which Four Tests Are Used to Determine Constants

Nadai for the helpful suggestions and criticism which have stimulated the application of various functional relations to practical problems and which, finally, led to the preparation of this paper.

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Discussion

K. Heindlhofer.[5] The author introduces his paper with emphasis upon "analysis, correlation, and interpretation" of creep data. He points out that the knowledge of the relation between creep variables obtained by analysis makes existing testing equipment more effective.

To this it may be added that planned experimental study accompanied by a sound theory, such as the modern chemical-rate theory, is bound to advance this field more effectively than many of the unco-ordinated tests made in the past. A chemical-rate theory, successful in many fields, was advanced by H. Eyring. It expresses rate (creep) in terms of temperature and pressure (stress). This theory leads to the conclusion that the variation of creep with stress is represented by a hyperbolic sine, which is in agreement with L. Prandtl's deduction arrived at earlier by independent considerations. This hyperbolic-sine law was first proposed by A. Nadai as an aid in co-ordinating creep data. Because of the fact that the hyperbolic-sine relationship rests upon a rational foundation, it should be given preference over the empirical functions which have no organic relation to the problem. Empirical functions are thus not entitled to the same high rating as the relation derived from well-founded axioms.

It is thus reassuring that the author's investigation supports this view, in favoring the hyperbolic-sine law. Nevertheless an important element in the applicability of the rate theory still remains to be investigated. This important element is the absolute

[5] Research Laboratory, United States Steel Corporation, Kearny, N. J.

value of the constants which appear in the rate function. Unless it is possible to determine these constants by means of measurements other than creep, the value of the hyperbolic function, derived from the rate theory is only slightly superior to the purely empirical functions such as the power function or parabola.

A concrete example will clarify this statement. Let us plot observed creep data, Fig. 11 of this discussion, in ordinary co-ordinates, rate being plotted against stress. The four observations, based upon tests of record duration (100,000 hr) by E. L. Robinson[6] may be used in this plot; a fifth point, the zero of the

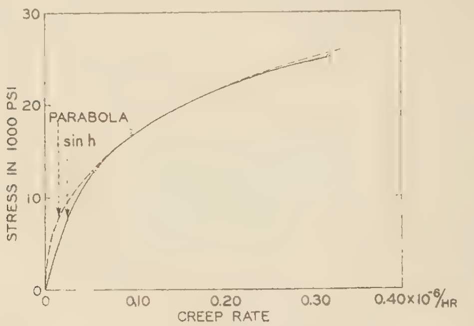

FIG 11 OBSERVED CREEP DATA

co-ordinates, is obtained without effort. If the constants of the hyperbolic sine were determinable by independent experiments, it would be possible to construct the "theoretical" curve and compare it with Mr. Robinson's points. This comparison would then decide at one stroke whether the theory is applicable to creep or not.

At the present status of the art, however, we are compelled "to lift ourselves by our bootstraps," e.g., we have no other alternative than to determine the constants from the information contained in any two data and to see how the curve satisfies the remaining points. This procedure, therefore, is no different from the purely empirical. The fact that the rate theory (hyperbolic sine) is in good agreement with the observations derived by the "bootstrap" method is not an ultimate proof of validity. A suitably chosen parabola runs so close to the hyperbolic sine that the two are almost indistinguishable within the range of the experiments. Extension of this range could hardly help matters, since on the one hand experimental accuracy is lower in the low range, in about the same proportion as the difference between the two curves increases; on the other hand, in the high stress range a new phenomenon sets in, e.g., acceleration of creep rate, resulting in an early destruction of the specimen.

This discussion thus shows the desirability of the independent determination of the creep constants as the most important step toward solving this difficult problem. The structural sensitiveness of creep suggests that the simple theory might not be adequate to interpret this phenomenon fully and that refinement or even modification might become necessary.

H. F. MOORE.[7] The writer has applied the author's method to several stress-creep rate diagrams for lead alloys. These diagrams do not follow a straight line on log-log paper but seem to be checked fairly well by the hyperbolic-sine relation.

Of course, it should be recognized that both the log-log method and the hyperbolic-sine method are empirical, and it is to be hoped someday that we can solve our creep problems in terms of physical constants which are more nearly fundamental than our present direct measurement of creep and stress.

Meanwhile, Mr. McVetty's hyperbolic-sine method seems worthy of a careful study, especially in the cases where the somewhat simpler log-log relation does not seem to be accurate. This is especially true in those cases where extrapolation is necessary. It is believed that such a procedure is thoroughly justified by the old adage, "Do the best you can, with what you have, where you are, today."

ERNEST L. ROBINSON.[8] This paper presents an admirable technique for evaluating the coefficients necessary to describe the relation between stress and creep rate by means of the hyperbolic-sine formula. As the author shows, in many cases, the formula represents actual behavior much better than a straight line on log-log paper. It fits particularly well the four 1000-hr constant-stress tests on 18 per cent chromium 8 per cent nickel steel at 1500 F, which the author quotes from the Creep Data Book.[3] Wisely, he does not claim any great generality of application, nor does he attempt to mark out a field of usefulness throughout which this rule could be depended upon.

However, to make effective use of such a rule in reducing the number of constant-stress tests required, it would be necessary to know in advance under what circumstances the rule holds good.

In the writer's opinion, a large number of low-alloy steels in the temperature range from the point at which creep is just perceptible up to around 900 F are best represented by a straight line on the log-log paper. At somewhat high temperatures these materials show a tendency to creep more rapidly at the lower stresses indicating some break in the line. Frequently, a new trend is established. In such cases it may very likely be that the hyperbolic-sine curve will give a better representation of the data than two straight lines with a knee.

It would also be desirable to outline the conditions under which constant-stress tests comply with the proposed rule. The author applies the rule to the analysis of creep rates at the end of 1000-hr constant-stress tests. A different rule would undubtedly be required to describe the relation between rate and stress under a different specified condition.

For example, the writer has presented[6] the results of a series of four 100,000-hr constant-stress creep tests. It seemed possible in this case to represent quite a variety of relationships by straight lines on the log-log plot. At the end of 2000 hr, one line described the relationship between stress and rate. A very different line represented the relationship at the end of 100,000 hr. A third line represented the relation between stress and rate for the minimum rate measured on each bar. It would be entirely possible to draw a hyperbolic-sine curve on this plot and then describe the required time or extension or both occurring in each bar to render the hyperbolic-sine rule valid.

Generally, when creep tests are run under relaxation conditions with a specified total elastic-plus-plastic extension maintained constant, a quite different relationship is observed between stress and creep rate than at the end of a specified time at constant stress. Here again at the more moderate temperatures the low alloys show a straight-line relationship but very much less steep than is given by constant-stress tests. At somewhat higher temperatures there is often a tendency for relaxation to proceed a little more rapidly than the straight-line relationship would indicate, and thus there may be a utility for the hyperbolic-sine rule under such circumstances, but it would be necessary to use a different set of coefficients from those which represent constant-stress behavior.

[6] "100,000-Hour Creep Test," by E. L. Robinson, *Mechanical Engineering*, March, 1943, pp. 166–168.

[7] Research Professor of Engineering Materials, University of Illinois, Urbana, Ill. Mem. A.S.M.E.

[8] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

Another type of behavior is typically presented in a paper by S. H. Weaver[9] on the creep strength of carbon-molybdenum steel. The results of these tests at 900 F are well represented by straight lines. However, at 1000 F a broken line is usually required to describe the results. If this broken line is just a single change of direction, it is possible that a hyperbolic-sine curve might also be adequate as a representation of the results. However, in some conditions this material shows a behavior which can only be represented by a jagged stroke-of-lightning-shaped line. Above and below a certain creep rate there appear to be two well-defined straight-line relationships. These are connected by a jagged offset, and it does not seem possible to represent such behavior easily by any analytical method.

With regard to the most effective use of high-temperature creep-test equipment, the writer is of the opinion that more useful information can be obtained by testing a single bar under step-down or relaxation conditions than from any other type of creep test. There does not seem to be any simple relationship between the relaxation test and the constant-stress test, but, with a given amount of equipment available, the largest amount of reliable comparative data can be obtained by testing single bars at successively lower loads under carefully defined conditions of extension.

Author's Closure

Dr. Heindlhofer's suggestion of a more fundamental study of the problem is worthy of further consideration. We have been much interested in the chemical-rate theory and its possible application to stress-creep rate relations. Reference may be made to a paper[10] on this subject, based on work at the Westinghouse Research Laboratories a few years ago. The development of the fundamental background requires many carefully conducted tests of each material in addition to the conventional type of creep tests. The general hyperbolic-sine relation developed by Eyring and his associates requires measurement of energy and entropy of activation, and the true value of the shearing stress which acts on the unit of flow. This unit may be a single molecule or a group of many molecules, and some practical difficulties in measuring the unit shear stress and the area on which it acts may be anticipated.

The chemical-rate theory was originally applied to the flow of ordinary liquids for which case application is simplified by measurement of the coefficient of viscosity. When applied to the plastic flow of crystalline solids, allowance must be made for the simultaneous action of widely different mechanisms of flow, and the dependence of the constants upon time and total strain. Consideration must be given also to Kanter's suggestion that stress may affect the entropy of activation. The many variables involved in this fundamental study require correlation with carefully conducted creep tests to insure that the necessary assumptions are applicable to the complex structures found in heat-resisting alloy steels.

The proposal to combine several fundamental factors into two easily determined empirical constants is similar to the use of the coefficient of viscosity to simplify application of the chemical-rate theory to the flow of liquids. Under present restrictions in the use of available equipment and personnel, an empirical relation may have practical value for use until the more complex fundamental relations have been explored.

Professor Moore's application of the proposed method to existing creep data for several lead alloys indicates what can be done to review previously determined stress-creep rate relations. We now have a considerable amount of published data on a wide variety of metals and alloys. In most cases it has been assumed that a parabolic type of power function expresses the relation between stress and creep rate for purposes of interpolation and extrapolation. A review of this material will permit a comparison with the proposed hyperbolic-sine function in many practical applications. If our experience is confirmed by this review, the appraisal of available alloys and the choice of working stresses will be affected.

Mr. Robinson's statement that we must know in advance which function to use for extrapolation to low stresses presents a serious problem to those who have accepted the power function without question. Under the usual condition that tests are conducted at stresses higher than working stresses, extrapolation of the power function represented by the straight-line log-log plot to lower creep rates always gives higher stress estimates than are obtained from similar use of the hyperbolic sine. For the determination of safe working stresses, it would appear that the hyperbolic-sine relation should be used if there is any doubt as to which of these two functions is applicable.

The use of creep rates measured at the end of 1000-hr tests was necessary to make a comparison with the log-log plots used in the Creep Data Book. We prefer to use minimum creep rates under conditions which give approximately constant rate over a large part of the life of the material in service. This subject was considered beyond the scope of a paper comparing two functions, and reference is made to a more general discussion of the interpretation problem in a recent A.S.T.M. paper.[11]

Mr. Robinson's recommendation that the step-down or relaxation type of test on one bar take the place of constant-stress tests on several bars is not supported by our experience. The step-down test may be adapted to bolting problems, but creep recovery after each stress change prevents reliable measurements of creep rate in the relatively short time at each of the successively lower test stresses. In the true relaxation test, the stress decrement is reduced to a minimum, and no direct measurement from this test is comparable with the minimum creep rate for a given combination of stress and temperature obtainable from the constant-stress test. Under these conditions, relaxation is related to the first stage of creep rather than to the second stage in which creep rate is approximately constant.

[9] "The Effect of Carbide Spheroidization Upon the Creep Strength of Carbon Molybdenum Steel," by S. H. Weaver, Proceedings A.S.T.M., vol. 41, 1941, pp. 608–628.

[10] "Flow of Solid Metals From the Standpoint of the Chemical-Rate Theory," by Walter Kauzman, "Metals Technology," A.I.M.M.E., vol. 8, June, 1941, technical publication No. 1301.

[11] "Interpretation of Creep Test Data," by P. G. McVetty, presented at the Annual Meeting of the A.S.T.M., Pittsburgh, Pa., July 1, 1943; Figs. 4 and 5.

Rate of Ice Formation

By A. L. London[1] and R. A. Seban[2]

A general approximate method for the solution of the freezing problem is presented with applications to ice formation at spherical, cylindrical, and plane boundaries. The degree of approximation is investigated, and the results of the analysis are indicated to be of satisfactory accuracy for the solution of engineering problems. Utilization of the analytical results in the prediction of freezing times for actual systems produces good agreement with quoted performance.

Nomenclature

THE following nomenclature is used in the paper:

C_i = ice thermal capacity at time τ, Btu per sq ft per deg F
c = ice-unit thermal capacity, Btu per lb per deg F
$d(\ )$ = differential operator
h_0 = unit conductance to surroundings at temperature t_0 (below the freezing point), Btu per sq ft per deg F per hr
h_l = unit conductance for thermal convection from liquid body to growing ice surface, Btu per sq ft per deg F per hr
k = unit thermal conductivity of ice, Btu per hr per sq ft (deg F per ft)
L = latent heat of fusion of ice, Btu per lb
log = logarithms to natural base
q = unit thermal current, Btu per per sq ft per hr, Btu per ft per hr, Btu per hr for slab, cylinder, and sphere, respectively
r = radial position of growing ice surface (sphere or cylinder), ft
R = unit thermal resistance, (hr sq ft deg F) per Btu, (hr ft deg F) per Btu, (hr deg F) per Btu for slab, cylinder, and sphere, respectively
t_0 = temperature of surroundings (freezing-point datum), deg F
t_l = temperature of liquid (freezing-point datum), deg F
t = temperature (freezing-point datum), deg F
x = position of growing slab-ice surface, ft
ϵ = difference functions defined by Equations [22], [23], [25], and [26]
ρ = ice density, lb per cu ft
τ = time, hr
ϕ = functional relation
$\cong$ = denotes approximate equivalence

Dimensionless Moduli:

$\dfrac{-t_0c}{L}$ = coefficient indicating significance of capacity effect (Equations [25] and [26])

ϵ_1, ϵ_2 = difference functions, defined by Equations [22] and [23]

$r^* = r/r_0$, a generalized radial position employed for cylinder and sphere problems

$R^* = \dfrac{h_0r_0}{k}, \dfrac{h_0x_0}{k}$, generalized surface resistance

$R\dagger t\dagger = \left(\dfrac{t_l}{-t_0}\right)\left(\dfrac{h_l}{h_0}\right)$, generalized potential-resistance ratio

$\tau^* = \left(\dfrac{-t_0k}{\rho L r_0^2}\right)\tau$, generalized time employed in cylinder and sphere problems

$\tau\dagger = \left(\dfrac{-t_0h_0^2}{\rho L k}\right)\tau$, generalized time employed in slab problem

$x^* = x/x_0$ generalized position employed in "double-slab" problem

$x\dagger = h_0x/k$ generalized position employed in slab problem

Introduction

The problem of ice formation in liquids in contact with boundaries of various geometries has received but little attention in the literature. Ingersoll and Zobel (1)[3] present an approximate solution for slab-ice formation on the surface of the liquid at the freezing temperature, with the free-surface temperature constant. Perkeris and Slichter (2) derive an approximate solution for the rate of ice formation on the outside of a cylinder for liquid at the freezing point and infinite surface conductance at the inside cylinder surface. In this solution, the inside surface temperature may be any function of the time. Elmer (3) derives a similar expression for the rate of ice formation on submerged pipes. Other more exact solutions for slab-ice formation are available (1), but these are of considerable complexity and limited to a constant free-surface temperature. A new consideration of the problem in general with emphasis on approximate solutions of such form as to be readily applied to typical problems therefore seems appropriate.

The objectives of this paper, then, are to accomplish the following:

1 Describe a general approximate method of analyzing the problem of freezing in liquids bounded by surfaces of various geometries. "Freezing" is hereafter called "ice formation" but the generality of the method will be apparent.

2 Present solutions for the rate of ice formation for boundary geometries which are of significance in applications, i.e., cylinders, spheres, plane surfaces. These solutions are presented algebraically and graphically and are expressed in terms of dimensionless variables.

3 Estimate the degree of approximation of these solutions.

4 Illustrate the application of the approximate solutions by specific applications in ice manufacture and quick freezing of food products.

Method of Analysis

Description of Idealized Systems. Fig. 1(*a*) reveals the temperature conditions in an ice layer on the surface of a liquid which is uniformly at the temperature of freezing. Ice formation occurs at the solid-liquid interface as a result of heat transfer through the ice to the surroundings at temperature t_0 ($-t_0$ deg F below the freezing point). This thermal current flows through the ice by

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Contributed by the Heat Transfer Division and presented at the Spring Meeting, Davenport, Iowa, April 26–28, 1943, of The American Society of Mechanical Engineers.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

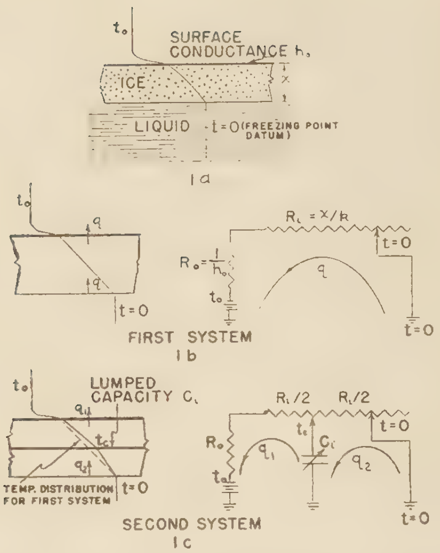

Fig. 1 Idealized Systems of Ice Formation

(a, Temperature conditions during ice formation. b, First idealized system and thermal-circuit representation. c, Second idealized system and thermal-circuit representation.)

conduction and from the ice surface to the surroundings by convection. The ice layer is subcooled except for the growing surface, which is in contact with the liquid at the freezing temperature.

As a first idealization, the thermal-energy abstraction in subcooling the ice may be considered as negligible relative to the latent-heat-of-freezing requirement. Then the thermal circuit of the system in Fig. 1(a) may be represented as in Fig. 1(b). In this circuit, the thermal current q (Btu per sq ft per hr), required for freezing, passes through the resistance of the ice layer $R_i = x/k$ (hr sq ft deg F) per Btu which is variable, depending upon the thickness x of the layer, and then through the resistance $R_0 = 1/h_0$ (hr sq ft deg F) per Btu for the convective transfer of this current to the cold surroundings at temperature t_0 (below the freezing point).

In this and succeeding developments, the ice thermal properties ρ, k, c are considered as constant and all temperatures are referred to the freezing point as the datum (Table 1).

The first idealized system and thermal circuit as described can be analyzed by elementary methods, and the resulting rate of formation is descriptive of that occurring in the actual system, providing the idealization of negligible thermal-capacity effect is valid. The magnitude of the thermal-capacity effect may be determined by a comparison of this first idealized system to the second idealized system, which considers the capacitance of the ice as lumped at the center of the ice layer, as shown in Fig. 1(c).

In this second system, the ice thermal capacitance $C_i = \rho c x$ Btu per sq ft per deg F depends upon the thickness x of the layer. The other circuit components are the same as those defined for the first idealized system. In this circuit, Fig. 1(c), the thermal current q_1 convected to the surroundings is in excess of that required to maintain the rate of freezing q_2, by the amount of thermal-energy abstraction required to subcool the ice.

Comparison of the solutions for the first and second idealized systems, Figs. 1(b) and 1(c), would provide a direct method of estimating the error introduced by neglecting the capacity effect. This, however, cannot be accomplished simply, and, as a consequence, the solution for the first system is employed to evaluate the terms of the differential equation describing the behavior of the second idealized system (see Appendix). The application of this method indicates that the first idealization yields results of sufficient accuracy for most engineering applications.

Cylindrical-Ice Formation. The first problem to be analyzed is that of ice formation in the direction of decreasing radius within a cylindrical boundary (right circular cylinder). The liquid is at the freezing point. Fig. 2 shows the system and the thermal circuit (the capacity effect is neglected). This circuit is similar to that for the slab of Fig. 1(b) except that the ice resistance $R_i = (\log r_0/r)/2\pi k$ (hr ft deg F) per Btu is for a unit length of the cylinder, and the surface resistance $R_0 = 1/(2\pi r_0 h_0)$ (hr ft deg F) per Btu is similarly expressed.

The rate equation for the thermal current q (Btu per hr per ft of length) flowing through the resistances offered by the ice and the surface, acting in series, as a result of the temperature potential drop $-t_0$ is

$$q = -t_0/(R_i + R_0) \qquad [1]$$

This thermal current q provides the extraction of the latent heat of fusion necessary for freezing at the surface r

$$q = -2\pi\rho L r \frac{dr}{d\tau} \qquad [2]$$

where $-2\pi r \dfrac{dr}{d\tau}$ is the volume rate of ice formation at the growing surface, cu ft per hr per ft, and ρL, the latent heat of fusion, Btu per cu ft.

Combination of Equations [1] and [2] to eliminate the thermal current q provides the differential equation expressing radius r as a function of time τ

$$-2\pi\rho L r \frac{dr}{d\tau} = -t_0 \Bigg/ \left(\frac{\log \dfrac{r_0}{r}}{2\pi k} + \frac{1}{2\pi r_0 h_0}\right) \qquad [3]$$

This equation may be readily arranged in the following dimensionless form, employing generalized radius r^*, resistance R^*, and time τ^*

$$\left(-\log r^* + \frac{1}{R^*}\right) r^* dr^* = -d\tau^* \qquad [3a]$$

The boundary conditions which will be considered as obtaining

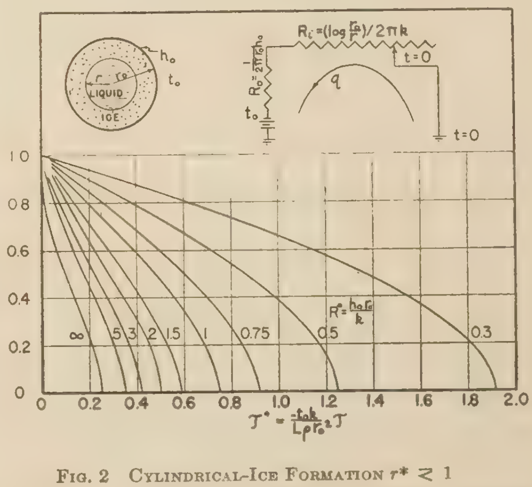

Fig. 2 Cylindrical-Ice Formation $r^* \leq 1$

(Thermal-circuit representation, and graphical results of the analysis, Equation [5].)

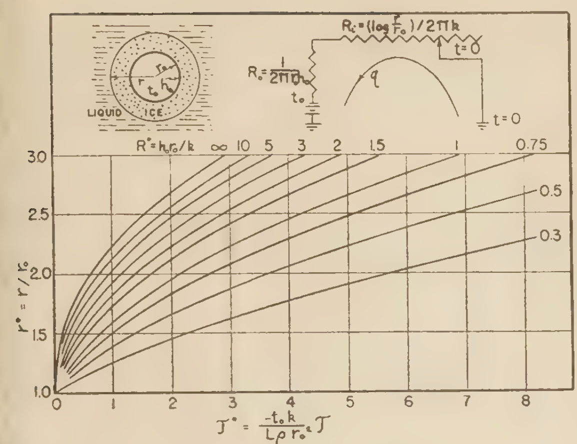

Fig. 3 Cylindrical-Ice Formation $r^* \gtrless 1$
(Thermal-circuit representation, and graphical results of the analysis, Equation [7].)

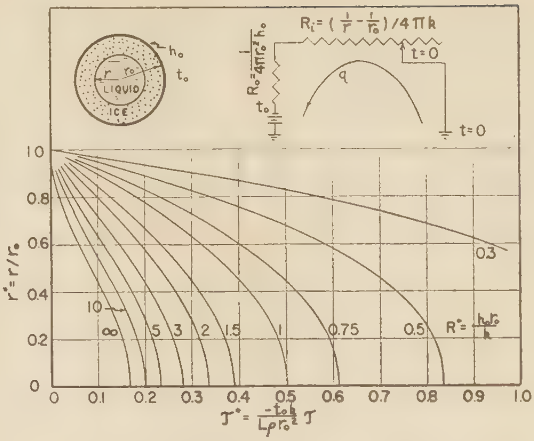

Fig. 4 Spherical-Ice Formation $r^* \lessgtr 1$
(Thermal-circuit representation, and graphical results of the analysis, Equation [9].)

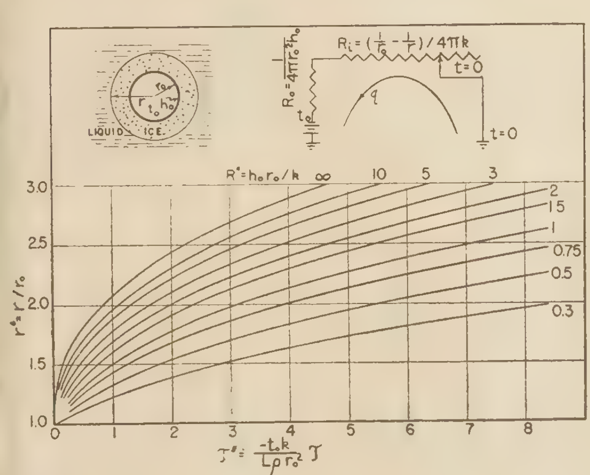

Fig. 5 Spherical-Ice Formation $r^* \gtrless 1$
(Thermal-circuit representation, and graphical results of the analysis, Equation [10].)

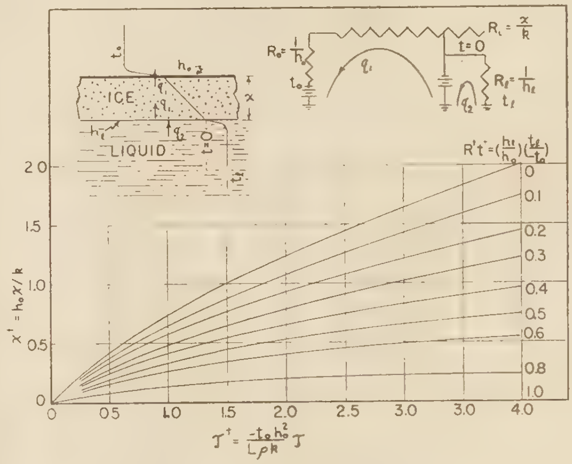

Fig. 6 Slab-Ice Formation
(Thermal-circuit representation, and graphical results of the analysis, Equation [16].)

are

$$\left.\begin{array}{lll} \text{at} & \tau = 0, & r = r_0 \\ \text{or} & \tau^* = 0, & r^* = 1 \\ \text{and for} & \tau = \tau, & r = r \\ \text{or} & \tau^* = \tau^*, & r^* = r^* \end{array}\right\} \qquad [4]$$

For these conditions, the solution of Equation [3a], obtained by elementary methods, becomes

$$\tau^* = \frac{r^{*2}}{2}\log r^* + \left(\frac{1}{2R^*} + \frac{1}{4}\right)(1 - r^{*2}) \qquad [5]$$

$$\tau^* = \phi(r^*, R^*)$$

In this equation $r^* \lessgtr 1$.

This result is plotted in dimensionless form as r^* versus τ^* employing R^* as a parameter, Fig. 2.

The time for complete solidification ($r^* = 0$) may be determined from Equation [5].

$$\tau^*_{r^*=0} = \left(\frac{1}{2R^*} + \frac{1}{4}\right) \qquad [6]$$

The solution of cylindrical-ice formation in the direction of increasing radius can be developed in a similar manner as

$$\tau^* = \frac{r^{*2}}{2}\log r^* + \left(\frac{1}{2R^*} - \frac{1}{4}\right)(r^{*2} - 1) \qquad [7]$$

In this equation $r^* \gtrless 1$. This solution is similarly plotted in dimensionless form as r^* versus τ^* employing R^* as a parameter, Fig. 3.

Spherical-Ice Formation. For spherical-ice formation in the direction of decreasing radius ($r^* \lessgtr 1$), the method of analysis parallels that employed for the problem of cylindrical-ice formation. In this problem, however, the resistance offered by the ice is

$$R_i = \left(\frac{1}{r} - \frac{1}{r_0}\right)/4\pi k$$

and the surface resistance

$$R_0 = 1/4\pi r_0^2 h_0$$

The resulting differential equation in dimensionless form is

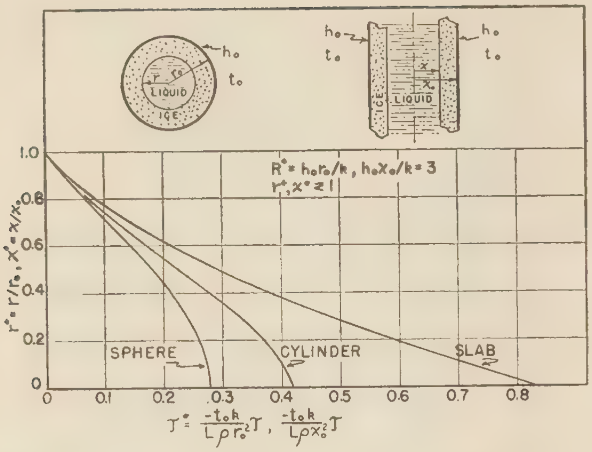

FIG. 7 COMPARISON OF SLAB-, CYLINDER-, AND SPHERICAL-ICE FORMATION, r^*, $x^* \lessgtr 1$

$$\left[r^*\left(\frac{1}{R^*}-1\right)+1\right]r^*dr^* = -d\tau^* \quad [8]$$

Here the generalized variables are identical in form to those previously employed. The boundary conditions obtaining are taken as

at $\qquad \tau^* = 0,\ r^* = 1$

and at $\qquad \tau^* = \tau^*,\ r^* = r^*$

The resulting solution then assumes the form

$$\tau^* = \frac{1}{3}\left(\frac{1}{R^*}-1\right)(1-r^{*3}) + \frac{1}{2}(1-r^{*2}) \quad [9]$$

where $r^* \lessgtr 1$.

The solution for spherical-ice formation in the direction of increasing radius, $r^* \gtrless 1$, assumes the similar form

$$\tau^* = \frac{1}{3}\left(\frac{1}{R^*}+1\right)(r^{*3}-1) - \frac{1}{2}(r^{*2}-1) \quad [10]$$

These expressions are represented graphically in Figs. 4 and 5 where the generalized radial position r^* (dimensionless) is plotted versus the generalized time τ^* (dimensionless) employing the generalized resistance R^* (dimensionless) as a parameter.

Slab-Ice Formation. The solution for slab-ice formation is developed more generally in that the liquid body is considered as existing at a constant temperature t_l in excess of the freezing point, and thus some heat transfer occurs from the liquid to the growing ice surface through the liquid surface resistance R_l. This system is illustrated in Fig. 6 with its corresponding idealized thermal circuit.

In this system the thermal current q_2 (Btu per sq ft per hr) is the heat transfer by thermal convection from the liquid body at temperature t_l (above the freezing point) to the growing ice surface at the freezing point, through the resistance $R_l = 1/h_l$ (hr sq ft deg F) per Btu. The rate equation for this transfer is

$$q_2 = t_l/R_l \quad [11]$$

The thermal current q_1 is the heat transfer by conduction from the growing ice surface through the ice layer of resistance $R_i = x/k$, and thence by thermal convection from the free ice surface to the surroundings at t_0 (below the freezing point) through the resistance $R_0 = 1/h_0$ (hr sq ft deg F) per Btu. The rate equation for this transfer is

$$q_1 = -t_0/(R_i + R_0) \quad [12]$$

From energy-balance considerations, it may be concluded that the thermal current q_1 is in excess of q_2 by the current required for the rate of ice formation $dx/d\tau$. Thus

$$q_1 - q_2 = \rho L\frac{dx}{d\tau} \quad [13]$$

Equations [11, 12, 13] may be readily combined to eliminate the thermal currents q_1 and q_2. The resulting differential equation, expressed in dimensionless form, becomes

$$\frac{(1+x\dagger)dx\dagger}{-(t\dagger R\dagger)(1+x\dagger)+1} = d\tau\dagger \quad [14]$$

where the dimensionless variables are defined in the nomenclature. The assumed boundary conditions are

$$\left.\begin{array}{ll}\text{at} & \tau\dagger = 0,\ x\dagger = 0\\ \text{at} & \tau\dagger = \tau\dagger,\ x\dagger = x\dagger\end{array}\right\} \quad [15]$$

For these conditions, the solution of Equation [14] becomes

$$\tau\dagger = -\frac{1}{(R\dagger t\dagger)^2}\log\left(1-\frac{R\dagger t\dagger x\dagger}{1-R\dagger t\dagger}\right) - \left(\frac{x\dagger}{(R\dagger t\dagger)}\right) \quad [16]$$

For the special case when the liquid body is at the freezing point (Fig. 1) $R\dagger t\dagger = 0$ as $t_l = 0$, so that the differential Equation [14] reduces to

$$(1+x\dagger)dx\dagger = d\tau\dagger \quad [14a]$$

and the solution is

$$x\dagger = -1 + \sqrt{1+2\tau\dagger} \quad [16a]$$

These results are shown graphically in Fig. 6, where the generalized thickness $x\dagger$ is plotted versus the generalized time $\tau\dagger$ employing the generalized potential-resistance ratio $t\dagger R\dagger$ as a parameter.

Comparison of Slab-, Cylindrical-, and Spherical-Ice Formation. The foregoing solutions for cylindrical, spherical, and slab ice formation, with the liquid body at the freezing point, may be compared graphically. For this purpose, the slab solution, Equation [16a], is rearranged in form to comply with the "double-slab" nomenclature shown in Figs. 7 and 8

$$1 - x^* = \frac{1}{R^*}\left(\sqrt{1+2\tau^*R^*}-1\right)$$

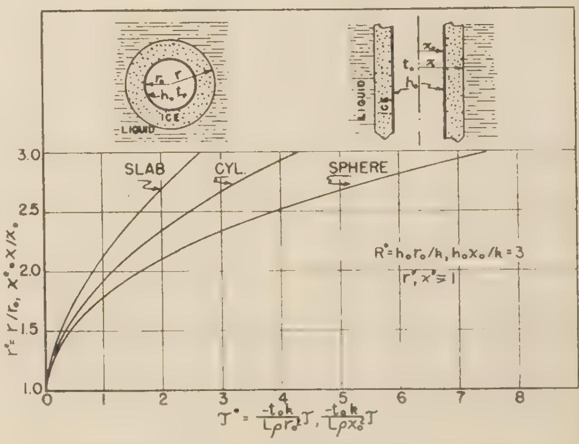

FIG. 8 COMPARISON OF SLAB-, CYLINDER-, AND SPHERICAL-ICE FORMATION, r^*, $x^* \gtrless 1$

for $x^* \lesssim 1$ and

$$(x^* - 1) = \frac{1}{R^*}\left(\sqrt{1 + 2\tau^* R^*} - 1\right)$$

for $x^* \gtrsim 1$ where

$$x^* = \frac{x}{x_0},\ R^* = \frac{h_0 x_0}{k},\ \tau^* = -\frac{t_0 k}{L\rho x_0^2}\tau$$

In this form, the dimensionless groupings for the slab system become comparable to those employed for the spherical and cylindrical problems. These solutions are compared graphically for a common magnitude of $R^* = h_0x_0/k$, $h_0r_0/k = 3$; in Fig. 7, for x^*, $\tau^* \lesssim 1$, and in Fig. 8, for x^*, $\tau^* \gtrsim 1$.

Surrounding Temperature a Function of Time. All the previously developed results are for the special case of the ambient temperature t_0, constant as well as constant-surface conductance h_0. However, the differential Equations [3*a*], [7], and [14] (written with t_0, h_0 outside the differentials) apply as well to the problem of t_0, a function of time. For the problem of h_0 constant and t_0, some known function of time (but always less than the freezing point) Equation [14] may be written in the form

$$\phi(x)dx = \phi(\tau)d\tau$$

where ϕ denotes a functional relation. The other differential equations may be similarly expressed. Thus, for given boundary conditions, it is readily possible to integrate these equations either graphically or analytically to obtain solutions for the more general problem described.

Applications

Can-Ice Manufacture. The time of freezing of water, contained in ice cans in the process of ice manufacture, may be determined from the solutions which have been presented. The standard ice can has dimensions of 11 in. × 22 in. × 50 in., with 1 in. side taper. These cans are immersed in brine having a temperature of about 10 F. Further system details and performance data may be obtained from the A.S.R.E. Data Book (6).

For analysis, the actual ice can is considered as an "equivalent" cylinder having the same cross-sectional area. End effects are neglected. Equation [6] gives the time of freezing of such a cylinder

$$\tau_{r^*=0} = \frac{L\rho r_0^2}{-t_0 k}\left(\frac{1}{2R^*} + \frac{1}{4}\right) \qquad [6]$$

r_0 = 0.706 ft for equivalent cylinder
t_0 = —22 F brine temperature (below freezing)
h_0 = 40 Btu per sq ft per deg F per hr, unit surface conductance at cylinder, including resistance of metal-can wall

Magnitudes of k, ρ, L are given in Table 1

TABLE 1 THERMAL PROPERTIES OF ICE[a]
(Averaged 0 to 32 F)

Density, ρ lb per cu ft	57.3
Unit heat capacity, c, Btu per deg F per lb	0.487
Thermal conductivity, k, Btu per hr per sq ft (deg F per ft)	1.34
Latent heat of fusion (at 32 F), L, Btu per lb	143.4

[a] Bibliography (4) and (5).

From these data and Equation [6], the time of complete freezing is 38 hr. At this rate 10.6 cans (300 lb per can) will be required per ton of ice-making capacity, which compares favorably with actual performance (6).

Similar calculations for various surface conductances yield the following results:

h_0	10	20	30	40	60	100	∞
$\tau_{r^*=0}$	47.9	41.3	39.1	38.0	36.8	36.0	34.6
Cans per ton	13.3	11.5	10.9	10.6	10.2	10.0	9.8

Circulation of the brine to produce higher surface conductances is effective in increasing the rate of freezing until conductances of about 40 have been obtained. With zero surface resistance, $h_0 = \infty$, the freezing rate is only 8 per cent greater than with $h_0 = 40$.

Flakice. Ice is produced in this process on a flexible rotating drum partially submerged in water. The cylinder is internally refrigerated with a brine spray. Ice formed on the exterior surface is peeled off as the revolving-drum surface emerges from the water (7). Data (7) are presented (Fig. 9), indicating the variation of thickness of ice layer with time of immersion. By means of the solution developed in this paper, a comparison of predicted and actual thickness is accomplished. Assumed operating conditions are as follows:

Brine temperature, t_0 = —20 F (below freezing).
Water liquid temperature, t_l = 8 F (above freezing).
Surface conductance, including metal-wall resistance, h_0 = 100 Btu per sq ft per deg F per hr.
Liquid conductance, h_l = 10 Btu per sq ft per deg F per hr.

Equation [16] defines the thickness of the layer as a function of time

$$\tau\dagger = -\frac{1}{(R\dagger t\dagger)^2}\log\left(1 - \frac{R\dagger t\dagger x\dagger}{1 - R\dagger t\dagger}\right) - \left(\frac{x\dagger}{R\dagger t\dagger}\right) \qquad [16]$$

$$R\dagger t\dagger = \frac{h_l}{h_0}\frac{t_l}{(-t_0)} = 0.040$$

$$x\dagger = \frac{h_0 x}{k} = 74.5x$$

The magnitude of x ranges from 0 to 0.02 ft and $R\dagger t\dagger x\dagger$ from 0 to 0.06; therefore, the second term in the argument of the logarithm is small relative to 1, and the first two terms of the power series expressing the logarithm serve to evaluate the function with satisfactory accuracy. Then

$$\tau\dagger = \frac{1}{(R\dagger t\dagger)^2}\left[\frac{R\dagger t\dagger x\dagger}{1 - R\dagger x\dagger} + \frac{1}{2}\left(\frac{R\dagger t\dagger x\dagger}{1 - R\dagger t\dagger}\right)^2\right] - \left(\frac{x\dagger}{(R\dagger t\dagger)}\right)$$

$$= \frac{x\dagger^2}{2(1 - R\dagger t\dagger)^2} + \frac{x\dagger}{(1 - R\dagger t\dagger)}$$

But

$$\tau\dagger = -\frac{t_0}{\rho L}\cdot\frac{h_0^2}{k}\tau,\ x\dagger = \frac{h_0 x}{k}$$

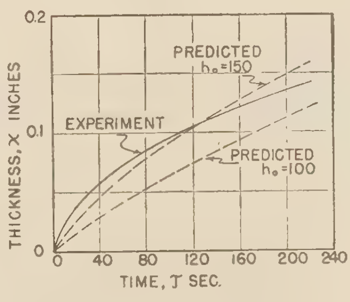

Fig. 9 Flakice Formation
(Comparison of predicted and experimental results.)

so after substitution of numerical magnitudes

$$\tau = 4.28x + 167x^2$$

This equation is represented graphically, together with the similar equation for $h_0 = 150$ and $h_l = 15$ Btu per sq ft per deg F per hr, and actual test data as given in (7), Fig. 9. The agreement is

not too satisfactory, but this may be partially accounted for by the possibility that the metal wall is considerably subcooled before contact with the liquid, which induces an initial rate of freezing higher than predicted by the ideal system analysis.

Tube Ice. A process for manufacturing ice in granulated form utilizes freezing of water flowing in small vertical tubes (8). The tubes in which the freezing occurs are 1.75 in. ID. Evaporation of ammonia on the exterior of the tubes (shell-and-tube-type evaporator) produces the refrigeration. The ammonia pressure is 30 psig (saturation temperature 17 F), and the liquid temperature is chosen at the freezing point ($t_l = 0$).

The freezing time may be computed in the same manner as used for can ice

$$\tau_{r^* = 0} = \left(\frac{\rho L}{-t_0}\right)\left(\frac{r_0}{2h_0} + \frac{r_0^2}{4k}\right) \ldots\ldots\ldots\ldots [6]$$

The unit surface conductance, ammonia condensing on vertical metal tubes, and including metal-wall and scale resistance, is estimated as $h_0 = 400$ Btu per sq ft per deg F per hr. $r_0 = 0.0729$ ft, $t_0 = -15$ F.

Then

$$\tau_{r^* = 0} = 0.59 \text{ hr} = 35 \text{ min}$$

The quoted freezing time for a unit of this type is 37 min (8).

Freezing of Foods. The time required for freezing foods may be determined from the solution for the time of complete freezing, as given for the various geometrical shapes. Perry (6) has investigated the freezing time of cylinders of salt gels, simulating foods, in an air stream. The air stream had a temperature of $t_0 = -28$ F (below the freezing point) and a velocity of 14.17 fps. Freezing times for the cylinders, as determined experimentally (6), are as follows:

Cylinder diam, in.	$\tau_{r^* = 0}$ min
1.7	80
1.1	38
0.7	22

Unit film conductances h_0 are determined from the equation $h = 0.026\, G^{0.6}/D^{0.4}$ (reference 9), and the times for complete freezing are determined by Equation [6]. The results of this calculation are as follows:

Cylinder diam, in.	h_0	$\tau_{r^* = 0}$ min
1.7	8.7	87
1.1	10.4	46
0.7	12.5	23

The foregoing times are comparable to the freezing times as obtained experimentally.

An analysis of this type may be used to determine the effect of size, air temperature, and air velocity upon the time of freezing for specimens of various shapes, as well as to provide a qualitative basis for extrapolating performance to changed operating conditions.

Summary and Conclusions

1 A general method of idealizing the ice-formation system is described.

2 The resulting solutions are presented for ice formation conforming to the several geometric boundaries: slab, sphere, and right circular cylinder.

3 An analysis of the degree of approximation introduced by the idealization of negligible capacity effect indicates that the resulting solutions are sufficiently accurate for most engineering calculations. The neglect of ice thermal capacity introduces no greater error than the uncertainty of the correct magnitude of the unit thermal conductivity k.

4 Application of the ideal system solutions to several specific problems of engineering significance serves to indicate substantial agreement with actual performance, as quoted in the references.

5 Data obtained under carefully controlled conditions would be welcomed, as it would allow further substantiation of the analysis presented herein.

Acknowledgments

The authors express their appreciation to Prof. J. V. Uspensky, of Stanford University, and to Prof. L. M. K. Boelter, of the University of California, for their interest in the problem.

Appendix

Estimation of Errors

To provide an estimate of the magnitude of the error introduced by the neglect of the ice thermal capacity, the second idealized circuit, Fig. 1(*c*), which includes a "lumped" ice capacity, will be considered.

Fig. 1(*c*) compares qualitatively the temperature distributions for the two systems. As can be seen, the current resulting in freezing q_2 for the second system, associated with the temperature gradient at the growing ice surface, is less than the corresponding current q for the first system, which does not include the capacity effect. Similarly, the thermal current q_1 to the surroundings (which exceeds q_2 by the rate of decrease of energy storage associated with the lumped ice capacity C_i) is greater than q for the first system.

The differential equation, describing the behavior of the second system, will be derived. The rate equations for the thermal currents q_1 and q_2 are, respectively, Fig. 1(*c*)

$$q_1 = (t_c - t_0)/\left(\frac{R_i}{2} + R_0\right) \ldots\ldots\ldots\ldots [17]$$

$$q_2 = -t_c/\frac{R_i}{2} \ldots\ldots\ldots\ldots [18]$$

By energy-balance considerations, the thermal current q_2 is related to the freezing rate $dx/d\tau$ by

$$q_2 = \rho L \frac{dx}{d\tau} \ldots\ldots\ldots\ldots [19]$$

while $(q_1 - q_2)$ is equal to the decrease of energy storage in the thermal capacity C_i

$$(q_1 - q_2) = -\frac{d}{d\tau}(C_i t_c) \ldots\ldots\ldots\ldots [20]$$

These equations may be directly combined so as to eliminate q_1, q_2, and t_c. The result is expressible as

$$\overbrace{\rho L \frac{dx}{d\tau}}^{q_2} = \overbrace{\left[\frac{-t_0}{(R_i + R_0)}\right]}^{q} - \overbrace{\left[\frac{(R_i/2 + R_0)}{(R_i + R_0)} \cdot \frac{\rho^2 Lc}{6k} \cdot \frac{d^2(x^2)}{d\tau^2}\right]}^{D} . [21]$$

This differential equation of the second order is nonlinear, and a direct solution for comparison with that for the first system (Equation [16*a*]) has not been obtained. However, it does provide the basis for quantitative conclusions as to the magnitude of the difference in the rate $\dfrac{dx}{d\tau}$ for a given thickness x, and the difference in the thickness for a given time τ. These conclusions are accomplished by associating the terms in Equation [21] with the thermal currents q and q_2 (freezing currents for the first and second system, respectively) for the same ice thickness x, as shown in Figs. 1(*b*) and 1(*c*).

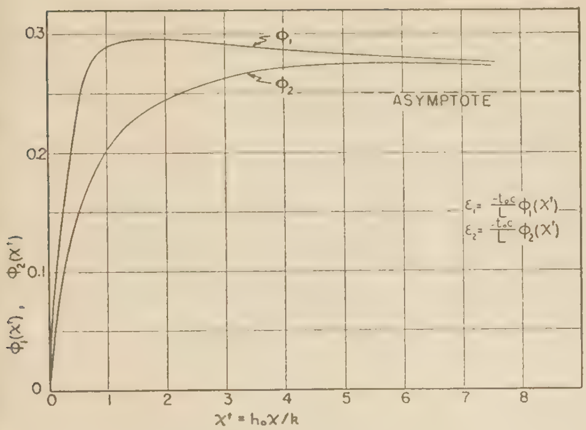

FIG. 10 DIFFERENCE FUNCTIONS
(Approximate comparison of first and second idealized systems, Equations [22, 23, 25, 26].)

The first term in Equation [21] is equal to the freezing current q_2 (Equation [19]), while the second term $-t_0/(R_i + R_0)$ is equal to the freezing current which would obtain were the capacity effect nil. Thus

$$q = -t_0/(R_i + R_0)$$

This conclusion results directly from a consideration of the thermal circuit, Fig. 1(*b*). Evidently, the third term D represents the difference in the freezing current as a result of the capacity effect. This quantity is proportional to the difference in temperature gradients at the growing ice surface for the first and second idealized systems, as shown in Fig. 1(*c*).

As $(q - q_2)$ is proportional to the difference in the freezing rates for the two systems, the ratio

$$\frac{q - q_2}{q} = \frac{\text{term } D}{q} = \epsilon_1 \ldots\ldots\ldots\ldots\ldots [22]$$

for a given thickness x may be employed as an expression of the percentage difference in the freezing rates. Similarly, the relation

$$\epsilon_2 = \frac{\int_0^\tau \epsilon_1 q d\tau}{\int_0^\tau q d\tau} \ldots\ldots\ldots\ldots\ldots [23]$$

expresses the percentage difference in the thickness x in the given time interval 0 to τ for the two systems. It will be demonstrated that ϵ_1 and ϵ_2 are small magnitudes, and the result provides the conclusion that the idealized thermal circuit which neglects the thermal-capacity effect adequately approximates the behavior of the actual ice system.

The term ϵ_1 may be expressed as

$$\epsilon_1 = -\left(\frac{R_i}{2} + R_0\right)\left(\frac{\rho^2 Lc}{kt_0}\right)\left[\frac{x^2}{2}\cdot\frac{d^2x}{d\tau^2} + x\left(\frac{dx}{d\tau}\right)^2\right] . [22a]$$

The derivatives are for the second system; however, to the first approximation, the magnitude of $dx/d\tau$ may be taken from

$$q = -\frac{t_0}{R_i + R_0} \cong \rho L \frac{dx}{d\tau} \ldots\ldots\ldots\ldots\ldots [24]$$

where $R_i = x/k$.

This approximation is conservative, as will be demonstrated. The substitution of $dx/d\tau$ and $d^2x/d\tau^2$ from Equation [24] into the expression for ϵ_1 results in a relation which may be rearranged to the following dimensionless form

$$\epsilon_1 \cong \left(-\frac{t_0c}{L}\right)\phi_1(x\dagger) \ldots\ldots\ldots\ldots\ldots [25]$$

where

$$\phi_1(x\dagger) = \frac{x\dagger}{4}\cdot\frac{(x\dagger + 2)^2}{(x\dagger + 1)^3}$$

Equation [24], together with Equation [25], may now be employed in Equation [23] to obtain an approximation to ϵ_2, namely

$$\epsilon_2 \cong \frac{\int_0^\tau \epsilon_1 dx\dagger}{\int_0^\tau dx\dagger} \cong \frac{\int_0^{x\dagger} \epsilon_1 dx\dagger}{\int_0^{x\dagger} dx\dagger} \cong \left(-\frac{t_0c}{L}\right)\cdot\phi_2(x\dagger) \ldots\ldots [26]$$

where

$$\phi_2(x\dagger) = \frac{1}{4x\dagger}\left[x\dagger + \log(x\dagger + 1) + \frac{1}{(x\dagger + 1)} + \frac{1}{2(x\dagger + 1)^2} - \frac{3}{2}\right]$$

The functions $\phi_1(x\dagger)$, $\phi_2(x\dagger)$ are plotted in Fig. 10. Both have an approximate magnitude of 0.25. The coefficient

$$-\frac{t_0c}{L} = -\frac{t_0}{294} \quad \text{(see Table 1)}$$

so even for $t_0 = -100$ F (below the freezing point) the difference in behavior of the two systems, with respect to both ice thickness and rate of freezing, does not exceed 10 per cent. For t_0 of -50 F (below the freezing point), the difference in x and $dx/d\tau$ behavior is less than 5 per cent for all ice thicknesses.

The approximation employed in Equation [24] which allowed evaluation of the magnitudes of ϵ_1 and ϵ_2 can be demonstrated as conservative; that is, ϵ_1 is actually somewhat smaller than $\left(-\frac{t_0c}{L}\right)\cdot\phi_1(x\dagger)$, as expressed by Equation [25]. The freezing rate $dx/d\tau$ in Equation [24] is for the first system, but for given thickness x, the second system would have a smaller magnitude than this as a result of the capacity effect.

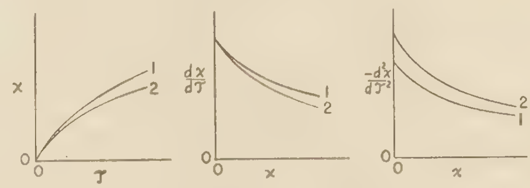

FIG. 11 QUALITATIVE COMPARISON OF $\frac{dx}{d\tau}$ AND $\frac{d^2x}{d\tau^2}$ FOR FIRST AND SECOND IDEALIZED SYSTEMS
(Refer to Fig. 1.)

Fig. 11 reveals qualitatively the relative magnitudes of $dx/d\tau$ and $d^2x/d\tau^2$ for the two systems. Reference to Equation [22*a*] will now reveal that the substitution of the derivatives for the first system, as an approximation to the derivatives of the second system, results in a prediction for ϵ_1 which is larger than actual.

The foregoing development demonstrates the closeness of behavior, with respect to the rate of freezing and thickness of ice

formation, of two idealized systems, the first of which neglects the ice capacity, while the second system has the capacity effect lumped at the center of the ice slab. Evidently, the capacity effect is small, and this conclusion obtains whether or not it is "lumped" at the center of the ice resistance, or distributed throughout, as in the actual system. This argument suggests that the behavior of the first idealized system adequately approaches the behavior of the actual system.

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Performance and Selection of Mechanical-Draft Cooling Towers

By JOSEPH LICHTENSTEIN,[1] NEW YORK, N. Y.

This paper indicates the need for standardization in cooling-tower practice and the adoption of a common basic theory for correlation of the vast amount of experimental data, coupled with practical experience now available. Such a theory would be invaluable in calculating performance at the guarantee point from test data taken under other conditions of service. From Merkel's differential equation for the cooling tower, the author derives a basic dimensionless equation for calculating cooling-tower performance. A graphic method of solution of this basic equation is then given in the form of curves which have been developed by the author's company as a result of extensive tests which are explained. It is further shown that the size of a cooling tower for a fixed set of conditions is not entirely determined by theory alone, but that economic conditions constitute the final criterion.

Nomenclature

The following nomenclature is used in the paper:

L = water flow, lb per hr
G = air flow, lb dry air per hr
a = interfacial surface, sq ft per cu ft
V = active tower volume, cu ft per sq ft ground area
α = coefficient of heat transfer by convection, Btu per sq ft per deg F per hr
K = coefficient of diffusion, based on vapor-content potential, lb per sq ft per hr (lb per lb)
T = water temperature, deg F
t = air temperature, deg F
h'' = enthalpy of saturated air at water temperature
h = enthalpy of main air stream
X'' = vapor content of saturated air film at water temperature, lb per lb dry air
X = vapor content of air-vapor mixture, lb per lb dry air
r = latent heat of evaporation, Btu per lb
C_p = specific heat of air, Btu per lb per deg F
H = total amount of heat transfer from water to air, Btu per hr
H_c = total amount of heat transfer by convection, Btu per hr
H_d = total amount of heat transfer by diffusion, Btu per hr

Subscripts 1 and 2 are used to designate values above and below tower packing, respectively.

Introduction

The last few years have shown a great increase in the demand for cooling towers by various industries, particularly the oil industry. This upward trend is still continuing and at the moment is magnified by such developments as the growth of the synthetic-rubber industry which requires cooling towers of considerable size. The need of the oil industry to recover lighter fractions has resulted in a demand for cooling water of lower temperature, and hence for cooling towers providing a closer approach to the surrounding wet-bulb temperature.

The upward trend in the demand for cooling towers is also connected with a change in the type of tower demanded. The atmospheric and natural-draft towers so prevalent only 10 or 15 years ago have practically disappeared and have been replaced exclusively by mechanical-draft towers. The development of more efficient fans and the reduction obtainable in pumping head with the mechanical-draft tower have resulted in a lowering of over-all power requirements. The ability to increase air flow through the mechanical-draft tower results in considerably smaller surface requirements, as compared with the natural or atmospheric tower. In short, the sum of amortized first cost and power cost of the mechanical-draft tower is lower than that of any other type.

The last few years have also brought out a number of theoretical and experimental studies of cooling-tower performance, particularly of towers of the counterflow type. This type lends itself more readily to mathematical analysis and to the development of mathematical methods for predicting performance.

The variety of methods used in analyzing tower performance in the earlier literature has given place today to a common method, with all authors employing the same basic differential equation for their studies.

The cooling-tower industry should, therefore, be able now to standardize and adopt a common basic theory which would be accepted for correlation of its vast amount of experimental data and practical experience, and which could also be used to calculate performance at the guarantee point from test data taken under other conditions of service.

It is unfortunate that today each cooling-tower manufacturer uses his own method of calculation. Such methods, which may be more or less scientific, are based mainly upon empirical rules resulting from previous experiences with towers of similar construction. The present-day cooling tower, however, can no longer be based upon the experiences with older designs. The conditions imposed are more stringent, involving greater ranges, closer approaches to wet bulb, greater capacities. Cooling-tower design always has been and still must be based mainly upon experiments and tests, but the variety of conditions is so great that a correlating method for all these tests is absolutely essential, in order that cautious extrapolations can be made with some degree of accuracy for cases where direct test results are not available.

It is equally important that the user of cooling towers be familiar with the essential laws of performance and their effect upon the economics of tower selection. Too many specifications for towers clearly show that the specifying engineer was not aware of the influence of various factors on tower size and so arrived at specifications which certainly could not be called economical.

It is not the purpose of this paper to introduce a new cooling-tower theory. On the contrary, it is intended to point out the existence of a basic theory, which by now has been adopted by practically all authors on the subject, and to urge its adoption by tower manufacturers and tower users as a basis for cooling-

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

tower design. This paper will also present tests to show that the theory does allow the correlation of a great variety of test points into a common curve, and therefore that it deserves the designation of a cooling-tower theory.

Basic Differential Equation of the Cooling Tower

In 1925, Merkel (1)[2] developed the differential equation for the cooling tower, which today forms the basis of analysis of cooling-tower performance. We refer to the literature, and particularly to Nottage (2), in which this equation is redeveloped. To summarize, Merkel's reasoning is as follows:

Considering a counterflow tower of 1 sq ft ground area through which G lb per hr of air are flowing upward and L lb per hr of water are flowing downward, the following amount of heat passing through a surface element adV of water surface $dH_e = \alpha (T - t)adV$ goes to the air by convection and thereby raises its temperature by $dt = \frac{dH_e}{GC_p}$. The same surface element evaporates an amount of water $dL = K(X'' - X)adV$ which corresponds to an amount of heat by evaporation

$$dH_d = r \cdot dL = r \cdot GdX$$

The total heat released by the water is $dH = dH_e + dH_d$ and its temperature is reduced by dT, so that

$$dH = L \cdot dT = GC_p dt + r \cdot G \cdot dX \quad [1]$$

$$L \cdot dT = [\alpha(T - t) + r \cdot K(X'' - X)]adV \quad [2]$$

Equations [1] and [2] determine water temperatures and conditions of air in the tower. Merkel makes the following assumptions:

1 The Lewis relationship $\frac{\alpha}{KC_p} = 1$ holds for the cooling tower. Koch (3) comes to the conclusion that $\frac{\alpha}{KC_p} = 0.9$ is more correct, whereas London, Mason, and Boelter (4) conclude from their tests that the Lewis formula is reasonably correct for cooling towers. Under all circumstances, the error cannot be very great and the question is difficult to decide because of the impossibility of making cooling-tower tests with laboratory precision.

2 The air enthalpy can be expressed as $h = C_p t + r \cdot X$ which is an approximation, neglecting the superheat in the vapor and the heat of the liquid corresponding to the vapor content.

Using these assumptions Equation [2] can be simplified to

$$LdT = K\left[\frac{\alpha}{K}(T - t) + r(X'' - X)\right]adV = K\left[(C_pT + r \cdot X'') - (C_pt + r \cdot X)\right]adV = K(h'' - h)\,adV \quad [3]$$

which is Merkel's basic cooling-tower equation with the enthalpy difference $(h'' - h)$ as potential and K the coefficient of vapor exchange, based upon a vapor-content potential.

It is advantageous to rearrange Equation [3] so as to make each side dimensionless and integrable

$$\frac{dT}{h'' - h} = \frac{KadV}{L} \quad [4]$$

so that integrating between inlet- and outlet-water temperatures gives

$$\int_{T_2}^{T_1} \frac{dT}{h'' - h} = \frac{KaV}{L} \quad [5]$$

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Performance Curves

Equation [5] is the basic equation for calculating cooling-tower performance. As mentioned already, both sides of the equation are dimensionless figures. The left side of the equation contains only the thermodynamic conditions for the cooling process. It is determined wholly by the initial and end temperatures of the water and by the initial and end conditions of the air flowing through the tower. The right side of the equation is independent of the thermodynamic conditions in the tower and is determined by the characteristic of the tower design KaV and the liquid flow L. The diffusion coefficient for a given G depends only upon the type of surface provided in the tower. The surface provided in the usual tower is a combination of drop and film surface. There is not sufficient information available for either type of surface, so that K could be considered as known.

It is, of course, possible from tower-filling design to determine the amount of film surface per cubic foot. Unfortunately, however, it is yet quite impossible to determine the amount of drop surface produced by a specific tower design. Considerable information on drop mechanics has been published lately by Nottage and Boelter (5) and it is, of course, the idea of a cooling theory to develop sufficient information so that K and the amount of drop surface produced by a specific tower design could be determined from the design itself. This may be achieved in the future. At the moment, however, only testing can determine the tower characteristic of a given design using Equation [5].

Tests must also prove that the tower characteristic really determines tower performance over the whole range of performance conditions. Test results which will be discussed later indicate that this condition is met with sufficient accuracy.

The left side of Equation [5], as mentioned already, is entirely determined by the thermodynamic cooling conditions. A graphic presentation of this left term is shown in Fig. 1 in which the water temperature T is selected as abscissa and the enthalpy as ordinate. Saturation line h'' gives the enthalpies of saturated air at water temperatures. Since $LdT = Gdh$, it follows that in a cooling tower the air enthalpy varies as a straight line with the water temperature. If the water is to be cooled from T_1 to T_2, then h_2 at T_2 represents the inlet-air enthalpy corresponding to the inlet-air wet-bulb temperature, and the air operating line $(h_1 - h_2)$ is a straight line with slope L/G.

The integration required on the left side of Equation [5] can now be graphically accomplished as shown in Fig. 1 and is represented by area $ABCD$, which determines the tower characteristic necessary to cool from T_1 to T_2 with inlet-air enthalpy of h_2 and a given weight-flow ratio L/G.

Approximate Methods of Solution

The graphic method of integration is somewhat cumbersome for daily routine work. More unsatisfactory, however, is the problem of determining the performance of a given tower with known characteristic under varying wet-bulb conditions. It would require assuming the temperature range, integrating to determine the tower characteristic and, if different from the actual tower characteristic, repeating the process by trial and error until equality is reached; obviously too tedious a method to be of practical value.

No wonder, therefore, that most authors attempt to introduce approximate methods. In accordance with the analogy of the heat exchanger, it is logical to try to introduce a "log-mean potential." The logarithmic-mean enthalpy potential would be mathematically correct if the enthalpy potential $h'' - h$ were a straight-line function of T. Obviously, this is true only if the saturation line is straight, and so it follows that the log-mean potential can give good results only for very small ranges over

which the saturation line could be considered as approximately straight. Sufficient accuracy is obtained when the cooling range is 15 F or less. As the range increases, the curvature of the saturation line causes an increasing error. As an example, let $L/G = 0.75$; inlet wet-bulb temperature = 75 F; $T_1 = 115$ F, and $T_2 = 80$ F. The log-mean-potential method gives $KaV/L = 2.29$. The graphic-integration method gives $KaV/L = 2.80$.

The use of the approximate method in this rather typical case, therefore, leads us to underestimate the required tower characteristic by 18 per cent. A tower so selected would obviously be inadequate for the required service.

Also, test data taken under one set of conditions cannot be used accurately to predict performance under other conditions if this method is employed, since the error changes with the curvature of the saturation line. For practical cooling-tower work, the use of a log-mean potential is therefore not adequate.

Merkel, after developing first the basic equation, introduced another mean potential, so defined that $(h'' - h)\ (T_1 - T_2) =$ area h_1'', h_1, h_2, h_2''. This leads immediately to great simplifications, since the saturation line can be integrated with T, once and for all. Unfortunately, this simplified method is as inaccurate for great ranges and close approaches as the log-mean method and suffers from the other serious defect that, by its use, cooling to wet-bulb temperature seems possible. Obviously, this is a physical impossibility.

The cooling-tower industry today is called upon to design towers requiring large ranges and close approaches to the wet bulb. The simplified methods cannot be used, since they lead to too small towers. Present cooling-tower practice requires a method which is exact and rapid enough to enable selections to be made without long and tedious integration work; a method which permits determination of the performance of the tower for all wet bulbs and for any variation in operating conditions, without entailing lengthy trial-and-error integrations.

The only solution to this demand would be to integrate graphically once and for all so that any possible selection problem which might arise would be subject to immediate solution, and to co-ordinate these results in curves, forming a curve book which would be to the cooling-tower engineer what the steam tables

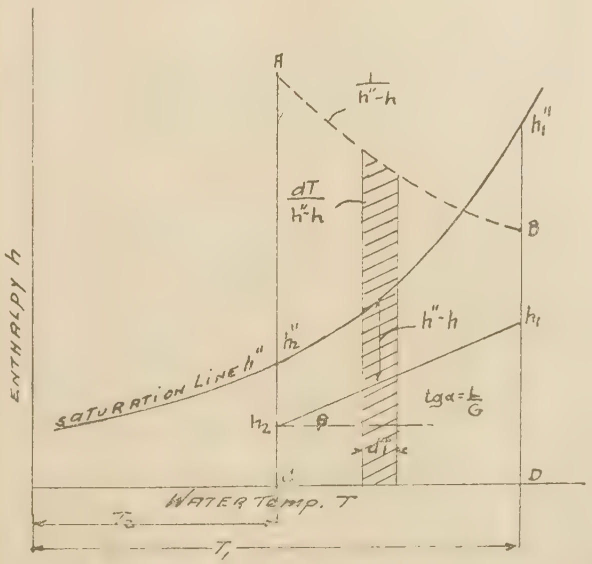

FIG. 1 GRAPHICAL PRESENTATION OF EQUATION $\dfrac{dT}{h'' - h} = \dfrac{KadV}{L}$

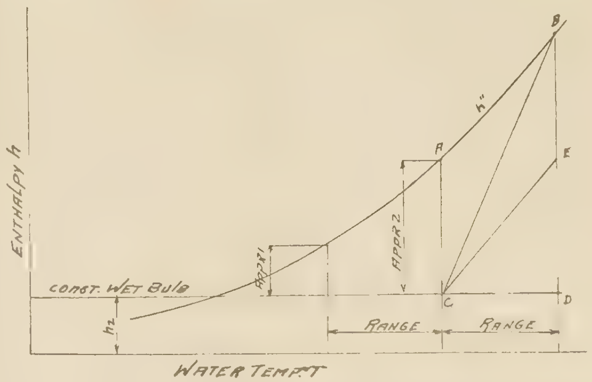

FIG. 2 METHOD FOR DEVELOPMENT OF PERFORMANCE CURVES USING RANGE AND WET BULB AS CONSTANTS AND APPROACH AS PARAMETER

are to the turbine engineer. This set of curves could also be used to predict the performance of a given cooling tower under widely varying conditions of service.

Such a curve book for cooling-tower work has just been completed by the author's company.

In order to construct these performance curves it is, of course, necessary to choose from among the many variables those which are to be used as co-ordinates, those to be used as parameters, and those to be held constant. This involves the following variables: approach to wet bulb, range, L/G, and wet-bulb temperature of the entering air. The tower characteristic KaV/L is the required factor and therefore it must be chosen as the ordinate. It seems logical to select L/G as abscissa, since it is one of the most important design characteristics. In cooling-tower practice, G is usually chosen from power-requirement considerations. If we find L/G, we then have L and therefore the necessary tower ground area for a given tower filling and a given tower capacity. As parameter for the performance curves, we have chosen the approach to the wet bulb; the wet bulb itself and the range are kept constant for each performance curve. Fig. 2 shows how these curves can now be calculated by graphic-integration methods.

Fig. 2 is shown for two approaches, 1 and 2, for constant enthalpy h_2, and constant range. Two points can immediately be indicated which limit the range of the constant-approach curves. The first is $L/G = 0$, corresponding to area $ABCD$ which gives the lowest tower characteristic for this performance; it corresponds to an infinite value of G. The next point is determined by the operating line CB and its corresponding slope or L/G. Since at B the potential is zero, the characteristic becomes infinite. In between, any number of operating lines such as CE can be drawn and KaV/L for the corresponding values of L/G determined. This is shown in Fig. 3.

Fig. 4 shows a typical performance curve for 80 F wet bulb, 35 F range.

These performance curves are calculated for ranges between 8 and 50 F in intervals of 2 deg up to 20 F and 5 deg from 20 to 50 F. Wet-bulb intervals are 5 deg from 35 to 60 F and 1 deg from 60 to 80 F. Each page covers a range of L/G from 0 to 3 and contains curves for various approaches from 2 to 30 F in steps of 1 deg from 2–8 F, 2 deg from 8–20 F and 5 deg from 20–30 F. All curves are based upon enthalpies of the saturation line and air-operating lines, corresponding to a normal atmospheric pressure of 29.92 in. of mercury.

Tower Characteristic Curves

Referring again to the basic tower Equation [5], we have con-

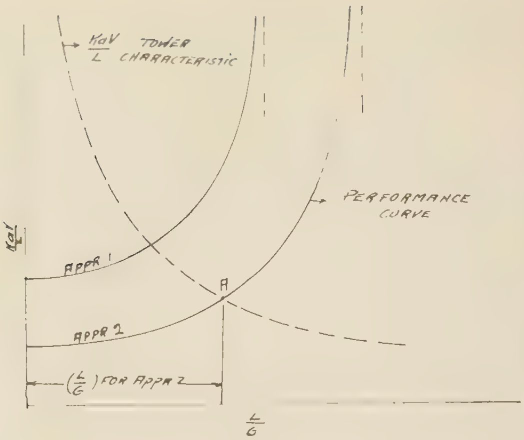

FIG. 3 EXAMPLE SHOWING INTERSECTION OF PERFORMANCE CURVE WITH TOWER CHARACTERISTIC, WHICH DETERMINES TOWER SELECTION

structed a set of curves which gives solutions for the left side of the equation, namely

$$\int_{T_2}^{T_1} \frac{dT}{h'' - h}$$

For any wet-bulb temperature range and approach, and for any chosen L/G these curves give the integrated value of

$$\int_{T_2}^{T_1} \frac{dT}{h'' - h}$$

The problem now is to find the corresponding tower which fulfills these conditions and has a characteristic equal to KaV/L. Since KaV is a function of L, as well as of G, we must construct, for each tower design, curves of its KaV/L in function of L/G. Equation [5] then states that a tower, in order to fulfill the specified conditions, is determined by the L/G ratio corresponding to the intersection of both curves, Fig. 3.

The tower characteristic curves are determined by the design features of the tower which fix the type and amount of surface provided (film surface, or drop surface, or both), the relative velocity of the two fluids, and their physical characteristics.

Unfortunately, no theory yet exists which permits us to calculate either K or aV from the design alone. Tests on KaV for various types of tower have been made and published but, because of lack of any general theory, such tests cannot be generalized and represent the characteristic of the specific design only.

Present tower practice is therefore still dependent upon testing of all tower designs, in order to find the tower characteristics. While published data concerning such tower tests are not yet sufficient to permit the deduction of a general theory of tower characteristics, they are still very useful because they at least permit us to visualize the general trend of variation of the main factors of the tower characteristic, the variation of KaV with L, G, and possibly T_1.

FILM-TYPE AND SPRAY-TYPE TOWERS

In this respect, two publications of experimental studies on cooling towers are particularly interesting. Both were made at the University of California (4, 6). The interest in these two studies lies in the fact that the authors tested two towers of a design forming the two opposite extremes of counterflow towers, namely, the pure film-type tower and the pure spray-type tower. For economic reasons, all commercial-tower designs are in between these two extremes. The characteristics of these towers should therefore be a good indication of what to expect of the commercial type of tower.

The film-type tower lends itself particularly to experimental analysis since the surface aV is known from design and is independent of L. Coefficient K alone can therefore be easily determined from test as a function of G. The authors (4) give the absolute value of K as a function of G and find that K varies as $G^{0.48}$.

The film-type tower is uneconomical, since all of its surface must be formed by lumber or other material, which makes the tower very costly.

The spray-type tower offers considerably greater difficulties of experimental analysis. In the first place, the surface aV is unknown and cannot be determined by experiment, since only the product KaV can be calculated from measurement. The total surface in the spray tower is made up of the surfaces of all the drops in the tower at a given time. The total surface therefore is proportional to the number of drops produced from L lb per hr of water, assuming an average drop diameter, and is also proportional to the time necessary for a drop to pass through the tower.

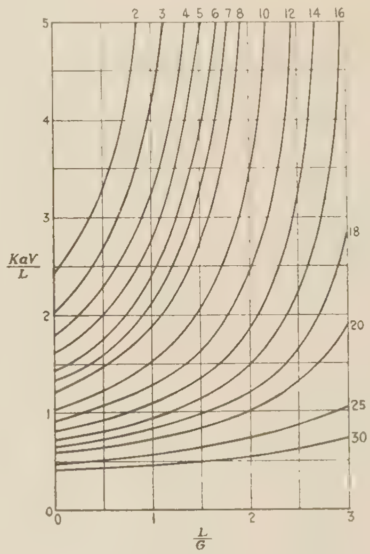

FIG. 4 TYPICAL CHART FROM PERFORMANCE-CURVE BOOK

Referring to the study on drop dynamics, mentioned previously, the authors (6) have graphically integrated the differential equations for water drops falling through air, taking air resistance into account, in order to determine the time of fall.

Although the cooling-tower industry is grateful for this most useful work, it is, however, not yet provided with the necessary theoretical means of calculating the drop surface of a spray tower from design features alone. Aside from uncertain assumptions of the drag coefficient of falling water drops in air, the determination of the average drop diameter cannot be safely predicted from the design. In the experimental determination of KaV, K cannot yet be separated from aV.

Another difficulty in spray-tower work is the fact that part of the water runs down along the walls forming a film type of surface, so that the actual performance is a combination of pure film- and pure spray-type surface. This effect will be the greater the smaller the tower ground area, and since test towers are necessarily small, a large percentage of the total liquid will flow along the tower walls.

The California experimenters circumvented this difficulty very cleverly by making over-all tests first, then shutting down the spray nozzles and measuring the performance of the wall surfaces separately. Deducting this from the over-all test permitted them to calculate the performance of the drop surface alone. The results are interesting. For the wall surface they found KaV to be dependent upon $L^{0.3}$ and to vary as $G^{0.5}$.

For the pure spray surface, KaV is independent of G and varies with the first power of L. For the combined surfaces, KaV varies of course with L to a power between 0.3 and 1 (0.76 in the California tests), and with G to a power between 0 and 0.5 (0.17 in the California tests).

The problem of the designer of a commercial tower is to provide the maximum amount of surface at a minimum cost. Drop surface is the cheapest surface but, since drop surface is proportional to the time it takes a drop to fall through the tower, the pure spray-type tower is not the most economical tower. The time of fall is small, the total surface small per square foot of ground area. In order to increase this surface, the designer tries to increase the time of fall of a drop through the tower by introducing decks which break up the distance of fall. The decks of course at the same time add film surface to the drop surface.

It is not the purpose of the decks so much to provide this film surface as to increase the amount of drop surface. The commercial tower is therefore also a combination of film and drop surface, with the latter, however, forming the greater part of the total surface. The characteristic of these towers depends of course upon the type of filling which forms the decks, the number of decks and their distance, and the method employed of dividing the water into drops. For purposes of this paper, we are not interested in surveying the characteristics of all types of towers but rather in the general trend of these characteristics. To this end, it will suffice to show the characteristics of a type built by the author's company.

Test Data on a Commercial Type of Tower

The Foster Wheeler Corporation has been building cooling towers for the last 35 years and has therefore accumulated a considerable amount of test data on all types and designs of towers. This test program has been intensified to a great extent in the last 2 years as a result of the greater demand for towers and particularly because of the rapid changes and improvements made in tower design and tower fillings.

Tests have been made, for example, on a tower of 6 ft × 6 ft ground area and 35 ft height, so that the number of decks and their spacing can be widely varied. Fig. 5 shows the general testing arrangement, and we shall omit here detailed description of the testing instruments which are standard for this type of work. Air is forced into the tower by a 7-ft-diam axial-flow fan through a long suction tunnel provided at the inlet with a calibrated nozzle. Pitot-tube and differential pressure measurements determine the amount of air handled. Wet-bulb measurements of the entering air are also taken at the nozzle inlet. A suction line from the tower basin conducts the cooling water to the centrifugal pump, circulating it through a steam heater, where the heat load is provided. Cold-water temperatures are taken at both the suction and discharge sides of the pump, and hot-water temperatures are taken at two check points in the discharge line from the heater to the tower. An orifice in the discharge line is used to measure water quantity. Wet-bulb measurements of the discharge air from the tower are taken so that heat balances between air and water can be established to check the accuracy of the test points. All test points having an error of more than 10 per cent on the heat balance are rejected. The number of points with an error as great as 10 per cent is, however, small. The whole arrangement is flexible enough to allow great variations in all the variables involved. Range, inlet-water temperature, inlet wet bulb, and L and G variations are reached by continuous testing all year round, so that wet bulbs between 27 and 84 deg can be observed.

In Table 1 is given a series of test points on a tower consisting of ten decks on 15 in. centers, and with a filling consisting of slats $^3/_8$ in. × 2 in. spaced $1^1/_8$ in. on centers. This filling has been used on Foster Wheeler towers but has been abandoned in

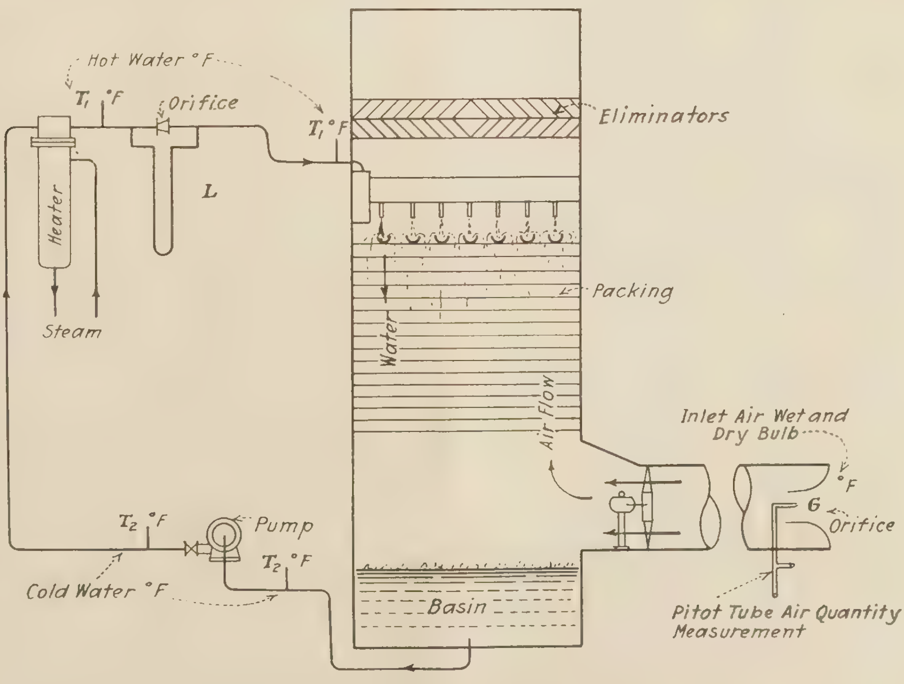

Fig. 5 Diagram of Test Arrangement of Model Tower

favor of later modifications. The test points selected are widely separated values of G to show the effect of G on KaV. Although G could not be held exactly constant, the variation around the average is small enough to keep the error within 4 per cent, which is well within the accuracy of determination of G.

Fig. 6 shows graphically these test points giving KaV as a function of L for constant values of G equal to 1680. 1220, and 644 lb per sq ft per hr. The variation of KaV with L is similar in trend to that of the spray tower KaV varying approximately as $L^{0.4}$ which was to be expected. Very different from the spray tower, however, is the variation with G at constant L. Expression KaV varies as $G^{0.5}$, the same variation as observed in the pure-film-type tower. The explanation may be that in the spray tower, due to greater height of fall, the drop velocity is high compared to the air velocity so that G has little effect whereas in the deck tower the drop velocity is small. The variation of KaV with $G^{0.5}$ has been observed in all types of deck towers and seems to be a well-established law for such towers.

Model and Full Execution

Cooling-tower tests are made on small towers, that is, small in ground area only, there being no scale factor involved, and the theory of similarity is not applicable. In fact the tower theory just outlined is based on the idea that a large tower is the sum of a number of small towers each of 1 sq ft ground area. If therefore each square foot receives the same amount of water and air, no difference in performance between a small and a large tower could be expected. However, in a small tower, the effect of the water running down the walls does reduce the over-all performance, and since with increased ground area the percentage of the water flowing along the walls becomes smaller, an improvement in performance of larger cells can be expected. Fig. 7 shows the effect on the tower characteristics of full-scale tower cells compared to the test model for an average G of 1220 lb per sq ft per hr.

TABLE 1 TYPICAL TEST POINTS OF MODEL TOWER FOR AIR WEIGHT FLOWS AVERAGING 1680, 1220, AND 644 LB PER HR

Test point	ΔT range	Approach	t_{wb} wet bulb	L	G	L/G	$\frac{KaV}{L}$	KaV
			G average = 1680					
16	29.0	55.6	30.4	2915	1721	1.69	0.76	2215
17	31.5	53.2	32.8	2645	1721	1.54	0.80	2105
18	34.6	52.8	33.2	2430	1705	1.42	0.82	1990
19	38.0	51.4	33.6	2220	1705	1.30	0.88	1950
23	16.0	27.6	37.4	1700	1710	0.99	1.08	1837
28	31.8	29.8	41.4	1250	1730	0.72	1.28	1600
29	34.5	28.5	42.0	1140	1712	0.66	1.37	1564
30	34.2	29.8	45.0	1350	1712	0.79	1.24	1672
32	41.8	25.6	45.6	970	1715	0.57	1.51	1463
33	48.8	25.4	44.8	830	1719	0.48	1.69	1402
35	24.0	16.8	49.2	905	1731	0.52	1.57	1420
36	24.0	18.2	50.8	1050	1668	0.63	1.46	1532
37	20.8	20.6	48.6	1200	1722	0.70	1.27	1525
44	31.4	18.8	52.8	930	1644	0.57	1.52	1413
45	39.6	18.6	51.8	765	1640	0.47	1.68	1284
46	14.4	7.4	65.2	835	1645	0.51	1.55	1293
47	17.7	7.1	65.2	695	1640	0.42	1.74	1210
50	34.6	2.6	69.8	345	1606	0.21	3.12	1077
51	32.2	5.8	67.0	490	1600	0.31	2.32	1136
53	29.4	12.6	59.0	780	1650	0.47	1.73	1350
54	40.2	17.2	57.6	835	1675	0.50	1.57	1310
55	42.2	15.0	57.8	725	1675	0.43	1.76	1275
56	45.0	14.4	53.6	585	1665	0.35	2.01	1175
57	44.0	10.0	53.0	415	1665	0.25	2.54	1055
68	35.0	27.0	60.0	1880	1634	1.15	1.01	1900
			G average = 1220					
1	33.8	27.2	61.0	1345	1260	1.07	0.95	1280
2	30.4	28.0	60.6	1490	1240	1.20	0.89	1328
3	30.6	24.8	61.6	1250	1235	1.01	0.98	1225
4	34.5	18.9	66.6	1110	1228	0.90	1.17	1300
5	27.5	22.1	53.4	970	1290	0.75	1.29	1250
6	31.5	20.5	54.0	835	1285	0.65	1.40	1170
8	16.0	17.4	52.6	1050	1280	0.82	1.24	1300
9	19.2	18.8	51.0	900	1285	0.70	1.26	1132
11	16.6	21.4	50.0	1195	1290	0.93	1.09	1300
14	19.2	12.8	56.0	680	1220	0.56	1.54	1048
17	32.2	6.8	61.0	350	1115	0.31	2.51	880
19	29.4	13.6	61.0	625	1100	0.57	1.66	1040
20	28.6	15.4	61.0	765	1090	0.70	1.51	1155
21	18.0	19.0	67.0	1460	1170	1.25	0.88	1285
34	21.0	28.0	65.0	2250	1180	1.90	0.73	1645
37	26.0	22.6	69.4	1600	1220	1.31	0.85	1360
38	23.0	23.6	69.4	1895	1205	1.57	0.80	1515
39	19.5	26.2	65.8	2225	1230	1.81	0.75	1670
40	20.0	29.6	64.4	2435	1240	1.96	0.67	1630
41	18.5	29.5	65.0	2640	1240	2.13	0.65	1720
			G average = 644					
7	20.0	27.6	52.4	835	650	1.29	0.95	794
8	24.6	25.8	52.6	695	650	1.07	1.11	773
9	12.4	15.6	58.0	710	632	1.12	1.13	803
11	11.6	20.2	58.2	970	617	1.57	0.87	845
12	10.6	21.2	58.2	1110	635	1.75	0.78	865
19	20.0	23.2	59.8	835	690	1.21	0.91	760
20	22.2	21.6	60.2	695	677	1.03	1.02	709
21	21.2	14.8	63.0	550	660	0.83	1.31	720
23	32.2	28.2	60.6	835	650	1.28	0.93	776
24	35.8	27.8	60.4	765	648	1.18	0.99	758
25	31.0	19.6	63.4	625	630	0.99	1.27	795
26	34.6	17.8	62.6	485	634	0.69	1.32	640
29	16.0	26.4	67.6	1460	625	2.33	0.68	994
30	16.0	29.8	68.2	1805	622	2.89	0.58	1050

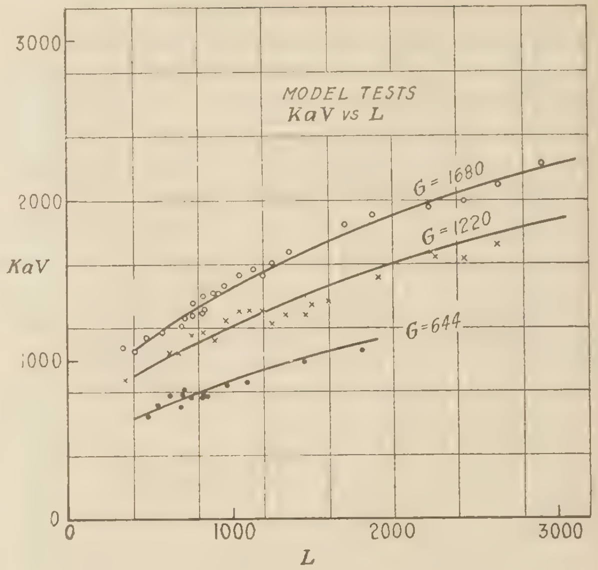

Fig. 6 Plot of Test Points From Table 1 Showing KaV Versus L for G = 1680, 1220 and 644 Lb per Hr

These field tests were made on a number of towers varying in cell size from 480 to 600 sq ft of ground area. A constant increase of about 13 per cent in KaV is observed. It is not probable that with yet larger cells the increase would be greater since, even at these sizes, the percentage of water flowing along the walls is negligible.

Tower Characteristic Curves

For each type of tower design, experiments must establish the tower characteristic giving KaV as a function of L for various values of G. Once these curves are available, they can be changed to form the tower characteristic by making the ordinate KaV/L instead of KaV and by making the abscissa L/G instead of L. Fig. 8 shows these tower characteristics using the test curves given in Fig. 7.

These tower characteristics vary with G but, as can be seen in Fig. 7, the characteristic is practically the same within the exactitude of testing, for G at 1680 and G at 1220, which is the range used in cooling-tower work. This simplifies selection considerably since there is only one curve to deal with for each type of tower.

In this connection the following reasoning could be applied: Equation [5] is already dimensionless. If K, aV, L, and G are the only supposed variables in cooling-tower performance, dimensional analysis would arrive at these two dimensionless groups. Accordingly, all test points should group themselves into one curve if the groups KaV/L and L/G are used. In other words, from the standpoint of dimensional analysis one tower

characteristic curve, independent of G, should exist. This would require the following relationship:

From tests we find

$$KaV = cL^X \cdot G^y$$

Divided by L

$$\frac{KaV}{L} = cL^{X-1} \cdot G^y$$

Dimensional analysis requires

$$\frac{KaV}{L} = c\left(\frac{L}{G}\right)^z = cL^z \cdot G^{-z}$$

Therefore

$$X - 1 = -y$$

or

$$X + y = 1 \qquad [6]$$

If the inlet temperature T_1 should be found also to influence tower performance (as in the spray tower of the California test),

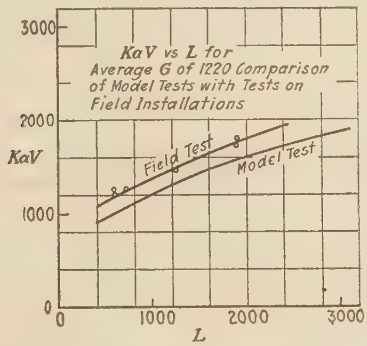

Fig. 7 Comparison of Performance of Model and Full-Size Towers

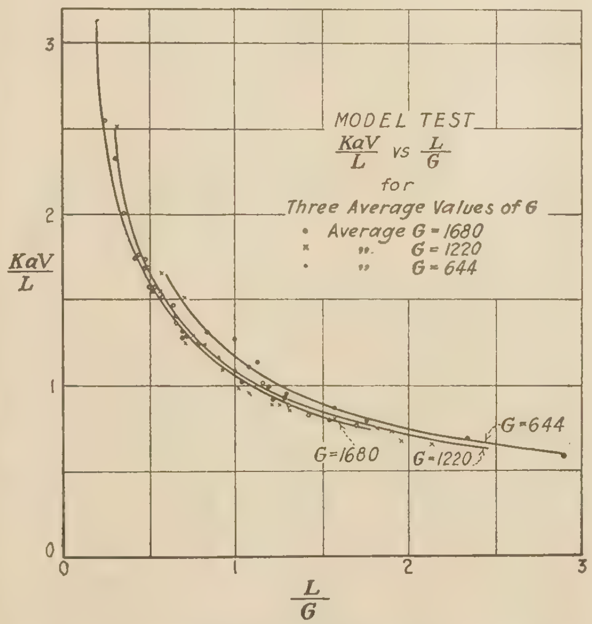

Fig. 8 Characteristic of Model Tower Calculated From Test Points of Table 1

it could only combine with K, which alone has temperature as a dimension. We would then have a family of tower characteristics with T_1 as parameter but for each of which Equation [6] would hold.

In our case $X + y = 0.9$. The California test on the spray tower gave $X = 1$, $y = 0$ for pure spray, $X + y = 0.93$ for the combination of spray and wall surface.

We are now making additional tests to determine experimentally whether Equation [6] holds and are inclined to believe from theoretical and other available test results that the equation is reasonably correct.

Should this be the case, the testing procedure for determining tower characteristics would be enormously simplified since two test points, varying either G or L, would be sufficient.

In accordance with basic Equation [5], we have the two sets of curves which represent the right and left sides and accordingly, if a specific tower design is selected, the solution of Equation [5] is determined by the intersection of both curves for a specific set of values of wet bulb, range, and approach. The solution gives the necessary ratio L/G in order to fulfill the specified cooling conditions.

Cooling-Tower Selection

The size of a cooling tower is determined when L, the value of pounds of water per square foot per hour, is known, whereas Equation [5] determines only the ratio L/G. The designer must therefore select G from other considerations before the cooling-tower size is fixed. Let us first consider extreme conditions. Assume that a tower is to be selected for a close approach to the wet bulb and a large range. This will lead to a small L/G ratio and, since G cannot be increased beyond a certain amount, L is necessarily small. Experiments on all cooling towers show that their characteristics break sharply when L reaches a critical point varying with design but roughly around L equal to 500 lb per sq ft per hr. This is due to the fact that the water is unable to spread sufficiently and cover evenly the whole available surface. Term L must be selected on the curve and not on the break. In such case, where L falls on the break, the tower design is unsuitable for the conditions. A design must be selected giving a larger L/G ratio and this leads to a tower having a great number of decks. Close approaches and large ranges require high towers with high pumping heads.

The opposite extreme is the selection of a tower for a large approach and a small range which leads to a large L/G ratio but, on this side too, the tower characteristic breaks, due to flooding, at a critical L, which depends upon the design and is roughly around 3000 lb per sq ft per hr. Since it is uneconomical to reduce G too much, and since L/G must be made smaller in order not to work on the break, a type of tower must be selected having a small number of decks. Such towers are low-head type with low pumping heads.

Average conditions are in between these two extremes. The theory establishes the ratio L/G for a fixed design. Even if a fixed design is selected, there is still an infinite number of sizes possible depending upon the selection of G. A small G means a small L with a large tower ground area, high first cost, but low fan-power requirements, whereas with a large G, the reverse is true. The solution naturally is fixed by selecting G so that the cost of fan power (plus pumping power if variation in design is also considered) plus the cost of capital charges is made a minimum; in other words G is determined by economic considerations.

It is not the purpose of this paper to discuss all the variables of cooling-tower design which affect its first cost. There are too many of them and any attempt to combine them in mathematical form in order to draw general conclusions only leads to long formulas from which no simple conclusions can be drawn. The

job has to be done by making a few selections, calculating total cost, and determining the minimum point. This fixes the economic selection for a given design. If the process is repeated for various designs an absolute optimum selection can be made. The experienced engineer quickly develops enough feeling in the matter to make the process much quicker and simpler than this description would indicate. For us the important conclusion is that, in the final analysis, the economics involved determines the tower size.

Many authors on the subject of cooling-tower theory feel compelled to introduce some scale of measurement of the degree of perfection of the tower. The ideal tower to some is one which heats the air to the inlet-water temperature, to others a tower which cools the water to the wet-bulb temperature. All towers are then compared to the ideal tower, and an effectiveness coefficient or "efficiency" of the cooling tower is introduced. We fail to see of what possible use such a coefficient could ever be for cooling-tower practice. The tower manufacturer is able to build towers of any degree of effectiveness or efficiency if one is willing to pay the price. Building such towers, however, would be no particular achievement. To the cooling-tower industry, the most efficient tower is simply the most economical tower.

The designer determines the most economical tower on the basis of fixed conditions given to him and over which he has no control. The wet bulb, the approach to the wet bulb, the range, and the capacity are all fixed. But these fixed conditions are themselves determined by economic considerations, except that this work belongs to another group of engineers, those who study the application of the tower to a particular problem.

If the cooling tower is to provide cooling water to a group of heat exchangers, it is a question whether it is economical to put more money into the tower and save heat-exchanger surface or vice versa. Here again the economic selection will be such that the sum of all costs involved will be at a minimum. The application engineer, however, is handicapped in his work because of lack of knowledge of how changes will affect the size of the tower.

Suppose the range is already fixed as well as the wet-bulb temperature; the engineer wishes to determine how the cooling-tower size is affected by different selections of approach to this wet bulb. The answer cannot be given to him in general terms. The curve of size versus approach depends upon the value of the wet bulb itself. The same is the case if he has fixed the wet bulb and the approach and wants to know the curve of size versus range; or his range and approach is fixed and he wants to know the variation of size with wet bulb itself. For economic calculations, this knowledge is necessary. The book of performance curves allows one, of course, to determine these functions easily for all possible conditions. Co-operation between purchaser and manufacturer for the best economic selection is the only answer. To show the trend of these curves, we have calculated them for a typical example, namely, 75 deg wet bulb, 25 deg range, and 10 deg approach.

Fig. 9 shows these curves, in which the size corresponding to the selected condition for a typical tower design is called 100 per cent. The curves then show percentage variation of the tower size if any two of the conditions are kept constant and the others varied. May we particularly point out the enormous increase in size of tower with closer approach to the wet bulb of 75 F. If a smaller wet bulb than 75 F had been selected the change would have been still more marked; less marked, however, for a higher wet bulb. The same general trend is true for the other two curves.

We have shown how the size of a cooling tower for a given set of conditions can be fixed. There are other questions entering into the selection of a tower which, however, are not the object

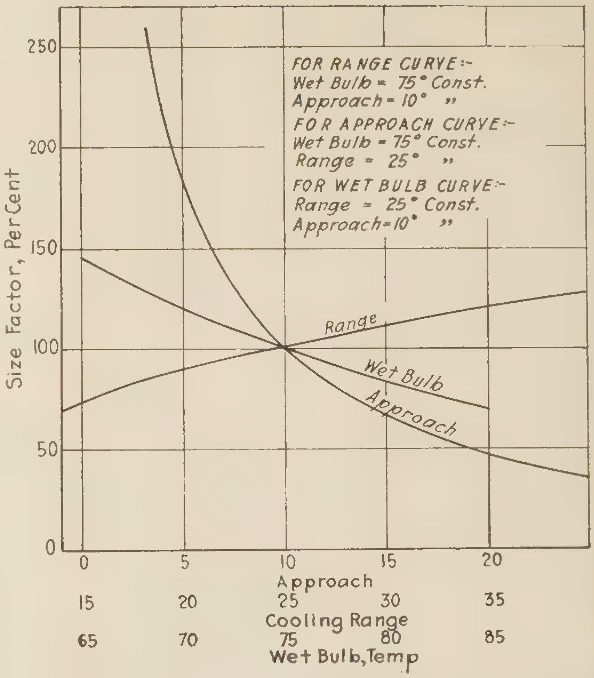

Fig. 9 Effect of Variations in Performance Requirement on Ground Area for Fixed Tower Design With Constant Unit Air Flow

of this paper and can be disposed of briefly. Once the size of the tower is fixed the question of fan arrangement comes up.

Forced-Draft Versus Induced-Draft Tower

In the forced-draft tower the fan equipment is located on the ground and is easily accessible; the fans blow the air through the tower and discharge it at a relatively low velocity. In the induced-draft tower the fan equipment is on top of the tower suctioning the air through the tower and discharging it with a relatively high velocity. This high-velocity discharge represents wasted energy in so far as the tower performance is concerned. The power requirements of the induced-draft tower are therefore greater than those of the forced draft.

However, the induced-draft tower, because of its high discharge velocity, reduces recirculation to a greater extent than the forced-draft tower. Recirculation occurs under unfavorable wind conditions when part of the hot discharge air of the tower is sucked back by the forced-draft fans into the tower, uniting with the fresh air, raising the air entering wet-bulb temperature, and so reducing the tower performance.

This question of selection can also be decided on economic considerations. The additional cost of fan power must be balanced against the loss of performance due to recirculation. If the number of days in the year in which recirculation occurs is known, and the loss of performance can be estimated, such economic calculations should enable one to decide this issue too.

In general the effect of recirculation is more important in the case of close-approach towers, justifying in general the selection of an induced-draft tower. In the case of larger approaches the advantages of forced draft carry more weight.

Summary

A graphic method of solution of the basic equation has been shown which, in the form of curves, allows quick and exact answers to all cooling-tower problems. The adoption of the

basic equation by the cooling industry is urged so that a common method of correlation of tests as well as a common method of translation of performance from one set of conditions to another may be available to the industry. It is further shown that the size of a cooling tower for a fixed set of conditions is not entirely determined by theory alone, but that economic considerations finally determine the tower size.

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Heat Transfer to a Fluid Flowing Periodically at Low Frequencies in a Vertical Tube

By R. C. MARTINELLI,[1] L. M. K. BOELTER,[2] E. B. WEINBERG,[3] and S. YAKAHI[4]

There are available in the literature many data for the transfer of heat from a tube to an enclosed fluid flowing with steady, unidirectional motion. Knowledge with respect to the applicability of such steady flow data to the analysis of heat transfer from a pipe to a fluid, the velocity of which is a periodic function of time, is necessary for the solution of certain types of heat-transfer problems. An outstanding example of a system in which such periodic heat transfer occurs is the internal-combustion engine under detonating and nondetonating conditions. The experiments (1)[5] described in this paper are the beginning of a long-range program on the subject of heat transfer in the cylinders of internal-combustion engines and, although the results of the tests are not immediately applicable to engine analysis, some light is shed on the phenomenon.

Nomenclature

The following nomenclature is used in the paper:

A = heat-transfer area, sq ft

A_p = pipe cross-sectional area, ft²

c_p = unit heat capacity of fluid, Btu/lb °F

D = tube diameter, ft

e = Naperian base of logarithms

f_{am} = unit thermal conductance at solid-fluid interface defined by

$$f_{am} = \frac{q_{avg}}{A\,\Delta t_{am}}, \quad \text{Btu/hr ft}^2\ \text{°F}$$

f_{lm} = unit thermal conductance at solid-fluid interface, defined by

$$f_{lm} = \frac{q_{avg}}{A\,\Delta t_{lm}}, \quad \text{Btu/hr ft}^2\ \text{°F}$$

g = gravitational force per unit mass = 32.2, $\text{lb}\Big/\left(\dfrac{\text{lb sec}^2}{\text{ft}}\right)$

k = thermal conductivity of fluid, Btu/ft² hr (°F/ft)

L = pipe length from beginning of heat-transfer section, ft

P = pressure, lb/ft²

Q = time average volume flow, cu ft/sec

q_0 = heat gained by fluid passing through heating section, Btu per hr

q_s = heat lost by steam in heat exchanger, Btu per hr

q_{avg} = average heat-transfer rate in tube length L, Btu per hr

rpm = revolutions per minute of pump = 60 × frequency in cycles per sec

$t_m = \dfrac{t_0 + t_{mL}}{2}$ = arithmetic average of mixed mean inlet- and outlet-fluid temperatures, °F

t_{mL} = mixed mean temperature of fluid leaving heat exchanger, °F

t_0 = inlet-fluid temperature, °F

t_w = tube-wall temperature, uniform with length and around periphery, °F

T = absolute temperature, °R

$\Delta t_{am} = \dfrac{(t_w - t_0) + (t_w - t_{mL})}{2}$ = arithmetic-mean temperature difference between tube wall and fluid, °F

Δt_{lm} = logarithmic-mean temperature difference between tube wall and fluid, °F

v = instantaneous mean velocity at any pipe section, fps

$\tilde{v}_m = \dfrac{Q}{A_p}$ = time average velocity at any pipe section fps

v_{max} = maximum value of instantaneous mean velocity at any pipe section, fps

v_m = average fluid velocity at any section (for steady unidirectional flow), fps

V = volume, cu ft

W = weight flow of fluid, lb per hr

x = any distance from beginning of heat-transfer section, ft

β = coefficient of expansion of fluid = $\dfrac{1}{V}\left(\dfrac{\partial V}{\partial T}\right)_P$, (°F)$^{-1}$

ν = kinematic viscosity of fluid, ft²/sec

ϕ = crank angle, radians

ρ = mass density of fluid, lb sec²/ft⁴

μ = viscosity of the fluid, lb sec/ft²

Dimensionless Moduli

F_1 = function of $\pi Nu_{am}/Gz_m$ which appears in Equation [3a], because of the utilization of the arithmetic-mean temperature difference in the definition of the unit conductance f_{am}, see Equation [3b]

F_2 = function of $\pi Nu_{am}/Gz_m$ which appears in Equation [3a], because the buoyant force acting on the fluid becomes smaller as the fluid approaches the tube-wall temperature, see Equation [3c]

F_3 = function of $\pi Nu_{lm}/Gz_m$ which enters Equation [4], because the buoyant force on the fluid decreases as the fluid approaches the pipe-wall temperature, see Equation [3e]

$Gr = \dfrac{D^3\rho^2\beta g(t_w - t_0)}{\mu^2}$ = Grashof modulus, utilizing the temperature difference $(t_w - t_0)$

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[5] Numbers in parentheses refer to the Bibliography at the end of the paper.

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$\left(\frac{GrPrD}{L}\right)_w$ = product of Grashof and Prandtl moduli and D/L

The fluid properties are evaluated at the tube-wall temperature, t_w

Gz_m = $(Wc_p/kL)_m$ = Graetz modulus, in which fluid properties are evaluated at arithmetic mixed mean fluid temperature, t_m

Gz_{mi} = instantaneous magnitude of Graetz modulus, properties evaluated at t_m

Nu_{am} = Nusselt modulus, based on unit conductance f_{am}, defined in terms of arithmetic-mean temperature difference between tube wall and mixed-mean temperatures of fluid

Nu_{lm} = $\frac{f_{lm}D}{k}$ = Nusselt modulus, in which unit conductance is based on logarithmic-mean temperature difference between tube wall and fluid

$\widetilde{Nu}_{lm}$ = Nusselt modulus for periodic fluid flow, based on logarithmic-mean temperature difference between tube wall and fluid

The thermal conductivity of the fluid, k, is evaluated at the average of the mixed mean inlet- and outlet-fluid temperatures, t_m

Nu_{lmi} = instantaneous magnitude of Nusselt modulus, which is obtained from steady-state unidirectional performance curve for each value of Re_{mi}

Nu_0 = Nusselt modulus calculated from observed rates of heat transfer for steady unidirectional flow

$\widetilde{Nu}_0$ = Nusselt modulus calculated from observed rates of heat transfer for periodic fluid flow

Nu_{oi} = Nusselt modulus, calculated from observed rates of heat transfer, each value chosen from steady-state unidirectional curve Nu_0 versus Re_m for magnitudes of Re_{mi}

Nu_R = Nusselt modulus equivalent to scale resistance for steady unidirectional flow

Pr = $\frac{3600\ \mu c_p g}{k}$ = Prandtl modulus

Re_m = $\left(\frac{v_m D}{\nu}\right)_m$ = Reynolds modulus for steady unidirectional flow, based on fluid properties at arithmetic average of mixed mean fluid temperatures at entrance and exit, t_m

$\widetilde{Re}_m$ = $\left(\frac{\tilde{v}_m D}{\nu}\right)_m$ = Reynolds modulus for periodic flow, properties evaluated at t_m

Re_{mi} = $\left(\frac{vD}{\nu}\right)_m$ = instantaneous magnitude of Reynolds modulus, properties evaluated at t_m

Experimental Equipment

In the periodic-flow tests to be described, the rate of heat transfer was measured from a vertical 0.422-in-diam tube to water being pumped through it by a low-speed (13–265 rpm) reciprocating pump.

The experimental equipment (2) consisted of a 4.44-ft vertical heat exchanger (a complete description of which is given in Appendix 1) to which the test fluid (water) was supplied either by means of a small centrifugal pump, or by means of a single-cylinder single-acting reciprocating pump with the air chamber removed. A schematic diagram of the experimental equipment is shown in Fig. 1.

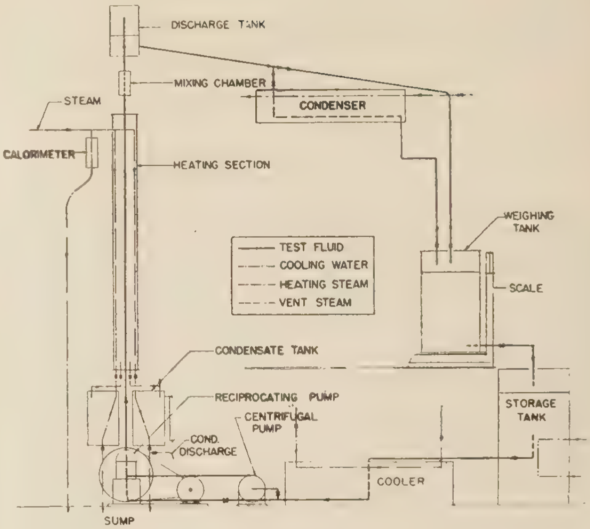

Fig. 1 Schematic Diagram of 4.44-Ft Heating Section

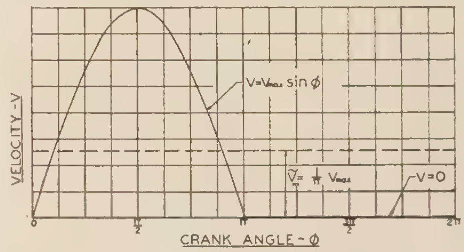

Fig. 2 Fluid Velocity Variation With Pump Crank Angle (Time)

(The fluid velocity is postulated proportional to piston velocity during the discharge stroke.)

As a basis for analysis, steady unidirectional fluid-flow heat-transfer data were first obtained on the vertical heat exchanger utilizing the centrifugal pump to transfer the test fluid (water). After the steady-unidirectional-flow performance of the exchanger had been well established, the reciprocating pump was utilized to obtain the periodic-flow data. This particular type of pump was chosen as one producing an extreme case of periodic flow, for the pump causes fluid to flow during one-half revolution of the crank, fluid flow being zero during the remaining half revolution. For all practical purposes, the higher harmonics introduced by the finite connecting-rod length can be neglected and the flow produced by the pump may be expressed as:

$$v = v_{\max} \sin \phi \qquad \text{for } 0<\phi<\pi \quad [1]$$

$$v = 0 \qquad \text{for } \pi<\phi<2\pi \quad [2]$$

A plot of this velocity variation is shown in Fig. 2. The maximum velocity of the cycle is thus related to the average rate of flow by the relation

$$v_{\max} = \pi\left(\frac{Q}{A_p}\right) \quad [3]$$

Experimental Data, Steady Unidirectional Flow

The data obtained for the steady-flow conditions are tabulated in Table 1. The magnitudes of the Nusselt modulus listed in Table 1 are greater than those directly observed, the latter being lower because of the presence of a thin oxide film on the inner surface of the copper tube (Appendix 2). A plot of Nu_{lm}, as a function of the Reynolds modulus based on the fluid properties at the arithmetic average mixed mean temperature is

shown in Fig. 3. Although the steady-state performance was studied mainly as a preliminary step in explaining the periodic-flow data, the points shown in Fig. 3 merit a brief discussion.

Between magnitudes of the Reynolds modulus (Re_m) of 1000 and 10,000, two distinct magnitudes of the Nusselt modulus obtain at a particular value of Re_m. The lower branch of the curve corresponds to flow conditions which tend to maintain viscous flow (i.e., low tube-wall temperature, very steady flow, etc.) while, in contrast, the points along the upper branch correspond to test conditions which tend to induce turbulence. Thus it may be stated that in nonisothermal systems, as in isothermal flow systems, there exists a transition region in which the rate of heat transfer is unstable and in which the magnitude of the Nusselt modulus depends upon flow conditions which are not described by the magnitude of the Reynolds modulus.

The rising portion of the lower branch of the curve yields a magnitude of Nu_{l_m} which rises almost linearly as Re_m is increased. In a recent paper Norris (3) notes finding a similar variation of Nu_{l_m} with Re_m, using a mineral oil and askarel. Norris concluded that "the surface-heat-transfer coefficient (in the transition region) is directly proportional to the velocity." The presence of two branches on the curve in Fig. 3 illustrates that

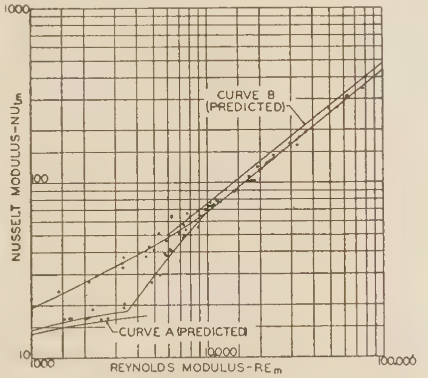

FIG. 3 EXPERIMENTAL NUSSELT MODULUS Nu_{l_m} PLOTTED AS A FUNCTION OF REYNOLDS MODULUS Re_m FOR STEADY UNIDIRECTIONAL FLOW

(Curve *A* represents the predicted behavior, reference 4, in the viscous region, and curve *B* represents the predicted behavior in the turbulent region, Equation [4]. The points represent experimental data.)

this conclusion is not universally applicable, for under certain flow conditions points may lie along the upper branch. However, for very viscous fluids with high vapor pressure and low coefficient of expansion, such as Norris utilized, the maintenance of turbulence at low Re_m, even at high wall temperatures, may be sufficiently difficult to eliminate possibility of performance along the upper curve.

In order to allow a plot of all data on one curve sheet, magnitudes of the Nusselt modulus in the viscous region have been plotted utilizing the log-mean temperature difference (l.m.t.d.) as the basis for calculation of the unit conductance. It should be emphasized that this temperature difference becomes useless when the fluid leaves the exchanger at almost the tube-wall temperature, for under this condition an error of a fraction of a degree in the outlet temperature will cause a large error in the l.m.t.d. The arithmetic-mean temperature difference is then the logical basis for the calculation of the Nusselt modulus in the viscous region, unless a direct comparison with turbulent-flow data is desired.

The equation

$$Nu_{l_m} = 1.75 \sqrt[3]{\left[Gz_m + 0.0722\, F_3 \left(\frac{GrPrD}{L} \right)_w^{0.84} \right]} \quad \ldots\ldots [4]$$

TABLE 1 EXPERIMENTAL DATA, STEADY FLOW

Run	W	q_o	q_s/q_o	t_w	t_o	t_{mL}	Re_m	Nu_{1m}
1F	458	33,000	102.5	213.5	80.5	152.5	11,690	80.0
3F	182	17,900	94.5	213.2	67.5	166.5	4,650	42.7
4F	70*	8,800	- - -	230.5	87.5	214.0	2,340	29.2
5F	764	42,400	94.5	202.0	79.5	135.5	17,800	105.0
6F	1130	53,700	89.5	195.0	87.5	134.0	27,100	151.0
7F	2280	66.100	98.0	168.2	85.5	114.5	49,300	207.0
8F	758	44,400	96.8	204.5	76.0	134.5	18,600	104.0
9F	2190	85,500	95.5	204.7	96.5	135.5	55,500	257.0
10F	433	33,200	- - -	213.5	71.0	147.5	10,250	73.8
12F	65	7,820	90.5	220.6	84.0	203.5	2,120	27.0
13F	341	28,900	96.1	226.0	77.2	161.0	9,000	65.0
14F	42*	4,900	- - -	216.5	92.0	208.0	1,420	24.0**
15F	2650	91,500	99.5	203.0	112.5	147.0	76,200	346.0
22F	1038	65,400	99.5	234.5	108.0	171.0	32,600	164.0
23F	1132	71,500	99.5	228.0	97.5	160.5	32,400	177.0
24F	800	64,400	98.5	244.0	93.5	174.0	23,800	139.0
29F	1100	63,600	99.5	214.0	89.7	147.5	28,600	163.0
1	938	47,500	101.0	196.4	92.0	142.7	24,380	144.0
2	887	43,700	100.6	194.0	92.5	141.7	23,000	134.0
3	756	42,700	98.0	194.6	87.5	144.0	19,400	128.5
4	427	32,100	98.0	204.1	73.5	148.7	10,480	78.5
5	1302	66,700	98.7	210.4	99.5	150.7	36,450	196.0
6	816	54,900	96.6	221.5	89.3	156.7	22,400	131.5
7	750	52,000	98.1	222.0	87.3	156.8	20,380	119.3
8	751	51,100	103.0	219.2	84.5	152.5	19,820	117.8
9	641	48,200	98.8	224.4	83.8	159.0	17,350	108.2
10	540	44,300	97.5	230.5	80.5	162.5	14,600	94.5
10b	237	15,250	94.5	209.8	71.0	135.2	5,360	29.5
11	493	30,500	94.0	191.9	74.5	136.5	11,460	78.4
12	483	29,450	97.5	193.3	76.5	137.5	11,400	73.8
13	600	34,900	97.5	194.9	79.8	138.0	14,460	88.8
14	312	19,950	97.8	201.8	83.0	147.0	7,940	40.9
15	159	9,950	90.0	213.1	79.0	141.7	3,880	20.0
16	137	9,400	88.5	212.8	77.5	146.5	3,370	19.4
17	2360	53,400	99.7	166.6	104.8	127.4	60,800	290.0
18	2360	54,800	97.7	166.3	105.5	128.7	61,300	312.0
19	2380	55,200	97.3	168.5	106.1	129.3	62,100	310.0
20	700	36,400	98.5	189.4	84.8	136.8	17,130	102.0
21	309	20,200	94.5	194.1	75.0	140.5	7,360	49.9
22	146	10,150	90.8	207.9	70.5	140.2	3,380	20.4
23	1348	57,100	96.8	194.7	95.8	138.2	33,600	183.0
24	876	48,400	98.7	200.6	84.0	139.3	21,650	129.2
25	454	33,600	95.5	210.2	72.0	146.0	10,930	74.2
26	454	33,600	96.5	209.3	70.3	144.3	10,740	73.7
27	346	28,100	93.8	219.6	75.5	156.7	8,900	55.2
28	284	23,900	95.5	222.6	73.8	158.0	7,300	48.5
29	247	20,900	95.2	224.1	71.7	156.3	6,230	40.6
30	246	20,950	95.5	223.3	70.0	155.8	6,160	41.3
31	191	15,080	92.6	231.1	74.2	153.3	4,810	26.8
34	231	19,670	92.5	223.9	71.8	157.2	5,820	38.3
36	485	35,000	94.6	212.7	75.1	147.3	11,900	77.6
37	239	19,820	93.8	225.8	73.7	156.8	5,740	39.3
38	105.7	13,460	95.0	229.1	73.6	201.3	3,305	37.8
39	105.5	13,530	96.1	227.6	71.7	199.8	3,255	34.8
45A	52.1	5,660	92.2	215.0	82.8	191.0	1,630	16.9
45B	50.1	5,690	93.5	213.1	79.5	190.5	1,540	16.8
46	57.5	6,500	91.5	213.8	81.5	177.7	1,680	16.8
47	108.1	8,150	92.0	210.3	75.2	150.6	2,700	16.9
48	94.5	7,630	91.3	212.3	76.2	156.8	2,460	16.5
49	74.6	6,810	92.0	216.4	80.6	172.0	2,105	15.8
50	68.8	6,610	88.2	216.1	82.3	178.8	2,030	16.7
51	76.7	7,400	84.0	216.1	81.7	178.5	2,250	18.8
52	229.0	27,700	94.0	246.4	76.7	197.5	7,150	57.3
53	153.6	22,000	93.5	252.3	77.3	220.5	5,270	51.4
54	188.0	26,700	100.6	248.3	75.7	217.5	6,350	64.5
55	187.5	26,650	101.5	247.5	75.5	217.4	6,350	65.0
56	252.0	31,500	99.3	242.4	72.3	197.3	7,720	68.3
57	342.0	35,700	100.1	240.4	74.8	179.2	9,750	70.0
58	257.0	31,000	99.0	247.0	75.0	195.5	7,910	63.7
59	148.5	16,900	100.7	230.0	75.7	189.2	4,440	37.8
60	169.0	18,300	99.5	228.6	75.7	183.7	4,940	41.2
61	211.0	21,000	101.1	224.0	74.1	173.5	5,830	46.5
62	269.0	24,300	102.3	218.7	70.3	160.7	6,840	51.9
63	297.0	25,400	102.3	218.2	72.1	157.4	7,550	53.8
64	365.0	32,300	92.5	214.6	70.3	150.6	8.850	69.5
66	102.2	13,020	103.8	234.1	77.3	204.4	3,300	32.9

*Estimated from heat balance
**Corrected to $t_w - t_o = 140°F$

presented in Appendix 3, represents fairly accurately the behavior of a system in which a steady unidirectional forced flow and free convection are superposed in the viscous region. By means of this equation, the Nusselt modulus, in the viscous region based on the logarithmic-mean temperature difference, may be predicted.

In Equation [4]

Gz_m = Graetz modulus, properties evaluated at the arithmetic-mean mixed temperature, t_m

$\left(\frac{Gr\ Pr\ D}{L}\right)_w$ = product of Grashof modulus, utilizing temperature difference between pipe wall and entering fluid, the Prandtl modulus and the ratio D/L. All properties evaluated at the tube-wall temperature, t_w

$$F_3 = \frac{5}{4}\int_0^L \left(\frac{x}{L}\right)^{1/4} e^{-3/4 \cdot \frac{\pi Nu_{lm}}{Gz_m} \cdot \left(\frac{x}{L}\right)^{3/4}} \frac{dx}{L}$$

F_3 is a function which enters the equation because buoyant force on fluid becomes smaller as fluid approaches pipe-wall temperature; see Appendix 3 for a plot of F_3 as a function of $\pi Nu_{lm}/Gz_m$

Equation [4] was utilized to predict curve A in Fig. 3. The experimental points are seen to fall, on the average, about 7 per cent above the predicted curve. It is of interest to note that, for the case of heating water, as shown by curve A, the magnitude of Nu_{lm} decreases with decreasing magnitudes of Re_m, mainly because of the decrease in F_3, which is a partial measure of the free convection forces, rather than being due entirely to a lower forced-flow velocity (see discussion in Appendix 3).

Application of the methods outlined in reference (4) indicated that the measured magnitudes of Nu_{lm} in the turbulent region were about 8 per cent below the predicted values (curve B, Fig. 3).

Experimental Data, Periodic Flow

Having established the steady-unidirectional-flow performance of the vertical heat exchanger, several runs were made utilizing the reciprocating pump to provide a periodic variation in velocity. The pump stroke was kept constant and the speed of the pump was varied in a series of runs from 13 rpm to 265 rpm. This procedure yielded a range of $\widetilde{Re}_m$ from 2660 to 77,300, as shown in Table 2. The magnitudes of $\widetilde{Re}_m$ are calculated by utilizing fluid properties evaluated at the arithmetic average mixed mean temperature of the fluid and a mean velocity determined by dividing the mean fluid-flow rate by the pipe cross-sectional area. The magnitudes of $\widetilde{Nu}_{lm}$ have been corrected for the presence of the oxide scale, as outlined in Appendix 2.

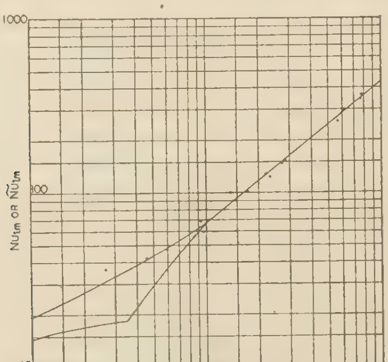

Fig. 4 Nusselt Modulus as a Function of Reynolds Modulus (The lines represent Nu_{lm} versus Re_m steady-state-unidirectional experimental data; the points represent experimental values of $\widetilde{Nu}_{lm}$ versus $\widetilde{Re}_m$ for periodic flow.)

The points are shown plotted in Fig. 4. Within the range of frequencies utilized (13 cycles per min to 265 cycles per min), the frequency per se apparently has no direct effect on the Nusselt modulus, the main variable being the velocity of the fluid. Further, if the Nusselt modulus $\widetilde{Nu}_{lm}$ is plotted against the Reynolds modulus $\widetilde{Re}_m$, as defined, the points almost coincide (average deviation, 10 per cent) with the steady-unidirectional-flow data. It should be noted particularly that at low magnitudes of $\widetilde{Re}_m$, the points fall along the upper or "turbulent" branch of the steady-flow curve. The latter phenomenon is to be expected, for, the flow being of a transient nature, the turbulence existing in the fluid stream cannot be damped immediately, even at low magnitudes of the Reynolds modulus. Further, some cross-flow must take place in order to establish the nonuniform velocity across the tube diameter each time the flow starts from rest.

As a preliminary conclusion, therefore, it may be stated that, at the low frequencies of velocity variation utilized in the tests, a fairly accurate estimate of the heat transfer may be made by utilizing directly the steady-flow data to obtain $\widetilde{Nu}_{lm}$ at the corresponding Reynolds modulus, $\widetilde{Re}_m$.

Closer inspection of the data, however, indicates the following general trends, which will be verified by a more careful analysis of the problem as follows:

(a) The magnitudes of $\widetilde{Nu}_{lm}$ are slightly higher than those of Nu_{lm} at low magnitudes of $\widetilde{Re}_m$.

(b) The magnitudes of $\widetilde{Nu}_{lm}$ are slightly lower than those of Nu_{lm} at higher magnitudes of $\widetilde{Re}_m$.

Analysis of Heat-Transfer Phenomenon During Periodic Flow

During the cycle of velocity variation, produced by the reciprocating pump (Fig. 2), the instantaneous magnitude of the Reynolds (Re_{mi}) modulus may be calculated. Thus

$$Re_{mi} = \frac{vD}{\nu_m} \qquad [5]$$

where

$$v = v_{\max} \sin\phi \qquad \text{for } 0<\phi<\pi$$
$$v = 0 \qquad \text{for } \pi<\phi<2\pi$$

$v_{\max}$ is defined by

$$\frac{Q}{A_p} = \frac{1}{2\pi}\int_0^{2\pi} v_{\max} \sin\phi\, d\phi \qquad [6]$$

and thus

$$v_{\max} = \pi\left(\frac{Q}{A_p}\right)$$

The postulate is made that at each instantaneous magnitude of the Reynolds modulus, Re_{mi}, the corresponding magnitude of the Nusselt modulus is given by the curve of Nu_{lm} versus Re_m obtained for the steady-unidirectional-flow conditions. This postulate has been used for the calculation of pressure drop in periodic flow (5) and is valid as long as the period of the cycle of velocity variation is not too short. The flow will vary from turbulent to viscous along the upper branch of the steady-flow curve, for reasons which have been discussed previously.

When, in the cycle of velocity variation, the fluid is brought to rest (or when it is flowing with very low velocity), free convec-

tion controls the rate of heat transfer. Under steady-flow conditions, with very low velocity, the fluid leaves the column at practically the tube-wall temperature, and the Nusselt modulus would be predicted by Equation [3*d*] of Appendix 3. In the case of periodic flow, however, the fluid (even though it is at rest) remains much cooler than the tube wall due to its heat capacity. A simple calculation and experimental evidence reveals that, due to the heat capacity of the fluid, the temperature difference between the tube wall and the fluid (mixed-mean temperature) does not vary appreciably with time. Thus, free convection is much more pronounced during the zero-velocity period in the periodic-flow system than during the equivalent condition under steady flow.

As shown in Appendix 3, the increase in Nu_{lm}, due to the large difference in temperature between the tube wall and the fluid, is readily accounted for by setting the function F_3 of Equation [4] equal to unity, for during periodic flow the decrease in buoyant force caused by the increase in fluid temperature is negligible. Thus the instantaneous Nusselt modulus for conditions of periodic flow in the viscous region may be expressed as

$$Nu_{lmi} = 1.75 \sqrt[3]{\left[Gz_{mi} + 0.0722 \left(\frac{Gr\, Pr\, D}{L} \right)_w^{0.84} \right]} \quad \ldots\ldots [7]$$

Fig. 6 illustrates the curve of Nu_{lm} versus Re_m utilized to evaluate instantaneous magnitudes of the Nusselt modulus for the case of periodic flow. It is to be noted that, (*a*) the "turbulent" branch of the steady-flow curve has been followed, and (*b*) the magnitude of the Nusselt modulus approaches an asymptote of 18.5, the magnitude predicted by Equation [7].

Having established the curve for the evaluation of Nu_{lmi}, it is a simple matter to plot Nu_{lmi} for various magnitudes of Re_{mi}. A typical curve of Nu_{lmi} and Re_{mi} versus crank angle is shown in Fig. 5.

Graphical integration of the Nu_{lmi} versus crank-angle curve will yield the time-average magnitude of the Nusselt modulus for periodic-flow conditions. This direct graphical integration is valid, since, as previously mentioned, the logarithmic-mean temperature difference between the tube wall and the fluid is independent of time. Thus

$$\widetilde{Nu}_{lm} = \frac{1}{2\pi} \int_0^{2\pi} Nu_{lmi}\, d\phi \ldots\ldots\ldots\ldots\ldots [8]$$

and

$$\widetilde{Re}_m = \frac{\left(\frac{Q}{A_p}\right) D}{\nu_m} = \frac{1}{2\pi} \int_0^{2\pi} Re_{mi}\, d\phi \ldots\ldots\ldots\ldots [9]$$

A plot of the resulting curve of $\widetilde{Nu}_{lm}$ versus $\widetilde{Re}_m$ is shown in Fig. 6, to which the experimental points have been added.

For the frequencies and velocity variations utilized in these experiments, it is noted that the effect of periodic flow is to increase the magnitude of the Nusselt modulus above the value obtained in steady flow, for a range of $300 < \widetilde{Re}_m < 4500$. For $\widetilde{Re}_m < 300$, since free convection controls the heat-transfer phenomenon, no effect of periodic flow is noted. The increase in the Nusselt modulus for $\widetilde{Re}_m > 300$ follows in the first place because, due to free convection, heat is transferred while there is no forced flow, and in the second place, although $\widetilde{Re}_m < 2000$, instantaneous magnitudes of Reynolds modulus may extend into the turbulent region during a cycle of operation. The decrease in the Nusselt modulus at high magnitudes of the Reynolds modulus ($\widetilde{Re}_m < 4500$) occurs because the instantaneous magnitude of the Nusselt modulus does not increase linearly with Reynolds modulus, but at a somewhat lower rate.

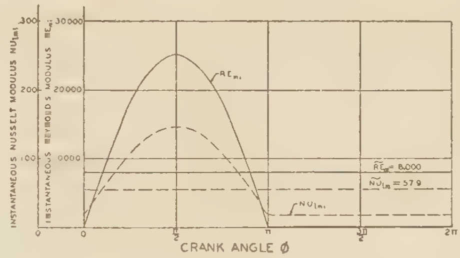

FIG. 5 CURVES REPRESENT VARIATION OF INSTANTANEOUS MAGNITUDES OF Nu_{lmi} AND Re_{mi} AS FUNCTION OF CRANK ANGLE (TIME) FOR 1 CYCLE

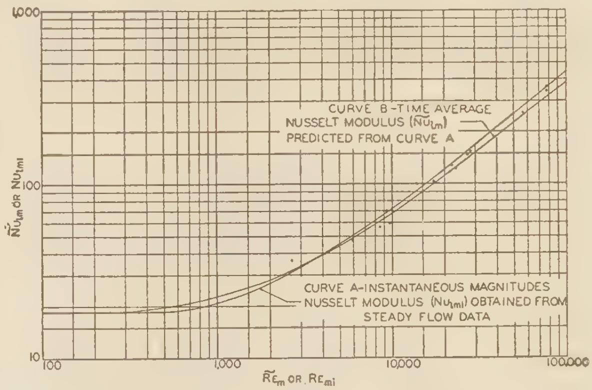

FIG. 6 NUSSELT MODULUS PLOTTED AS FUNCTION OF REYNOLDS MODULUS

(Curve *A* represents Nu_{lmi} versus Re_{mi} obtained from the steady-unidirectional-flow test results, i.e., Nu_{lm} versus Re_m; and curve *B* represents $\widetilde{Nu}_{lm}$ versus $\widetilde{Re}_m$ for the periodic case and predicted by utilizing curve *A*. The points are experimental data for periodic fluid flow.)

Although fairly satisfactory predictions result from the concept of an instantaneous Nusselt modulus determined by the steady-flow curve, it must not be concluded, without further evidence, that the same reasoning would be applicable to cases in which the frequency of velocity fluctuation was much higher than that used in these experiments.

CONCLUSIONS

For the type of velocity variation utilized in these experiments it has been shown that steady-unidirectional-flow heat-transfer results may be utilized to predict heat-transfer performance under conditions of periodic flow:

(*a*) As a first approximation the steady-flow data may be utilized directly, replacing Re_m by $\widetilde{Re}_m$ and obtaining $\widetilde{Nu}_{lm}$ from the steady-unidirectional-flow curve.

(*b*) For a more precise and rational method, the steady-unidirectional-fluid-flow data may be utilized to predict instantaneous magnitudes of the Nusselt modulus at any point in the cycle of velocity variation. The time-average Nusselt modulus may then be obtained by graphical integration.

Appendix 1

DESCRIPTION OF HEAT EXCHANGER

The main element of the experimental equipment (2) was a small vertical heat exchanger, Fig. 7. The test section consisted of a 0.422-in-ID copper tube with an effective heating length of 4.44 ft. A 3-in-diam tin-plated steel tube separated the inner and outer sections of the heat exchanger, allowing the outer por-

TABLE 2 EXPERIMENTAL DATA, PERIODIC FLOW

Run	W	q_o	q_s/q_o	t_w	t_o	t_{ml}	Pump RPM	$\widetilde{Re}_m$	$\widetilde{Nu}_o$	$\widetilde{Nu}_{lm}$
1S	964	64,000	97.0	240.5	78.0	144.5	100	23,400	100	123
3S	1820	72,900	107.0	221.5	118.5	158.5	185	56,400	170	158
4S	366	30,000	109.0	217.5	75.5	157.5	55	9,360	61.4	69.5
5S	2840	85,200	106.0	199.5	107.5	137.5	265	77,300	218	362
6S	810	57,100	99.0	234.0	88.0	158.5	98	22,000	104	127
7S	1080	64,700	106.0	223.5	83.5	143.5	125	27,100	118	148
8S	2850	84,000	104.0	201.0	106.0	135.5	265	76,300	205	341
9S	2180	77,500	107.0	204.5	107.0	142.5	220	60,300	192	298
10S	564	42,200	94.0	213.0	74.5	149.5	70	13,800	86.0	100
16F	1042	57,900	98.5	221.0	95.0	150.5	150	28,200	121	153
17F	333	23,500	93.2	212.0	79.5	150.0	48	8,520	50.2	56.2
18F	820	36,900	94.3	186.5	75.5	120.5	90	17,350	87.3	103
25F	79	10,220	98.0	227.0	83.5	213.0	13	2,660	34.8	36.5
26F	162	15,950	95.5	218.0	79.0	117.5	24	4,570	39	42.1
27F	224	18,900	98.0	215.0	79.0	163.5	33	5,950	42.9	47.1
28F	415	24,700	97.5	205.5	78.5	138.0	60	9,700	52.7	59.6

tion to act as a thermal jacket. The steam which condensed on the copper tube was collected and weighed. The steam which condensed on the outer shell was drained into the sump.

Temperatures were measured at 12-in. intervals along the copper tube by means of five thermocouples constructed of No. 24 iron-constantan spun-glass-covered duplex wire. The wire was inserted in an 0.08 × 0.08-in. milled groove, 2.5 in. in length, cut longitudinally in the wall of the copper tube. The end of the thermocouple was peened into the copper wall at one end of the groove and the wire was covered with a bar of copper which was soldered in place and then filed flush with the outer surface of the pipe wall. The thermocouple leads emerged from the tube wall and the end of the groove opposite that at which the junction was made. This construction reduced errors caused by conduction of heat away from the thermocouple junction.

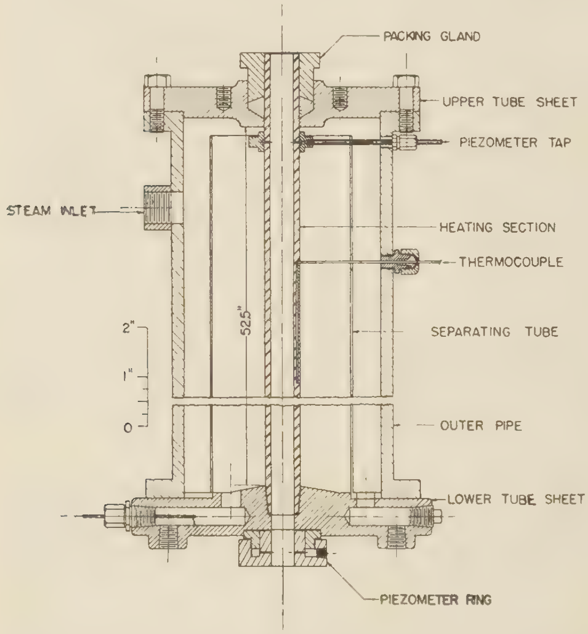

FIG. 7 CROSS SECTION OF 4.44-FT HEAT EXCHANGER

The inlet fluid temperature was measured by means of a No. 24 iron-constantan thermocouple placed in the fluid stream before the calming section.

The outlet fluid temperature was measured similarly in a mixing chamber provided at the discharge side of the heating section. Steam pressure was measured by means of calibrated Bourdon gages. Steam quality was measured with a throttling calorimeter. The steady-state velocity distribution in the test section was established by the use of a 60-diam fiber-tube calming section.

Appendix 2

CORRECTION FOR OXIDE SCALE

The formation of a thin layer of oxide on the inner surface of the copper tube was unavoidable. For the steady-unidirectional-flow runs, the value of this added resistance was established by plotting the reciprocal of the observed Nusselt modulus (Nu_0) versus the reciprocal of the Reynolds modulus $(Re_m)^{0.80}$. Extrapolation of the resulting straight line yielded the scale resistance (6). The equivalent Nusselt modulus Nu_R of the scale resistance is tabulated in Table 3.

TABLE 3 EQUIVALENT NUSSELT MODULUS OF SCALE RESISTANCE

Runs	Nu_R
1F−29F; 1S−10S	.830
1−66	.695

The corrected Nusselt modulus is then calculated by means of the relation

$$Nu_{lm} = \frac{Nu_0\, Nu_R}{Nu_R - Nu_0} \qquad [2a]$$

Magnitudes of Nu_{lm} are tabulated in Table 1.

For the periodic flow, the corrections for scale resistance are slightly more complicated. A graphical integration of the instantaneous observed magnitudes of the Nusselt modulus is necessary. Then

$$\widetilde{Nu}_{lm} = \widetilde{Nu}_0 \frac{\int_0^{2\pi} Nu_{lmi}\, d\phi}{\int_0^{2\pi} Nu_{0i}\, d\phi} \qquad [2b]$$

where

$$Nu_{lmi} = \frac{Nu_{0i}\, Nu_R}{Nu_R - Nu_{0i}} \qquad [2c]$$

Magnitudes of $\widetilde{Nu}_{lm}$ are tabulated in Table 2.

Appendix 3

EQUATION FOR HEAT TRANSFER IN THE VISCOUS REGION UTILIZING LOG-MEAN TEMPERATURE DIFFERENCE

It is usually customary to utilize the arithmetic-mean temperature difference in defining the Nusselt modulus in the viscous region, but occasions arise in which the log-mean temperature difference must be utilized.

In reference (7), an equation is derived for the Nusselt modulus in the viscous régime, utilizing the arithmetic-mean temperature difference. The resulting equation is

$$Nu_{am} = 1.75\, F_1 \sqrt[3]{\left[Gz_m + 0.0722\, F_2 \left(\frac{Gr\,Pr\,D}{L}\right)_w^{0.84}\right]} \quad \ldots [3a]$$

where

Nu_{am} = Nusselt modulus based on arithmetic-mean temperature difference

$$F_1 = \frac{\left(\dfrac{\pi\, Nu_{am}}{Gz_m}\right)}{\log_e \left(\dfrac{2 + \dfrac{\pi\, Nu_{am}}{Gz_m}}{2 - \dfrac{\pi\, Nu_{am}}{Gz_m}}\right)} \quad \ldots\ldots\ldots\ldots [3b]$$

F_1 is a function which enters the equation because of the use of the arithmetic-mean temperature difference in evaluating the unit thermal conductance

$$F_2 = \frac{5}{4}\int_0^L \left(\frac{x}{L}\right)^{1/4} e^{-\frac{3}{4}\cdot\frac{\pi Nu_{am}}{F_1 Gz_m}\left(\frac{x}{L}\right)^{2/3}} \frac{dx}{L} \quad \ldots\ldots [3c]$$

F_2 is a function which enters the equation because the buoyant force on the fluid becomes smaller as the fluid approaches the pipe-wall temperature

$\left(\frac{Gr\,Pr\,D}{L}\right)_w$ = product of Grashof modulus, utilizing temperature difference between pipe wall and entering fluid, the Prandtl modulus of fluid, and the ratio of D/L. All properties are evaluated at wall temperature

Gz_m = Graetz modulus based on mixed mean temperature

A slightly modified analysis readily reveals that, if the logarithmic-mean temperature difference is utilized instead of the arithmetic mean, the magnitude of the Nusselt modulus is given by

$$Nu_{lm} = 1.75 \sqrt[3]{\left[Gz_m + 0.0722\, F_3 \left(\frac{Gr\,Pr\,D}{L}\right)_w^{0.84}\right]} \quad \ldots [3d]$$

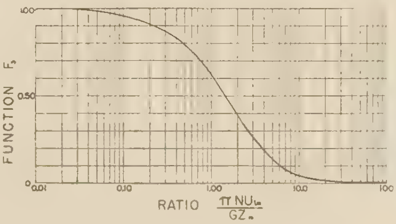

FIG. 8 FUNCTION F_3, EQUATIONS [3d] AND [3e], IN TERMS OF $\frac{\pi\, Nu_{lm}}{Gz_m}$

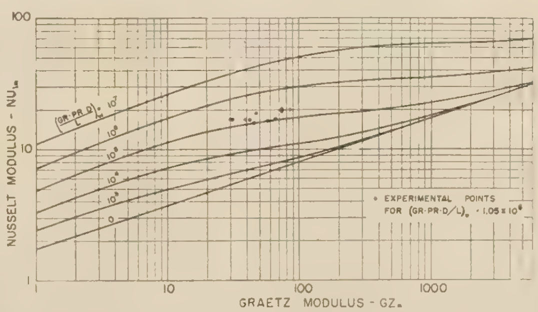

FIG. 9 NUSSELT MODULUS Nu_{lm} BASED ON LOGARITHMIC TEMPERATURE DIFFERENCE PRESENTED AS FUNCTION OF GRAETZ MODULUS, Gz_m WITH $\left(\frac{Gr\;Pr\;D}{L}\right)_w$ AS PARAMETER

where

$$F_3 = \frac{5}{4}\int_0^L \left(\frac{x}{L}\right)^{1/4} e^{-\frac{3}{4}\cdot\frac{\pi Nu_{lm}}{Gz_m}\left(\frac{x}{L}\right)^{2/3}} \frac{dx}{L} \quad \ldots\ldots [3e]$$

Term F_3 is similar to F_2 and represents a function which enters the equation because the buoyant force on the fluid becomes smaller as the fluid approaches the pipe wall temperature.

A plot of F_3 as a function of $\pi\, Nu_{lm}/Gz_m$ is shown in Fig. 8 and a plot of Equation [3d] is shown in Fig. 9. Magnitudes of the function F_3 are tabulated in Table 4.

TABLE 4 MAGNITUDES OF FUNCTION F_3

$\frac{\pi\, Nu_{lm}}{Gz_m}$	F_3
0.00	1.000
0.10	0.952
0.20	0.910
0.30	0.869
0.40	0.830
0.50	0.790
0.60	0.760
0.70	0.730
0.80	0.690
0.90	0.655
1.00	0.620
1.50	0.490
2.00	0.390
3.00	0.265
5.00	0.140
7.00	0.074
10.0	0.038
20.0	0.010
30.0	0.005
50.0	0.002
∞	0.000

A study of the curves shows that the decrease in Nu_{lm} at lower magnitudes of Gz, observed by many experimenters, evidently is not wholly due to the effect of decreasing forced velocity but also due to the decrease in the buoyant force of free convection (i.e., decrease in F_3 at low magnitudes of Gz_m). The predicted curve, shown in Fig. 3, is based on Equation [3d] for a magnitude of $\left(\frac{Gr\,Pr\,D}{L}\right)_w = 1.05 \times 10^5$ which is the average of the observed experimental values. Several experimental points are shown in Fig. 9.

As discussed in the text of the paper, when periodic flow exists, not enough time is available during the low-velocity portion of the cycle for the fluid to reach steady-flow temperatures. Thus the decrease in the buoyant force, which results at low steady velocities, due to the heating of the fluid to a temperature approaching that of the tube wall, does not take place during periodic flow.

As a good approximation, in periodic flow, even at low magnitudes of Graetz modulus $(Gz)_{mi}$ (in the limit $Gz_{mi} = 0$), F_3 can be assumed to remain unity. In other words, because of the heat capacity of the fluid, the value of Nu_{lmi} at low magnitudes of Graetz modulus (Gz_{mi}) does not follow the curves shown in Fig. 9 but moves along a curve given by

$$Nu_{lmi} = 1.75 \sqrt[3]{\left[Gz_{mi} + 0.0722 \left(\frac{Gr\,Pr\,D}{L}\right)_w^{0.84}\right]} \quad \ldots [3f]$$

ACKNOWLEDGMENTS

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Discussion

J. H. MARCHANT.[6] In this paper there are certain questions such as the assumption that the liquid velocity is a sinusoidal function of the pump crank angle, the possible effects of cavitation, etc., which limit any far-reaching conclusions from such experimental work. While at first sight, this work might not appear to be directly connected with the periodic heat-transfer problems of the internal-combustion engine, it is a sound, straightforward step toward that goal, and we all would like more information of this sort from the authors using compressible fluids.

If there is any criticism of this paper which the writer might offer, it is that it is too concise to be of greatest usefulness to the average engineer who might desire to use the results contained therein.

So, rather than criticize, the writer should like to present some experimental results which he obtained some years ago at Columbia University, which serve to confirm the authors' conclusion that in both isothermal and nonisothermal systems there is a transition region within which the Nusselt modulus depends upon flow conditions which are not described completely by the magnitude of the Reynolds modulus.

It might be added that the purpose of the preliminary experiments herein described was to introduce a third mode (forced convection) into the mechanism of heat transfer through a controlling film resistance.

Consider the case of heat transfer from the outer surface of a steam-condenser tube to the water flowing through it. It is generally conceded that the heat transfer through the boundary layer on the water side (inside) of the tube is effected by means of the conduction and radiation modes only. In other words, the convection mode plays little or no part in the transfer of heat through this inner boundary layer which is immediately in contact with the inner surface of the tube wall and effectively at rest with respect to it.

If a uniform pressure is suddenly applied across the inner cross section of this condenser tube in an opposite direction to that in which water is flowing, each of the various concentric annuli of water will be accelerated in a direction opposite to that of the flow for the duration of the application of this pressure. This backward acceleration of the various annuli of water will diminish the forward speed of some of them, bring others to rest, and give still others a definite speed in the opposite direction, depending, among other things, upon the magnitude of the initial speed of these various annuli before application of the pressure. Because of the large velocity gradient existing in the water near the inner wall of the tube, the annuli of liquid near this wall will tend to be stripped off by the application of the pressure.

This stripping of the outer annuli of liquid in the neighborhood of the inner surface of the condenser tube, due to the interruption of the stream of water, can be demonstrated readily for the case of laminar flow. In other words, if the flow through a standard Reynolds apparatus is interrupted by opening and closing the outlet valve leading from it, this stripping effect is made perfectly apparent by the convolutions of the threads of color which develop near the inner wall of the tube when the outlet valve is suddenly closed.[7]

Due to the small temperature differences which exist between the tube wall and the water within, it is reasonable to assume that only a small fraction of the heat is transferred by radiation. Moreover, since the boundary layer of liquid immediately in contact with the inner surface of the tube is sensibly at rest with respect to the tube, the convection mode plays no part in the heat transfer which takes place through this inner-surface boundary layer, that is, in the case of steady flow, the heat transfer through the inner-surface boundary layer to the liquid within is effected almost entirely by conduction through this boundary layer.

In attempting to introduce the convection mode into the heat transfer which takes place through this inner-surface boundary layer, pressure was periodically applied to the stream of water flowing through the tube by interrupting the flow of the water itself. Effectively, this increases the pressure head at a given section in the liquid by an amount approximately equivalent to the corresponding velocity head which obtained at this section when the flow was steady (before interruption). These periodic pressure variations produce a forced circulation among those annuli of water in the neighborhood of the inner surface of the condenser tube. The efficiency of this stripping action in promoting forced convection in the water-side boundary layer is, among other things, evidently a function of the frequency and mode of interruption of the stream of water, the boundary-layer thickness, and the vibration characteristics of the system as a whole.

Whether or not these annuli include part or all of the inner-surface boundary layer is for the present a matter of conjecture. For certain flow patterns, however, there is evidence which seems to indicate that at least an inner portion of the inner-surface boundary layer is stripped.

The apparatus used in these preliminary experiments consisted of a horizontal concentric double-tube heat exchanger. The inner tube (test section) was of 1.25-in-ID wrought-iron pipe, 60 in. long. Temperatures were measured to within 1 deg F; and the data were checked by suitable heat balances which compared heat rejected to the water with latent heat of condensate.

After passing through the heat exchanger, the stream of cooling water passed through two inlet valves of a Model "A" Ford automobile engine which operated in parallel and in phase, and thence through the single inlet port of the engine block to a weighing tank. The valves, which were spring-loaded, were upstream of the flow which they were used to interrupt, and the cams which actuated them were of the quick-opening-and-closing type, designed for sharp cutoff.

Over-all heat-transfer coefficients were determined over a range of a Reynolds moduli of from approximately 400 to 100,000 (consistent units) under steady-flow conditions and at water pulsation rates of 10, 25, and 60 cycles per min.

The over-all heat-transfer coefficients and the Reynolds moduli were computed in terms of the arithmetical mean of the inlet- and outlet-water temperature. Mean mass-flow-rate velocities

[6] Engineer, Pratt and Whitney Aircraft, East Hartford, Conn. Mem. A.S.M.E.

[7] This stripping phenomenon, which was demonstrated to the writer by Dr. Hunter Rouse, suggested this problem to him.

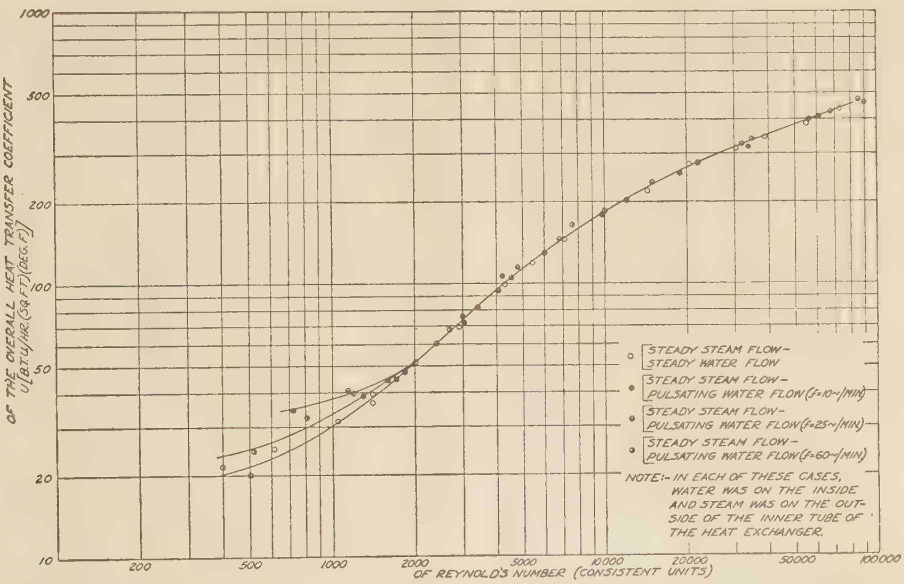

FIG. 10 RESULTS OF EXPERIMENTS TO DETERMINE RELATIONSHIP BETWEEN OVER-ALL HEAT TRANSFER COEFFICIENTS AND REYNOLDS NUMBER

were also used in calculating the Reynolds moduli. The results[8] of these experiments are shown in Fig. 10 of this discussion.

Some conclusions which may be drawn from these data are as follows:

1 There is a transition range within which the over-all heat-transfer coefficient depends upon flow conditions which are not entirely described by the magnitude of the Reynolds modulus.

2 In heat exchangers in which the flow pattern is laminar under steady-flow conditions and the controlling thermal resistance is on the liquid side, it might be practical to pulsate the liquid and take advantage of the larger over-all heat-transfer coefficients in spite of the larger pumping losses associated with the pulsating flow.

3 The over-all heat-transfer coefficient was not affected by pulsation of the water over the range of frequencies investigated when the flow was turbulent.

4 The increase in over-all heat-transfer coefficient observed when the water was pulsated at low Reynolds moduli is probably due to the addition of forced convection to the two normal modes (conduction and radiation) by means of which heat is transferred through what would correspond to the inner-surface boundary layer under steady-flow conditions.

W. R. WYKOFF.[9] In Fig. 3 the authors have presented considerable steady flow data showing two separate curves for values of the Reynolds modulus below 10,000. On these tests, the tube-wall temperature varied from 166 F to 252 F. These data have been replotted by the writer giving special marking to points on which the tube-wall temperature exceeded 220 F. It is seen that the upper branch of the replotted curve is composed mainly of high wall-temperature points, and the lower branch is composed of low wall-temperature points.

The test fluid (water) was under 15 lb psi gage pressure which corresponds to a boiling point of about 250 F. A tube wall temperature of 220 F seems to affect the turbulence, however. Thus it seems that high tube-wall temperatures are conducive to turbulence, but the explanation is not clear.

[8] The low values of the over-all heat-transfer coefficients were due to very heavy scale deposits on both sides of the inner tube.

[9] Pratt & Whitney Aircraft, East Hartford, Conn.

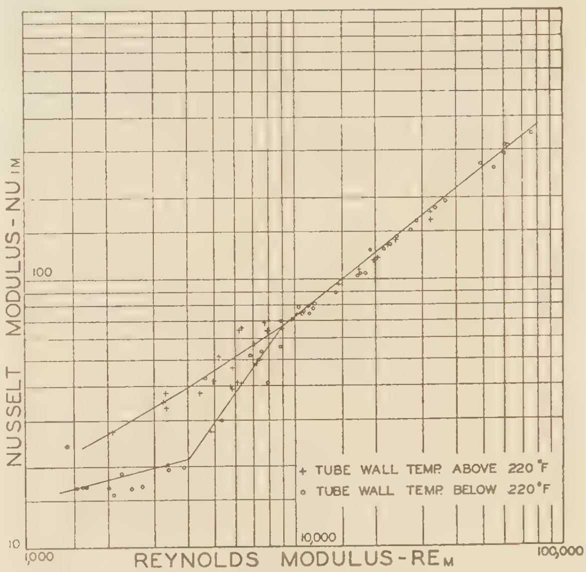

FIG. 11

(Experimental Nusselt Modulus Nu_{lm} as a function of Reynolds Modulus Re_m for steady unidirectional flow, showing an effect of tube wall temperature upon turbulence.)

AUTHORS' CLOSURE

The results presented by Mr. Marchant are very interesting and it is gratifying that his data on the whole substantiate the main conclusions reached by the authors.

The details of the mechanism of heat transfer during pulsating flow presented by Mr. Marchant agree well with those visualized by the authors, with the following exceptions:

1 Radiation in liquids is negligible, since liquids are opaque to infrared radiation.

2 For the "periodic" flow, utilized by the authors, in contrast with the "pulsating" flow utilized by Mr. Marchant, the laminar sublayer is not subjected to a sudden stripping action but varies

in thickness gradually, due to the changes in the mean velocity of the fluid passing through the pipe.

An important limitation to the applicability of the methods of analysis presented in this paper must be mentioned. If the mean velocity of flow in a pipe at any time becomes negative (i.e., flow reversal) the velocity calculated from the ratio Q/A can no longer be used to estimate the Nusselt modulus from steady-flow data. This limitation becomes clear when flow with zero mean velocity, but large fluctuations from the mean, is considered. The estimate of Nu from the steady-flow curve at the zero mean velocity would obviously be much too low.

The graphical method, on the other hand, may be utilized readily when the flow is largely turbulent, if the velocity is always plotted as a positive quantity, even during flow reversal, for the rate of heat transfer depends on the magnitude and not on the direction of the flow. The latter statement does not hold for viscous flow (and in the transition and turbulent region—of low Reynolds modulus), however, for then free convection plays an important role in the heat-transfer mechanism.

Mr. Wykoff has focused attention on a very interesting point concerning the test results. An inspection of the original data reveals that runs 1 F to 29 F were made with the water at atmospheric pressure, and runs 1 to 66 with the water under a 15-lb psi gage pressure. This fact explains the presence of the two low-wall-temperature points on the upper branch of the curve in Fig. 11. A possible explanation of the pronounced effect of wall temperature on the Nusselt modulus may be the following: A calculation of the Reynolds modulus of the water entering the heat-exchange section for the steady-flow runs, reveals that this "entrance" Reynolds modulus is 2500 or less for all runs whose Re_m is less than 6000. Thus for the majority of the points in the transition region, the water entered the heating section flowing in the viscous region. If the flow was carefully controlled, and in particular if the tube-wall temperature was kept relatively low, the viscous flow pattern persisted for an appreciable distance down the tube before transition into turbulent flow. Data obtained for such runs lie in the lower branch of the curve. It is noted that for some of the runs, at the lowest magnitudes of the Re_m, viscous flow existed in the major portion of the tube even though the Re_m was over 2000. As the Reynolds modulus at entrance is increased in magnitude, the transition to turbulent flow occurs closer and closer to the entrance of the heating section, and therefore the points along the lower branch of the curve approach the completely turbulent-flow asymptote.

When the wall temperature was very high, however, even though the Reynolds modulus at entrance was less than 2000, the rapid decrease of viscosity in the neighborhood of this tube wall apparently caused the viscous flow to break into turbulent flow very soon after the water had entered the heating section. This rapid transition from viscous to turbulent flow is evidenced by the higher magnitude of Nu_{lm} obtained for runs in which the latter phenomenon occurred. It is evident that the mean Reynolds number Re_m, per se is not completely significant in the so-called transition region, since the magnitude of the Nusselt modulus in this range depends greatly on factors which cause the transition from viscous flow, existing at the entrance to the heating section, to turbulent flow as the fluid passes through the tube.

The Nusselt modulus as employed in this paper is a mean over the heat-transfer area. The unit conductance, however, is a point function and varies with the distance from the beginning of the heat-transfer section (an isothermal hydrodynamic calming section was provided), (1) because of the infinite temperature gradient existing in the fluid at the fluid-wall interface at the entrance to the heating section, and (2) by virtue of the variation of fluid properties (viscosity) which in turn change the flow distribution from point to point along the tube. Utilizing mean Nusselt and Reynolds moduli masks these effects.

Theoretical and Experimental Investigations of Thin-Webbed Plate-Girder Beams

By H. L. LANGHAAR,[1] SAN DIEGO, CALIF.

A simple, semirational theory for the design of webs and flange rivets of thin-webbed rectangular plate-girder shear beams is presented. Calculations of shear loads to cause web rupture and flange rivet failure are compared with test data from twenty-seven beams.

Nomenclature

The following nomenclature is used in the paper:

X, Y = rectangular co-ordinates along and perpendicular to beam center line. The beam center line is defined to be the centroidal axis of the two flange cross sections
A_s = cross-sectional area of a web stiffener
t = web thickness
L = length of the beam
h = beam depth between the centroids of flange cross sections
b = spacing of web stiffeners
d_s = diameter of a web-stiffener rivet
p_s = pitch of web-stiffener rivets
E_f, E_w, E_s = Young's moduli for flange, web, and web-stiffener metals, respectively
I_{ft}, I_{fc} = moments of inertia of cross sections of tension and compression flanges about their respective horizontal centroidal axes
I = moment of inertia of both flange cross sections about beam center line
Q = moment of area of either flange cross section about beam center line
A_r = cross-sectional area of all flange rivets per flange per bay, including stiffener end rivets
λ = dimensionless constant, depending upon beam dimensions and materials
F_{tu} = ultimate tensile stress for web metal
F_{su} = ultimate shearing stress for web metal
α = angle of principal stress, measured from beam center line
K_r = experimentally determined web-stiffener rivet factor
ϵ = principal tensile strain in web
$\epsilon_x, \epsilon_y, \gamma$ = strain tensor for web
δ = deflection of free end of beam
S = shear load carried by beam
P_r = load per bay carried by flange rivets
σ_x, σ_y, τ = stress tensor resulting from shear load S
σ = diagonal tensile stress due to shear load S
τ_{cr} = buckling shear stress for a panel
r = τ_{cr}/τ (a stress-ratio factor)
τ_{tu} = value of τ to cause diagonal-tension rupture of web
τ_{su} = value of τ to cause shear rupture of web
τ_{al} = the lesser of two values τ_{tu} and τ_{su}

Introduction

The theory of the diagonal-tension type of beam, which was initiated by H. Wagner in 1929, and which has been modified and extended by numerous other investigators,[2] is yet not adequate to give complete and accurate design information. An exhaustive theory must await the determination of the stress distribution in a rectangular panel that is maintained in a buckled state by opposing couples applied to its bordering frame. Unfortunately, the solution of this problem is obstructed by the nonlinearity of the differential equations for a deflected plate, although, due to the developable character of the wrinkle pattern in the center portion of a panel, the nonlinear terms become significant only in the boundary region. This fact possibly accounts for the so-called "gusseting effect," which is exhibited by a transition from diagonal tension to a more complex stress distribution near the boundary.

In the present study, no account has been taken of the variation of stress within a panel. However, fair agreement with experimental results is obtained and, in contrast to most of the existing theories on thin-webbed beams, no modification is needed for those cases in which the stiffener spacing exceeds the depth.

Shear in the Web

The acting shearing[3] stress τ in the web is approximately equal to QS/It, in which the section constants Q and I refer to the flange cross sections alone. Since, for the beams used in aircraft structures, I/Q is practically equal to the depth h, it follows that the stress τ is also given by

$$\tau = \frac{S}{ht} \qquad [1]$$

It is evident that the section constants I and Q for the flanges of a thin-webbed beam differ but slightly from the corresponding constants for the entire beam cross section. Consequently, the shearing stress given by Equation [1] is approximately correct, even though the web does not buckle.

Web Rupture

Web rupture is the phenomenon of failure under the action of combined stresses, and, accordingly, an exact analysis must call upon one of the several theories of the cause of rupture in metals. However, in the present study, failure is attributed either to shear or to diagonal tension.

Rupture by Shear. The allowable shearing stress τ_{su}, as determined by the ultimate shearing stress F_{su}, is obtained by reducing F_{su} by a rivet factor. The empirical factor 0.90 $(1 - d_s/p_s)$ is recommended for 24S-T aluminum alloy. This may be regarded as the product of a rivet-pattern factor $(1 - d_s/p_s)$ and a rivet-hole stress-concentration factor 0.90. It follows

$$\tau_{su} = 0.90(1 - d_s/p_s)F_{su} \qquad [2]$$

[1] Structural Research Engineer, Consolidated-Vultee Aircraft Corporation.

Contributed by the Aviation Division and presented at the Semi-Annual Meeting, Los Angeles, Calif., June 14–17, 1943, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

[2] Refer to Bibliography at the end of the paper.

[3] See reference (10). Due to Vierendeel-truss action of the frame of the beam, this formula is slightly conservative.

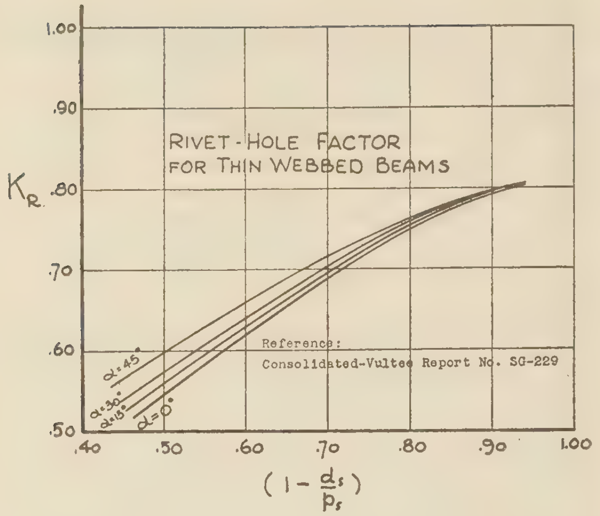

FIG. 1 RIVET-HOLE FACTOR FOR THIN-WEBBED BEAMS

In exceptional cases, the inner row of flange rivets has a smaller spacing factor than the stiffener rivets, and, in these cases, the flange-rivet factor should be used for determining τ_{su}. It is to be noted that τ_{su} is the ultimate shearing stress for the web, only when this stress is more critical than the diagonal tensile stress.

Rupture by Diagonal Tension. Before the web buckles, the diagonal compression stress is equal to the shearing stress τ. It is assumed that the diagonal compression stress remains constant after the buckling. Then the principal stresses in the buckled web are σ and $-\tau_{cr}$. Hence by rotating axes through an angle α, the following equations are obtained:

$$\left.\begin{aligned} \sigma_x &= \tau \cot \alpha - \tau_{cr} \ldots\ldots\ldots\ldots\ldots\ldots (a) \\ \sigma_y &= \tau \tan \alpha - \tau_{cr} \ldots\ldots\ldots\ldots\ldots\ldots (b) \\ \tau &= (\sigma + \tau_{cr}) \sin \alpha \cos \alpha \ldots\ldots\ldots\ldots (c) \end{aligned}\right\} \ldots\ldots [3]$$

The ultimate value for the tensile stress σ is obtained by reducing F_{tu} by a rivet factor K_r. It follows from Equation [3c] that the allowable shearing stress determined by diagonal tension is

$$\tau_{tu} = (K_r F_{tu} + \tau_{cr}) \sin \alpha \cos \alpha \ldots\ldots\ldots\ldots [4]$$

The allowable shearing stress τ_{al} for web design is the smaller of the two values τ_{su} and τ_{tu} given, respectively, by Equations [2] and [4].

A chart (Fig. 1) for the stiffener rivet-hole factor K_r has been constructed from test data obtained by tensile ruptures of 0.030-gage 24S-T alclad sheets with single inclined rows of $^1/_8$-in-diam holes. In exceptional cases, the inner row of flange-rivet holes becomes critical, and then α should be replaced by (90 deg $-$ α) for the determination of K_r from Fig. 1.

The buckling stress τ_{cr} is given by the well-known formula $\tau_{cr} = K_s E (t/b)^2$. Values of the factor K_s may be obtained from Fig. 1-6, ANC-5, 1942 edition. The assumption of simply supported panel edges will cause only a small amount of conservatism in the results. If the stress τ_{cr} exceeds the shear yield stress, then a reduced modulus E should be used. However, in these cases, it is usually found that τ_{su} is less than τ_{tu}.

SELECTION OF FLANGE RIVETS

The flange rivets are subjected to horizontal and transverse loads with respective intensities τt and $\sigma_y t$. Consequently, in view of Equation [3b], the load per inch carried by the flange rivets is

$$t \sqrt{\tau^2 + (\tau \tan \alpha - \tau_{cr})^2}$$

and the net load carried by the flange rivets in one bay is

$$P_r = bt \sqrt{\tau^2 + (\tau \tan \alpha - \tau_{cr})^2} \ldots\ldots\ldots\ldots [5]$$

It is assumed that the stiffener end rivets may be included with the flange rivets in the calculation of the rivet area required to support this load.

ANGLE OF PRINCIPAL STRESS

For the application of Equations [4] and [5], the angle α of principal stress must be determined. Accordingly, attention is focused on a single line Q-Q of diagonal tension in the web, Fig. 2. If the flange deflections at opposite ends of the line of tension are δ_1 and δ_2, then the average strain on the line of tension is

$$\epsilon = h^{-1}(\delta_2 - \delta_1) \sin^2 \alpha \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

The deflections δ_1 and δ_2 result from the shear deflection of the beam[4] and from secondary deflections due to sagging of the flanges and compression of the web stiffeners.

Neglecting end effects, the shear deflection at the section X, Fig. 2, is $\delta L^{-1} X$, where δ is the shear deflection of the free end of the beam, and L is the length of the beam. Hence

$$\delta_1 = \delta L^{-1} X + \delta_1', \quad \delta_2 = \delta L^{-1}(X + h \cot \alpha) - \delta_2' \ldots\ldots [7]$$

in which δ_1' and δ_2' are the secondary deflections of the respective flanges. It follows from Equation [6]

$$\epsilon = [\delta L^{-1} \cot \alpha - h^{-1}(\delta_1' + \delta_2')] \sin^2 \alpha$$

or

$$\epsilon = {}^1/_2 \delta L^{-1} \sin 2\alpha - h^{-1} \delta' \sin^2 \alpha \ldots\ldots\ldots\ldots\ldots [8]$$

where

$$\delta' = \delta_1' + \delta_2'$$

Since the direction of principal stress is that of maximum strain, the derivative of ϵ with respect to α must vanish. Accordingly,

[4] Deflection caused by bending of the beam is neglected, since bending stresses are assumed to be carried by the flanges alone.

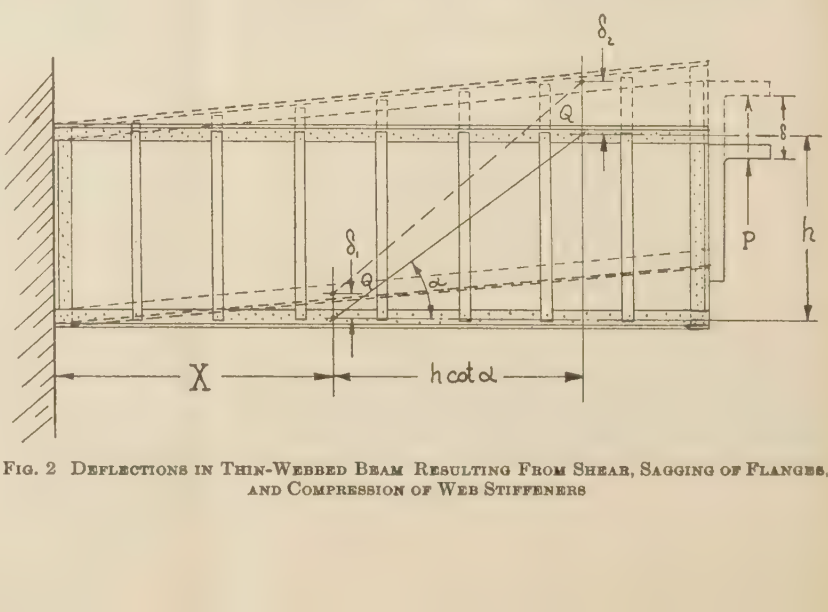

FIG. 2 DEFLECTIONS IN THIN-WEBBED BEAM RESULTING FROM SHEAR, SAGGING OF FLANGES, AND COMPRESSION OF WEB STIFFENERS

Equation [8] yields

$$\tan 2\alpha = \frac{\delta h}{\delta' L} \qquad [9]$$

The term δ' in Equation [9] represents the mean value of the lateral contraction of the beam. Hence δ'/h represents the mean lateral strain $-\epsilon_y$. The ratio δ/L represents the mean shearing strain 2γ. Accordingly, Equation [9] may be written

$$\tan 2\alpha = \frac{-2\gamma}{\epsilon_y} \qquad [9a]$$

This result might have been anticipated at the outset, since Equation [9a] is merely the elementary formula which expresses the angle of principal strain in terms of the components of the strain tensor when the component ϵ_x is zero. However, the derivation of this result, in the present case, serves as a justification for the assumptions that are implied in the derivation of Equation [8].

For the application of Equation [9], the lateral contraction δ' is broken into two components, δ'' due to compression of the web stiffeners, and δ''' due to sagging of the flanges. The first component is

$$\delta'' = \frac{hbt\sigma_y}{A_s E_s} \qquad [10]$$

since $bt\sigma_y$ is the compression load in a stiffener. The second component is

$$\delta''' = \frac{b^4 t\sigma_y}{720 E_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right) \qquad [11]$$

This equation is obtained by the evaluation of the mean bending deflection of a flange, each flange being regarded as a continuous beam supported by the web stiffeners and uniformly loaded by the lateral web tension $\sigma_y t$.

A characteristic beam constant λ is defined by

$$\lambda = btE_w\left[\frac{b^3}{720hE_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right) + \frac{1}{E_s A_s}\right] \qquad [12]$$

It then follows from Equations [10], [11], and [12]

$$\delta' = \frac{h\lambda\sigma_y}{E_w} \qquad [13]$$

Since $\sigma = E_w \epsilon$, it follows from Equation [8]

$$\delta/\delta' = 2L\left(\frac{\sigma}{E_w \delta'} + \frac{\sin^2\alpha}{h}\right)\csc 2\alpha \qquad [14]$$

Hence by Equations [9] and [14]

$$\tan 2\alpha = \frac{\sigma h}{E_w \delta' \sin\alpha\cos\alpha} + \tan\alpha \qquad [15]$$

With Equations [3c] and [13] this may be expressed

$$\tan 2\alpha = \frac{2\csc 2\alpha - r}{\lambda(\tan\alpha - r)\sin\alpha\cos\alpha} + \tan\alpha$$

in which $r = \tau_{cr}/\tau$. This equation may be reduced to

$$(\lambda + 1)\tan^3\alpha = \frac{1 - r\tan\alpha}{\tan\alpha - r} \qquad [16]$$

Equations [12] and [16] determine α. To facilitate the solution of Equation [16], Fig. 3 is given. Since r is a rather passive factor, it is recommended that the angle α at the ultimate web stress τ_{al} be determined from Fig. 3 by an estimated value of r; say $r = \tau_{cr}/25{,}000$.

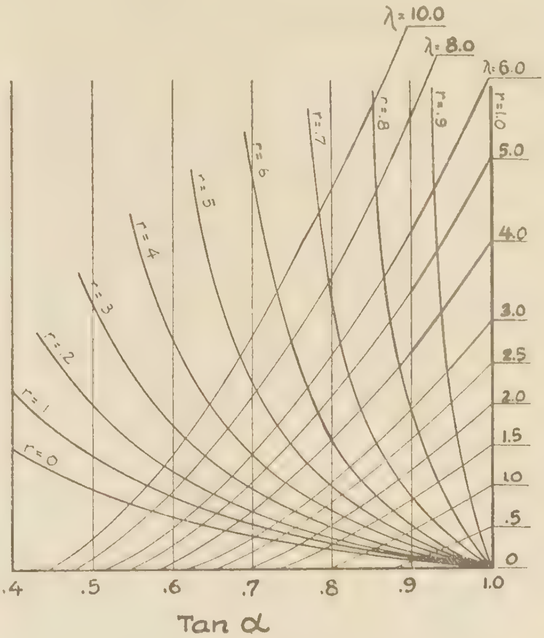

Fig. 3 Wagner Angle Chart

In particular, if the web is a membrane, $r = 0$, and Equation [16] reduces to

$$\cot\alpha = \sqrt[4]{\lambda + 1} \qquad [16a][5]$$

Test Results

Tables 1 and 3 give descriptive constants for twenty-seven test beams, while Tables 2 and 4 give test results and computations. All beams were 24S-T alclad, excepting web stiffeners, as noted in Table 1. For the determination of τ_{su}, an average shear ultimate of 39,000 psi was assumed for 24S-T alclad, rather than the specified minimum of 34,000 psi. Values of F_{tu} were obtained from test coupons. The so-called observed shearing stresses τ_{obs} were obtained from the observed ultimate loads S, by means of Equation [1].

With the exceptions of beams Nos. 12 and 14, the data of Table 2 show close agreement between observed ultimate shearing stresses τ_{obs} and calculated allowable shearing stresses τ_{al}.

TABLE 1 WEB-RUPTURE SPECIMENS; DESCRIPTIVE CONSTANTS

Beam no.	L	h	t	b	A_s	I_f	Stiffener rivets	F_{tu}
1	80	30.1	0.025	9	0.116	0.232	5/32 @ 1.00	66750
2	96	40.0	0.026	9	0.168	0.232	5/32 @ 1.00	66400
3	60	16.1	0.050	4	0.115	0.428	3/16 @ 1.00	64800
4	52.5	10.4	0.052	8.5	0.134	0.108	5/32 @ 1.06	62100
5	106	10.3	0.051	24	0.111	0.194	5/32 @ 0.94	65000
6	66	10.0	0.050	14	0.116	0.128	5/32 @ 0.94	64100
7	166	14.0	0.039	6	0.113	0.820	5/32 @ 1.00	63600
8*	72.2	14.0	0.041	17.8	0.359*	0.495	3/16 @ 1.00	63100
9	80	10.5	0.040	9	0.113	0.299	3/16 @ 1.10	65050
10	80	13.2	0.025	9	0.199	0.299	3/16 @ 1.04	65000
11	80	10.6	0.052	12	0.262	0.385	3/16 @ 1.10	67200
12*	62.5	10.7	0.050	8.5	0.359*	0.495	1/4 @ 1.00	66000
13	79	11.9	0.027	15	0.125	0.385	3/16 @ 1.00	64400
14	73	14.3	0.027	3.25	0.125	0.299	3/16 @ 1.04	64000
15	40	12.1	0.080	3	0.104	0.231	3/16 @ 1.00	65000
16	40	14.2	0.089	3.25	0.148	0.288	3/16 @ 1.00	65000
17	40	12.2	0.080	3	0.089	0.300	3/16 @ 0.88	65000

* Steel stiffeners.

[5] Refer to the Appendix.

TABLE 2 WEB-RUPTURE SPECIMENS; TEST RESULTS AND COMPUTATIONS

No.	S_{obs}	τ_{cr}	r	λ	$\tan\alpha$	$1 - \frac{d_s}{p_s}$	K_r	τ_{su}	τ_{tu}	τ_{al}	τ_{obs}
1	18400	420	0.017	2.00	0.762	0.845	0.777	29600	25200	25200	24400
2	27100	440	0.017	1.45	0.795	0.845	0.777	29600	25300	25300	26000
3	22500	8350	0.300	1.73	0.815	0.812	0.763	28500	28300	28300	27800
4	12100	3100	0.139	4.06	0.687	0.852	0.780	30000	24000	24000	22300
5	9900	1740	0.093	37.60	0.401	0.834	0.773	29200	17900	17900	18700
6	10580	1990	0.094	10.60	0.560	0.834	0.773	29200	21900	21900	21100
7	13750	2840	0.113	2.09	0.770	0.845	0.777	29600	25200	25200	25200
8	12200	790	0.037	2.64	0.725	0.812	0.763	28500	23200	23200	21200
9	10650	1780	0.070	3.46	0.695	0.830	0.770	29200	24400	24400	25400
10	8100	570	0.023	1.27	0.825	0.820	0.767	28800	24800	24800	24400
11	13300	2490	0.104	3.28	0.710	0.830	0.770	29200	25600	25600	24000
12	15850	2930	0.099	0.58	0.900	0.750	0.735	26300	25500	25500	29600
13	7110	490	0.022	4.23	0.665	0.812	0.763	28500	22800	22800	22100
14	12100	3700	0.118	0.71	0.885	0.820	0.767	28800	26200	26200	31400
15	26900	38000	...	...	...	0.812	...	28500	...	28500	27800
16	36800	40200	...	...	...	0.812	...	28500	...	28500	29100
17	27400	38000	...	...	...	0.786	...	27700	..	27700	28000

TABLE 3 FLANGE-RIVET-SHEAR SPECIMENS; DESCRIPTIVE CONSTANTS

Beam no.	L	h	t	b	A_s	I_f	Flange rivets per bay	Stiffener end rivets
18	84	20.2	0.050	16	0.150	0.235	29 1/8	2 3/16
19	78	30.1	0.040	9	0.156	0.180	15 1/8	2 3/16
20	57.5	20.4	0.050	3.5	0.132	0.385	5 1/8	2 3/36
21	57.5	20.4	0.051	3.5	0.130	0.385	5 5/32	2 3/16
22	57.5	18.5	0.051	3.5	0.130	0.385	5 1/8	5/32 and 3/16
23	68	20.2	0.051	5	0.162	0.299	5 1/8	2 3/16
24	78	20.2	0.054	7	0.194	0.299	13 3/32	2 3/16
25	80	20.2	0.042	9	0.195	0.299	15 3/32	2 3/16
26	80	30.2	0.024	9	0.126	0.299	9 3/32	2 3/16
27	80	30.2	0.062	9	0.266	0.299	11 1/8	2 3/16

TABLE 4 FLANGE-RIVET-SHEAR SPECIMENS; TEST RESULTS AND COMPUTATIONS

Beam no.	S_{obs}	τ_{obs}	τ_{cr}	r	λ	$\tan\alpha$	P_r	A_r	P_r/A_r
18	16200	16000	740	0.046	7.44	0.595	14600	0.411	35400
19	24000	19900	1060	0.053	2.45	0.740	8680	0.239	36300
20	27000	26400	10500	0.398	1.32	0.857	5080	0.116	43800
21	30800	29500	10900	0.369	1.37	0.850	5850	0.151	38700
22	22900	24300	13400	0.550	1.52	0.865	4590	0.108	42400
23	17800	17000	7200	0.422	1.59	0.843	4700	0.116	40500
24	16800	15350	4270	0.278	2.00	0.793	6540	0.145	45000
25	12000	14150	1630	0.115	2.05	0.772	6390	0.159	40100
26	13300	18100	500	0.028	1.75	0.778	4950	0.117	42200
27	24600	13100	3300	0.252	2 23	0.780	8230	0.190	43300

Since beams Nos. 12 and 14 are the only beams whose characteristic constants λ are less than unity, it is conjectured that the proposed method for determining the allowable shearing stress becomes conservative in this case. The reason for this circumstance is not clear, but since the condition $\lambda < 1$ is unusual, the conservatism that it occasions in beam design is not important.

All beams had A17S-T flange rivets. The ANC-5 allowable shearing stress for these rivets is 27,000 psi, whereas tests of lap joints with heavy sheets indicate ultimate rivet shearing stresses in the range 32,000 to 34,000 psi. By comparing these values with the ultimate rivet shearing stresses (P_r/A_r), computed in Table 4, it is realized that the proposed method of rivet selection (Equation [5]) averages 23 per cent conservatism, a margin of safety which is usually permissible in rivet design. The discrepancy may be partly due to gusseting effect, although, in most cases, it is observed that some margin of safety exists even when the lateral load $\sigma_y t$ is entirely neglected.

Conclusions

1 The acting shearing stress in a thin-webbed beam is closely approximated by Equation [1].

2 The angle α of principal stress in a diagonal tension web is given by Equation [12] and Fig. 3.

3 The allowable shearing stress τ_{al} in a thin-webbed beam is the smaller of the two values τ_{su} and τ_{tu}, given, respectively, by Equations [2] and [4]. The rivet factor K_r is given by Fig. 1.

4 The load per bay carried by the flange rivets is conservatively approximated by Equation [5]. In the use of this equation, the web-stiffener end rivets are included among the flange rivets.

Appendix

It is interesting to see that Equation [16a] may also be derived from the principle that the angle of principal stress is that which entails the least strain energy. The strain energy of a membrane under tension σ is

$$U_w = \frac{t}{2E_w}\int\int \sigma^2\, dX\, dY$$

Consequently, since σ is constant, the strain energy of the web in one bay is

$$U_w = \frac{bht\sigma^2}{2E_w} = \frac{bht\tau^2}{2E_w \sin^2\alpha\cos^2\alpha}$$

The flanges may be regarded as continuous beams, supported by the web stiffeners. Therefore, the strain energy of bending in the flanges in one bay is

$$U_f = \frac{t^2b^5\sigma_y^2}{1440E_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right) = \frac{t^2b^5\tau^2}{1440E_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right)\tan^2\alpha$$

The strain energy in a web stiffener is

$$U_s = \frac{b^2t^2h\sigma_y^2}{2E_sA_s} = \frac{b^2t^2h\tau^2}{2E_sA_s}\tan^2\alpha$$

Hence the total strain energy in one bay is

$$U = \frac{2bht\tau^2 csc^2 2\alpha}{E_w} + \left[\frac{t^2b^5\tau^2}{1440E_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right) + \frac{t^2b^2h\tau^2}{2E_sA_s}\right]\tan^2\alpha \quad [17]$$

The condition $\partial U/\partial\alpha = 0$ yields

$$\lambda = btE_w\left[\frac{b^3}{720hE_f}\left(\frac{1}{I_{fc}} + \frac{1}{I_{ft}}\right) + \frac{1}{E_sA_s}\right] = \frac{\cos 2\alpha}{\sin^4\alpha} \quad ..[18]$$

It is to be noted that the expression λ of Equation [18] is the same as that of Equation [12]. Furthermore by solving Equation [18] for α, one is again led to Equation [16a].

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Application of Turbine-Supervisory Instruments to Power-Generating Equipment

By J. L. Roberts[1] and H. M. Dimond,[2] Schenectady, N. Y.

Proper application and use of turbine-supervisory instruments promote efficient operation and prolong the life of power-generating equipment. Equipment is available to record turbine conditions in terms of shaft eccentricity, bearing vibration, shell expansion, speed, and camshaft position. A nonrecording interference detector may also be provided. The interpretation of typical records made by these instruments is a feature of the paper. Not the least important function of the records is the indication given of mechanical trouble in turbine or generator. Case examples of detecting incipient failures are included by the authors.

It is becoming increasingly important that full advantage be taken of any equipment which will make possible the more efficient operation and prolong the life of power-generating equipment. Some years ago, the General Electric Company developed a line of turbine-supervisory instruments to indicate to the turbine operator, and to make a permanent record of, various mechanical conditions of the turbine.[3]

During the past few years many improvements and additions have been made to these original instruments. The equipment now records turbine conditions in terms of shaft eccentricity, bearing vibration, shell expansion, speed, and camshaft position. Also provided is a nonrecording interference detector. All these values are important in the proper operation of a turbine during starting, running, and shutdown periods. When unusual conditions are shown, the turbine operator will be able to make changes in his operating procedure to prevent unnecessary wear which might otherwise occur.

Records of Starting Sequence

In Fig. 1 is shown a set of records taken at the time of a normal starting sequence on an 80,000-kw turbine. For easy reference, the various chart scales are printed at frequent intervals along the 4-in-wide turbine chart. The shaft-eccentricity and shell-expansion scales are linear. The bearing-vibration scale is logarithmic to enable accurate reading at the low-vibration end and still enable vibrations as high as 15 mils to be recorded on the same scale. The speed and camshaft-position recorder chart is marked with two sets of values:

1 Speed in revolutions per minute.
2 Camshaft position in per cent open.

[1] Turbine Engineering Department, General Electric Company. Mem. A.S.M.E.

[2] General Engineering Laboratory, General Electric Company.

[3] "Turbine Supervisory Instruments and Records," by J. L. Roberts and C. D. Greentree, Trans. A.S.M.E., vol. 58, 1936, pp. 607–614.

Contributed by the Power Division and presented at the Annual Meeting, New York, N. Y., Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

As the machine is being brought to speed the record made is referred to the speed scale for interpretation. After the machine is brought to synchronism and connected to the line, the speed is determined by the system frequency, and a speed record during this period would be of little value. Therefore upon closing of the line switch the recording of the speed and camshaft-position recorder is automatically transferred to camshaft position. The load on the machine and load effects of system disturbances are then clearly indicated by the per cent rotation of the inlet-valve camshaft. Opening of the line breaker for any reason will automatically return the recorder to speed indication. This is a valuable feature because in the case of an emergency opening of the circuit breaker, the amount of overspeed and the subsequent stopping of the turbine are permanently recorded.

Referring again to the normal starting sequence, shown in Fig. 1, the conditions at 4:00 a.m. are shown to be:

(*a*) Shaft eccentricity on turning gear—4 mils (shaft-eccentricity record).

(*b*) Some heat from previous operation is retained by the turbine shell as evidenced by the expansion (shell-expansion record).

The speed record shows that steam was first applied to the turbine at 4:03 a.m. The speed was raised to approximately 350 rpm and held during the warm-up period. During this time, the shell expansion changed very little and the eccentricity was constant at 3 mils. At 4:40 the turbine speed was gradually increased until full speed was reached at approximately 5:20. The eccentricity and shell-expansion changes were normal. The turbine was run at full speed for 30 min without load. The notation indicates an oil-trip test of the emergency governor after the first 20 min of this period

The machine was connected to the line at 5:48 a.m. and the speed and camshaft-position recorder automatically switched from speed recording to camshaft-position recording. The machine was then gradually loaded until full load was reached at 9:00 a.m. During the loading period, the expansion was uniform and the eccentricity and bearing vibration normal. The time, from the first steam application to the start of the loading period, was 1 hr 45 min.

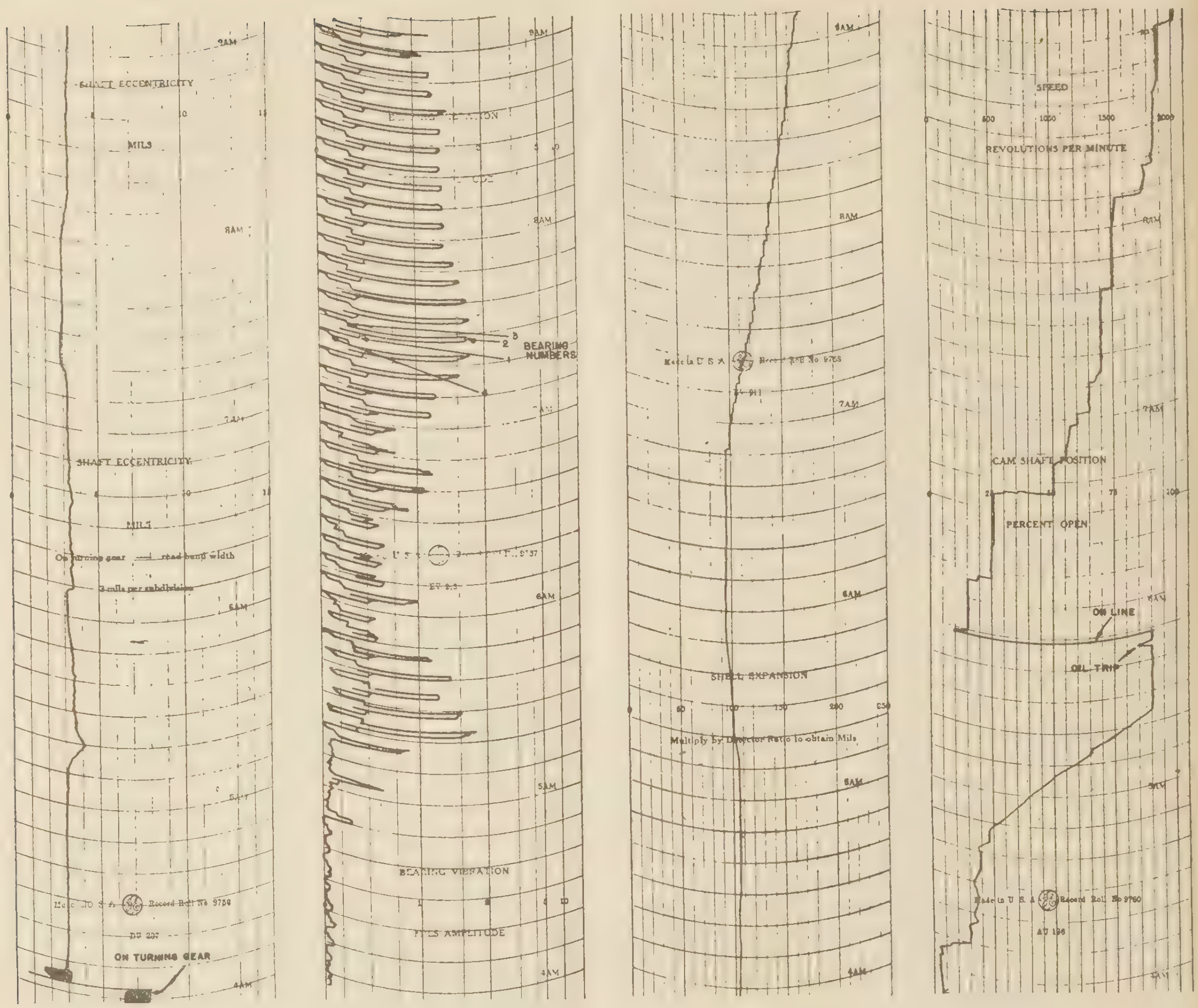
The starting records for the same machine under abnormal conditions, as shown in Fig. 2, indicate initial conditions to be:

(*a*) Shaft eccentricity on turning gear—6 mils (shaft-eccentricity record).

(*b*) The turbine shell retained more heat from previous operations than in the first case (shell-expansion record).

Steam was first applied at 7:15 p.m. and the speed increased to about 300 rpm and held for 20 min. As the speed was increased to 1500 rpm, the eccentricity and bearing vibration increased much too far and the speed was reduced to 700 rpm. The speed was again increased to 1200 rpm and the eccentricity and vibration were still excessive, so speed was then reduced to 500 rpm and held until temperature equalization had reduced the eccentricity to about 5 mils.

The speed was again increased and at about 1400 rpm the



SHAFT ECCENTRICITY | BEARING VIBRATION | SHELL EXPANSION | SPEED AND CAM SHAFT POSITION

Fig. 1 Records of Starting Sequence of 80,000-Kw Turbine

eccentricity and vibration started to increase rapidly. The speed was then reduced slightly and held at 1300 rpm until the vibration and eccentricity were both reduced. The machine was then brought to synchronous speed and connected to the line. The loading then proceeded as usual.

If the machine had been brought up to speed under initial conditions, unnecessary wear on packings, bearings, etc. would have occurred, resulting in decreased machine efficiency. By the indications of the instruments, it was possible to know the condition of the machine and to allow necessary time for working out excessive eccentricity and vibration. During both of these starting sequences, the interference detector gave the operator a highly amplified "listening-rod" signal.

Warning of Mechanical Trouble

In addition to providing information essential to the operation of a turbine for minimum wear and strain, the turbine-supervisory instruments will, in many cases, give warning of the development of some mechanical trouble. Of considerable interest are the cases herein described in which unnecessary expense and serious damage to the turbine-generator, due to mechanical failure of some part, has been averted by heeding the warning given by the turbine-supervisory-instrument records.

Fig. 3 shows the vibration and eccentricity records from a turbine in which certain conditions of load, oil temperature, etc. produced considerable shaft whip. The gradually increasing band width of the eccentricity record indicates that there is some movement of the shaft other than the cyclic movement caused by a constant value of eccentricity. This erratic movement gradually increased until it broke into a definite whip. At point *A* on the record, this increasing eccentricity indication enabled the operator to change operating conditions, thus preventing the whip. Forewarning of the shaft whip was given only by the indication of the turbine-supervisory instruments. In this case the turbine-supervisory instrument not only indicated the unsatisfactory condition but provided a means for studying the possible cause.

The eccentricity record shown in Fig. 4 was taken on a 40,000-

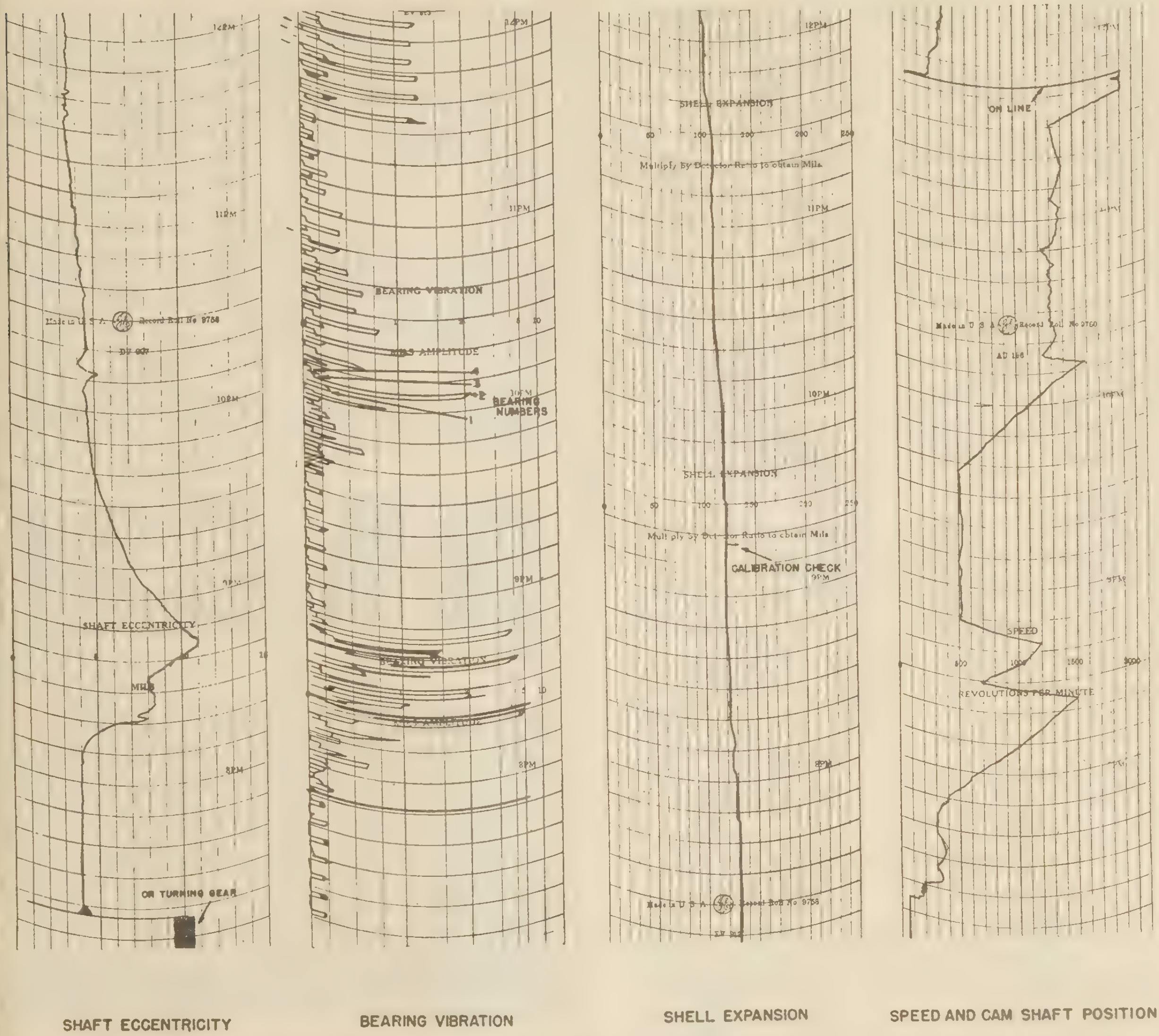


Fig. 2 Starting Records for 80,000-Kw Turbine Under Abnormal Conditions

kw turbine and illustrates another warning of approaching trouble. The eccentricity recorder suddenly began to draw a wide band as shown in the first part of section *A*. At intervals during the day, the records show sudden changes in eccentricity and also increased band width of the record. The machine was shut down for a preliminary inspection and nothing was found wrong. An attempt to load the machine again resulted in a record shown at *C* in Fig. 4. The machine was again shut down and a more thorough inspection revealed that several buckets were broken. It is concluded that the sudden changes in the eccentricity records were caused by the breaking of the buckets. Had it not been apparent from the chart records that some unusual condition existed, it is probable that extensive damage would have resulted.

The sections of turbine-supervisory records shown in Fig. 6 were taken on a large turbine. The records show that although the eccentricity, shell expansion, and load were very nearly constant, the vibration gradually increased. For illustration, small sections of the charts taken at 24-hr intervals are shown. This gradual increase continued for several days until the engineers ordered the machine shut down for inspection. The machine was stopped and checked, and a subsequent attempt to start the machine resulted in the high vibration and eccentricity shown in the last section of the record. Upon a more detailed inspection, it was found that a crack was developing in the generator-field shaft.

Without the warning given by the supervisory instruments, this crack could have progressed until complete failure resulted. The vibration increase was so gradual as to be unnoticed except for the indication of the recorder. The permanent record also showed that the increase was steady. Such a set of data could not have been obtained by any of the other vibration-checking procedures commonly used. It is the opinion of the engineers concerned with this particular incident that considerable expense was saved by the prevention of serious damage to the generator and accompanying equipment.

Test Panels

A typical panel arrangement of the complete set of instruments Fig. 5 illustrates their neat, compact appearance. Electronic

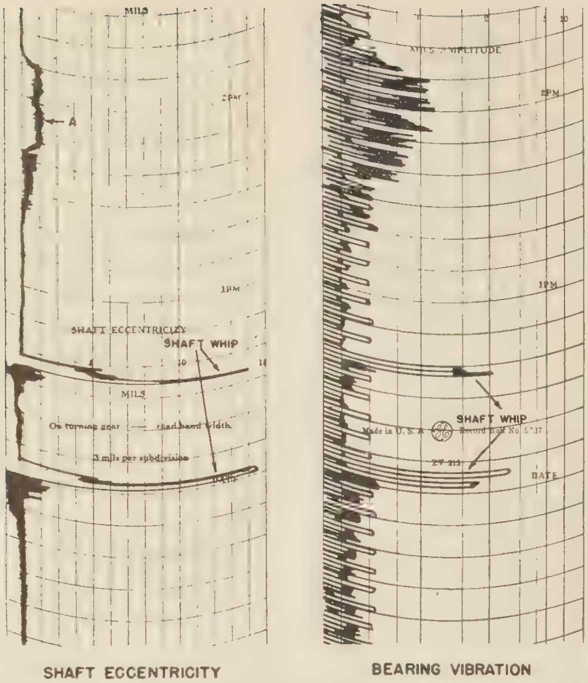


Fig. 3 Vibration and Eccentricity Records of Turbine Experiencing Shaft Whip

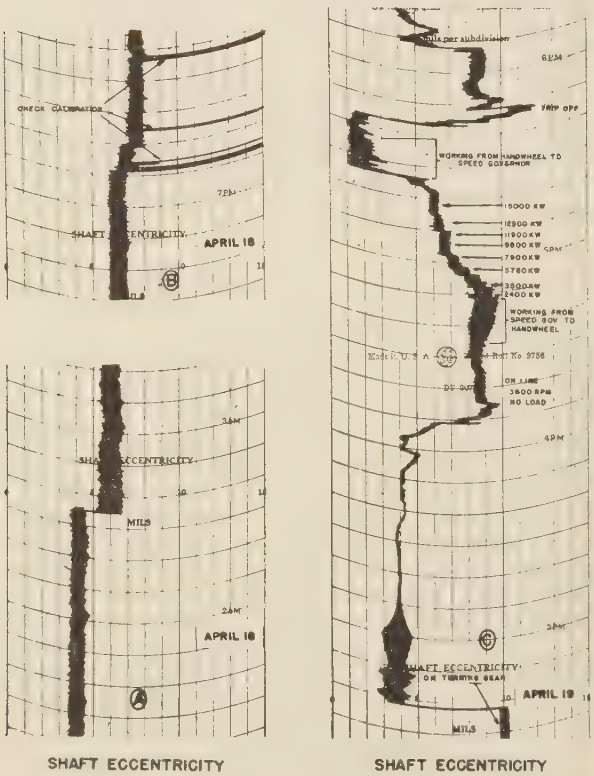


Fig. 4 Eccentricity Record Taken on a 40,000-Kw Turbine in Which Buckets Were Breaking

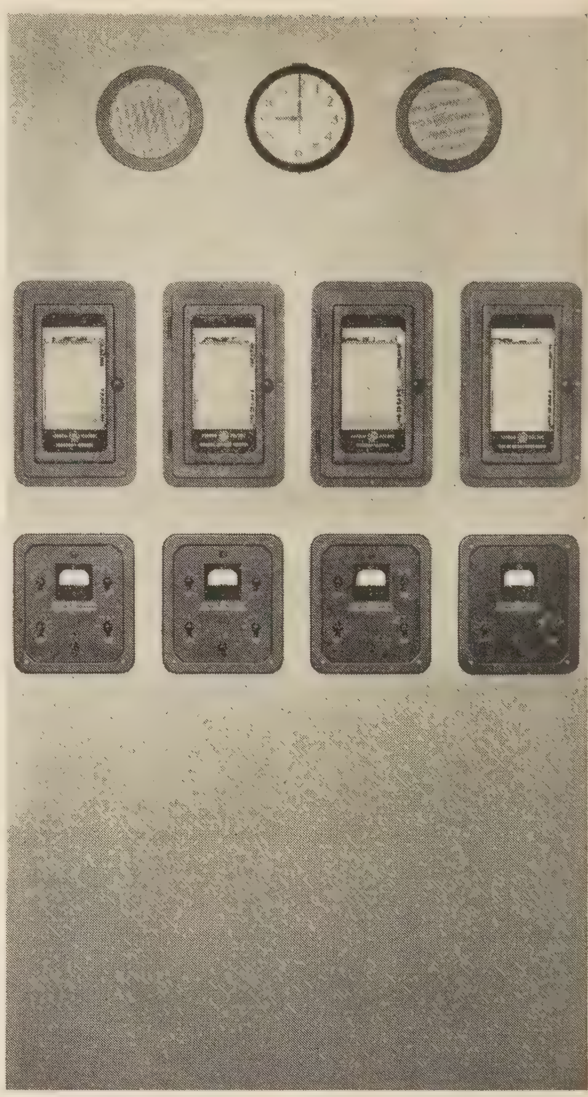


Fig. 5 Turbine-Supervisory-Instrument Panel

circuits are employed in the equipment. Each power unit includes a small test instrument and test-point selector switch semiflush mounted on a small panel directly below the recorder. By means of this test instrument and test switches, the calibration of the recording equipment may be easily checked. In case of tube deterioration or other abnormal condition of the circuit, it may be found by the test-point indications, even by persons not familiar with electronic devices. By means of the test point and the calibration adjustments provided, the accuracy of the instruments may be maintained within five per cent of full scale values.

Each equipment consists of an appropriate detector or detectors, mounted at the proper location of the turbine generator set, and a power unit and recorder mounted on a panel near the operating board. These equipments have been expressly designed for use in power plants and therefore have the required ruggedness and reliability to perform their operations satisfactorily.

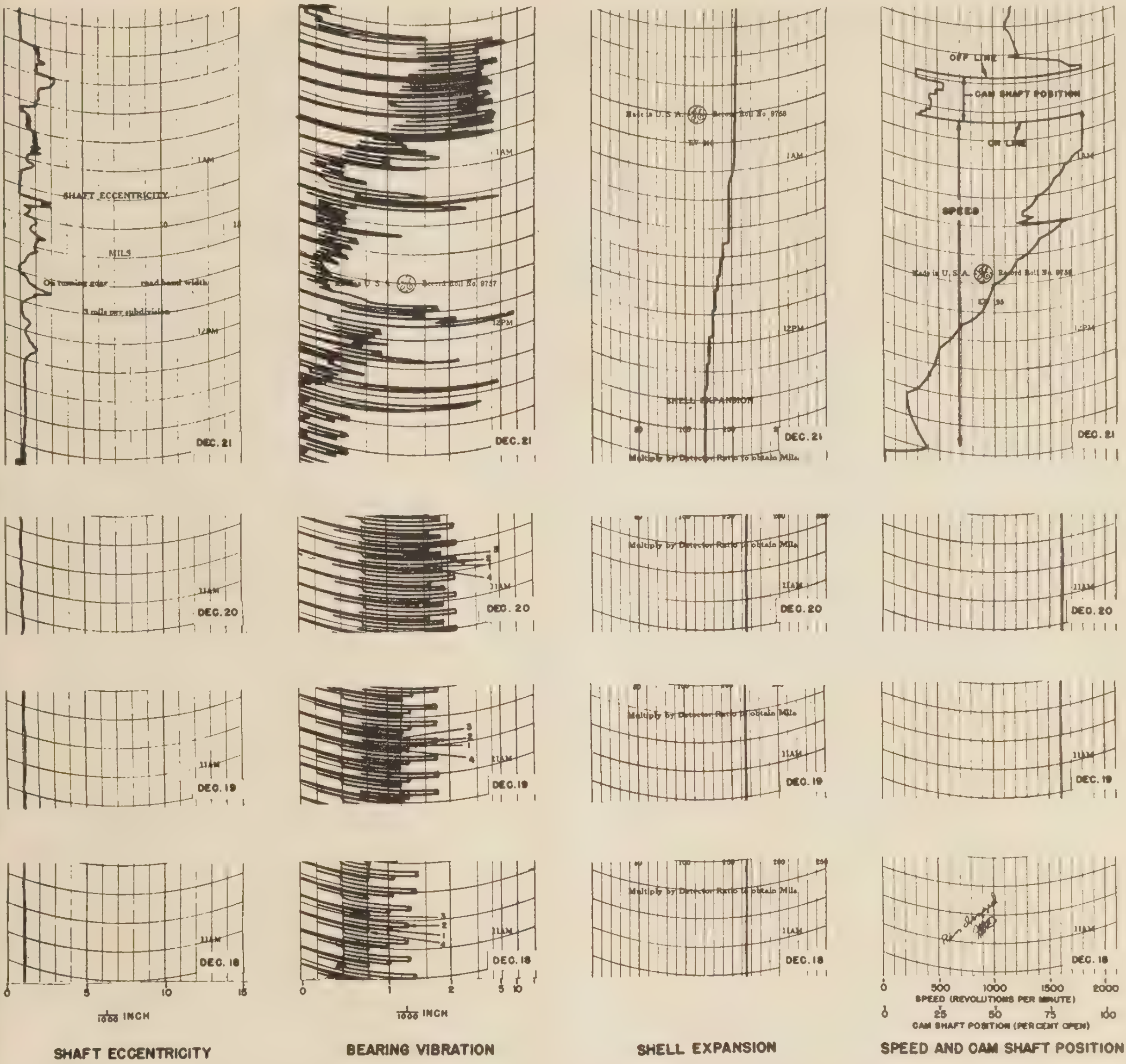


Fig. 6 Records Taken on Large Turbine Showing a Crack to Be Developing in the Generator-Field Shaft

As more experience is being acquired, the importance of these instruments to proper turbine operation is becoming increasingly evident. Approximately 80 per cent of the large turbines now manufactured by the authors' company are being equipped with one or more of these instruments. The instruments recommended depend upon turbine characteristics, power-plant arrangement, etc.

One large power company makes a practice of keeping a file of photostatic copies of all starting-sequence records on their machines. These records and an accompanying brief "case history" are invaluable in the study of turbine performance, training new operators, etc.

By careful observation of the turbine-supervisory-instrument records during starting and operating periods, either "on the spot" or by review at a later date, much valuable information concerning machine performance and operation may be obtained. From intelligent use of such information, the maximum life and efficiency of the power-generating equipment may be realized.

Discussion

C. C. Franck.[4] We wish to take this opportunity to congratulate the authors on an interesting presentation, especially for the excellent charts.

The writer's company has been engaged in the development of supervisory instruments for large-turbine practice since 1932. The first supervisory instruments were developed for use with the large 1800-rpm machines. The original group of instruments included (1) expansion meter, (2) shaft-eccentricity meter, (3)

[4] Engineer in Charge of Central Station Turbines, Westinghouse Electric & Manufacturing Company, South Philadelphia Works, Philadelphia, Pa. Mem. A.S M.E.

noise meter, and (4) bearing-pedestal vibration meter. With the wider use of the more recent 3600-rpm type of unit, experience dictated that it would be necessary to reconsider our original ideas in regard to the suitability of the instruments at hand.

For example, we found that although a bearing pedestal might be reasonably quiet, the turbine shaft could be vibrating considerably. In other words, the bearing-pedestal vibrometer was indicating that the unit was smooth while the readings on the shaft might show that excessive vibration existed within the spindle proper. Following this experience, a decision was made to develop a shaft vibrometer to replace the heretofore satisfactory pedestal vibrometer. This has proved to be a very satisfactory type of supervisory instrument and has given reliable results over a long period of operation.

The expansion meter has proved to be a very valuable instrument when used during warm-up periods prior to placing the unit on the line. A careful check of the record will indicate whether or not all parts are responding in a normal way to the changes in temperature when coming from a "cold condition" to the normal running condition. This instrument will give a good indication of conditions existing if water is carried over from the boiler into the turbine.

Based on our experience we have not deemed it advisable to combine the speed and governor-valve position indicator. It is true that the speed recorder is of greatest value when starting the machine and warming it up prior to putting it on the line. Once the unit has been placed on the line the speed recorder merely draws a straight line unless some unusual condition is experienced. However, there have been cases in which the speed of the unit exceeded synchronous speed while carrying load. The release of a good part of the load from the unit would naturally be accompanied by an increase in speed. We are of the opinion that by placing the dual duty on the instrument it might result in complications and a valuable bit of information would be lost. In addition, the governor-valve position indicator is of greater value to the control room than it would be to the operators on the turbine-room floor. In our arrangement of governor position indicator the meter itself is placed in the control room and remotely indicates the position of the governor valves while the turbine is in operation.

In addition to the previously mentioned supervisory instruments, we have developed another rather interesting and useful instrument known as the spindle-position meter. This is used to give an indication of the relation between the spindle and the cylinder while the machine is in operation. It has been our practice to thoroughly and comprehensively chart the relative motion of the rotating and stationary parts; during the warming-up period, periods of operation with changes in load, and in shutting down the unit. Once having made a complete check of the internal changes in the machine under the various conditions of operation, we feel safe in assuming that as long as the relation between the fixed portions of the machine remains constant, no internal interference will be experienced.

A complete description of the supervisory instruments is contained in an article by H. C. Werner and G. V. Krenikoff.[5]

[5] "Instrument Aids for Turbine Operation," by H. C. Werner and G. V. Krenikoff, *Power Plant Engineering*, vol. 46, Nov., 1942, pp. 84–87.

Automatic Temperature-Recording Control System

By M. E. Moore,[1] Santa Monica, Calif.

The automatic temperature-recording system developed by the Douglas Aircraft Company, Inc., is described. Designed primarily for use in flight-test work, the system may be readily adapted to test stands and the like. The system provides for the measurement of any number of points in groups of twenty-four each. The adaptation of this system to any automatic temperature-recording device is discussed, and the requirements of such a device are stated. Design considerations are covered, and the construction and operation of the system described.

Modern airplanes have made high-speed recording of flight-test information more necessary than ever before. Because of their high rate of climb less time is available in which to obtain the required information due to rapidly changing outside-air temperature, altitude, power, and other conditions. Automatic recording is necessary because test pilots and flight engineers of modern airplanes simply have too much to do. In addition, a constantly increasing number of temperature measurements are necessary because engines are continually being built with more and more power packaged into a single unit.

With the advent of a successful high-speed automatic temperature recorder, it became apparent that the problem of flight-test temperature measurement could be established on a more substantial basis than had been previously possible. Prior to the development of such a recorder, temperature measurements were made with the aid of a manually balanced laboratory-type potentiometer and a group of manually operated switches. It was necessary for the flight engineer to switch the potentiometer from one thermocouple circuit to another, and then manually to balance the instrument and to record the reading upon a suitable data sheet. Such an installation was extremely time-consuming in point of operation; therefore the amount of data which could be obtained was severely limited, often below basic necessity. The appearance of an automatic high-speed temperature recorder removed the restrictions of the manual system, with regard to the time required for the operation of the potentiometer. It then remained to devise a suitable automatic-control system for use with the recorders to avoid the necessity of switching manually from point to point and recording the readings upon a data sheet. The development of the system to be described has largely eliminated the remaining difficulties. It has become possible to obtain a greater amount of temperature information in a much shorter time with equal if not greater accuracy than formerly. At the same time, the flight engineer may spend the major portion of his time at other duties.

Details of Control System

The system developed by the author's company is designed to work in conjunction with a suitable automatic temperature recorder to enable a series of temperatures to be measured automatically in a progressive sequence. It consists basically of a number of switching units and a master control panel. The switching units are used to select the thermocouples for measurement in order, 24 couples to each switching unit. The control panel is for the purpose of controlling the selection of switching units in sequence, and indicating the couple being measured at any given time. The system is controlled by an impulse obtained from the automatic recorder which is supplied by the instrument at the time it has stabilized and recorded the temperature of the point under measurement. The system thus provides for the selection of groups of 24 thermocouples, corresponding to the 24 couples of each switching unit in any desired sequence of such groups, and will accommodate 8 switching units.

The system may be adapted for use with any automatic temperature recorder provided one condition can be realized, i.e., the recorder must provide an impulse after the instrument has been stabilized and the temperature recorded. On the recorders used by the author's company, this impulse is obtained from a switch which is actuated by the printing mechanism each time the printing head is lowered to record a temperature. The purpose of the impulse thus obtained is to advance the switching units one position to the next thermocouple in order, thus making it possible to go through the points one by one in order. A typical record obtained in flight is shown in Fig. 1. It will be noted that two important principles are employed:

1 All temperatures falling within a reasonably narrow temperature range are grouped together. This speeds up the recording as the printing head has but a short distance to travel from one point to the next, rather than to travel from one end of the scale to the other each time a new couple is selected.

2 A certain pattern is employed to identify the various switching units or "banks." Thus, starting from the reference pattern, the temperatures are recorded in time sequence on the chart, and hence may be recognized. Were this not done it would be virtually impossible to tell from the record what point of measurement a given print represented.

In the development of the system, several basic design considerations presented themselves. The system must obviously be fully automatic if it is to be completely successful. It must be dependable. To this end it must be thoroughly foolproof and must not rely for its proper operation upon a large number of interdependent units. It must consist of units of such size that space for them can be found even in small airplanes. The system must, further, be flexible. This has been accomplished by breaking the system down into a master control panel and a number of separate switching units, any number up to 8 of which may be used with one panel. These units may be located where convenient. To this end a previous design, having 6 switching units built into the control panel, Fig. 11, was abandoned in favor of the more flexible system herein described. It is generally possible to find space for a large number of relatively small units, while it is practically impossible in many cases to find space for one large unit. By locating the switching units near the source of temperatures, the length of thermocouple leads may be re-

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[1] Engineering Laboratories, Douglas Aircraft Company, Inc.

Contributed by the Heat Transfer Division and presented at the Semi-Annual Meeting, Los Angeles, Calif., June 14–17, 1943, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

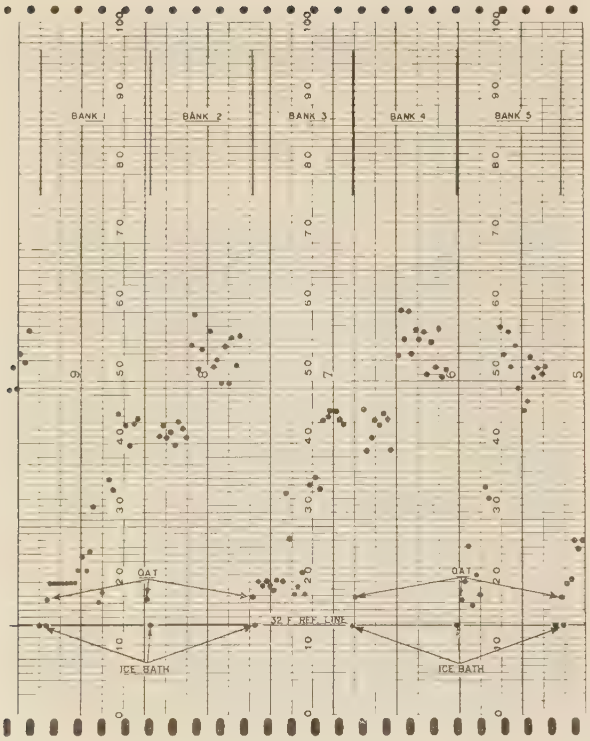

Fig. 1 Typical Temperature Record Obtained in Flight

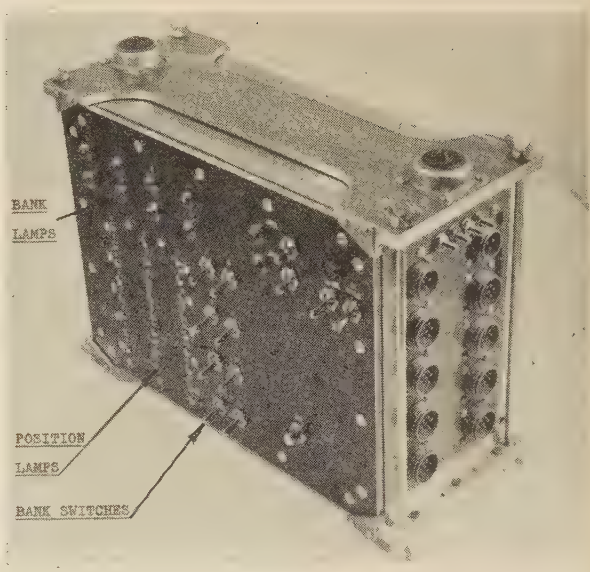

Fig. 2 Control Panel Mounted in Cradle

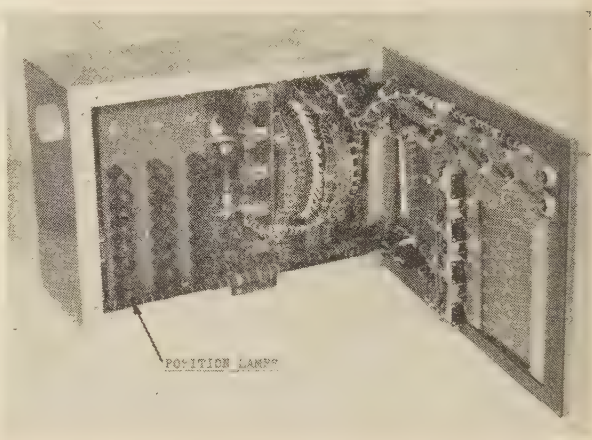

Fig. 3 Control Panel With Front Open

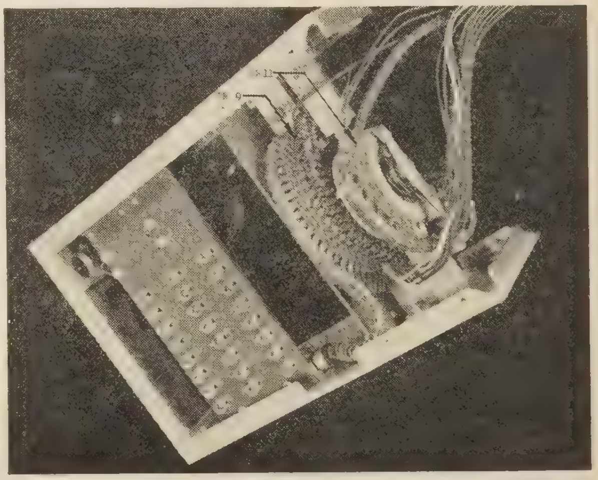

Fig. 4 Front View of Panel During Construction

duced. Large bundles of thermocouple wires, etc., are not desirable as the length of such leads changes with each installation, and furthermore space is not always available for a large bundle of such wires.

The control panel provides the means by which the switching units are caused to operate automatically. It further indicates the point under measurement at all times and also gives a positive check on the operation of the system. These functions are covered by the basic control circuit. An electronic amplifier, associated with and built into the control panel, provides a further check on the system and also provides a means of synchronizing the switching units with the control panel following servicing of the units, or in the event that the system drops out of synchronism, due to temporary power failure, or any other such cause.

The general arrangement of the control panel and switching units is shown in Figs. 2 to 8, inclusive. Fig. 2 shows the control panel mounted in its cradle and illustrates the general arrangement of the unit. Fig. 3 illustrates the control panel with the front swung open showing the arrangement of the position lamp bank and general arrangement of equipment. Figs. 4 and 5 are views of the front and back, respectively, of the control panel during construction. Figs. 6 and 7 are views of the switching units. Figs. 8, 9, and 10 show the detailed wiring diagram of the control panel, electronic amplifier, and switching units, respectively.

Functions of System Parts

For the purpose of explaining the system employed, the functions of the components and their operation will be described. First, some means of switching the recorder from one point of measurement to the next is required. This is taken care of by a number of switching units each handling 24 thermocouples, as

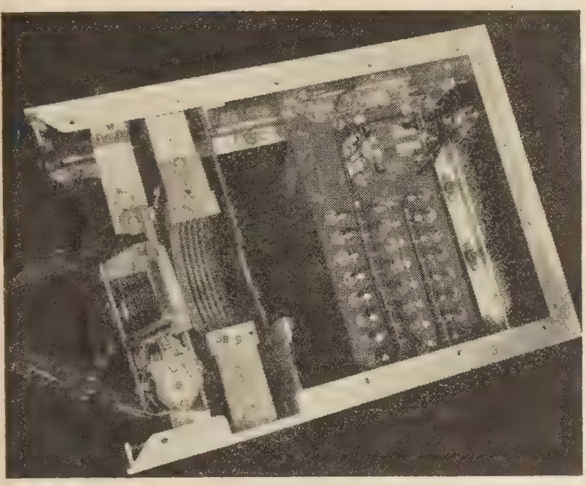

FIG. 5 BACK OF PANEL DURING CONSTRUCTION

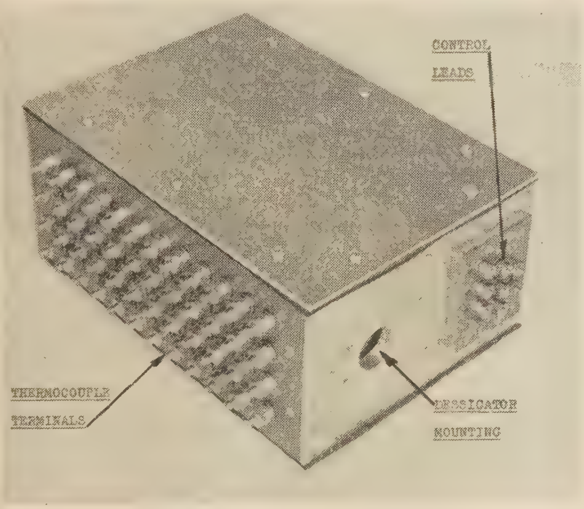

FIG. 6 SWITCHING UNIT

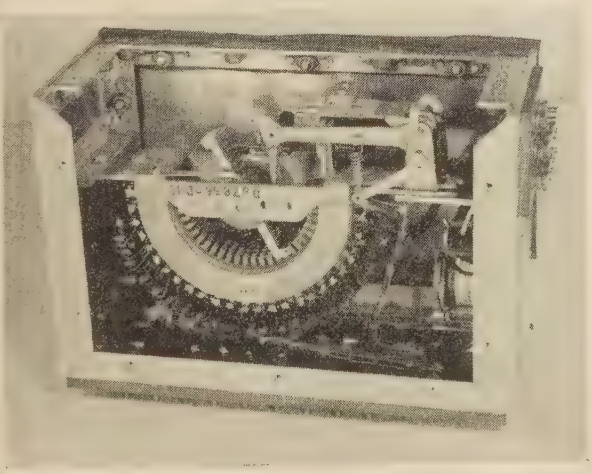

FIG. 7 INTERIOR VIEW OF SWITCHING UNIT

indicated in Fig. 10. The switching units in turn must provide the following:

1 A method of switching both sides of the thermocouple circuit in order to eliminate common returns.

2 A method of identifying at the control panel which switching unit is in operation.

3 A means by which it is possible to tell if the switching unit is in step with the position-indicator lamps on the control panel, and a means by which it is possible to bring them into step automatically if they are out of step.

4 A means by which the unit is cleared from the recorder-thermocouple circuit after the control panel has gone through the cycle of operation of the particular switch unit in question, thus preventing a "jammed" switching unit from making the rest of the system inoperative also.

These functions are accomplished by the four levels of the 25-point rotary switch in order from top to bottom, and the auxiliary relay R 41, as shown in Fig. 10. The 25th position of the switching unit is the "off" position, in which the switching unit is left when it is not in operation. The third level of the rotary switch runs directly to the bank-lamp circuit on the control panel; when the switch is in operation the corresponding bank lamp on the control panel is lighted. This feature provides a positive check on the operation of the system, for if the bank lamp is not extinguished when the control panel advances to the next switch unit in order, the switch unit is obviously jammed. The fourth level of the rotary switch forms one pair of arms of a bridge circuit. The other pair of arms consists of a similar resistor bank on the position-indicating switch of the control panel. The bridge circuit thus formed in connection with an electronic control amplifier fulfills the third function mentioned. The operation of this portion of the system will be described elsewhere.

In the construction of the switching units, iron and constantan wire is used throughout the thermocouple portion of the circuit between the rotary switch and the terminal blocks, etc. This has been found necessary to minimize errors arising from possible temperature gradients throughout the box. Switching units constructed in this manner have been subjected to cold-room tests to simulate the temperature gradient encountered in a rapid descent from high altitude as in a dive. The maximum error caused through an initial temperature difference of 140 F was found to be approximately 3 F. Approximately the same amount of error was obtained with standard manually operated thermocouple switches subjected to the same test conditions.

As was mentioned previously, the control panel allows for the proper selection of the switching units and also provides the means by which the point under measurement may be identified. The 11-point rotary switch, S.R. 11 of Fig. 8, selects the proper switching units. The terminals 1 to 7 of the switching unit connect in order to terminals *A* to *G* of the control-panel switch-unit plug. S.R. 11 switches, in order from top to bottom level, the signal input lead to the grid of the amplifier obtained from the two arms of the bridge circuit of the switching units, the coil circuit of the switching-unit motor, and completes the coil circuit of the auxiliary relay R 41 of the switching unit. The fourth level of S.R. 11, in conjunction with the bank selector and recycle switches, provides a portion of the circuit by which S.R. 11 itself is controlled. The impulse circuit, shown in the upper left-hand corner of Fig. 8 when closed, completes the coil circuit of the motor of the switching unit and the 25-point rotary switch of the control panel (S.R. 9), thus energizing the motors of these rotary switches, and causing the switches to advance one point for each impulse from the recorder. The 25-point rotary switch of the control panel switches the remaining pair of arms of the

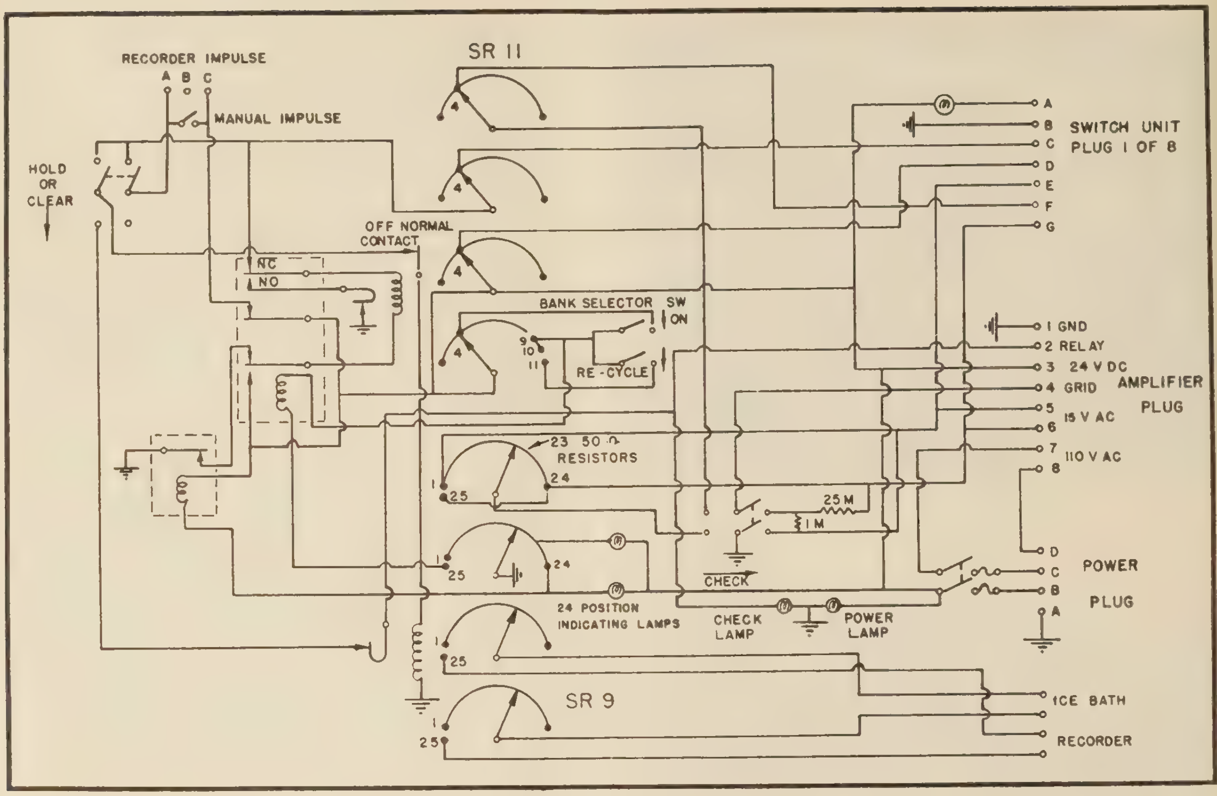

Fig. 8 Wiring Diagram of Control Panel

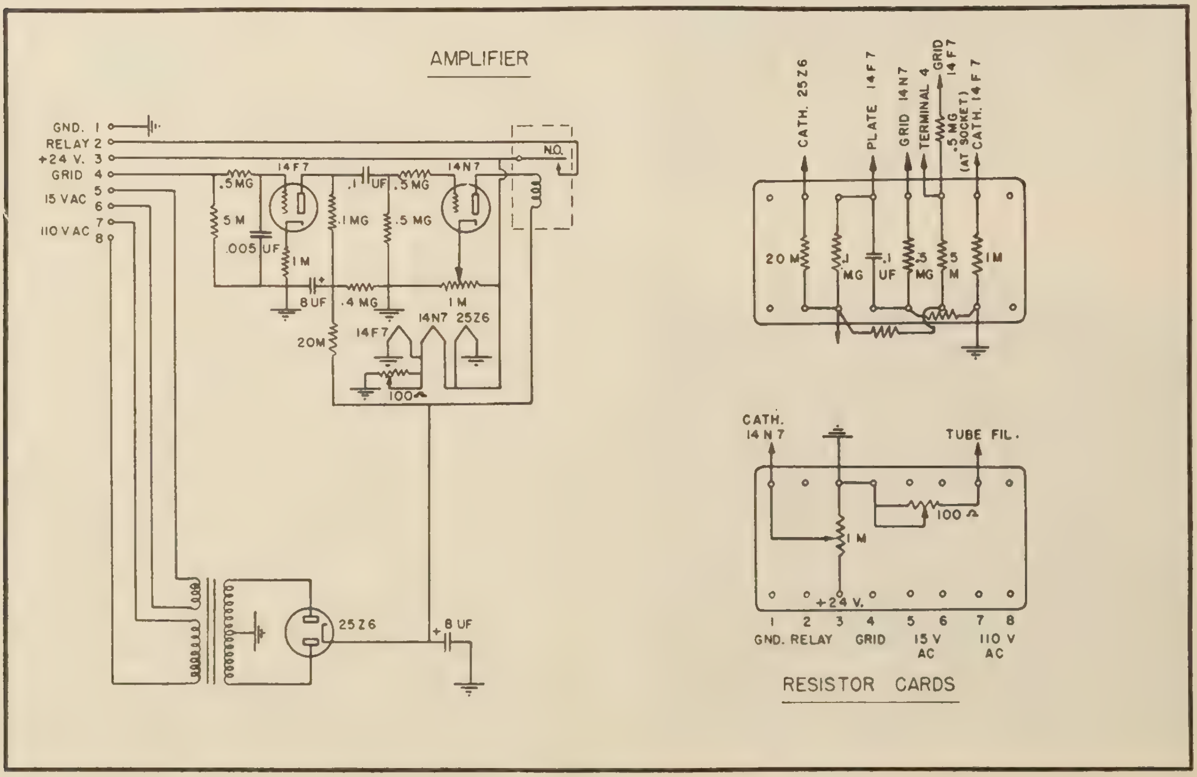

Fig. 9 Wiring Diagram of Electronic Amplifier

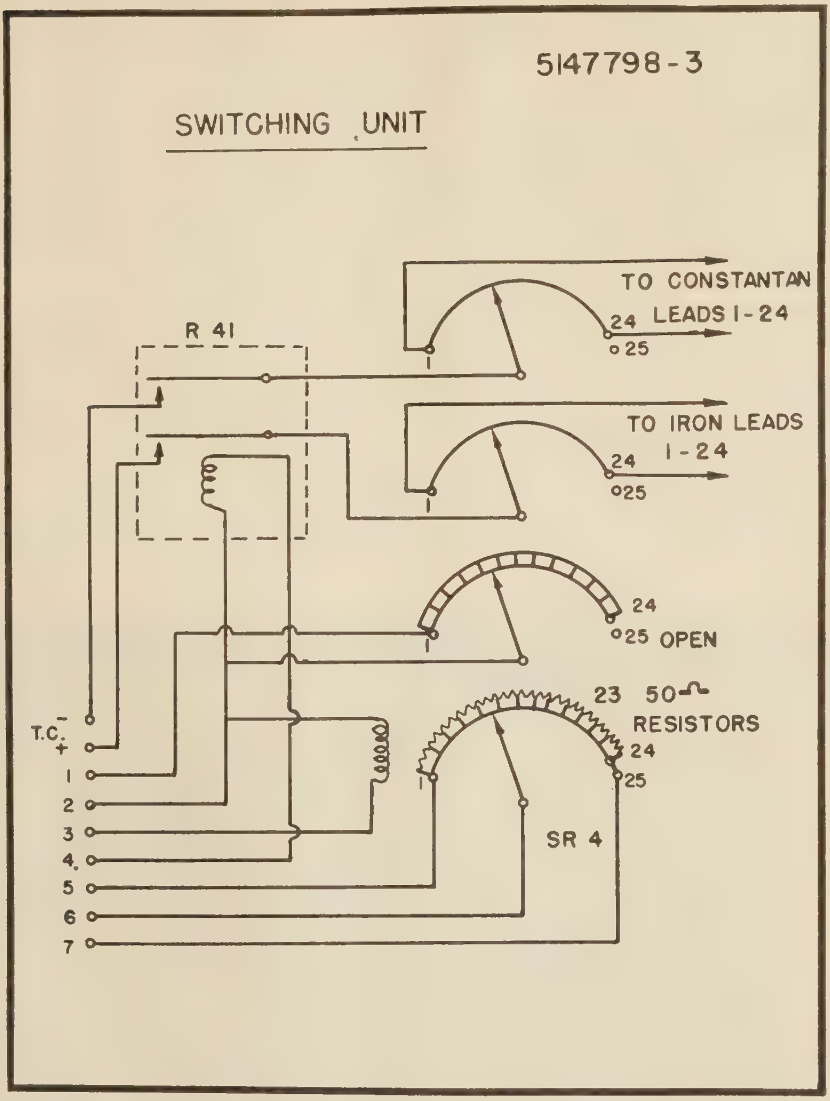

Fig. 10 Wiring Diagram of Switching Units

bridge circuit, the position-indicating lamps of the control panel, and the reference-thermocouple ice-bath circuit.

Henceforth the switching unit of Fig. 10 will be referred to as switching unit No. 4, and is one of 8 provided for in the system. By making use of two spare points of S.R. 11, up to 10 switching units may be handled by the control panel. However, it was felt that 8 switching units were about the maximum number which the flight engineer could be expected to familiarize himself with, and that this number was about the maximum which could be handled by one automatic recorder successfully.

When in proper operation, switching unit No. 4 having been selected, is in step with S.R. 9 on the control panel. Thus, the position-indicator lamp on the control panel indicates to which point of measurement the switching unit is connected. The impulse which advances S.R. 4 and S.R. 9 from the 24th to the 25th position also advances S.R. 11 one position. This is provided for by the single-pole single-throw relay on the control panel. The three-pole double-throw relay of the control panel is then caused to operate and accomplishes two purposes:

1 It prevents any of the 25-point rotary switches from advancing, until S.R. 11 has reached the position of the next switch unit selected.

2 If one or more of the following banks are omitted, it also causes S.R. 11 to advance one position upon the impulse which advances S.R. 4 and S.R. 9 to the 25th position. The recycle switch is placed in the 11th position of S.R. 11, at which time the off-normal controls of S.R. 11 are open, thus preventing S.R. 9 from being impulsed by the recorder when the system is thus rendered otherwise inoperative.

By opening the recycle switch, the system may be stopped at the completion of one sequence of operation. Otherwise, the system is in continuous operation.

The electronic control amplifier performs two functions:

1 It gives an instantaneous indication of whether or not the switching unit is out of step with the control panel.

2 If this is the case, it provides the means by which they may be brought into step.

It may be remarked at this time that if the switches are properly adjusted, it is practically impossible to get them out of step even when trying deliberately to do so, regardless of the rapidity or duration of the impulses employed. The operation of the electronic control amplifier is dependent upon an unbalanced condition of the aforementioned bridge, which only occurs if the switching unit and control panel are out of step. If this is the case, then a signal voltage is applied to the input of the amplifier, placing an amplified signal on the grid of the second tube. This tube is biased to cutoff for zero applied signal voltage. Thus when a signal is applied to the amplifier, the amplifier relay is closed, which lights the warning lamp on the control panel, indicating that the switching unit and control panel are out of step. When this occurs, merely by throwing the "clear" switch, the rotary switch S.R. 9 of the control panel is caused to home around until it is once more in step with the switching unit. This homing operation is controlled by the amplifier, for as soon as the rotary switches are in step, the bridge being once more balanced, the relay in the amplifier opens, thus stopping the homing operation. The "clear" switch may also be used to hold the system on any desired point, since, by putting the switch in the "clear" position; the impulse circuit is opened, preventing further impulses from reaching the system.

Precautions to Be Observed

A word of caution is in order concerning the bridge circuit. The 50-ohm resistors used were of the $^1/_2$-watt wire-wound type. In order to assure balance, the resistors were sorted into ranges of $\pm ^1/_2$ ohm from established mean values. All resistors measuring accurately certain key values as 49, 50, and 51 ohms were set aside as "precision" resistors. This stock was then drawn upon in building up the control panels, which thus contained 23 resistors of equal values to precision tolerances. In the switching units there is a random variation of $\pm ^1/_2$ ohm from the mean value. The average, however, will be statistically close to the value which would be obtained if all resistors were precision-selected. The reason for using precision-sorted resistors in the control panel is to minimize the effect of these random values of resistances in the switching-unit section of the bridge circuit. It is thus unnecessary to buy bulky precision-wound resistors.

There is one further precaution which must be mentioned at this time. The power to operate the rotary switches, the filament power of the tubes, and the direct-current bias of the second tube of the amplifier are obtained from a suitable direct-current source, in this case, 24-v direct-current storage batteries. The 110-v alternating current to operate the bridge circuit and to

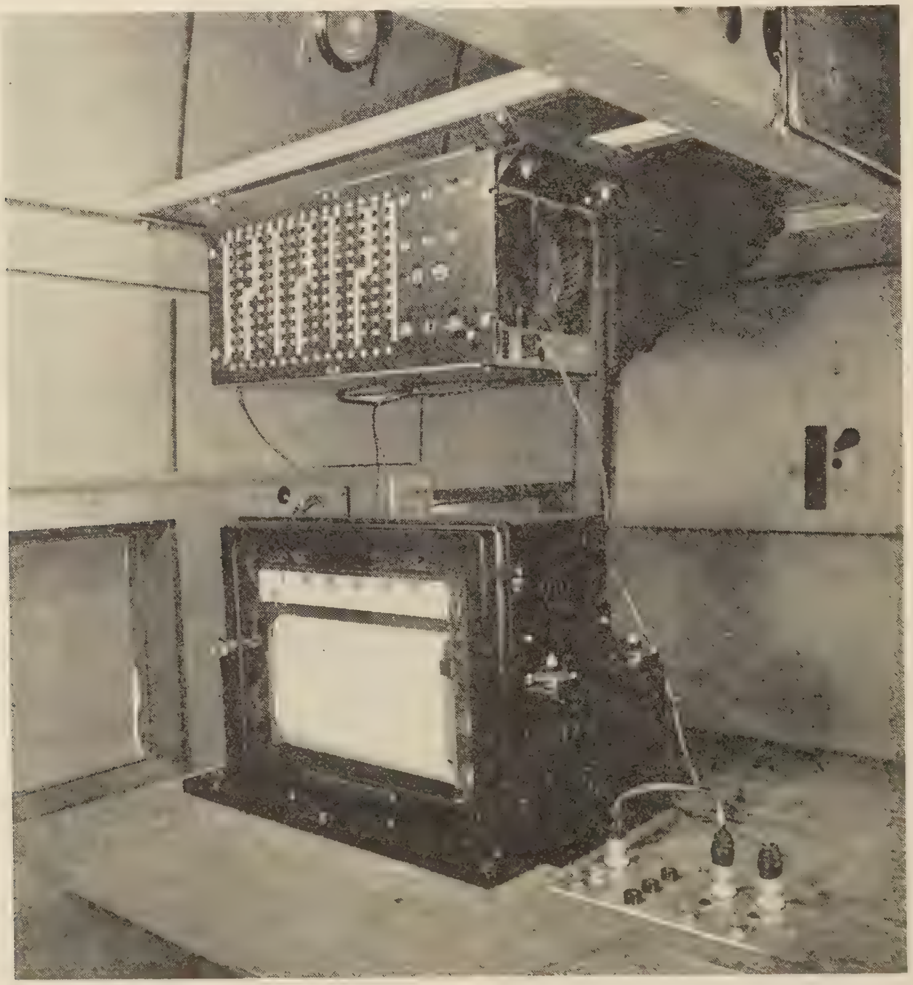

Fig. 11

supply the plate voltage of the amplifier must be obtained from this same source, either through a rotary converter, a vibrator supply, or the equivalent. This is necessary to provide a constant ratio between the direct-current bias voltage and the plate voltage of the amplifier. If this is not done, the amplifier may not operate properly, while, if this precaution is adhered to, the direct-current voltage may be varied between 18 and 30 v with satisfactory operation of the amplifier. One further point of note is that the system, as wired with a common ground to chassis, and so on, is predicated upon a negative grounding of the direct-current source, which is necessary to obtain the proper bias on thel ast tube of the amplifier.

Basic Operating Features

The basic features of operation of the system may be summarized as follows:

1 Two checks on the operation of the system are available. One of these is provided by the electronic amplifier and associated bridge circuit. If the control panel and switching unit drop out of step, an unbalanced condition of the bridge circuit is created and the warning lamp is lighted. The second of these checks is provided by the bank-lamp circuit. When the system has reached the 24th position, the following impulse puts the switching unit on the 25th or "off" position and advances the control panel to the next switching unit selected. If the bank lamp of the first switching unit remains lighted, it simply indicates that the switching unit was not left on the 25th or off position and was therefore out of step with the control panel. This is an absolutely positive check on the operation of the system.

2 A reference-temperature position is provided once every 25th impulse by the 25-point rotary switch in the control panel. This may be used for ice-bath or hot-bath reference as desired to provide a check on the recorder at all times.

3 As the control panel advances from one switching unit to another the relay, as R 41 in switching unit No. 4, is opened in the one switching unit and the corresponding relay is closed in the following switching unit. This positively prevents more than one switching unit from being placed across the recorder circuit at any one time.

4 When the recycle switch is opened, the control panel will go through one sequence of operation and stop. If the recycle switch is closed, the system is placed in continuous operation.

5 By throwing the clear switch to the "clear" position, it is possible to stop the system at any desired point for a detailed inspection of this temperature. If the system is out of step, this will also automatically bring the control panel into step with the switching unit.

Mechanical Construction

The mechanical construction of the system has been made very rugged. This is evidenced by the illustrations of the equipment. Experience has shown that from the standpoint of practical operation under typical service conditions, this is an absolute necessity. In general, regardless of how careful instrumentation personnel may be, over a period of time, the equipment is invariably subjected to considerable abuse. The cases are hermetically sealed and a desiccator is employed to exclude moisture (corrosive salt air in locations near the seacoast) from the compartments. This has been found useful in that it prevents the switch contacts from excessive corrosion and results in increased freedom from servicing. In this connection, it has further been found that the application of a small amount of lubricant applied to the stationary switch contacts is useful in preventing them from becoming dirty and has been found to reduce greatly the amount of servicing required. The lubricant used for this purpose is known as Beacon Lubricant M-285.

Automatic temperature-recording systems, as described in this paper, have been used in both flight test and other applications by the author's company for a sufficient length of time to eliminate the "bugs" usually found in newly developed equipment. The apparatus in its present form is performing in a highly satisfactory manner. The successful conclusion of this development is due in no small measure to the patient co-operation and helpful advice of the staff of the Douglas Research Laboratories, who built the apparatus, and the flight engineers who have used it.

Mechanics of Sheet-Metal Bending

By WILLIAM SCHROEDER,[1] BURBANK, CALIF.

The author discusses the basic mechanical phenomena occurring during the bending of sheet material. Analytical methods are given whereby all aspects of behavior during and after bending, such as forming pressure and springback, can be accurately predicted on the basis of the stress-strain diagram of the material. An experimental and analytical study of the strain distribution in the bend is also included.

Introduction

THE airplane as at present designed is made up almost wholly of sheet-metal parts; hence the subject of sheet-metal forming has become of the greatest importance in modern aircraft fabrication. Sheet-metal forming was previously viewed as a craft, the work being done largely by hand by skilled workers whose art was based on long experience. However, mass production, required by the present emergency, has necessitated a transition from the stage of hand work to the stage of mechanical production of parts by methods controllable to give identical results in one quick operation. This transition from handcraft to technical control has necessitated a careful study of the basic mechanics involved in forming sheet-metal parts.

This general subject has been discussed in a comprehensive technical paper by F. R. Shanley (1)[2] of the Lockheed Aircraft Corporation. It was stated in that paper that supplementary papers covering the various phases in more detail would be published from time to time. The present paper discusses the first and most fundamental field in sheet-metal forming, namely, the simple bending of the material, with particular reference to rectilinear bends. This analysis may then form the basis for more advanced studies on bending parts with curved flanges, or having contours in more than one dimension.

The bending of sheet metal, while at first glance of a simple nature, involves several problems, the chief of which is the tendency of the material to spring back to its original form after the bending force is removed. This effect was of relatively little importance in earlier days of aircraft manufacture when bending was done by hand and material of soft temper was used; but when bending is to be done on presses, with the necessity of having the part come out correctly shaped at the first operation, and when materials of hard temper are to be used, the subject of "springback" becomes a very important one. Time and money can be saved by forming materials of heat-treated aluminum alloys rather than from annealed stock which has to be heat-treated after the part is formed. However, the production gains involved in such a procedure have often, in practice, been partly or totally submerged by the tide of shop troubles arising from inability to allow for the springback of the hard-tempered material, or to predict forming pressures required.

The paper by F. R. Shanley already alluded to (1) contains a concise description of the mechanics of bending, without going into details on exact calculations. Pure bending of bars with rectangular cross sections is also discussed by A. Nadai (2), C. V. Bach (3), and S. Timoshenko (4). A method for evaluating one phase of springback, namely, radius change in bending structural shapes, has been presented by R. G. Sturm and B. J. Fletcher (5). Empirical data on the springback of straight flanges have been gathered in several quarters and published information has appeared on this subject (6). An analysis of the conditions in the bend, with a graphical method of determining springback, was made by E. I. Ryder and R. B. Glassco, of Lockheed Aircraft Corporation, and is included in the present paper.

Stress and Strain Conditions During Bending

As a first step, a simple discussion will be presented showing the basic strain conditions in a bend. For convenience, the simplest kind of bend is assumed, namely, an ideal or circular bend in a homogeneous material. Bends resulting from actual forming operations, and in nonhomogeneous materials such as Alclad, will be treated later. Fig. 1 shows such a bend. The

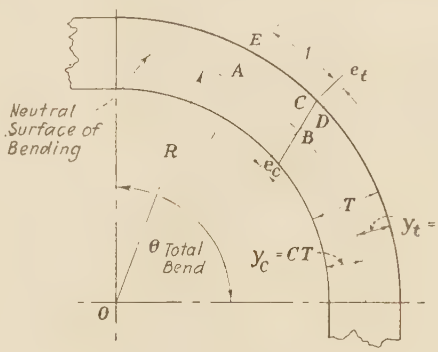

Fig. 1 Geometry of a Bend

outer fibers are elongated during the bending process by the amount indicated by e_t, while the inner fibers are compressed. At some point between, there is a surface whose length is not changed in the bending process, and this is called the neutral surface or "neutral axis." In accordance with the classical beam theory (7), the strain distribution between the neutral axis and the outer fiber may be assumed to be linear, as indicated by the line BC, drawn parallel to line AE. Such an assumption will be correct if all cross sections which were plane before bending remain plane after bending.

Messrs. Bach and Baumann (3) and Meyer (8) have investigated the strain distribution during the bending of bars and have found slight distortion of the planes, but for practical purposes the assumption of plane sections has been found to give satisfactory results.

In Fig. 1 assume that the line AB represents a unit length measured along the neutral axis. Then, from the similar triangles OAB and BCD, the following simple relationship is obtained

$$\frac{e_t}{y_t} = \frac{1}{R} \quad \ldots\ldots [1]$$

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[1] Research Engineer, Lockheed Aircraft Corporation. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Contributed by the Aviation Division and presented at the Semi-Annual Meeting, Los Angeles, Calif., June 14–17, 1943, of The American Society of Mechanical Engineers.

or

$$e_t = \frac{y_t}{R} \qquad [1a]$$

or for any other distance y from the neutral axis

$$e = y/R \qquad [1b]$$

If the neutral surface is assumed to be the mid-surface of the sheet, $y = T/2$ and Equation [1a] becomes

$$e_t = \frac{T}{2R} \qquad [1c]$$

For cases in which the neutral axis is not at the mid-surface of the sheet, the formulas may be generalized by expressing the distance from the innermost compression fiber to the neutral axis by CT. Then $y_c = CT$ and $y_t = (1 - C)T$, giving the equation

$$e_t = (1 - C)\frac{T}{R} \qquad [1d]$$

For the compression side

$$e_c = C\frac{T}{R} \qquad [1e]$$

(C will usually be very nearly equal to 0.5 but will vary somewhat depending upon the nature of the material and bending process.)

The foregoing relations are imposed by the geometry of the bend and will always apply either before or after springback.

The equations merely give the average strain, or average unit elongation existing between the points E and D on the bend. For the purposes of preliminary analysis, it will be assumed that the distribution of the strain along any fiber of the bend is uniform, and hence equal at any point to the average value, as derived from the equations. Actually, it is known that the strain distribution around a bend is far from uniform, but this phenomenon will be reserved for discussion at a later point in the paper.

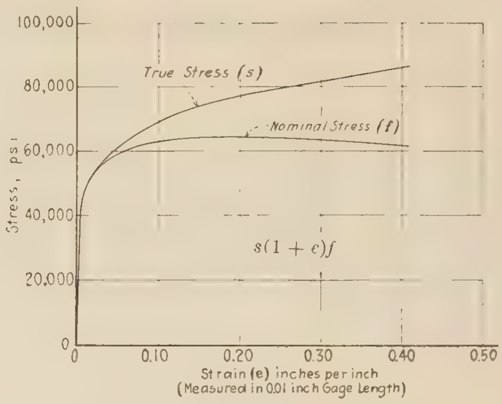

FIG. 2 STRESS-STRAIN DIAGRAMS

It is also clear that, since distance along the neutral axis is given by $R\theta$, and the neutral axis by definition does not change its length during the bending process, $R\theta$ will always be a constant. Thus

$$R\theta_{\text{total bend}} = \text{const} \qquad [2]$$

Another assumption which has to be made in order to get a simple basic picture of the process is that the stress-strain curve for each fiber being deformed around the bend is the same as that derivable from the physical test of the piece as a whole. This may be accepted for the time being as a reasonable assumption, subject to the reservation that it may be re-examined later if necessary to explain observed results.

Fig. 2 shows a typical stress-strain diagram for material such as sheet metal, in this case 24S-T. Both the nominal stress-strain and true stress-strain (2, 9) diagrams are shown.[3] For an ac-

[3] True stress, as the term is used here, is determined by dividing the applied load by the corresponding actual deformed area of the specimen; while nominal stress is determined by dividing the applied load by the original area of the specimen.

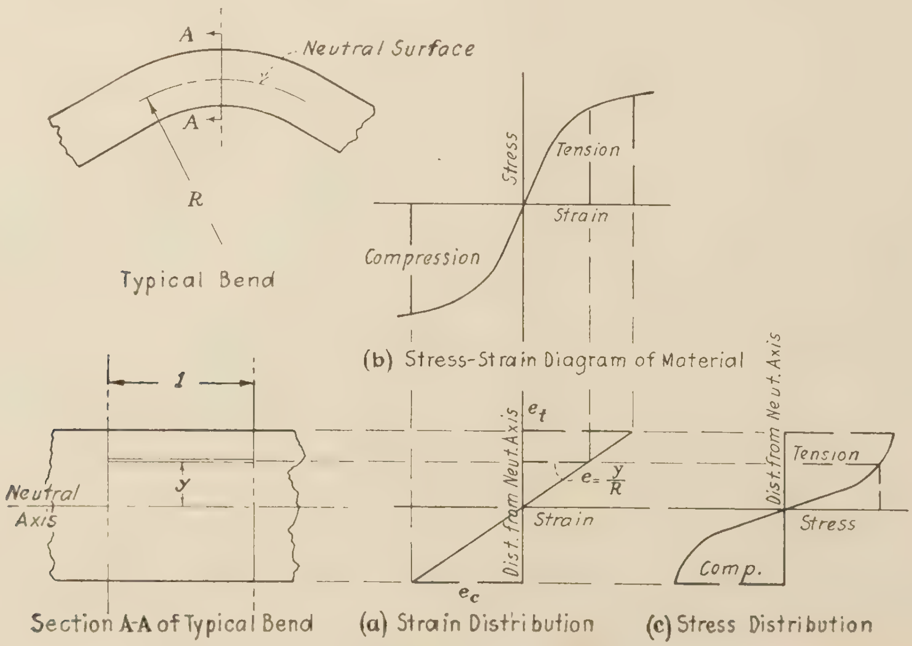

FIG. 3 STRESS AND STRAIN IN BEND

curate analysis of bending, the true stress-strain diagram should be used; however, if the strains are low the more familiar nominal stress-strain diagram may be used without serious error.

It will be observed that this curve is approximately linear only in a very limited region, usually referred to as the elastic range, and becomes nonlinear as it enters the plastic range. Since in the bending operation the material must be deformed into the plastic range in order that the bend may remain after the deforming force has been removed, it can be stated in general that the stress-strain diagram applicable during bending is of a nonlinear nature. Hence if the strain distribution from B to D in Fig. 1 is linear, as has been assumed, the stress distribution cannot be linear but must be as indicated in Fig. 3. Note that in this figure a transition is made from the curve of strain versus distance from neutral axis, to that of stress versus distance from neutral axis, by means of the stress-strain diagram. The construction will be conveniently used at a later point in discussing springback conditions.

Springback

The basic relations described in the foregoing section apply at all times, although numerical values will change after the bending force has been removed, owing to the tendency of the bent

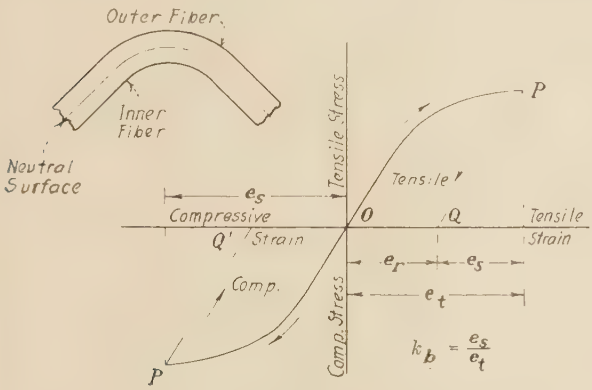

Fig. 4 Elastic Recovery From Large Initial Strain

piece, through elastic recovery, to return toward its original flat shape. This tendency is called "springback" and is usually measured by the angular change occurring when the bending force is released. There is also an associated change in bend radius.

If all stresses during bending were to remain within the elastic range, the strains would return to zero when the bending force is removed in accordance with Hooke's law, the inner and outer fibers (Fig. 1) would resume their original lengths, and the piece would become completely flat. However, when deformation has extended into the plastic region, removal of the force results in return of the material to zero stress by "elastic recovery" along the line PQ in Fig. 4 assumed to be parallel to the initial elastic portion of the curve although usually not strictly so. The permanent strain or "set" OQ (called e_r) remains. Hence in Fig. 1 the strains e_t and e_c do not entirely vanish and therefore the piece stays in a bent shape although at a different angular position than it had while the bending force was being applied.

In order to obtain a basic conception of the mechanical action occurring during springback, it is convenient to start with the simplifying assumption that only the outer fibers are to be considered. The diagram Fig. 4 would then apply, the tensile and compressive portions being considered equal for the sake of simplicity. Each outer fiber during bending would follow its respective curve indicated by the arrows, and would reach the condition represented by P or P' when the material is forced against the form block or other stop. When the pressure is released, the "elastic recovery" along the line PQ (and $P'Q'$) would occur, and the final position would be such that the outer-fiber strains would each correspond to the residual strain e_r. If a final part angle, corresponding to an outer-fiber strain e_r is desired, the part must be overformed to give a total strain of e_t before the deforming force is released. This is commonly done by making the form-block radius less than the part radius and by undercutting the form block to give "springback allowance."

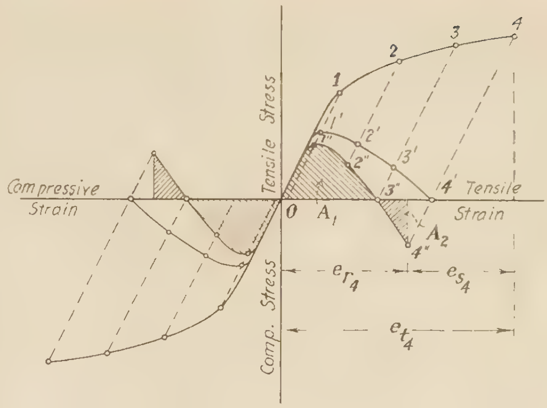

Fig. 5 Equilibrium Conditions for All Fibers After Springback

Actually, there are many intermediate fibers involved besides the outer ones, and these all have their influence on the final equilibrium position assumed by the bent part. Fig. 5 has been drawn to indicate qualitatively what happens within the specimen.

In Fig. 5 we consider five different fibe s, starting with "number zero" at the neutral axis and equally spaced. Since strain has been assumed proportional to distance from the neutral axis, Fig. 1 and Equation [1 b], if the outer fiber is deformed to cause a total strain e_{t_4}, the other fibers must have strains proportioned thereto in direct ratio to their distance from the neutral axis, as indicated by the projections of points 1, 2, 3, and 4 on the horizontal axis in Fig. 5. This geometrically determined ratio of strains on the different fibers must always be maintained (assuming that planes before bending remain planes after bending) even when the deforming force is removed and the fibers return toward the condition of zero stress along their respective lines of elastic recovery. But if each actually returned to a condition of zero stress along its elastic line it is clear from Fig. 5 that the intercepts on the horizontal axis, representing strains, would not be evenly spaced. Hence it must be that the individual fibers take up positions of varying residual stress, along some such curve as 1′, 2′, 3′, 4′ or 1″, 2″, 3″, 4″ in Fig. 5. This curve must be of such shape as to satisfy the following conditions:

1 The strains remain in direct proportion to the distances of the fibers from the neutral axis (i.e., the points must be evenly spaced along the strain scale).

2 The residual stresses represent a resultant force and bending moment, each equal to zero (so that equilibrium may result).

If the final positions were along curve 1′, 2′, 3′, 4′, with the outer fiber at zero stress (as was assumed in Fig. 4), the residual stresses represented by 1′, 2′, and 3′ would each exert a bending moment equal to its own magnitude times its distance from the neutral axis. (Such distances are in turn proportional to the abscissas of the points in question, since all strains are assumed

proportional to linear distances from the neutral axis.) These moments would tend to move the piece back toward the flat position, and would be aided by corresponding moments exerted by residual compressive stresses in the inner portion of the bend (since the moments arising in the two halves of the diagram are additive). Thus the moments within each half of the diagram must add up to zero if equilibrium is to be achieved. The stress-distribution curve must therefore assume some such final position as that indicated at 1″, 2″, 3″, 4″, the outer fibers on the tension side actually being in compression, and vice versa. The moment conditions are shown in Fig. 6, wherein stress s is plotted against distance y from the neutral axis. For equilibrium

$$0 = M = w(s_1y_1\Delta y + s_2y_2\Delta y + \ldots s_ny_n\Delta y) \ldots\ldots [3]$$

where w is the width of the strip bent (length of bend along mold line).

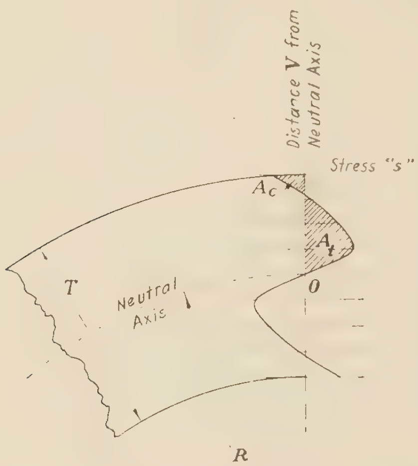

Fig. 6 Stress Distribution on Cross Section Through Bend After Springback

In the foregoing expression, some of the terms must have opposite signs in order that reduction of M to zero be possible; hence the curve must cross the axis in each half, as shown.

In differential form

$$0 = M = w\int_0^{T/2} sydy \ldots\ldots\ldots\ldots [3a]$$

If the neutral axis is in the center of the sheet, and the compression and tension portions of the stress-strain diagram are identical, the summation of forces will always add to zero by symmetry.

Equation [3a] means that, referring to the tension half of the total stress-distance diagram, Fig. 6, the area A_c in compression times its mean distance from the neutral axis must be equal to the area A_t in tension times its mean distance from the neutral axis. Applying this to curve 1″, 2″, 3″, 4″, Fig. 5, distances along the strain axis are by definition proportional to distances of the points from the neutral axis; hence area A_1 times its mean abscissa must be equal to area A_2 times its mean abscissa.

The stress distribution for equilibrium may be determined graphically by trial and error using the method just suggested. An analytical method may also be used.

The foregoing paragraphs have shown how to find the residual strain e_r and the springback strain $e_s = e_t - e_r$, when the initial bending strain (before springback) e_t is known. The ratio

$$k_b = \frac{e_s}{e_t} \ldots\ldots\ldots\ldots [4a]$$

is a significant index of the amount of springback occurring in any given case, expressed as a fraction of the total strain before springback, and is called the "springback ratio." An analogous ratio is also convenient, namely

$$k_p = \frac{e_s}{e_r} \ldots\ldots\ldots\ldots [4b]^4$$

The strains referred to are all measured in the outermost fiber. By the application of the simple relations of Equations [1] and [2] the geometrical conditions of angular change and radius change can readily be worked out (see Figs. 4 and 7, for nomenclature). Specifically, applying Equation [1d] and assuming $C = 0.5$

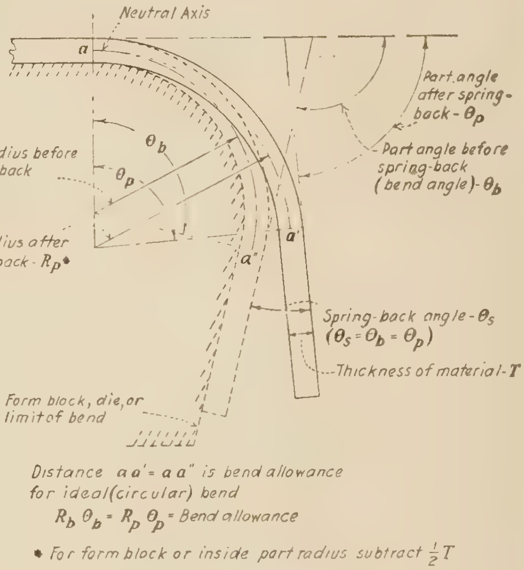

Fig. 7 Nomenclature for Springback

Outer fiber strain before springback

$$= e_t = (1 - C)\frac{T}{R_b} = \frac{T}{2R_b} \ldots\ldots\ldots\ldots [5]$$

$$\text{Outer fiber strain after springback} = e_r = \frac{T}{2R_p} \ldots\ldots\ldots\ldots [6]$$

Therefore

$$\frac{e_r}{e_t} = \frac{T}{2R_p}\cdot\frac{2R_b}{T} = \frac{R_b}{R_p} \ldots\ldots\ldots\ldots [7]$$

Also rewriting Equation [2] with the terminology of Fig. 7

$$R_b\theta_b = R_p\theta_p \ldots\ldots\ldots\ldots [8]$$

from which

$$\frac{\theta_p}{\theta_b} = \frac{R_b}{R_p} = \frac{e_r}{e_t} \ldots\ldots\ldots\ldots [9]$$

from which

$$\frac{\theta_b - \theta_p}{\theta_b} = \frac{R_p - R_b}{R_p} = \frac{e_t - e_r}{e_t} \ldots\ldots\ldots\ldots [10]$$

[4] This gives the springback strain as a fraction of the residual strain or strain remaining in the part after forming; hence the subscript p.

Substituting $\theta_s = \theta_b - \theta_p$, and $e_s = e_t - e_r'$

$$\frac{\theta_s}{\theta_b} = \frac{R_p - R_b}{R_p} = \frac{e_s}{e_t} = k_b \qquad [11]$$

and by a similar process

$$\frac{\theta_s}{\theta_p} = \frac{R_p - R_b}{R_b} = \frac{e_s}{e_r} = k_p \qquad [12]$$

From these equations, two very important points may be noted, as follows:

1 The strain in any bend is a function only of R and T, not[5] of the total angle (Equations [5] and [6].

2 The springback ratio or "unit springback" $\frac{\theta_s}{\theta_b}$ being proportional to the strain (Equation [11]) is also only a function of R and T, not of the total angle of bend. Thus the springback per degree of bend is constant, regardless of the total angle of bend.

If the initial conditions θ_b and R_b are known (R_b would be the radius to the neutral axis of bend before springback and would therefore be the radius of the form block plus one half the thickness of the material), e_t is found from Equation [5], e_r (outer fiber) by the area-moment method already described, R_p from Equation [6], and θ_p from Equation [9]. Terms R_p and θ_p then constitute the resulting part radius and angle.

If (as is often the case) the desired part radius and angle are known, and the form-block dimensions are to be calculated, the foregoing process should theoretically be worked in exact reverse. However, there is a practical difficulty here since it is hard to operate the area-moment computation backward to find e_t from a given e_r. To overcome this, sets of curves have been developed showing springback ratios k_b or k_p for various ratios of T/R.

Analytical Solution of Springback

The graphical analysis described serves well to illustrate the basic principles involved in springback but suffers somewhat in accuracy due to some of the simplifying assumptions which were made. For routine calculations, an analytical solution is usually more desirable. General equations which may be applied to many materials can be derived.

For a homogeneous material the equation found for springback ratio k_b is

$$k_b = \frac{\theta_s}{\theta_b} = \frac{\int_{-C(T/R_b)}^{(1-C)(T/R_b)} sede}{E\left[\frac{T}{R_b} - \frac{\left(\frac{T}{R_b}\right)^2}{\ln A} + \frac{1-2C}{2}\left(\frac{T}{R_b}\right)^2\right]} \qquad [13]$$

where

k_b = springback ratio = $\frac{\theta_s}{\theta_b}$ for simple bend, degrees springback per degree of bend

s = stress co-ordinate of true stress-strain diagram

e = strain co-ordinate of true stress-strain diagram

C = position of neutral axis of bending expressed as a ratio of distance between neutral axis and inside surface to thickness of sheet. If the true stress-strain diagram is equal in tension and compression, $C = 0.5$

E = modulus of elasticity, psi

T = sheet thickness, in.

R_b = radius of curvature of neutral axis before springback = block radius plus CT, in.

$$A = \frac{1 + \frac{(1-C)T}{R_b}}{1 - \frac{CT}{R_b}}$$

$$\frac{T}{R_b} = e_t + e_c$$

It may be noted that the numerator of Equation [13] is the moment of the area under the true stress-strain diagram for the material between the limits e_t and $-e_c$ taken about the origin. In most cases the true stress-strain diagrams in tension and compression may be considered equal, and in such case $C = 0.5$. In general, values of k_b may be solved and plotted against values of T/R_b as an independent variable.

Some materials, such as Alclad, cannot be considered as homogeneous material. Alclad or other similar aluminum-alloy-sheet materials consist of an inner layer "core" of high-strength alloy which is covered with two surface layers "coats" of aluminum which is relatively a weak material. For these materials it may be assumed that the true stress-strain diagram is equal in tension and compression, and therefore $C = 0.5$.

For Alclad, the equation for k_b is as follows

$$k_b = \frac{\theta_s}{\theta_b} = \frac{2\int_0^{0.45\,(T/R_b)} s_a e de - 0.0475(f_{yt})_a(T/R_b)^2}{E_a\left[0.9\frac{T}{R_b} - \frac{0.81\left(\frac{T}{R_b}\right)^2}{\ln A'}\right]} \qquad [14]$$

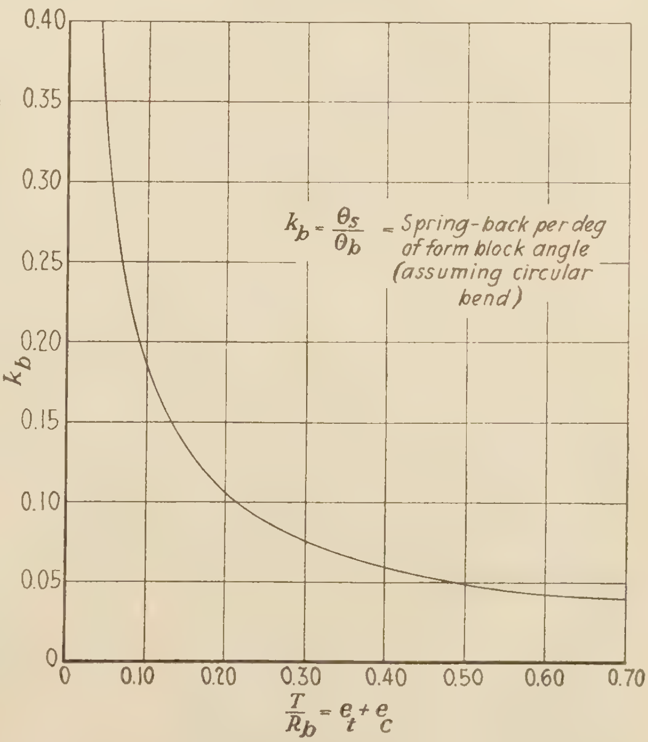

Fig. 8 Values of Springback Ratio (k_b) for 24S-T Alclad

[5] Not strictly true in actual bends due to nonuniformity of strain distribution around the bend and effect of flow from adjacent straight sections.

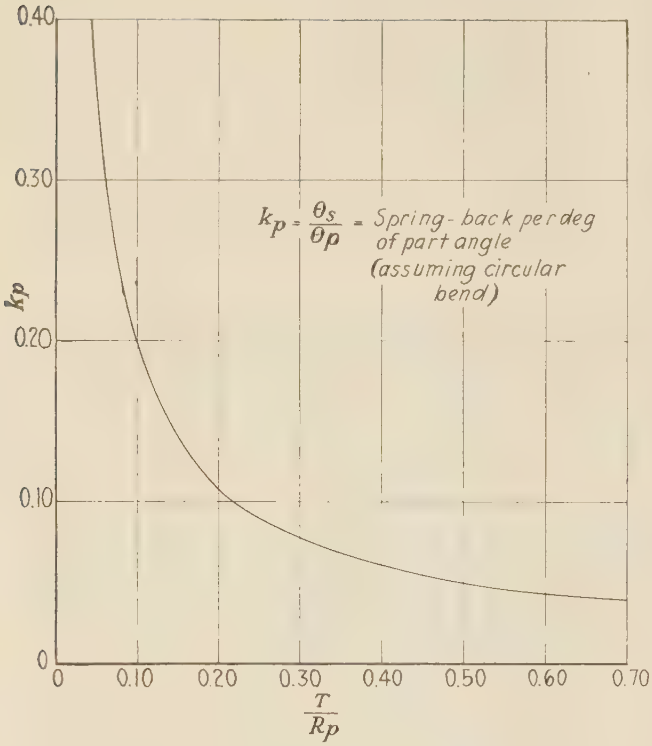

Fig. 9 Values of Springback Ratio (k_p) for 24S-T Alclad

where

s_d = stress co-ordinate of true stress-strain diagram for core material, psi

e = strain co-ordinate of true stress-strain diagram for core material, in. per in.

$(f_{yt})_a$ = yield strength of coat material, psi

T = total sheet thickness, in.

R_b = radius of curvature of neutral surface = block or die radius plus one-half sheet thickness, in.

E_d = modulus of elasticity of core material, psi

$$A' = \frac{1 + 0.45\,\dfrac{T}{R_b}}{1 - 0.45\,\dfrac{T}{R_b}}$$

$$\frac{T}{R_b} = e_t + e_c$$

Results computed for 24S-T Alclad are shown in Table 1. In Fig. 8 values of k_b are shown plotted against T/R_b. Values of $k_p = \frac{\theta_s}{\theta_p}$ have also been computed and are plotted against T/R_p in Fig. 9. The value of k_b is useful for computing θ_s when the form block angle θ_b is known; while the value k_p is used when the final part angle θ_p is known.

Bending by Rubber-Pressure Hydropress

The foregoing discussion has been directed toward conditions in an idealized bend in which it is assumed that the radius of curvature is constant and the bend is circular. When bends are made in the production shop such ideal bends do not in general result. Of the various methods of bending or forming, the rubber-pressure hydropress will be discussed and analyzed, for the reason that it is a widely used method of sheet-metal forming in the aircraft industries; and its use involves the necessity of predicting springback in order to construct the required dies or "form blocks." While other methods of forming are not considered in this paper it is believed that most of them could be analyzed in a manner similar to the one discussed herein.

Rubber-pressure-hydropress forming of flanged parts is usually performed over the male half of a die, commonly called a "form block," in a specially designed press. The ram of the press is fitted with a rubber pad of suitable hardness enclosed on the edges with a retaining ring. In operation this retaining ring indexes accurately with the platen of the press and in this way serves to restrain the rubber when under pressure. The blank is placed on the form block and formed to the shape of the form block by forcing the rubber pad down on the form block and onto the platen, in the manner shown in Fig. 10.

Bends made in the rubber-pressure hydropress differ from the ideal in that the bend radius cannot be considered to be entirely circular. The approximate conditions resulting when a part is formed are shown in Fig. 10. In the instance illustrated the presence of the spaces between the form block and the part is explained by the well-known principle that the applied bending

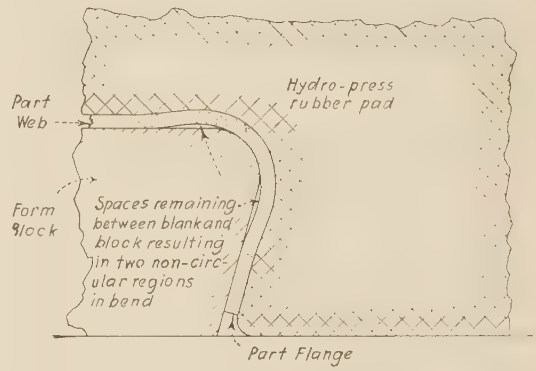

Fig. 10 Nonconformity of Part to Form Block

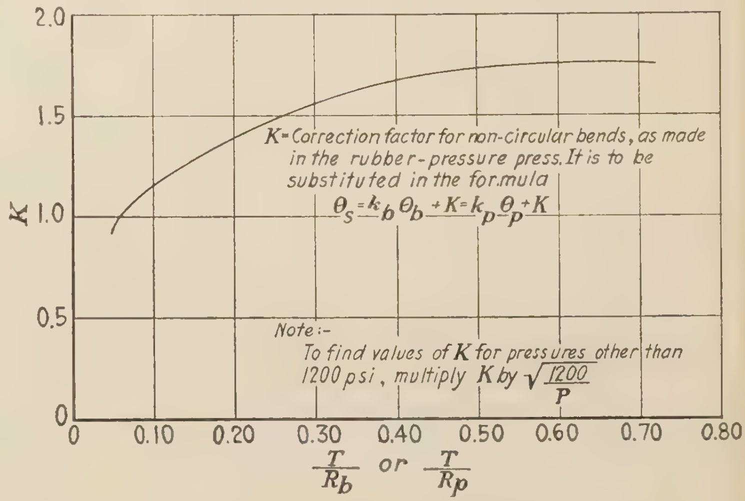

Fig. 11 Values of K for 24S-T Alclad

TABLE 1 COMPUTED VALUES OF $\frac{M}{T^2}$, k_b, AND k_p FOR 24S-T ALCLAD

$e_t + e_c = \frac{T}{R_b}$ in. per in.	$e_t = \frac{T}{2R_b}$ in. per in.	s_d psi	S_a psi	$\int_0^{0.9e_t} s_d de$	$\int_{0.9e_t}^{e_t} s_a de$	$\frac{M}{T^2}$	$0.0475(f_{yt})_a\left(\frac{T}{R_b}\right)^2$	E_d[*]	k_b deg per deg	$k_p = \frac{k_b}{1 - k_b}$ deg per deg	$\frac{T}{R_p} = (1 - k_b)\frac{T}{R_b}$ in. per in.
0.02	0.01	53500	5850	1.72	0.05	8900	0.095	4.98	0.675	2.080	0.0065
0.04	0.02	59500	6550	8.60	0.25	11000	0.38	41.9	0.399	0.663	0.0240
0.08	0.04	65200	7600	38.1	1.11	12250	1.52	335.0	0.223	0.287	0.0622
0.12	0.06	69600	8600	91.0	2.77	13070	3.42	1110.0	0.161	0.192	0.1007
0.16	0.08	73100	9400	170.5	5.30	13760	6.10	2560.0	0.131	0.151	0.1390
0.20	0.10	75600	10200	276.0	8.84	14260	9.50	5110.0	0.106	0.118	0.1788
0.24	0.12	77500	11100	409.0	13.53	14650	13.70	9010.0	0.0890	0.098	0.216
0.32	0.16	81000	12400	762.0	26.0	15410	24.3	21100.0	0.0711	0.0770	0.297
0.40	0.20	83800	13700	1240.0	43.4	16070	38.1	40600.0	0.0599	0.0635	0.376
0.48	0.24	86500	14900	1820.0	65.6	16350	54.7	70400.0	0.0510	0.0539	0.455
0.56	0.28	89200	15900	2540.0	92.9	16800	74.4	112400.0	0.0445	0.0468	0.535
0.64	0.32	91900	16800	3410.0	124.0	17250	97.1	168000.0	0.0400	0.0416	0.614
0.72	0.36	94500	17700	4430.0	161.0	17710	123.2	238000.0	0.0366	0.0380	0.693

$* = 0.9\,(T/R_b) - \frac{0.81\,(T/R_b)^2}{\ln A'}$

TABLE 2 COMPUTED QUANTITIES FOR NONCIRCULAR TRANSITION REGION OF RUBBER-PRESSURE HYDROPRESS BENDS IN 24S-T ALCLAD

$e_t + e_c = \frac{T}{R_b}$ in. per in.	$\frac{M}{T^2}$ psi	k_b deg per deg	$k_b(e_t + e_c)$	$\int_0^{\frac{M_0}{T^2}} (e_t+e_c)d\left(\frac{M}{T^2}\right)$	$\int_0^{\frac{M_0}{T^2}} (e_t + e_c) \times \left(\frac{M_0}{T^2} - \frac{M}{T^2}\right)d\left(\frac{M}{T^2}\right)$	θ_t p = 1200 psi radians	$\int_0^{\frac{M_m}{T^2}} (e_t+e_c)d\left(\frac{M}{T^2}\right)$	$\frac{M_m}{T^2}$	$\int_0^{\frac{M_0}{T^2}} k_b \times (e_t+e_c)d\left(\frac{M}{T^2}\right)$	$\int_0^{\frac{M_m}{T^2}} k_b \times (e_t+e_c)d\left(\frac{M}{T^2}\right)$	$(\theta_s)_t$ radians	K deg	$\frac{L_e}{T}$	$\frac{L_c}{T}$
0.027	10000	0.570	0.0154	96.8	697	0.017	13.6	4500	74.5	15.1	0.0121	0.27	0.78	0.30
0.070	12000	0.247	0.0173	194.0	1780	0.0225	22.8	5700	107.4	24.2	0.0155	1.13	1.10	1.56
0.180	14000	0.115	0.0207	444.0	5070	0.0614	41.0	7250	146.0	39.2	0.0184	1.30	1.36	2.04
0.406	16000	0.059	0.0240	930.0	12400	0.0975	77.0	9220	191.0	63.4	0.0206	1.70	1.60	2.72
0.575	17000	0.043	0.0253	1350.0	20510	0.1380	107.0	10350	216.0	79.8	0.0213	1.76	1.71	2.94
0.791	18000	0.033	0.0262	2130.0	32870	0.212	160.0	11400	242.0	97.0	0.0221	1.73	1.81	3.08

moment cannot in practice vary in a discontinuous manner.[6] Also, the radius of curvature of the bend, like the bending moment upon which it is dependent, cannot vary in a discontinuous manner, that is to say, in practice the shape cannot change abruptly from circular in one region to straight in an adjoining one. Actually the bend may be thought of as consisting of a circular central portion bounded by noncircular transitional regions in which the curvature $1/R$ varies uniformly from the constant value applying in the circular portion, to zero in the straight portion. The length of the transitional region depends upon the forming pressure, the form-block radius, and the strength and thickness of the material.

The circular portion of the bend may be considered to have the properties of the simple ideal bend which has already been discussed, but the bend as a whole has somewhat modified properties. The principal effects of the noncircular nature of bends are as follows:

1 The length around the bend measured on the neutral axis is not simply $R_b\theta_b = R_p\theta_p$. It is greater than the value which would be expected for a truly circular bend.

2 The springback angle θ_s of the bend is greater than the value $k_b\theta_b$ computed for a circular bend.

3 The strain in any fiber is nonuniformly distributed around the bend. Its magnitude is quite uniform in the circular region and varies gradually from this value to zero in the transitional region.

A method of computing springback under the foregoing conditions has been developed in which it is found that the springback of a rubber-pressure-hydropress bend may be expressed as

$$\left.\begin{array}{l} \theta_s = k_b\theta_b + K \\ \text{or} \\ \theta_s = k_p\theta_p + K \end{array}\right\} \qquad [15]$$

in which

θ_s = springback for bend as a whole

$k_b\theta_b = k_p\theta_p$ = angular springback in bend if it is assumed to be ideal and entirely circular (from Figs. 8 and 9)

K = correction necessary due to nonideal nature of actual bend

For a specific material and forming pressure, the value of K depends upon the ratio T/R. (No appreciable error is made in the value of K by assuming $\frac{T}{R_b} = \frac{T}{R_p}$.) Values of K were computed for 24S-T Alclad and a forming pressure of 1200 psi and are shown in Fig. 11 and Table 2. (Since the factor K varies inversely as the forming pressure, it is a simple matter to find the value of K at any forming pressure by multiplying the value of K taken from Fig. 11 by $\sqrt{\frac{1200}{p}}$.)

Evaluation of General Properties of Bends

The foregoing discussion has been directed mainly at the important subject of springback. However, a number of other quantities which are useful in the analysis of bending have been evaluated. Expressions have been derived for bending moment, residual stress, and bend allowance. While space does not permit a detailed discussion of these factors, they will be mentioned briefly.

Bending Moment. In order to make a bend, it is necessary to

[6] The basis for this statement is the elasticity equation $\frac{1}{R} = \frac{M}{EI}$ (reference 10), where R is radius of curvature, M is bending moment, E is modulus of elasticity, and I is moment of inertia of the section. R cannot vary in a discontinuous manner along the periphery of the bend unless M also varies in a discontinuous manner. In sheet-metal forming M cannot be discontinuous as long as the pressures are finite (as assumed). For material in the plastic state the foregoing equation becomes $\frac{1}{R} = \frac{e_t + e_c}{T}$, but the same reasoning applies.

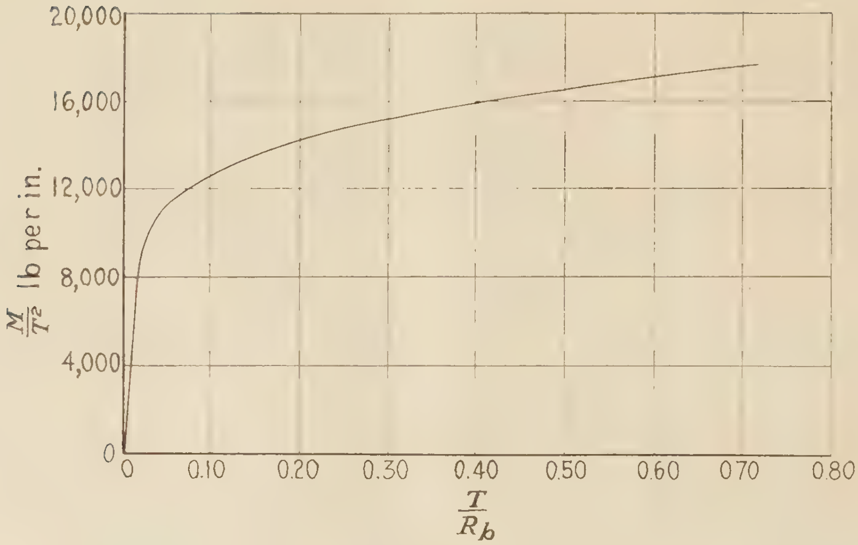

Fig. 12 Value of M/T^2 for 24S-T Alclad

apply forces sufficient to overcome the resistance of the material to be bent. This resistance of the material to bending is termed the bending moment. For expressing bending moment, the most significant parameters are M/T^2 and T/R_b; where M is bending moment, T is sheet thickness, and R_b the block or die radius measured to the neutral axis of the sheet. Values of M/T^2 plotted against T/R_b are shown for 24S-T Alclad in Table 1 and Fig. 12.

Residual Stress. By "residual stresses" are meant the stresses which remain in the material after all the forming forces are removed (after springback). While it is not the purpose of this paper to discuss residual stresses in detail, such stresses are significant in several respects. They explain the well-known fact that it is easier to unbend a bend slightly than it is to make the original bend. In some materials high residual stresses lower the resistance to intergranular corrosion to such an extent that "season cracking" results, or formed parts which are not stress-relieved may crack in a period of several hours after forming. It is also quite safe to say that residual stresses affect the impact and fatigue strength of the materials.

The distribution of residual stresses for a typical bend are shown by the ordinates of the curve in Fig. 6.

Bend Allowance and Strain Distribution. The term "bend allowance," as used here, signifies the distance measured around the bend on the neutral axis from one point of tangency to the other point of tangency with the flat material. In the flat pattern or blank, an amount of material equal to this length must be allowed for making the bend. The bend allowance may also be considered as the distance over which deformation takes place due to bending.

Extremely accurate knowledge of the bend allowance and strain distribution are of no great practical value. However, these quantities are characteristics of the bend which may be measured. As such they serve the useful purpose of offering a means for checking the general theory of bending against results obtained in practice.

Comparison of Computed Results With Measurements

Springback Angles. For the purpose of comparison, parts of the approximate size and shape illustrated in Fig. 13 were formed on the rubber-pressure hydropress. The form blocks used in the investigation were made of steel. The radii on the blocks were produced by experienced form-block makers; however, when examined critically they could not be considered as precision-made. It was considered at the time that it would be better to observe conditions as they existed in the factory, and in this respect the radii could be considered as well made. The local imperfections were well within a $^1/_{128}$-in. tolerance. The springback angle was determined to the nearest $^1/_4$ deg by measuring the difference of the exterior block angle and the exterior part angle. Table 3 shows a comparison of typical measured values with the values computed by the method given in this paper. Although the figures in the column marked "measured θ_s" differ from the computed values for the same nominal pressure (1200 psi) by from

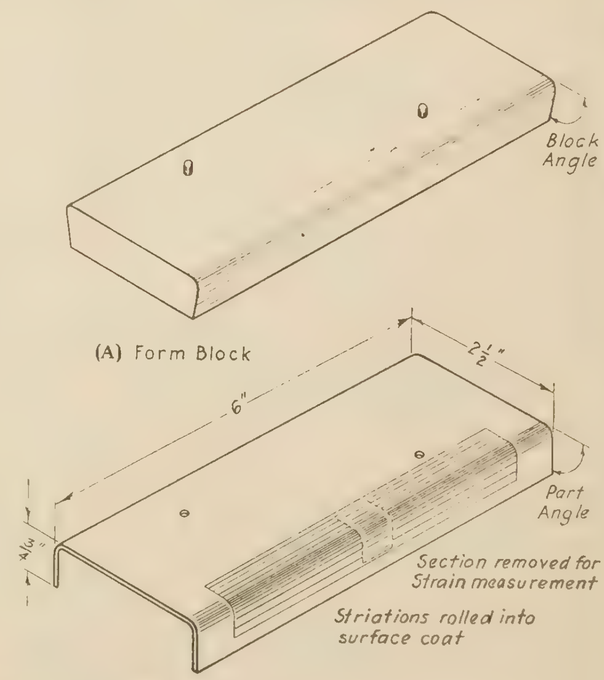

Fig. 13 Typical Form Block and Part for Experimental Determination of Springback

TABLE 3 COMPARISON OF COMPUTED AND EXPERIMENTALLY DETERMINED VALUES OF SPRINGBACK FOR 24S-T ALCLAD FORMED ON THE RUBBER-PRESSURE HYDROPRESS

Bend values			Computed values						Measured†
T in.	R_b in.	θ_b deg	$\frac{T}{R_b}$	k_b	$k_b\theta_b$ deg	K* (1200 psi) deg	θ_s (1200 psi) deg	θ_s (2000 psi) deg	θ_s (1200 psi) $\pm$ 1/4 deg
0.020	1/32	98	0.49	0.050	4.9	1.7	6.4	6.0	7.0
	1/16	98	0.28	0.076	7.5	1.5	9.0	8.7	9.5
	1/8	98	0.15	0.139	13.6	1.3	14.9	14.6	13.5
	1/4	98	0.077	0.225	22.0	1.1	23.1	22.9	23.0
0.032	1/32	98	0.68	0.038	3.7	1.8	5.5	5.1	4.2
	1/16	98	0.410	0.059	5.8	1.7	7.5	7.1	7.2
	1/8	98	0.23	0.093	9.1	1.4	10.5	10.1	9.7
	1/4	95	0.120	0.163	15.5	1.2	16.7	16.4	14.5
0.040	1/16	98	0.49	0.050	4.9	1.6	6.5	6.1	6.5
	1/8	98	0.28	0.076	7.5	1.5	9.0	8.7	9.2
	1/4	95	0.15	0.139	13.2	1.3	14.5	14.2	13.5
0.064	1/8	98	0.41	0.059	5.8	1.7	7.5	7.1	7.5
	1/4	95	0.23	0.093	8.8	1.4	10.2	9.9	9.5
	1/8	50	0.41	0.059	3.0	1.7	4.7	4.3	4.0
	5/32	50	0.34	0.067	3.3	1.6	4.9	4.5	4.5
	1/8	70	0.41	0.059	4.1	1.7	5.8	5.4	6.0
	5/32	70	0.34	0.067	4.7	1.6	6.3	5.9	6.5
	1/4	75	0.23	0.093	7.0	1.4	8.4	8.1	8.2
	3/8	75	0.16	0.133	10.0	1.3	11.3	11.0	10.5
	1/4	105	0.23	0.093	9.8	1.4	11.2	10.8	10.5
	3/8	105	0.16	0.133	14.0	1.3	15.3	15.0	18.5
0.020	1/8	50	0.15	0.139	7.0	1.3	8.3	8.0	6.5
	5/32	50	0.12	0.165	8.2	1.2	9.4	9.1	7.0
	1/16	60	0.28	0.076	4.6	1.5	6.1	5.8	5.0
	3/32	60	0.19	0.110	6.6	1.4	8.0	7.7	6.7
	1/4	75	0.08	0.225	23.6	1.1	17.9	17.7	15.0
	3/8	75	0.05	0.300	22.5	0.9	23.4	23.2	21.0
	1/4	105	0.08	0.225	23.6	1.1	24.7	24.5	22.0
	3/8	105	0.05	0.300	31.5	0.9	32.4	32.2	29.2
0.032	1/8	50	0.23	0.093	4.7	1.4	6.1	5.8	5.0
	5/32	50	0.19	0.113	5.7	1.4	7.1	6.8	5.5
	1/16	60	0.41	0.059	3.5	1.7	5.2	4.8	4.0
	3/32	60	0.29	0.075	4.5	1.5	6.0	5.7	4.5
	1/4	75	0.12	0.163	12.2	1.2	13.4	13.1	10.7
	3/8	75	0.08	0.220	16.5	1.1	17.6	17.3	15.0
	1/4	105	0.12	0.163	17.1	1.2	18.3	18.0	15.0
	3/8	105	0.08	0.220	23.0	1.1	24.1	23.8	20.0
0.040	1/8	50	0.28	0.076	3.8	1.5	5.3	5.0	5.0
	5/32	50	0.23	0.092	4.6	1.4	6.0	5.7	5.5
	1/8	70	0.28	0.076	5.3	1.5	6.8	6.5	7.0
	5/32	70	0.23	0.092	6.4	1.4	7.8	7.5	8.0
	1/4	80	0.15	0.139	11.1	1.3	12.4	12.1	11.0
	3/8	80	0.10	0.019	15.2	1.2	16.4	16.1	15.0
	1/4	105	0.15	0.139	14.6	1.3	15.9	15.6	14.0
	3/8	105	0.10	0.019	20.0	1.2	21.2	20.9	18.2

* K at 2000 psi = K at 1200 psi multiplied by $\sqrt{\frac{1200}{2000}}$.

† Measurements are for specific parts and not average values for a number of parts (accuracy of reading is ±1/4 per cent).

1/2 to 4 deg, this discrepancy is no greater than the variation which experience shows will occur in the actual values under production conditions, even when the nominal pressure remains at 1200 psi. Actually, with such a nominal pressure the localized pressure at various locations on the press platen may differ considerably. The column of computed values of θ_s for 2000 psi has been added to show the approximate effect of such pressure variation.

Measured Strain and Bend Allowance of Bends. In connection with this investigation of springback, the strain distribution in representative bends in 24S-T Alclad was also determined. The technique for strain determination consisted of rolling into the Alclad surface of the blank, prior to bending, shallow striations accurately spaced 0.01 in. apart. These striations did not penetrate the Alclad coat and hence did not affect the properties of the sheet to an appreciable extent.

The blanks were formed on the form blocks previously mentioned. After bending the blanks in the manner indicated in Fig. 13, a section (shown dotted) was carefully cut from the part,[7] mounted in Lucite as if for micrographic examination, polished etched to remove burrs due to polishing, and photographed. A suitable enlargement of the photograph was used for making measurements. In order to avoid the uncertainty that may arise regarding the degree of enlargement, a control strip, cut from the marked but unformed blank, was mounted alongside the bend section and therefore appeared in the enlargement of the photograph. This view of the control strip served to establish the scale for measuring strains in the bend.

Measurements of strain were made by establishing bisectors of the striation and measuring the distance between adjacent bisectors with dividers. By making measurements continuously from point to point, using the end point of the previous measurement for the starting point of the next measurement, cumulative errors were practically eliminated.

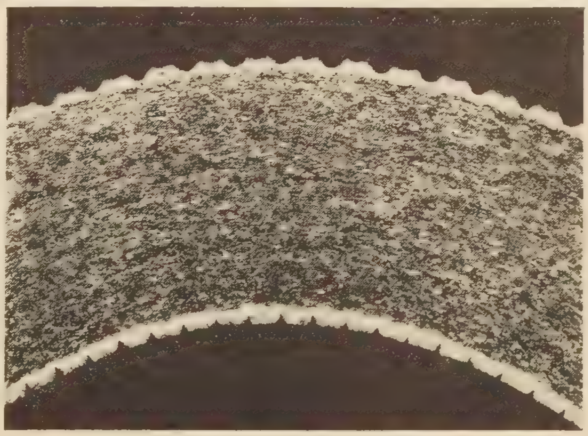

FIG. 14 ENLARGEMENT OF A CROSS SECTION OF A TYPICAL BEND OF 24S-T ALCLAD PREPARED FOR MEASURING FORMING STRAINS

[7] No appreciable change in shape of the strip occurred when it was cut out.

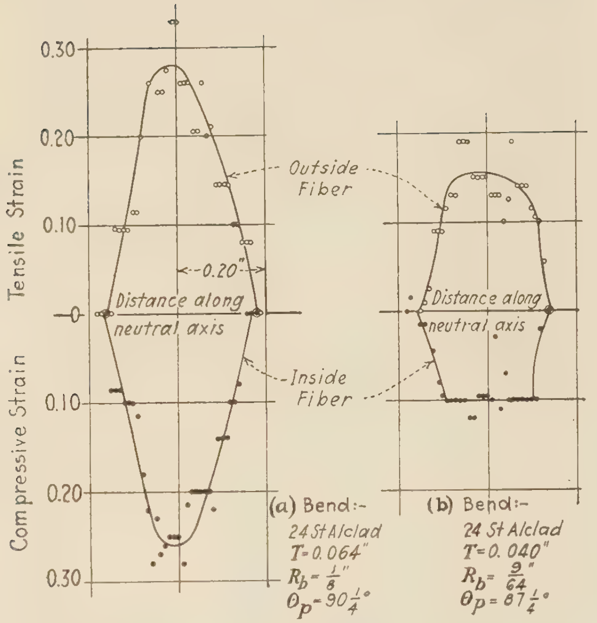

Fig. 15 Typical Strain-Distribution Curves

By comparing duplicate measurements by different individuals, it is estimated that an accuracy of approximately ± 2 per cent was obtained.

An enlarged illustration of a sample bend is shown in Fig. 14. Strain distributions for a number of bends are shown in Fig. 15.

The bend allowance was determined by noting the distance between the two intersections of the tensile- and compressive-strain distribution curves. Typical measured values are compared with the corresponding computed values in Table 4.

Results. As a result of the investigation outlined, the following properties of bends were observed:

1 The strain is in general nonuniformly distributed around the bend, approaching zero more or less gradually at the points of tangency of the bend with the flat unformed sheet.

2 The extent of the bend (bend allowance) measured on the neutral surface from point of tangency to point of tangency is greater than the value computed by the theoretical formula based on the assumption of a circular bend. If the transition regions are considered, good agreement exists.

3 The radius of curvature of the bend at any point[8] is very nearly equal to $\frac{e_t + e_c}{T}$. (This tends to verify the assumption that cross sections of the material normal to the neutral axis remain approximately plane and normal to the neutral axis during bending.)

4 For bends made on the rubber-pressure hydropress, the tensile strains are consistently larger than the compressive strains. This is not in agreement with results predicted on the assumption that the neutral axis lies close to the center of the material being bent. However, for bends made on the power brake, the measured tensile strains in the outer surface and the compressive strains on the inner surface are approximately equal. This, it is believed, indicates that the rubber-pressure hydropress subjects the material to some tension in addition to bending. (Other observations tend to bear this out.) This tension probably arises from a frictional "gripping" action of the rubber on the outer surface.

5 Fairly large abrupt variations in strain are observed. In some instances these can be attributed in part to irregularities in the form block, but in most instances these results must be

[8] From Equations [1*d*] and [1*e*]

$$e_t + e_c = (1 - C)\frac{T}{R} + C\frac{T}{R} = \frac{T}{R}$$

$$R = \frac{e_t + e_c}{T} \qquad [1f]$$

Equations [1*d*] and [1*e*] were based on the assumption that all cross sections which were plane before bending, and normal to the neutral axis, remain so after bending. Hence, if Equation [1*f*] is found to hold in an actual bend, the assumption tends to be verified. Note that Equation [1*f*] holds at any given point, but the value of R does not necessarily remain the same from point to point, as already discussed.

TABLE 4 COMPARISON OF COMPUTED AND EXPERIMENTALLY DETERMINED VALUES OF BEND ALLOWANCE FOR 24S-T ALCLAD FORMED ON THE RUBBER-PRESSURE HYDROPRESS

Bend values			Computed values						
T in.	R_b in.	θ_b deg	$\frac{T}{R_b}$	$\frac{L_c}{T}$ (1200 psi)	$\theta_b R_b$	L_c (1200 psi) in.	L_b (1200 psi) in.	L_b (2000 psi)[a] in.	Measured L_b ± 0.02 in.
0.032	3/64	98	0.51	2.8	0.11	0.09	0.20	0.18	0.21
	3/64	98	0.51	2.8	0.11	0.09	0.20	0.18	0.19
	1/16	98	0.41	2.7	0.14	0.09	0.23	0.21	0.21
	1/16	98	0.41	2.7	0.14	0.09	0.23	0.21	0.24
	5/64	98	0.34	2.6	0.16	0.08	0.24	0.22	0.25
	5/64	98	0.34	2.6	0.16	0.08	0.24	0.22	0.23
	3/32	98	0.29	2.5	0.18	0.08	0.26	0.24	0.24
	3/32	98	0.29	2.5	0.18	0.08	0.26	0.24	0.24
	7/64	98	0.26	2.4	0.22	0.08	0.30	0.28	0.28
	7/64	98	0.26	2.4	0.22	0.08	0.30	0.28	0.29
	1/8	95	0.23	2.3	0.24	0.07	0.31	0.29	0.31
	1/4	95	0.12	1.9	0.44	0.06	0.50	0.49	0.46
0.040	1/16	98	0.49	2.8	0.14	0.11	0.25	0.23	0.29
	1/16	98	0.49	2.8	0.14	0.11	0.25	0.23	0.27
	3/32	98	0.35	2.6	0.18	0.10	0.28	0.26	0.27
	3/32	98	0.35	2.6	0.18	0.10	0.28	0.26	0.27
	3/32	90	0.35	2.6	0.16	0.10	0.26	0.24	0.23
	7/64	98	0.31	2.5	0.22	0.10	0.32	0.30	0.26
	7/64	98	0.31	2.5	0.22	0.10	0.32	0.30	0.28
	1/8	98	0.28	2.5	0.24	0.10	0.34	0.32	0.30
0.064	7/64	98	0.45	2.8	0.22	0.18	0.40	0.36	0.34
	1/8	98	0.41	2.7	0.26	0.17	0.43	0.39	0.35
	1/8	98	0.41	2.7	0.26	0.17	0.43	0.39	0.40
	1/8	90	0.41	2.7	0.25	0.17	0.42	0.38	0.35
	9/64	98	0.37	2.6	0.29	0.17	0.46	0.42	0.39
	9/64	98	0.37	2.6	0.29	0.17	0.46	0.42	0.42
	1/4	95	0.23	2.3	0.45	0.15	0.60	0.57	0.52

[a] Values of L_c at 2000 psi = $\sqrt{\frac{1200}{2000}}$ × values of L_c at 1200 psi.

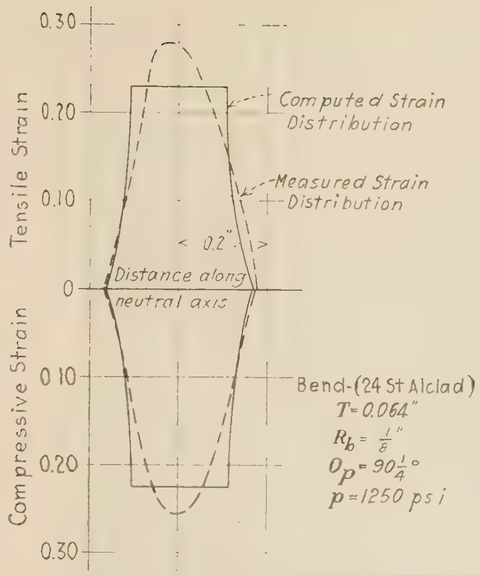

Fig. 16 Comparison of Computed and Measured Strain Distribution of a Typical Rubber-Pressure-Hydropress Bend

attributed to the nonhomogeneous or granular nature of the material.

The measured strain distribution in a bend made on a rubber pressure hydropress is not greatly different from the computed strain distribution of an ideal bend, Fig. 16. If it is borne in mind that the die radii were not extremely accurate, it may be said that there is fair agreement within the accuracy of measurement between computed and measured values.

It is found that for accuracy in computing springback it is very important to predict closely the extent of the bend. This was done in the developed theory by introducing the correction K for the transition region. It is much less important to predict the exact values of strain. The error in the computed springback due to a moderate error in the assumed strain is relatively unimportant because the value of k_b or k_p varies only slightly with changes in strain. The exception to this exists when the strain $(e_t' + e_c)$ itself is quite low, 5 per cent or less.

Accuracy of Computed Bend Moments and Residual Stresses of Sheet-Metal Bends

It is recognized that the material in a sheet-metal bend is subjected to lateral restraint resulting in biaxial stresses which are probably different from those encountered in the standard sheet-metal tensile coupon which was used to obtain the stress-strain diagrams, and upon which the computations in this paper were based. Due to transverse tension there is an increase in strength of as much as perhaps 15 per cent (11, 12) as predicted by the maximum elastic shear strain-energy theory of strength. However, it is also recognized that some lateral restraint (in the direction of the width) exists in the tensile coupon during test. The latter is evidenced by the fact that reductions in width less than the reduction in thickness were observed in the tensile tests for the stress-strain diagram. Taking the possible error due to the foregoing difference between the bend and tensile test into consideration, the computed values of stress and moment are probably somewhat less than 15 per cent too low. It is doubtful, on the basis of the agreement which exists with experimental values and on the basis of theoretical considerations, that the error in the computed values of springback is nearly so great. This is mainly because there is a change in the effective modulus of elasticity approximately equal to the change in strength due to the transverse component of the biaxial stress acting on the bending. It is seen from Equation [13] that if s and E are both changed by the same ratio the value of k_b is unaffected.

Greater accuracy of the computed values for residual stress and moments could be obtained if the stress-strain diagram for a condition of combined stresses similar to that in the bend were known; but for general practical use this refinement seems unnecessary.

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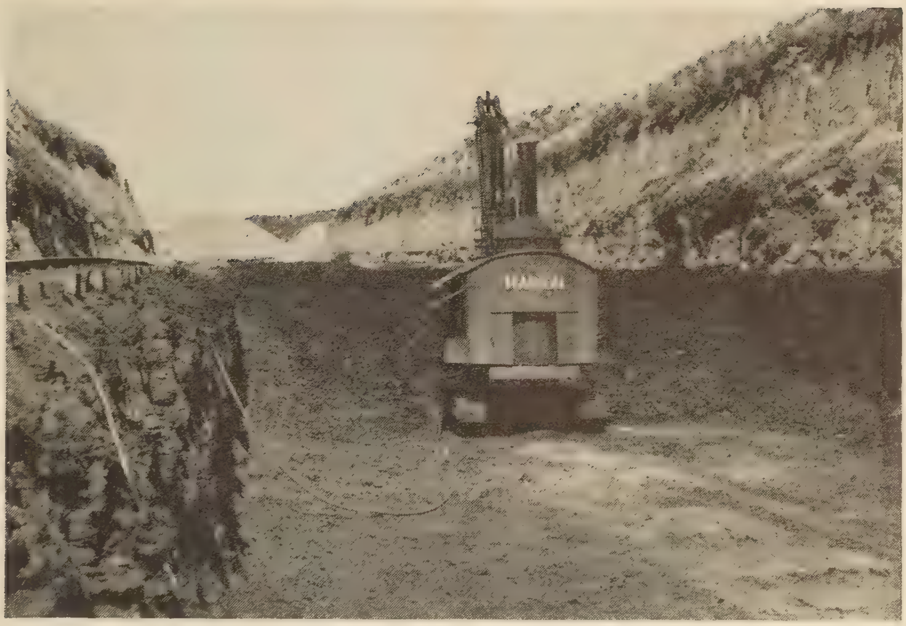

FIG. 1 VIEW OF LIGNITE MINE LOCATED AT ROCKDALE, TEXAS, SHOWING SHALLOW OVERBURDEN, THICKNESS OF LIGNITE BED, AND THE EASE WITH WHICH THIS FUEL MAY BE MINED

Lignite—Influence of Storage Conditions Upon Size Degradation, Size Stability, and Friability

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This paper is presented for the purpose of focusing attention on the little known possibilities of lignite as a direct source of potential energy for power-generating and industrial purposes, for the manufacture of gaseous fuels, and for the manufacture of liquid fuels. More than 18 per cent of the nation's reserve of mineral fuels consists of lignite, but only an insignificant fraction of this low-rank solid fuel is being utilized. Test data are presented to dispel misconceptions concerning the characteristics of lignite which today stand in the way of its more extended use. The most serious of these relates to its tendency to undergo size-reduction processes while in storage and being handled. Nevertheless, it has been demonstrated that it is possible to store this fuel with complete satisfaction, the conclusions reached from the tests described giving factual support to the possibilities in this direction. Size stability, or measure of the ability of lignite to withstand a reduction in size of particles when being handled, and friability tests are also reported.

In America, the term lignite is used to denote the amorphous coals which are woody or claylike in their appearance. The low rank of this fuel must be attributed in a large measure to the absence of great thrust pressures in the course of the dynamochemical period of its formation. Thus the paste or binder, formed in the earlier biochemical period, was never dehydrated and devolatilized to an extent sufficient to produce a coal of higher rank. In its "as-mined" state, this fuel generally possesses a moisture content of 30 to 40 per cent, an ash content of 6 to 15 per cent, and a heating value of 6000 to 8000 Btu per lb. When subjected to rapid and repetitive moisture-absorption or rejection phases, this fuel undergoes more extended size-reduction processes than the coals of higher rank.

The principal deposits of lignite are found in those sections of the nation in which no mountain-making movements of the earth's crust have been experienced. An estimate of the nation's reserves of lignite at the end of 1936 is given in Table 1. This in-

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

TABLE 1 LIGNITE RESERVES

State	Reserve, net tons
North Dakota	599,945,525,000
Montana	315,473,043,000
Texas	22,928,747,000
South Dakota	1,019,628,000
Others	90,500,000
Total	939,457,443,000

formation establishes at once the fact that the deposits of lignite in this country represent a source of potential energy of great magnitude. Fig. 1 shows a lignite-mine operation, indicating the ease with which this type of fuel may be obtained.

More Than One Sixth of Mineral-Fuel Reserve

Despite the fact that more than 18 per cent of the nation's reserve of mineral fuels consists of lignite, this low-rank solid fuel is being utilized to a very small extent at the present time. Inasmuch as the magnitude of the reserves of mineral fuels (Table 2) with which the nation has been endowed is relatively very large, a traditional disregard exists for sources of potential energy of the lower orders. Thus the pattern of its fuel-utilization program has become greatly unbalanced. The magnitude of the reserves of the more admirable fuels though large is by no means of a sufficient order to justify unconcern over their prudent utilization either in time of peace or in the course of a national emergency.

TABLE 2 PRODUCTION AND RESERVES OF MINERAL FUELS AND THEIR IMPORTANCE IN TERMS OF HEATING VALUE

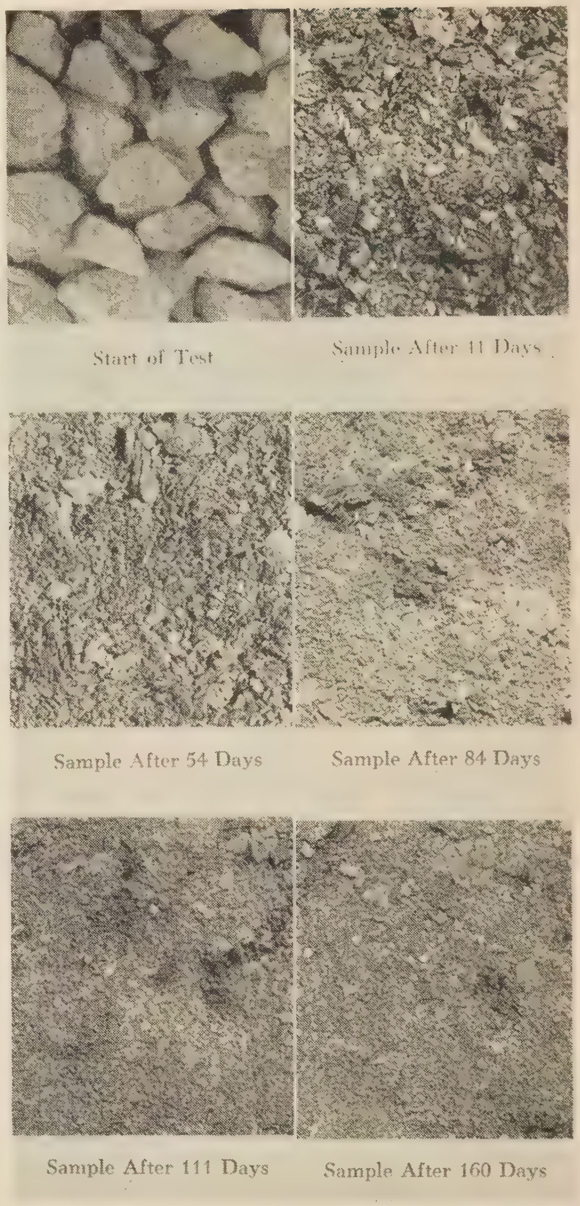
	Production in 1940		Estimated reserves	
Fuel	Trillions of Btu	Per cent of total	Trillions of Btu	Per cent of total
Bituminous coals	12500	48.0	54100000	79.7
Lignite	40	0.2	12580000	18.5
Oil in shales	...	...	580000	0.9
Anthracite coal	1300	5.0	390000	0.6
Petroleum	8500	32.6	120000	0.2
Natural gas	3700	14.2	100000	0.1
Total	26040	100.0	67870000	100.0

The unbalance in the nation's fuel-utilization program results from the lush, frequently wasteful, and in many cases extravagant production of the fuels of the higher order. Such fuels of a liquid and gaseous nature are mined, transported, and distributed with great ease. Their prolific production makes them available at costs which tax the imagination. They may, moreover, be utilized with consummate proficiency. Their period of adequacy is, however, of a very much smaller order than that of the solid fuels. The frugality with which these liquid and gaseous fuels are produced, and the care with which their use is confined to the processes for which their use approaches indispensability may yet determine the welfare of the nation in the not distant future.

The failure to utilize lignite to a greater extent may be attributed in part to rather widespread misconceptions with regard to the heat losses which attend its use as a fuel. The disproportionality which exists between the per cent by weight of the fuel which consists of moisture, and the loss resulting from the presence of that moisture has not been very generally understood. The failure in the case of many installations to use appropriate grate surfaces has caused this fuel to be maligned with regard to sifting losses resulting from the size reduction of the fuel particles as heat is applied and the moisture is driven off. Yet it can be shown that the water losses of this fuel are no greater than those of some of the more admirable fuels and that the size reduction while this fuel burns can be rendered inconsequential. The most serious misconceptions with regard to lignite relate to its tendency to undergo size-reduction processes while in storage and while being handled.

Moisture Content Influences Physical Properties

The moisture content of lignite, as well as the rate at which, the extent to which, and the frequency with which this moisture



Start of Test — Sample After 11 Days — Sample After 54 Days — Sample After 84 Days — Sample After 111 Days — Sample After 160 Days

Fig. 2 Record of Size Degradation of Lignite Stored Under Adverse Conditions

content is changed, influence to a very great extent the physical properties of this fuel. Inasmuch as very little specific information with regard to the size degradation, size stability, and friability of lignite is available, a study has been made in this instance to establish these physical properties of lignite and to determine the manner in which they are influenced by conditions normally encountered in the storage of such fuel.

Evidence to support the common belief that lignite particles approach a state of almost complete comminution when stored for extended periods of time under unfavorable conditions abounds. Fig. 2 demonstrates this fact quite effectively. It should not be inferred, however, that it is not possible to store lignite in a satisfactory manner.

No standard procedure has as yet been established for the de-

termination of specific values for the size degradation of solid fuels. The accelerated laboratory-test procedures which have been developed for the determination of the slacking characteristics of coal may be used for the higher ranks of solid fuel. They cannot, however, be used in the case of such low-rank solid fuel as lignite in an effective and purposeful manner. In the work which is reported in this instance, no effort has been made to develop or to use an accelerated procedure for this determination. Instead, tests were conducted for a period of time comparable to that normally involved in the storage of such fuel. Although this method is laborious, it is nevertheless positive. The results obtained eliminate all doubt but involve a time-consuming procedure. It is by no means inconceivable that an accelerated process may be developed when the properties of lignite are better understood.

The term size degradation is used in this instance to denote that tendency or property of a solid fuel to undergo a reduction in the size of its particles while in storage. For that reason, the test samples of lignite were very carefully sized and placed in the various kinds of storage. After being placed in storage, this fuel was subjected to no handling other than that involved in removing it from storage at the end of the given test period and that required to resize it after its removal from storage. Great care was exercised to reduce the nature and extent of the handling to a minimum in both cases. By virtue of the fact that the initial size of the fuel particles was known and that at the end of the period could be determined, it was possible to secure, from a comparison of these data, an indication of the extent of the size-degradation tendencies in the case of each type of storage.

Tests for Size Degradation of Lignite

In order to conduct suitable tests in this case, an aggregate of 22,500 lb of lignite, consisting only of particles passing through a 3-in. round-hole screen and retained upon a 2-in. round-hole screen was placed into the following types of storage:

1 Samples exposed completely to the atmosphere
2 Samples exposed to the atmosphere but protected from sunshine
3 Samples exposed to the atmosphere but protected from rain
4 Samples exposed to the atmosphere but protected from sunshine and rain
5 Samples exposed to the atmosphere in a basement
6 Samples sealed in metal cans and subjected to atmospheric temperature.
7 Samples submerged under water in sealed metal cans and subjected to atmospheric temperature.

The amount of lignite indicated was sufficient to permit the removal of samples from storage for test purposes with regard to size degradation and other tests mentioned hereinafter at the end of 0, 6, 13, 20, 27, 34, 41, 54, 84, 111, and 160 days.

In order to compare the size of the fuel particles at the end of each test period with the initial size, an appropriate average-of-screen-openings factor was applied to each size of fuel particle screened at the end of the tests. The factors selected are given in Table 3.

TABLE 3 TABLE OF AVERAGE-OF-SCREEN-OPENINGS FACTORS

Retained on; screen size, in.	Passing; screen size, in.	Average-of-screen openings, in.	Average-of-screen-openings factors
2	3	2.500	1.00
1¹/₂	2	1.750	0.70
1	1¹/₂	1.250	0.50
³/₄	1	0.875	0.35
¹/₂	³/₄	0.625	0.25
	¹/₂	0.250	0.10

The screens used to make these size determinations were round-hole screens which had been constructed to conform in detail to the American Society for Testing Materials Standard Specifications, "Sieves for Testing Purposes," A.S.T.M. Designation: E-11.

Inasmuch as the same samples were used for size-stability and friability tests as well as for size-degradation tests, it was necessary to vary the size of the samples to take into consideration the period of storage involved in each instance. The size-degradation results were computed in each case in the manner indicated by the data shown in Table 4.

TABLE 4 SIZE-DEGRADATION TEST FOR SOLID FUELS

Date of Test: June 24, 1942 (160)

Fuel Index No. 11 Fuel Storage No. 1

Screen Analysis of Fuel Using Round-Hole Screens

Retained on; screen size, in.	Passing; screen size, in.	Weight, lb	Weight, per cent (1)	Average of screen Inches (2)	Average of screen Factor (3)	Product of (1) × (3)
			Sample			
2	3	699³/₄	100.0	2.500	1.00	100.0 = *S*
			Tested Fuel			
2	3	20¹/₂	2.9	2.500	1.00	2.900
1¹/₂	2	5¹/₂	0.8	1.750	0.70	0.560
1	1¹/₂	20¹/₂	2.9	1.250	0.50	1.450
³/₄	1	38¹/₂	5.5	0.875	0.35	1.925
¹/₂	³/₄	149³/₄	21.4	0.625	0.25	5.350
	¹/₂	465	66.5	0.250	0.10	6.650

Total [sum of products (1) × (3) for dropped fuel] = 18.835

Size degradation = 100.0 — 18.8 = 81.2 per cent

Size Stability Tested by Drop-Shatter Apparatus

The size stability of the lignite was determined with the use of an appropriate drop-shatter apparatus. Whereas the term "size degradation" was regarded as one which denotes the tendency of this fuel to undergo a reduction in the size of its particles while in storage, the term "size stability" is used here as a measure of the ability of this fuel to withstand a reduction in the size of its particles when being handled after being removed from storage.

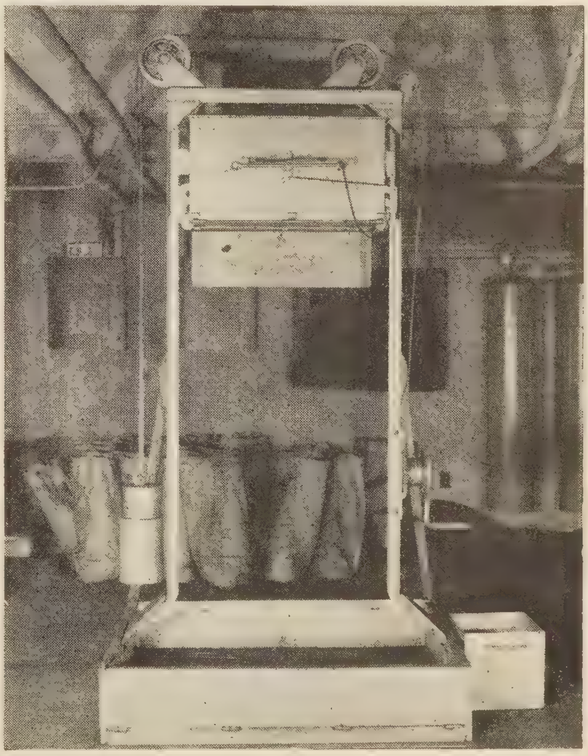


Fig. 3 Actual Drop-Shatter-Test Apparatus

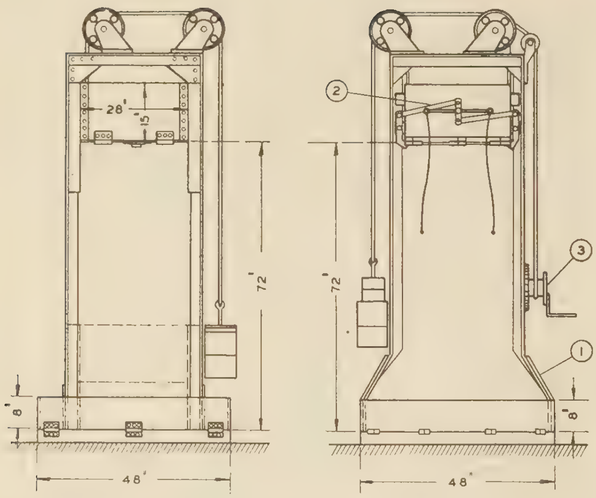


Fig. 4 Arrangement of Conventional Drop-Shatter Apparatus (Left) and Modified Apparatus (Right)

The drop-shatter apparatus, Fig. 3, was designed to conform to the provisions of the American Society for Testing Materials, Tentative Method of "Drop Shatter Test for Coal," A.S.T.M. Designation: D 440-37T, in all but three details. The changes made in the construction of this apparatus could not be expected to alter in any manner the results obtained. It was believed that these changes would expedite the test procedure. An inspection of Fig. 4, in which both the conventional and modified drop-shatter-test apparatus are shown will reveal the fact that the following modifications were made:

1 The vertical standards by which the coalbox is supported were bent outward at the bottom and extended to the sides of the bottom steel plate so that the obstruction otherwise imposed might be eliminated. This action greatly expedited the fuel-removal and recharging operations, because these operations could then be conducted from one side only.

2 A more complicated latching mechanism was fashioned on the bottom of the coalbox in order that the bottom of that member might be opened with less possibility of retarding the movement of the bottom half of the box to which the ordinary latch is usually attached.

3 A hand-operated winch was incorporated in the design of the actual apparatus so that the coalbox might be returned to its lower or recharging position with dispatch after the coal had been dropped.

In the determination of the size stability of lignite, a 50-lb sample of this fuel, all of which passed through a 3-in. screen and all of which was retained upon a 2-in. screen at the end of each storage period, was given two drops with the use of the drop-shatter apparatus in the manner prescribed in A.S.T.M. D 440-37T mentioned. These lignite particles were then rescreened and the results were computed in the manner indicated in Table 5.

TABLE 5 DROP-SHATTER TEST FOR SOLID FUELS

Date of Test: June 24, 1942 (160)

Fuel Index No. 11 — Fuel Storage No. 3

Screen Analysis of Fuel Using Round-Hole Screens

Retained on; screen size, in.	Passing; screen size, in.	Weight, lb	Weight, per cent (1)	Average of screen: Inches (2)	Average of screen: Factor (3)	Product of (1) × (3)
			Sample			
2	3	50	100.0	2.500	1.00	100.0 = S
			Dropped Fuel			
2	3	14 1/4	28.5	2.500	1.00	28.500
1 1/2	2	10	20.0	1.750	0.70	14.000
1	1 1/2	10 1/2	21.0	1.250	0.50	10.500
3/4	1	5 1/2	11.0	0.875	0.35	3.850
1/2	3/4	4 1/2	8.8	0.625	0.25	2.200
	1/2	5 1/4	10.5	0.250	0.10	1.050

Total [sum of products (1) × (3) for dropped fuel] 60.100 = S

$$\text{Size stability} = \frac{100 \times S}{S} = \frac{100 \times S}{100} = S = 60.1 \text{ per cent}$$

Testing Lignite for Friability

The friability of the lignite samples, removed from storage after the expiration of the various storage periods, was determined in order to measure the ability of this fuel to withstand breakage when subjected thereafter to repeated handling. The tumbler-test apparatus, Fig. 5, used to conduct these tests was constructed to conform in every detail to the American Society for Testing Materials, Standard Tentative Method of "Tumbler Test for Coal," A.S.T.M. Designation: D 441-37T.

In the conduct of the friability tests, carefully machined cast-iron tumbler jars provided with appropriate steel frames having suitable lifting shelves were used. Two sets of sieves were employed in the conduct of this work. Both sets were of the square-hole type arranged to conform to the American Society for Testing Materials, Standard Specification for "Sieves for Testing Purposes, A.S.T.M. Designation, E-11. Those employed for sample-preparation purposes measured 2 ft square and were provided, in one case, with openings of 1.5 in. between the wires, and in the other with openings of 1.06 in. For sizing the lignite particles after the tumbling operations had been completed in the case of the 1000-g sample in each case, screens having an 8-in-diam metal frame were employed. The openings between the wires of these square-hole sieves were 1.06, 0.750, 0.531, 0.375,

TABLE 6 TUMBLER TEST FOR SOLID FUELS

Date of Test: Feb. 13, 1942 (27)

Fuel Index No. 5 — Fuel Storage No. 7

Sieve Analysis of Fuel Using Square-Hole Sieves

Retained on; screen size, in.	Passing; screen size, in.	Weight recorded, g	Weight, per cent (1)	Average of sieve openings: Inches (2)	Average of sieve openings: Factor (3)	Product of (1) × (3)
			Sample			
1.06	1.50	1000	100.0	1.280	1.00	100.00 = S
			Tumbled Fuel			
1.06	1.50	650	65.0	1.280	1.00	65.0
0.750	1.06	154	15.4	0.905	0.70	10.78
0.530	0.750	53	5.3	0.640	0.50	2.65
0.375	0.530	31	3.1	0.452	0.35	1.09
0.0469	0.375	26	2.6	0.211	0.16	0.416
0.0117	0.0469 (No. 16)	37	3.7	0.029	0.023	0.085
	0.0117 (No. 50)	49	4.9	0.006	0.005	0.024

Total [sum of products (1) × (3) for tumbled fuel] 80.045 = s

$$\text{Friability} = \frac{100\,(S - s)}{S} = \frac{100\,(100 - 80.0)}{100} = 20 \text{ per cent}$$

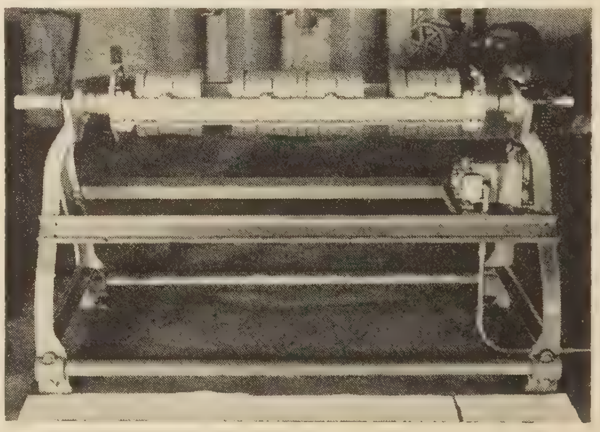


Fig. 5 Multiple-Jar Tumbler Mill Used for Lignite-Friability Tests

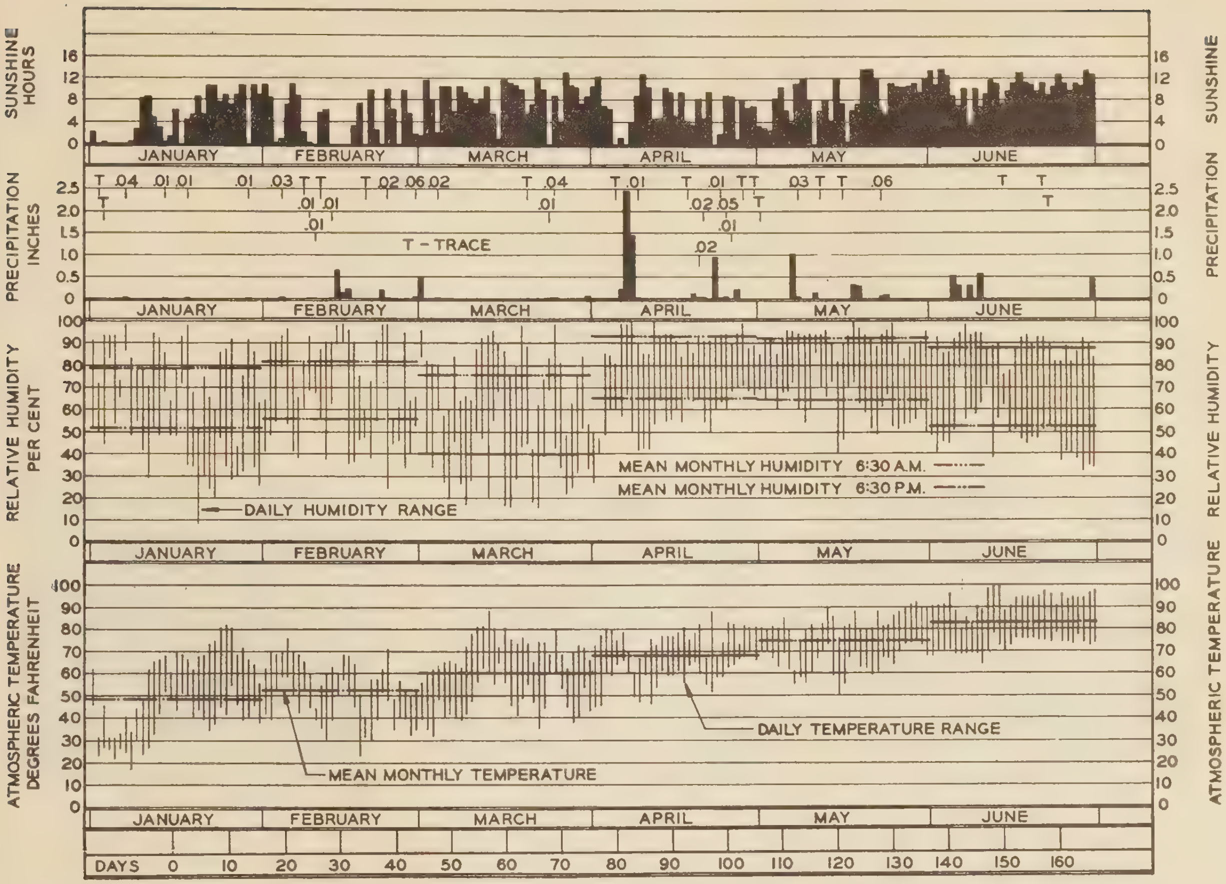


Fig. 6 Graph of Meteorological Data Which Relates to Tests Conducted

0.0469, and 0.0117 in., respectively. By virtue of the fact that all lignite particle sizes before tumbling were known and those after tumbling could be determined, a comparison of these results gave an appropriate index to the friability of the fuel. The manner in which the results were computed is shown in Table 6.

Influence of Meteorological Conditions on Lignite in Storage

In order to evaluate the results obtained in the course of this investigation appropriate consideration must be given to the meteorological conditions which prevailed in the course of the test period which involved a span of 160 days. Among other facts, the following data should be borne in mind in the consideration of the results hereinafter presented:

1 The tests were begun in midwinter and completed in the summer. Thus a wide variety of weather was experienced.

2 Temperatures as low as 17 F, and as high as 100 F were experienced in the course of the test period. The mean monthly temperature increased steadily as the test period was extended, from a value of only 48.5 F in January to 82.8 F in June.

3 The relative humidity varied through a very wide range in this interval. The lowest value recorded was 8 per cent and the highest was 98 per cent. The mean daily relative-humidity range approximated 25 per cent.

4 Rains of small magnitude were experienced upon frequent occasions, whereas those of considerable magnitude were less frequent. The frequency of the heavier rains was, moreover, greater in the later stages of the tests than in the earlier periods.

5 The percentage of possible sunshine per month ranged from 50 to 77 per cent. The greatest irregularity in the case of this factor occurred in the early part of the test period.

The extent to which meteorological conditions influenced the size degradation, size stability, and friability of lignite in this case very naturally varied to a considerable extent with the nature of the storage in which the fuel was placed. Some of the characteristics of lignite hereinafter presented can only be understood when viewed in the light of the information thus provided. In other cases, the meteorological influences were indeed very small. A graphical representation, Fig. 6, of these meteorological data is presented to assist in understanding the results reported.

The results of the size-degradation tests are shown in graphical form in Fig. 7. Curves for seven classes of storage are provided. Three facts in particular may be established from an inspection of these data. They are as follows:

1 The extent of the size degradation of lignite varies through a very wide range depending upon the class of storage involved:

Class of storage	Size degradation at end of 160-day storage period, per cent
Sample exposed completely to atmosphere.	81.1
Sample exposed to atmosphere but protected from sunshine	70.0
Sample exposed to atmosphere but protected from rain	28.2
Sample exposed to atmosphere but protected from sunshine and rain	17.1
Sample stored in basement	9.6
Sample sealed in metal cans	8.8
Sample submerged under water	5.2
Sample at beginning of test	3.2

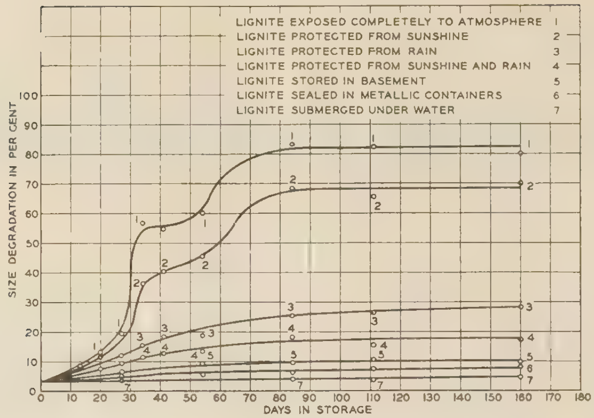
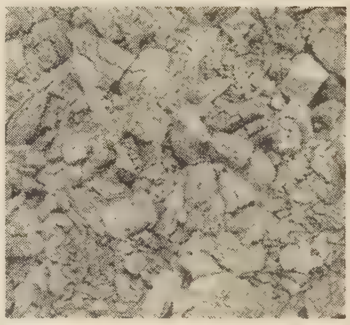


Fig. 7 Results of Size-Degradation Tests

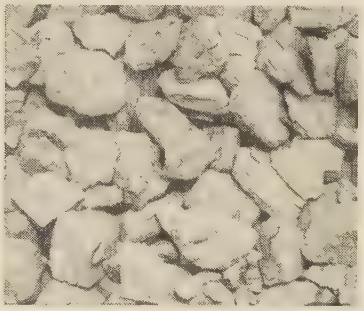
Typical sample, at beginning of tests

Sample completely exposed, after 160 days in storage

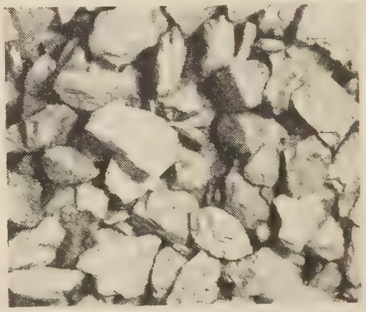


Sample protected from sunshine, after 160 days in storage

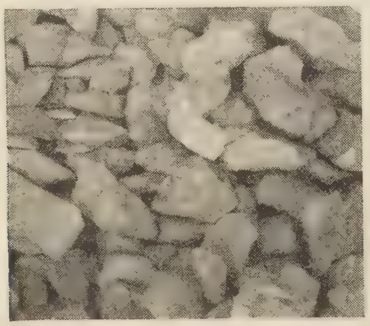
Sample protected from rain, after 160 days in storage



Sample protected from sunshine and rain, after 160 days in storage



Sample stored in basement, after 160 days in storage



Sample sealed in metal cans, after 160 days in storage

Sample submerged under water, after 160 days in storage

Fig. 8 Views Showing Initial Condition of Lignite and Extent of Its Degradation at End of 160 Days in Storage in Case of Seven Different Types of Storage

2 Under certain classes of storage, the duration of the storage period is of significance. In other cases, it appears to be of comparatively small importance. The sample stored in a basement reached the 10 per cent degradation point in 160 days. The same point was reached by the completely exposed sample in 15 days. For all classes of storage, the extent of degradation appears to be greater in the early storage period than in the later stages.

3 Lignite can be stored for extended periods of time without incurring serious size-degradation tendencies under favorable circumstances.

The foregoing data reveal the fact that the rate at which moisture is transported from the exterior of the lignite particle to the interior of that particle in moisture-absorption phases, and the rate at which it is transported from the interior of the particle to the exterior in moisture-rejection phases, is of tremendous importance in the size-degradation processes. Inasmuch as the fuel particle expands upon absorbing moisture and contracts upon rejecting it, it becomes apparent that, in the absence of an equilibrium between the rate of absorption or rejection at the surface and the rate of transportation through the fuel particle, size degradation will accompany both phases.

The extent to which this degradation process took place in the case of fuels in the various types of storage may be observed in Fig. 8, in which are shown views of lignite in its initial condition upon being placed into storage and in the final condition at the end of 160 days of seven different types of storage. Only a casual inspection is required to reveal the fact that the visual data complement the statistical and graphical data previously presented in this paper.

The data obtained in the course of this conduct of size-stability tests is presented in Fig. 9, which shows the results for seven different classes of storage. From an inspection of the data presented the following conclusions can be drawn:

1 As in the case of size degradation, storage conditions have a very great influence upon the size stability of this fuel. The range and extent of this influence are as follows:

Type of Storage	Size stability at end of 160-day storage period, per cent
Sample completely exposed	..[a]
Sample exposed to atmosphere but protected from sunshine	48.8
Sample exposed to atmosphere but protected from rain	59.9
Sample exposed to atmosphere but protected from sunshine and rain	58.8
Sample stored in basement	69.2
Sample sealed in metal cans	81.0
Sample submerged under water	87.5
Sample at beginning of tests	88.3

[a] It was not possible to secure an appropriate sample after 84 days.

2 The general pattern of size-stability curves, which show the tendency of the fuel particles to break when handled after being removed from storage, is much like that of the size-degradation curves, which show this property with regard to size reduction in the course of the storage period.

3 In the case of samples which were protected from rain and to which no moisture was added, the size-stability characteristics appear to bear a very close relationship to the moisture content of the fuel.

An additional index to the fragility of this lignite can be obtained by comparing the graphical data presented in Fig. 10, showing how the moisture content of the samples varied, with the size-stability curves, Fig. 9.

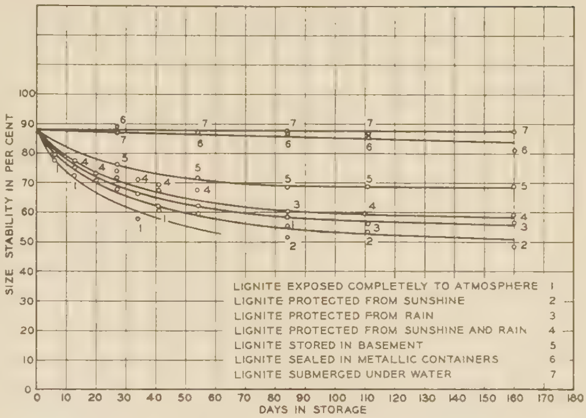


Fig. 9 Results of Size-Stability Tests

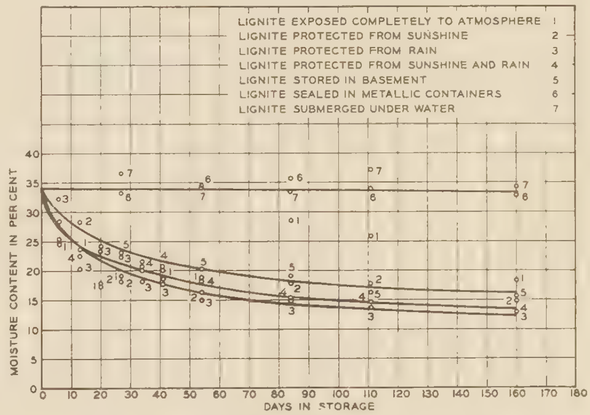


Fig. 10 Graph Showing Loss of Moisture of Lignite While in Storage

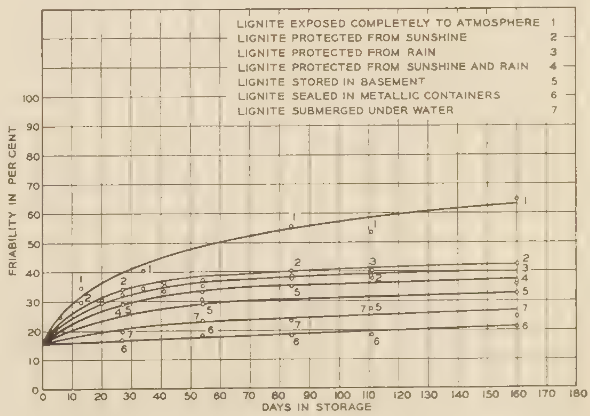


Fig. 11 Graph Showing Friability of Lignite

Class of Storage	Reduction in moisture content in 160 days in storage, per cent	Reduction in size stability in 160 days in storage, per cent
Samples protected from rain	21.0	28.4
Samples protected from sunshine and rain	21.0	29.5
Samples stored in basement	18.7	19.1
Samples sealed in metal cans	1.3	7.3

The results of the tumbler tests by means of which the friability of this lignitic fuel was obtained are shown in graphical form in Fig. 11. These data justify the formulation of the following conclusions:

1 The character of the storage into which lignite is placed influences to a very great extent its friability as well as its size degradation and size stability.

Class of Storage	Friability at the end of 160-day storage period, per cent
Sample completely exposed	64.9
Sample exposed to atmosphere but protected from sunshine	42.4
Sample exposed to atmosphere but protected from rain	36.5
Sample exposed to atmosphere but protected from sunshine and rain	35.5
Sample stored in basement	32.6
Sample sealed in metal cans	21.1
Sample submerged under water	24.7
Sample at beginning of storage period	15.7

2 The magnitude of the friability values were greatest where the least control was exercised over moisture absorption-rejection cycles and they were least where the frequency of these cycles was limited to the greatest extent.

3 The values for the friability of lignite submerged under water were found to be higher consistently than those of lignite stored in sealed metal cans. This circumstance must be attributed to the fact that the lignite particle structure was weakened when it was removed from storage and was permitted to undergo a sufficient amount of drying to remove the surface moisture. The presence of this surface moisture made it necessary to withhold screening operations for a reasonable period of time after removal from the water.

Conclusions

In the future, it seems reasonable to presume, lignite will be used as a direct source of potential energy for power-generating and industrial purposes, for the manufacture of gaseous fuels, and for the manufacture of liquid fuels. It is, moreover, not unreasonable to suppose that, for some of these processes a knowledge of the size-degradation, size-stability, and friability characteristics of this low-rank solid fuel may make possible the advantageous use of properties which as yet are regarded as undesirable and deleterious.

The work which this paper represents has been done with the hope that through this enlargement of knowledge with regard to the properties of lignite, some contribution, however small, may be made to the conception of the processes which must be used in the future. The authors have the very definite conviction that a very great amount of work remains yet to be done before any measurable improvement can be expected in the case of the nation's present unbalanced fuel-utilization program.

Drying Characteristics of Vegetables—Riced Potatoes

By A. H. Brown[1] and P. W. Kilpatrick,[2] Albany, Calif.

The food necessities of war conditions have brought into prominence the dehydration of foods on a scale never before undertaken. Many agencies are studying the problems involved, but comparatively little information is as yet available in the literature. The present paper takes into account the current shortage of construction materials in its approach to developing experimental data upon which basis dehydrators may be designed and built. The equipment involved, the conditions of the investigation, as applied to the dehydration of riced potatoes, and the analysis of experimental results lead to the presentation of a rational theory of dehydrator design and operation.

The wartime food-production program calls for a great increase in the nation's capacity to produce dehydrated vegetables. This expansion must take place under conditions of critical shortage of structural steel, piping, heaters, boilers, blowers, and other materials and equipment which enter into the construction of dehydrators. It therefore becomes essential to apply the best of engineering knowledge and judgment, to the end that the limited bank of critical equipment and material shall be invested so as to produce the greatest possible quantity of high-grade dehydrated food. It is almost equally important that means be available to assess the usefulness of existing driers not now being fully used for essential production. These considerations emphasize the need for a rational theory of dehydrator design and operation, because economy in the utilization of equipment can be assured only on the basis of sound theory.

Although the dehydrator is the heart of a dehydrating plant, the designer must not lose sight of the fact that he is dealing with a perishable material. If dehydration is to be successful as a method of food preservation, and if the dehydration industry is to become permanent, the product must be of a high quality. The proper quality of raw material, preparation, blanching, holding time before drying, and other practices are as important to the production of a stable, high-quality product as is the dehydrating process itself. A short drying time contributes to the retention of quality, but only if the temperatures encountered by the food material are low enough to prevent excessive rates of deterioration. This type of quality degradation must be considered for each vegetable as an individual. Proper packaging and storage conditions must be provided for the product after its dehydration. In all cases, the dehydrator design must be a compromise between a satisfactory drying rate from the engineering or economic viewpoint and sufficiently low temperatures to preserve the quality of the food product during the removal of its moisture.

Fundamental information required by the designer includes (*a*) a material balance, (*b*) a heat balance, (*c*) accurate data on the properties of air and water vapor, and (*d*) information about the rate at which a specific material dries under different conditions. The first two items can be set up readily by the engineer, and the third is available in humidity tables, but a search of the literature reveals little information concerning drying rates. Limited data are available on the drying of some fruits, but practically none on the drying of vegetables.

The need for information regarding the drying rates of vegetables has been emphasized in recent articles by Marshall (1, 2, 3, 4, 5)[3] and by Van Arsdel (6, 7, 8, 9). The only available information (4) is concerned with relative drying rates and is not readily applicable to the efficient design of dehydrating equipment. This paper presents data on the drying characteristics of the most important vegetable (white potato), when converted to one of its most important commercial forms, namely, cooked and riced.

Equipment Used in Investigation

These investigations were conducted in a cabinet drier which was locally designed and constructed. Ten trays, each 24 in. wide and 40 in. long, were included in the original design, but the experimental work utilized only a single tray. The long dimension of the tray was perpendicular to the air stream within the drier. Heat was supplied by steam coils, and the air was humidified by the direct injection of steam. Both the dry-bulb and wet-bulb temperatures of the air stream were controlled through motor-operated valves, actuated by a throttling type of temperature controller. A desired temperature could be maintained within ±1 F. Mercury thermometers were inserted into the air stream for use as a continuous check on the temperature controller.

The air velocity within the drier was adjusted by means of baffles and screens, and the variation across the tray in use was held to within ±5 per cent of a desired value. Velocity measurements were made with a Biram anemometer resting on the tray and were taken at a number of points along the mid-line of the tray, perpendicular to the air stream.

The drying progress of a tray of vegetables was followed by weighing at intervals on an external balance. This system was found preferable to the internal suspension of a tray because of errors introduced into the latter method by the moving air stream. Runs were made to compare weight losses when the tray of material was allowed to dry for a definite interval of time without interruption and when the drying process was interrupted by frequent weighings. The results of these comparison runs, and the behavior of a trayload of material upon the balance, indicated that the drying process is not appreciably affected by the removal of the tray from the drier for weighing.

Conditions of Experiments

The material used for these investigations consisted of white potatoes, Oregon Russet variety, No. 1 grade, purchased on the open market about the middle of the normal storage season. They were held in common storage for a maximum of two weeks

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Contributed by the Heat Transfer Division and presented at the Semi-Annual Meeting, Los Angeles, Calif., June 14–17, 1943, of The American Society of Mechanical Engineers.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

after their purchase. (Oregon Russet is a regional name for the Russet Burbank variety.) The potatoes were washed; the skin was removed in an abrasive peeler; and the eyes and unsound portions were removed by hand trimming. The clean potatoes were quartered by hand, washed in cold water, and steam-cooked at atmospheric pressure for 20 minutes. During the entire period between peeling and cooking, the potatoes were held in cold water to prevent darkening. Immediately after cooking, the material was riced through $^1/_8$-in. openings and fell directly on a wooden slat tray. The tray was similar in construction to those used commercially. A uniform tray loading was secured by moving the tray beneath the ricer as required, and the material was not disturbed after loading. The trayload was weighed, immediately placed in the drier, and dried under a desired set of conditions. After a run, the product was held overnight in a friction-top can to permit equalization of moisture content. Samples were taken the following morning and were ground to pass a 2-mm screen. Moisture determinations were made by heating the ground material in a vacuum oven, at a pressure of approximately 12 mm of mercury for 16 hr at 70 C. The conditions were similar to those specified in the tentative method for fruit and fruit products of the Association of Official Agricultural Chemists (10), except that a longer drying period was used, and that dry air was not admitted to the oven during the drying period.

Experimental Data

The variables used for this work were those most simple and useful. The use of observed quantities such as dry-bulb and wet-bulb temperatures eliminates the necessity of a humidity chart, and the use of the total moisture content of the material eliminates the necessity of knowing the equilibrium moisture content. Consequently, the following nomenclature will be used:

T = moisture ratio, total water/dry solids
θ = elapsed drying time, hr
t = temperature, deg F
L_0 = tray loading, lb prepared material per net sq ft
V = air velocity, linear fpm

Subscripts:

$_0$ = initial conditions (for $\theta = 0$ and $T = T_0$)

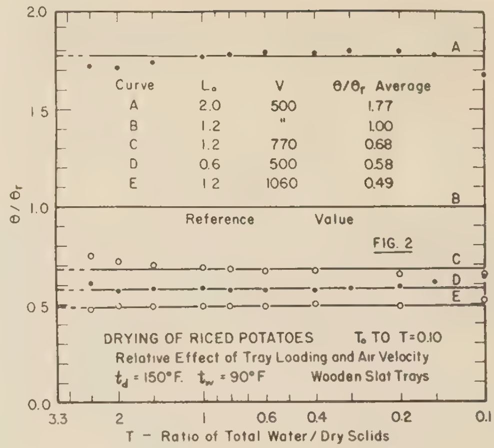


Fig. 2 Relative Effect of Tray Loading and Air Velocity

$_f$ = final conditions
$_d$ = dry-bulb temperature
$_w$ = wet-bulb temperature
$_r$ = reference conditions (of L_0 = 1.2 and V = 500)

Constant-temperature runs were made under different conditions of temperature, humidity, tray loading, and air velocity in order to determine the influence of the variables involved in the drying process. The data which were obtained show that the drying process takes place in two major stages:

1 From T_0 to T = 0.1, approximately, the drying rate is nearly proportional to the moisture content.

2 From T = 0.1 to T_f the drying rate decreases much more rapidly than the moisture content.

Consequently, the data were handled separately for each stage, and the total drying time is the sum of the drying times in both stages. Fig. 1 shows the drying curves for riced potatoes from T_0 to T = 0.1 under various temperature and humidity conditions. Fig. 2 shows the effect of air velocity and tray loading,

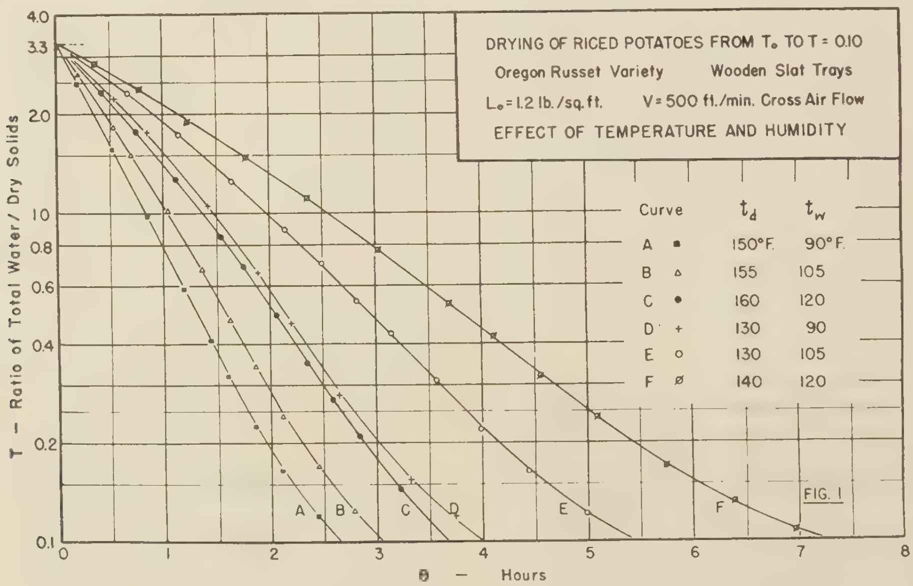


Fig. 1 Effect of Temperature and Humidity on Drying of Riced Potatoes

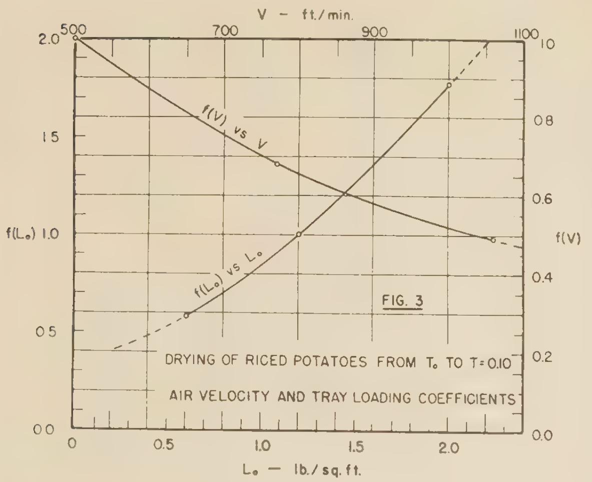


Fig. 3 Air-Velocity and Tray-Loading Coefficients

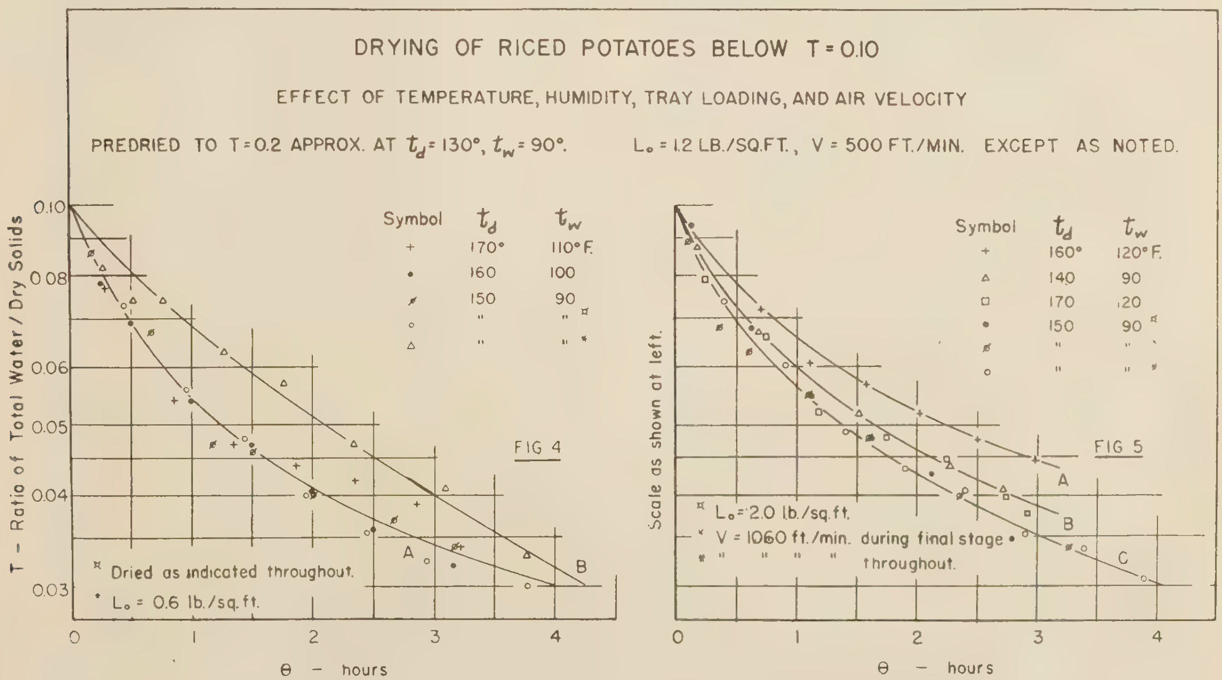
using a run at t_d = 150 F, t_w = 90 F, L_0 = 1.2, and V = 500 as a reference value. The effect of these variables is expressed as a drying-time ratio, comparing the elapsed time from T_0 to T with the reference run. The average effect of these variables over the range of T_0 to T = 0.1 is plotted in Fig. 3. Slight initial time corrections were applied to the data in order to base them on a common value of T_0 = 3.3 for the riced material as loaded on the trays. The experimental range of T_0 encountered was from 3.1 to 3.7, averaging 3.3.

In order to eliminate possible effects of severe drying conditions during the early stages, riced potatoes were predried at t_d = 130 F and t_w = 90 *F* to a value of T = 0.2, approximately, and further dried under various experimental conditions. The data, showing the drying curves for values of T of less than 0.1, are shown in Figs. 4 and 5. A study of these curves indicates that the original density of tray loading (between 1.2 and 2 lb per sq ft) and the air velocity (between 500 and 1000 fpm) cease to have any appreciable influence upon the rate of drying when the moisture content of the material has been reduced to T = 0.1 or less (see curve A, Fig. 4, and curve C, Fig. 5). This contrasts sharply with the strong influence of tray loading and air velocity at higher levels of moisture content, which is shown in Fig. 3.

Further study indicates that the only significant variable over the dry-bulb temperature range of 140 to 170 F is the wet-bulb depression, or the difference between the dry-bulb and wet-bulb temperatures (see curve A, Fig. 4, and curves A and B, Fig. 5). Other data indicate that the previous drying history of a trayload of riced potatoes has no significant effect upon the point drying rate at a later stage. An apparent exception is shown in curve B, Fig. 4, where a light tray loading was used. This deviation is readily explained by the shape of the drying-rate curve at low moisture contents and by the relative heterogeneity of light and heavy tray loadings when the average moisture content is in the region of T = 0.1. The range of T encountered at different depths on a lightly loaded tray is much smaller than that on a heavily loaded tray. Since a longer time is required to reach an average moisture content of T = 0.1 for the heavier tray loading, the upper layers of the material must be dried on the more heavily loaded tray. The rapid decrease in drying rate as the lower moisture contents are reached (Figs. 4 and 5) permits the average moisture content to decrease more rapidly (i.e., shorter times are required, taking θ = 0 at T = 0.1) when heavier tray loading is used.

Treatment of Data

In the earlier stages of this work, the data were converted into empirical equations, both integral and differential. The accuracy of these equations was not satisfactory, the form was bulky, and the differential forms required graphical integration for other than constant-temperature conditions. A simpler treatment resulted from the construction of empirical nomographs. It was



Figs. 4 and 5 Effect of Temperature, Humidity, Tray Loading, and Air Velocity on Drying of Riced Potatoes

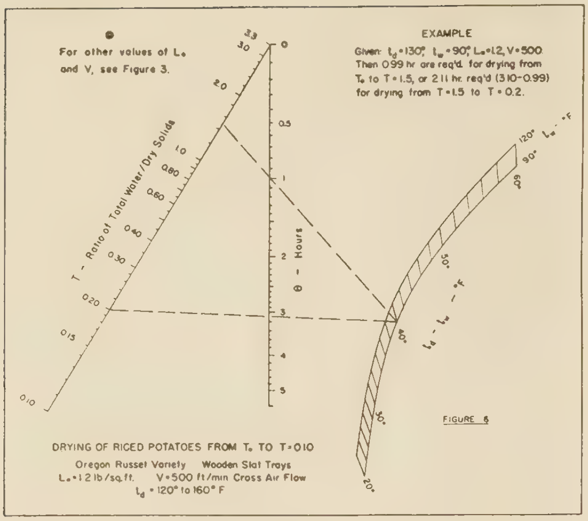


FIG. 6[4] NOMOGRAPH FOR MOISTURE RANGE T_0 TO T = 0.1

also felt that a nomograph is less likely to be misused by excessive extrapolation than an equation. The nomograph for the moisture range of T_0 to $T = 0.1$ is presented in Fig. 6, and for the moisture range of $T = 0.1$ to $T = 0.03$ in Fig. 7. These nomographs, in combination with Fig. 3 (which assumes the effect of air velocity and tray loading to be independent of temperature), permit the solution of any problem within the range of the experimental data. An illustration of the method follows.

APPLICATION OF DATA TO DESIGN PROBLEMS

It has been pointed out that the change in air temperature from one point to another along a tunnel dehydrator is proportional to the change in moisture content of the product between the two points (7). This leads to the fact that the drop in air temperature through the entire dehydrator is related linearly to the difference in moisture content between the entering material and the product. This principle permits the application of the data in Figs. 3, 6, and 7, to the design of a tunnel dehydrator.

Consider the following problem where a product containing about 10 per cent moisture is desired, the remainder to be removed in a second stage, and a counterflow tunnel is to be used:

Entering-air temperature	150 F
Exhaust-air temperature	120 F
Wet-bulb temperature	90 F
Moisture content of material entering dehydrator	T_0 = 3.3
Product moisture content	T_f = 0.1

For the present, a tray loading of 1.2 lb per sq ft and an air velocity of 500 fpm will be used. The relationship between the air temperature and the moisture content of the material within the tunnel is shown by the solid line in Fig. 8. This continuous relationship may be closely approximated by a series of steps as shown by the dotted line in the same figure. The stepwise procedure assumes that the drying process occurs in a series of constant-temperature stages, each holding over a certain change in moisture content. Good accuracy of estimate will be attained if the number of steps taken corresponds to an average temperature change between steps of no more than 5 deg; the steps should, however, be graded in size. They should be smallest where the drying rate is expected to be slowest and may be larger where the drying rate will be more rapid. In Fig. 8, for

[4] Information sheets containing reproductions of Figs. 6 and 7 on a larger scale, and more detailed information regarding the use of drying nomographs, may be obtained from the authors upon request.

TABLE 1 TIME REQUIRED IN EACH STAGE OF DRYING

Step No.	T	Air temperature, deg F	$(t_d - t_w)$, deg F	Time required for step, hr	Drying time, hr
	3.30				0.00
1		124.5	34.5	0.53	
	2.33				0.53
2		133	43	0.50	
	1.48				1.03
3		140	50	0.47	
	0.84				1.50
4		145	55	0.53	
	0.40				2.03
5		148	58	0.53	
	0.20				2.56
6		150	60	0.73	
	0.10				3.29

example, the smallest steps are shown at the hot end of the tunnel, since that will be the slowest-drying section of the tunnel under these particular conditions. Each step of Fig. 8 may be evaluated from the nomograph in Fig. 6 to give the time required for each, as shown in Table 1.

The drying curve of moisture content versus drying time is now established and is shown in Fig. 9. For values of L_0 and V other than those selected, the drying times may be multiplied by the proper coefficients from Fig. 3, i.e.,

$$\theta \text{ (at } L_0, V) = \theta_r \cdot f(L_0) \cdot f(V)$$

The values of $f(L_0)$ and $f(V)$ are those corresponding to L_0 and V, selected from Fig. 3. Note, however, that no coefficient is required for drying below $T = 0.10$.

Equations relating capacity with drying time, tray surface, and tray loading and relating the temperature drop in a tunnel with moisture content, capacity, and air circulation are available (7). Combination of these equations with the nomographs presented in this paper permits the solution of problems ranging from the equipment requirements to obtain a certain capacity to a prediction of the performance of an existing piece of equipment.

LIMITATIONS OF DATA

A number of limitations must be involved in the use of data such as are presented in this paper. Differences in variety, maturity, and storage history have some significance. The temperature at which potatoes are riced is of considerable importance. While very hot, the potatoes may be riced with a light pressure,

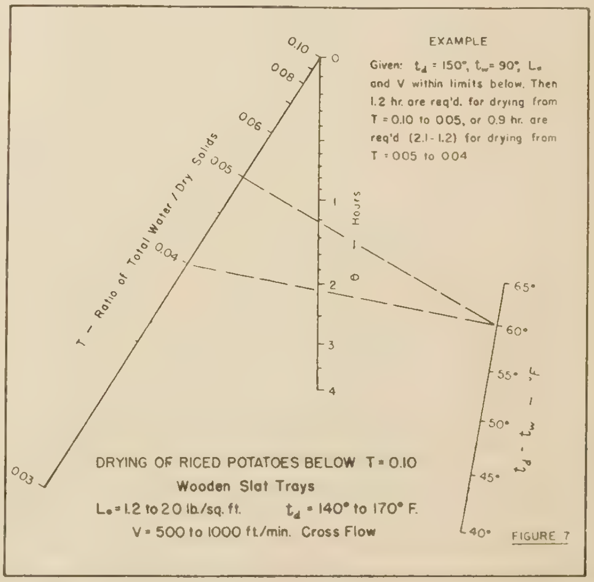
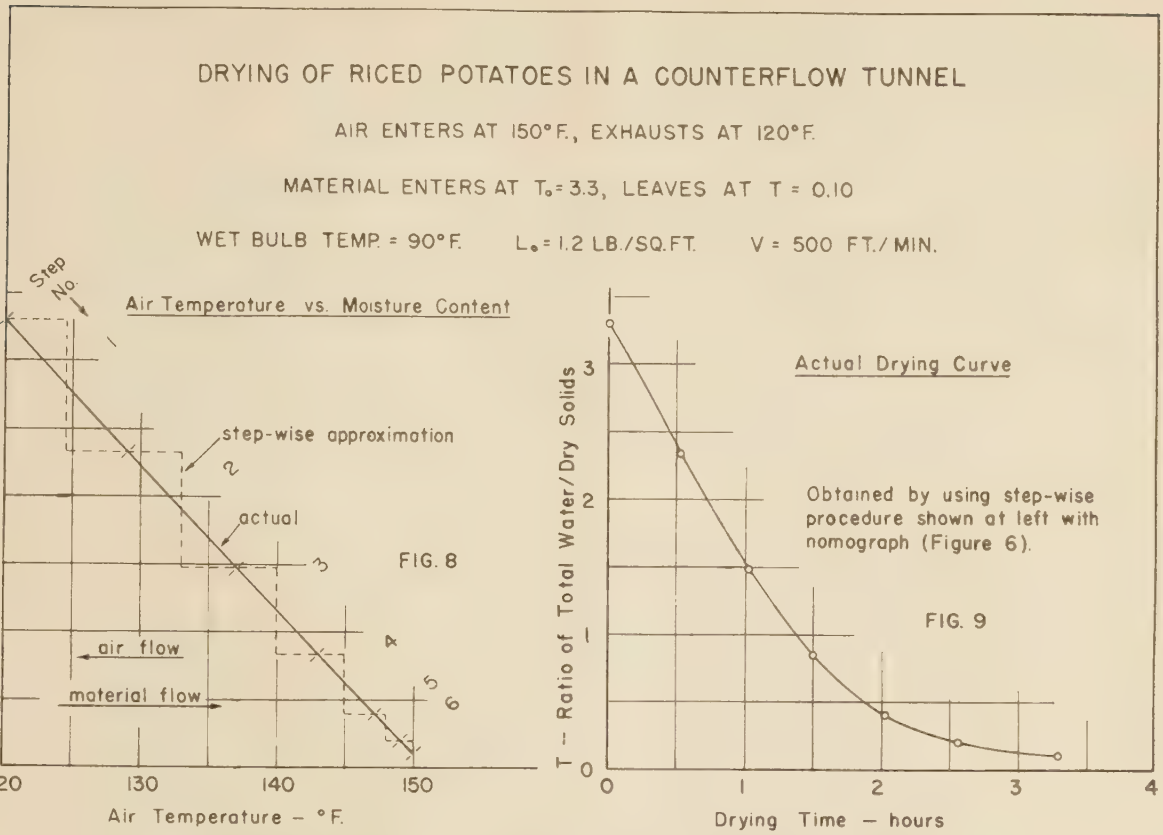


FIG. 7[4] NOMOGRAPH FOR MOISTURE RANGE T = 0.10 TO T = 0.03



FIGS. 8 AND 9 DRYING DATA FOR RICED POTATOES IN A COUNTERFLOW TUNNEL

yielding an open-grained string. After cooling, ricing is much more difficult, and the higher pressure required produces a dense, slower-drying string. A material such as riced potatoes is very difficult to load evenly on a tray, and any attempt to spread the material results in a destruction of the desirable, fast-drying open structure of the loosely piled strings. Consequently, the data presented herein express the behavior of riced potatoes under different drying conditions but must be used with full recognition of the factors which cannot be fully controlled.

ACKNOWLEDGMENTS

The authors are indebted to the following members of the staff of the Western Regional Research Laboratory: W. B. Van Arsdel, who supervised this work, and L. R. Leinbach, who made the many moisture determinations which were required.

APPENDIX

Supplementing the general information presented in the paper Tables 2 and 3 (page 842) give detailed drying data developed during the investigation, while Fig. 10 shows the tray arrangement used, and Table 4, the physical characteristics of riced potatoes.

TABLE 4 APPROXIMATE PHYSICAL CHARACTERISTICS OF RICED POTATOES

	Initial	Final
Average height of load on tray, in.	0.67	0.43
True specific gravity	1.10	1.54[a]
Apparent specific gravity on tray	0.35	0.11
Percentage of voids	68	93
Approximate decrease in shred diameter, per cent	50	
Ratio of surface area to volume,[b] sq ft/cu ft	32	

[a] True specific gravity of dry material determined by immersion in kerosene. The potatoes were ground to —20 +35 mesh in size.
[b] Potato shreds are shaped like bent rods initially, but dry into rod and ribbon shapes. A tendency toward mealiness produces a greater surface-to-volume ratio than that of a true cylinder, but overlapping of the shreds decreases this ratio.

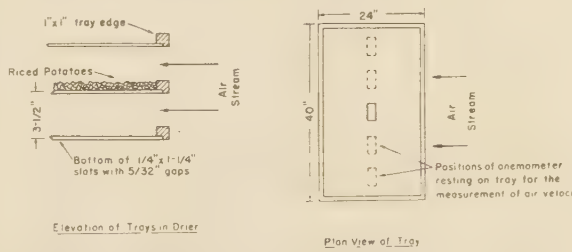


FIG. 10 ARRANGEMENT OF TRAY FOR DEHYDRATING RICED POTATOES

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TABLE 2 DRYING DATA FOR RICED POTATOES FROM T_0 TO $T = 0.10$

Figure No.	1		1		1		1		1		1	
Curve	A		B		C		D		E		F	
t_d (° F.)	150		155		160		130		130		140	
t_w (° F.)	90		105		120		90		105		120	
L_o (lb./net sq.ft.)	1.2		1.2		1.2		1.5		1.2		1.2	
V (linear ft./min.)	500		500		500		500		500		500	
θ Correction (hr.)**	+0.02		+0.03		-0.08		-0.13		+0.07		-0.14	
	θ	T	θ	T	θ	T	θ	T	θ	T	θ	T
Observed Data***	0	3.10	0	3.19	0	3.47	0	3.68	0	3.28	0	3.47
	0.17	2.47	0.17	2.64	0.50	2.33	0.67	2.23	0.60	2.31	0.50	2.86
	0.50	1.56	0.50	1.82	0.83	1.76	0.98	1.75	1.08	1.73	0.92	2.39
	0.83	0.98	0.67	1.50	1.20	1.27	1.55	1.05	1.58	1.26	1.42	1.91
	1.17	0.59	1.00	1.01	1.62	0.85	2.02	0.66	2.08	0.89	1.92	1.48
	1.42	0.41	1.33	0.67	1.83	0.69	2.33	0.46	2.42	0.70	2.50	1.11
	1.58	0.32	1.58	0.47	2.15	0.49	2.78	0.28	2.75	0.54	3.17	0.77
	1.83	0.225	1.83	0.34	2.42	0.35	3.45	0.156	3.08	0.43	3.83	0.53
	2.08	0.165	2.08	0.24	2.67	0.27	3.87	0.120	3.50	0.31	4.25	0.42
	2.42	0.120	2.42	0.17	2.92	0.21	4.41	0.093	3.92	0.23	4.70	0.32
	2.75	0.094	2.75	0.124	3.30	0.145			4.37	0.164	5.23	0.24
			3.12	0.097	3.72	0.098			4.92	0.121	5.88	0.17
									5.42	0.099	6.53	0.132
											7.10	0.108

Figure No.	2		2*		2		2		2	
Curve	A		B		C		D		E	
t_d (° F.)	150		150		150		150		150	
t_w (° F.)	90		90		90		90		90	
L_o (lb./net sq.ft.)	2.0		1.2		1.2		0.6		1.2	
V (linear ft./min.)	500		500		770		500		1060	
θ Correction (hr.)**	-0.02		-0.06		0.0		0.0		-0.03	
	θ	T	θ	T	θ	T	θ	T	θ	T
Observed Data***	0	3.27	0	3.56	0	3.39	0	3.27	0	3.51
	0.52	2.15	0.17	2.88	0.15	2.58	0.17	2.18	0.17	2.51
	1.02	1.45	0.50	1.97	0.45	1.54	0.33	1.47	0.33	1.63
	1.47	1.01	1.00	1.10	0.72	0.97	0.50	0.94	0.50	1.10
	1.92	0.71	1.25	0.80	0.85	0.74	0.67	0.60	0.67	0.70
	2.47	0.46	1.50	0.59	1.02	0.52	0.83	0.39	0.83	0.46
	2.82	0.35	1.75	0.42	1.18	0.37	1.00	0.27	1.00	0.30
	3.25	0.25	2.00	0.30	1.35	0.26	1.17	0.193	1.25	0.16
	3.62	0.182	2.45	0.176	1.60	0.16	1.33	0.152	1.50	0.11
	4.17	0.128	3.00	0.108	1.85	0.098	1.58	0.115	2.00	0.074
	4.83	0.084					2.08	0.087		

* Curve A, Fig. 1 (1941 growing season) is reference run (Curve B, Fig. 2) for curves A and D, Fig. 2. Data marked with asterisk (1942 growing season) are reference data for curves C and E, Fig. 2. These two runs give some indication of the material variations which may be expected.

** θ_o correction obtained by the following process:

(1) First few points of data for each run were plotted linearly as T vs. θ and a smooth curve drawn through them, ignoring the first point.

(2) Intercepts of the curves at θ = 0 were averaged, weighting those in which the "warm-up" period had a minimum effect, i.e., those in which t_w was low. The average value of T_o was 3.3.

(3) The θ_o correction is the correction required to place all runs on a common value of T_o = 3.3.

*** The tabulated values of θ are uncorrected, and the values of T_o are those observed experimentally.

TABLE 3 DRYING DATA FOR RICED POTATOES FROM $T = 0.10$ TO T_f

Figure No.	4		4		4		4		4		5	
Curve	A		A		A		A		B		A	
t_d (° F.)	170		160		150*		150		150		160	
t_w (° F.)	110		100		90		90		90		120	
L_o (lb./net sq.ft.)	1.2		1.2		1.2		1.2		0.6		1.2	
V (linear ft./min.)	500		500		500		500		500		500	
	θ	T	θ	T	θ	T	θ	T	θ	T	θ	T
Observed Data	0	0.10	0	0.10	0	0.10	0	0.10	0	0.10	0	0.10
	0.28	0.077	0.24	0.078	0.17	0.086	0.44	0.073	0.26	0.082	0.70	0.072
	0.85	0.054	0.49	0.069	0.58	0.067	0.96	0.056	0.51	0.074	1.11	0.061
	1.35	0.047	0.99	0.054	1.17	0.047	1.44	0.048	0.76	0.074	1.58	0.057
	1.85	0.044	1.49	0.047	1.50	0.046	1.94	0.040	1.26	0.063	2.02	0.052
	2.35	0.042	1.99	0.041	2.00	0.040	2.44	0.036	1.76	0.057	2.50	0.048
	2.85	0.039	2.49	0.037	2.67	0.037	2.94	0.033	2.33	0.047	2.98	0.045
	3.21	0.034	3.16	0.032	3.17	0.034	3.77	0.030	3.09	0.041		
									3.76	0.033		

Figure No.	5		5		5		5		5	
Curve	B		B		C		C		C	
t_d (° F.)	140		170		150		150		150*	
t_w (° F.)	90		120		90		90		90	
L_o (lb./net sq.ft.)	1.2		1.2		2.0		1.2		1.2	
V (linear ft./min.)	500		500		500		1060		1060	
	θ	T	θ	T	θ	T	θ	T	θ	T
Observed Data	0	0.10	0	0.10	0	0.10	0	0.10	0	0.10
	0.18	0.088	0.24	0.079	0.12	0.094	0.10	0.089	0.40	0.074
	0.68	0.067	0.74	0.066	0.62	0.068	0.35	0.068	0.90	0.061
	1.51	0.052	1.14	0.052	1.12	0.055	0.60	0.063	1.40	0.049
	2.26	0.044	1.74	0.048	1.62	0.048	1.10	0.055	1.90	0.044
	2.71	0.041	2.24	0.045	2.12	0.043	1.60	0.048	2.40	0.041
			2.74	0.040	2.79	0.035	2.35	0.040	2.90	0.036
			2.91	0.038			3.27	0.034	3.40	0.034
									3.90	0.031
									4.40	0.030

* Note: All runs in this table were made at t_d = 130°, t_w = 90°, V = 500 ft./min. cross flow from T_o to T = 0.2 approx. and finished as indicated except those marked with an asterisk. The latter were dried as indicated throughout.

A Thermal Anemometer for Low Velocity Flow

By R. A. SEBAN,[1] W. H. HILLENDAHL,[2] E. J. GALLAGHER,[3] and A. L. LONDON[4]

A thermal anemometer for low velocity flow, which indicates air velocities in terms of the cooling rate of a heated sphere, is described in this paper. The calibration presented reveals an adequate consistency for velocities from 15 fpm to 3000 fpm. Beyond 100 fpm, free-convection effects become small and the instrument indication is substantially nondirectional because of its spherical form(4) (5).[5] Operation with transient conditions enables the use of small temperature differences, so that free-convection effects may be made small. Further, heater input need not be controlled or metered, as is necessary with steady-state thermal anemometers (6).

Nomenclature

The following nomenclature is used in the paper:

A = sphere surface area, sq ft
c = unit heat capacity (sphere), Btu/(lb °F)
c_p = unit heat capacity at constant pressure (air), Btu/(lb °F)
C = sphere thermal capacity, Btu/°F
D = sphere diameter, ft
G = air mass velocity, lb/(min ft²)
h = film conductance for thermal convection, Btu/(hr ft² °F)
k = thermal conductivity (air), Btu/(hr ft² °F/ft)
k_m = thermal conductivity (sphere), Btu/(hr ft² °F/ft)
M = mass of the sphere, lb
qt = hermal current, Btu/hr
R = sphere surface resistance, (hr °F)/Btu
t = temperature of the sphere (air datum), °F, μv
T = absolute air temperature, deg R
T_0 = standard air temperature, 535 R
V = air velocity, fpm
ρ = air density, lb per cu ft
ρ_0 = standard air density, 0.075 lb per cu ft
τ = time, hr
μair = viscosity, centipoises
μv = microvolts thermocouple response

Dimensionless moduli:

$\frac{hD}{k}$ = Nusselt number

$\frac{GD}{\mu}$ = Reynolds number

$\frac{h}{Gc_p}$ = heat-transfer group

Consistent units must be employed to secure the nondimensionality assumed for these moduli.

Objectives of Investigation

The objectives of this paper are as follows:

1 To describe the thermal anemometer for low velocity flow and to present a calibration for it.

2 To present an analysis of the ideal system and thereby indicate the method of design of such an instrument for a desired response.

Description of the Instrument

The primary element of the anemometer consists of two bronze spheres (commercial ball bearings), one containing an electric heater, and each having a single junction of a copper-constantan thermocouple circuit imbedded within it. The secondary element is a potentiometer for the measurement of the difference in temperature between the spheres as indicated by the thermocouple. Fig. 1(a) shows the spheres in detail.

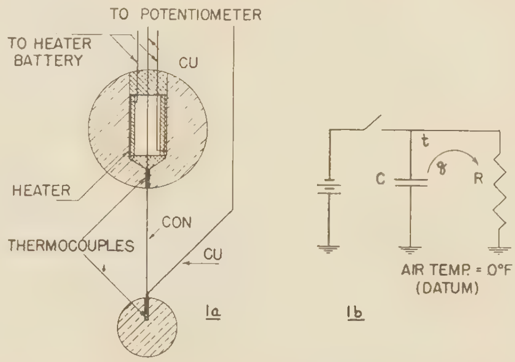

Fig. 1 Anemometer Detail and Thermal Circuit
(For dimensions and physical constants see Table 1.)

The larger sphere of 1 in. diam contains a heater coil of No. 28 constantan wire, having a resistance of 5.7 ohms. External connection of this heater, through a switch to a 3-v source, provides a heating current of sufficient intensity to raise rapidly the temperature of the sphere above that of the air around it. Elimination of this heating current is followed by cooling of the sphere and it is the measurement of the rate of this cooling which gives an indication of the air velocity.

The small $^1/_2$-in-diam sphere provides sufficient thermal capacity so that high-frequency air-temperature fluctuations do not affect the average air temperature as indicated by the thermocouple within this sphere. The small sphere has no other function and the subsequent discussion will be of the anemometer primary element as consisting of the heated sphere alone.

The temperature of the heated sphere is measured with respect to air temperature as datum and it is stated either in degrees F, or in the microvolt response of the copper-constantan thermocouple. The cooling range used is sufficiently small so that the thermocouple electromotive force, microvolts per degree Fahren-

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[5] Numbers in parentheses refer to the Bibliography at the end of the paper.

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heit, is constant over the range. Thus the temperature ratio in microvolts is sensibly identical to the temperature ratio in degrees Fahrenheit.

A Leeds and Northrup portable precision potentiometer is employed as the secondary element of the anemometer.

In operation, the heater is used to raise the sphere temperature to about 15 F (above air temperature), at which point the heater circuit is opened and the sphere begins to cool. The potentiometer is set at 300 μv (about 13 F) and timing is begun at the null point. A second potentiometer setting is made at 150 μv and the timing period is ended when the null point is obtained. Application of the measured cooling rate, expressed as $\frac{\Delta \log t}{\Delta \tau}$, to the anemometer calibration yields the air velocity (Figs. 5, 6).

The Ideal System

Assumption of negligible thermal resistance within the sphere, with all thermal resistance residing in the air film external to the sphere, yields an ideal system for which performance is readily predicted. This assumption is valid when the thermal resistance of the average internal-conduction heat-flow path (approximately $D/4$) is less than 10 per cent of the surface resistance, i.e., $\frac{hD}{4k_m} < 10$ per cent. For the sphere in question, the surface conductance is always less than $h = 20$ in the range of calibration, and for this value $\frac{hD}{4k_m} = 7.6$ per cent.

The ideal system thus obtained, Fig. 1(b), is composed of the thermal capacitor, C (the sphere), a thermal resistance, R (the air film), and a source of energy for charging the capacitor (the heater of the actual system). A switch is provided so that the source can be removed from the circuit. Electrical symbolism is employed for convenience.

With the source in the circuit there is a thermal current into the capacitor C, which produces a temperature rise. When the switch is opened the temperature drops as the energy stored in the capacitor flows out through the resistance. An energy balance on the capacitor for these conditions indicates that the thermal current through the resistance is equal to the rate of change of energy storage in the capacitor. This current is, in turn, defined by the rate equation as the quotient of the temperature difference and the resistance.

Thus from the energy equation

$$q = -\frac{d(Ct)}{d\tau} \quad \text{[1]}$$

and from the rate equation

$$q = \frac{t}{R} \quad \text{[2]}$$

where $C = Mc$ and $R = \frac{1}{hA}$.

Together, Equations [1] and [2] yield

$$-\frac{d(Ct)}{d\tau} = \frac{t}{R} \quad \text{[3]}$$

Sphere capacity C is not a function of t for the small temperature range involved and thus

$$-\frac{\frac{dt}{t}}{d\tau} = -\frac{d(\log t)}{d\tau} = \frac{1}{CR} = \frac{hA}{C} \quad \text{[4]}$$

The magnitude of h, the effective film conductance, is a function of the slope of the cooling curve when that curve is plotted on log t, τ co-ordinates.

When h is not a function of t, Equation [4] integrated with boundary conditions of

$$t = t_0 \quad \text{at} \quad \tau = 0$$

$$t = t \quad \text{at} \quad \tau = \tau$$

yields

$$\frac{\log \frac{t_0}{t}}{\tau} = \frac{1}{RC} = \frac{hA}{C} \quad \text{[5]}$$

This indicates that the cooling curve of the ideal system is a straight line on log t, τ co-ordinates and that the slope of this line depends upon the conductance h for given sphere dimensions.

The performance of an ideal system having the actual system properties (Table 1) may be obtained if it is assumed that the conductance h is a function of air velocity only. Then Equation [5] may be combined with an equation empirically defining the conductance to obtain the predicted performance. This will relate cooling rate to air-mass rate of flow. For forced convection from spheres, McAdams (1) recommends the equation

TABLE 1 PHYSICAL CONSTANTS FOR THE HOT SPHERE

Surface area, A, sq ft	0.0218
Diameter, D, ft	0.0833
Mass, M, lb	0.136
Unit heat capacity, c, Btu/(lb °F)	0.087
Sphere capacity, C, Btu/°F	0.0118
Thermal conductivity, k_m, Btu/(hr ft² °F/ft)	55

$$\frac{hD}{k} = 0.33 \left(\frac{DG}{\mu}\right)^{0.6} \quad \text{[6]}$$

This may also be written, with $\frac{c_p \mu}{k} = 0.74$, as

$$\frac{h}{Gc_p} = 0.45 \left(\frac{DG}{\mu}\right)^{-0.4} \quad \text{[7]}$$

Equation [6] becomes, for air at 75 F

$$h = 0.378 \frac{G^{0.6}}{D^{0.4}} \quad \text{[7a]}$$

so that combination of Equations [5] and [7a] will yield prediction of the anemometer performance.

Calibration of Anemometer

Calibration of the anemometer for velocities to 1000 fpm was made by means of a rotating-boom mechanism which carried the anemometer through a tunnel, Fig. 2. The tunnel was necessary for the elimination of extraneous air currents. In the range from 900 to 3000 fpm calibration was accomplished in the jet of a 6-in. wind tunnel, Fig. 3. Air velocities were determined from radius and speed-of-rotation measurements for the rotating boom and by a pitot tube for the tunnel jet. Rotating-boom air velocities were determined within 2 per cent; pitot-tube determinations were less accurate.

Operation of the instrument was as previously described, except that intermediate temperatures were determined as a function of time (null-point time values at 250, 200, and 150 μv) after the start of the timing at 300 μv. The substantial linearity of the response curves obtained (some are shown in Fig. 4) justifies the assumptions of negligible thermal resistance within the sphere, and an effective surface conductance a function of velocity only,

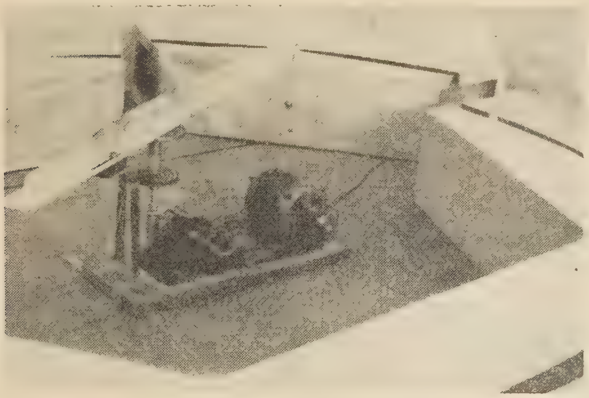

Fig. 2 Rotating Boom and Tunnel

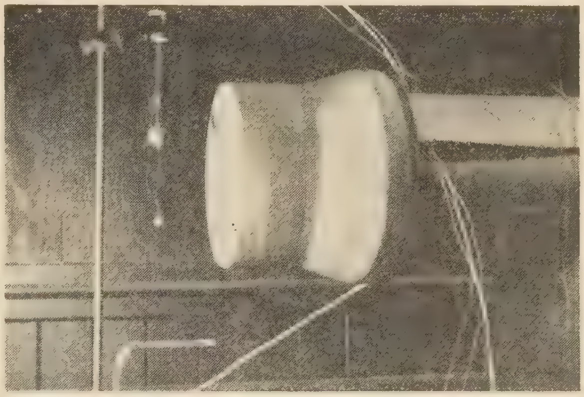

Fig. 3 Nozzle of 6-In. Wind Tunnel
(For calibration, the spheres were mounted more centrally than shown.)

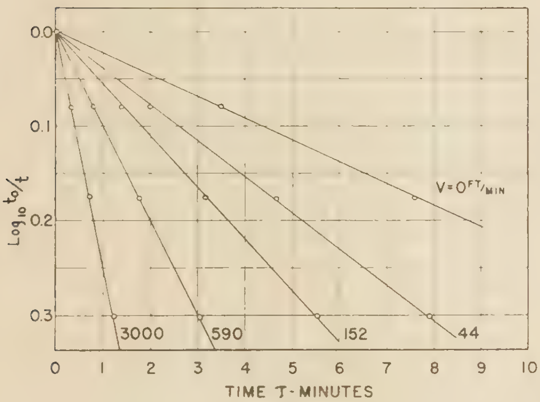

Fig. 4 Cooling Curves for Anemometer Calibration
(Air temperature 75 F.)

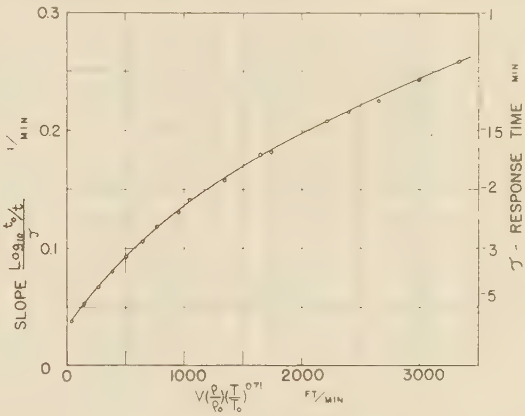

Fig. 5 Calibration Curve
(V is the air velocity with conditions ρ and T. For the calibration conditions, air at 75 F: ρ_0 = 0.075 lb per cu ft; T_0 = 535 deg R. The right-hand ordinate gives response time for a temperature decrease from 300 μv to 150 μv.)

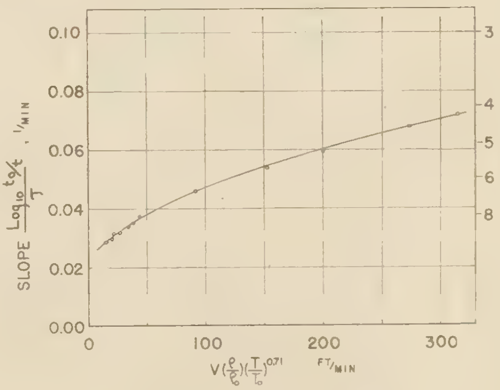

Fig. 6 Calibration Curve
(This is an enlargement of the lower range of Fig. 5.)

as were made for the ideal system. This is expected since the radiation and free-convection effects are relatively small for the small change in sphere temperature used.

The calibration curve of the anemometer, Figs. 5 and 6 (Fig. 6 reveals in more detail the calibration for the lower velocities), presents these slopes as a function of the air velocity. The abscissa of the calibration curve is the velocity multiplied by a factor $\left(\frac{\rho}{\rho_0}\right)\left(\frac{T}{T_0}\right)^{0.71}$ which compensates for variation of air density and temperature from calibration conditions. The factor is derived on the basis of a film-conductance variation as described by Equation [8], arranged in the form of Equation [7], which assumes a constant Stanton's number. It will become less applicable when free convection contributes significantly to the effective conductance ($V < 100$ fpm). In this range calibration should be made for pressure and temperature conditions similar to those under which the anemometer is to be employed.

Since the linearity of the response curves, Fig. 4, indicates a close approximation of actual to ideal system behavior, discrepancies between predicted and experimental behavior of the anemometer will be due to differences between the actual and the empirically defined film conductances. Such a comparison is made in Fig. 7, where film conductances derived from the experimental data and Equation [5] are compared with those indicated by Equation [6] and with those indicated by another equation (2)

$$h = 0.336 \frac{G^{0.52}}{D^{0.48}} \qquad [8]$$

as obtained by the transient method for similar spheres in the same wind tunnel. The correlation of the experimental data from 100 to 3000 fpm is within 25 per cent with the McAdams equation and is much better with Equation [8].

Errors

The calibration curve, Figs. 5 and 6, represents the data points with an accuracy of 5 per cent and this accuracy is to be expected

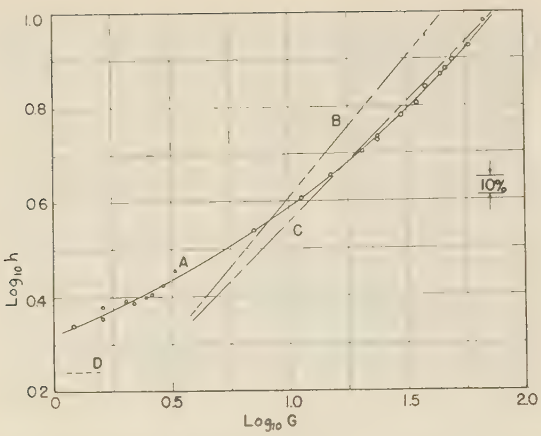

Fig. 7 Comparison of Experimental and Available Sphere-Conductance Data

(For air at 75 F ρ = 0.075 lb per cu ft, μ = 0.018 centipoises.)

Curve A Experimentally determined film conductances
Curve B Evaluation from Equation [7a] (1)
Curve C Evaluation from Equation [8] (2)
Curve D Magnitude of experimental "free-convection" conductance
Units
$[h]$ = Btu/(hr ft² °F)
$[G]$ = lb/(min ft²)

from the instrument when it is used under conditions similar to those of the calibration. This similarity requires the same air-flow and radiation conditions and a potentiometer of adequate sensitivity.

Reproduction of air-flow and radiation conditions is necessary at low velocities because the effective film conductance, as experienced by the sphere, is composed of radiation and convection conductances acting in parallel. The convection conductance is in turn affected by free-convection velocities as well as the velocity to be measured. Provided that the cooling range is not greater than 13 F and the sphere "sees" surroundings which are substantially at air temperature, the radiation conductance does not exceed 5 per cent of the total conductance in any case and becomes less than 2 per cent at velocities in excess of 100 fpm. Operation of the anemometer under conditions in which the sphere would "see" surfaces at temperatures appreciably different from that of the air would change the effective conductance and so produce error with respect to a calibration not obtained for such conditions.

The measured "free convection" ($V = 0$) conductance was $h = 1.74$ Btu/(hr ft^2 ° F), a value 75 per cent in excess of the empirically defined magnitude. Similarity is not expected because velocity distributions for the transient case are not similar to those for steady state, nor were air conditions completely static when this magnitude was obtained. At velocities below 100 fpm, the relative contribution of free convection to the total conductance is sufficient to require retention of the same cooling range and horizontal air flow (as in the calibration), if the calibration is to be used with confidence. At higher velocities this effect becomes small and the instrument calibration should be independent of the direction of flow.

Since timing is made in terms of null-point measurements on the potentiometer, and a cooling range of 150 μv is employed, an instrument having a sensitivity and accuracy of 5 μv is required. With high cooling rates, greater sensitivity is desirable, but since both initial and final time measurements are taken with the temperature decreasing, errors due to the length of the galvanometer period of oscillation and galvanometer insensitivity tend to cancel. Use of a larger cooling range would enable utilization of a less sensitive and less accurate potentiometer, but free-convection effects would become more significant. Installation of a multiple-junction thermopile instead of the single-junction difference couple would be preferable.

For low velocities and the attendant low cooling rates, use of a potentiometer of higher sensitivity would reduce the time required for measurement and also minimize the free-convection effect, since lower maximum temperatures could be employed.

Applications

The anemometer described is useful for the measurement of low air velocities, such as are encountered in air-conditioning and refrigeration practice. It operates with a smaller temperature difference between instrument and air than do most steady-state thermal meters and thus minimizes directional errors due to free-convection effects.

The reasonable check of experimental and predicted film conductances suggests the method for the direct determination of these conductances when the velocity is above 100 fpm. There are many cases where the conductances are of primary, and the velocities of secondary, importance (4) and such a direct determination would be possible if the objects under consideration could be reproduced in a form satisfying the ideal system requirements (3).

Conclusions

1 A thermal anemometer has been described and a calibration has been given for the range 15 to 3000 fpm.

2 The ideal system analysis, in conjunction with available conductance data, provides a basis for the design of such an instrument for a desired response.

Acknowledgment

The authors gratefully recognize the contribution of the Safeway Stores, Inc., Fellowship, which served to encourage this work.

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Investigation of Large Diesel-Engine Wrist-pins, Pistons, and Crankcase Explosions

By F. E. FAAST,[1] TAMPA, FLA.

The causes of crankcase explosions in large Diesel engines have been somewhat obscure, although it has been suspected that wristpins might be the principal source. This paper contains the results of a research undertaken to solve certain wristpin and piston difficulties in large marine Diesels which had experienced disastrous explosions of this nature. Experiments were carried out on slotted-end wristpins having 0.001 in. clearance in the piston bosses, for the purpose of studying the effects of different equivalent amounts of wristpin and boss interference on (*a*) piston deflections; (*b*) strained shape of the boss; (*c*) holding power in boss. The possibility is demonstrated that large wristpin deflections in fixed-end wristpins can cause excessive bearing pressures and temperatures, rupture of oil film, large skirt deflections, all of which constitute explosion hazards and may cause rupture of the piston skirt. It is suggested that these and other difficulties noted in the paper could be eliminated by the use of full-floating wristpins with suitable piston skirts.

WRISTPINS have long been suspected as the source of most large Diesel-engine crankcase explosions. The exact relation between the wristpin, the piston, and the conditions immediately responsible for the explosion has been more or less obscure. The purpose of this paper is to present some findings of a research undertaken to solve certain wristpin and piston difficulties in large Diesels in some of which crankcase explosions had been experienced. The engines were of 21 in. bore, 4 cycle, with trunk pistons.

The wristpins in these engines were originally force-fitted in the piston bosses. This caused the pistons to distort considerably out of round and frequently caused excessive scoring in the bosses. It was found on disassembly after long operation that the piston-boss ends of the wristpin were tapered about $^1/_{64}$ in. per ft, caused apparently by scraping and burnishing during pressing-in at assembly. However, when new wristpins were made up with this amount of taper, even though force-fitted and securely doweled to the boss, they loosened after unreasonably short periods of operation.

Experiments With Slotted-End Wristpins

Experiments were conducted with slotted-end wristpins having 0.001 in. clearance in the piston bosses. These were secured in the bosses by drawing together two cone-shaped plugs in the ends of the wristpin, by means of a drawbolt, in order to expand the pin ends into the bosses, as shown in Fig. 1. This also proved an ideal method for studying the effects of different equivalent amounts of wristpin and boss interference on (*a*) piston deflections, (*b*) the strained shape of the boss, and (*c*) the holding power of the wristpin in the boss, or its resistance to turning due to rubbing on the journal. Several thousand precision measurements were taken at many predetermined points on the piston, as a basis for the graphs accompanying this paper and the conclusions reached.

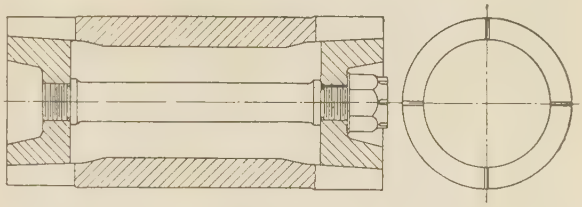

Fig. 1 Expanding Pin Ends Into Bosses by Means of Cone-Shaped Plugs Drawn Together by Drawbolt

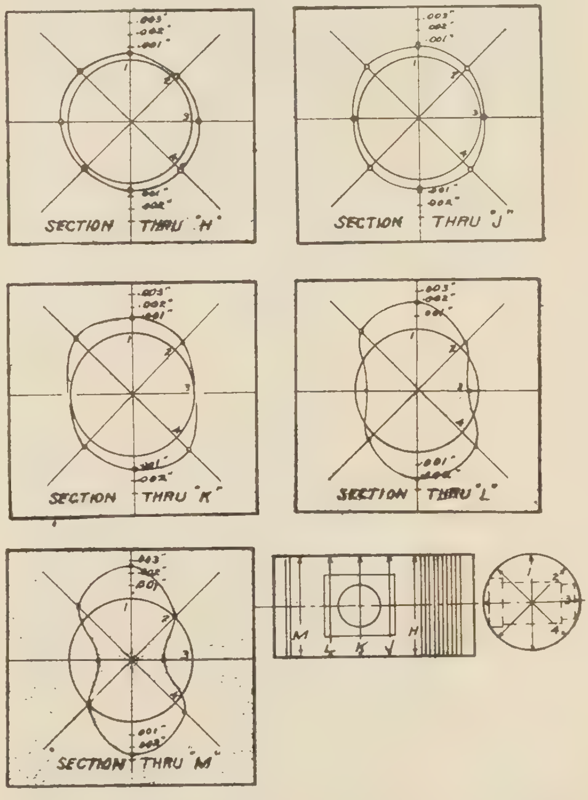

Fig. 2 Deflection of Piston Caused by Expansion of Bosses; Plotted on Circumference
(For plug, pull-in = 0.130 in.)

This slotted-end wristpin and drawbolt arrangement when tightened produced deflections in the piston skirt which were exactly similar to those produced in pistons by straight-end force-fitted wristpins. These distortions are shown in Fig. 2 and are considered highly significant, for the following reasons:

1 They show that the pressing-in operation of force-fitted

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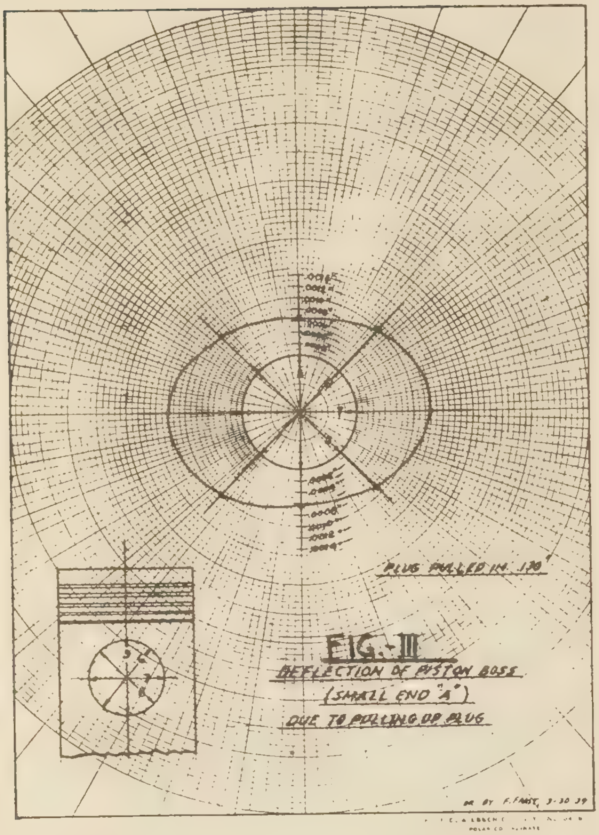

FIG. 3 DEFLECTION OF PISTON BOSS DUE TO TIGHTENING CONE-PLUGS
(Small end A.)

wristpins widely used in large trunk-piston engines has much less, if any, effect on piston deformation than is popularly supposed. The usual assumption has been that the opposite sides of the piston skirt were squeezed together in the pressing operation, causing the skirt to ovalize in a manner similar to the exaggerated shape in Fig. 2 (*e*). Elimination of the pressing-in operation proved that the hoop stresses alone, and the resulting expansion of the bosses, caused the skirt deformations shown in Fig. 2. Consideration of that portion of the boss hoop stresses, acting in planes approximately parallel to the piston rings, would indicate that this would necessarily be so. The hoop stresses parallel to the piston axis, however, cause no skirt deflection except lengthening of the skirt. The strained shape of the boss, Fig. 3, confirms this.

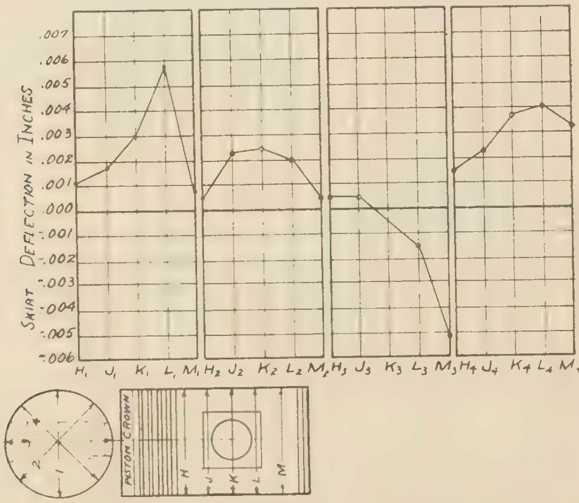

FIG. 4 DIAMETRAL DEFLECTION OF PISTON SKIRT CAUSED BY EXPANSION OF BOSSES
(Plotted for axial elements of piston wall. Average boss expansion = 0.0023 in.)

2 Fig. 2 (*c*) merits special study. This is the deflected shape of the piston periphery in a section through the wristpin and parallel to the piston rings. It will be noted that here high spots make an appearance on the periphery at about 45 deg from the wristpin axis, or roughly just outside the piston bosses. These high portions do not appear very prominent, but after the wristpin-pressing operation, when the skirt took a shape somewhat like Fig. 2 (*e*), exaggerated, the piston skirt was supposed to be rounded up again by hammering simultaneously the humps on the opposite sides, until diameters 1 and 3 were equal. This is regular shop-assembly procedure for pressed piston pins for large engines. Actually, this operation aggravates the formation of high spots at the wristpin level at planes 2 and 4 and goes unnoticed because the diameters are ordinarily checked after "rounding up" only at points parallel and perpendicular to the wristpin. The condition is again further aggravated during operation of the engine, as the wristpin expands longitudinally with operating temperatures, carrying the bosses and adjacent high spots toward the cylinder walls. The strained shape of the piston along axial elements is shown in Fig. 4.

The resistance of the wristpin to turning in the bosses, for different equivalent interferences, was measured with the aid of the rig shown in Fig. 5. The twisting arm was a heavy, specially built clamp-on wrench, secured to the wristpin by a heavy steel through-dowel.

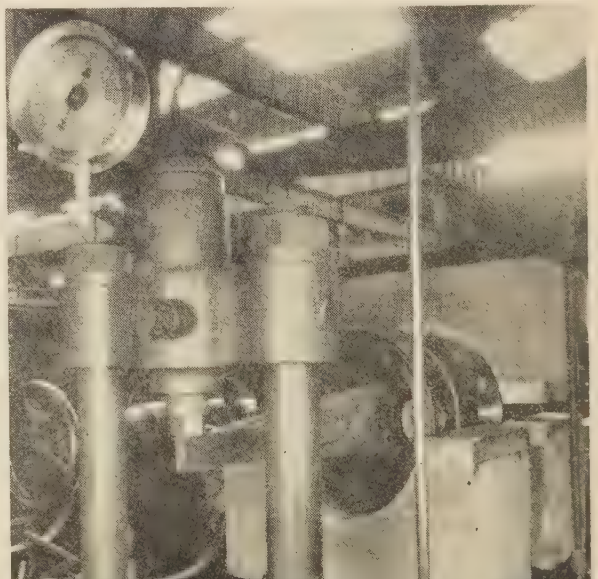

FIG. 5 RIG FOR MEASURING RESISTANCE OF WRISTPIN TO TURNING IN BOSSES

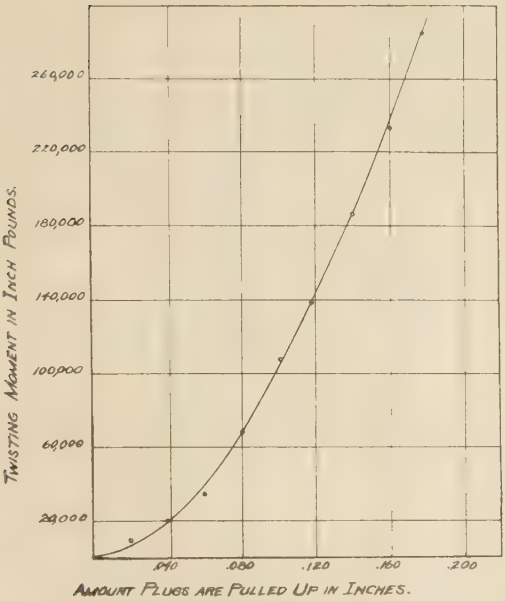

FIG. 6 TURNING MOMENT REQUIRED TO TWIST WRISTPIN IN BOSSES

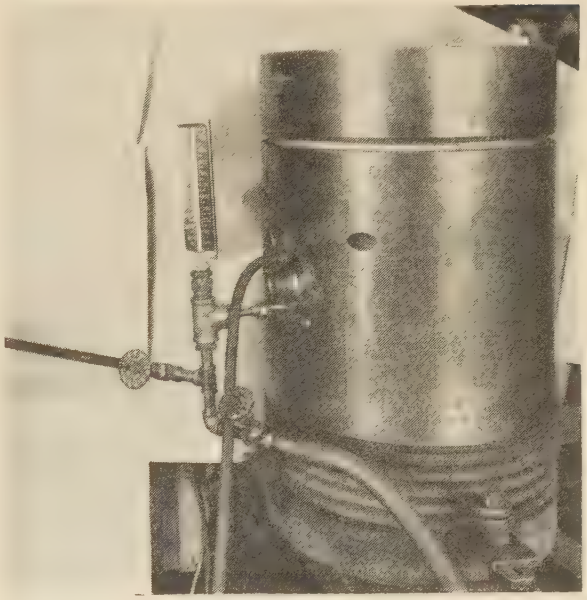

FIG. 7 APPARATUS FOR HEATING SLOTTED-END WRISTPINS TO STUDY DEFLECTIONS IN PISTON SKIRT

In Fig. 6 are plotted the twisting moments which were required to turn the wristpin. Piston iron deflects about twice as much as wristpin steel for equal pressures, since the elasticity moduli vary inversely in approximately that proportion. For a wristpin interference of 0.003 in., therefore, and with pin-wall thickness equal to or slightly less than boss thickness, the boss expands approximately 0.002 in. in diam. For this amount, a moment of 160,000 lb-in. was required to initiate turning in the bosses. In this connection, if engine operating conditions are fair, with a frictional coefficient of 0.008 (Marks), then the tangential frictional moment will be

$$M = fpA\,\frac{d}{2} = 8400 \text{ lb-in.}$$

where f = coefficient of friction; p = cylinder pressure, 800 psi; A = piston area; d = wristpin diameter.

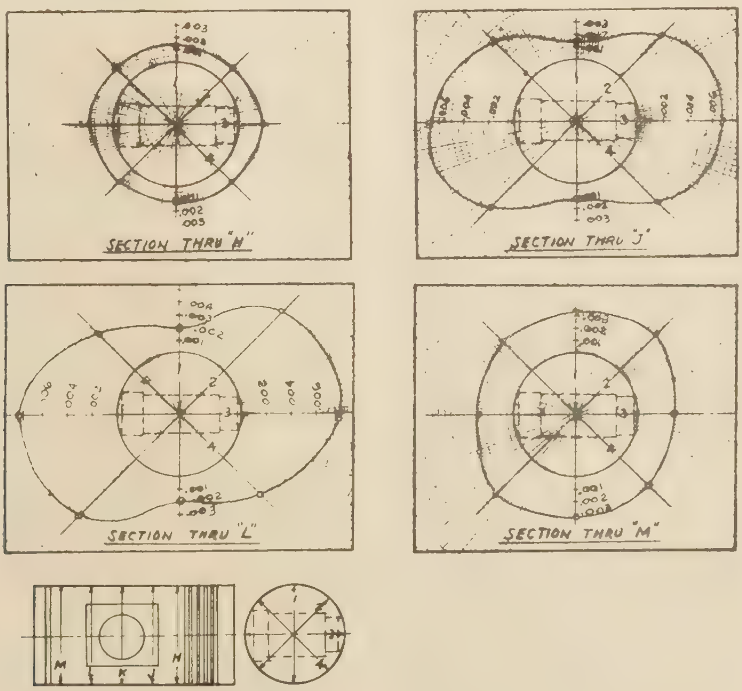

FIG. 8 DEFLECTION OF PISTON PLOTTED ON CIRCUMFERENCE FOR WRISTPIN DIFFERENTIAL-TEMPERATURE RISE OF 119 DEG. F

$$\text{Factor of safety} = \frac{160{,}000}{8400} = 19.1$$

The 0.003-in. interference between the wristpin and boss is, therefore, ample.

Thermal Expansion of Wristpin

It was felt that the effect on piston distortion of possible sudden temperature rises during engine operation, in fixed-end wristpins, whether due to temporary failure or reduction of lubrication, or other reasons, might be profitably scrutinized.

For this purpose the slotted-end wristpin, with tapered draw-plugs was assembled in the piston and heated with hot water. This was accomplished by drilling and tapping the taper plugs for pipe connections for admitting and discharging water from inside the pin. The water temperature was controlled by mixing steam with cold water in the supply pipe. Several hundred precision dimensions were taken in order to study the deflections of the piston skirt, bosses, and wristpin, for various temperature differentials. The apparatus is shown in Fig. 7.

It is pointed out here that the conditions studied under this heading refer to that expansion of the wristpin and piston which is due only to a differential temperature rise above the normal operating temperatures of the wristpin. The conclusions which follow would apply, for example, if the engine were operated until average pin and piston temperatures become constant, and then some sudden rise in temperature occurred in the wristpin, from whatever cause. In this connection, the following comments are offered:

1 Fig. 8 shows the resulting deformations at different levels on the piston skirt. The maximum piston deformation, caused by differential heating of the wristpin, occurred near the pin ends at the skirt undercuts. This amounted to about 0.0136 in. per 100 F wristpin temperature rise. Of course, much of this expansion is absorbed in the skirt undercuts and is usually prevented thereby from becoming dangerous. The expansion parallel to the wristpin decreased toward the bottom of the skirt. The expansion below the undercut (at $M3$) was about 0.005 in. per 100 F temperature differential, or approximately one third the amount within the undercut.

2 At 45 deg to the wristpin axis, the skirt deformation amounted to +0.008 in. per 100 F rise. This value was noted just outside the skirt undercuts and about level with the bottom of the undercut. This particular deformation is considered highly undesirable and dangerous, especially in the light of the condition that, unlike skirt deformations occurring at points well removed from the bosses, its proximity thereto renders it rigid and incapable of flexing to conformity with the liner shape. It should be noted that most cases of piston seizures and cylinder scoring, as far as could be determined from the experience of operators of all types of engines, have occurred in the planes described in this section.

3 At M_1 near the bottom of the skirt and perpendicular to the wristpin, the piston took a diametral increase up to 0.0055 in. per 100 F rise. This was contrary to expectations, as increase of the piston diameter in line with the pin should ordinarily cause shortening of the diameter perpendicular to the pin. The paradox can be readily understood, however, by virtue of the fact that, as previously explained, circumferential expansion of the piston bosses will cause such a skirt deformation, and in this instance, the taper plugs and bosses were caused to expand by conducted heat from the hot water in the wristpin.

The customary pressed or expanded-end wristpin designs used in large trunk pistons, as has been shown, creates high spots along axial skirt elements just outside of the outer boss diameter, or undercut. Comparatively small wristpin temperature-differential rises can then spread the rigid bosses apart and cause immediate scoring, or seizure; or, when the scoring shaves off small pieces of cylinder-wall metal combined with sufficiently high friction or burnishing action to cause the pieces to melt or become red hot and drop into the hot crankcase gases, immediate explosion may result. Paradoxically, well-worn-in engines may be particularly vulnerable in this respect, for the following reasons: In multicylinder in-line engines, bellmouthing of the connecting-rod small-end bearing may take place from local connecting-rod misalignments due to:

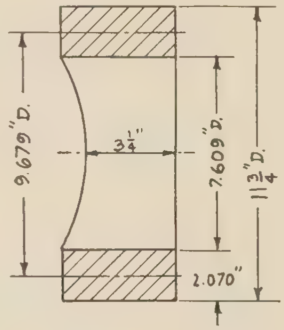

Fig. 9 Detail of Piston Boss

1 Unequal expansion of the engine frame, caused by high heat conditions at the upper end of the structure, as compared with the crankshaft temperature.

2 The lower part of the frame is bolted to the rigid foundations and is not free to expand with the upper structure.

3 In marine applications, weaving of the engine frame with the ship's structure.

4 Local cyclical crankpin misalignments near crankshaft torsional-vibration nodal points, and from gas loading on adjacent cranks.

The eventual effect of bellmouthing of the upper-end bearing is to concentrate the gas loading at the high (center) portion of the bearing, with bearing pressure and temperature rising with increasing bellmouth taper. From a study of the section through M, Fig. 8, it is apparent that even with piston-skirt undercuts, or reliefs in the skirt surface, dangerous high points will appear just outside or underneath the corners of the undercuts. At elevated wristpin operating temperatures, these points are doubly dangerous.

If the piston boss is assumed for the sake of simplicity to have the dimensions shown in Fig. 9, and the approximate amount of expansion of the boss caused by pressing-in the wristpin, on the diameter, is taken at 0.002 in., then by Timoshenko's thick-cylinder formula[2] the increase of the inner radius is

$$d_1 = \frac{aq_i}{E}\left(\frac{b^2 + a^2}{b^2 - a^2}\right) + \frac{1}{m}$$

where a = inner radius; b = outer radius; q_i = internal pressure; E = elastic modulus, 19,000,000; $\frac{1}{m}$ = Poisson's ratio for cast iron. Then the internal pressure

$$q_i = \frac{Ed_1}{a\left(\frac{b^2 + a^2}{b^2 - a^2}\right) + \frac{1}{m}} = 1840 \text{ psi}$$

[2] Bibliography at the end of the paper, reference (4), p. 252.

and the tensile stress in the boss

$$P_\Phi = \frac{a^2 q_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

where, for $r = a$, $P_\Phi = 4500$ psi. (Considered as a thin-walled cylinder, this stress would be 3930 psi.) Similarly, the compressive stress for $r = a$ is $P_r = 1840$ psi.

The compressive boss stress due to gas loading is

$$S_2 = \frac{PA}{2a} = 5600 \text{ psi}$$

where A = piston area, 342 sq in.; P = gas pressure, 800 psi; a = projected area of boss bore, 24.5 sq in. Hence the total stress at the top section of the boss is

$$\text{Total stress} = \sqrt{(5600 + 1840)^2 + 4500^2} = 8640 \text{ psi}$$

This does not take into account the additional stressing due to cantilever action of the wristpin on the boss acting against the skirt wall, the tangential twisting moment of the pin on the bosses during operation, and bellying out of the wristpin at the boss edges during the firing stroke. Under certain conditions, this bellying can easily rupture the boss and skirt.[3] The ultimate stress of the boss and piston iron is about 30,000 psi. The calculated stress, plus these variables, probably makes the true factor of safety less than 2.

As far as the wristpin itself is concerned, if it is considered as a beam supported at the ends, the following load and deflection conditions are possible:

(*a*) For uniform loading and fixed ends, the deflection is $\frac{Pl^3}{384EI}$, where P is the total load

(*b*) For concentrated loading and fixed ends, the deflection is $\frac{Pl^3}{192EI}$

(*c*) For uniform loading with free ends not fixed, the deflection is $\frac{5}{384} \cdot \frac{Pl^3}{EI}$

(*d*) For concentrated loading, with simple ends not fixed, the deflection is $\frac{Pl^3}{48EI}$

These deflections bear the following relation to each other:

$$(a) : (b) : (c) : (d) = 1 : 2 : 5 : 8.$$

For the particular wristpins studied in this case, the deflection in case (*a*) would be

$$\frac{274{,}000 \times 12.75^3}{384 \times 30{,}000{,}000 \times 130} = 0.0004 \text{ in.}$$

For case (*d*), the deflection would be 0.0032 in., which would be excessive in fixed-end wristpins. The actual value, due to gas loading, is somewhere between these points and has been calculated at 0.0014 in. for the fixed-end wristpins under investigation, by assuming the load to be uniformly distributed over only the center half of the connecting-rod upper bearing length, with proper allowance for partial flexibility of the pin-end fixation due to the relatively thin skirt walls.

For this same load distribution and rigidly fixed ends, the maximum fiber stress is

$$S = \frac{Mc}{I} = 19{,}600 \text{ psi}$$

where

$$M = \frac{P}{2}\left(\frac{l}{2} - \frac{l}{4}\right)$$

Streeter and Lichty (5)[4] say this stress should fall below 12,000 psi. Lucke says the stress should not exceed 15,000 psi. In a great many engines, the stresses are well over 20,000 psi when similarly calculated. The actual stresses are believed to be considerably higher than these values, because of the additional twisting stress caused by tangential rubbing of the journal and the tendency of the bearing pressure to become concentrated near the center of the pin as the bearing bellmouths with wear.

Professor Enselin,[5] in carrying out certain experiments for the K. Schmidt & Company interests in Germany, found the gas loads were cracking piston bosses, and ascribed this to the wristpin filling up the running clearance in the connecting-rod bearing by bellying of the pin under load, since this condition always occurred when the piston bosses cracked.

Collier's fine analysis (7) points to a more satisfying explanation. He shows that the wristpin fiber stresses are greatest between the connecting-rod bearing ends and the boss ends, running as much as 8 to 9 times the stress at the center of the pin. If a fixed wristpin is considered as a double-ended cantilever beam, it is easily seen that maximum stress must occur at the point of fixation, or edge of the boss, as he claims, and that the tendency of the boss in constraining the highly stressed wristpin from bellying out is what places the boss in the condition of stress that may rupture it.

Preventing Crankcase Explosions

As the research and experiments summarized by this paper (and carried out in the beginning for the sole purpose of solving certain wristpin operating and manufacturing difficulties) progressed, nearly all signs pointed to a purely coincidental solution of the why of many large engine crankcase explosions, and how they might be prevented. For this reason the wristpin-heating experiment previously referred to was carried out. The following calculations and statements are made to attempt further to definitely link fixed-end wristpin designs to crankcase explosions.

The melting point of wristpin-bearing babbitt is approximately 460 F. The $12^3/_4$ in. length of wristpin journal will expand $12^3/_4 \times 0.073 \times 10^{-4} = 0.000093$ in. per F deg rise, or it will expand 0.001 in. for each rise of $10^3/_4$ deg. The designed piston clearance was 0.014 to 0.017 in. This is much reduced at operating temperatures, since the water-cooled liner will not expand as much as the piston. Even if the operating clearance is taken as high as 0.010 in., this clearance would be destroyed for a temperature-differential rise, from foregoing wristpin expansion figures of only

$$\frac{0.010}{0.008} \times 100 = 125 \text{ deg F}$$

If the wristpin operating temperature is about 225 deg, a rise of 460 minus 225 deg, or 235 deg, would be required to fuse the bearing metal. Piston-seizure tendencies and actual seizure would therefore occur long before fusion of the babbitt.

[3] "Wristpin Calculations for Automotive Diesel Engines," by G. Rothmann, *Zeitschrift des Vereines deutscher Ingenieure*, no. 13, vol. 81, 1937, p. 384.
"Influence of the Material and Wall Thickness of Wristpins on the Strength of the Piston," by Professor Enselin, Bulletin of K. Schmidt & Co., Neckarsulm, Germany, 1935.

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

[5] Footnote 3, second reference.

It is submitted that any appreciable slowing down of engines without change in fuel supply may indicate a hazard and should warrant immediate shutdown and investigation. For example, in the engines studied, a reduction from normal operating speed, corresponding to a 15 per cent reduction of power output without change of throttle setting, may reduce the engine speed 52.5 rpm, and the output approximately 352 hp. This represents a power loss of

$$\frac{352 \times 33{,}000}{778} = 15{,}000 \text{ Btu per min}$$

If this loss is represented by friction at a high spot on the piston, caused by a temporary temperature rise in the wristpin due to, say, temporary partial failure of lubrication or rupture of oil film, the piston-wall area heated thereby may be considered, for the sake of conservatism, to be as large as 80 in. square, in order to offset heat losses through radiation and conduction. The cylinder liner and piston were semisteel.

Density of semisteel = 470 psf
Thickness of liner = 1.098 in.
Thickness of piston skirt = 0.530 in.
Specific heat of semisteel = 0.120

$$\text{Weight of 80 sq in. of skirt area} = \frac{0.530 \times 100 \times 470}{1728} = 11.5 \text{ lb}$$

In view of the relative wall thicknesses, the greater heat conductivity of the water-cooled liner, and periodicity of contact, only one third of the available generated heat is taken for heating the skirt wall. The theoretical temperature rise of the skirt section would then be

$$\frac{15{,}000}{3 \times 11.5 \times 0.120} = 3630 \text{ deg F per min}$$

above the initial temperature of about 300 F. This gives a total temperature of 3930 F. In other words, since the melting point of semisteel is approximately 2200 F, the piston iron at the high spot would melt easily in about $^1/_2$ min. It should be further noted here that the assumptions made in reaching this figure lean so far to the side of conservatism, it is believed that, for the conditions assumed, melting would occur in much less time than this. Seizure and resultant stalling tendencies, or slowdown, of the engine are more often accompanied by local fusion in much smaller hot spots on the piston and liner surfaces. Actually, it is believed that the first few hot sparks of molten metal that can fly past the bottom of the piston skirt and into the crankcase vapors at the beginning of the scoring action will start off an explosion, if the crankcase has any tendency to run hot and an appreciable amount of air is present to support combustion, or rather detonation.

In view of the possibility that large pin deflections in fixed-end wristpins can cause excessive bearing pressures and temperatures, rupture of oil film, large skirt deflections from resultant heating, all of which constitute explosion hazards, and even rupture (1) of the piston skirt, it is submitted that most of these and other vulnerable features noted in this paper could be eliminated by the use of full-floating wristpins with suitable piston skirts. The advantages of such a design would be:

1 Boss stresses would be maintained at reasonable limits under all operating conditions. Collier's analysis sets the wristpin stresses between the edge of the boss and the edge of the bearing at 8 to 9 times the stresses at the center of the journal. Most of the resulting strain will be prevented from being transmitted to the boss if the latter is provided with floating clearance.

2 Reduced lubrication for short periods of time and high wristpin operating temperatures would not cause hazardous piston deformations, since the resulting thermal pin expansion would merely be absorbed, or "float," in the bosses.

3 Elimination of piston distortions which accompany fixed-end wristpins, whether force-fitted or expanded-end types.

4 Elimination of permanent distortion of the piston due to the rounding-up operation in shop assembly. This distortion can only be removed by machine-turning the piston after assembly with the piston pin, and upon disassembly the skirt would again distort.

5 Reduction of hazards from cylinder deformations and local high spots on the cylinder wall or piston skirt, due to the flexibility of the piston skirt and its ability to change shape to suit conditions. This is impossible with fixed-end pins, as the latter makes the skirt a rigid structure at exactly the skirt height where it will do the most damage.

6 Less precise manufacturing required for floating journals and bearings.

7 Less complicated means needed for securing the wristpin.

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Proposed Expressions for Roots' Supercharger Design and Efficiencies

By F. A. HIERSCH,[1] ANN ARBOR, MICH.

This paper defines certain proposed positive-displacement-blower efficiencies in Equations [1], [2], [3], [5] in terms of slippage air, i.e. (1 — vol. eff/100), which is taken as a means for rating the design of the blower. Equation [6] may be used to obtain n_s, which is defined as the "slip" polytropic-compression exponent. The first example shows actual details of computation of optimum rotor-bore diameter and axial length for a given delivery volume, speed, impeller clearance, and head based on the methods given by K. Schopper.[2] The volumetric efficiency for the given blower design is then compared with that of the optimum-design blower. These results are then applied in Equations [1] and [2]. In the second example volumetric efficiency is plotted against the ratio of axial length to bore diameter with conditions fixed as in the first example. These results may be applied to Equations [1] and [2].

Since we may determine the optimum rotor diameter[2, 3, 4] for maximum volumetric efficiency in a Roots' blower of a given capacity at a given speed, and since we may also calculate the temperature-rise effect due to slippage air,[5] it is therefore possible to establish a theoretical design efficiency for such a blower.

1 It is suggested that the ratio of the adiabatic temperature rise, plus the temperature rise due to slippage air for the ideal design (optimum rotor diameter), to the adiabatic temperature rise, plus the temperature rise due to slippage air for the actual design, be designated the "Roots' adiabatic design efficiency."

2 Likewise, it is suggested that the ratio of the sum of the work of adiabatic compression plus the work equivalent of the temperature rise, due to slippage based on optimum rotor diameter, to the work of Roots' type compression plus the work equivalent of the temperature rise, due to slippage based on actual rotor design diameter, be designated as "Roots' type design efficiency."

For case 1:

"Roots' adiabatic design efficiency" $= \eta_{\text{Ad.d.}} =$

$$\frac{[(T_2' + \Delta T_{os}) - T_1]}{[(T_2' + \Delta T_{ds}) - T_1]} = \frac{\Delta T[1 + s_{os} + s_{os}^2]}{\Delta T[1 + s_{ds} + s_{ds}^2]} = \frac{(1 + s_{os} + s_{os}^2)}{(1 + s_{ds} + s_{ds}^2)} \quad [1]$$

where

T_1 = inlet air temperature

T_2' = calculated adiabatic temperature rise

$\Delta T_{ds} = \Delta T(s_{ds} + s_{ds}^2) = (T_2' - T_1)(s_{ds} + s_{ds}^2)$

s_{ds} = slippage air in parts per hundred, calculated from actual design data

$\Delta T_{os} = \Delta T(s_{os} + s_{os}^2)$

s_{os} = slippage air in parts per hundred calculated from idea design data.

The standard form for Roots' type efficiency is

$$\eta_{\text{type}} = \frac{k}{k-1} \cdot \frac{p_1 v_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]}{(P_2 - P_1)\, v_1}$$

Applying the second definition this becomes the "Roots' type design efficiency," or

$$\eta_{\text{R.t.d.}} = \frac{n_{os}}{n_{os} - 1} \left\{ \frac{p_1 v_1 \left[\frac{{}_{os}T_2'}{T_1} - 1\right]}{(P_2 - P_1)v_1 + wJc_p({}_{ds}T_2' - T_2')} \right\}$$

$$= \frac{n_{os}}{n_{os} - 1} \left\{ \frac{wR({}_{os}T_2' - T_1)}{(P_2 - P_1)v_1 + wJc_p({}_{ds}T_2' - T_2')} \right\} \ldots [2]$$

where

$${}_{os}T_2' = T_2' + \Delta T(s_{os} + s_{os}^2)$$
$${}_{ds}T_2' = T_2' + \Delta T(s_{ds} + s_{ds}^2)$$

and

n_{os} = polytropic-compression exponent

For the case of built-in compression, the numerator of Equation [1] remains the same. The denominator may be applied in the same form if an over-all flow coefficient k is determined which will take care of design slippage loss through both the built-in adiabatic compression and the Roots' type compression stages.

Taking care of each stage separately, Equation [1] becomes

$$\eta_{\text{Ad.d.}} = \frac{\Delta T(1 + s_{os} + s_{os}^2)}{\underbrace{[({}_1T_x' + {}_1\Delta_x T_{ds}) - T_1]}_{\text{(Built-in compression)}} + \underbrace{[({}_xT_2' + {}_x\Delta_2 T_{ds}) - T_x]}_{\text{(Root's type compression)}}} \ldots [3]$$

where $({}_1T_x' - T_1)$ and $({}_xT_2' - T_x)$ represent the computed adiabatic temperature rise for the respective stages.

The application of this expression is evidently simple after the volumetric efficiency and k[2, 3] has been fixed for each stage of compression. It might prove more convenient in practice to use test values in Equation [1] in the denominator and to apply a casing-loss coefficient to the numerator. This would result in an efficiency expression, based upon adiabatic temperature rise, design slip loss, and a casing enthalpy-loss coefficient c in the numerator, and an adiabatic temperature rise and measured slip loss s in the denominator, or

$$\eta_{\text{Ad.d.}} = \frac{c(1 + s_{os} + s_{os}^2)}{(1 + s + s^2)} \ldots\ldots [4]$$

For case 2, terms must be added to the denominator because of the mixed type compression. Thus for Case 2 the expression becomes

[1] Assistant Professor, University of Michigan; at present Mechanical Engineer, N.A.C.A., Engine Research Laboratory, Cleveland, Ohio.

[2] "Das Rootsgebläse als Ladungsverdichter an Mercedes-Benz Motoren," by K. Schopper, *Automobiltechnische Zeitschrift*, vol. 38, 1935, pp. 28–32.

[3] "Problems in the Design of Roots' Type Superchargers," by P. M. Heldt, *Automotive Industries*, vol. 72, 1935, pp. 450–452.

[4] "Internal Combustion Engines," by L. C. Lichty, fifth edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1939, p. 471.

[5] "Inlet-Air-Temperature Correction in a Roots' Supercharger," by F. A. Hiersch, presented at the Annual Meeting, New York, N. Y., Nov. 30-Dec. 4, 1942, of The American Society of Mechanical Engineers.

Presented at the National Meeting of the Oil and Gas Power Division, Baltimore, Md., June 14–16, 1943, of The American Society of Mechanical Engineers.

$$\eta_{\text{R.t.d.}} = \left\{ \frac{\dfrac{n_{os}}{n_{os}-1}(p_1v_1)\left[\dfrac{{}_{os}T_2'}{T_1}-1\right]}{\dfrac{n_{os}}{n_{os}-1}(p_1v_1)\left[\dfrac{T_x'}{T_1}-1\right] + wJc_p({}_{ds}T_x' - T_x) + (p_2 - p_x)v_x + wJc_p({}_{ds}T_2' - T_2)} \right\}$$

$$= \frac{\dfrac{n_{os}}{n_{os}-1}\left[wR({}_{os}T_2' - T_1)\right]}{\left\{\dfrac{n_{os}}{n_{os}-1} wR(T_x' - T_1) + (p_2 - p_x)v_x + wJc_p[({}_{ds}T_x' - T_x) + ({}_{ds}T_2' - T_2)]\right\}} \quad \ldots [5]$$

where the subscript x indicates the state at the end of adiabatic compression and the beginning of Roots' type compression. The first two terms in the denominator give the work of adiabatic and Roots' type compression, while the last terms account for work resulting from slippage computed from design values.

A simple relationship giving an index of the design efficiency and making use of volumetric efficiency may be had by solving for n_s the slip polytropic-compression exponent in the equation

$$\left(\frac{{}_sT_2'}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n_s-1}{n_s}} \quad \ldots\ldots\ldots\ldots [6]$$

where

$${}_sT_2' = T_2' + \Delta T(s + s^2)$$

T_2' = calculated adiabatic temperature

$\Delta T = (T_2 - T_1)$ observed outlet- and inlet-air temperatures

s = slippage air in parts per hundred

Computations

Example 1:

Q_{th} = 0.351 cu ft per rev = theoretical displacement

a = 9.637 in. = bore diameter

$l = ca$ = 7.572 in. = rotor length

$$m^2 = \frac{\text{Rotor cross-sectional area}}{\text{Circle of bore diameter}} = 0.45, \text{ a constant}$$

q = 0.006 in. = impeller clearance

n = 2500 rpm = speed

$p = (p_2 - p_1)$ = 9 in. Hg (pressure head)

Q_a = actual air delivered in cu in. per min

$K = 150\, qpK$ = leakage factor

k = experimental flow coefficient = 400 (when p = 9 in. Hg)

$$z = \frac{\pi}{2}(1 - m^2)$$

Calculation of Q_a:

$$Q_a = zca^3n - 3K(ca + a) \quad \ldots\ldots\ldots\ldots [7]$$

$$z = \frac{\pi}{2}(1 - 0.45) = 0.865$$

$$K = 150(0.006)(9)(400) = 3240$$

Substituting values

$$Q_a = (0.865)(7.572)(9.637)^2(2500) - 3(3240)(7.572 + 9.637) \quad [8]$$

$= 1.354 . 10^6$ cu in. per min

Volumetric efficiency for the design given may now be computed as follows

$$\eta_{\text{vol}} = 10^2\left[1 - \frac{3za^3nK + 3KQ_a}{za^2nQ_a + 3Ka^3n}\right] \quad \ldots\ldots\ldots [9]$$

substituting values

$$= 10^2\left[1 - \frac{3(0.865)(9.637)^3(2500)(3240) + 3(3240)(1.354\cdot10^6)}{(0.865)(9.637)^2(2500)(1.354\cdot10^6) + 3(3240)(9.637)^3(2500)}\right.$$

$$\eta_{\text{vol}} = \underline{89.08} \text{ per cent} \quad \ldots\ldots\ldots\ldots [10]$$

The optimum design may be determined as follows:

a_o = optimum bore diameter, in.

$$a_o = \sqrt[3]{\frac{Q_a}{zn} + \sqrt{\frac{Q_a^2}{z^2n^2} - \frac{27K^3}{z^3n^3}}} + \sqrt[3]{\frac{Q_a}{zn} - \sqrt{\frac{Q_a}{zn} - \frac{27K^3}{z^3n^3}}} \quad \ldots [11]$$

substituting values

$$a_o = \sqrt[3]{\frac{1.354\cdot10^6}{(0.865)(2500)} + \sqrt{\frac{(1.354\cdot10^6)^2}{(0.865)^2(2500)^2} - \frac{27(3240)^3}{(0.865)^3(2500)^3}}} + \sqrt[3]{\frac{1.354\cdot10^6}{(0.865)(2500)} - \sqrt{\frac{(1.354\cdot10^6)^2}{(0.865)^2(2500)^2} - \frac{27(3240)^3}{(0.865)^3(2500)^3}}} \quad \ldots [12]$$

a_o = 11.47 in.

From Equation [7]

$$c_o = \frac{Qa + 3Ka}{za^3n - 3Ka} \quad \ldots\ldots\ldots\ldots [13]$$

$$c_o = \frac{1.354\cdot10^6 + 3(3240)(9.637)}{(0.865)(9.637)^3(2500) - 3(3240)(9.637)} = 0.4649 \quad \ldots [14]$$

$c_oa_o = l_o$ = 5.33 in. (optimum rotor axial length)…. [15]

The optimum volumetric efficiency with fixed values of Q_a, q, n, p, and K may be calculated from Equation [3] by using the optimum dimensions a_o and l_o

$$\eta_{\text{vol (optimum)}} = 10^2\left[1 - \frac{3(0.865)(11.47)^3(2500)(3240) + 3(3240)(1.354\cdot10^6)}{(0.865)(11.47)^2(2500)(1.354\cdot10^6) + 3(3240)(11.47)^3(2500)}\right]$$

$$= \underline{\underline{89.33}} \text{ per cent} \quad \ldots\ldots\ldots\ldots [16]$$

From Equation [1], Roots' adiabatic design efficiency may be determined as follows:

$$s_{os} = \text{optimum slip} = \left[1 - \frac{\eta_{\text{vol}}\,(\text{optimum})}{100}\right] = (1 - 0.8933)$$

$$= 0.1067$$

$$s_{ds} = \text{design slip} = \left[1 - \frac{\eta_{vol}\,(\text{design})}{100}\right] = (1 - 0.8908] = 0.1092$$

$$\eta_{\text{Ad·d}} = 10^2 \frac{(1 + 0.1067 + 0.1067^2)}{(1 + 0.1092 + 0.1092^2)} = \underline{\underline{99.73}} \text{ per cent } [17]$$

"Roots' type design efficiency" is calculated with the use of Equation [2] and the following values:

$$k_1 = \frac{c_p}{c_v} = 1.397 \text{ for air}$$
R = gas constant = 53.34 for air
J = 778 ft-lb
c_p = 0.241
p_1 = 2116 lb per sq ft
p_2 = (29.92 + 9)144·0.491 = 2752 lb per sq ft
v_1 = 0.351 cu ft
T_1 = 520 deg R

$$\frac{T_1}{T_2'} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$$

$$n = k_1$$

$$\frac{520}{T_2'} = \left(\frac{2116}{2752}\right)^{\frac{0.397}{1.397}}$$

T_2' = 560 deg R

$$_{os}T_2' = 560 + (560 - 520)(0.118) = 564.72 \text{ deg R}$$

$$_{ds}T_2' = 560 + (560 - 520)(0.121) = 564.84 \text{ deg R}$$

$$p_1 v_1 = wRT_1$$

$$w = \frac{(2116)(0.351)}{(53.34)(520)} = 0.02678 \text{ lb}$$

$$\frac{564.72}{520} = \left(\frac{2752}{2116}\right)^{\frac{n_{os}-1}{n_{os}}}$$

$$n_{os} = 1.458$$

$$\eta_{\text{R.t.d.}} = \frac{1.458}{0.458} \left[\frac{(0.02678)(53.34)[564.72 - 520]}{(2752 - 2116)(0.351) + (0.02678)(778)(0.241)(564.82 - 560)}\right] 10^2$$

$$= \underline{82.1} \text{ per cent} \ldots\ldots\ldots\ldots [18]$$

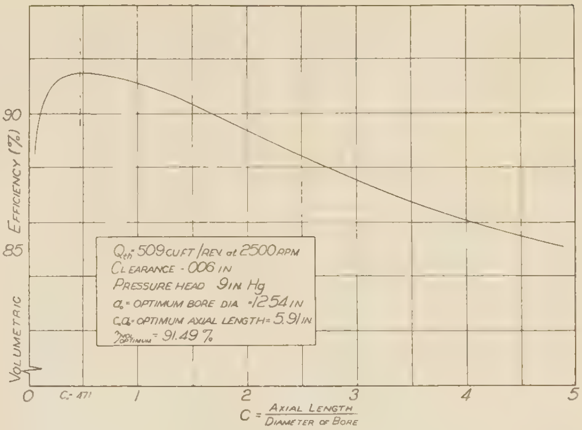

FIG. 1 EFFECT ON VOLUMETRIC EFFICIENCY OF DEVIATION FROM OPTIMUM DESIGN

Example 2:

Equations used in Example 2 appear in Example 1. The curve, Fig. 1, shows the effect on volumetric efficiency of deviation from optimum design. These results may immediately be applied to Equations [7] and [8].

The curve is calculated for a blower operating under the following conditions:

Q_a = 1.947·10⁶ cu in. per min
q = 0.006 in.
K = 3240
z = 0.865 (m^2 — 0.45)
n = 2500 rpm
p = (p_2 — p_1) = 9 in. Hg
η_{vol} (optimum) = 91.49 per cent (calculated)
a_o = 12.54 in. (optimum bore diameter; calculated)
$c_o a_o$ = l_o = 5.91 in. (optimum axial length; calculated)

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Portable Oil-Well Drilling and Servicing Equipment

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This paper deals mainly with descriptions of certain oil-well drilling and servicing equipment carrying its own derrick as well as the necessary hoisting and rotary drilling equipment. Such equipment is mounted permanently and integrally on wheels, making it completely mobile over highway and across country. The equipment described is of two types, i.e., self-propelled or trailer-mounted. In either case, the entire machine is transportable, virtually with no disassembly; and moving from one well to another has proved to be a matter of minutes. Design and design requirements of this equipment and records showing its usefulness are discussed. Figures are given concerning the saving of steel. Safety requirements and their effect upon the design are also considered, as well as the requirements concerning the use of such equipment over state highways.

History and Development

In 1939 Laurence O'Donnell[2] mentioned that one's first impression of an oil field is a large area covered with derricks. In fact, the expression "a forest of derricks" is often heard from individuals as they first view a producing area. He further stated that newer oil fields, with depths of 4000 ft and possibly greater, will not in the future be marked by permanent derricks. Time has borne out Mr. O'Donnell's prophecy, as portable derrick structures are beginning to enjoy increasing popularity. In California, the Bolsa property of the Signal Oil Company is not equipped with a single permanent derrick. Wells 11,500 ft in depth have been serviced with portable derrick structures, and wells to 6000 ft have been drilled with the same kind of equipment.

Prior to 1939, wells had been drilled and serviced without permanent derricks, it is true, but the equipment was ill suited to the service needs and wells of greater than 4000 ft depth required some form of permanent structure. For shallow-depth wells, single- and double-pole masts, integrally mounted with a service hoist on a truck or trailer, had been used.

Cardwell, Hopper, Wilson, and Franks all had portable hoist machines of this type, some built from the ground up, others built on standard truck and trailer chassis. Also, for about 3 years prior to 1939, Franks had been building short one-piece open-face portable derricks, carried as an integral part of the complete machine. These derricks were power-raised and were used for drilling and servicing shallow-depth wells. However, since they were only 47 to 56 ft high, doubles of range 2 could not be stacked, nor was it possible to hang rods in doubles. Taller pole masts could be designed, but the problems connected with this design became somewhat acute with the greater heights. Therefore, the Shell Oil Company, after extensive collaboration with this manufacturer decided upon an entirely new idea in well-servicing equipment for East Texas. This involved the design of a completely portable integrally mounted hoist and an 84-ft 100,000-lb-capacity telescoping derrick mounted upon a Marmon-Herrington all-wheel-drive truck of 50,000 lb gross vehicle weight. The complete machine weighed 42,000 lb and had a 140-hp gas-gasoline engine. It was delivered early in 1939.

About the same time, there were being developed for several major oil companies, portable "slim-hole" and exploratory drilling machines, capable of going to 4000 ft and also carrying their own power-raised derricks, pumps, two engines, draw works, and rotary table. Such rigs cut moving time by 60 per cent, according to a paper presented by E. E. Davis, and H. V. Steadman.[3] Hence it might be said that 1938 saw the birth of a trend toward portable drilling and servicing equipment, completely and integrally mounted, transportable as a single piece, and capable of handling the deeper wells.

Since 1938, there has been a definite trend by the oil companies toward the use of such equipment, and there are now on the market a considerable number of designs, all incorporating the portable-derrick idea. Time does not permit going into the details of all of these, nor is the author thoroughly familiar with all the various makes and types now in use. However, some interesting features of a few designs will be discussed.

Usefulness of Equipment

Naturally, the purchase of any piece of equipment must be justified by the economics of its use. It is extremely doubtful if any portable derricks have been or shall be sold just because some oil company does not desire to have its property cluttered up by a lot of permanent derricks. An investment, such as required by one of these machines, is considerable, and an oil company must be assured of its financial soundness.

Therefore in making a cost analysis and comparison, we find several important items. First is the cost of the permanent derrick, which Carl White, Jr.,[4] in a recent paper estimates to be as high as $1750, complete with a two-sheave crown block and tubing board. If the well requires frequent servicing, a lighting system as well as a tubing line will probably also be installed. These must be added for each and every well. The second consideration is rig-up time. If the derrick is left without a crown, one must be installed, together with a line and block each time the tubing or rods are pulled. The third item is maintenance and taxes. Every derrick standing must be maintained against weather and general oil-field depreciation. Taxes must be paid each year on each derrick structure. The fourth is erection cost, which will run beween $400 and $650.

Offsetting this in the portable hoist-and-derrick unit, there is but one piece of equipment, one lighting system, one crown block, one pulling line and block, and one machine to maintain and pay

[1] Franks Manufacturing Corporation, Tulsa, Okla.; Hillman-Kelley, Inc., Los Angeles, Calif.

[2] "A Revolutionary Derrick Development," by Laurence O'Donnell, *Shell News* of the Shell Oil Company, May, 1939.

Contributed by the Petroleum Division and presented at the Semi-Annual Meeting, Los Angeles, Calif., June 14–17, 1943, of The American Society of Mechanical Engineers.

[3] "The Development of Slim Hole Drilling," by E. E. Davis and H. V. Steadman, Proceedings of the American Petroleum Institute, 9th Annual Meeting, November, 1938, sec. 4, "Production," pp. 79–84.

[4] "Why Slim Hole Drilling?" by Carl White, Jr., paper presented at the American Petroleum Institute, district meeting, Pampa, Texas, April 8, 1940.

taxes upon. The derrick is arranged to carry the pulling lines permanently strung. Erection of the structure is by power and consumes but a few minutes. The structure can be easily taken down in bad weather.

The price of the largest of these machines, which is capable of servicing 10,000-ft wells and drilling to 5000 ft, is approximately $30,000, and such a machine is capable of servicing from 50 to 100 wells, depending upon the nature of the field. The cost of 16 or 17 permanent derricks alone, disregarding the costs of erection, extras, and maintenance, is sufficient to justify the purchase of such equipment.

As an example of the use of these machines for drilling, a paper by T. A. Atkinson[5] on a portable drilling rig is quoted. The machine used was the Ideco Hopper trailer-mounted portable derrick and hoist and is used by the General Petroleum Corporation for drilling $12^1/_4$-in. holes to 2000 ft. Table 1 gives an idea of the performance of this rig.

TABLE 1 AVERAGE TIME FOR WELL-DRILLING WITH PORTABLE RIG, BASED ON DRILLING PERFORMANCE FOR FIRST TEN WELLS

Operation	Time required, hr	Per cent of time
Move, rig up, drill rat hole, spud in	26	10
On bottom drilling and reaming	43	16
Round trips to drill and ream	13	5
Round trips to core and service core	12	5
Core	6	2
Formation testing and electric logging	12	5
Running casing, liners, and cementing	8	3
Standing cemented and recemented	69	26
Miscellaneous (oil machinery, repairs, circulate, etc.)	31	12
Bail, run rods and tubing	43	16
Total per well	263	100
or 10 days 23 hr		

In the panhandle district of Texas, another major oil company using a Franks' self-propelled drilling machine, complete with integral derrick, drilled a total of 105,000 ft of $6^3/_4$-in. hole in 11 months for a total of 33 wells.

It has been proved repeatedly that portable drilling machines, complete with derrick, can be moved in and set up and drilling can be begun in from 4 to 8 hr. Servicing machines, carrying their own derricks, have been moved in and erected, and begun pulling rods or tubing in less than 1 hr. Furthermore, the same crew doing the moving and erecting is used for the drilling or pulling operations.

A typical example of the usefulness of these units is the experience of a drilling contractor in Texas, who in 1 year with one of these rigs worked on 58 wells, including 16 cleanouts, 11 deepenings, 10 rotary follow-ups, and 21 jobs of running tubing. Moves in excess of 100 miles and rig-up the same day were not unusual.

Another example of the use of a truck-mounted servicing machine is reported from California where one of these units moved from Santa Maria to Ventura in a driving time of 6 hr, and moving onto a small mountainside location after negotiating a curving mountain road, and with an inexperienced crew, pulled 8475 ft of 3-in. tubing in 7 hr, and ran it back in slightly less than 5 hr.

Still another example is that of a trailer-mounted machine being used for both drilling and servicing. In slightly less than 1 year, it has drilled 16 wells (2000 ft average depth), redrilled two others, and is being used to service 28 wells without derricks, which range in depth from 9000 to 11,500 ft. During the year it made seven long moves, two being more than 100 miles.

Besides the saving in cost of new derricks, there is also the saving of steel. It is estimated that approximately 90,000 tons of new steel derricks were used in 1941 and left standing over the holes of nearly 10,000 new wells. Taking as an average example the 94-ft servicing derrick, complete with crown block, tubing board, and pulling line, but without substructure, we find an expenditure of steel amounting to approximately 9 tons. A brief analysis made last year of portable telescoping derricks, engaged in servicing work, showed that eight such machines had replaced or would replace a total of 452 permanent derricks (many of which had never been erected) with a total saving of 4068 tons of steel. Since that time, many more portable telescoping servicing derricks have been placed in service.

[5] "Portable Drilling Rig," by T. A. Atkinson, paper presented at the Los Angeles Meeting of the American Institute of Mining Engineers, October 29 and 30, 1941; also abstract, *Oil and Gas Journal*, vol. 40, Nov. 13, 1941, Digest of A.I.M.E. papers, p. 44.

Design and Design Requirements

The design requirements of portable drilling and servicing units are many and varied. Some types must serve the double duty of both drilling and servicing oil wells. Other types are designed for a single purpose, either that of servicing or drilling. Each type has its place. Also, there are several types of skid derricks on the market used for drilling only, which are dismantled in several pieces. Some of these derricks are part of complete skid drilling machines, such as the Franks or the Brewster skid type. Others, such as the Lee C. Moore cantilever type, may be used with conventional spark-plug or steam drilling units. Usually they are used on deeper drilling where setup time is not such a high percentage of total location time. All are removed after completion of the well.

Since the design of a drilling or servicing machine involves the machine as a whole, the general arrangement is usually the first consideration. The size of the machine is decided upon and a general-arrangement drawing made. Selection of hoist, pump, and derrick size should be matched to provide a balanced unit, as weight is at a premium and it is undesirable to team up a 200,000-lb derrick with a hoist designed to handle only 100,000-lb loads.

The controls should be placed at a convenient point where vision is at its best. In many designs, controls are now air-actuated and are placed at the derrick front leg where the operator not only has perfect vision but can also assist in operations at the well head. There is a tendency toward simplification of control on the newer machines, and it is hoped that future controls will be even better.

Lubrication should be kept simple with as few grease points as possible. All chains should have oil-bath, force-feed, or drip lubrication. All grease points should be easily accessible.

If lighting is to be provided, it should be of the explosion-proof type with quick-type connections. The lights should be located to illuminate properly the derrick floor, tubing board, and crown, without glare.

The derrick should be arranged to provide ample clearance around the well head. This can be accomplished by tilting the derrick or by spreading the legs, although the latter usually causes complications with the road laws. There have been portable derricks on the market which were only 8 ft wide to conform with the road laws and which did not tilt, but in these derricks, working room around the rotary table was virtually nil. A straight up-and-down derrick will usually work all right if the crown is set forward far enough with respect to the front legs and if the internal bracing is properly arranged.

Portability is a feature also to be considered. A weight-and-balance estimate should be run to determine the center of gravity and the tire, axle, and frame loading. The ideal setup, especially for servicing, is to carry the complete machine in one integral piece, ready to do the job when it arrives at the well. The type of foundations and guy-line anchors needed should also be considered.

In connection with the derrick, the requirements of A.P.I. Standards No. 4 are usually followed. The derricks are designed

to stand in a 70-mph wind and the gross allowable load is calculated by using the A.P.I. column formula and design conditions. Member loads are obtained by graphical analysis. Leg size and material are determined and if other than an A.P.I. or similar column formula is used, a safety factor is selected, as the A.P.I. column formula provides a factor of safety of approximately 2.

The tubing board should be adjustable over a range of at least 15 ft on a 90-ft combination drilling-and-servicing derrick to accommodate the various lengths of range-2 and range-1 tubing. The crown block should have a sufficient number of properly grooved sheaves to accommodate a catline, a sand line, and the pulling line, without unstringing any of them. The sheaves should have as large a diameter as is compatible with the design and should be flame-hardened manganese steel, mounted on antifriction bearings. Lubrication should be from the tubing platform to avoid the necessity of climbing the derrick.

Alloy steels for the legs and other derrick members should be used to keep the weight down. Also, a satisfactory method of raising must be provided as well as a method of locking the two sections. A position must be selected for a dead-line anchorage, which will permit the operator to see the weight indicator.

The number of guys, if any, must be decided upon. Some 90-ft derricks may not need guys from a structural standpoint, but it is not believed to be safe practice to eliminate guying of a 90-ft structure, resting on an 8-ft base, unless that structure is securely bolted down to the base. Naturally, the fewer guys the better as they consume handling time and are a nuisance to carry. However, properly designed guys, with a means of quick takeup and a proper anchor, form one of the most efficient methods of tiedown yet devised.

In the design of the hoist, the drums should be selected with consideration of the rest of the design. The sandreel should carry as much line as the deepest hole the main drum and derrick will handle. Friction clutches should be used in the ends of the drums to absorb shock loads. Fully equalized brakes of ample capacity for the job must be provided. An analysis of the bending and bearing loads imposed on all shafting should be made. Small heavily loaded fast-running chains should have hardened sprockets.

The rotary table and rotary-table drive also call for considerable attention. Their size, too, must be considered with the rest of the machine. The location of the automatic and regular catheads with relation to the derrick legs and the rotary table is important. Speed and capacity of the rotary table must be considered in laying out its design and installation.

The size and mounting of the pumps to be used must also be considered. Many times these are carried separately from the drilling unit on a special trailer of their own. The amount of horsepower required, pressure, strokes and gallons per minute, size of hole to be cut, all enter into the design of the portable pump unit.

There are two design requirements which add nothing at all to the equipment, yet which must be kept in mind while laying out the design; (*a*) the many and varied state road laws and (*b*) the state safety laws. Neither was devised to apply to such special types of equipment as portable well-drilling and servicing machines, yet both must be considered. Some safety codes demand a crow's nest of certain size be added, even though the derrick may be lowered to lubricate the sheaves (or the sheaves may be lubricated from a point on the tubing platform), or to string the crown, or to do any repair work on the crown. This crow's nest adds weight which it is desirable to eliminate. Also, safety-code ladder requirements with offset platforms are very nearly impossible to comply with in telescoping derrick design.

As for highway requirements, it might be and is possible to design a machine complying with the road laws of just one state. But to design a machine to comply with the road laws of all the oil-producing states is a practical impossibility. The greatest service the various state governments could perform for the designer of portable industrial equipment would be to adopt a uniform set of highway regulations. Fortunately, most state highway departments are very liberal with permits.

Brief descriptions of the design of typical portable drilling and servicing equipment now in use will be presented in the following:

Hopper-Ideco Blitzrig

The Hopper-Ideco Blitzrig, Fig. 1, is a semi-trailer-mounted portable telescoping derrick and hoist. It may be used as a combination drilling-and-servicing unit by the addition of a bolted-on type rotary drive and a conventional rotary table. The trailer is Hopper-designed and fabricated especially for the equipment. Leveling jacks are permanently attached to the trailer frame. Pumps and pump engines are carried separately on another semi-trailer. The semi-trailer is equipped with three axles, mounting

Fig. 1 Hopper Ideco "Blitzrig" Raising Portable Derrick by Means of Hydraulic Jacks

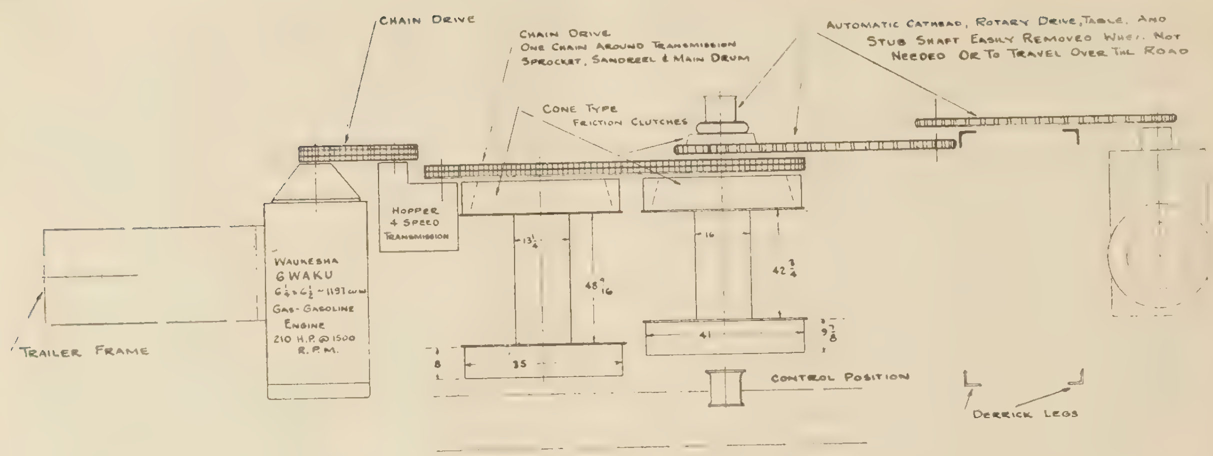


Fig. 2 Power-Drive Layout of Hopper Combination Drilling-and-Servicing Unit

7.50 × 20 tires. The entire machine, without the rotary-drilling-equipment extras, conforms to California highway regulations of 8 ft maximum allowable width, 13 ft 6 in. maximum allowable height, and 60 ft maximum allowable length; the gross weight is approximately 67,500 lb.

The power-drive layout, Fig. 2, consists of a Waukesha WAK engine, mounted across the frame, and a drive chain runs from the engine power-takeoff shaft to the input shaft of the Hopper 4-speed transmission. The output shaft of the transmission then drives both drums and the rotary-drive sprocket by means of a single (double-strand) drive chain, which goes around both drum-drive sprockets. There is no hoist countershaft going completely across the frame. The rotary table is of the conventional side-driven type and is driven by a drive chain from a sprocket on the end of the drum shaft. This chain drive is broken about halfway to the rotary table, a short distance behind the rear derrick leg by a bolted-in-type shaft-and-sprocket arrangement.

The hoist is of the double-drum type with main drum and sand reel. Drumshafts are of the live-shaft type, mounted on roller bearings. The drums are driven through cone-type friction clutches of unique design. Catheads, both regular and automatic, are mounted by means of bolted-on flanges on the ends of the drumshafts. The dimensions and capacities of the main drum and sand reel are given in Tables 2 and 3.

TABLE 2 MAIN-DRUM DIMENSIONS AND CAPACITIES OF HOPPER RIG

		Capacity main drum	
Main drum	Size, in.	Line size, in.	Ft of line
Brake flange diameter..........	41	$^9/_{16}$	12600
Barrel diameter................	16	$^5/_8$	10150
Distance between flanges.......	$42^3/_4$	$^3/_4$	7100
Brake width..................	$9^7/_8$	1	4000

TABLE 3 SAND REEL DIMENSIONS AND CAPACITIES OF HOPPER RIG

		Capacity sand reel	
Sand reel	Size, in.	Line size, in.	Ft of line
Brake flanges, diameter..........	35	$^9/_{16}$	11270
Barrel diameter..................	$13^1/_4$	$^5/_8$	9250
Distance between flanges.........	$48^9/_{16}$	$^3/_4$	6450
Brake width.....................	8	$^7/_8$	4450
		1	3550

Power for the Hopper hoist is supplied by a Waukesha 6WAK engine, as previously mentioned. This engine has a $6^1/_4$-in. bore and a $6^1/_2$-in. stroke, giving it 1197 cu in. displacement. It may be gas, gasoline, or butane operated, and is rated at 210 hp at 1500 rpm.

Using the Hopper 4-speed transmission and the conventional sprocket reduction, the range of line speeds and pulls on the main drum is given in Table 4.

TABLE 4 RANGE OF LINE SPEEDS AND PULLS

	Speed rpm	Bare barrel line pull, lb
1	236	30200
2	440	16300
3	590	12100
4	1000	7170

Controls are located on the right-hand side of the hoist (when facing the well) and include a starter button for the prime-mover and hoist-operating levers.

The derrick is of the two-section telescoping type, having the following characteristics:

1 Live-load capacity, 200,000 lb with a factor of safety of 2 (other capacities are available).

2 Vertical distance, ground to center of crown sheaves, 91 ft.

3 Collapsed length in horizontal position, $51^1/_2$ ft.

4 Over-all height over the highway, 13 ft 5 in.

5 Electric-welded construction throughout.

6 Derrick raised from horizontal to vertical position by two hydraulic jacks, through the power of a hydraulic pump driven by a $12^1/_2$-hp gasoline engine. Two bumper springs are mounted horizontally on the end of the trailer chassis, projecting a sufficient distance to contact the bottom of the derrick, just before the hydraulic jacks push the top of the derrick past the vertical center. The force of the jacks is required to compress the bumper springs and push the derrick over the center to a tilted position. At the proper tilt point, lock nuts are engaged which hold the bumper springs in the compressed position. These lock nuts are operated by remote control by a crank at the operator's station. The upper section of the mast is raised by a cable and power winch. Safety latches at all four corners are designed to engage with stops at each girt in the lower section. A warning gong is tripped as the derrick nears its extended height. The two sections are bolted together at the extended height of the derrick.

7 Derrick leans $^7/_8$ in. per ft when in place, although this may vary to a minor extent with different locations.

8 Ten guy lines are used. Two main back guys from the top are $^3/_4$ in. All others are $^1/_2$ in., including four from the center, two from the pipe-racking platform, and two from the top.

9 Legs are of 5-in. × 5-in. × $^3/_8$-in. silicon-steel angles, approximately equally loaded by the crown, fast line, and guy arrangement. It is assumed that the derrick was analyzed graphi-

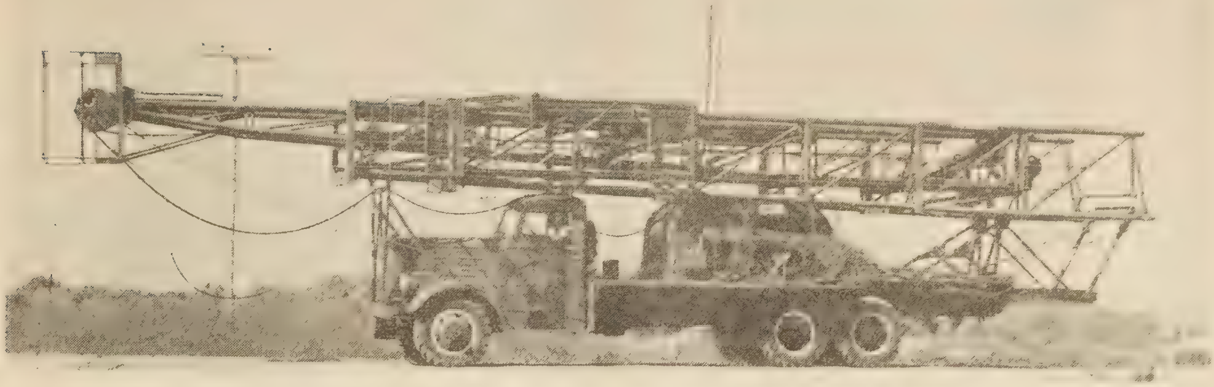


FIG. 3 FRANKS SELF-PROPELLED TELESCOPER TRAVELING IN "OVER-THE-ROAD" POSITION WITH LINES AND BLOCKS STRUNG

cally, and that the A.P.I. Standards No. 4 was consulted for wind-loading and pipe set-back requirements, as well as for the silicon-steel-column formula.

This formula is:

Allowable unit stress with factor of approximately 2 over failure

$$= \frac{27,000}{1 + \dfrac{L^2}{12,000\, r^2}}$$

where L = length of column (measured between girts)

r = radius of gyration for section used

The derrick is divided into 17 bays (sections between girts) with a column length of 63 in.

10 The crown block is welded integrally with the derrick and consists of four 24-in-OD steel sheaves grooved for 1-in. line.

11 A folding tubing platform is provided, hinged to the derrick legs of the upper section. Capacity for racking 13,000 ft of $2^1/_2$-in. tubing or its equivalent is provided, racked in 60-ft stands.

12 Leveling screws are provided in each of the four derrick legs.

When used as a drilling unit, a $7^1/_4$-in-bore × 12-in-stroke double-acting pump, driven by a 6WAK Waukesha engine and mounted upon a semi-trailer has been used with the performance given in Table 5.

TABLE 5 PUMP PERFORMANCE

Strokes per min	Pounds per sq in.	Calculated gal per min	Calculated hydraulic hp	Depth, ft
53	400	420	100	736
52	450	418	110	1037
47	525	380	120	1598
40	600	315	115	1813

The weight of the pumping equipment mounted on a two-axle semi-trailer is 36,400 lb, and has a width of 8 ft and a height of 13 ft 5 in.

Any conventional rotary table may be used. A typical hookup provides a maximum speed of 256 rpm and average rotating speeds of 175 to 210 rpm.

According to one major oil company now drilling with identical equipment to that just described, such a unit is easily capable of drilling 3000 ft of $12^1/_4$-in. hole, or of servicing 11,500-ft wells.

FRANKS SELF-PROPELLED TELESCOPER

The Franks self-propelled model 7000 "telescoper" combination drilling-and-servicing unit, Figs. 2, 3, and 4, consists of a double-drum hoist, a tubular telescoping derrick and complete rotary equipment, all integrally mounted upon an all-wheel drive special 6 × 6 chassis. The same motor used to drive the hoist is used to drive the machine over the road. The chassis (including cab, engine, radiator, and frame) is built by several of the leading truck companies to Franks' specifications. This chassis is of the 3-axle, 6-wheel-drive type with 11.00 × 24 tires. Since the equipment is manufactured for a world market, it does not conform to all the highway requirements of all the states. Over-all length of the complete machine is 57 ft, over-all height is 13 ft 10 in. and over-all width is 8 ft. Total bare weight without the rotary equipment is 57,100 lb, with approximately 13,150 lb on the front axle and 43,950 lb on the two rear axles. Total

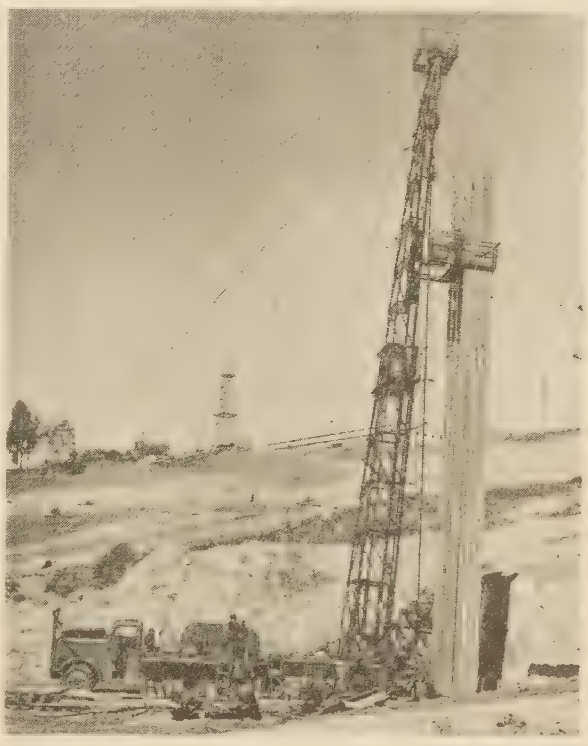


FIG. 4 TELESCOPER SERVICING 8000 TO 10,000-FT WELLS IN MOUNTAIN COUNTRY

(9788 ft of 3-in. tubing is shown racked in the derrick.)

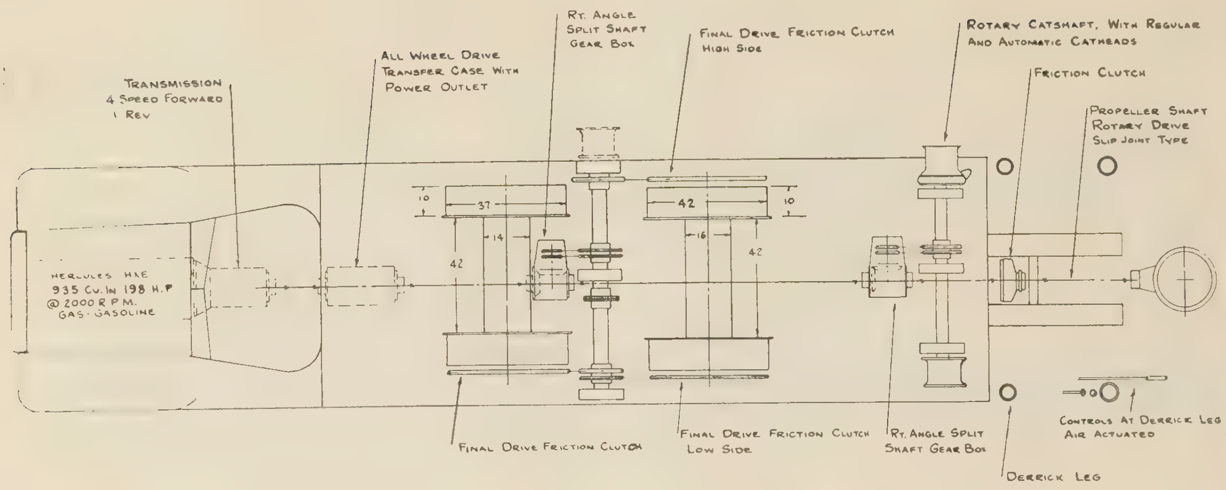


Fig. 5 Power-Drive Layout of Franks Combination Drilling-and-Servicing Unit

weight, complete with all extras and with line and block is approximately 68,000 lb.

The power-drive layout, Fig. 5, consists of a Hercules HXE engine, mounted in a conventional position in the chassis, parallel to the frame rails and driving through a Fuller 4B86 transmission to a transfer case which diverts the power of the engine to the forward and two rear axles for tractive effort over the road. A drive straight through the transfer case provides a power-take-off arrangement for driving the hoist; a propeller-shaft drive is provided from the transfer case to a bevel-gear split-shaft gearbox, which allows the drive to be diverted at right angles or to drive straight through. The right-angle drive then drives the hoist countershaft by means of a chain-and-sprocket arrangement; the drums are driven from the hoist countershaft by sprockets and roller chain through single-plate two-faced friction clutches mounted in the end of the drum, the drive, continuing straight back out of the split-shaft bevel-gear box, drives the rear-end rotary catshaft through another gearbox arrangement of the same type and continues on to drive a propeller-shaft-driven rotary table.

The double-drum hoist consists of a main drum and a sandreel driven by friction clutches in the end of each drum. The main drum has a high-low friction-clutch drive arrangement, there being a high-sprocket-ratio friction-clutch drive on one side of the drum and a low-sprocket-ratio friction-clutch drive on the other. This increases the range of speeds and pulls considerably. The friction clutches are built into the end of each drum and are of the single-plate two-friction face type. Control action is a light over-center arrangement.

Drum sizes and capacities are given in Table 6, and those for the sand reel in Table 7.

TABLE 6 MAIN-DRUM DIMENSIONS AND CAPACITIES OF FRANKS TELESCOPER

Main drum	Size, in.	—Capacity main drum— Line size, in.	Ft of line
		$^1/_2$	15500
Brake-flange diameter..............	42	$^9/_{16}$	12500
Barrel diameter....................	15	$^5/_8$	10170
Distance between flanges..........	42	$^3/_4$	7020
Brake width.......................	10	$^7/_8$	5160
		1	3960

TABLE 7 SAND REEL DIMENSIONS AND CAPACITIES OF FRANKS TELESCOPER

Sand reel	Size, in.	—Capacity sandreel— Line size, in.	Ft of line
		$^1/_2$	12120
Brake-flange diameter..............	37	$^9/_{16}$	9560
Barrel diameter....................	14	$^5/_8$	7760
Distance between flanges..........	42	$^3/_4$	5370
Brake width.......................	10	$^7/_8$	3940
		1	3030

The Hercules HXE engine used to power the hoist and to drive the unit over the road has 935 cu in. displacement with a bore and stroke of $5^3/_4$ in. × 6 in. It is a gas-gasoline engine rated at 198 hp at 2000 rpm or 187 hp at 1800 rpm, where it is usually governed for oil-field duty. It is equipped with a combination carburetor, water-cooled manifold, front-end exhaust and flame arrester.

Conventional line speeds and pulls of the hoist with this engine are given in Table 8. The line pulls are calculated at 70 per cent engine efficiency. Maximum usable bare-barrel line pull with the Franks friction clutch is 35,000 lb.

TABLE 8 LINE SPEEDS AND PULLS OF FRANKS TELESCOPER

			1	2	3	4	Rev.
Main Drum	Low side	Speed	134	228	423	745	113
		Pull	32300	19000	10250	5820	38300
	High side	Speed	208	350	650	1140	174
		Pull	20800	12340	6650	3800	24900
Sand reel		Speed	191	325	605	1052	161
		Pull	22670	13323	7157	4065	26894

Controls for the hoist are located at the derrick leg nearest the well and on the right-hand side. These controls actuate the various clutches by air through selector-valve assemblies and are extremely simple and easy to operate. The brake is manual and the throttle is hydraulic. A starter button and remote gear shift are also provided.

The Franks rotary table is of the propeller-shaft-driven type with a rated static capacity of 212,000 lb. Being driven directly from the engine by a propeller shaft beneath the floor, it utilizes all the speed changes of the transmission. The maximum quill opening is $13^1/_4$ in. diam. Lubrication is oil-bath sealed into the case. The slip-joint propeller shaft allows complete removal of the table simply by unscrewing the locking collar.

A rear-end rotary catshaft is furnished, driven through a bevel-gear split-shaft gearbox. It is equipped with both automatic and regular catheads located immediately behind the derrick legs.

The derrick, which is integrally mounted, is of the two-piece telescoping type, having the following characteristics:

1 Gross load capacity, 200,000 lb. When properly guyed with 4 guys, the minimum factor of safety is 3.34. Minimum factor of safety with no guys is 1.5.

2 Vertical distance ground to crown beam (actual clearance), 88 ft.

3 Collapsed length in horizontal position is 57 ft.

4 Over-all height over the highway is 13 ft 10 in.

5 Electric-welded construction throughout.

6 Derrick raised to vertical position by a power-driven screw mechanism, driven from the main-propeller-shaft drive, and located alongside the frame below the hoist and main bed. The raising device consists of two large-diameter raising screws of S.A.E. 3140 steel, operated by two ball-bearing-mounted internally lubricated drive nuts, the mechanism being irreversible except under power. The least factor of safety in any part of the raising mechanism is 8.

Extending the upper section is accomplished by a small extending winch of 45,000 lb ultimate capacity, also driven from the main propeller shaft, using a $^5/_8$-in. wire line anchored in the derrick. The derrick is locked in either a high or low position by means of a screw-drive pawl-and-lug mechanism operable from the ground. Design of the pawl and lug is such that the load is transferred by direct bearing from the leg of the upper section to the leg of the lower section.

An electrical signaling device warns the operator when the derrick has been extended to its full height and indicates when the pawls are locked. The derrick may be tilted over the center as much as 6 deg to allow its use on wells having various cellar sizes; the degree of tilt can be easily selected by use of a built-in spirit-level protractor. Throwing out the main friction clutch stops and locks the raising operation in any position of the derrick.

7 Four guy lines of $^3/_4$-in. improved plow-steel 6 × 19 wire rope are used. The two back guys are carried straight back to within 10 ft of the nose of the machine. The two forward guys are set at an angle of approximately 120 deg to the center line of the two back guys, and about 15 to 25 ft from the center line of the well. Slack in each guy is quickly taken up by means of a 6-ton roller-chain-type Coffing hoist, located on the end of each. Erection and tiedown time with this arrangement can easily be held to $^1/_2$ hr.

8 Legs are S.A.E. X4130 chrome-moly cold-drawn seamless-steel tubing in the normalized condition, having a minimum yield point of 85,000 psi. Front leg size is 5 in. OD × $^9/_{32}$ in. wall (maximum in lower section). Rear legs size is 4 in. OD × $^3/_{16}$ in. wall. Wall thickness is reduced in front legs in upper section. Legs are unequally loaded. The derrick was analyzed graphically with loads being introduced for the guy lines, hook load, pipe setback, and a 70-mph wind upon the structure. Also, the derrick was analyzed in both the guyed and unguyed conditions. Pipe-setback loads and wind loading conform to A.P.I. Standards No. 4; but since the leg material used was of better quality than the A.P.I. column formulas allow for, a column formula was selected from the ANC-5, a government handbook on the strength of aircraft elements. This column formula is used for material having a 75,000-lb or better yield point and is as follows:

$$\text{Allowable unit stress: } 79{,}500 - 51.9\left(\frac{L}{r}\right)1.5$$

where L = unbraced length of column between girts
r = radius of gyration

All joints are treated as pin joints.

Interior bracing is of S.A.E. 1015 cold-drawn seamless-steel tubing in the finish-annealed condition. This material has a minimum yield point of 55,000 psi. The column formula used was supplied by the Bethlehem Steel Company and is as follows:

Allowable unit stress with a factor of 1.8 over failure = 22,000 $- 0.55\left(\frac{L}{r}\right)^2$ for short columns and

$$\frac{200{,}000{,}000}{(L/r)^2} \text{ for long columns}$$

The transitional L/r is 118.

Average column length for the legs is 72 in. Maximum column length for the legs is about 88 in. There are 14 bays in the derrick.

9 The crown block consists of four 25-in-OD flame-hardened manganese-steel sheaves, three grooved for 1-in. line and one for $^5/_8$-in. or $^9/_{16}$-in. sandline, and one 21-in-OD sheave grooved for $1^1/_2$-in. catline. All sheaves are ball-bearing mounted in line on a common shaft. The entire crown is integrally welded to the derrick.

10 A folding tubing platform is provided capable of racking 11,000 ft of $2^1/_2$-in. tubing or 5000 ft of $3^1/_2$-in. drill pipe. The tubing platform is hinged to a sliding track in the upper section of the derrick which allows easy adjustment of the tubing platform through a range of 15 ft, to compensate for the differences in a stand of 3 joints of range 1 and two joints of range 2.

11 Leveling screws are provided in all four legs. Also, should it be desired to use the hoist equipment under a standard derrick, an arrangement is provided to leave the portable derrick standing by itself by the removal of two capscrews and two nuts. This feature is often used in areas where the oil company has both standard and portable derricks.

When used as a drilling unit, a separate pump, mounted either on skid, truck, or trailer is employed. This pump is usually a $7^1/_4$-in. bore × 14 in. stroke and is powered by a 220- to 250-hp engine. This pump is rated to deliver 577 gpm at 550 lb pressure with $7^1/_4$-in. liners at 60 strokes per min.

Conclusion

In conclusion, it may be said that such equipment as has just been described has a definite place in the oil industry because of the following features:

1 Eliminates individual permanent derrick cost, since the portable derrick and hoist equipment will service up to 100 wells and also can be used for shallow- and medium-depth drilling.

2 Saves rig-up and tear-down time. Portable derricks are power-raised, and either rig-up time or tear-down time is reduced to a matter of minutes.

3 It is 100 per cent portable and is available in self-propelled or trailer mountings, permitting complete mobility of the derrick as part of the drilling or servicing unit. The derrick, crown block, draw works, and power plant are all unitized into a complete machine.

4 Carries lines and blocks strung. Lines and blocks may be carried strung at all times, including highway travel, thus reducing rig-up time.

5 Open-face tilted design permits greater block clearance with more room around the well head or rotary table.

6 Portable derrick can be left standing full of tubing in the field while the hoist is used under a standard derrick or with other equipment.

7 May be used as either a servicing machine for the deepest wells, or for rotary drilling by the use of the rotary-drilling attachment extras.

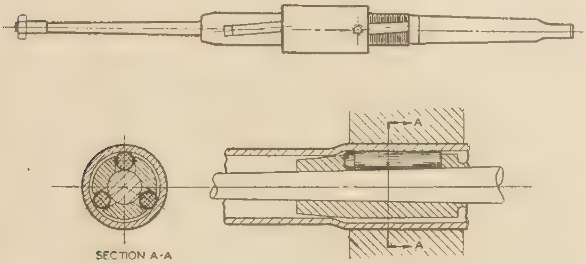
Theory of the Expanding of Boiler and Condenser Tube Joints Through Rolling

By A. Nadai,[1] East Pittsburgh, Pa.

Tubes in industrial water heaters, steam boilers, and condensers of turbines are fitted in the holes of adjoining drums or head plates by expanding the tube ends. These are slightly enlarged by means of small revolving rolls. In one large steam condenser many thousands of such tube joints have to be rolled, and in high-pressure boilers these joints must remain tight under several thousands of pounds pressure at high temperatures. The investigation reported in this paper, of the conditions under which tube joints are expanded and made pressure-tight, was undertaken at the suggestion of R. A. Bowman, manager of condenser engineering, of the South Philadelphia Works of the Westinghouse Electric and Manufacturing Company, early in 1942. The pressure of the revolving rolls creates a radial plastic distribution of stress in the tube wall and around the hole in the adjoining heavy steel plate. After the tube end has been rolled, a system of residual stresses remains locked up near the joints which is essential for its pressure tightness. These plastic states of stress have been investigated for various types of the stress-strain characteristics of the tube metal and steel of the head plates. Simple rules are used for computing the stresses in a moderately thick-walled tube under external and internal pressure, either in the elastic or in the plastic state of stress.

1 Tube-Joint Problem

The tube ends are expanded to the required small amount by means of devices consisting usually of three hardened rolls with a slight taper which are mounted symmetrically around a long tapered pin. An example of a roller expander used for the rolling of condenser tubes is shown in Fig. 1. The device with the three rolls is inserted in the tube; the rolls are rotated by a motor while the tapered pin is slowly advanced. The radial enlargement of the tube ends depends upon the pressure for which the joint



TUBE EXPANDER

Fig. 1

has to be designed and has empirically been determined. Condenser tubes have to operate under less than 1 atm pressure difference; tubes in feedwater heaters perhaps under 2000 psi or more at 400 to 500 F; and boiler tubes at several thousands of pounds pressure and much higher temperatures.

The pressures must be determined in the contact surface of the tube and the steel plate in function of the small permanent increase of the inner or outer tube diameters. This is not a simple problem, even if the stress-strain behavior of the tube and plate metals in the plastic state is known. The reasons are quite understandable. The distribution of the stresses in a rolled joint is not of a simple nature. Joints which must remain tight at an elevated temperature introduce further considerable difficulties for a mechanical treatment, because the locked-up stresses must gradually change with time. The only hope for obtaining some quantitative information on these stresses can be based on simplified assumptions. When attempting to describe the state of deformation in an expanded joint, three questions are raised, as follows:

(*a*) The first to be considered refers to the plastic expansion process, during which the wall thickness of the tube is reduced and the diameter of the hole in the steel plate is permanently enlarged to a slight degree. The assumption is made that the flow in the tube and in the plate is radially symmetrical. In other words, it is assumed that all points at equal distances from the axis of the tube move uniformly in radial directions by the same amounts. In addition, small axial movements will also occur, which must be considered. Under these circumstances, the stresses will also be radially symmetrical and the pressures at any instant constant on the inner and outer surfaces of the tube. It should be noted that the deformation during the actual rolling of a tube is not produced in this continuous manner, but through three concentrated forces in a succession of infinitesimal steps under the localized pressures of the three rolls.

After the tube and plate have been plastically deformed, we shall assume that the external pressure which acted along the inner surface of the tube is gradually removed. In the afterwake, a radial distribution of residual stresses remains around the tube, which will be determined.

(*b*) The second question refers to the behavior of tube joints at elevated temperatures. If the joint is exposed continuously to heat, which is the case in boilers and water heaters, the system of residual stresses, remaining after the cold expansion, must undergo further changes due to creep. From long-time experiments, it is known that creep rates increase much faster than the stresses. As a consequence of these laws of creep deformation, we must expect that the high values of the residual stresses, remaining after the cold-rolling, will be reduced, and the peak stresses leveled off with time. The contact pressure required for the maintenance of a tight joint must be reduced. This is a special case of the relaxation of a system of radial stresses under prolonged heating, which deserves to be investigated separately.

(*c*) In a tube joint, which has been expanded through rolling, an additional system of stress is present. Although it may not affect the tightness of the joint greatly, it can become detrimental. During rolling, the portion of the tube which was inserted in the head plate is supported. Since this has a limited length

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

within which the tube diameter has been enlarged, such a deformation of the tube wall must be accompanied by a concentration of high axial and peripheral bending stresses near the end of the expanded zone. These bending stresses almost certainly reach the plastic limit and constitute an undesirable addition to the stresses, the presence of which may facilitate corrosion attacks. If the operating temperatures are sufficiently high, creep may reduce the peaks of these superposed bending stresses; at lower temperatures they may be detrimental.

The observations furthermore show that the compressed portion of the tube becomes longer in the axial direction. This motion is resisted by friction forces in the contact surface, which produce an additional disturbance in the distribution of the stresses.

When the essential parts of the following computations were submitted in a report dated February 11, 1942, only case (*a*) had been considered.[2] Treatment of the time-dependent case (*b*), of relaxation of the contact pressures, and of case (*c*) is contemplated in a subsequent investigation.

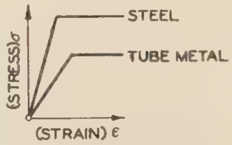
For an investigation of this kind, it is necessary to know the mechanical behavior of the tube metal and of the steel plates, or their stress-strain relations. The head plates in condensers are made of ordinary low-carbon steel, for which a stress-strain curve in the range of strains which are here of interest can be assumed, consisting of an inclined straight line in the elastic and of a horizontal straight line in the plastic range of the strains. If other or alloy steels are used, other stress-strain relations should be considered. The stress-strain characteristics of the 70 per cent copper and 30 per cent nickel alloy (Admiralty metal), which is used in condensers, were not known to the writer at the time when the report was written. To facilitate the subsequent computations, two extreme cases were considered for the mechanical behavior of the materials, namely, (*a*) both the tube metal and the steel have well-defined yield stresses which are σ_0 and σ_0' for pure tension, respectively, assuming that $\sigma_0 < \sigma_0'$. Stress-strain curves have the shapes shown in Fig. 2. (*b*) The steel plate has the same stress-strain curve as shown in Fig. 2, but that for the tube metal consists of a curve which becomes tangent to the straight line expressing the elastic behavior at small stresses, Fig. 3. A suitable expression would perhaps be for softer metals

$$\epsilon = \alpha\sigma + \beta\sigma^n \qquad [1]$$

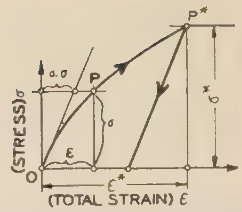
where ϵ designates the strain, σ the stress, and α, β, n are material constants. The first term represents the elastic strain, assuming that the constant $\alpha = 1/E$, where E is the modulus of elasticity of the tube metal. The second term in Equation [1] is the plastic strain. The functional relation $\epsilon = f(\sigma)$, according to Equation [1], is valid as long as the load or stress increases. After loading is interrupted and during unloading, in most cases, a straight-line relation

$$\epsilon = \epsilon^* - \frac{(\sigma^* - \sigma)}{E} \qquad [1a]$$

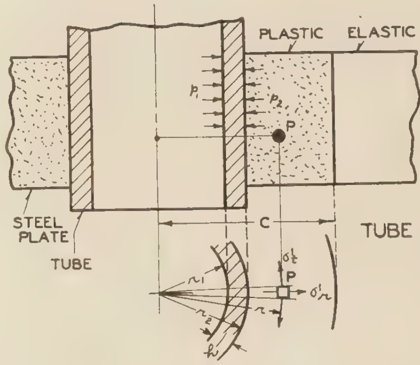
can be used, where ϵ^*, σ^* are the values of the strain and stress from which unloading started, Fig. 2. With these expressions, it is assumed that the contact pressures depend only upon the



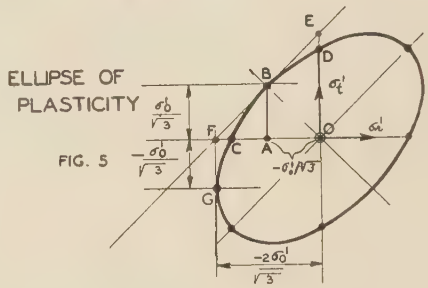
STRESS-STRAIN CURVES
FIG. 2



STRESS-STRAIN CURVE FOR TENSION FOR TUBE MATERIAL.
FIG. 3



TUBE AND STEEL PLATE
FIG. 4



FIGS. 2, 3, 4, AND 5

elasticity, and the stress-strain curves of the tube and plate materials, but not upon the time during which stress acts, or upon the rate of flow.

We shall assume that, initially, the tube fits without clearance in the hole. Under a hydrostatic external pressure which is applied on the inner surface of the tube, this latter and the steel plate will be stressed in the radial and tangential directions. They will expand first elastically, if the tube metal has a stress-strain curve similar to the one shown in Fig. 2. However, if the stress-strain curve has the shape shown in Fig. 3, permanent strains soon start to develop in the tube walls. Experience shows and the theory of plastic deformation predicts that, during expanding of a tube by a roller expander, the portion of the tube which is radially deformed increases its length by a small amount. This axial lengthening is resisted by the friction of the steel plate. Neglecting this friction, we can assume that during expansion the axial normal stress (σ_z) vanishes both in the tube wall and in the steel plate. Therefore $\sigma_z = 0$.

As mentioned, two phases of the expansion process must be considered: (1) The plastic expansion during which all radii increase with the increasing external pressure which is applied to the inner surface of the tube. The plastic zone in the steel plate grows during this phase of the process. (2) An elastic contraction of the tube and plate when the external pressure is reduced to zero, in consequence of which a residual pressure appears in the contact surface.

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[2] The author was at that time not aware that related work was under consideration and in progress under the auspices of the Research Committee and the Power Division of the Society. At the 1942 Annual Meeting, three papers were presented on the subject of tube expansion: namely, E. D. Grimison and G. H. Lee, "Experimental Investigation of Tube Expanding;" C. A. Maxwell, "Practical Aspects of Making Expanded Joints;" and J. N. Goodier and G. J. Schoessow, "The Holding Power and Hydraulic Tightness of Expanded Tube Joints, Analysis of the Stress and Deformation." The third paper in this series deals with the same subject as this present investigation. [These three papers are published in the Transactions of the A.S.M.E., vol. 65, 1943, pp. 489–522.]

We shall designate the quantities which refer to the steel plate with a dash and those referring to the tube without a dash. The subscript "one" shall refer to the inner and the subscript "two" to the outer surface of the tube. The subscript zero refers to the yield stress (σ_0 for the tube and σ_0' for the steel plate) in pure tension.

The following designations will be used:

r = radial distance of a point P from axis
u = radial displacement at point P
$\epsilon_r, \epsilon_t, \epsilon_z$ = radial, tangential, and axial strain, respectively
$\sigma_r, \sigma_t, \sigma_z$ = radial, tangential, and axial stress, respectively
r_1 = inner radius of tube
r_2 = outer radius of tube
h = wall thickness of tube
p_1 = pressure on inside of tube
p_2 = pressure on outside of tube
ν = Poisson's ratio of material
E, E' = moduli of elasticity of materials
G, G' = moduli of rigidity of materials
ν_1 = r_1/h, $\nu_2 = r_2/h$ = abbreviations

2 Spreading of Plastic Distortion in Steel Plate Under Radial Pressure

Assuming a flat infinite steel plate having a circular hole of radius r_2, our first task consists in determining the function which expresses how the pressure p_2 exerted at the hole increases with the radial displacement u_2 at the bore, while plastic flow spreads in the plate. Small pressures p_2 deform the plate elastically. Assuming a steel having a well-defined yield point, at a certain value of the pressure p_2, yielding will start along the circle $r = r_2$. If the pressure p_2 increases further, the steel plate will yield in a zone $r_2 < r < c$ but will remain partially stressed elastically beyond it ($r > c$).

As long as the strains are purely elastic, the following distribution of the stresses exists in the plate:

$$\left.\begin{aligned} &\text{Radial stress} \quad \sigma_r' = -p_2 r_2^2/r^2 \\ &\text{Tangential stress} \quad \sigma_t' = p_2 r_2^2/r^2 \end{aligned}\right\} \qquad [2]$$

$$\text{Tangential strain} \quad \epsilon_t' = (\sigma_t' - \nu'\sigma_r')/E' = (1 + \nu')p_2 r_2^2/E' r^2 \qquad [3]$$

The radial displacement is

$$u' = r\epsilon_t' \qquad [4]$$

At the bore $r = r_2$ this displacement amounts to

$$u_2' = (1 + \nu')p_2 r_2/E' \qquad [5]$$

After yielding has started in the plate, the shearing stress in the plastic zone $r_2 < r < c$

$$\tau_0 = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad [6]$$

remains constant and equal to $\sqrt{2}\cdot\sigma_0'/3$. In Equation [6], σ_1 σ_2, σ_3 designate the principal stresses.

Assuming for $\sigma_1 = \sigma_t'$, $\sigma_2 = \sigma_r'$, $\sigma_3 = \sigma_z' = 0$, this equation simplifies to the following condition of flow or of plasticity:

$$\underline{\underline{\sigma_t'^2 - \sigma_t'\sigma_r' + \sigma_r'^2 = \sigma_0'^2 = \text{const.}}} \qquad [7]$$

where σ_0' is the yield stress for pure tension. In addition to Equation [7] in the plastic zone, we must satisfy the equilibrium condition

$$\frac{d}{dr}(r\sigma_r') - \sigma_t' = 0 \qquad [8]$$

The solutions of these two equations have been expressed previously by the author in the parametric form

$$\left.\begin{aligned} \sigma_t' &= \frac{2\sigma_0'}{\sqrt{3}}\sin\left(\theta + \frac{\pi}{6}\right) \\ \sigma_r' &= \frac{2\sigma_0'}{\sqrt{3}}\sin\left(\theta - \frac{\pi}{6}\right) \end{aligned}\right\} \qquad [9]$$

$$r^2 = c^2 \cdot \frac{e^{\sqrt{3}\theta}}{\cos\theta} \qquad [9a]$$

Equation [7] represents "the ellipse of plasticity," Fig. 5. It contains the following six points: $\sigma_r' = \pm\sigma_0'$; $\sigma_t' = 0$; $\sigma_r' = \sigma_t' = \pm\sigma_0'$; $\sigma_r' = 0$; $\sigma_t' = \pm\sigma_0'$.

It is of importance for the following application to note now that all the plastic states in an "infinite" plate which can be practically generated by a radial pressure p_2 are found in the branch BCG of this ellipse.

Point B, having the co-ordinates $\sigma_r' = -p_2$, $\sigma_t' = p_2 = \sigma_0'/\sqrt{3} = 0.577\ \sigma_0'$, evidently represents the plastic state at the bore just when yielding starts in the plate, since according to Equation [2] at this instant $-\sigma_r' = \sigma_t'$, the values of which after being substituted in Equation [7] must equal $\sigma_0'/\sqrt{3}$ Point B of the ellipse of plasticity, Fig. 5, represents first yielding in the plate, and furthermore we must have $r_2 = c$ in Equation [9a] or $\theta = 0$. Let us designate by θ_2 the value of the parameter θ when $r = r_2$. We note that when $r_2 = c$, $\theta_2 = 0$. The radius c to which the plastic zone penetrates is found from Equation [9a].

$$c^2 = r_2^2 e^{-\sqrt{3}\theta_2}\cos\theta_2 \qquad [10]$$

When plotted in function of θ_2, c^2 is represented by a portion of the well-known curve of damped oscillations. The corresponding pressure p_2 for which the plastic zone extends to the radius c is given by

$$\sigma_r' = -p_2 = \frac{2\sigma_0'}{\sqrt{3}}\sin\left(\theta_2 - \frac{\pi}{3}\right) \qquad [11]$$

The following table illustrates corresponding values of θ_2 and p_2:

θ_2	$\dfrac{\sqrt{3}p_2}{2\sigma_0'}$	Interval for θ
0	0.5000	0
−15°	0.7071	$-15° < \theta < 0$
−30°	0.8660	$-30° < \theta < 0$
−45°	0.9659	$-45° < \theta < 0$
−60°	1.000	$-60° < \theta < 0$

Equation [9a] may be rewritten after using Equation [10] also as

$$\frac{1}{r^2} = \frac{e^{-\sqrt{3}\theta}\cos\theta}{c^2} = \frac{e^{\sqrt{3}(\theta_2-\theta)}\cos\theta}{r_2^2\cos\theta_2} \qquad [12]$$

A few curves showing the distributions of the radial and the tangential stresses σ_r', σ_t' in the steel plate, corresponding to the values of θ_2, given in the preceding table were plotted in Fig. 6. They show how these stresses vary with the radius after partial yielding when the steel plate is expanded under an increasing pressure p_2.

From the curves and by looking to the ellipse of plasticity, Fig. 5, we must conclude that the pressure p_2 soon must reach the maximum value

$$p_{2\max} = \frac{2\sigma_0'}{\sqrt{3}} = 1.155\sigma_0' \qquad [13]$$

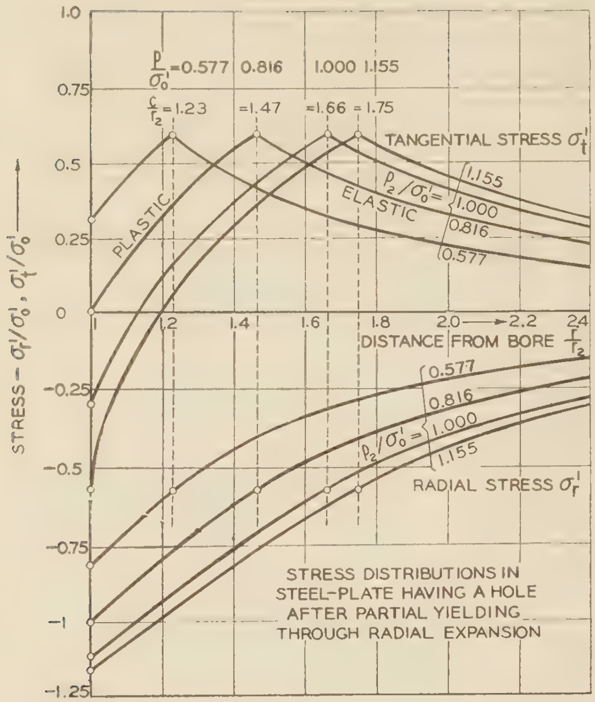

Fig. 6

This is a consequence of our assumption that the material of the steel plate has a definite yield point. Of further interest is that, when the radial pressure approaches the value given by Equation [13], the plastic zone does not increase more appreciably. The radius c of the plastic zone, corresponding to $p_{2\max}$ can be computed from Equation [10] by taking $\theta_2 = -\pi/3$. This gives a radius $c = 1.75r_2$. Not much is gained in carrying the expansion of the steel plate beyond this point.

We must now establish the required function $p_2 = f(u_2)$, expressing the dependence of the pressure p_2 on the radial displacement u_2. We can simplify considerably the further treatment by assuming that the straight line EBF in Fig. 5, which is the tangent to the ellipse of plasticity in point B, represents the plastic states instead of the branch BCG of this curve. Instead of using Equation [7] as the condition of plasticity, we assume that the steel yields when the stresses σ_r' and σ_t' satisfy a linear equation

$$\sigma_t' = \frac{2\sigma_0'}{\sqrt{3}} + \sigma_r' \qquad [14]$$

which is the equation of the tangent to the ellipse at point B. After combining Equation [14] with Equation [8], the following approximate expressions for the stresses σ_r and σ_t are easily verified

$$\left.\begin{aligned} \sigma_r' &= \frac{\sigma_0'}{\sqrt{3}}\left(-1 + 2\log_e\frac{r}{c}\right) \\ r_2 \leq r \leq c, \qquad & \\ \sigma_t' &= \frac{\sigma_0'}{\sqrt{3}}\left(1 + 2\log_e\frac{r}{c}\right) \end{aligned}\right\} \qquad [15]$$

We note that an approximate expression for the pressure p_2 required to make the steel plate yield within a circle of the radius c is given now by

$$p_2 = \frac{\sigma_0'}{\sqrt{3}}\left(1 + 2\log_e\frac{c}{r_2}\right) \qquad [16]$$

After making use of the formulas, Equations [2], [3], and [6], we see also that in the elastic zone

$$\left.\begin{aligned} \sigma_t' = -\sigma_r' &= \frac{\sigma_0'}{\sqrt{3}}\cdot\frac{c^2}{r^2} \\ u &= (1 + \nu')\sigma_0' c^2/\sqrt{3}E'r \end{aligned}\right\} \text{ For } c < r \ldots [17]$$

At the boundary of the plastic region $r = c$, we shall designate the radial displacement by u_c which is therefore given by

$$u_c = (1 + \nu')\sigma_0' c/\sqrt{3}E' \qquad [18]$$

Let u_c^* be the value of u_c when $c = r_2$, i.e. just when yielding in the steel plate starts. Then

$$u_c^* = (1 + \nu')\sigma_0' r_2/\sqrt{3}E' \qquad [19]$$

and

$$u_c = u_c^*\cdot\frac{c}{r_2} \qquad [20]$$

Equation [19] expressing u_c^* will be used in the following computations. Finally, we need an expression of the radial displacements u within the plastic zone $r_2 < r < c$.

For the plastic strains we must have (condition of volume constancy)

$$\epsilon_r' + \epsilon_t' + \epsilon_z' = 0 \qquad [21]$$

$$\left(\begin{gathered}\text{stress-strain}\\ \text{relations}\end{gathered}\right) \quad \left.\begin{aligned} \epsilon_r' &= \frac{du'}{dr} = k\left(\sigma_r' - \frac{\sigma_t'}{2}\right) \\ \epsilon_t' &= \frac{u'}{r} = k\left(\sigma_t' - \frac{\sigma_r'}{2}\right) \\ \epsilon_z' &= -k(\sigma_r' + \sigma_t')/2 \end{aligned}\right\} \qquad [22]$$

From these relations and Equations [15], we obtain for the ratio of the strains ϵ_z' and ϵ_t'

$$\frac{\epsilon_z'}{\epsilon_t'} = -\frac{2\log\frac{r}{c}}{\frac{3}{2} + \log\frac{r}{c}} \qquad [23]$$

After combining Equations [23] with [22], we obtain the differential equation for the radial displacement u' in the plastic zone

$$\frac{du'}{dr} + \frac{\frac{3}{2} - \log\frac{r}{c}}{\frac{3}{2} + \log\frac{r}{c}}\cdot\frac{u'}{r} = 0 \qquad [24]$$

This equation can be integrated and is satisfied by

$$u' = \left(\frac{3}{2}\right)^3 \frac{u_c r}{c\left(\frac{3}{2} + \log\frac{r}{c}\right)^3} \qquad [25]$$

which function is valid for $r_2 < r < c$. When $r = c$, we obtain $u = u_e$ as we should expect and for $r = r_2$ at the bore

$$u_2' = \left(\frac{3}{2}\right)^3 \frac{u_e r_2}{c\left(\frac{3}{2} + \log\frac{r_2}{c}\right)^3} \quad \ldots\ldots\ldots\ldots [26]$$

or remembering Equation [20]

$$u_2 = \frac{u_e^*}{\left(1 + \frac{2}{3}\log_e\frac{r_2}{c}\right)^3} \quad \ldots\ldots\ldots\ldots [27]$$

In the denominator, we will now substitute the pressure p_2 taken from Equation [16], which gives

$$u_2 = \frac{u_e^*}{\left(\frac{4}{3} - \frac{p_2}{\sqrt{3}\sigma_0'}\right)^3} \quad \ldots\ldots [28]$$

and after solving this for the pressure p_2 this is found to be equal to

$$p_2 = \frac{\sigma_0'}{\sqrt{3}}\left[4 - 3\left(\frac{u_e^*}{u_2}\right)^{1/3}\right] \quad \ldots\ldots\ldots\ldots [29]$$

Summing up the preceding, we see that the pressure in function of the radial displacements u_2 at the bore r_2 is expressed by the following:

In the elastic region

$$p_2 = \frac{\sigma_0'}{\sqrt{3}} \cdot \frac{u_2}{u_e^*} \quad \text{for} \quad u_2 < u_e^* \ldots\ldots\ldots\ldots [30]$$

In the plastic region

$$p_2 = \frac{\sigma_0'}{\sqrt{3}}\left[4 - 3\left(\frac{u_e^*}{u_2}\right)^{1/3}\right] \quad \text{for} \quad u_2 > u_e^* \ldots [31]$$

where u_e^* is the radial enlargement of the hole in the steel plate just when the yield point σ_0' is first reached in the plate and is given by

$$u_e^* = (1 + \nu')\sigma_0' r_2/\sqrt{3}E' \quad \ldots\ldots\ldots\ldots [32]$$

This function is shown in Fig. 7, indicating that the pressure increases first linearly with u_2 and after the yield point in the steel plate has been reached (at the pressure $p_2 = \sigma_0'/\sqrt{3}$), according to a curve which very slowly approaches a horizontal asymptote at a pressure which is 4 *times* the pressure at which yielding started.

We know from Equation [13] that the maximum pressure was $p_{2max} = 2\sigma_0'/\sqrt{3} = 1.155\sigma_0'$. This latter then defines the horizontal asymptote of the curve $p_2 = f(u_2)$, or the horizontal asymptote which the pressure line, in Fig. 7, should really approach. It should have the height $\pi = 2$ in the scale used in Fig. 7. That our approximation works pretty well, in so far as the curve in Fig. 7 has been traced, has been proved by comparing it with some values which have been computed through additional methods which cannot be reproduced here in detail.[3] Values of p_2 beyond those shown in Fig. 7 should, however, not be computed by using the approximate form given by Equation [31].

We can summarize the preceding developments very briefly as follows: If we introduce instead of the radial displacements

[3] The writer is indebted to Mr. R. K. Carlson who recently carried out this computation at his request.

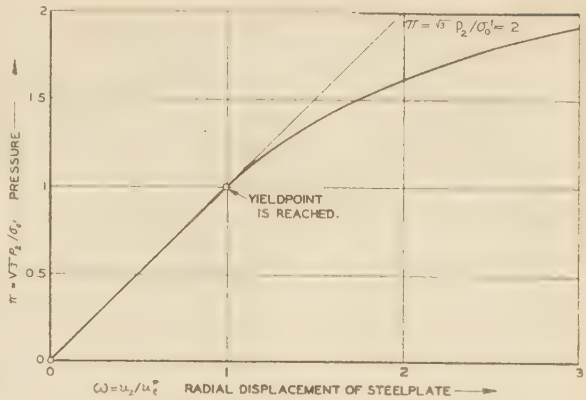

FIG. 7 RADIAL PRESSURE p_2 IN FUNCTION OF RADIAL DISPLACEMENT u_2 OF STEEL PLATE

u_2 at the bore $r = r_2$ of the steel plate and instead of the pressure two new variables ω and π defined by

$$\omega = \frac{u_2}{u_e^*}, \quad \pi = \frac{\sqrt{3}p_2}{\sigma_0'} \quad \ldots\ldots\ldots\ldots [33]$$

we can rewrite the pressure function $\pi = F(\omega)$, given by Equations [30] and [31], in the simple form

$$\left.\begin{array}{ll} \text{when } 0 < \omega < 1 \text{ (elastic range)} \ldots\ldots & \pi = \omega \\ \text{and when } 1 < \omega \text{ (plastic range)} \ldots\ldots & \pi = 4 - 3\omega^{-1/3} \end{array}\right\} \ldots [34]$$

This pressure-expansion curve can be used for steel plates having a definite or fairly well-defined yield point σ_0' in pure tension. The second Equation [34] should not be used for values of the variable ω which are much larger than perhaps $\omega = 3$, and it is not recommended to go much beyond this expansion ratio $u_2/u_e^* = 3$ for the reasons already stated; u_e^* is the elastic expansion of the plate given by Equation [32].

3 Expansion of Tube Joint Assuming a Definite Yield Point and a Stress-Strain Curve According to Fig. 2 for Tube Material

After having considered the deformation of the steel plate, we will now treat the plastic yielding of the tube material. The ends of the tubes are strained under the simultaneous action of an internal pressure p_1 and of an external pressure p_2. As long as the tube is stressed elastically, the following well-known distribution of stresses acts in the tube wall:

$$\left.\begin{array}{ll} \text{Radial stress} & \sigma_r = -p_1 \dfrac{\dfrac{r_2^2}{r^2} - 1}{\dfrac{r_2^2}{r_1^2} - 1} - p_2 \dfrac{1 - \dfrac{r_1^2}{r^2}}{1 - \dfrac{r_1^2}{r_2^2}} \\ \text{Tangential stress} & \sigma_t = p_1 \dfrac{\dfrac{r_2^2}{r^2} + 1}{\dfrac{r_2^2}{r_1^2} - 1} - p_2 \dfrac{1 + \dfrac{r_1^2}{r^2}}{1 - \dfrac{r_1^2}{r_2^2}} \end{array}\right\} \ldots\ldots [35]$$

If we designate the ratio r_1/r_2 by

$$k = \frac{r_1}{r_2} \quad \ldots\ldots\ldots\ldots [36]$$

we obtain from Equation [35]

$$\text{for} \quad r = r_1 \ldots \begin{cases} \sigma_r = -p_1 \\ \sigma_t = \dfrac{(1+k^2)p_1 - 2p_2}{1-k^2} \end{cases} \ldots [37]$$

$$\text{for} \quad r = r_2 \ldots \begin{cases} \sigma_r = -p_2 \\ \sigma_t = \dfrac{2p_1k^2 - (1+k^2)p_2}{1-k^2} \end{cases} \ldots [38]$$

and the elastic radial expansion u is given by

$$u = \epsilon_t r = \frac{(\sigma_t - \nu\sigma_r)r}{E} \ldots [39]$$

For $r = r_1$ we obtain

$$u_1 = \frac{[1 + k^2 + \nu(1-k^2)]p_1 - 2p_2}{(1-k^2)E} \cdot r_1 \ldots [40]$$

and for $r = r_2$

$$u_2 = \frac{2p_1k^2 - [1 + k^2 - \nu(1-k^2)]p_2}{(1-k^2)E} \cdot r_2 \ldots [41]$$

It is useful in many practical applications which can be made of these formulas for the thick-walled tube to develop the functions appearing in Equation [35] in powers of a new variable x defined by

$$x = r - r_m \ldots [42]$$

where $r_m = (r_1 + r_2):2$ is the mean radius of the tube. If the thickness of the tube wall h is not too large, compared to the mean radius r_m, it is sufficient when the expressions are developed to retain only the first power of x. Equations [35, 40, 41] when expressed in terms of the new variable x become linear functions of x. From Equation [35], we obtain

$$\left.\begin{aligned} \sigma_r &= -p_1\left(\frac{1}{2} - \frac{x}{h}\right) - p_2\left(\frac{1}{2} + \frac{x}{h}\right) \\ \sigma_t &= p_1\frac{(r_1 - x)}{h} - p_2\frac{(r_2 - x)}{h} \end{aligned}\right\} \ldots [43]$$

Taking $x = 0$, we note that the mean tangential stress in a tube with a finite wall thickness h is

$$\sigma_{tm} = \frac{p_1r_1 - p_2r_2}{h} \ldots [43a]$$

If we introduce the abbreviations

$$\nu_1 = \frac{r_1}{h}, \quad \nu_2 = \frac{r_2}{h}$$

$$\text{noting that } \nu_1 - \nu_2 = 1 \ldots [44]$$

σ_{tm} is given by the expression

$$\underline{\sigma_{tm} = p_1\nu_1 - p_2\nu_2} \ldots [45]$$

This latter expression is valid in a tube in which the strains may either be elastic or also plastic and is exact with inclusion of linear terms of x.

Now, making use of Equation [39], we find that the corresponding approximate expressions for the radial displacements are

$$\left.\begin{aligned} &\text{for } r = r_1 \quad u_1 = [(1 + 2\nu + 2\nu_1)p_1 - (1 + 2\nu_2)p_2]\frac{r_1}{2E} \\ &\text{for } r = r_2 \quad u_2 = [(2\nu_1 - 1)p_1 - (2\nu_2 - 1 - 2\nu)p_2]\frac{r_2}{2E} \end{aligned}\right\} . [46]$$

If, for example, the ratio of inner to outer radius $k = r_1/r_2 = 4/5 = 0.8$ from the exact expressions Equations [40] and [41] we find

$$\left.\begin{aligned} u_1 &= (4.86p_1 - 5.56p_2)\frac{r_1}{E} \\ u_2 &= (3.56p_1 - 4.26p_2)\frac{r_2}{E} \end{aligned}\right\} \ldots [47]$$

while from the approximate expressions Equation [46], we compute

$$\left.\begin{aligned} u_1 &= (4.80p_1 - 5.50p_2)\frac{r_1}{E} \\ u_2 &= (3.50p_1 - 4.20p_2)\frac{r_2}{E} \end{aligned}\right\} \ldots [48]$$

showing that these "linearized" expressions, Equations [42] [45], and [46], can serve very well instead of the exact terms. By taking the expression of u_2 given by Equations [46] and equating it to the elastic radial expansion of the steel plate, Equation [5], we obtain

$$\left.\begin{aligned} &[(2\nu_1 - 1)p_1 - (2\nu_2 - 1 - 2\nu)p_2]\frac{r_2}{2E} = \frac{(1+\nu')p_2r_2}{E'} \\ &\text{or} \\ &\underline{\underline{p_1 = \frac{\left\{2\nu_2 - 1 - 2\nu + \dfrac{2E}{E'}(1+\nu')\right\}p_2}{(2\nu_1 - 1)}}} \end{aligned}\right\} \ldots [49]$$

For the same example ($k = r_1/r_2 = 0.8$) we find $p_1 = \dfrac{4.85}{3.50}\,p_2$

or

$$\underline{\underline{p_2}} = \frac{3.50}{4.85}\,p_1 = \underline{\underline{0.722p_1}} \ldots [50]$$

for the elastic range of the strains in the tube below the yield point.

When the inner pressure p_1 increases, the yield stress will first be reached in the tube wall. In the following computation it is assumed (1) that the material of the tube has for pure tension a definite yield point at a stress σ_0 and (2) that the steel plate is only stretched elastically. It is required to find the function $p_1 = f(u_1)$, connecting the pressure p_1 (under which the tube is rolled) with the permanent increase of the inner radius u_1. Also, the residual pressure p_2 is required between the tube and the steel plate after the pressure p_1 has been reduced to zero.

We can again make use of the plasticity condition Equation [7] or

$$\sigma_r^2 - \sigma_r\sigma_t + \sigma_t^2 = \sigma_0^2 \ldots [51]$$

where σ_0 is the yield stress of the tube material for pure tension.

We shall make use of this flow condition along a cylinder whose radius is equal to the mean radius $r_m = (r_1 + r_2)/2$ of the tube. We note from Equation [45] that for $x = 0$ in the plastic state of stress, we must have

$$\sigma_r = -\frac{(p_1 + p_2)}{2}, \quad \sigma_t = p_1\nu_1 - p_2\nu_2 \ldots [52]$$

After substituting these values in Equation [51] we obtain

$$(p_1\nu_1 - p_2\nu_2)^2 + \frac{(p_1 + p_2)}{2}(p_1\nu_1 - p_2\nu_2) + \frac{(p_1 + p_2)^2}{4} = \sigma_0^2 . [53]$$

or

$$\left[\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right] p_2^2 - 2\nu_1\nu_2 \cdot p_1p_2 + \left[\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right] p_1^2 = \sigma_0^2 \qquad [54]$$

The bracketed expressions are constants. If we plot p_1 and p_2 as rectangular co-ordinates, this is the equation of an ellipse. This ellipse has been represented in Fig. 8, assuming the same numerical values which were used previously; namely, for $r_1 = 0.4$ in., $r_2 = 0.5$ in., $h = 0.1$ in. or for $\nu_1 = r_1/h = 4$, $\nu_2 = r_2/h = 5$. Its equation is

$$22.75p_2^2 - 40p_1p_2 + 18.25p_1^2 = \sigma_0^2 \qquad [55]$$

In Fig. 8 has also been plotted a straight line ORP whose equation was given by Equation [50]. If the inner pressure p_1

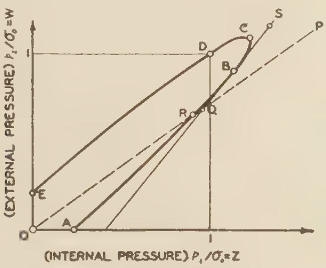

Fig. 8 Internal and External Pressures p_1 and p_2 for the Expansion of a Tube Having a Stress-Strain Curve With a Definite Yield Point

is increased the tube is first elastically stretched and the outer pressure p_2 of the tube increases along this inclined straight line. In point R, the yield point of the tube material is reached. If the inner pressure p_1 changes further, the corresponding outer pressure p_2 is given by the ordinate of a point situated along the ellipse. The co-ordinates of the upper apex of the ellipse can easily be computed from Equation [55]. They were found equal to $p_1 = 1.220\sigma_0$, $p_2 = 1.091\sigma_0$. The ellipse crosses the p_1 axis in point A at $p_1 = 0.234\sigma_0$ and the p_2 axis in point E at $p_2 = 0.209\sigma_0$. It passes also through the point D having the co-ordinates $p_1 = p_2 = \sigma_0$.

If we wish to investigate the flow within the tube, we need to consider, in addition to the plasticity condition Equation [54], also the following relations; the strains being all plastic strains

$$\epsilon_r + \epsilon_t + \epsilon_z = 0 \qquad [56a]$$

$$\epsilon_r = \frac{du}{dr}, \quad \epsilon_t = \frac{u}{r} \qquad [56b]$$

therefore

$$\frac{du}{dr} + \frac{u}{r} = -\epsilon_z = \text{const} \qquad [56c]$$

or

$$u = \frac{c}{r} - \frac{\epsilon_z r}{2} \qquad [56d]$$

this gives for

$$r = r_1, \quad u_1 = \frac{c}{r_1} - \frac{\epsilon_z r_1}{2} \qquad [56e]$$

and for

$$r = r_2, \quad u_2 = \frac{c}{r_2} - \frac{\epsilon_z r_2}{2} \qquad [56f]$$

or

$$u_1r_1 - u_2r_2 = \frac{\epsilon_z}{2}(r_2^2 - r_1^2) \qquad [56g]$$

From the stress-strain relations in the plastic state

$$\left.\begin{aligned} \epsilon_r &= S\left(\sigma_r - \frac{\sigma_t}{2}\right) \\ \epsilon_t &= S\left(\sigma_t - \frac{\sigma_r}{2}\right) \\ \epsilon_z &= -S\left(\frac{\sigma_r + \sigma_t}{2}\right) \end{aligned}\right\} \qquad [56h]$$

we compute for the ratio of the plastic strains

$$\frac{\epsilon_t}{\epsilon_z} = \frac{\sigma_r - 2\sigma_t}{\sigma_r + \sigma_t} \qquad [56i]$$

or after using the expressions for σ_r and σ_t given by Equation [52] also

$$\frac{\epsilon_t}{\epsilon_z} = \frac{(1 + 4\nu_1)p_1 + (1 - 4\nu_2)p_2}{(1 - 2\nu_1)p_1 + (1 + 2\nu_2)p_2} \qquad [56j]$$

Here we introduce the value of ϵ_z taken from Equation [56g] or

$$\epsilon_z = \frac{2(u_1r_1 - u_2r_2)}{r_2^2 - r_1^2} \qquad [56k]$$

and for ϵ_t we can also write

$$\epsilon_t = \frac{u_m}{r_m} = \frac{u_1 + u_2}{r_1 + r_2} \qquad [56l]$$

This gives finally for the left side of Equation [56j]

$$\frac{\epsilon_t}{\epsilon_z} = \frac{u_1 + u_2}{2(\nu_1u_1 - \nu_2u_2)} \qquad [56m]$$

By equating Equations [56m] with [56j], an equation is obtained containing the unknowns u_1, u_2, and p_1, p_2. Two more relations for the same quantities have already been determined, namely, Equation [54] and Equation [5] or

$$u_2 = \frac{(1 + \nu')p_2r_2}{E'} \qquad [57]$$

Therefore by means of these three equations, a relation $u_1 = f(p_1)$ has been established, since from three equations two quantities can always be eliminated.

Instead of using Equation [54], we will now simplify the computation considerably by a further step. We shall replace the plasticity ellipse, Fig. 8, Equation [54], by its tangent in a point B. Fig. 8 shows that the ellipse is quite flat. Since we need only a small portion of it near the points R or B, we can replace it by one of its tangents. We choose for the point of tangency, a point B in which the mean tangential stress σ_{tm} in the tube vanishes. The co-ordinates of B are designated by p_1^*, p_2^*. They must satisfy the condition

$$p_1^*\nu_1 = p_2^*\nu_2 \qquad [58]$$

Combining this with Equation [54], we obtain for p_1^*, p_2^* the expressions

$$p_1^* = \frac{2\nu_2\sigma_0}{\nu_1 + \nu_2}, \quad p_2^* = \frac{2\nu_1\sigma_0}{\nu_1 + \nu_2} \qquad [59]$$

and for the tangent of the ellipse in point B the equation

$$-\nu_1p_2 + \nu_2p_1 - 2\sigma_0 = 0 \qquad [60]$$

Instead of the plasticity condition as originally expressed by Equation [54], we can now make use of Equation [60] in which p_1, p_2 are the running co-ordinates of this straight line which has been represented in Fig. 8 by the line QBS. Point B is the point of tangency and point Q is the intersection of the tangent with the extension of the straight line OR. Using again for an example the values $\nu_1 = 4$, $\nu_2 = 5$, we have for the plasticity condition Equation [60]

$$-4p_2 + 5p_1 - 2\sigma_0 = 0 \qquad [61]$$

and the co-ordinates of point Q are equal to $p_1 = 0.947\sigma_0$, $p_2 = 0.683\sigma_0$. The point R is situated on the straight line (representing the elastic states) and on the ellipse (representing the plastic states of stress). In point R, the tube starts to yield completely. The co-ordinates of R were found to be equal to $p_1 = 0.906\sigma_0$, $p_2 = 0.654\sigma_0$, showing that the point Q whose co-ordinates have just been computed is in the proximity of R.

Summing up the preceding, we find the following results: The pressure p_1 for expanding the tubes creates a pressure p_2 between the tube wall and the steel plate. Until the yield point is reached, the pressure p_2 can be computed from Equation [49]. For $\nu_1 = 4$, $\nu_2 = 5$ in the elastic range

$$p_2 = 0.722p_1 \qquad [62]$$

When $p_1 = 0.906\sigma_0$ and $p_2 = 0.654\sigma_0$ the yield point in the tube is reached (σ_0 yield stress for tension in tube material).

When the tube is further expanded beyond the plastic limit, we must simultaneously satisfy the three equations

$$\frac{u_1 + u_2}{2(\nu_1 u_1 - \nu_2 u_2)} = \frac{(1 + 4\nu_1)p_1 + (1 - 4\nu_2)p_2}{(1 - 2\nu_1)p_1 + (1 + 2\nu_2)p_2} \qquad [63]$$

$$-\nu_1 p_2 + \nu_2 p_1 - 2\sigma_0 = 0 \qquad [64]$$

$$u_2 = \frac{(1 + \nu')p_2 r_2}{E'} \qquad [65]$$

establishing a functional relation between the inner expanding pressure p_1 and the outer radial expansion u_2 of the tube

To solve these three equations we write for

$$u_1 = \frac{(1 + \nu')\sigma_0 r_2}{E'} \cdot y \qquad [66]$$

$$p_1 = z\sigma_0, \quad p_2 = w\sigma_0 \qquad [67]$$

and find

$$\frac{y + w}{2(\nu_1 y - \nu_2 w)} = \frac{(1 + 4\nu_1)z + (1 - 4\nu_2)w}{(1 - 2\nu_1)z + (1 + 2\nu_2)w} \qquad [68]$$

$$-\nu_1 w + \nu_2 z - 2 = 0 \qquad [69]$$

These two equations represent a common hyperbola $y = f(z)$, if w is eliminated from them. In our example, inner radius of tube $r_1 = 0.4$ in., outer radius $r_2 = 0.5$ in., wall thickness $h = 0.1$ in., $\nu_1 = 4$, $\nu_2 = 5$, $\nu' = 0.3$, $E' = 3 \times 10^7$ psi (steel), $E = 1.5 \times 10^7$ psi (tube material), $\sigma_0' = 30{,}000$ psi (steel) $\sigma_0 = 15{,}000$ psi (tube material), this reduces to finding points along the hyperbola

$$\frac{y + 1.250z - 0.500}{4y - 6.250z + 2.500} = \frac{-13.500z + 19}{6.750z - 5.50}$$

or

$$y = \frac{75.9z^2 - 142.1z + 44.8}{60.8z - 81.5} \qquad [70]$$

Table 1 gives a few points along this hyperbola.

TABLE 1 RADIAL EXPANSIONS u_1 AND RADIAL PRESSURES p_1, p_2 OF A TUBE

	$z = \frac{p_1}{\sigma_0}$	$y = \frac{E'u_1}{(1 + \nu')r_2\sigma_0}$	Inner pressure, psi, p_1	Outer pressure, psi, p_2	Inner radical expansion of tube, in. u_1
Elastic	0	0	0	0	0
range	0.947	0.965	14200	10400	0.000315
	0.947	0.91[a]	14200	10400	0.000296[a]
	1	1.03	15000	11200	0.000334
	1.10	1.33	16500	13100	0.000432
	1.20	1.90	18000	15000	0.000628
Plastic					
range	1.25	2.58	18750	16500	0.000838
	1.30	4.60	19500	16900	0.001500
	1.34		(20100)	(17600)	

[a] Note that the two values of y for $z = 0.947$ are different. $y = 0.965$ was computed from the theory of elasticity and the other value $y = 0.91$ using our plastic-flow condition. Evidently ϵ_t/ϵ_z jumps at the yield point, due to our assumptions, and this explains the smaller value $u_1 = 0.000296$ in the last column of the table, compared to $u_1 = 0.000315$ at $p_1 = 14{,}200$ psi.

The increase of the inner pressures p_1 with the radial expansions u_1 is shown in Fig. 9. Very soon after the yield point in the tube has been reached, the inner diameter of the tube expands by several mills without much further increase of the pressures p_1.

However, it should be noted that during this expansion the outer diameter of the tube will not change much. The volume of material displaced in the radial direction appears in the increased length of the tube. The conditions during this flow approach those of a state of pure radial compression, since both the peripheral and the axial stresses in the tube remain practically at zero. Strain hardening of the tube material, which has not been considered, naturally would upset our conclusions. In the following section this case has been considered in detail.

4 Expansion of Tube Joint Assuming a Gradual Increase of Yield Stresses With Permanent Strains in Tube Material

Since the stress-strain behavior of the tube material was not known, it is impossible to make a statement concerning the actual values of the pressures which remain in a tube joint after a

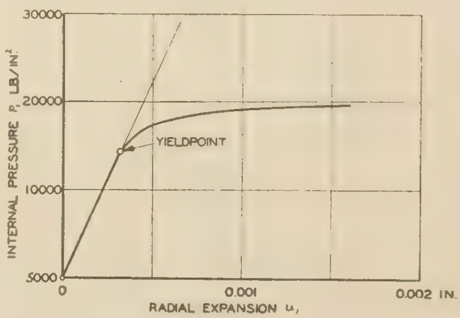

Fig. 9 Expansion Curve for a Tube
(The metal has a definite yield point σ_0 = 15,000 psi.)

permanent expansion of the tube. A general method for obtaining these residual stresses is developed in this section.

Suppose that the steel has a definite yield point. The function connecting the radial pressure p_2 in the hole $r = r_2$ with the radial expansion u_2 has been obtained in Section 2. We found, according to Equations [30] and [31] in the elastic range

$$p_2 = \frac{\sigma_0'}{\sqrt{3}} \cdot \frac{u_2}{u_e^*} \quad (u_2 < u_e^*) \ldots \ldots \ldots \ldots [71]$$

in the plastic range

$$p_2 = \frac{\sigma_0'}{\sqrt{3}} \left\{ 4 - 3\left(\frac{u_e^*}{u_2}\right)^{1/3} \right\} \quad (u_2 > u_e^*) \ldots \ldots \ldots [72]$$

where

$$u_e^* = (1 + \nu')\sigma_0' r_2/\sqrt{3}E' \ldots \ldots \ldots [73]$$

is a constant. After introducing the variables ω_2 and π_2 defined by

$$\omega_2 = \frac{u_2}{u_e^*}, \ \pi_2 = \frac{\sqrt{3}p_2}{\sigma_0'} \ldots \ldots \ldots \ldots [74]$$

we can rewrite these equations again as follows

$$\left.\begin{array}{lll} \text{Elastic} & 0 < \omega_2 < 1, & \pi_2 = \omega_2 \\ \text{Plastic} & 1 < \omega_2, & \pi_2 = 4 - 3\omega_2^{-1/3} \end{array}\right\} \ldots \ldots \ldots [75]$$

When the metal of the tubes is deformed in a tension test in the plastic range, in general, $\sigma = f(\epsilon)$ or $\epsilon = g(\sigma)$ expresses its behavior during strain hardening. In a three-dimensional radial symmetrical distribution of stress (σ_r, σ_t, σ_z) the "intensity" of stress can be measured by the octahedral shearing stress

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_t)^2} \ldots [76]$$

and the plastic distortion ϵ_r, ϵ_t, ϵ_z by the octahedral shearing strain, defined by

$$\gamma_{oct} = \frac{2}{3}\sqrt{(\epsilon_t - \epsilon_r)^2 + (\epsilon_r - \epsilon_z)^2 + (\epsilon_z - \epsilon_t)^2} \ . [77]$$

and the property of a metal to strain-harden under comparatively not too large strains (of an order of magnitude of a few per cent) can be expressed by assuming that

$$\gamma_{oct} = F(\tau_{oct}) \ldots \ldots \ldots \ldots \ldots \ldots [78]$$

is a known function which can be given either by a graph or by an analytical expression.

We have already stated that, in the case of the expansion of a tube by rolling, we can neglect the axial stress σ_z so that $\sigma_z = 0$ in Equation [76]. Since in the plastic state also $\epsilon_r + \epsilon_t + \epsilon_z = 0$ we obtain the Equations [76] and [77] in the simpler form

$$\tau_{oct} = \frac{1}{3}\sqrt{2(\sigma_r^2 - \sigma_r\sigma_t + \sigma_t^2)} \ldots \ldots \ldots [79]$$

$$\left.\begin{array}{l} \gamma_{oct} = \dfrac{2}{3}\sqrt{6(\epsilon_t^2 + \epsilon_t\epsilon_r + \epsilon_r^2)} \\ \text{or} \\ \gamma_{oct} = \dfrac{2}{3}\sqrt{6(\epsilon_t^2 + \epsilon_t\epsilon_z + \epsilon_z^2)} \end{array}\right\} \ldots \ldots \ldots [80]$$

In the first of these two expressions, we can now substitute for the stress components σ_r and σ_t in the tube

$$\sigma_t = \nu_1 p_1 - \nu_2 p_2, \ \sigma_r = -(p_1 + p_2)/2 \ldots \ldots \ldots [81]$$

so that

$$\tau_{oct} = \frac{\sqrt{2}}{3} \cdot \sqrt{\left(\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right)p_2^2 - 2\nu_1\nu_2 p_1 p_2 + \left(\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right)p_1^2} \ldots [82]$$

Similarly, we can introduce in Equation [80] the expressions which have already been derived (Equations [56k] and [56l]) for ϵ_t and ϵ_z, namely

$$\epsilon_t = \frac{u_1 + u_2}{2r_m}, \ \epsilon_z = \frac{\nu_1 u_1 - \nu_2 u_2}{r_m} \ldots \ldots \ldots [83]$$

so that

$$\gamma_{oct} = \frac{2}{r_m} \cdot \sqrt{\frac{2}{3}} \cdot \sqrt{\left(\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right)u_2^2 - 2\nu_1\nu_2 u_1 u_2 + \left(\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right)u_1^2} \quad [84]$$

We note that in these two expressions for τ_{oct} and γ_{oct}, Equations [82] and [84], the same factors appear under the square-root sign at corresponding places.

If we now use four dimensionless variables

$$\omega_1 = \frac{u_1}{u_e^*}, \ \omega_2 = \frac{u_2}{u_e^*}, \ \pi_1 = \frac{\sqrt{3}p_1}{\sigma_0'}, \ \pi_2 = \frac{\sqrt{3}p_2}{\sigma_0} \ldots [85]$$

(the quantity u_e^* was given by Equation [19]), and define the ratios

$$X = \frac{\omega_1}{\omega_2}, \ Y = \frac{\pi_1}{\pi_2} \ldots \ldots [86]$$

we can rewrite Equations [82] and [84] as follows:

$$S = 3\sqrt{\frac{3}{2}} \cdot \frac{\tau_{oct}}{\sigma_0'} = \pi_2\sqrt{\left(\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right) - 2\nu_1\nu_2 Y + \left(\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right)Y^2} \quad [87]$$

$$e = \sqrt{\frac{3}{2}} \cdot \frac{r_m\gamma_{oct}}{2u_e^*} = \omega_2\sqrt{\left(\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right) - 2\nu_1\nu_2 X + \left(\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right)X^2} \ldots [88]$$

The special strain-hardening law, which we will consider is

$$\gamma_{oct} = \alpha\tau_{oct} + c\tau_{oct}^n \ldots \ldots \ldots [89]$$

Copper and alloys containing copper as their major constituent, in a tension test, usually have a stress-strain curve which departs quite gradually from an inclined straight line representing the elastic behavior of the metal. Equation [89] expresses such a behavior for small strains. The constant α in Equation [89] is equal to $1/G$ where G is the modulus of rigidity of the tube material, and c and the exponent n are further material constants. The first term $\alpha\tau_{oct}$ in Equation [89] expresses the elastic strain, and the second term on the right side of Equation [89], the plastic strain. For the case of pure tension (stress σ and strain ϵ), we have $\tau_{oct} = \sqrt{2}\cdot\sigma/3$ and $\gamma_{oct} = \sqrt{2}\cdot\epsilon$. The constant c in Equation [89] can, therefore, be computed from a tension test.

$$e = \frac{\sqrt{2}\epsilon}{\left(\dfrac{\sqrt{2}\,\sigma}{3}\right)^n} \ldots\ldots\ldots\ldots\ldots\ldots [90]$$

We can finally rewrite the strain-hardening Equation [89], using e and S as the variables as follows

$$e = \alpha^* S + C^* S^n \ldots\ldots\ldots\ldots\ldots\ldots [91]$$

with the new material constants

$$\alpha^* = \frac{\sigma_0' r_m}{\sigma u_e^*} \cdot \alpha,\ c^* = \frac{1}{3^n}\left(\frac{2}{3}\right)^{\frac{n-1}{2}} \cdot \frac{\sigma_0'^n r_m}{2u_e^*} \cdot c \ldots\ldots [92]$$

and the kinematic flow condition based on Equation [63] which is valid also for this more general type of flow

$$(1 - 4\nu_1 - 8\nu_1^2)\omega_1\pi_1 + (3 + 8\nu_1\nu_2)(\omega_1\pi_2 + \omega_2\pi_1) + (1 + 4\nu_2 - 8\nu_2^2)\omega_2\pi_2 = 0 \ldots\ldots\ldots [93]$$

as follows:

$$[1 - 4\nu_1 - 8\nu_1^2]XY + [3 + 8\nu_1\nu_2](X + Y) + [1 + 4\nu_2 - 8\nu_2^2] = 0 \ldots\ldots [94]$$

We note that this last equation represents an equilateral hyperbola. Solving Equation [94] for

$$X = \frac{[8\nu_2^2 - 4\nu_2 - 1] - [8\nu_1\nu_2 + 3]Y}{[8\nu_1\nu_2 + 3] - [8\nu_1^2 + 4\nu_1 - 1]Y} \ldots\ldots [94a]$$

shows that the horizontal asymptote of this hyperbola is given by

$$Y = \frac{8\nu_1\nu_2 + 3}{8\nu_1^2 + 4\nu_1 - 1} \ldots\ldots\ldots\ldots\ldots [94b]$$

The four Equations [87], [88], [91], and [94] solve the problem of the flow in the expanded tube joint.

The procedure for finding corresponding values of the radial pressure p_1 and the radial expansion u_1 of the tube consists of computing the corresponding values of π_1 and ω_1 from the four equations mentioned. We start with the conditions, Equations [75], for the deformation of the steel plate and choose a value of ω_2. If $0 < \omega_2 < 1$, we take for $\pi_2 = \omega_2$, but if $\omega_2 > 1$ (i.e. if the steel plate has been stretched beyond the yield point), we have to compute π_2 from

$$\pi_2 = 4 - 3(\omega_2)^{-1/3} \qquad [95]$$

Given ω_2 and π_2, Equation [91] expresses a function of X and Y. A second equation for X and Y is Equation [94]. From these two equations, we can compute X and Y, and finally $\omega_1 = X\omega_2$ and $\pi_1 = Y\pi_2$.

An example for the application of the preceding computation follows. Only the results of the numerical computations are given: We assume that the steel plate has a modulus of elasticity $E' = 3 \times 10^7$ psi, a yield stress in tension $\sigma_0' = 30{,}000$ psi, and a Poisson's ratio $\nu' = 0.3$. For the tube, we assume an inner radius $r_1 = 0.4$ in., an outer radius $r_2 = 0.5$ in., a wall thickness $h = 0.1$ in., E 1.5×10^7 psi, $\nu = 0.3$, $G = 5{,}770{,}000$ psi. The constants α and ν_1, ν_2 for the tube were: $\alpha = 1/G = 1.733 \times 10^{-7}$, $\nu_1 = r_1/h = 4$, $\nu_2 = r_2/h = 5$. For the strain-hardening function of the tube metal, according to Equation [89], an exponent $n = 4$ and

$$\gamma_{oct} = 1.733 \times 10^{-7}\tau_{oct} + 2.866 \times 10^{-18}\tau_{oct}^4 \ldots\ldots [96]$$

was assumed. This would represent a material, which in pure tension would stretch 0.1, 1.6, 8.1, and 25.8 per cent at a stress of 1, 2, 3, and 4 × 10,000 psi, respectively. Results of the numerical computation are shown in Fig. 10 and Table 2. In Fig. 10 are drawn two curves representing the pressures p_1 at the inner surface $r = r_1$ of the tube, and the pressures p_2 at the contact surface $r = r_2$ between the tube and plate. These pressures were plotted in function of the radial expansion u_1. Table 2 contains a few values of the inner and outer pressure p_1 and p_2 of the tube, and the corresponding radial displacements u_1 and u_2.

TABLE 2 EXPANSION OF A TUBE

(Pressures p_1 and p_2 in function of the radial expansions u_1 and u_2 using stress-strain relation Equation [89], and $n = 4$.)

Variables				Pressures, Psi		Radial expansions, in.	
				inner	outer	inner	outer
ω_2	π_2	ω_1	$\pi_1 = 1.140\pi_2$	p_1	p_2	u_1	u_2
0	0	0	0	0	0	0	0
0.6	0.6	0.83	0.684	11800	10400	0.000312	0.000225
0.8	0.8	1.55	0.912	15800	13800	0.000580	0.000300
1	1	3.00	1.140	19730	17320	0.00113	0.000375
1.2	1.175	5.48	1.340	23200	20350	0.00205	0.000450
1.4	1.318	8.87	1.502	26000	22830	0.00333	0.000525
1.6	1.433	13.68	1.635	28300	24830	0.00513	0.000600

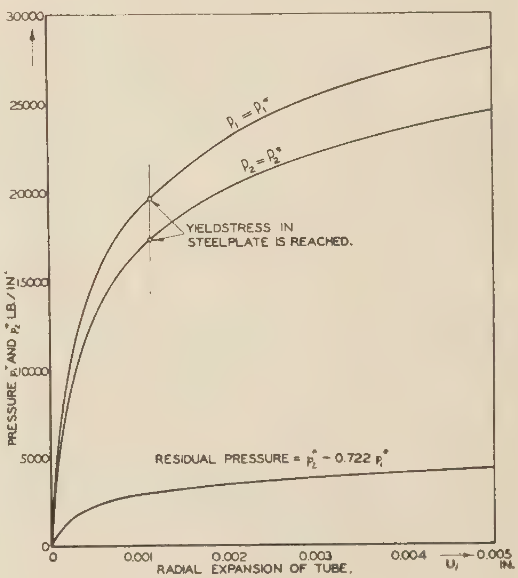

FIG. 10 PRESSURES REQUIRED FOR EXPANSION OF A TUBE (r_1 = 0.4 IN., r_2 = 0.5 IN.) AND RESIDUAL PRESSURE IN TUBE JOINT

5 METHOD OF COMPUTING RESIDUAL CONTACT PRESSURE AT INTERFACE BETWEEN STEEL PLATE AND TUBE

After the pressures p_1 and p_2 have been computed by any of the methods which were described in the preceding sections, depending upon the type of plastic flow to be expected in the tubes during rolling and in the steel plate, it is easy to find the residual contact pressure for $r = r_2$ for a given radial expansion of the tube.

Suppose that p_1^* and p_2^* are the pressures at which the radial

expansions u_1^* and u_2^* were reached in the tube joint at radii r_1 and r_2, respectively. When the inner pressure p_1 is reduced to zero after the permanent expansion of the tube, this latter and the steel plate contract. This contraction is due to an elastic state of stress in the tube joint. Since this state has been fully described in section 4, all that is needed to find the residual pressure in the joint after the external pressure p_1^* has been released is to superimpose on the system p_1^*, p_2^* an elastic-stress distribution which together with the former one will just reduce the pressure to zero at the inner surface $r = r_1$ of the tube.

If we designate by σ_1 and σ_2 any two corresponding simultaneous values of the radial stresses which are transmitted in the tube joint at $r = r_1$ and $r = r_2$ by a purely elastic distribution of stresses, we know from Section 4 that it is expressed by the following set of equations (see Equation [43])

$$\text{In the tube}\quad \left\{\begin{aligned} \sigma_r &= \sigma_1\left(\frac{1}{2}-\frac{x}{h}\right)+\sigma_2\left(\frac{1}{2}+\frac{x}{h}\right)\\ \sigma_t &= -\sigma_1\frac{(r_1-x)}{h}+\sigma_2\frac{(r_2-x)}{h}\end{aligned}\right\}\quad\ldots\ldots\ [97]$$

$$\text{In the steel plate}\quad \left\{\begin{aligned} \sigma_r' &= \sigma_2\frac{r_2^2}{r^2}\\ \sigma_t' &= -\sigma_2\frac{r_2^2}{r^2}\end{aligned}\right\}\quad\ldots\ldots\ [98]$$

As long as the tube and the steel plate remain in contact and have not separated, we know also that σ_2 must be proportional to σ_1 (see Equation [49])

$$\frac{\sigma_2}{\sigma_1} = \frac{2\nu_1 - 1}{2\nu_2 - 1 - 2\nu + \dfrac{2E}{E'}(1+\nu')}\quad\ldots\ldots\ [99]$$

where $\nu_1 = r_1/h$, $\nu_2 = r_2/h$ and ν, ν', E, E' are Poisson's ratio and the moduli of elasticity of the tube material and the steel plate. Taking $\nu_1 = 4$, $\nu_2 = 5$ and the values which were used in the previous examples, we obtain from Equation [49]

$$\frac{\sigma_2}{\sigma_1} = 0.722\quad\ldots\ldots\ [100]$$

The radial stresses σ_r remaining in the tube joint must therefore be equal to

$$\left.\begin{aligned} &\text{for} && r = r_1 && \sigma_r = -p_1^* + \sigma_1 = 0\\ &\text{and for} && r = r_2 && \sigma_r = -p_2^* + \sigma_2\end{aligned}\right\}\quad\ldots\ldots\ [101]$$

Since $\sigma_1 = p_1^*$ we see that the residual pressure in the contact surface $r = r_2$ of the tube joint must be equal to

$$p_{2\ \text{res}} = p_2^* - 0.722p_1^*\quad\ldots\ldots\ [102]$$

These values have been plotted in Fig. 10, in function of the radial expansion u_1 for the same numerical example which has been computed in section 4. In this figure were already plotted the curves for the pressure p_1^* and p_2^* required for expanding the tube versus the radial expansions u_1.

The curve for the residual pressures shows that but little is gained by rolling or expanding the tube much beyond a radial expansion of 2 mills in this particular example. The residual pressures reached in the joint are of the order of 3000 to 4000 psi.

6—Short Cut Through General Method Previously Described

The pressure p_1 required for a permanent radial expansion of a tube joint is computed as follows:

First the stiffness of the steel plate has to be expressed. For a mild-steel plate, having a definite yield point, we can compute the pressure p_2 at the bore $r = r_2$ from the expressions:

below the yield point

$$p_2 = \frac{\sigma_0'}{\sqrt{3}}\cdot\frac{u_2}{u_e^*},\ (u_2 < u_e^*)\quad\ldots\ldots\ [a]$$

after yielding has started

$$p_2 = \frac{\sigma_0'}{\sqrt{3}}\left[4 - 3\left(\frac{u_e^*}{u_2}\right)^{1/3}\right],\ (u_2 > u_e^*)\quad\ldots\ldots\ [b]$$

where u_e^* designates the radial expansion of the hole $r = r_2$ just when the yield point σ_0' is reached in the steel plate and

$$u_e^* = (1+\nu')\sigma_0' r_2/\sqrt{3}E'\quad\ldots\ldots\ [c]$$

A series of corresponding values of p_2 and u_2 are therefore known.

Next, we have to express the conditions of yielding in the tube. The mean values of the radial and tangential stresses in the tube wall are given by

$$\sigma_r = -\frac{(p_1+p_2)}{2},\quad \sigma_t = \nu_1 p_1 - \nu_2 p_2\quad\ldots\ldots\ [d]$$

and the mean values of the strains are

$$\epsilon_r = -\frac{(1+2\nu_1)u_1 + (1-2\nu_2)u_2}{2r_m},\quad \epsilon_t = \frac{u_1+u_2}{2r_m},$$

$$\epsilon_z = \frac{\nu_1 u_1 - \nu_2 u_2}{r_m}\quad\ldots\ldots\ [e]$$

The octahedral stress and strain can be computed from

$$\tau_{\text{oct}} = \frac{\sqrt{2}}{3}\cdot\sqrt{\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2}\quad\ldots\ldots\ [f]$$

$$\gamma_{\text{oct}} = 2\sqrt{\frac{2}{3}}\cdot\sqrt{\epsilon_t^2 + \epsilon_t\epsilon_r + \epsilon_r^2}\quad\ldots\ldots\ [g]$$

Since we note that the Equations [d] for σ_t and σ_r in terms of p_1 and p_2 are the same as the Equations [e] for ϵ_t and ϵ_s in terms of u_1 and u_2, we must obtain τ_{oct} and γ_{oct} in the symmetrical forms

$$\tau_{\text{oct}} = \frac{\sqrt{2}}{3}p_2\sqrt{\left[\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right] - 2\nu_1\nu_2\cdot\frac{p_1}{p_2} + \left[\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right]\frac{p_1^2}{p_2^2}}\quad\ldots\ldots\ [h]$$

$$\gamma_{\text{oct}} = 2\sqrt{\frac{2}{3}}\cdot\frac{u_2}{r_m}\cdot\sqrt{\left[\nu_2^2 - \frac{\nu_2}{2} + \frac{1}{4}\right] - 2\nu_1\nu_2\cdot\frac{u_1}{u_2} + \left[\nu_1^2 + \frac{\nu_1}{2} + \frac{1}{4}\right]\frac{u_1^2}{u_2^2}}\quad [i]$$

The kinematic condition of plastic flow is expressed in first approximation by assuming that

$$y = \frac{p_1}{p_2} = \frac{8\nu_1\nu_2 + 3}{8\nu_1^2 + 4\nu_1 - 1} = \text{const.}\quad\ldots\ldots\ [k]$$

indicating that during flow the ratio of the internal pressure p_1 to the outer pressure p_2 is a constant whose value depends only upon the dimensions of the tube. Since p_1/p_2 = constant, we see that τ_{oct} is proportional to the pressure p_2

$$\tau_{\text{oct}} = kp_2\quad\ldots\ldots\ [l]$$

The numerical factor k appearing in Equation [l] has to be computed from Equation [h]. If the stress-strain curve is given as a graph or by means of a function

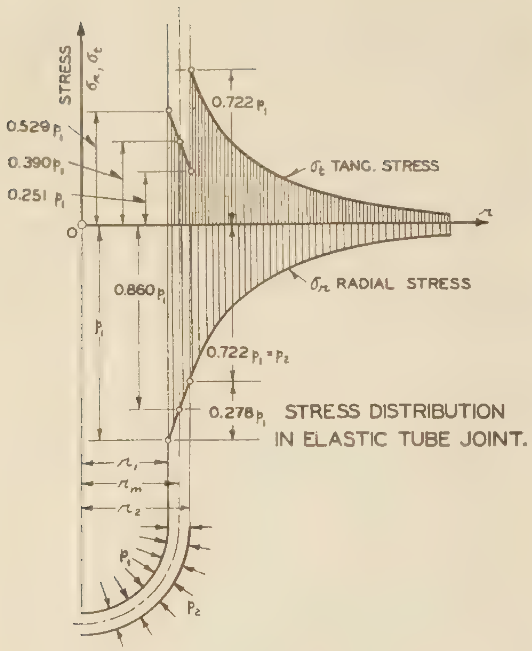

FIG. 11

$$\gamma_{oct} = F(\tau_{oct}) \qquad [m]$$

the corresponding values of γ_{oct} can numerically be computed for a number of points and Equation [i] is a quadratic equation for the variable

$$X = u_1/u_2 \qquad [n]$$

with numerical constants. The positive root X_r of this equation furnishes the unknown radial expansion $u_1 = X_r u_2$ corresponding to u_2 and p_2. Thus, finally p_1, p_2, u_1, u_2 are known, and the curves p_1 or p_2 in function of the radial expansion can be plotted, Fig. 10.

If we wish to compute the residual pressure in the contact cylinder between the tube and the steel plate, we must superpose, on the plastic distribution, an elastic distribution of stress of the tube joint carrying at $r = r_2$ an external normal stress $\sigma_1 = p_1^*$ (see Equations [97], [98], and [99]) equal but opposite in sign to the normal stress ($-p_1^*$) under which the joint was permanently expanded. Fig. 11 shows this purely elastic fictitious state of stress for the dimensions which were previously used: $\nu_1 = r_1/h = 4$, $\nu_2 = r_2/h = 5$. For these tube dimensions, we found that the stress σ_2 in the contact surface $r = r_2$ was $\sigma_2 = 0.722\sigma_1$ (Equation [100]). If we designate, in general, the ratio of these two elastic stresses by $m = \sigma_2/\sigma_1$ (= 0.722 in our special example), the residual pressure p in the contact surface $r = r_2$ is found to be equal to

$$p = p_2^* - mp_1^* \qquad [o]$$

As was just stated p_1^* and p_2^* are here the maximum pressures at $r = r_1$ and $r = r_2$ under which the tube joint was plastically enlarged.

Fig. 12 shows what might be expected in a tube joint after partial yielding in the steel plate has progressed to a radius $r = c$. The two curves designated by σ_r^*, σ_t^* refer to the stresses just before unloading. The curves σ_r, σ_t represent the distribution given by Equation [43], using the values just expressed, and the shaded ordinates represent the distribution of the residual stresses in the tube joint remaining after its plastic expansion. The residual pressure in the contact surface $r = r_2$ is given by the ordinate BC in Fig. 12. The residual radial pressures are all compressions, but the residual tangential stresses change their sign once. They consist of compressions only in the inner portion of the joint.

7 Conclusions

Assuming that the steel in the head plate or boiler drum has a definite yield stress, the pressures p_2 required to enlarge the radius r_2 of the bore were determined in function of the radial displacements u_2 at the bore. This function $p_2 = f(u_2)$ for partial yielding is represented in Fig. 7 and expressed by Equations [30] and [31]. The distribution of the radial and tangential stresses around the hole of the steel plate is shown in Fig. 6.

If the tube metal is steel, having a similar stress-strain behavior as the head plate, the stress curves, as shown in Fig. 6, may include also the tube walls and be extended to the inner surface with the radius r_1 of the tube. If the tube metal is much softer and yields itself at lower stresses than the steel plate, it appears that the effect of the rolling of a tube joint in first approximation is equivalent in the tube wall to a state of pure compression in the radial direction. In other words, the tangential and the axial stresses in the tube wall can be assumed to be zero and the mean value of the radial stresses is of the order of the radial pressure p_2 required to expand the steel plate. (In first approximation one can take $\frac{d}{dr}(r\sigma_r') = 0$ because $\sigma_t' = 0$ and obtains in the tube wall a radial stress $\sigma_r' = C/r$ from Equation [8].)

The state of plastic strain in the tube wall can therefore, in first approximation, be described by assuming that the radial strain ϵ_r is equal to the strain which would be observed in an ordinary compression test of the tube metal under the pressure p_2. Strain hardening for the tube metal can be expressed, therefore, by $\epsilon_r = F(p_2)$, using for this function the ordinary strain-hardening curve for compression. Furthermore, the permanent changes of the dimensions of the tube during rolling can be estimated from the relations $\epsilon_t = \epsilon_z = -\epsilon_r/2$ valid for a state of pure com-

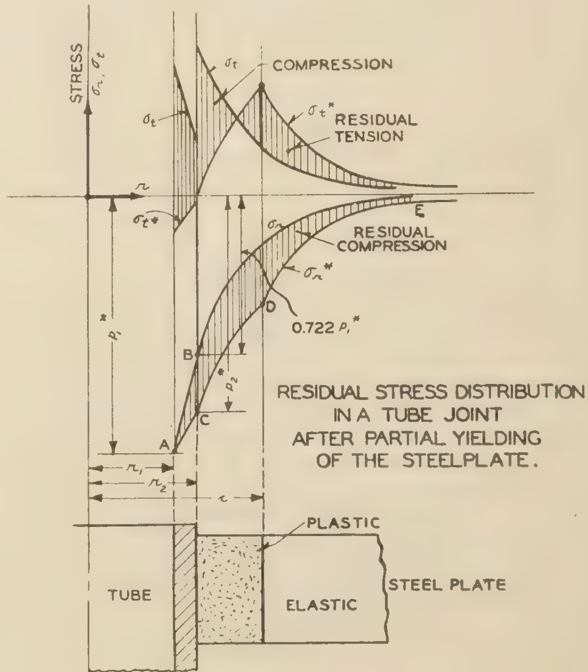

FIG. 12

pression strain, where ϵ_t and ϵ_s designate the strain in tangential and in axial direction in the tube wall. For ϵ_t can be taken the mean value

$$\epsilon_t = \frac{u_1 + u_2}{r_1 + r_2} \qquad [103]$$

Since also $\epsilon_t = -\epsilon_r/2$, Equation [103] permits us to compute the corresponding permanent increase of the inner radius of the tube from

$$u_1 = -\frac{(r_1 + r_2)\epsilon_r}{2} - u_2 \qquad [104]$$

It is recommended that tubes be rolled not much beyond a pressure $p_2^* = \sigma_0'$ where σ_0' is the yield stress in pure tension or compression in the steel plate. The residual pressure p in the contact surface $r = r_2$, remaining after rolling of a tube joint, is of the order of $(1 - m)\sigma_0'$ where $m = \sigma_2/\sigma_1$ has to be computed from Equation [99].

BIBLIOGRAPHY

A series of papers is available, most of which deal with the practical aspects of tube expanding. The three papers[2] mentioned in the text already contain this information, so that it is perhaps unnecessary to add a list of references to the present paper

Discussion

E. T. Cope.[4] The paper by Dr. Nadai is a welcome addition to the growing bibliography of papers written on this subject in English. Until recently these writings dealt only with the practical phases of the problem, and it is encouraging and gratifying to witness the beginning of a theoretical treatment of many features which so far have been treated only in an empirical manner. It is, of course, recognized that the erecting crews that build boilers and other heat exchangers in shops and in the field may not be able to make direct use of analytical studies such as this of Dr. Nadai, yet rigorous analysis accompanying and supporting empirical procedures usually proves to be the best for construction work.

One of the most difficult questions to settle in the rolling of any heat-exchanger tube joint is, "How much should a tube be expanded?" The author gives an answer to this question in his Fig. 10 in which is shown the change in value of the residual pressure between the outside surface of the tube and the inside surface of the tube; the change being produced by the expanding operation. The curve representing residual pressure shows a distinct decline in slope after a small increase in tube diameter. This indicates that, after a relatively small amount of expansion, further rolling does not produce a proportional increase in residual pressure.

If it is assumed that the holding strength of the joint is a function of the residual pressure (a reasonable assumption if the surfaces are of uniform finish), Fig. 10 indicates that a relatively small amount of expansion will develop by far the greater portion of the possible holding strength that can be developed. That this is true has been shown by Fisher and Cope in several illustrations of an earlier paper.[5] In all of these, the holding strength, expressed in pounds per square inch hydrostatic pressure necessary to cause the tube to slip in the hole, increased to a maximum value and then decreased as the amount of expansion progressed. Thum and Jantscha[6] observed the same phenomenon and mentioned it as follows:

> "The holding strength of the rolled joint reaches a maximum for a certain degree of rolling, and it decreases again after that more or less but always very definitely. From this the conclusion must be drawn that too much rolling has a distinctly unfavorable effect upon holding strength."

Lieberherr[7] states his observation of the same phenomenon in these words:

> "It was found that the moderately rolled tubes, . . ., remained tight or sweated only slightly, while those which had been rolled excessively showed leaks."

The explanation for this phenomenon noted by these observers is not difficult.

It has been observed by Fisher and Cope in their tests and by Siebel[8] that the surfaces of tube and tube hole, originally either ground to commercial finish or machined with shallow parallel grooves, were materially smoothed out when the joint was rolled heavily. This smoothing-out reduced the sliding friction between the surface of the tube and the tube hole, with the result that even though the residual pressure might be somewhat greater as a result of excessive rolling as indicated by the author in Fig. 10, the product of coefficient of friction and residual pressure was less when the joint was rolled heavily than when it was not rolled so much. It must be concluded from Fig. 10 and from the observation of the other investigators mentioned, that in the matter of holding strength of the joint, the best results are obtained when the joint is given a light to medium amount of rolling.

Such a mild treatment of the metals which constitute the joint is favorable from another point of view, namely, the effect of work-hardening and disturbance of the metal, especially that on the inside of the tube, which may result from excessive rolling. C. H. Fellows,[9] in his discussion of the Fisher and Cope paper,[5] calls attention to the fact that the excessive rolling of tubes may be the basic cause for corrosion failure of boiler drums and tubes. This same fact is discussed by J. H. Walker in his report of Subcommittee No. 6 of the Joint Research Committee on Boiler Feedwater Studies. He calls attention to this same condition in the following statement:[10]

> " . . . but there seems to be complete agreement on the idea that caustic embrittlement can occur only when the following three conditions exist simultaneously:
>
> (a) "Certain chemical conditions of the boiler water.
>
> (b) "Physical conditions which permit high concentration of dissolved solids.
>
> (c) "Contact of the concentrated solution with highly stressed boiler metal."

and further:

> "Contact of the concentrated solution with highly stressed metal is a requirement which has been thoroughly demonstrated. Cracking has not been observed unless the stress is near or above the yield point. . . . Previous cold-working of the steel, renders it more susceptible. . . . The rolled-in tube end, however, remains a vulnerable point."

A striking example of the condition resulting from excessive or incorrect rolling is shown in the author's closure to Fisher and Cope.[5] Here photographs of tubes which failed after a short

[4] Research Department, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[5] "Rolling-In of Boiler Tubes," by F. F. Fisher and E. T. Cope, Trans. A.S.M.E., vol. 57, 1935, paper FSP-57-7, Figs. 3, 4, 6, 7, 8, and 9.

[6] "The Rolling and Pressing-In of Boiler and Superheater Tubes Made From Various Metals," by A. Thum and R. Jantscha, *Archiv für Wärmewirtschaft*, vol. 11, 1930, pp. 397–401.

[7] "The Stress in the Drum of a Water Tube Boiler," by A. Lieberherr, *Schweizerische Bauzeitung*, vol. 102, 1933, pp. 87–91.

[8] "Rolled Joints," by Eric Siebel, *Stahl und Eisen*, vol. 53, 1933, pp. 1205–1215.

[9] Chemist, Research Department, The Detroit Edison Company, Chairman Joint Research Committee on Boiler Feedwater Studies.

[10] "Caustic Embrittlement Research Brings Results," by J. H. Walker, *Mechanical Engineering*, vol. 64, 1942, pp. 891–893.

period of service are shown in Figs. 3 and 4. In Figs. 5 and 6 of that closure, the results of a laboratory test are shown in which the excessive cold-working of the tube due to excessive or incorrect rolling is demonstrated. While it is true that Figs. 5 and 6 were used to show a condition resulting from the use of improperly shaped rollers, the effect of excessive movement of the metal is evident.

Dr. Nadai has dealt only with the pressures on the inside and outside of the tube in the rolled joint. Such values would be very difficult to measure even in the laboratory and probably could not be measured at all in the field or shop during the erection of a boiler or a condenser. These relationships are very interesting but what is needed by the one responsible for correctly rolling-in a joint is an easily usable measure of the correctness, that is, adequacy and uniformity, of his tube rolling. Two empirical means have been described by Fisher and Cope[6,11] in their papers on tube rolling. The earlier of these papers described the "elongation method," and the latter the method in which the power input was used, employing a current-limiting relay to stop the rolling-in at a predetermined value of current input.

The authors of these two papers would welcome an investigation of the validity of these empirical criteria, as determined by mathematical analyses of their procedures. Whether or not such analyses can be made does not affect the urgent need for some tangible, easily applied measure for determining the correctness of any tube-rolling procedure. This might be listed as question (*a*1) in the list of questions in section 1 of the present paper, inasmuch as, in the writer's opinion, it surely deserves a higher priority than either question (*b*) or question (*c*).

After the author has developed the analytical proof supporting some readily usable criterion for the optimum amount of rolling and has answered questions (*b*) and (*c*), the writer would ask permission to pose some other difficult questions which have arisen out of experiences with boiler tubes.

There are three criticisms which the writer wishes to make, the correction of which would facilitate the reading and enhance the comprehensibility of this splendid paper. These are as follows:

1 In the presentation of Equations [7] and [8], the author states: "The solutions of these two equations have been expressed previously by the author in the parametric form . . ." Could the author conveniently insert the name and date of the publication in which this material was shown?

2 The use of the Greek letter π to signify some value other than 3.1416, the ratio of the circumference to the diameter of a circle, is strange to most engineers. Would not some other symbol have been equally satisfactory? Certainly the use of some symbol other than π would be less confusing.

3 In the last paragraph of "Conclusions," the new symbol m is introduced. It is not given in the list of designations. Even Equation [99], referred to in this paragraph, does not offer a clue. After a search the definition of m is found in the paragraph following Equation [n], in section 6.

The writer wishes to thank the author for presenting this paper before the Society. He urged the author to make this presentation after the existence of this paper was made known during the 1942 Annual Meeting of the Society. This work brings the tube-rolling problem nearer to a rational solution and is a valuable contribution to knowledge of the subject.

[11] "Automatic Uniform Rolling-In of Small Tubes," by F. F. Fisher and E. T. Cope, Trans. A.S.M.E., vol. 65, 1943, pp. 53–60.

J. N. Goodier[12] and G. J. Schoessow.[13] This paper will be of the greatest interest to all engineers concerned with the design of tube joints made by the expanding process. It is one of the few contributions to a thorough theoretical analysis.

In 1941, The Babcock & Wilcox Company developed work on this subject, which had been in progress many years, into a three-way investigation which resulted in the presentation of three papers.[14]

In order to treat this problem theoretically, so far it has been necessary to make certain assumptions, three of which are basic in importance.

1 That the expander has a large number of small-diameter rolls, or is in some other way equipped to produce an action inside the tube which can be assumed to result in the same stress distribution through the tube seat as would be accomplished by a uniform hydrostatic internal pressure applied to the inside surface of the tube end.

2 That the longitudinal stresses set up in the tube material within the longitudinal limits contained by the tube-plate thickness are zero, that is, there is no longitudinal restraint of the tube end or the tube plate material, and it is free to flow longitudinally.

3 That the analysis for a single hole in a large plate will serve also when the plate has many holes.

These basic assumptions are also made in the writers' paper,[14] and it appears that the results in so far as they cover the same ground are in agreement. In view of the necessity of such assumptions, however, theoretical results must be regarded as suggestive rather than conclusive, and the writers' work was especially aimed at comparison with measured residual pressures in order to gain some idea of the significance of the assumptions.

The author has introduced strain hardening and calculations of radial displacement. Here, again, it will be very valuable to have a comparison with measurements, whenever suitable measurements are available.

The question of how far the plastic zone should be extended into the tube plate by the expanding process is defined by two limits, an upper limit of 1.75 times the radius of the hole, a lower limit of yielding just beginning in the tube-seat plate. The range between these two limits is large. However, detailed consideration applying the theoretical formulas now available to any specific application will help to narrow the range and arrive at a limit for that particular construction. Theoretical help of this kind was heretofore not available to the designing engineers.

In section 1 of the paper, certain questions are raised, two of which, namely, (*b*) the time-temperature relaxation of the forces in the expanded joint, and (*c*) the axial and peripheral bending stresses at the end of the expanded zone in the tube, are worthy of serious study. Generally, they play a part of minor importance in an expanded joint. However, there may be some unusual applications of expanded joints in which these items would assume first importance.

Another item might be added, which appears to be of real importance, namely, (*d*) the exact nature of the stress produced under each roll of a 3-roll expander (or multiroll expander), as compared with the results predicted by these current papers, which are on the basis of uniform hydrostatic pressure producing the expanding force.

On these three possible investigations, the writers might comment as follows:

(*b*) The problem of the time-temperature relaxation of forces

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[13] Engineer, The Babcock & Wilcox Company, Barberton, Ohio. Mem. A.S.M.E.

[14] Refer to footnote 2 of the paper.

in the expanded joint is to some extent the same one which occurs for bolted-flange joints. In order to predict the relaxation of the stresses in the expanded joint when it is first made, it is necessary to have a mathematical expression representing the creep properties of the materials involved. This mathematical expression is generally not coincident with actual results of test data, and assumptions have to be made to cover this situation. Having set up the proper assumptions, however, it would be possible to continue through the data which would be of real help to designers in predicting the relaxation of the expanded joints. The writers have had this subject under consideration for some time and believe that it will be possible to accomplish theoretical results which would be worth the effort involved.

In the ordinary construction involved in boiler-tube expanded joints for low-pressure and low-temperature service, relaxation is not of primary importance. However, as temperatures and pressures continue upward, this factor becomes very important and has much to do with determining the limit at which an expanded joint may still be satisfactory or whether the joint must be welded or some other construction resorted to.

(*c*) The axial and peripheral bending stresses at the end of the expanded zone in the tube:

In Fig. 13 of this discussion, it is apparent that when sufficient expanding of the tube end has taken place to produce a satisfactory joint, the enlargement at some point such as A will have been such that the peripheral stresses are just at the yield-point value. To the right of this point, the stresses decrease and to the left of this point they are in the zone of plasticity.

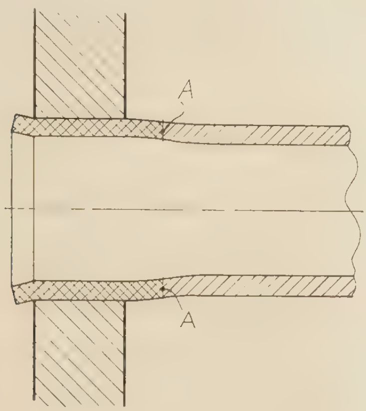


Fig. 13 Diagram Relating to the Axial and Peripheral Bending Stresses at End of Expanded Zone in Tube

In 1937, the writers made calculations on this problem using the elastic-foundation theory and arrived at a formula giving the value of the bending stresses at the maximum value to the right of point A. This formula is

$$s = \frac{0.55\, e\, E}{r\sqrt{1-u^2}}$$

in which

s = longitudinal bending stress, psi
e = elastic increase in radius at point A, in.
E = modulus of elasticity.
r = mean radius of tube at point A
u = Poisson's ratio of tube material

Applying this formula to a steel tube in which $u = 0.3$; $E = 30{,}000{,}000$ psi; $e = 0.001\, r$, we find that $s = 17{,}000$ psi maximum to the right of point A and is independent of diameter or thickness of tube.

(*d*) The exact nature of the stress produced under each roll of a 3-roll expander (or multiroll expander), as compared with the results predicted by the current papers which are on the basis of uniform hydrostatic pressure producing the expanding force:

The results of experiments, compared with the theory in the current series of papers, indicate that the hydrostatic-pressure assumption is a fairly safe one for tube joints that are expanded to a large degree, in which the plastic zone is carried well into the tube-seat plate. However, the writers are not certain that the results of these papers would agree with experimental data covering lightly expanded tubes; for instance, those in which the plastic zone is not carried into the tube sheet at all, but in which the expanding is stopped exactly at the moment when the tube-seat surface has just reached the plastic state. For such lightly expanded tubes, the action of the cylindrical roll on the inside surface of the tube end produces stresses which are very different from those contained in the assumption of hydrostatic pressure.

The writers have made calculations on the basis of three or more individual rolls with their concentrated loading on the inner surface of the tube end and the subsequent yielding of the tube immediately under the roll, and how this plastic zone extends through the tube. It would appear that detailed attention must be focused on the nature of the stresses under the individual roll when establishing a theory which is to apply to the lightly expanded tubes.

Another important consideration which must not be overlooked is the relation between the inner-face pressure between the tube and the seat, and the hydraulic tightness and structural strength of the joint. This also must receive considerable study before the predicted residual pressures can be applied to a particular design of expanded joint.

It is apparent from the foregoing that much work remains to be done on the theoretical analysis of this problem. However, considering the progress which has been made in this direction in the last few years, the writers are hopeful that it may not be too long before the investigations outlined will be completed and published for the use of the designing engineer.

Author's Closure

The valuable comments and observations by Messrs. E. T. Cope, J. N. Goodier, and G. J. Schoessow raise a number of interesting questions and points which need further investigation or discussion. Some of these remarks refer to what may be termed an oversimplification of the actual conditions of flow present under which a tube joint is produced. This has to be admitted; however, it seemed hopeful to consider the flow around a tube joint only under the simplifying assumption of a purely radial symmetry. If three rolls are used in the expanding tool, it is evident that the pressure is exerted in three equally spaced concentrated strips along the surface of the hole. The true local pressures under which these rolls act must be higher than the mean pressure which would be needed to expand the steel plate radially by the same amount uniformly. But it must also be conceded that the concentration of the stresses produced around the three local impressions of the rolls must disappear very quickly around each impression. The more rolls possible to use in the expanding tool and the more gradually they would be loaded, the nearer would the cases converge to the uniform distribution which was assumed in the paper.

It is rather important that Mr. Cope and other observers found that excessive rolling of the tubes accelerates corrosion attack. Although the paper mainly deals with the problem of determining the distribution of the plastic stresses during yielding

and of the residual stresses remaining after rolling, Mr. Cope raises a question concerning the amount of expansion. This may be judged by means of the permanent distortion produced in the joint, i.e., by measuring the accompanying permanent radial increase of the inner tube diameters. The mathematical analysis included also these radial changes of the inner or outer diameter of the tubes accompanying the expansion and thus may assist in predicting permissible amounts of rolling. Equations [9] and [9*a*] of the paper, containing the solution of the two Equations [7] and [8], quoted by Mr. Cope, were given in an earlier text by the author.[15] Use of the Greek letter π may be excused by stating that it was chosen as the Greek equivalent of the Latin letter p universally used for designating hydrostatic pressures and because π should designate a dimensionless variable containing the pressures p. Omission of the symbol m was an oversight.

Messrs. Goodier and Schoessow emphasize once more the simplifying assumptions on which the computations had to be based. The writer believes that, for example, assumption 2 could well be justified since the comparatively small length of the rolled portion of the tube has to carry a secondary equilibrium distribution of stresses in the axial direction. The normal stresses σ_z vanish at or near the two plane surfaces of the steel plate both in the tube wall and in the steel plate. The secondary stress σ_z could, therefore, reach its maximum value in the plane bisecting the plate, and the mean value of σ_z was a fraction of the maximum value. It is known that in similar cases also, shearing stresses τ_{rz} must be present which are caused through the friction which opposes the change of the tube dimensions in the axial direction. These stresses again reach their maximum values near the two plane surfaces of the steel plate and are probably quite small over the central portion of it, so that it appeared justified to neglect both stresses.

As to the third assumption it must be admitted that it was not even mentioned in the paper. It is true that if the plate has many holes and the thickness of material remaining between two neighboring holes is a small fraction of the hole diameter, the resistance of the steel plate against any radial expansion must be diminished considerably. Some idea about this effect could, if necessary, be obtained from tests. Such tests were run by E. A. Davis in Pittsburgh a few years ago in which the "apparent" elasticity modulus was experimentally determined for perforated steel plates which were tested in tension. For the weakened elasticity of a plate having many holes, certain correction formulas could, if needed, be established. It is, doubtful however, whether something corresponding could be done also for the plastic behavior of a perforated plate.

The author agrees with the last quoted discussers that among the unsolved questions the one referring to the relaxation of the locked-up pressure distribution in an expanded tube joint is of great importance if the temperatures are high (boiler tubes). Whether a theory can be worked out for the relaxation of a tube joint is to be seen. Conditions cannot be easily compared with those in bolted flange joints because the relaxation laws of residual stresses after cold work and after an intermediate heating are unexplored and very little has yet been done by experimenting in this field. To the last two points brought up by Messrs. Goodier and Schoessow, the author cannot add any observation because he has had no occasion to investigate them.

[15] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, p. 192.

Fatigue Characteristics of Rubber

By F. L. YOST,[1] EVANSTON, ILL.

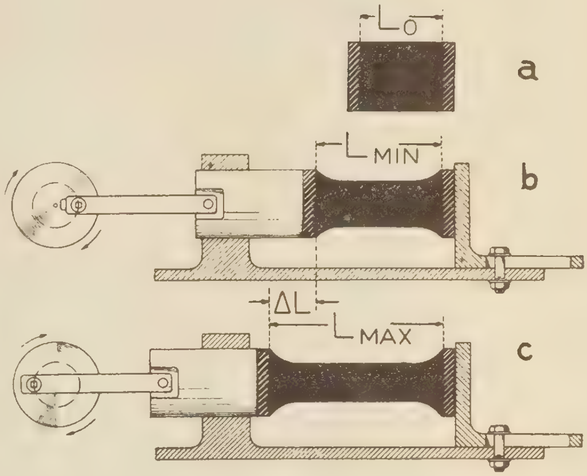
Repeated oscillations eventually will cause rubber or synthetics to deteriorate and crack from "dynamic fatigue." If maintained under constant stress just below its tensile strength, rubber will break from the high stress. This phenomenon is characterized as "static fatigue." Both vibration and loading are involved in practical applications of rubber and synthetics, and the material fails generally from a combination of these two causes. The author presents data resulting from quantitative studies made on rubber and synthetic samples covering both types of failure. These are considered separately, and then practical conclusions are drawn from the combination of the two.

If a rubber member is continuously vibrated it will crack and ultimately rupture as a result of the repeated oscillations to which it has been subjected. The gradual deterioration of physical and chemical properties which accompanies such vibration is called "dynamic fatigue." The number of complete vibrations required to rupture the rubber is referred to as the "dynamic fatigue life" of the member for the particular condition of vibration imposed. The time period of vibration depends upon the fatigue life in cycles and the frequency of vibration.

If a rubber sample is not vibrated but is kept under a static stress slightly less than its ordinary tensile strength it will break in a very short time due to the high static stress to which it has been subjected. If smaller loads are placed upon similar samples it is found that the time elapsing between loading and rupture varies with the load. In other words, the load required at any particular time to break a sample decreases with time as the sample is kept under a static load. Ultimately, the load required to break the sample decreases until it becomes equal to the load which is acting on the sample and rupture results. This phenomenon of failure caused by a static load is referred to as "static fatigue."

Both vibration and loading are ordinarily involved in practical applications of rubber, and hence actual fatigue is a combination of these two types. We shall discuss dynamic and static fatigue separately and then consider how the two combine in practice.

If a rubber part is vibrated back and forth along its own length between constant-strain limits we say that it is undergoing constant-strain linear dynamic fatigue. Fig. 1 serves to define certain oscillation variables which are of fundamental importance for that type of vibration. The free unstrained length of the rubber is L_0; its maximum and minimum vibration lengths are L_{max} and L_{min}; and its oscillation stroke is ΔL. Other things being equal, the two variables which determine the dynamic fatigue life of the rubber are the percentage linear strain in the rubber at its minimum length and the oscillation stroke, expressed as a percentage of the free length. These are, respectively, $\frac{L_{min}-L_0}{L_0} \times 100$ and $\frac{\Delta L}{L_0} \times 100$. It is clear that for rubber there are wide limits of variability for these two quantities.



$$\frac{\Delta L}{L_0}(100) = \text{per cent oscillation stroke}$$

$$\frac{L_{min} - L_0}{L_0}(100) = \text{per cent minimum strain} \begin{cases} +\text{extension} \\ -\text{compression} \end{cases}$$

Fig. 1 Rubber Sample in Idealized Test Machine

Investigation of Linear Dynamic Fatigue Life of Rubber

The two questions which have been investigated quantitatively are:

1 If per cent ΔL is kept constant how does the fatigue life of a rubber unit depend upon the percentage linear strain at minimum length for strains ranging from high compression to high extension?

2 If the percentage linear strain at minimum length is kept

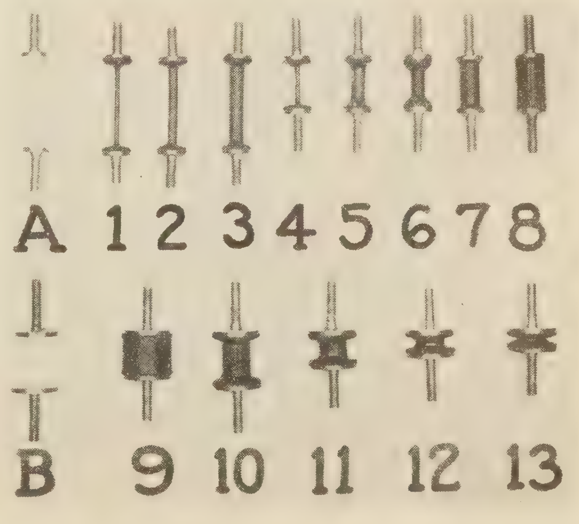


Fig. 2 Types of Rubber Samples Used in Linear Dynamic Fatigue Tests

[1] At the time of delivering this paper, the author was connected with the U. S. Rubber Company, Detroit, Mich.

Contributed by the Rubber and Plastics Group and presented at the Annual Meeting, Nov. 30–Dec. 4, 1942, of The American Society of Mechanical Engineers.

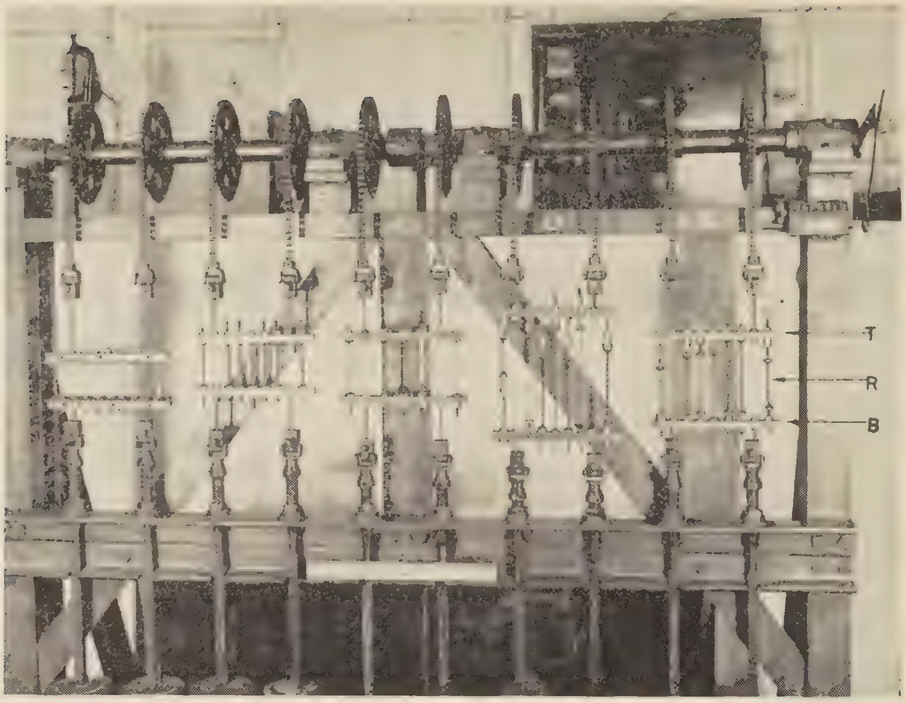


Fig. 3 Rubber Samples Mounted in Slow-Speed Fatigue Machine

[Fig.] 4 Rubber Samples Mounted in High-Speed Dynamic Fatigue Machine

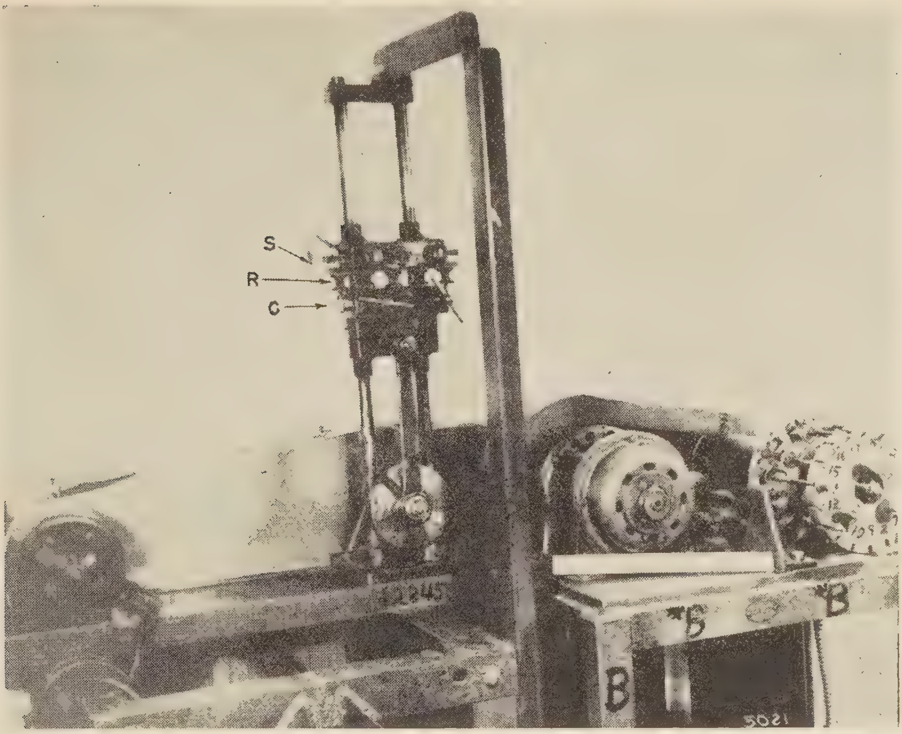


Fig. 5 Two Types of Fatigue Machines
(Left, DeMattia fatigue machine with rubber samples; right, high-speed dynamic fatigue machine, $^1/_4$-in. stroke

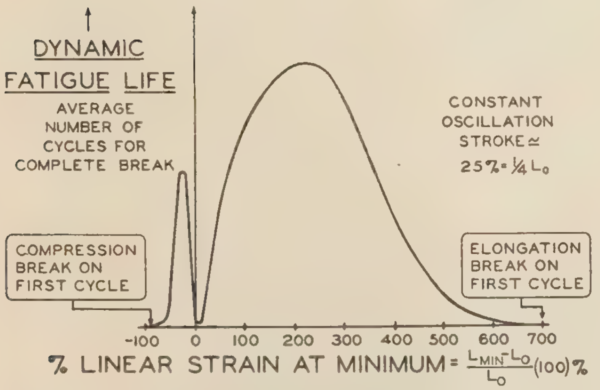


Fig. 6 Dynamic Fatigue Life Curve for Small Oscillation Strokes

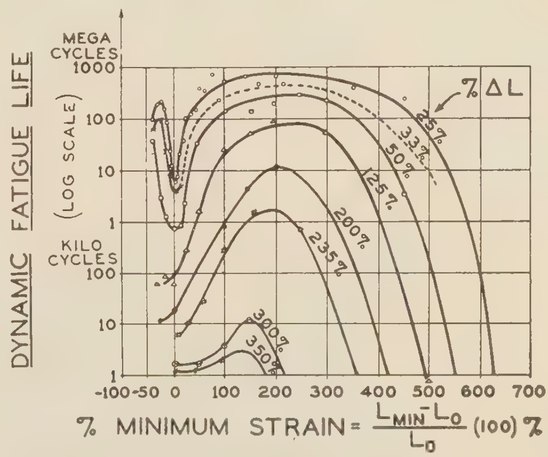


Fig. 7 Dynamic Fatigue Life Curves for 50-Durometer Stock

constant how does the dynamic fatigue life depend upon the magnitude of per cent ΔL?

A number of different types of samples and test machines were used in this investigation. Fig. 2 shows the types of tension samples, which were cylindrical dumbbells of rubber bonded to metal-bolt ends. Effective rubber lengths varied from 2 to $^1/_8$ in., and the ratio of diameter to length varied from $^1/_{16}$ to 8 in.

Fig. 3 shows a low-speed fatigue machine in which the bottom bars are held fixed and the top bars are moved up and down. Samples fastened between the bars are vibrated 180 times per min through oscillation cycles which can have an arbitrary minimum length and a stroke up to $3^1/_2$ in.

Fig. 4 shows a high-speed fatigue machine. The end heads are fixed and the inner heads are "wobble plates" which vibrate the samples linearly. Samples fastened between the two plates are vibrated 3600 times per min through oscillation cycles which have a stroke of $^1/_2$ in. and an arbitrary minimum length.

Fig. 5 shows a low-speed fatigue machine which operates at 660 cycles per min. The stroke can be varied up to 2 in. and the minimum length is arbitrary. The fatigue machine in the lower right-hand corner is similar to the one in Fig. 4 except that its stroke is $^1/_4$ in. instead of $^1/_2$ in.

Fig. 6 shows the general nature of the dynamic fatigue life curve for small oscillation strokes, say of the order of per cent $\Delta L = 25$. The average number of vibrations required to rupture the rubber completely, i.e., its dynamic fatigue life, is plotted

against the percentage strain at minimum length, both in arbitrary units. Rubber exhibits a minimum dynamic fatigue life where the minimum oscillation length is approximately equal to the free length. Fatigue lives for both compression and extension are higher. The general nature of the curve remains the same if we plot the number of cycles to produce the first visible crack rather than the number of cycles to produce failure.

Fig. 7 is a plot of experimental results for a 50-durometer stock. A 50-durometer stock is one of medium hardness, somewhat softer than an ordinary tread stock. The data are for rubber worked indoors, in artificial light and in the absence of oil or similar deteriorating chemical agents. The average dynamic fatigue life in cycles is plotted logarithmically against the percentage linear strain at minimum length. The data are for per cent ΔL varying from 25 to 350. On a linear scale the difference in height between each maximum and its corresponding minimum would be greatly enhanced. For example, for the 25 per cent oscillation the fatigue life at the minimum is about 6×10^6 cycles, whereas that at the extension maximum is over 6×10^8 cycles, or more than 100 times as great. This graph contains results for some 500 samples and each point represents the numerical average of from 1 to 20 sample breaks. For small per cent ΔL there are two maxima. For large per cent ΔL there is no compression maximum. For a given percentage strain at minimum length the fatigue life falls off with increase of per cent ΔL. As per cent ΔL increases, the fatigue-life hump in the extension region shifts toward the origin, because for a given stock the percentage elongation at break is a fairly definite quantity.

Corrections were not made in this graph for rubber temperature differences resulting from fatiguing samples of different sizes and shapes at different frequencies. If all temperatures were corrected to, say, 100 F the general nature of the curves would remain the same, but the compression humps would be raised considerably. For small per cent ΔL the compression hump would be higher than the extension hump.

All stocks considered (which ranged from 30 to 80 durometer) have similar constant-strain linear dynamic fatigue life curves. In general, for the same strain conditions of oscillation, a harder rubber stock will have a lower life than a softer stock.

The size of sample and the frequency of vibration ordinarily affect the fatigue life only in so far as they affect the rubber temperature. Fig. 8 shows in a general way how the temperature of the rubber affects its fatigue life. The graph is a composite one intended to cover most of the stocks with which we worked. The ratio of fatigue life at a given temperature to that at 100 F is plotted logarithmically against the temperature. The effects of high temperature endured for any length of time depend upon the aging properties of the stock; and low temperature brings in the freezing properties. These and their interaction with the effect of vibration vary for different stocks so that it is impossible to be too definite about the course of the curve for high and low temperatures, which accounts for the broad bands in this attempt at generalization.

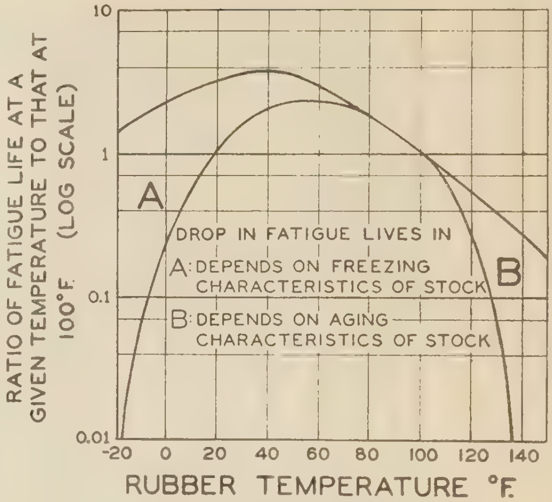


Fig. 8 Effect of Temperature on Dynamic Fatigue Life

Fig. 9 shows a high-speed "constant-load" fatigue machine. In this machine the lower end of a sample is loaded with weights. The top of the rubber sample is vibrated up and down while the weight remains stationary due to mismatching of forced and

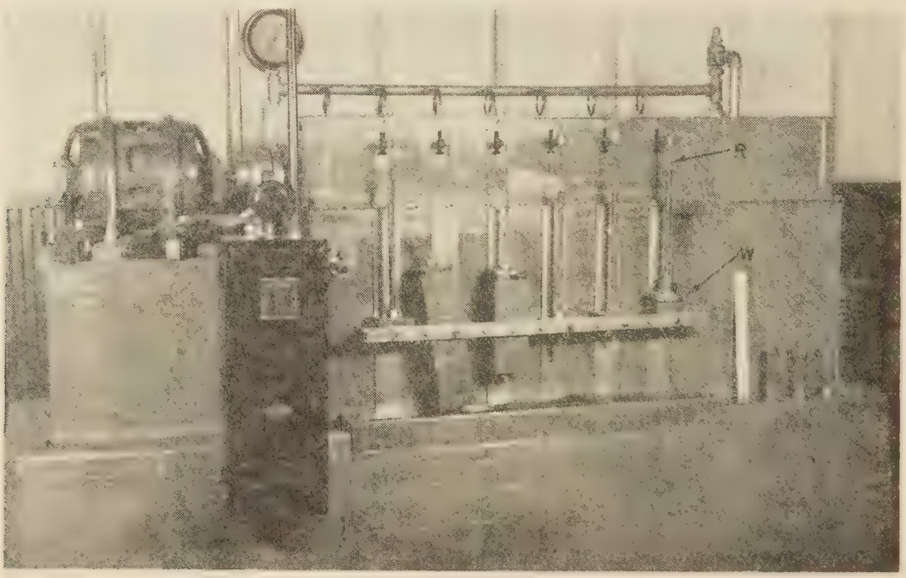


Fig. 9 Rubber Samples Mounted in High-Speed Constant-Load Fatigue Machine

natural frequencies. In this type of testing the rubber samples can drift as they are being fatigued. This machine vibrates the samples 1800 times per min through a $^1/_2$-in. stroke.

The general nature of the constant-load fatigue-life curve is similar to that for constant strain, except that for constant load the extension fatigue-life hump is narrower, for a reason which will be fairly obvious when we consider static fatigue. An interesting feature of constant-load testing is that when the elongation under load is sufficiently small so that the sample is in compression during part of the stroke, we find the fatigue lives starting up on a constant-load compression hump.

The size and shape of a practical rubber part ordinarily bear little resemblance to one of these test pieces. That is usually true of fatigue-testing of any material. For that reason the practical application of fatigue data partakes more of the nature of an art than of a science. However, the knowledge obtained by the use of these dumbbell samples becomes a powerful tool in reasoning concerning more complex cases.

Dynamic Fatigue Tests in Shear

As an example, we want to show the results of fatigue testing in shear and to demonstrate how the results are in agreement with what one would conclude should be the case from linear testing.

Fig. 10 shows results obtained with shear samples of the 50-durometer stock previously discussed. The center plate of the

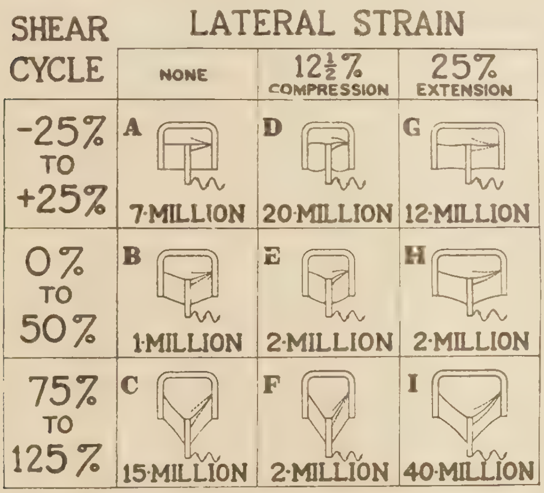


Fig. 10 Dynamic Fatigue Results on Shear Samples of 50-Durometer Stock

shear sandwich was vibrated while the outer plates were held fixed. The oscillation displacement was $^1/_2$ the thickness of the rubber, representing a 50 per cent shear vibration. In shear mountings, the rubber may be put in lateral strains which are normal to the center plate. Three conditions of lateral strain are shown: 0, $12^1/_2$ per cent compression, and 25 per cent extension. The first row corresponds to a —25 to +25 per cent shear cycle; the second, to a 0 to 50 per cent shear cycle; and the third, to a 75 to 125 per cent cycle.

Both *B* and *C* underwent a 50 per cent shear oscillation, but *B* had a minimum shear of 0 per cent whereas *C* had a minimum shear of 75 per cent. Essentially the rubber elements in *C* were being worked up on the linear-extension fatigue hump whereas those in *B* were being vibrated at the linear-fatigue minimum. As a result *C* had a life 15 times that of *B*.

For each complete 50 per cent shear oscillation, sample *A* was actually subjected to two alternating 25 per cent shear cycles. A shear unit of this same stock vibrating from 0 to 25 per cent shear has a life of about 14×10^6 cycles, which explains the 7×10^6 cycles here. Even though the sample returns twice each cycle to a condition of zero strain the life is greater than that for *B* because the effective stroke is less, being 25 rather than 50 per cent.

Samples *D* and *E* have higher minimum strains than *A* and *B*, respectively, and accordingly have higher lives. Sample *F* falls below both *C* and *I* because its minimum strain is less than that of either of the others. Sample *I* has the greatest life because its minimum strain is greatest.

The whole conception of shear, therefore, conforms well with that which we know for the linear case.

Results Obtained With Synthetic Stocks

When we were doing most of our work on fatigue we were not concerned with thoughts of a rubber shortage and the properties of rubber itself offered a wide field for investigation. Only a little work was done on synthetics. Tests were carried out on synthetic stocks of 35, 45, and 55 durometer. Roughly, results were as follows: For constant-strain fatigue the synthetics had a minimum near the origin, as does rubber. For these stocks the fatigue-life minimum was displaced somewhat from the origin toward the extension region, probably due to the acquisition of a permanent set. For constant-load testing the fatigue-life hump in the extension region was much narrower for these stocks than for comparable rubber stocks. In general the fatigue lives of the synthetics were lower than those for rubber of similar hardness. This was particularly true for the constant-load tests where differences were pronounced. Considering the volume of work which has been done on synthetics since these tests, it is probable that there is less difference between their fatigue behavior and that of rubber.

Static Fatigue Tests Conducted

We will now consider static fatigue, which has previously been defined as failure under a static load. Fig. 11 shows the type of

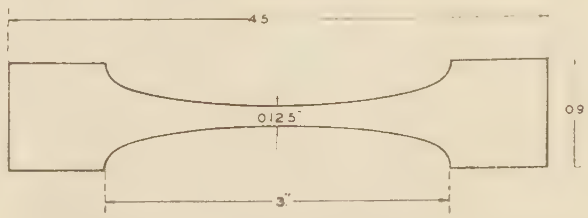


Fig. 11 Sample Used for Determining Tension Static Fatigue of Rubber Stocks

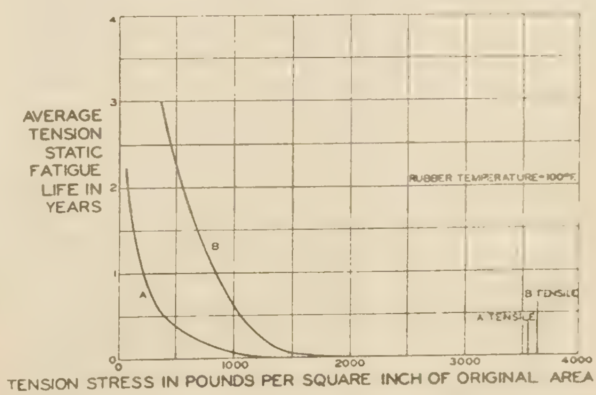


Fig. 12 Tension Static Fatigue Data for Two Typical Rubber Stocks at 100 F

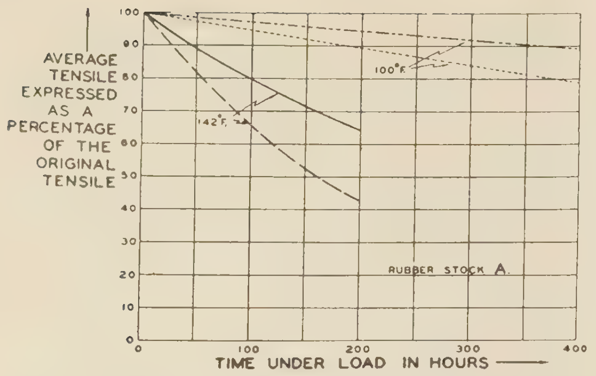


Fig. 13 Effect on Average Room-Temperature Tensile Strength of Samples of Stock *A* When Samples Are Kept Under Load at 100 F or 142 F for Different Periods of Time

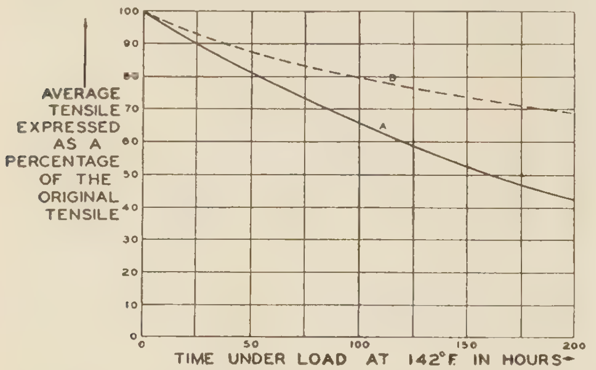


Fig. 14 Relative Loss in Tensile Strength for Samples *A* and *B* Stressed at About 32 Per Cent of Their Tensiles at 142 F

sample we used for tension static fatigue tests. The sample had a uniform width of $^1/_8$ in. for a length of about $^1/_2$ in. at the center. This shape was chosen to insure breaks near the center. The samples were cut from cured rubber slabs. In what follows, stresses on the samples are expressed in pounds per square inch of original area at the centers of the samples, the stress for each sample being corrected for the actual gage of the sample. In a tension static fatigue test a sample was subjected to a constant load at a constant temperature, and time under load to failure was noted.

Fig. 12 shows tension static fatigue data for two typical motor products stocks at 100 F. Stock *A* is 54 durometer with an average tensile strength of 3540 psi of original area. Stock *B* is 39 durometer with an average tensile strength of 3650 psi of original area. The average tension static fatigue lives of the samples in years are plotted against the stresses at the centers of the samples in pounds per square inch of original area. The curves show that the static fatigue lives of samples are functions of the stresses acting on them; that the lives fall off rapidly with increasing stress; and that the dependence of life on stress is a function of the stock, among other things. For stresses higher than 1500 to 2000 psi, the lives become smaller and smaller for increasing stresses and decrease to minutes or seconds in the region of the tensile value; and in the complete course of the curves *B* is superior to *A*.

This dependence of life on stress is to be expected from the fact that average tensiles of samples which have been supporting loads for some time are lower than the average tensiles of control samples which were not under load. Fig. 13 shows how average room-temperature tensiles of samples of stock *A* are affected when samples are kept under load at 100 F or 142 F for different periods of time. Groups of samples were subjected to the same stresses. At different time intervals a group was unloaded and the average room-temperature tensile was determined. These tensiles, expressed as percentages of the corresponding tensiles of samples not subjected to stress, are plotted against time under load previous to tensile testing. The upper of each pair of curves corresponds to a stress of about 15 per cent of the original tensile of the samples; the lower, to a stress of about 30 per cent. Actual loads on the samples were 5 and 10 lb.

These curves show that the tensiles of samples under load actually decrease and that the decrease is greater the greater the load and the greater the length of time under the load. They also show that static fatigue takes place much more rapidly at high temperature than at low.

Fig. 14 shows the relative loss in tensile for samples of stock *A* and *B* stressed at about 32 per cent of their tensile at 142 F. The behavior of the two curves is in qualitative agreement with the static fatigue curves already shown, that is, *A* is worse than *B*.

Static fatigue occurs not only in rubber stocks but also in the neighborhood of rubber-to-metal bonds. We refer to a failure in this region as a "bond" failure, even though a thin layer of rubber remains coating the metal. Fig. 15 is a plot of data for static fatigue of the bond for a stock with a shear modulus of about 80 psi used in the type of shear sandwich shown. During the tests the samples were kept at 100 F and at various constant-shear deflections. The average static fatigue lives of the bonds, in days, are plotted against the percentage constant-shear deflections.

Static fatigue life curves for either rubber stocks or rubber-to-metal bonds and for all types of deformations have this same gen-

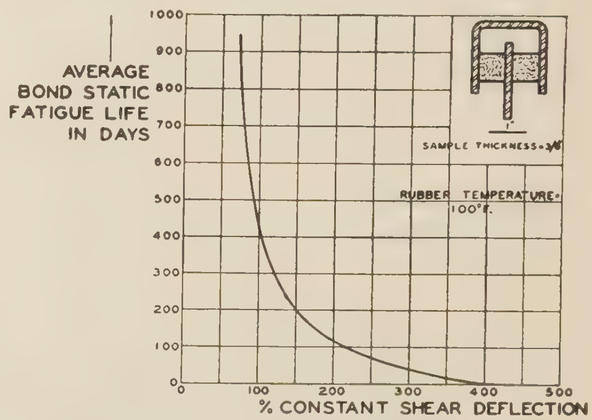


Fig. 15 Static Fatigue of Rubber-to-Metal Bond for a Stock With a Shear Modulus of About 80 Psi

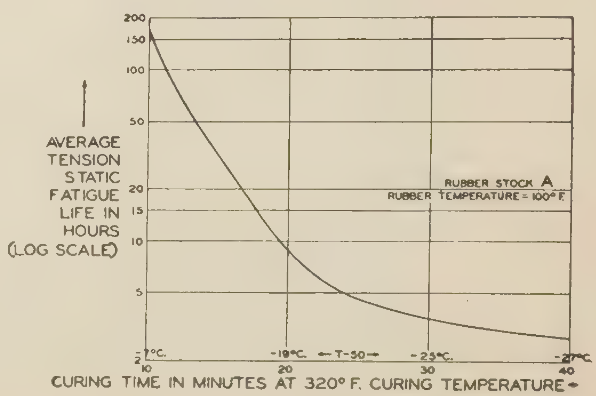


Fig. 16 Effect of Degree of Cure on Static Fatigue of Stock *A*

eral form whether plotted as functions of constant stress or of constant deflection. For any particular case the actual magnitudes of the lives, which may be greater or less than those shown in this graph, depend upon many factors, i.e., the shape of the unit, the stock, the cure, the rubber-to-metal bond, the temperature of the rubber, and the type and amount of deformation. For example, there was no lateral pressure on the rubber in the units from which data were taken to plot this curve. If the rubber had been under 10 or 20 per cent lateral compression strain during test the average fatigue lives would have been much greater.

As is shown in Fig. 16 the static fatigue life of a stock varies with the degree of cure. Average tension static fatigue lives, in hours, are plotted logarithmically against times of cure, in minutes, at a curing temperature of 320 F. T-50's which are a measure of the degree of cure are indicated for certain times. These samples were kept at 100 F and were stressed at about 1570 psi of original area at the center, corresponding to 44 per cent of the tensile for normal cure (i.e., maximum tensile). Resistance to static fatigue decreases as degree of cure is increased beyond a certain optimum value which varies with the stock. In particular the long-time static fatigue lives for slight undercures are many times greater than those for normal cures or overcures. Of course, for degrees of cure low enough for the stock to be practically raw the static-fatigue lives would fall off to negligible values.

It would be concluded from this graph that a static fatigue life curve for a stock highly overcured would lie below that for the same stock normally cured or slightly undercured; and that reduction in tensile under load would be more pronounced for high overcure than for normal cure. Tests have shown this to be the case. This dependence of fatigue on cure is an important consideration because it is often the practice to overcure stocks slightly in order to reduce their drift in use. Such overcures unquestionably reduce the static fatigue resistance.

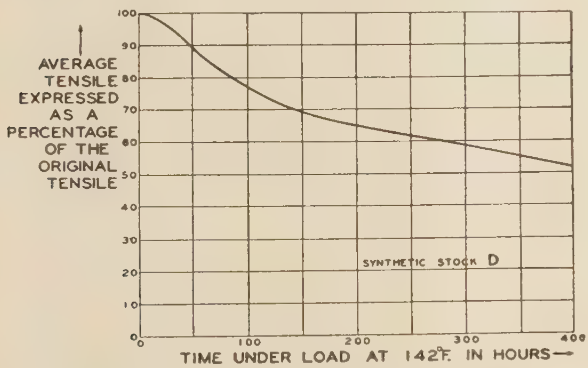


Fig. 17 Reduction of Average Tensile With Time Under Stress at 142 F for Synthetic Stock *D*

Synthetic rubbers, like natural rubber, undergo static fatigue. Fig. 17 shows how time under stress reduces the tensile of a synthetic stock. Samples were stressed at about 26 per cent of their tensile at 142 F. In going from room temperature to 142 F, the tensile for rubber will ordinarily not decrease below 80 to 90 per cent of its room temperature, whereas the tensile of this synthetic fell to about 10 per cent of its room-temperaure value. Therefore, although this curve and the previous reduction in tensile curves for rubber represent roughly the same relative loadings, these samples were supporting loads of 10 oz, whereas the rubber samples were supporting loads of about 10 lb. This radical reduction in tensile with increase of temperature must be of importance in fatigue units working at high temperatures. In cases of use at room temperature and where cracking would lower the fatigue life this synthetic would be expected to be somewhat better than rubber. Otherwise it should be about the same at room temperature.

Effect of Static Fatigue on Dynamic Fatigue Life of Rubber

We now want to consider how static fatigue effects limit the use of high stresses to get good dynamic fatigue. Fig. 18 shows static and constant-load dynamic fatigue life curves for linear extension for a given stock at a given temperature. As ordinate is plotted either the static or dynamic fatigue life in arbitrary time units. As abscissa is plotted in arbitrary units either the static stress or the equilibrium stress for the dynamic case.

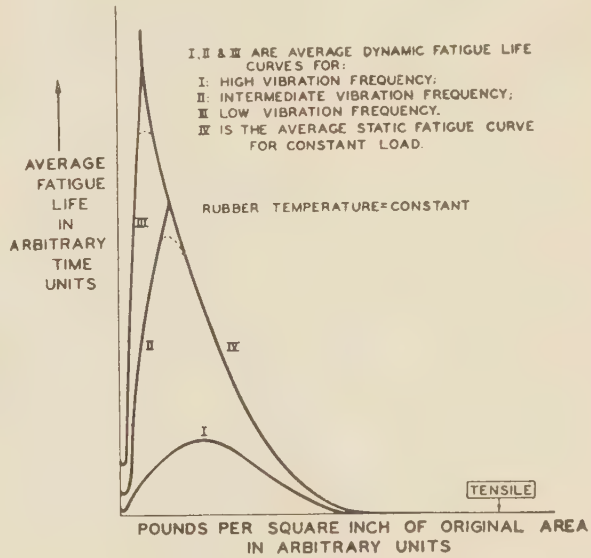


Fig. 18 Superimposed Static and Dynamic Fatigue Life Curve for a Hypothetical Stock

For the dynamic fatigue curves, the per cent ΔL and temperature are the same. They differ only in the frequency of vibration. Points on the curves corresponding to the same stress represent the same number of cycles but different time lives. The frequency of vibration for curve I is so high that the complete dynamic curve falls well within the static curve. Hence it represents the true dynamic fatigue case. Curves II and III are for lower vibration frequencies. Out to their intersection with IV, these curves represent multiples of the time intervals for curve I. The dashed lines at the intersections show the trend actually taken when the stresses are of such magnitude that static fatigue starts to become the predominating factor.

Therefore, if rubber is worked in extension and if the conditions of vibration are such that static fatigue can become an important consideration there is a limit to the time life which can be obtained by increasing the stress in the rubber; this limit is roughly the time corresponding to the intersection of the dynamic and static fatigue curves, because at that time static fatigue becomes the determining factor. The lower the vibration frequency the lower will be the stress at which this intersection takes place.

The same general relation between static and dynamic fatigue holds for all other types of deformations. In practical designing allowance must be made for both types of fatigue. Any choice of working stresses must be so determined that both the dynamic and static fatigue lives are adequate.

Discussion

J. H. Dillon.[2] This paper constitutes an excellent summary of the results of several papers published previously by Dr. Yost and his co-workers. In particular, the first[3] of these papers set forth for the first time in sound quantitative form the basic dependence of dynamic fatigue life upon the minimum strain and strain amplitude. In a second paper[4] the principles of static fatigue were given and their relationship to those of dynamic fatigue was discussed. The present paper should be of great value to the mechanical engineer since it reviews previous published results and groups them in a convenient form for engineering application.

Dr. Yost has clearly established the important fact that the dynamic fatigue life of rubber has a minimum value at zero minimum linear strain; strain amplitude, frequency, and temperature being held constant. He points out that this principle applies to a shear support even though the rubber may be in a state of static compression or tension. In the case of a shear support where the rubber is in static compression, for example, the minimum fatigue life occurs at some positive shear deflection for which the minimum linear strain is small.

Most of the results of this paper were obtained with rubber cylinders in forced axial tension and compression vibration. This relatively simple experimental arrangement is ideal for bringing out the basic facts. In actual applications of rubber in vibration service, however, the state of strain is often complex and considerable caution should be exercised in using the conclusions derived from the simpler experimental system. For example, consider the case of a test piece of rectangular cross section small compared to its length so that bending takes place when the jaws of the vibration apparatus are separated by a distance greater than the mimimum length. To illustrate this case, some preliminary results were obtained with a Buna-S tire-tread stock. As indicated in Fig. 19, bending actually took place with values of the minimum static tensile strain as high as 30 per cent. The reason for this behavior, of course, is that the elastic time constants of the Buna-S stock are sufficiently large to prevent complete relaxation within the vibration period of the machine which operates at 400 cycles per min. The minimum fatigue life is seen to occur at positive values of the minimum static strain sufficiently large to prevent formation of the dynamic loop. Otherwise stated, the minimum fatigue life appears to occur for the

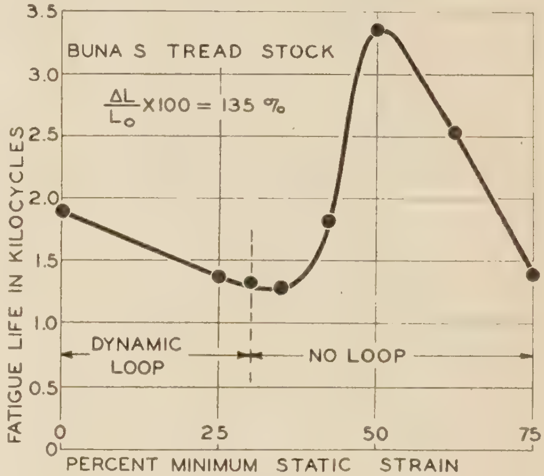


Fig. 19

condition where the minimum dynamic strain is equal to the minimum static strain. A general conclusion of this type is, of course, not justified in view of the limited number of data here presented. Nevertheless, it is clear that the fatigue life is not uniquely determined by the value of the minimum static strain at a fixed strain amplitude in this important but more complex system. It is also clear that the existence of permanent set in the test pieces does not directly explain the results. The permanent set, measured on the broken test pieces, ranged from 4 to 8 per cent, whereas the fatigue life minimum occurred at about 35 per cent.

In conclusion, the results presented here are not in disagreement with those of Dr. Yost's paper. They simply constitute an extension of the conclusions of his paper to include the case where static and dynamic values of the minimum strain are not identical.

[2] Firestone Tire & Rubber Co., Akron, Ohio.

[3] "Dynamic Fatigue Life of Rubber," by S. M. Cadwell, R. A. Merrill, C. M. Sloman, and F. L. Yost, *Industrial and Engineering Chemistry*, Analytical Edition, vol. 12, January, 1940, pp. 19–23.

[4] "Rubber in the Automotive Industry," by S. M. Cadwell, R. A. Merrill, C. M. Sloman, and F. L. Yost, *Industrial and Engineering Chemistry*, vol. 33, March, 1941, pp. 370–374.

Indexes to A.S.M.E. Papers and Publications

THIS and the following pages will serve as a guide to the current publications of the A.S.M.E. during the calendar year 1943 and also to publications developed by technical committees.

Regular Society Publications, 1943

Mechanical Catalog and Directory, 1944 edition

Special Publications Issued in 1943

Reflections on the Motive Power of Heat

American Standards

Circular and Dovetailed Forming Tool Blanks and Holding Elements
Spindle Noses and Arbors
Tool Shanks and Tool Posts
Shafting and Stock Keys
Machine Tapers
Markings for Grinding Wheels
Socket Set Screws and Socket-Head Cap Screws
Air Gaps and Backflow Preventers in Plumbing Systems
Threaded C.I. Pipe for Drainage, Vent, and Waste Services
Ferrous Plugs, Bushings, Locknuts, and Caps
Engineering and Scientific Graphs for Publications
Letter Symbols for Heat and Thermodynamics
Letter Symbols for Gear Engineering
Safety Code for Cranes, Derricks, and Hoists
Safety Code for Jacks

1943 API-ASME Code for Unfired Pressure Vessels

Boiler Construction Code

1943 editions of:
- Locomotive Boiler Code
- Low-Pressure Heating Boiler Code
- Miniature Boiler Code
- Power Boiler Code
- Unfired Pressure Vessel Code
- Specifications for Materials
- Suggested Rules for Care of Power Boilers
- Welding Qualifications

Power Test Codes

Appendix to the Test Code for Steam Turbines

Auxiliary Sections

Part 8, Heat of Combustion

How to Find Papers Presented at 1943 A.S.M.E. Meetings

THE technical programs of the meetings of the Society and of its Professional Divisions have been published in *Mechanical Engineering* and may be located by consulting the index on pages RI-83–RI-90. A majority of these papers were published, or will be published, in *Mechanical Engineering* or the Transactions (including the *Journal of Applied Mechanics*) and may be located by reference to the indexes of these publications. Several additional papers and reports included in these 1943 programs were not published during the year in Transactions or *Mechanical Engineering* but were issued in mimeographed or photo-offset form.

Complete sets of these are on file for reference purposes at the office of the Society and the Engineering Societies Library, under the title of "Miscellaneous Papers Presented at A.S.M.E. Meetings, 1943." Photostat copies of any of the papers may be secured from the Library at twenty-five cents a page to members, or thirty cents a page to nonmembers.

Publications Developed by the Technical Committees

THE Society's technical committees, the first of which was organized many years ago and all of which have been continuously at work on codes, standards, research, and other special reports, have developed a series of publications of permanent value to the membership. The following list is presented here for record and for ready reference. This list covers the entire group of publications of these committees completed to date which are now available.

To assist members in securing copies of these publications the sale price is also given. A discount of 20 per cent is allowed to A.S.M.E. members on all publications except where otherwise noted.

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Small Rivets (B18a—1927), $0.30
Socket Set Screws and Socket-Head Cap Screws (B18.3—1943), $0.40
Tinners', Coopers', and Belt Rivets (B18g—1929), $0.35
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Wrench-Head Bolts and Nuts and Wrench Openings (B18.2—1941), $0.65

Piping and Pipe Fittings

Air Gaps and Backflow Preventers in Plumbing Systems (A40.4—1942 and A40.6—1943) $0.45.
Brass Fittings for Flared Copper Tubes (A40.2—1936), $0.35
Cast-Iron Pipe Flanges and Flanged Fittings for 25 Lb Maximum Saturated Steam Pressure (B16b2—1931), $0.40
Cast-Iron Pipe Flanges and Flanged Fittings for 125 Lb Maximum Saturated Steam Pressure (B16a—1939), $0.60
Cast-Iron Pipe Flanges and Flanged Fittings for 250 Lb Maximum Saturated Steam Pressure (B16b—1928), $0.50
Cast-Iron Pipe Flanges and Flanged Fittings for 800 Lb Maximum Hydraulic Pressure (B16b1—1931), $0.35
Cast-Iron Soil Pipe and Fittings (A40.1—1935), $0.65
Cast-Iron Long Turn Sprinkler Fittings for 150 and 250 Lb Maximum Saturated Steam Pressure (B16g—1929) and Addendum (B16g1—1937), $0.50
Cast-Iron Screwed Fittings for 125 and 250 Lb Maximum Saturated Steam Pressure (B16d—1941), $0.40
Cast-Iron Screwed Drainage Fittings (B16.12—1942), $0.45
Code for Pressure Piping (B31.1—1942), $2.00
Face-to-Face Dimensions of Ferrous Flanged and Welding End Valves (B16.10—1939), $0.55

Ferrous Plugs, Bushings, Lock Nuts, and Caps (B16.14—1943) $0.40
Malleable-Iron Screwed Fittings for 150 Lb Maximum Saturated Steam Pressure (B16c—1939), $0.50
Pipe Plugs (B16e2—1936), $0.35
Pipe Threads (B2.1—1942), $0.75
Scheme for the Identification of Piping Systems (A13—1928), $0.50
Steel Pipe Flanges and Flanged Fittings for 150 to 2500 Lb Maximum Steam Service Pressure (B16e—1939), $1.25
Soldered-Joint Fittings (A40.3—1941), $0.45
Steel Butt-Welding Fittings (B16.9—1940), $0.40
Threaded Cast-Iron Pipe for Drainage, Vent, and Waste Services (A40.5—1943), $0.25
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Letter and Graphical Symbols and Charts

Abbreviations for Scientific and Engineering Terms (Z10.1—1941), $0.35
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Engineering and Scientific Graphs for Publications (Z15.3—1943), $0.75
Graphical Symbols for use on Drawings in Mechanical Engineering (Z32.2—1941), $0.50
Letter Symbols for Gear Engineering (B6.5—1943), $0.25
Letter Symbols for Hydraulics (Z10.2—1942), $0.35
Letter Symbols for Mechanics of Solid Bodies (Z10.3—1942), $0.25
Letter Symbols for Heat and Thermodynamics (Z10c—1943), $0.35
Time Series Charts (Z15.2—1938), $1.25

MISCELLANY

Fire-Hose Coupling Screw Thread (B26—1925), $0.25
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Hose Coupling Screw Threads (B33.1—1935), $0.25
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Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases (C44—1931), $0.35
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Spur Gear Tooth Form (B6.1—1932), $0.45

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Acme and Other Translating Threads (B1.3—1941), $0.45
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- Welding Qualifications, $0.65

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A.S.M.E. Transactions,* including *Journal of Applied Mechanics*, Annual subscription rate in United States, $12, elsewhere, $12.75

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Volume 65, January—December, 1943

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Index to A.S.M.E. Transactions

Volume 65, 1943

The A.S.M.E. Transactions for 1943 was issued monthly. Four of the twelve issues are the *Journal of Applied Mechanics*, the page numbers of which are preceded by the letter A.

The Society Records for the year 1943 appeared as supplements, one in February, 1943, the Memorial Biographics in October, 1943, and the index section in January, 1944. The page numbers for these supplements are designated by the symbol RI, and the February supplement contains its own index.

(AC) denotes author's closure; (BR) book review; (D) discussion of a paper.

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